Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.8-P-x-c-x-^m-a+b-x^n-^p

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September 20, 2021 Compiled on September 20, 2021 at 5:28am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [427]. This is test number [16].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (427)	0.00 (0)
Mathematica	100.00 (427)	0.00 (0)
Mupad	99.53 (425)	0.47 (2)
Maple	99.30 (424)	0.70 (3)
Maxima	98.83 (422)	1.17 (5)
Giac	98.36 (420)	1.64 (7)
Fricas	79.86 (341)	20.14 (86)
Sympy	66.74 (285)	% 33.26 (142)
IntegrateAlgebraic	3.98 (17)	96.02 (410)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

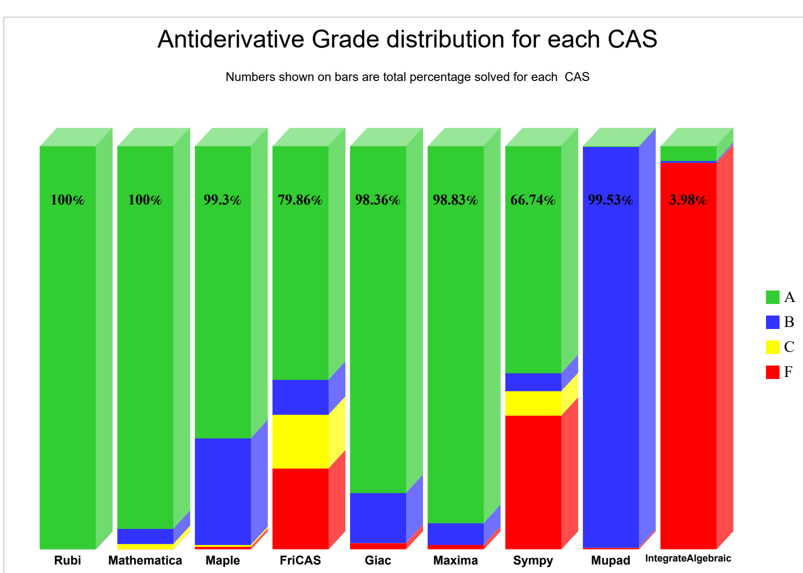
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

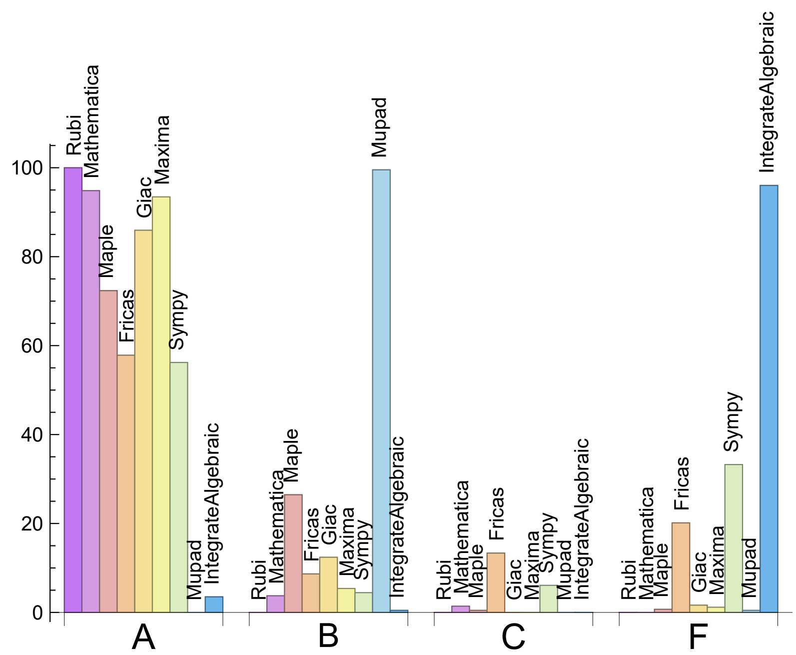
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	94.85	3.75	1.41	0.00
Maxima	93.44	5.39	0.00	1.17
Giac	85.95	12.41	0.00	1.64
Maple	72.37	26.46	0.47	0.70
Fricas	57.85	8.67	13.35	20.14
Sympy	56.21	4.45	6.09	33.26
IntegrateAlgebraic	3.51	0.47	0.00	96.02
Mupad	N/A	99.53	0.00	0.47

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	3	66.67 %	33.33 %	0.00 %
Fricas	86	0.00 %	98.84 %	1.16 %
IntegrateAlgebraic	410	100.00 %	0.00 %	0.00 %
Giac	7	42.86 %	57.14 %	0.00 %
Maxima	5	100.00 %	0.00 %	0.00 %
Sympy	142	1.41 %	97.89 %	0.70 %
Mupad	2	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

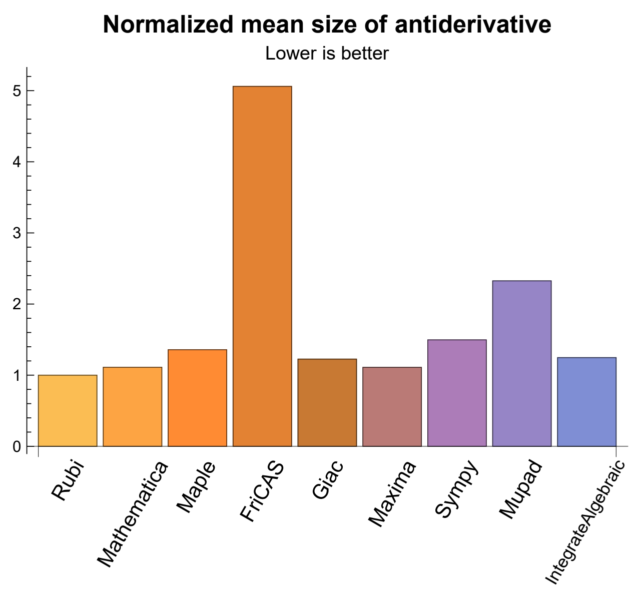
1.3 Performance

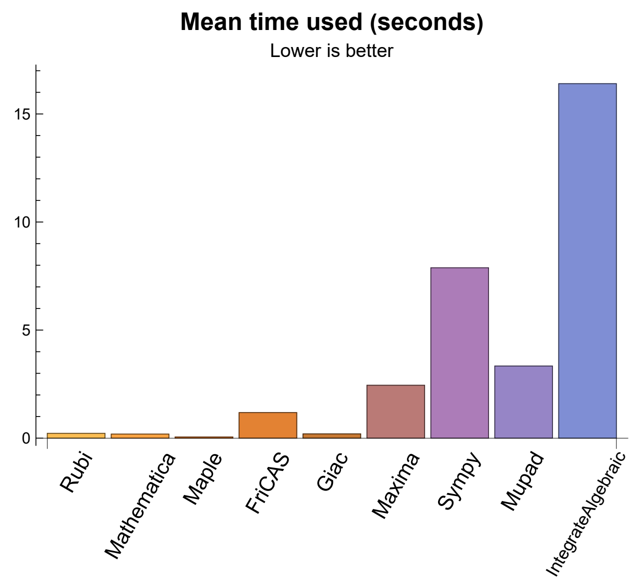
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.22	176.22	1.00	158.00	1.00
Mathematica	0.19	181.74	1.11	168.00	0.99
Maple	0.06	254.36	1.36	194.50	1.21
Maxima	2.45	190.42	1.11	173.00	0.99
Fricas	1.18	1179.66	5.06	160.00	1.12
Sympy	7.88	181.73	1.50	109.00	1.05
Giac	0.20	216.82	1.23	189.50	1.03
Mupad	3.34	502.68	2.33	193.00	1.01
IntegrateAlgebraic	16.40	233.71	1.25	38.00	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {41, 408, 423}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

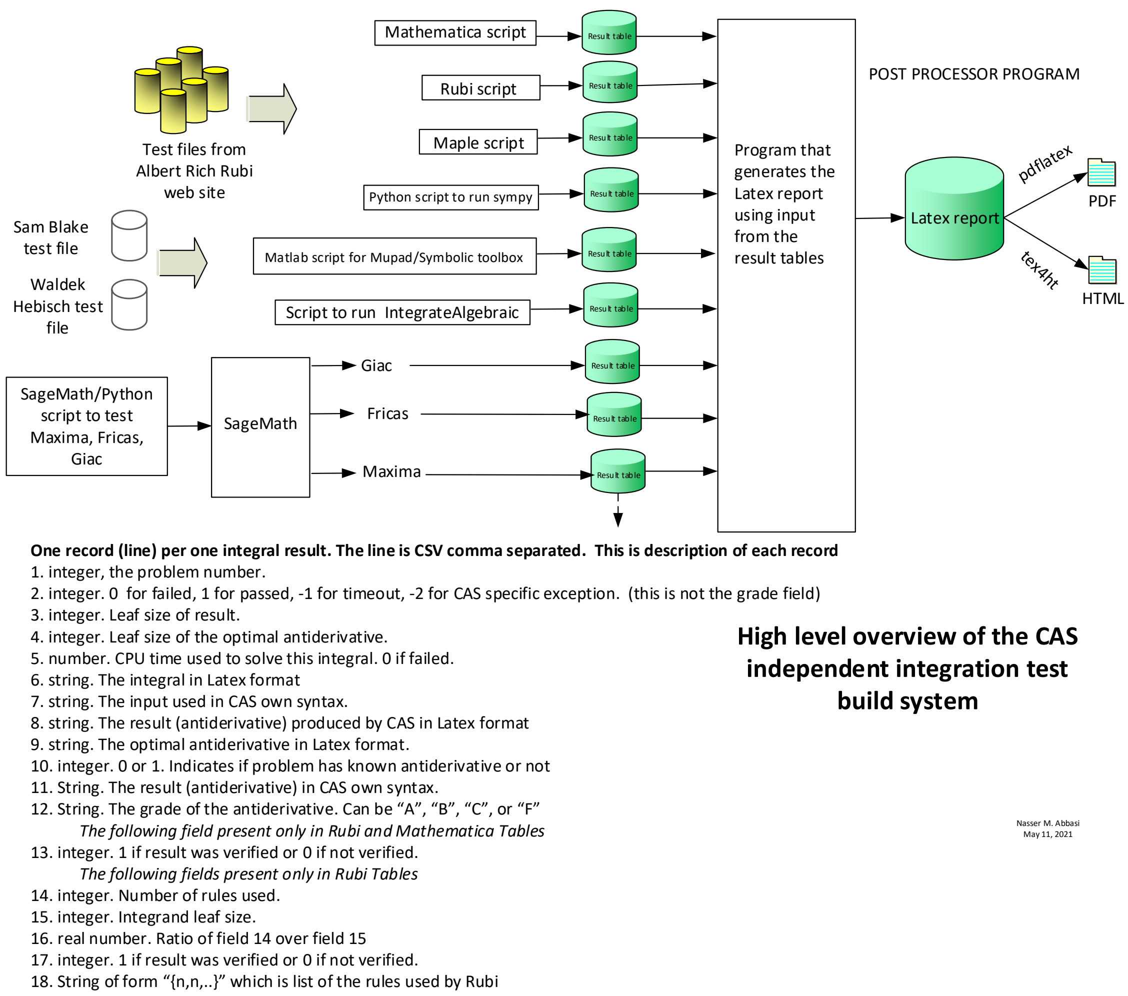
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 38, 39, 40, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 424, 425, 426, 427 }

B grade: { 21, 32, 33, 34, 35, 36, 41, 44, 45, 46, 47, 316, 317, 318, 319, 398 }

C grade: { 77, 114, 168, 169, 408, 423 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 33, 36, 39, 42, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 104, 105, 106, 107, 108, 110, 112, 113, 114, 115, 116, 118, 119, 120, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 142, 148, 151, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 369, 370, 371, 372, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 426 }

B grade: { 6, 20, 21, 29, 30, 31, 32, 34, 35, 37, 38, 40, 41, 43, 45, 46, 62, 63, 64, 76, 78, 80, 82, 102, 103, 109, 111, 117, 121, 122, 124, 125, 139, 140, 141, 143, 144, 145, 146, 147, 149, 150, 152, 153, 168, 169, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 234, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 317, 318, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 398 }

C grade: { 425, 427 }

F grade: { 421, 423, 424 }

2.1.4 Maxima

A grade: { 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 36, 39, 42, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 425, 426 }

B grade: { 3, 6, 20, 21, 31, 32, 34, 35, 37, 38, 40, 41, 43, 45, 46, 68, 114, 132, 138, 317, 318, 398, 427 }

C grade: { }

F grade: { 168, 169, 421, 423, 424 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 65, 66, 67, 76, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 112, 114, 120, 133, 134, 135, 136, 137, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 377, 378, 379, 380, 381, 382, 383, 384, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 412, 413, 414, 415, 416, 419, 420, 421, 422, 426, 427 }

B grade: { 40, 41, 44, 45, 46, 47, 105, 108, 109, 113, 116, 117, 121, 132, 138, 168, 169, 230, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 398, 410, 411, 417, 418, 424, 425 }

C grade: { 7, 8, 9, 10, 11, 12, 24, 25, 26, 57, 58, 59, 60, 61, 62, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375 }

F grade: { 63, 64, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 102, 103, 104, 107, 110, 111, 115, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 367, 376, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 423 }

2.1.6 Sympy

A grade: { 1, 2, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 29, 31, 37, 38, 39, 42, 43, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 81, 83, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 107, 108, 109, 110, 112, 113, 115, 116, 117, 118, 120, 121, 122, 123, 133, 134, 135, 136, 137, 165, 166, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 298, 299, 300, 305, 306, 307, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 377, 378, 379, 380, 381, 382, 383, 384, 389, 390, 391, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420 }

B grade: { 78, 80, 82, 84, 93, 102, 103, 111, 119, 132, 138, 168, 169, 350, 351, 352, 353, 354, 398 }

C grade: { 18, 19, 20, 21, 27, 28, 30, 32, 33, 34, 35, 36, 49, 76, 114, 163, 164, 167, 312, 313, 314, 315, 316, 317, 318, 319 }

F grade: { 3, 5, 6, 40, 41, 44, 45, 46, 47, 62, 63, 64, 104, 124, 125, 126, 127, 128, 129, 130, 131, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 177, 178, 179, 193, 194, 195, 196, 197, 203, 204, 205, 206, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 288, 289, 290, 294, 295, 296, 297, 301, 302, 303, 304, 308, 309, 310, 311, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 385, 386, 387, 388, 392, 393, 394, 421, 422, 423, 424, 425, 426, 427 }

2.1.7 Giac

A grade: { 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 35, 36, 39, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 123, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 142, 143, 144, 148, 149, 150, 154, 155, 156, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 419, 420 }

B grade: { 3, 6, 29, 30, 31, 32, 33, 34, 44, 45, 68, 70, 72, 74, 76, 78, 80, 82, 84, 102, 103, 104, 114, 122, 124, 125, 126, 132, 139, 140, 141, 145, 146, 147, 151, 152, 153, 157, 158, 159, 201, 316, 317, 318, 385, 386, 398, 417, 418, 424, 425, 426, 427 }

C grade: { }

F grade: { 37, 38, 40, 41, 421, 422, 423 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 424, 425, 426, 427 }

C grade: { }

F grade: { 421, 423 }

2.1.9 IntegrateAlgebraic

A grade: { 1, 2, 4, 5, 56, 132, 133, 134, 163, 164, 165, 166, 167, 421, 423 }

B grade: { 3, 6 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 424, 425, 426, 427 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	53	53	77	53	223	78	58	62
N.S.	1	1.00	0.74	0.74	1.07	0.74	3.10	1.08	0.81	0.86
time (sec)	N/A	0.033	0.157	0.060	0.923	0.405	11.050	0.157	4.715	0.038

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	155	194	237	192	644	237	149	229
N.S.	1	1.00	0.96	1.20	1.47	1.19	4.00	1.47	0.93	1.42
time (sec)	N/A	0.105	0.292	0.048	0.900	0.412	85.150	0.167	4.762	0.085

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	274	274	294	495	525	457	0	526	299	592
N.S.	1	1.00	1.07	1.81	1.92	1.67	0.00	1.92	1.09	2.16
time (sec)	N/A	0.194	1.029	0.052	0.978	0.413	0.000	0.226	0.098	0.172

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	82	91	128	90	354	129	103	107
N.S.	1	1.00	0.72	0.80	1.12	0.79	3.11	1.13	0.90	0.94
time (sec)	N/A	0.071	0.175	0.045	0.833	0.410	45.834	0.174	4.814	0.057

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	320	320	303	447	500	417	0	516	316	544
N.S.	1	1.00	0.95	1.40	1.56	1.30	0.00	1.61	0.99	1.70
time (sec)	N/A	0.244	0.565	0.047	1.002	0.415	0.000	0.200	4.698	0.163

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	708	708	678	1417	1360	1221	0	1414	896	2128
N.S.	1	1.00	0.96	2.00	1.92	1.72	0.00	2.00	1.27	3.01
time (sec)	N/A	0.625	2.894	0.053	1.094	0.420	0.000	0.287	0.242	0.424

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	124	186	135	1931	76	141	127	0
N.S.	1	1.00	0.77	1.16	0.84	11.99	0.47	0.88	0.79	0.00
time (sec)	N/A	0.111	0.083	0.050	1.912	1.148	1.187	0.166	5.511	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	180	238	169	2088	105	174	169	0
N.S.	1	1.00	0.95	1.26	0.89	11.05	0.56	0.92	0.89	0.00
time (sec)	N/A	0.139	0.230	0.047	2.000	1.166	2.146	0.178	4.872	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	205	272	203	2215	146	194	206	0
N.S.	1	1.00	0.95	1.27	0.94	10.30	0.68	0.90	0.96	0.00
time (sec)	N/A	0.187	0.254	0.055	1.965	1.195	2.470	0.226	0.268	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	229	306	238	2308	185	218	241	0
N.S.	1	1.00	0.95	1.28	0.99	9.62	0.77	0.91	1.00	0.00
time (sec)	N/A	0.223	0.229	0.053	2.616	1.191	3.638	0.230	4.931	0.001

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	125	186	135	1961	76	132	127	0
N.S.	1	1.00	0.78	1.16	0.84	12.18	0.47	0.82	0.79	0.00
time (sec)	N/A	0.121	0.073	0.053	2.509	1.187	1.433	0.175	4.847	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	125	188	132	1905	78	115	124	0
N.S.	1	1.00	0.78	1.17	0.82	11.83	0.48	0.71	0.77	0.00
time (sec)	N/A	0.100	0.058	0.046	2.678	1.183	1.492	0.181	0.213	0.001

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	17	16	16	26	16	16	0
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84	0.00
time (sec)	N/A	0.015	0.007	0.045	2.485	0.397	0.370	0.189	4.699	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	17	16	16	26	16	16	0
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84	0.00
time (sec)	N/A	0.013	0.006	0.044	2.426	0.405	0.217	0.172	4.670	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	16	17	17	16	0
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.77	0.73	0.00
time (sec)	N/A	0.012	0.004	0.047	2.441	0.401	0.253	0.164	0.063	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	19	18	18	17	19	18	0
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.86	0.82	0.00
time (sec)	N/A	0.013	0.005	0.043	2.469	0.400	0.225	0.170	0.112	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	33	32	32	44	33	46	0
N.S.	1	1.00	1.00	0.80	0.78	0.78	1.07	0.80	1.12	0.00
time (sec)	N/A	0.027	0.009	0.050	2.433	0.405	0.468	0.147	0.140	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	31	35	34	28	54	28	28	0
N.S.	1	1.00	1.07	1.21	1.17	0.97	1.86	0.97	0.97	0.00
time (sec)	N/A	0.022	0.012	0.074	2.901	0.406	0.397	0.206	0.054	0.001

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	34	33	26	53	26	28	0
N.S.	1	1.00	1.00	1.17	1.14	0.90	1.83	0.90	0.97	0.00
time (sec)	N/A	0.020	0.025	0.043	2.980	0.402	0.465	0.165	0.045	0.001

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	35	195	163	107	88	48	49	0
N.S.	1	1.00	0.90	5.00	4.18	2.74	2.26	1.23	1.26	0.00
time (sec)	N/A	0.027	0.016	0.063	2.977	0.450	0.588	0.223	4.821	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	129	228	174	114	105	58	49	0
N.S.	1	1.00	3.15	5.56	4.24	2.78	2.56	1.41	1.20	0.00
time (sec)	N/A	0.043	0.066	0.058	2.950	0.454	0.854	0.214	0.231	0.001

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	90	94	159	310	26	103	98	0
N.S.	1	1.00	0.76	0.80	1.35	2.63	0.22	0.87	0.83	0.00
time (sec)	N/A	0.126	0.033	0.048	3.028	0.435	0.477	0.186	4.944	0.001

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	90	94	159	305	22	115	96	0
N.S.	1	1.00	0.76	0.80	1.35	2.58	0.19	0.97	0.81	0.00
time (sec)	N/A	0.106	0.016	0.049	2.994	0.431	0.214	0.209	5.015	0.001

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	124	186	188	1961	76	147	127	0
N.S.	1	1.00	0.77	1.16	1.17	12.18	0.47	0.91	0.79	0.00
time (sec)	N/A	0.166	0.047	0.045	2.958	1.180	1.282	0.198	4.923	0.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	122	108	145	1043	75	110	158	0
N.S.	1	1.00	0.91	0.81	1.08	7.78	0.56	0.82	1.18	0.00
time (sec)	N/A	0.113	0.026	0.049	2.924	1.206	0.713	0.176	0.189	0.001

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	123	111	144	1267	70	95	178	0
N.S.	1	1.00	0.92	0.83	1.07	9.46	0.52	0.71	1.33	0.00
time (sec)	N/A	0.089	0.039	0.045	3.045	1.188	0.590	0.173	5.009	0.001

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	72	43	42	36	60	37	84	0
N.S.	1	1.00	1.95	1.16	1.14	0.97	1.62	1.00	2.27	0.00
time (sec)	N/A	0.058	0.023	0.053	2.992	0.410	0.500	0.169	4.809	0.001

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	71	45	44	36	60	38	86	0
N.S.	1	1.00	1.82	1.15	1.13	0.92	1.54	0.97	2.21	0.00
time (sec)	N/A	0.042	0.026	0.058	2.966	0.409	0.697	0.154	0.093	0.001

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	76	117	47	134	58	115	147	0
N.S.	1	1.00	1.58	2.44	0.98	2.79	1.21	2.40	3.06	0.00
time (sec)	N/A	0.037	0.022	0.056	2.991	0.449	0.625	0.416	5.137	0.001

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	72	84	36	40	85	111	145	0
N.S.	1	1.00	1.53	1.79	0.77	0.85	1.81	2.36	3.09	0.00
time (sec)	N/A	0.034	0.029	0.045	3.003	0.426	0.745	0.204	5.024	0.001

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	99	122	122	182	58	91	176	0
N.S.	1	1.00	1.74	2.14	2.14	3.19	1.02	1.60	3.09	0.00
time (sec)	N/A	0.069	0.033	0.051	2.943	0.448	0.987	0.306	5.272	0.001

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	106	110	93	43	95	98	142	0
N.S.	1	1.00	2.26	2.34	1.98	0.91	2.02	2.09	3.02	0.00
time (sec)	N/A	0.061	0.043	0.052	2.988	0.430	0.934	0.214	0.328	0.001

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	146	87	51	52	100	166	172	0
N.S.	1	1.00	2.92	1.74	1.02	1.04	2.00	3.32	3.44	0.00
time (sec)	N/A	0.077	0.054	0.051	3.030	0.431	0.738	0.224	5.098	0.001

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	150	135	167	53	110	162	172	0
N.S.	1	1.00	2.83	2.55	3.15	1.00	2.08	3.06	3.25	0.00
time (sec)	N/A	0.081	0.102	0.049	3.031	0.425	0.843	0.221	5.402	0.001

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	149	132	168	56	109	91	173	0
N.S.	1	1.00	2.76	2.44	3.11	1.04	2.02	1.69	3.20	0.00
time (sec)	N/A	0.060	0.067	0.043	3.149	0.432	0.771	0.176	5.270	0.001

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	147	90	52	53	102	85	171	0
N.S.	1	1.00	2.77	1.70	0.98	1.00	1.92	1.60	3.23	0.00
time (sec)	N/A	0.058	0.052	0.050	2.999	0.430	0.791	0.215	5.191	0.001

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	95	117	162	160	70	0	193	0
N.S.	1	1.00	1.56	1.92	2.66	2.62	1.15	0.00	3.16	0.00
time (sec)	N/A	0.041	0.021	0.056	2.869	0.456	0.732	0.000	5.307	0.001

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	116	122	173	205	73	0	221	0
N.S.	1	1.00	1.66	1.74	2.47	2.93	1.04	0.00	3.16	0.00
time (sec)	N/A	0.072	0.032	0.054	3.022	0.447	1.241	0.000	5.239	0.001

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	50	32	31	31	42	32	46	0
N.S.	1	1.00	1.25	0.80	0.78	0.78	1.05	0.80	1.15	0.00
time (sec)	N/A	0.030	0.030	0.046	2.988	0.428	0.294	0.321	0.155	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	122	310	236	430	0	0	386	0
N.S.	1	1.00	1.74	4.43	3.37	6.14	0.00	0.00	5.51	0.00
time (sec)	N/A	0.067	0.052	0.056	3.120	3.299	0.000	0.000	6.232	0.001

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	88	88	238	345	252	470	0	0	444	0
N.S.	1	1.00	2.70	3.92	2.86	5.34	0.00	0.00	5.05	0.00
time (sec)	N/A	0.112	0.655	0.054	3.041	2.718	0.000	0.000	6.324	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	12	12	12	12	7	13	12	0
N.S.	1	1.00	1.09	1.09	1.09	1.09	0.64	1.18	1.09	0.00
time (sec)	N/A	0.011	0.002	0.045	1.359	0.392	0.244	0.362	0.036	0.001

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	218	210	17	20	16	15	0
N.S.	1	1.00	1.00	10.38	10.00	0.81	0.95	0.76	0.71	0.00
time (sec)	N/A	0.015	0.003	0.049	2.989	0.421	0.263	0.312	4.903	0.001

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	247	121	78	429	0	242	436	0
N.S.	1	1.00	3.48	1.70	1.10	6.04	0.00	3.41	6.14	0.00
time (sec)	N/A	0.094	0.334	0.051	2.950	1.857	0.000	0.204	6.077	0.001

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	288	345	238	459	0	235	456	0
N.S.	1	1.00	3.79	4.54	3.13	6.04	0.00	3.09	6.00	0.00
time (sec)	N/A	0.102	0.254	0.049	3.010	1.780	0.000	0.214	6.479	0.001

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	253	340	239	450	0	133	453	0
N.S.	1	1.00	3.24	4.36	3.06	5.77	0.00	1.71	5.81	0.00
time (sec)	N/A	0.108	0.356	0.050	3.031	1.779	0.000	0.192	6.053	0.001

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	244	124	78	450	0	125	435	0
N.S.	1	1.00	3.25	1.65	1.04	6.00	0.00	1.67	5.80	0.00
time (sec)	N/A	0.105	0.322	0.051	3.140	1.721	0.000	0.179	6.357	0.001

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	31	36	26	26	24	27	35	0
N.S.	1	1.00	0.97	1.12	0.81	0.81	0.75	0.84	1.09	0.00
time (sec)	N/A	0.034	0.014	0.054	2.972	0.395	0.869	0.155	4.778	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	62	87	47	47	323	52	87	0
N.S.	1	1.00	1.13	1.58	0.85	0.85	5.87	0.95	1.58	0.00
time (sec)	N/A	0.057	0.038	0.049	2.992	0.424	1.885	0.169	4.948	0.001

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	5	7	6	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75	0.00
time (sec)	N/A	0.006	0.001	0.054	1.270	0.385	0.134	0.154	0.023	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	33	32	32	5	33	63	0
N.S.	1	1.00	1.00	1.10	1.07	1.07	0.17	1.10	2.10	0.00
time (sec)	N/A	0.030	0.010	0.053	2.958	0.417	0.336	0.156	4.930	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	16	16	15	17	16	0
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.94	0.89	0.00
time (sec)	N/A	0.019	0.006	0.050	2.926	0.388	0.161	0.149	0.041	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	113	98	97	97	117	97	97	0
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.04	0.86	0.86	0.00
time (sec)	N/A	0.099	0.005	0.045	1.386	0.353	0.731	0.165	0.061	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	88	75	74	74	90	74	74	0
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.02	0.84	0.84	0.00
time (sec)	N/A	0.060	0.003	0.050	1.387	0.355	0.155	0.155	0.036	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	51	50	50	58	50	50	0
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.83	0.00
time (sec)	N/A	0.038	0.003	0.041	1.402	0.354	0.097	0.165	0.028	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	14
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	1.17
time (sec)	N/A	0.014	0.001	0.043	1.322	0.388	0.127	0.198	0.018	0.026

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	124	186	135	1931	76	141	127	0
N.S.	1	1.00	0.77	1.16	0.84	11.99	0.47	0.88	0.79	0.00
time (sec)	N/A	0.098	0.051	0.046	2.985	1.179	1.338	0.184	5.094	0.001

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	180	238	169	2088	105	174	169	0
N.S.	1	1.00	0.95	1.26	0.89	11.05	0.56	0.92	0.89	0.00
time (sec)	N/A	0.127	0.173	0.046	3.016	1.187	1.851	0.213	5.084	0.001

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	200	211	192	5014	156	175	357	0
N.S.	1	1.00	1.08	1.13	1.03	26.96	0.84	0.94	1.92	0.00
time (sec)	N/A	0.178	0.088	0.049	2.925	1.204	1.384	0.176	0.262	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	222	214	325	240	7245	245	214	370	0
N.S.	1	1.00	0.96	1.46	1.08	32.64	1.10	0.96	1.67	0.00
time (sec)	N/A	0.319	0.238	0.051	2.960	1.816	5.714	0.186	5.143	0.001

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	282	280	277	446	303	8787	325	294	513	0
N.S.	1	0.99	0.98	1.58	1.07	31.16	1.15	1.04	1.82	0.00
time (sec)	N/A	0.443	0.365	0.051	3.044	5.152	60.245	0.193	4.973	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	272	270	269	444	314	12827	0	264	769	0
N.S.	1	0.99	0.99	1.63	1.15	47.16	0.00	0.97	2.83	0.00
time (sec)	N/A	0.490	0.422	0.071	3.033	1.764	0.000	0.212	5.134	0.001

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	416	416	439	837	520	0	0	432	1700	0
N.S.	1	1.00	1.06	2.01	1.25	0.00	0.00	1.04	4.09	0.00
time (sec)	N/A	0.700	0.577	0.052	3.049	0.000	0.000	0.233	4.913	0.001

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	645	643	678	1339	833	0	0	723	2971	0
N.S.	1	1.00	1.05	2.08	1.29	0.00	0.00	1.12	4.61	0.00
time (sec)	N/A	1.097	0.476	0.056	3.126	0.000	0.000	0.210	5.047	0.001

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	54	38	37	37	44	38	49	0
N.S.	1	1.00	1.26	0.88	0.86	0.86	1.02	0.88	1.14	0.00
time (sec)	N/A	0.078	0.016	0.052	2.858	0.409	0.251	0.151	0.098	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	54	38	37	37	46	38	51	0
N.S.	1	1.00	1.17	0.83	0.80	0.80	1.00	0.83	1.11	0.00
time (sec)	N/A	0.084	0.029	0.052	2.900	0.408	0.308	0.157	0.093	0.001

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	38	37	37	48	38	49	0
N.S.	1	1.00	1.00	0.86	0.84	0.84	1.09	0.86	1.11	0.00
time (sec)	N/A	0.043	0.010	0.047	2.809	0.397	0.327	0.153	4.697	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	F(-1)	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	134	101	126	0	126	225	182	0
N.S.	1	1.00	1.54	1.16	1.45	0.00	1.45	2.59	2.09	0.00
time (sec)	N/A	0.065	0.039	0.046	2.882	0.000	1.219	0.181	5.012	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	184	151	207	0	124	213	160	0
N.S.	1	1.00	0.84	0.69	0.95	0.00	0.57	0.97	0.73	0.00
time (sec)	N/A	0.171	0.090	0.045	3.042	0.000	1.030	0.186	4.798	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	168	142	157	0	156	254	283	0
N.S.	1	1.00	1.53	1.29	1.43	0.00	1.42	2.31	2.57	0.00
time (sec)	N/A	0.082	0.202	0.050	3.038	0.000	1.798	0.173	4.919	0.001

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	224	188	238	0	155	238	282	0
N.S.	1	1.00	0.93	0.78	0.99	0.00	0.64	0.99	1.17	0.00
time (sec)	N/A	0.202	0.279	0.052	2.932	0.000	1.511	0.170	4.944	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	193	180	186	0	194	272	315	0
N.S.	1	1.00	1.42	1.32	1.37	0.00	1.43	2.00	2.32	0.00
time (sec)	N/A	0.110	0.208	0.049	3.015	0.000	1.970	0.188	4.979	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	249	222	269	0	192	256	315	0
N.S.	1	1.00	0.94	0.83	1.01	0.00	0.72	0.96	1.18	0.00
time (sec)	N/A	0.230	0.264	0.049	3.061	0.000	1.989	0.181	4.989	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	217	177	223	0	231	296	351	0
N.S.	1	1.00	1.34	1.09	1.38	0.00	1.43	1.83	2.17	0.00
time (sec)	N/A	0.130	0.215	0.064	2.974	0.000	2.062	0.278	4.975	0.001

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	274	225	304	0	231	280	350	0
N.S.	1	1.00	0.94	0.77	1.04	0.00	0.79	0.96	1.20	0.00
time (sec)	N/A	0.268	0.338	0.070	3.198	0.000	1.807	0.180	0.309	0.001

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	42	44	35	35	313	37	100	0
N.S.	1	1.00	1.75	1.83	1.46	1.46	13.04	1.54	4.17	0.00
time (sec)	N/A	0.018	0.021	0.044	3.042	0.412	0.923	0.147	4.918	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	99	68	86	0	83	86	71	0
N.S.	1	1.00	1.01	0.69	0.88	0.00	0.85	0.88	0.72	0.00
time (sec)	N/A	0.067	0.125	0.046	2.998	0.000	0.705	0.172	0.092	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	187	161	153	0	471	263	725	0
N.S.	1	1.00	1.61	1.39	1.32	0.00	4.06	2.27	6.25	0.00
time (sec)	N/A	0.095	0.062	0.045	2.911	0.000	11.044	0.179	5.143	0.001

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	277	277	229	280	257	0	466	275	712	0
N.S.	1	1.00	0.83	1.01	0.93	0.00	1.68	0.99	2.57	0.00
time (sec)	N/A	0.197	0.115	0.046	3.042	0.000	10.540	0.174	5.086	0.001

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	211	228	191	0	508	311	477	0
N.S.	1	1.00	1.45	1.56	1.31	0.00	3.48	2.13	3.27	0.00
time (sec)	N/A	0.128	0.283	0.055	2.936	0.000	13.740	0.182	4.982	0.001

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	308	308	305	344	294	0	505	306	472	0
N.S.	1	1.00	0.99	1.12	0.95	0.00	1.64	0.99	1.53	0.00
time (sec)	N/A	0.255	0.545	0.049	3.102	0.000	11.548	0.177	0.333	0.001

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	244	286	230	0	563	340	826	0
N.S.	1	1.00	1.36	1.60	1.28	0.00	3.15	1.90	4.61	0.00
time (sec)	N/A	0.167	0.289	0.052	3.124	0.000	45.341	0.235	5.111	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	341	341	337	396	336	0	558	336	826	0
N.S.	1	1.00	0.99	1.16	0.99	0.00	1.64	0.99	2.42	0.00
time (sec)	N/A	0.311	0.417	0.051	3.089	0.000	40.860	0.191	5.047	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	276	274	279	0	612	377	874	0
N.S.	1	1.00	1.31	1.30	1.32	0.00	2.90	1.79	4.14	0.00
time (sec)	N/A	0.211	0.284	0.061	3.019	0.000	59.744	0.223	5.220	0.001

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	372	372	369	394	383	0	610	373	873	0
N.S.	1	1.00	0.99	1.06	1.03	0.00	1.64	1.00	2.35	0.00
time (sec)	N/A	0.379	0.585	0.065	3.108	0.000	63.470	0.191	5.137	0.001

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	27	24	25	24	27	25	25	0
N.S.	1	1.00	0.96	0.86	0.89	0.86	0.96	0.89	0.89	0.00
time (sec)	N/A	0.011	0.002	0.045	1.372	0.360	0.119	0.145	4.674	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	32	27	27	27	29	27	27	0
N.S.	1	1.00	0.97	0.82	0.82	0.82	0.88	0.82	0.82	0.00
time (sec)	N/A	0.014	0.002	0.041	1.396	0.350	0.073	0.169	0.034	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	51	50	50	58	50	50	0
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.83	0.00
time (sec)	N/A	0.062	0.002	0.048	1.356	0.365	0.110	0.150	0.025	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	27	27	27	31	27	27	0
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.94	0.82	0.82	0.00
time (sec)	N/A	0.013	0.001	0.042	1.357	0.366	0.132	0.195	0.039	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	51	50	50	60	50	50	0
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.00	0.83	0.83	0.00
time (sec)	N/A	0.027	0.003	0.044	1.317	0.367	0.083	0.151	0.025	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	54	53	53	61	53	53	0
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.94	0.82	0.82	0.00
time (sec)	N/A	0.098	0.003	0.039	1.349	0.367	0.135	0.149	0.027	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	92	77	76	76	90	76	76	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83	0.00
time (sec)	N/A	0.053	0.004	0.043	1.430	0.367	0.163	0.150	0.038	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	33	27	15	27	29	16	26	0
N.S.	1	1.00	1.94	1.59	0.88	1.59	1.71	0.94	1.53	0.00
time (sec)	N/A	0.005	0.001	0.044	1.386	0.380	0.244	0.143	0.033	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	60	51	50	50	58	50	50	0
N.S.	1	1.00	1.33	1.13	1.11	1.11	1.29	1.11	1.11	0.00
time (sec)	N/A	0.019	0.003	0.043	1.314	0.347	0.078	0.201	0.024	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	65	54	53	53	60	53	53	0
N.S.	1	1.00	1.30	1.08	1.06	1.06	1.20	1.06	1.06	0.00
time (sec)	N/A	0.020	0.003	0.039	1.320	0.359	0.116	0.163	0.027	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	92	77	76	76	88	76	76	0
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.14	0.99	0.99	0.00
time (sec)	N/A	0.051	0.004	0.044	1.363	0.367	0.085	0.150	0.038	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	65	54	53	53	61	53	53	0
N.S.	1	1.00	1.30	1.08	1.06	1.06	1.22	1.06	1.06	0.00
time (sec)	N/A	0.023	0.004	0.043	1.360	0.358	0.085	0.146	0.027	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	92	77	76	76	90	76	76	0
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.17	0.99	0.99	0.00
time (sec)	N/A	0.042	0.005	0.043	1.337	0.371	0.146	0.201	0.038	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	97	80	79	79	92	79	79	0
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.12	0.96	0.96	0.00
time (sec)	N/A	0.084	0.005	0.038	1.417	0.353	0.112	0.161	0.041	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	124	103	102	102	121	105	102	0
N.S.	1	1.00	1.14	0.94	0.94	0.94	1.11	0.96	0.94	0.00
time (sec)	N/A	0.080	0.008	0.040	1.393	0.347	0.089	0.167	4.677	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	180	151	150	150	180	154	150	0
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.19	1.02	0.99	0.00
time (sec)	N/A	0.106	0.005	0.049	1.339	0.343	0.133	0.168	4.863	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	220	248	200	0	520	320	483	0
N.S.	1	1.00	1.42	1.60	1.29	0.00	3.35	2.06	3.12	0.00
time (sec)	N/A	0.118	0.229	0.052	3.002	0.000	24.169	0.231	0.415	0.001

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	253	326	249	0	583	358	832	0
N.S.	1	1.00	1.35	1.73	1.32	0.00	3.10	1.90	4.43	0.00
time (sec)	N/A	0.155	0.262	0.054	2.964	0.000	116.916	0.204	5.185	0.001

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	220	220	286	280	297	0	0	395	880	0
N.S.	1	1.00	1.30	1.27	1.35	0.00	0.00	1.80	4.00	0.00
time (sec)	N/A	0.189	0.502	0.058	3.061	0.000	0.000	0.187	5.246	0.001

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	78	114	123	284	88	97	36	0
N.S.	1	1.00	0.77	1.13	1.22	2.81	0.87	0.96	0.36	0.00
time (sec)	N/A	0.096	0.034	0.045	2.937	0.447	0.441	0.196	0.125	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	16	15	15	19	15	15	0
N.S.	1	1.00	1.00	0.73	0.68	0.68	0.86	0.68	0.68	0.00
time (sec)	N/A	0.013	0.011	0.039	2.875	0.412	0.134	0.166	4.774	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	107	129	147	0	88	115	119	0
N.S.	1	1.00	0.87	1.05	1.20	0.00	0.72	0.93	0.97	0.00
time (sec)	N/A	0.103	0.055	0.049	2.907	0.000	0.717	0.191	0.200	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	78	114	123	278	88	97	32	0
N.S.	1	1.00	0.77	1.13	1.22	2.75	0.87	0.96	0.32	0.00
time (sec)	N/A	0.077	0.017	0.043	3.038	0.413	0.430	0.202	4.974	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	113	226	167	2278	68	131	315	0
N.S.	1	1.00	0.80	1.60	1.18	16.16	0.48	0.93	2.23	0.00
time (sec)	N/A	0.098	0.061	0.043	3.038	0.478	0.573	0.204	5.110	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	99	129	147	0	85	114	162	0
N.S.	1	1.00	0.80	1.05	1.20	0.00	0.69	0.93	1.32	0.00
time (sec)	N/A	0.118	0.054	0.047	3.090	0.000	0.770	0.195	0.217	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	129	241	187	0	292	143	270	0
N.S.	1	1.00	0.79	1.48	1.15	0.00	1.79	0.88	1.66	0.00
time (sec)	N/A	0.124	0.083	0.048	3.062	0.000	5.070	0.216	5.519	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	11	10	11	9	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.69	0.00
time (sec)	N/A	0.004	0.004	0.046	1.323	0.392	0.094	0.164	0.030	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	108	125	149	359	51	109	117	0
N.S.	1	1.00	0.95	1.10	1.31	3.15	0.45	0.96	1.03	0.00
time (sec)	N/A	0.099	0.034	0.048	3.056	0.434	0.417	0.196	0.283	0.001

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	65	28	113	27	53	93	25	0
N.S.	1	1.00	1.81	0.78	3.14	0.75	1.47	2.58	0.69	0.00
time (sec)	N/A	0.031	0.045	0.047	3.062	0.386	0.411	0.181	0.056	0.001

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	128	140	171	0	199	125	307	0
N.S.	1	1.00	0.94	1.03	1.26	0.00	1.46	0.92	2.26	0.00
time (sec)	N/A	0.118	0.096	0.045	3.053	0.000	1.684	0.197	5.496	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	108	125	152	272	70	109	117	0
N.S.	1	1.00	0.95	1.10	1.33	2.39	0.61	0.96	1.03	0.00
time (sec)	N/A	0.121	0.030	0.041	3.026	0.447	0.422	0.276	0.374	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	148	237	195	2326	148	137	286	0
N.S.	1	1.00	0.96	1.54	1.27	15.10	0.96	0.89	1.86	0.00
time (sec)	N/A	0.117	0.106	0.047	2.993	0.489	1.379	0.209	5.810	0.001

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	125	140	174	0	189	124	300	0
N.S.	1	1.00	0.92	1.03	1.28	0.00	1.39	0.91	2.21	0.00
time (sec)	N/A	0.140	0.083	0.046	3.017	0.000	1.991	0.197	5.388	0.001

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	164	252	207	0	580	149	1168	0
N.S.	1	1.00	0.93	1.43	1.18	0.00	3.30	0.85	6.64	0.00
time (sec)	N/A	0.144	0.187	0.046	2.993	0.000	13.068	0.216	5.636	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	5	7	6	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75	0.00
time (sec)	N/A	0.008	0.003	0.043	1.347	0.401	0.073	0.150	0.023	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	50	102	76	145	73	70	156	0
N.S.	1	1.00	0.94	1.92	1.43	2.74	1.38	1.32	2.94	0.00
time (sec)	N/A	0.042	0.042	0.046	3.004	0.426	0.427	0.152	0.402	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	203	171	160	0	187	290	312	0
N.S.	1	1.00	1.64	1.38	1.29	0.00	1.51	2.34	2.52	0.00
time (sec)	N/A	0.087	0.064	0.049	3.024	0.000	2.278	0.173	5.034	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	277	277	283	286	296	0	187	270	305	0
N.S.	1	1.00	1.02	1.03	1.07	0.00	0.68	0.97	1.10	0.00
time (sec)	N/A	0.196	0.240	0.047	2.975	0.000	2.268	0.209	5.041	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	249	244	202	0	0	303	5082	0
N.S.	1	1.00	1.68	1.65	1.36	0.00	0.00	2.05	34.34	0.00
time (sec)	N/A	0.203	0.092	0.047	3.104	0.000	0.000	0.194	5.507	0.001

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	221	289	224	0	0	344	1393	0
N.S.	1	1.00	1.28	1.68	1.30	0.00	0.00	2.00	8.10	0.00
time (sec)	N/A	0.165	0.425	0.052	3.114	0.000	0.000	0.184	5.560	0.001

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	221	263	328	284	0	0	393	1002	0
N.S.	1	1.00	1.19	1.48	1.29	0.00	0.00	1.78	4.53	0.00
time (sec)	N/A	0.263	0.772	0.072	3.004	0.000	0.000	0.195	5.436	0.001

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	313	368	345	0	0	442	1056	0
N.S.	1	1.00	1.18	1.38	1.30	0.00	0.00	1.66	3.97	0.00
time (sec)	N/A	0.320	0.390	0.059	3.179	0.000	0.000	0.189	5.662	0.001

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	319	319	311	429	328	0	0	340	5042	0
N.S.	1	1.00	0.97	1.34	1.03	0.00	0.00	1.07	15.81	0.00
time (sec)	N/A	0.351	0.407	0.055	3.028	0.000	0.000	0.273	5.588	0.001

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	341	341	319	482	350	0	0	365	1383	0
N.S.	1	1.00	0.94	1.41	1.03	0.00	0.00	1.07	4.06	0.00
time (sec)	N/A	0.305	0.232	0.053	3.003	0.000	0.000	0.203	5.592	0.001

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	394	394	366	519	412	0	0	416	1001	0
N.S.	1	1.00	0.93	1.32	1.05	0.00	0.00	1.06	2.54	0.00
time (sec)	N/A	0.439	0.392	0.062	3.029	0.000	0.000	0.197	0.705	0.001

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	437	437	411	560	472	0	0	466	1053	0
N.S.	1	1.00	0.94	1.28	1.08	0.00	0.00	1.07	2.41	0.00
time (sec)	N/A	0.531	0.520	0.064	3.120	0.000	0.000	0.188	5.562	0.001

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	9	8	15	15	15	15	15	9
N.S.	1	1.00	0.82	0.73	1.36	1.36	1.36	1.36	1.36	0.82
time (sec)	N/A	0.013	0.002	0.046	1.295	0.397	0.093	0.242	0.031	0.027

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	14	8	12	12	8	12	11	9
N.S.	1	1.00	1.27	0.73	1.09	1.09	0.73	1.09	1.00	0.82
time (sec)	N/A	0.012	0.001	0.043	1.292	0.400	0.082	0.159	0.024	0.025

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	8	7	7	5	7	6	11
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.56	0.78	0.67	1.22
time (sec)	N/A	0.009	0.001	0.041	1.295	0.409	0.070	0.154	0.019	0.021

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	5	7	6	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75	0.00
time (sec)	N/A	0.007	0.001	0.045	1.372	0.404	0.073	0.166	0.002	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	8	7	7	5	7	7	0
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00	0.00
time (sec)	N/A	0.016	0.001	0.042	1.296	0.397	0.107	0.213	0.031	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	9	8	12	12	10	7	7	0
N.S.	1	1.00	0.82	0.73	1.09	1.09	0.91	0.64	0.64	0.00
time (sec)	N/A	0.018	0.003	0.038	1.302	0.396	0.210	0.157	4.837	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	9	8	17	17	17	7	7	0
N.S.	1	1.00	0.82	0.73	1.55	1.55	1.55	0.64	0.64	0.00
time (sec)	N/A	0.019	0.001	0.045	1.325	0.393	0.150	0.156	4.806	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	256	296	222	0	0	342	2478	0
N.S.	1	1.00	1.55	1.79	1.35	0.00	0.00	2.07	15.02	0.00
time (sec)	N/A	0.261	0.427	0.047	3.039	0.000	0.000	0.197	5.544	0.001

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	301	367	240	0	0	541	3810	0
N.S.	1	1.00	1.60	1.95	1.28	0.00	0.00	2.88	20.27	0.00
time (sec)	N/A	0.327	0.547	0.054	3.025	0.000	0.000	0.213	5.074	0.001

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	318	393	257	0	0	556	5673	0
N.S.	1	1.00	1.55	1.92	1.25	0.00	0.00	2.71	27.67	0.00
time (sec)	N/A	0.313	0.512	0.052	3.074	0.000	0.000	0.215	5.161	0.001

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	337	337	342	462	351	0	0	375	2469	0
N.S.	1	1.00	1.01	1.37	1.04	0.00	0.00	1.11	7.33	0.00
time (sec)	N/A	0.399	0.487	0.047	3.047	0.000	0.000	0.189	5.542	0.001

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	384	384	427	603	399	0	0	562	3798	0
N.S.	1	1.00	1.11	1.57	1.04	0.00	0.00	1.46	9.89	0.00
time (sec)	N/A	0.565	0.371	0.056	3.077	0.000	0.000	0.209	5.054	0.001

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	402	402	445	627	429	0	0	578	5664	0
N.S.	1	1.00	1.11	1.56	1.07	0.00	0.00	1.44	14.09	0.00
time (sec)	N/A	0.567	0.418	0.049	3.156	0.000	0.000	0.204	5.203	0.001

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	257	340	243	0	0	380	1626	0
N.S.	1	1.00	1.40	1.85	1.32	0.00	0.00	2.07	8.84	0.00
time (sec)	N/A	0.204	0.284	0.053	3.077	0.000	0.000	0.193	5.615	0.001

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	302	409	260	0	0	583	2611	0
N.S.	1	1.00	1.49	2.01	1.28	0.00	0.00	2.87	12.86	0.00
time (sec)	N/A	0.274	0.279	0.051	3.064	0.000	0.000	0.200	5.671	0.001

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	338	431	299	0	0	610	3943	0
N.S.	1	1.00	1.50	1.92	1.33	0.00	0.00	2.71	17.52	0.00
time (sec)	N/A	0.310	0.251	0.060	3.149	0.000	0.000	0.225	5.909	0.001

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	353	353	359	515	374	0	0	398	1623	0
N.S.	1	1.00	1.02	1.46	1.06	0.00	0.00	1.13	4.60	0.00
time (sec)	N/A	0.340	0.302	0.056	3.194	0.000	0.000	0.186	5.578	0.001

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	395	395	415	654	416	0	0	589	2605	0
N.S.	1	1.00	1.05	1.66	1.05	0.00	0.00	1.49	6.59	0.00
time (sec)	N/A	0.492	0.465	0.054	3.194	0.000	0.000	0.598	5.702	0.001

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	417	417	460	675	458	0	0	617	3939	0
N.S.	1	1.00	1.10	1.62	1.10	0.00	0.00	1.48	9.45	0.00
time (sec)	N/A	0.536	0.440	0.063	3.214	0.000	0.000	0.219	5.844	0.001

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	309	389	316	0	0	440	1687	0
N.S.	1	1.00	1.28	1.61	1.31	0.00	0.00	1.83	7.00	0.00
time (sec)	N/A	0.339	0.416	0.062	2.963	0.000	0.000	0.208	5.732	0.001

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	359	472	343	0	0	652	2680	0
N.S.	1	1.00	1.34	1.76	1.28	0.00	0.00	2.43	10.00	0.00
time (sec)	N/A	0.435	0.400	0.063	3.083	0.000	0.000	0.280	5.801	0.001

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	285	285	380	488	377	0	0	684	2696	0
N.S.	1	1.00	1.33	1.71	1.32	0.00	0.00	2.40	9.46	0.00
time (sec)	N/A	0.391	0.338	0.061	3.129	0.000	0.000	0.213	5.910	0.001

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	413	413	411	561	446	0	0	459	1686	0
N.S.	1	1.00	1.00	1.36	1.08	0.00	0.00	1.11	4.08	0.00
time (sec)	N/A	0.486	0.425	0.058	3.068	0.000	0.000	0.204	5.691	0.001

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	463	463	473	716	497	0	0	661	2680	0
N.S.	1	1.00	1.02	1.55	1.07	0.00	0.00	1.43	5.79	0.00
time (sec)	N/A	0.686	0.677	0.061	3.175	0.000	0.000	0.226	5.749	0.001

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	480	480	500	731	535	0	0	693	2695	0
N.S.	1	1.00	1.04	1.52	1.11	0.00	0.00	1.44	5.61	0.00
time (sec)	N/A	0.666	0.508	0.059	3.093	0.000	0.000	0.221	5.788	0.001

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	360	434	389	0	0	501	1747	0
N.S.	1	1.00	1.23	1.48	1.33	0.00	0.00	1.71	5.96	0.00
time (sec)	N/A	0.431	0.487	0.064	3.178	0.000	0.000	0.263	5.994	0.001

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	422	522	429	0	0	727	2747	0
N.S.	1	1.00	1.27	1.58	1.30	0.00	0.00	2.20	8.30	0.00
time (sec)	N/A	0.567	0.545	0.059	3.163	0.000	0.000	0.209	6.140	0.001

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	349	349	439	538	463	0	0	759	2764	0
N.S.	1	1.00	1.26	1.54	1.33	0.00	0.00	2.17	7.92	0.00
time (sec)	N/A	0.524	0.525	0.060	3.085	0.000	0.000	0.207	6.396	0.001

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	462	462	461	607	517	0	0	521	1743	0
N.S.	1	1.00	1.00	1.31	1.12	0.00	0.00	1.13	3.77	0.00
time (sec)	N/A	0.619	0.581	0.070	3.126	0.000	0.000	0.204	6.075	0.001

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	516	516	530	767	579	0	0	735	2741	0
N.S.	1	1.00	1.03	1.49	1.12	0.00	0.00	1.42	5.31	0.00
time (sec)	N/A	0.850	1.010	0.066	3.160	0.000	0.000	0.207	6.084	0.001

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	534	534	555	783	613	0	0	767	2757	0
N.S.	1	1.00	1.04	1.47	1.15	0.00	0.00	1.44	5.16	0.00
time (sec)	N/A	0.824	0.706	0.067	3.228	0.000	0.000	0.206	6.480	0.001

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	80	12	12	14
N.S.	1	1.00	1.00	0.93	0.86	0.86	5.71	0.86	0.86	1.00
time (sec)	N/A	0.006	0.010	0.047	1.764	0.414	9.598	0.198	5.036	0.368

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	27	24	25	34	104	23	23	29
N.S.	1	1.00	0.93	0.83	0.86	1.17	3.59	0.79	0.79	1.00
time (sec)	N/A	0.023	0.103	0.048	1.767	0.417	12.387	0.245	4.905	15.155

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	27	24	23	33	109	22	20	27
N.S.	1	1.00	1.08	0.96	0.92	1.32	4.36	0.88	0.80	1.08
time (sec)	N/A	0.028	0.043	0.048	1.828	0.420	17.799	0.201	4.901	15.855

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	35	44	44	133	31	29	38
N.S.	1	1.00	1.00	0.92	1.16	1.16	3.50	0.82	0.76	1.00
time (sec)	N/A	0.030	0.046	0.044	1.850	0.413	21.515	0.224	4.836	34.846

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	58	10	10	12
N.S.	1	1.00	1.00	0.92	0.83	0.83	4.83	0.83	0.83	1.00
time (sec)	N/A	0.003	0.006	0.048	3.244	0.407	5.208	0.181	4.847	0.206

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	51	173	0	835	1287	101	64	0
N.S.	1	1.00	0.47	1.59	0.00	7.66	11.81	0.93	0.59	0.00
time (sec)	N/A	0.064	0.012	0.122	0.000	1.282	1.202	0.221	4.918	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	47	169	0	799	1287	101	65	0
N.S.	1	1.00	0.43	1.55	0.00	7.33	11.81	0.93	0.60	0.00
time (sec)	N/A	0.041	0.011	0.112	0.000	1.268	1.284	0.185	4.981	0.001

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	187	266	209	210	216	246	237	0
N.S.	1	1.00	0.90	1.28	1.00	1.01	1.04	1.18	1.14	0.00
time (sec)	N/A	0.316	0.106	0.046	1.374	0.408	1.318	0.173	4.920	0.002

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	154	218	169	170	172	197	189	0
N.S.	1	1.00	0.91	1.28	0.99	1.00	1.01	1.16	1.11	0.00
time (sec)	N/A	0.242	0.093	0.046	1.383	0.412	1.345	0.186	4.959	0.001

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	119	170	129	130	128	148	141	0
N.S.	1	1.00	0.90	1.29	0.98	0.98	0.97	1.12	1.07	0.00
time (sec)	N/A	0.183	0.073	0.050	1.374	0.410	1.045	0.168	4.927	0.001

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	88	124	91	92	88	101	96	0
N.S.	1	1.00	0.92	1.29	0.95	0.96	0.92	1.05	1.00	0.00
time (sec)	N/A	0.140	0.051	0.044	1.389	0.412	1.126	0.205	4.827	0.001

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	75	97	77	80	70	79	76	0
N.S.	1	1.00	0.94	1.21	0.96	1.00	0.88	0.99	0.95	0.00
time (sec)	N/A	0.120	0.037	0.050	1.383	0.463	5.262	0.205	4.925	0.001

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	77	94	77	85	70	95	74	0
N.S.	1	1.00	0.95	1.16	0.95	1.05	0.86	1.17	0.91	0.00
time (sec)	N/A	0.116	0.048	0.056	1.335	0.477	14.561	0.177	4.973	0.001

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	88	116	93	101	85	126	92	0
N.S.	1	1.00	0.93	1.22	0.98	1.06	0.89	1.33	0.97	0.00
time (sec)	N/A	0.129	0.075	0.052	1.356	0.458	74.001	0.163	4.993	0.001

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	128	161	125	127	0	184	123	0
N.S.	1	1.00	1.00	1.26	0.98	0.99	0.00	1.44	0.96	0.00
time (sec)	N/A	0.162	0.093	0.049	1.358	0.445	0.000	0.178	5.025	0.001

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	164	210	166	168	0	235	161	0
N.S.	1	1.00	1.00	1.28	1.01	1.02	0.00	1.43	0.98	0.00
time (sec)	N/A	0.181	0.087	0.057	1.359	0.492	0.000	0.168	5.073	0.001

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	194	260	208	210	0	287	200	0
N.S.	1	1.00	0.95	1.27	1.01	1.02	0.00	1.40	0.98	0.00
time (sec)	N/A	0.209	0.237	0.053	1.415	0.516	0.000	0.170	0.257	0.001

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	348	348	351	592	351	342	469	454	358	0
N.S.	1	1.00	1.01	1.70	1.01	0.98	1.35	1.30	1.03	0.00
time (sec)	N/A	0.333	0.088	0.049	3.015	0.435	4.350	0.182	0.311	0.001

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	311	554	313	321	513	441	313	0
N.S.	1	1.00	0.98	1.75	0.99	1.02	1.62	1.40	0.99	0.00
time (sec)	N/A	0.306	0.107	0.050	2.946	0.425	4.057	0.185	5.162	0.001

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	312	312	306	544	311	304	423	401	311	0
N.S.	1	1.00	0.98	1.74	1.00	0.97	1.36	1.29	1.00	0.00
time (sec)	N/A	0.298	0.113	0.045	2.968	0.424	3.243	0.179	5.187	0.001

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	266	502	269	281	469	386	267	0
N.S.	1	1.00	0.95	1.80	0.96	1.01	1.68	1.38	0.96	0.00
time (sec)	N/A	0.273	0.117	0.047	3.025	0.424	2.481	0.182	5.147	0.001

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	274	274	264	492	267	249	376	346	264	0
N.S.	1	1.00	0.96	1.80	0.97	0.91	1.37	1.26	0.96	0.00
time (sec)	N/A	0.267	0.109	0.046	2.929	0.436	2.511	0.185	5.101	0.001

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	231	450	225	568	427	291	225	0
N.S.	1	1.00	0.94	1.84	0.92	2.32	1.74	1.19	0.92	0.00
time (sec)	N/A	0.216	0.179	0.046	3.014	0.447	2.380	0.196	5.135	0.001

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	229	442	223	600	342	253	222	0
N.S.	1	1.00	0.95	1.84	0.93	2.50	1.42	1.05	0.92	0.00
time (sec)	N/A	0.153	0.168	0.043	3.014	0.447	3.414	0.187	5.174	0.001

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	227	224	419	217	560	408	269	204	0
N.S.	1	1.00	0.99	1.85	0.96	2.47	1.80	1.19	0.90	0.00
time (sec)	N/A	0.193	0.205	0.049	2.962	0.469	4.724	0.181	5.372	0.001

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	218	414	214	565	326	232	201	0
N.S.	1	1.00	0.97	1.85	0.96	2.52	1.46	1.04	0.90	0.00
time (sec)	N/A	0.170	0.161	0.055	2.982	0.452	4.364	0.217	0.284	0.001

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	227	220	412	217	556	411	261	209	0
N.S.	1	1.00	0.97	1.81	0.96	2.45	1.81	1.15	0.92	0.00
time (sec)	N/A	0.186	0.157	0.057	3.037	0.461	11.527	0.184	5.164	0.001

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	220	410	214	584	328	220	207	0
N.S.	1	1.00	0.98	1.82	0.95	2.60	1.46	0.98	0.92	0.00
time (sec)	N/A	0.170	0.122	0.051	3.080	0.442	19.683	0.483	5.091	0.001

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	231	440	234	610	432	275	219	0
N.S.	1	1.00	0.95	1.82	0.97	2.52	1.79	1.14	0.90	0.00
time (sec)	N/A	0.188	0.145	0.048	3.067	0.448	46.610	0.213	5.200	0.001

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	231	441	234	595	348	297	220	0
N.S.	1	1.00	0.95	1.81	0.96	2.44	1.43	1.22	0.90	0.00
time (sec)	N/A	0.174	0.245	0.061	3.009	0.449	88.516	0.196	5.126	0.001

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	277	277	266	491	260	262	0	376	253	0
N.S.	1	1.00	0.96	1.77	0.94	0.95	0.00	1.36	0.91	0.00
time (sec)	N/A	0.222	0.135	0.062	3.054	0.431	0.000	0.194	5.329	0.001

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	280	280	266	493	260	295	0	338	253	0
N.S.	1	1.00	0.95	1.76	0.93	1.05	0.00	1.21	0.90	0.00
time (sec)	N/A	0.199	0.163	0.054	2.974	0.433	0.000	0.185	5.153	0.001

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	308	546	307	317	0	419	286	0
N.S.	1	1.00	0.98	1.74	0.98	1.01	0.00	1.34	0.91	0.00
time (sec)	N/A	0.238	0.135	0.053	2.988	0.433	0.000	0.183	5.228	0.001

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	315	315	311	548	307	335	0	393	287	0
N.S.	1	1.00	0.99	1.74	0.97	1.06	0.00	1.25	0.91	0.00
time (sec)	N/A	0.229	0.153	0.056	3.114	0.428	0.000	0.195	5.171	0.001

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	351	351	346	600	353	355	0	474	323	0
N.S.	1	1.00	0.99	1.71	1.01	1.01	0.00	1.35	0.92	0.00
time (sec)	N/A	0.258	0.146	0.056	3.029	0.433	0.000	0.190	5.164	0.001

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	220	220	205	288	222	303	236	300	356	0
N.S.	1	1.00	0.93	1.31	1.01	1.38	1.07	1.36	1.62	0.00
time (sec)	N/A	0.341	0.209	0.065	1.303	0.412	14.416	0.249	4.988	0.001

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	180	167	240	180	257	189	248	233	0
N.S.	1	1.00	0.93	1.33	1.00	1.43	1.05	1.38	1.29	0.00
time (sec)	N/A	0.265	0.142	0.059	1.381	0.400	12.381	0.183	4.995	0.001

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	129	192	138	202	141	217	155	0
N.S.	1	1.00	0.92	1.37	0.99	1.44	1.01	1.55	1.11	0.00
time (sec)	N/A	0.199	0.124	0.056	1.397	0.398	12.812	0.197	4.930	0.001

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	93	142	98	143	100	206	103	0
N.S.	1	1.00	0.90	1.38	0.95	1.39	0.97	2.00	1.00	0.00
time (sec)	N/A	0.145	0.068	0.066	1.352	0.391	11.614	0.189	0.085	0.001

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	95	125	100	145	95	125	100	0
N.S.	1	1.00	0.95	1.25	1.00	1.45	0.95	1.25	1.00	0.00
time (sec)	N/A	0.125	0.181	0.062	1.319	0.439	41.960	0.168	5.033	0.001

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	97	132	116	172	0	131	109	0
N.S.	1	1.00	0.89	1.21	1.06	1.58	0.00	1.20	1.00	0.00
time (sec)	N/A	0.142	0.151	0.059	1.434	0.436	0.000	0.213	5.046	0.001

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	118	167	138	208	0	201	130	0
N.S.	1	1.00	0.91	1.28	1.06	1.60	0.00	1.55	1.00	0.00
time (sec)	N/A	0.154	0.137	0.066	1.353	0.433	0.000	0.168	5.014	0.001

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	160	229	181	261	0	275	175	0
N.S.	1	1.00	0.91	1.31	1.03	1.49	0.00	1.57	1.00	0.00
time (sec)	N/A	0.203	0.139	0.063	1.433	0.469	0.000	0.196	5.084	0.001

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	198	282	226	310	0	331	216	0
N.S.	1	1.00	0.93	1.32	1.06	1.45	0.00	1.55	1.01	0.00
time (sec)	N/A	0.234	0.264	0.069	1.438	0.484	0.000	0.171	5.087	0.001

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	369	369	364	622	369	488	500	451	481	0
N.S.	1	1.00	0.99	1.69	1.00	1.32	1.36	1.22	1.30	0.00
time (sec)	N/A	0.467	0.435	0.053	2.982	0.427	15.903	0.199	0.347	0.001

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	335	335	319	584	325	455	539	442	362	0
N.S.	1	1.00	0.95	1.74	0.97	1.36	1.61	1.32	1.08	0.00
time (sec)	N/A	0.705	0.196	0.054	3.021	0.421	58.232	0.200	5.278	0.001

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	328	328	315	567	321	423	449	394	358	0
N.S.	1	1.00	0.96	1.73	0.98	1.29	1.37	1.20	1.09	0.00
time (sec)	N/A	0.369	0.304	0.055	3.016	0.422	14.983	0.183	5.201	0.001

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	282	529	277	920	490	344	287	0
N.S.	1	1.00	0.95	1.78	0.93	3.09	1.64	1.15	0.96	0.00
time (sec)	N/A	0.463	0.299	0.058	3.078	0.454	51.285	0.187	5.222	0.001

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	277	514	270	946	401	295	280	0
N.S.	1	1.00	0.96	1.78	0.94	3.28	1.39	1.02	0.97	0.00
time (sec)	N/A	0.326	0.193	0.053	2.985	0.469	12.901	0.179	0.311	0.001

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	255	495	259	874	461	318	246	0
N.S.	1	1.00	0.94	1.83	0.96	3.23	1.70	1.17	0.91	0.00
time (sec)	N/A	0.289	0.187	0.054	3.122	0.469	22.482	0.220	5.232	0.001

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	264	264	251	482	254	861	377	273	241	0
N.S.	1	1.00	0.95	1.83	0.96	3.26	1.43	1.03	0.91	0.00
time (sec)	N/A	0.264	0.200	0.055	3.045	0.446	7.023	0.208	5.177	0.001

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	265	265	255	474	258	860	457	305	244	0
N.S.	1	1.00	0.96	1.79	0.97	3.25	1.72	1.15	0.92	0.00
time (sec)	N/A	0.253	0.212	0.058	3.015	0.447	32.216	0.185	5.390	0.001

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	260	260	250	463	258	902	381	261	245	0
N.S.	1	1.00	0.96	1.78	0.99	3.47	1.47	1.00	0.94	0.00
time (sec)	N/A	0.246	0.219	0.061	2.971	0.453	77.381	0.178	5.222	0.001

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	255	486	267	902	473	310	247	0
N.S.	1	1.00	0.95	1.81	0.99	3.35	1.76	1.15	0.92	0.00
time (sec)	N/A	0.288	0.274	0.066	2.930	0.447	177.026	0.197	5.177	0.001

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	253	477	268	897	0	264	248	0
N.S.	1	1.00	0.94	1.77	0.99	3.32	0.00	0.98	0.92	0.00
time (sec)	N/A	0.272	0.192	0.055	2.927	0.449	0.000	0.177	5.127	0.001

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	297	281	529	292	982	0	333	274	0
N.S.	1	1.00	0.95	1.78	0.98	3.31	0.00	1.12	0.92	0.00
time (sec)	N/A	0.384	0.258	0.069	3.053	0.449	0.000	0.231	5.184	0.001

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	297	280	520	292	959	0	347	274	0
N.S.	1	1.00	0.94	1.75	0.98	3.23	0.00	1.17	0.92	0.00
time (sec)	N/A	0.370	0.240	0.063	3.031	0.444	0.000	0.199	5.200	0.001

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	334	334	319	575	323	442	0	437	310	0
N.S.	1	1.00	0.96	1.72	0.97	1.32	0.00	1.31	0.93	0.00
time (sec)	N/A	0.457	0.206	0.063	3.133	0.422	0.000	0.371	5.406	0.001

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	335	335	317	566	323	475	0	391	310	0
N.S.	1	1.00	0.95	1.69	0.96	1.42	0.00	1.17	0.93	0.00
time (sec)	N/A	0.434	0.325	0.058	3.065	0.430	0.000	0.180	5.121	0.001

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	375	375	370	631	374	507	0	482	348	0
N.S.	1	1.00	0.99	1.68	1.00	1.35	0.00	1.29	0.93	0.00
time (sec)	N/A	0.534	0.413	0.063	2.974	0.420	0.000	0.207	5.118	0.001

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	246	361	275	396	0	349	449	0
N.S.	1	1.00	0.92	1.36	1.03	1.49	0.00	1.31	1.69	0.00
time (sec)	N/A	0.436	0.190	0.061	1.542	0.391	0.000	0.184	4.958	0.001

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	226	226	208	313	233	353	0	298	293	0
N.S.	1	1.00	0.92	1.38	1.03	1.56	0.00	1.32	1.30	0.00
time (sec)	N/A	0.331	0.202	0.061	1.423	0.398	0.000	0.246	4.969	0.001

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	170	266	191	295	0	236	204	0
N.S.	1	1.00	0.91	1.43	1.03	1.59	0.00	1.27	1.10	0.00
time (sec)	N/A	0.269	0.165	0.058	1.352	0.398	0.000	0.216	4.924	0.001

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	145	213	147	225	0	146	152	0
N.S.	1	1.00	0.99	1.46	1.01	1.54	0.00	1.00	1.04	0.00
time (sec)	N/A	0.201	0.103	0.072	1.395	0.405	0.000	0.184	0.105	0.001

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	105	156	109	158	0	100	112	0
N.S.	1	1.00	0.96	1.43	1.00	1.45	0.00	0.92	1.03	0.00
time (sec)	N/A	0.152	0.061	0.062	1.384	0.396	0.000	0.196	4.939	0.001

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	104	147	129	187	0	128	123	0
N.S.	1	1.00	0.91	1.29	1.13	1.64	0.00	1.12	1.08	0.00
time (sec)	N/A	0.155	0.126	0.062	1.369	0.435	0.000	0.241	0.175	0.001

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	121	163	144	250	0	173	135	0
N.S.	1	1.00	0.90	1.22	1.07	1.87	0.00	1.29	1.01	0.00
time (sec)	N/A	0.171	0.109	0.059	1.355	0.425	0.000	0.183	5.068	0.001

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	149	213	182	316	0	189	167	0
N.S.	1	1.00	0.91	1.31	1.12	1.94	0.00	1.16	1.02	0.00
time (sec)	N/A	0.200	0.133	0.059	1.398	0.428	0.000	0.194	5.099	0.001

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	200	293	232	396	0	324	222	0
N.S.	1	1.00	0.92	1.34	1.06	1.82	0.00	1.49	1.02	0.00
time (sec)	N/A	0.264	0.166	0.067	1.459	0.473	0.000	0.188	5.171	0.001

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	258	258	238	349	280	448	0	380	265	0
N.S.	1	1.00	0.92	1.35	1.09	1.74	0.00	1.47	1.03	0.00
time (sec)	N/A	0.304	0.267	0.063	1.465	0.500	0.000	0.201	0.307	0.001

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	416	416	411	706	424	667	0	500	575	0
N.S.	1	1.00	0.99	1.70	1.02	1.60	0.00	1.20	1.38	0.00
time (sec)	N/A	0.742	0.688	0.066	3.056	0.443	0.000	0.203	5.241	0.001

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	384	384	380	668	380	634	0	491	425	0
N.S.	1	1.00	0.99	1.74	0.99	1.65	0.00	1.28	1.11	0.00
time (sec)	N/A	1.050	0.554	0.068	3.004	0.429	0.000	0.202	5.339	0.001

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	375	375	362	651	376	602	0	443	420	0
N.S.	1	1.00	0.97	1.74	1.00	1.61	0.00	1.18	1.12	0.00
time (sec)	N/A	0.606	0.468	0.064	3.033	0.436	0.000	0.197	5.351	0.001

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	345	345	329	611	330	1278	0	391	338	0
N.S.	1	1.00	0.95	1.77	0.96	3.70	0.00	1.13	0.98	0.00
time (sec)	N/A	0.763	0.365	0.061	3.075	0.448	0.000	0.203	5.530	0.001

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	323	596	326	1318	0	345	335	0
N.S.	1	1.00	0.96	1.77	0.97	3.92	0.00	1.03	1.00	0.00
time (sec)	N/A	0.509	0.368	0.053	3.035	0.446	0.000	0.197	5.302	0.001

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	300	574	311	1224	0	365	295	0
N.S.	1	1.00	0.95	1.82	0.98	3.87	0.00	1.16	0.93	0.00
time (sec)	N/A	0.505	0.292	0.063	3.096	0.446	0.000	0.201	5.271	0.001

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	307	307	294	561	305	1213	0	319	290	0
N.S.	1	1.00	0.96	1.83	0.99	3.95	0.00	1.04	0.94	0.00
time (sec)	N/A	0.412	0.308	0.065	3.061	0.467	0.000	0.216	5.143	0.001

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	301	301	284	550	296	1158	0	339	280	0
N.S.	1	1.00	0.94	1.83	0.98	3.85	0.00	1.13	0.93	0.00
time (sec)	N/A	0.368	0.321	0.059	2.942	0.444	0.000	0.206	5.268	0.001

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	292	292	279	539	291	1184	0	295	275	0
N.S.	1	1.00	0.96	1.85	1.00	4.05	0.00	1.01	0.94	0.00
time (sec)	N/A	0.307	0.235	0.062	3.068	0.453	0.000	0.198	5.196	0.001

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	303	303	286	547	300	1206	0	341	276	0
N.S.	1	1.00	0.94	1.81	0.99	3.98	0.00	1.13	0.91	0.00
time (sec)	N/A	0.339	0.293	0.116	2.961	0.459	0.000	0.210	5.198	0.001

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	301	301	283	539	302	1217	0	312	279	0
N.S.	1	1.00	0.94	1.79	1.00	4.04	0.00	1.04	0.93	0.00
time (sec)	N/A	0.329	0.295	0.061	3.073	0.463	0.000	0.215	5.164	0.001

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	317	317	303	574	317	1254	0	357	293	0
N.S.	1	1.00	0.96	1.81	1.00	3.96	0.00	1.13	0.92	0.00
time (sec)	N/A	0.370	0.320	0.066	3.031	0.463	0.000	0.244	5.232	0.002

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	299	566	318	1247	0	310	293	0
N.S.	1	1.00	0.95	1.79	1.01	3.95	0.00	0.98	0.93	0.00
time (sec)	N/A	0.368	0.275	0.063	3.051	0.466	0.000	0.231	5.196	0.001

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	328	611	343	1340	0	380	321	0
N.S.	1	1.00	0.96	1.78	1.00	3.91	0.00	1.11	0.94	0.00
time (sec)	N/A	0.569	0.312	0.069	3.046	0.465	0.000	0.315	5.262	0.001

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	341	341	324	603	343	1317	0	394	321	0
N.S.	1	1.00	0.95	1.77	1.01	3.86	0.00	1.16	0.94	0.00
time (sec)	N/A	0.547	0.346	0.062	2.949	0.459	0.000	0.232	5.217	0.001

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	381	381	366	659	376	621	0	486	359	0
N.S.	1	1.00	0.96	1.73	0.99	1.63	0.00	1.28	0.94	0.00
time (sec)	N/A	0.714	0.588	0.075	3.199	0.439	0.000	0.202	5.279	0.001

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	380	380	376	651	376	654	0	440	359	0
N.S.	1	1.00	0.99	1.71	0.99	1.72	0.00	1.16	0.94	0.00
time (sec)	N/A	0.669	0.577	0.066	3.286	0.447	0.000	0.350	5.176	0.001

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	424	424	419	716	427	686	0	531	397	0
N.S.	1	1.00	0.99	1.69	1.01	1.62	0.00	1.25	0.94	0.00
time (sec)	N/A	0.853	0.666	0.071	3.055	0.438	0.000	0.220	5.304	0.001

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	59	45	44	44	53	45	56	0
N.S.	1	1.00	1.09	0.83	0.81	0.81	0.98	0.83	1.04	0.00
time (sec)	N/A	0.072	0.015	0.047	2.897	0.404	0.180	0.162	0.099	0.001

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	24	25	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.83	0.80	0.00
time (sec)	N/A	0.040	0.007	0.046	3.006	0.399	0.117	0.163	0.032	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	53	38	37	37	44	38	49	0
N.S.	1	1.00	1.20	0.86	0.84	0.84	1.00	0.86	1.11	0.00
time (sec)	N/A	0.060	0.011	0.048	2.962	0.414	0.230	0.152	4.961	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	50	35	34	34	42	35	63	0
N.S.	1	1.00	1.22	0.85	0.83	0.83	1.02	0.85	1.54	0.00
time (sec)	N/A	0.042	0.009	0.052	2.868	0.409	0.270	0.148	0.080	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	53	37	36	36	46	38	48	0
N.S.	1	1.00	1.26	0.88	0.86	0.86	1.10	0.90	1.14	0.00
time (sec)	N/A	0.049	0.009	0.051	2.879	0.420	0.208	0.164	4.960	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	60	44	43	48	49	45	55	0
N.S.	1	1.00	1.22	0.90	0.88	0.98	1.00	0.92	1.12	0.00
time (sec)	N/A	0.050	0.018	0.049	3.034	0.404	0.223	0.163	0.080	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	28	33	27	29	25	0
N.S.	1	1.00	1.00	0.84	0.88	1.03	0.84	0.91	0.78	0.00
time (sec)	N/A	0.033	0.005	0.046	2.996	0.385	0.128	0.147	0.069	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	47	35	34	34	42	35	63	0
N.S.	1	1.00	1.15	0.85	0.83	0.83	1.02	0.85	1.54	0.00
time (sec)	N/A	0.041	0.035	0.051	2.991	0.401	0.184	0.151	4.956	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	53	33	32	32	41	33	63	0
N.S.	1	1.00	1.36	0.85	0.82	0.82	1.05	0.85	1.62	0.00
time (sec)	N/A	0.040	0.020	0.063	2.988	0.400	0.159	0.152	0.089	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	44	43	43	49	45	43	0
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.89	0.82	0.78	0.00
time (sec)	N/A	0.055	0.025	0.041	1.334	0.347	0.075	0.199	0.028	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	44	43	43	49	45	43	0
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.89	0.82	0.78	0.00
time (sec)	N/A	0.038	0.003	0.048	1.359	0.354	0.074	0.177	0.025	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	41	40	40	46	42	40	0
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.92	0.84	0.80	0.00
time (sec)	N/A	0.025	0.002	0.045	1.345	0.367	0.074	0.147	0.024	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	39	38	38	44	41	38	0
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.96	0.89	0.83	0.00
time (sec)	N/A	0.025	0.005	0.048	1.326	0.401	0.139	0.153	0.029	0.001

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	39	38	45	41	41	38	0
N.S.	1	1.00	1.00	0.89	0.86	1.02	0.93	0.93	0.86	0.00
time (sec)	N/A	0.035	0.006	0.058	1.335	0.409	0.164	0.149	0.030	0.001

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	39	38	45	44	41	38	0
N.S.	1	1.00	1.00	0.89	0.86	1.02	1.00	0.93	0.86	0.00
time (sec)	N/A	0.034	0.011	0.056	1.351	0.408	0.247	0.163	0.028	0.001

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	97	80	79	79	92	82	79	0
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.12	1.00	0.96	0.00
time (sec)	N/A	0.059	0.004	0.049	1.352	0.345	0.089	0.163	0.040	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	97	80	79	79	94	82	79	0
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.15	1.00	0.96	0.00
time (sec)	N/A	0.051	0.003	0.046	1.292	0.353	0.086	0.150	0.038	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	92	77	76	76	88	79	76	0
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.14	1.03	0.99	0.00
time (sec)	N/A	0.061	0.004	0.043	1.404	0.365	0.087	0.178	0.038	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	88	75	74	74	88	78	74	0
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.00	0.89	0.84	0.00
time (sec)	N/A	0.052	0.010	0.040	1.371	0.409	0.190	0.151	0.042	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	83	74	73	81	82	77	73	0
N.S.	1	1.00	1.00	0.89	0.88	0.98	0.99	0.93	0.88	0.00
time (sec)	N/A	0.063	0.014	0.058	1.297	0.410	0.248	0.150	0.042	0.001

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	84	75	74	81	87	78	74	0
N.S.	1	1.00	1.00	0.89	0.88	0.96	1.04	0.93	0.88	0.00
time (sec)	N/A	0.064	0.009	0.052	1.311	0.393	0.308	0.165	0.038	0.001

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	139	116	115	115	138	119	115	0
N.S.	1	1.00	1.26	1.05	1.05	1.05	1.25	1.08	1.05	0.00
time (sec)	N/A	0.079	0.005	0.043	1.332	0.354	0.091	0.157	0.078	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	139	116	115	115	138	119	115	0
N.S.	1	1.00	1.26	1.05	1.05	1.05	1.25	1.08	1.05	0.00
time (sec)	N/A	0.069	0.005	0.040	1.371	0.371	0.089	0.169	0.074	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	134	113	112	112	134	116	112	0
N.S.	1	1.00	1.28	1.08	1.07	1.07	1.28	1.10	1.07	0.00
time (sec)	N/A	0.097	0.037	0.052	1.329	0.366	0.139	0.149	0.073	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	127	110	109	109	131	114	109	0
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.03	0.90	0.86	0.00
time (sec)	N/A	0.074	0.012	0.048	1.286	0.395	0.294	0.152	0.079	0.001

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	125	110	109	117	128	114	109	0
N.S.	1	1.00	1.00	0.88	0.87	0.94	1.02	0.91	0.87	0.00
time (sec)	N/A	0.092	0.016	0.050	1.350	0.385	0.289	0.181	0.081	0.001

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	126	111	110	117	131	115	110	0
N.S.	1	1.00	1.00	0.88	0.87	0.93	1.04	0.91	0.87	0.00
time (sec)	N/A	0.086	0.010	0.046	1.325	0.407	0.362	0.153	4.896	0.001

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	181	152	151	151	184	156	151	0
N.S.	1	1.00	1.31	1.10	1.09	1.09	1.33	1.13	1.09	0.00
time (sec)	N/A	0.099	0.006	0.040	1.310	0.358	0.107	0.165	5.073	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	181	152	151	151	185	156	151	0
N.S.	1	1.00	1.31	1.10	1.09	1.09	1.34	1.13	1.09	0.00
time (sec)	N/A	0.093	0.005	0.041	1.336	0.352	0.113	0.158	0.131	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	173	148	147	147	178	152	147	0
N.S.	1	1.00	1.33	1.14	1.13	1.13	1.37	1.17	1.13	0.00
time (sec)	N/A	0.145	0.005	0.040	1.314	0.359	0.102	0.167	0.152	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	166	145	144	144	175	150	144	0
N.S.	1	1.00	1.00	0.87	0.87	0.87	1.05	0.90	0.87	0.00
time (sec)	N/A	0.109	0.010	0.044	1.301	0.412	0.336	0.153	0.141	0.001

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	162	145	144	153	168	150	144	0
N.S.	1	1.00	1.00	0.90	0.89	0.94	1.04	0.93	0.89	0.00
time (sec)	N/A	0.133	0.012	0.052	1.311	0.393	0.379	0.159	4.994	0.001

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	166	147	146	153	175	152	146	0
N.S.	1	1.00	1.00	0.89	0.88	0.92	1.05	0.92	0.88	0.00
time (sec)	N/A	0.125	0.010	0.051	1.334	0.408	0.443	0.166	4.993	0.001

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	191	231	190	4798	178	208	319	0
N.S.	1	1.00	0.93	1.13	0.93	23.40	0.87	1.01	1.56	0.00
time (sec)	N/A	0.261	0.111	0.044	2.942	1.270	1.639	0.180	5.067	0.002

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	184	221	181	4261	150	195	340	0
N.S.	1	1.00	0.95	1.15	0.94	22.08	0.78	1.01	1.76	0.00
time (sec)	N/A	0.248	0.096	0.049	2.936	1.236	1.490	0.211	5.133	0.001

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	200	209	173	4628	160	178	266	0
N.S.	1	1.00	1.09	1.14	0.95	25.29	0.87	0.97	1.45	0.00
time (sec)	N/A	0.226	0.060	0.046	2.943	1.235	1.432	0.208	5.162	0.001

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	176	200	159	4671	160	163	274	0
N.S.	1	1.00	0.99	1.13	0.90	26.39	0.90	0.92	1.55	0.00
time (sec)	N/A	0.132	0.105	0.046	3.013	1.202	1.423	0.180	0.257	0.001

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	176	207	176	4588	0	179	716	0
N.S.	1	1.00	0.96	1.12	0.96	24.93	0.00	0.97	3.89	0.00
time (sec)	N/A	0.206	0.099	0.053	3.018	1.383	0.000	0.184	5.247	0.002

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	184	216	186	4524	0	201	723	0
N.S.	1	1.00	0.96	1.12	0.97	23.56	0.00	1.05	3.77	0.00
time (sec)	N/A	0.214	0.254	0.049	2.981	1.431	0.000	0.230	5.056	0.001

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	192	225	177	4279	0	204	701	0
N.S.	1	1.00	0.95	1.11	0.87	21.08	0.00	1.00	3.45	0.00
time (sec)	N/A	0.194	0.231	0.218	3.032	1.343	0.000	0.183	0.131	0.001

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	174	219	163	2077	110	180	180	0
N.S.	1	1.00	0.92	1.15	0.86	10.93	0.58	0.95	0.95	0.00
time (sec)	N/A	0.166	0.203	0.053	3.033	1.209	2.333	0.204	0.220	0.001

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	186	228	185	2358	124	190	194	0
N.S.	1	1.00	0.93	1.14	0.92	11.79	0.62	0.95	0.97	0.00
time (sec)	N/A	0.153	0.204	0.052	2.840	1.219	1.853	0.182	5.167	0.001

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	189	253	179	2118	116	184	175	0
N.S.	1	1.00	0.95	1.27	0.90	10.64	0.58	0.92	0.88	0.00
time (sec)	N/A	0.132	0.282	0.045	3.025	1.203	1.385	0.183	0.251	0.001

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	222	199	274	203	5018	0	217	490	0
N.S.	1	1.00	0.90	1.23	0.91	22.60	0.00	0.98	2.21	0.00
time (sec)	N/A	0.313	0.236	0.058	2.938	1.420	0.000	0.183	0.380	0.001

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	213	275	222	4976	0	237	488	0
N.S.	1	1.00	0.92	1.19	0.96	21.54	0.00	1.03	2.11	0.00
time (sec)	N/A	0.343	0.336	0.061	3.099	1.471	0.000	0.180	5.468	0.001

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	221	276	220	4774	0	248	733	0
N.S.	1	1.00	0.91	1.14	0.91	19.73	0.00	1.02	3.03	0.00
time (sec)	N/A	0.345	0.216	0.059	2.862	1.409	0.000	0.197	5.394	0.001

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	262	262	225	289	236	5373	0	269	537	0
N.S.	1	1.00	0.86	1.10	0.90	20.51	0.00	1.03	2.05	0.00
time (sec)	N/A	0.404	0.298	0.058	3.066	1.560	0.000	0.181	5.484	0.001

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	198	255	203	2163	148	208	216	0
N.S.	1	1.00	0.92	1.19	0.94	10.06	0.69	0.97	1.00	0.00
time (sec)	N/A	0.197	0.208	0.057	3.020	1.279	6.261	0.205	0.232	0.001

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	214	256	223	2519	170	215	232	0
N.S.	1	1.00	0.90	1.07	0.93	10.54	0.71	0.90	0.97	0.00
time (sec)	N/A	0.204	0.404	0.054	3.055	1.324	3.991	0.197	0.225	0.001

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	213	308	219	2251	163	210	212	0
N.S.	1	1.00	0.95	1.37	0.97	10.00	0.72	0.93	0.94	0.00
time (sec)	N/A	0.188	0.375	0.049	2.992	1.185	2.277	0.213	0.262	0.001

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	257	257	229	331	246	5229	0	253	540	0
N.S.	1	1.00	0.89	1.29	0.96	20.35	0.00	0.98	2.10	0.00
time (sec)	N/A	0.414	0.206	0.073	3.004	1.436	0.000	0.244	5.440	0.001

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	248	334	266	5112	0	273	793	0
N.S.	1	1.00	0.93	1.25	1.00	19.15	0.00	1.02	2.97	0.00
time (sec)	N/A	0.462	0.330	0.061	3.094	1.531	0.000	0.207	5.460	0.001

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	253	337	265	4911	0	282	778	0
N.S.	1	1.00	0.92	1.22	0.96	17.79	0.00	1.02	2.82	0.00
time (sec)	N/A	0.500	0.334	0.065	3.106	1.428	0.000	0.193	5.358	0.001

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	255	351	283	5550	0	305	870	0
N.S.	1	1.00	0.86	1.18	0.95	18.62	0.00	1.02	2.92	0.00
time (sec)	N/A	0.589	0.460	0.063	3.027	1.686	0.000	0.259	0.465	0.001

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	230	275	248	2364	201	242	253	0
N.S.	1	1.00	0.93	1.11	1.00	9.53	0.81	0.98	1.02	0.00
time (sec)	N/A	0.242	0.288	0.056	3.006	1.525	17.943	0.212	0.267	0.001

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	241	278	260	2646	214	244	265	0
N.S.	1	1.00	0.89	1.03	0.96	9.80	0.79	0.90	0.98	0.00
time (sec)	N/A	0.254	0.427	0.055	3.029	1.454	8.787	0.238	0.239	0.001

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	250	239	360	254	2344	202	234	247	0
N.S.	1	1.00	0.96	1.44	1.02	9.38	0.81	0.94	0.99	0.00
time (sec)	N/A	0.222	0.279	0.055	2.991	1.217	4.470	0.205	0.279	0.001

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	259	394	293	5370	0	290	871	0
N.S.	1	1.00	0.89	1.35	1.01	18.45	0.00	1.00	2.99	0.00
time (sec)	N/A	0.517	0.310	0.063	3.037	1.547	0.000	0.238	5.402	0.001

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	301	301	279	397	313	5250	0	310	840	0
N.S.	1	1.00	0.93	1.32	1.04	17.44	0.00	1.03	2.79	0.00
time (sec)	N/A	0.601	0.313	0.067	2.995	1.556	0.000	0.184	5.434	0.001

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	284	400	312	5049	0	320	825	0
N.S.	1	1.00	0.92	1.29	1.01	16.29	0.00	1.03	2.66	0.00
time (sec)	N/A	0.656	0.315	0.067	3.104	1.453	0.000	0.185	5.375	0.001

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	340	340	284	415	330	5670	0	333	918	0
N.S.	1	1.00	0.84	1.22	0.97	16.68	0.00	0.98	2.70	0.00
time (sec)	N/A	0.773	0.602	0.066	3.081	2.030	0.000	0.197	0.525	0.001

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	57	29	26	26	54	27	26	0
N.S.	1	1.00	1.97	1.00	0.90	0.90	1.86	0.93	0.90	0.00
time (sec)	N/A	0.058	0.019	0.052	2.892	0.398	0.179	0.155	4.970	0.001

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	57	29	26	26	54	27	26	0
N.S.	1	1.00	1.97	1.00	0.90	0.90	1.86	0.93	0.90	0.00
time (sec)	N/A	0.035	0.007	0.052	2.967	0.415	0.168	0.154	0.027	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	58	29	28	28	54	29	27	0
N.S.	1	1.00	1.87	0.94	0.90	0.90	1.74	0.94	0.87	0.00
time (sec)	N/A	0.056	0.015	0.049	2.923	0.435	0.173	0.154	4.946	0.001

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	58	29	28	28	54	29	27	0
N.S.	1	1.00	1.87	0.94	0.90	0.90	1.74	0.94	0.87	0.00
time (sec)	N/A	0.037	0.007	0.058	2.838	0.397	0.169	0.151	0.030	0.001

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	146	87	51	52	100	174	154	0
N.S.	1	1.00	2.92	1.74	1.02	1.04	2.00	3.48	3.08	0.00
time (sec)	N/A	0.087	0.044	0.049	3.068	0.435	0.322	0.480	5.223	0.001

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	149	135	166	53	110	165	156	0
N.S.	1	1.00	2.81	2.55	3.13	1.00	2.08	3.11	2.94	0.00
time (sec)	N/A	0.093	0.084	0.052	3.021	0.443	0.349	0.205	5.250	0.001

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	148	132	167	56	109	97	155	0
N.S.	1	1.00	2.74	2.44	3.09	1.04	2.02	1.80	2.87	0.00
time (sec)	N/A	0.077	0.051	0.056	2.994	0.443	0.319	0.190	5.224	0.001

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	147	90	52	53	102	90	155	0
N.S.	1	1.00	2.77	1.70	0.98	1.00	1.92	1.70	2.92	0.00
time (sec)	N/A	0.076	0.063	0.051	3.018	0.427	0.372	0.196	5.234	0.002

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	80	79	85	90	87	82	0
N.S.	1	1.00	1.00	0.82	0.81	0.88	0.93	0.90	0.85	0.00
time (sec)	N/A	0.121	0.043	0.046	1.353	0.356	0.086	0.155	0.051	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	80	79	85	90	87	82	0
N.S.	1	1.00	1.00	0.82	0.81	0.88	0.93	0.90	0.85	0.00
time (sec)	N/A	0.101	0.036	0.048	1.346	0.356	0.087	0.161	0.042	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	80	79	85	90	87	82	0
N.S.	1	1.00	1.00	0.82	0.81	0.88	0.93	0.90	0.85	0.00
time (sec)	N/A	0.091	0.032	0.046	1.337	0.372	0.085	0.164	0.044	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	80	79	85	90	87	82	0
N.S.	1	1.00	1.00	0.82	0.81	0.88	0.93	0.90	0.85	0.00
time (sec)	N/A	0.080	0.027	0.046	1.374	0.374	0.084	0.147	0.043	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	92	77	76	82	87	84	79	0
N.S.	1	1.00	1.00	0.84	0.83	0.89	0.95	0.91	0.86	0.00
time (sec)	N/A	0.073	0.015	0.040	1.405	0.368	0.084	0.148	0.041	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	88	81	74	74	85	83	77	0
N.S.	1	1.00	1.00	0.92	0.84	0.84	0.97	0.94	0.88	0.00
time (sec)	N/A	0.058	0.065	0.049	1.344	0.405	0.222	0.152	0.047	0.001

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	86	81	74	81	82	83	77	0
N.S.	1	1.00	1.00	0.94	0.86	0.94	0.95	0.97	0.90	0.00
time (sec)	N/A	0.066	0.073	0.051	1.352	0.396	0.235	0.163	0.048	0.001

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	78	78	74	81	83	80	76	0
N.S.	1	1.00	0.91	0.91	0.86	0.94	0.97	0.93	0.88	0.00
time (sec)	N/A	0.072	0.072	0.051	1.342	0.412	0.306	0.162	0.043	0.001

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	76	76	75	81	83	79	75	0
N.S.	1	1.00	0.88	0.88	0.87	0.94	0.97	0.92	0.87	0.00
time (sec)	N/A	0.071	0.069	0.048	1.363	0.400	0.669	0.169	0.041	0.002

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	77	76	75	81	83	77	74	0
N.S.	1	1.00	0.90	0.88	0.87	0.94	0.97	0.90	0.86	0.00
time (sec)	N/A	0.073	0.076	0.051	1.341	0.399	2.569	0.152	4.976	0.001

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	163	152	151	157	167	160	151	0
N.S.	1	1.00	1.00	0.93	0.93	0.96	1.02	0.98	0.93	0.00
time (sec)	N/A	0.210	0.050	0.043	1.374	0.370	0.101	0.154	0.102	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	163	152	151	157	167	160	151	0
N.S.	1	1.00	1.00	0.93	0.93	0.96	1.02	0.98	0.93	0.00
time (sec)	N/A	0.159	0.032	0.042	1.349	0.370	0.105	0.153	0.088	0.001

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	150	152	151	157	167	160	151	0
N.S.	1	1.00	0.95	0.96	0.96	0.99	1.06	1.01	0.96	0.00
time (sec)	N/A	0.126	0.082	0.042	1.376	0.342	0.104	0.182	0.091	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	163	152	151	157	167	160	151	0
N.S.	1	1.00	1.03	0.96	0.96	0.99	1.06	1.01	0.96	0.00
time (sec)	N/A	0.129	0.026	0.037	1.277	0.367	0.104	0.188	0.091	0.001

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	125	149	148	154	163	157	148	0
N.S.	1	1.00	0.82	0.97	0.97	1.01	1.07	1.03	0.97	0.00
time (sec)	N/A	0.127	0.087	0.043	1.318	0.370	0.103	0.164	0.092	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	154	153	146	146	162	156	146	0
N.S.	1	1.00	1.03	1.03	0.98	0.98	1.09	1.05	0.98	0.00
time (sec)	N/A	0.106	0.046	0.045	1.314	0.403	0.341	0.151	0.096	0.001

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	152	152	146	153	156	155	145	0
N.S.	1	1.00	1.03	1.03	0.99	1.04	1.06	1.05	0.99	0.00
time (sec)	N/A	0.128	0.070	0.052	1.398	0.412	0.360	0.152	0.098	0.001

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	127	150	146	153	158	153	145	0
N.S.	1	1.00	0.86	1.02	0.99	1.04	1.07	1.04	0.99	0.00
time (sec)	N/A	0.128	0.095	0.050	1.347	0.403	0.446	0.153	5.008	0.001

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	123	149	147	153	158	153	145	0
N.S.	1	1.00	0.81	0.98	0.97	1.01	1.04	1.01	0.95	0.00
time (sec)	N/A	0.118	0.101	0.049	1.322	0.398	0.880	0.172	0.076	0.001

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	125	149	147	153	156	152	145	0
N.S.	1	1.00	0.82	0.98	0.97	1.01	1.03	1.00	0.95	0.00
time (sec)	N/A	0.117	0.115	0.054	1.366	0.410	3.235	0.170	0.066	0.001

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	223	224	217	229	246	233	205	0
N.S.	1	1.00	1.00	1.00	0.97	1.03	1.10	1.04	0.92	0.00
time (sec)	N/A	0.293	0.062	0.037	1.373	0.365	0.124	0.152	0.174	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	223	224	217	229	246	233	205	0
N.S.	1	1.00	1.00	1.00	0.97	1.03	1.10	1.04	0.92	0.00
time (sec)	N/A	0.228	0.055	0.045	1.375	0.370	0.118	0.170	5.165	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	223	224	217	229	246	233	205	0
N.S.	1	1.00	1.05	1.06	1.02	1.08	1.16	1.10	0.97	0.00
time (sec)	N/A	0.179	0.063	0.048	1.356	0.376	0.125	0.176	0.162	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	223	224	217	229	246	233	205	0
N.S.	1	1.00	1.05	1.06	1.02	1.08	1.16	1.10	0.97	0.00
time (sec)	N/A	0.177	0.037	0.039	1.345	0.356	0.114	0.166	0.157	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	207	170	221	214	226	243	230	202	0
N.S.	1	1.00	0.82	1.07	1.03	1.09	1.17	1.11	0.98	0.00
time (sec)	N/A	0.177	0.109	0.042	1.338	0.373	0.121	0.152	0.157	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	214	224	212	212	240	228	199	0
N.S.	1	1.00	1.07	1.12	1.06	1.06	1.20	1.14	1.00	0.00
time (sec)	N/A	0.146	0.135	0.047	1.424	0.423	0.541	0.159	5.113	0.001

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	172	224	212	219	236	228	199	0
N.S.	1	1.00	0.87	1.13	1.07	1.11	1.19	1.15	1.01	0.00
time (sec)	N/A	0.182	0.208	0.052	1.292	0.412	0.511	0.155	5.045	0.001

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	174	222	212	219	238	226	199	0
N.S.	1	1.00	0.88	1.12	1.07	1.11	1.20	1.14	1.01	0.00
time (sec)	N/A	0.197	0.148	0.058	1.383	0.420	0.593	0.158	0.137	0.001

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	172	220	212	219	236	225	199	0
N.S.	1	1.00	0.82	1.05	1.01	1.05	1.13	1.08	0.95	0.00
time (sec)	N/A	0.180	0.146	0.050	1.360	0.421	1.043	0.185	0.122	0.001

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	170	220	212	219	235	224	199	0
N.S.	1	1.00	0.81	1.05	1.01	1.05	1.12	1.07	0.95	0.00
time (sec)	N/A	0.177	0.160	0.046	1.394	0.405	3.137	0.155	5.027	0.001

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	334	533	378	0	881	380	1271	0
N.S.	1	1.00	1.01	1.61	1.14	0.00	2.66	1.15	3.84	0.00
time (sec)	N/A	1.070	0.558	0.049	2.978	0.000	60.517	0.200	5.086	0.001

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	299	505	332	0	845	353	1236	0
N.S.	1	1.00	0.96	1.61	1.06	0.00	2.70	1.13	3.95	0.00
time (sec)	N/A	0.988	0.292	0.049	2.896	0.000	73.527	0.184	4.992	0.001

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	294	294	290	483	313	0	790	333	1170	0
N.S.	1	1.00	0.99	1.64	1.06	0.00	2.69	1.13	3.98	0.00
time (sec)	N/A	0.976	0.313	0.047	2.997	0.000	88.696	0.244	5.023	0.001

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	275	275	272	455	300	0	811	295	1161	0
N.S.	1	1.00	0.99	1.65	1.09	0.00	2.95	1.07	4.22	0.00
time (sec)	N/A	0.921	0.480	0.044	3.029	0.000	63.001	0.284	4.989	0.001

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	259	257	254	429	266	0	804	272	1150	0
N.S.	1	0.99	0.98	1.66	1.03	0.00	3.10	1.05	4.44	0.00
time (sec)	N/A	0.373	0.388	0.049	3.037	0.000	59.388	0.190	5.033	0.001

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	258	256	258	426	290	0	0	281	1731	0
N.S.	1	0.99	1.00	1.65	1.12	0.00	0.00	1.09	6.71	0.00
time (sec)	N/A	0.471	0.314	0.051	3.019	0.000	0.000	0.189	5.097	0.001

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	257	423	290	0	0	277	1802	0
N.S.	1	1.00	1.02	1.67	1.15	0.00	0.00	1.09	7.12	0.00
time (sec)	N/A	0.454	0.321	0.056	3.025	0.000	0.000	0.187	5.087	0.001

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	260	258	257	423	271	0	0	269	6948	0
N.S.	1	0.99	0.99	1.63	1.04	0.00	0.00	1.03	26.72	0.00
time (sec)	N/A	0.380	0.464	0.050	2.996	0.000	0.000	0.275	5.205	0.001

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	274	264	442	302	0	0	291	1842	0
N.S.	1	0.99	0.96	1.60	1.09	0.00	0.00	1.05	6.67	0.00
time (sec)	N/A	0.436	0.547	0.056	3.079	0.000	0.000	0.226	5.870	0.001

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	337	337	334	562	364	0	0	357	1241	0
N.S.	1	1.00	0.99	1.67	1.08	0.00	0.00	1.06	3.68	0.00
time (sec)	N/A	0.717	0.574	0.060	3.054	0.000	0.000	0.199	5.109	0.001

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	311	311	294	533	329	0	0	330	1229	0
N.S.	1	1.00	0.95	1.71	1.06	0.00	0.00	1.06	3.95	0.00
time (sec)	N/A	0.640	0.222	0.058	3.136	0.000	0.000	0.214	0.152	0.001

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	290	288	280	506	283	12153	0	307	816	0
N.S.	1	0.99	0.97	1.74	0.98	41.91	0.00	1.06	2.81	0.00
time (sec)	N/A	0.498	0.251	0.056	3.027	1.814	0.000	0.190	0.137	0.001

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	285	502	311	12617	0	318	827	0
N.S.	1	1.00	0.99	1.74	1.08	43.66	0.00	1.10	2.86	0.00
time (sec)	N/A	0.505	0.260	0.050	2.963	2.035	0.000	0.199	5.390	0.001

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	268	462	292	12636	0	302	835	0
N.S.	1	1.00	0.97	1.67	1.06	45.78	0.00	1.09	3.03	0.00
time (sec)	N/A	0.370	0.209	0.048	2.953	1.869	0.000	0.194	5.540	0.001

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	287	269	507	302	12541	0	319	1660	0
N.S.	1	0.99	0.93	1.75	1.04	43.39	0.00	1.10	5.74	0.00
time (sec)	N/A	0.559	0.223	0.061	3.045	35.287	0.000	0.203	5.601	0.001

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	301	301	285	517	329	12556	0	328	1684	0
N.S.	1	1.00	0.95	1.72	1.09	41.71	0.00	1.09	5.59	0.00
time (sec)	N/A	0.593	0.402	0.061	3.135	35.588	0.000	0.201	5.768	0.001

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	306	304	292	527	316	12231	0	336	1632	0
N.S.	1	0.99	0.95	1.72	1.03	39.97	0.00	1.10	5.33	0.00
time (sec)	N/A	0.577	0.539	0.066	3.063	24.670	0.000	0.191	5.712	0.001

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	338	336	303	561	365	0	0	363	1924	0
N.S.	1	0.99	0.90	1.66	1.08	0.00	0.00	1.07	5.69	0.00
time (sec)	N/A	0.727	0.622	0.062	3.079	0.000	0.000	0.229	5.958	0.001

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	345	345	342	619	391	12967	0	385	916	0
N.S.	1	1.00	0.99	1.79	1.13	37.59	0.00	1.12	2.66	0.00
time (sec)	N/A	0.891	0.376	0.064	3.129	2.693	0.000	0.207	0.579	0.001

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	325	315	515	366	12939	0	363	908	0
N.S.	1	1.00	0.97	1.58	1.13	39.81	0.00	1.12	2.79	0.00
time (sec)	N/A	0.642	0.338	0.060	3.119	2.467	0.000	0.777	5.659	0.001

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	297	287	490	308	6926	0	320	627	0
N.S.	1	1.00	0.97	1.65	1.04	23.32	0.00	1.08	2.11	0.00
time (sec)	N/A	0.430	0.302	0.057	3.051	1.936	0.000	0.230	5.690	0.001

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	323	323	297	498	344	7190	0	340	640	0
N.S.	1	1.00	0.92	1.54	1.07	22.26	0.00	1.05	1.98	0.00
time (sec)	N/A	0.482	0.358	0.056	3.067	2.229	0.000	0.207	5.365	0.001

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	295	506	327	6984	0	330	630	0
N.S.	1	1.00	0.94	1.62	1.04	22.31	0.00	1.05	2.01	0.00
time (sec)	N/A	0.429	0.282	0.057	3.107	1.940	0.000	0.218	0.432	0.001

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	347	345	311	618	368	12815	0	376	1716	0
N.S.	1	0.99	0.90	1.78	1.06	36.93	0.00	1.08	4.95	0.00
time (sec)	N/A	0.723	0.349	0.068	3.112	35.656	0.000	0.265	5.705	0.001

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	362	362	336	622	400	12951	0	390	1747	0
N.S.	1	1.00	0.93	1.72	1.10	35.78	0.00	1.08	4.83	0.00
time (sec)	N/A	0.830	0.779	0.064	3.074	36.805	0.000	0.220	5.747	0.001

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	357	337	626	390	12435	0	399	1697	0
N.S.	1	0.99	0.94	1.74	1.08	34.54	0.00	1.11	4.71	0.00
time (sec)	N/A	0.814	0.713	0.066	3.102	26.933	0.000	0.226	5.659	0.001

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	395	392	352	680	444	0	0	431	1994	0
N.S.	1	0.99	0.89	1.72	1.12	0.00	0.00	1.09	5.05	0.00
time (sec)	N/A	1.006	0.789	0.070	3.089	0.000	0.000	0.203	6.321	0.001

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	55	54	54	63	56	54	0
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.93	0.82	0.79	0.00
time (sec)	N/A	0.044	0.006	0.043	1.319	0.336	0.078	0.151	0.035	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	58	57	57	66	59	57	0
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.90	0.81	0.78	0.00
time (sec)	N/A	0.064	0.004	0.045	1.334	0.356	0.073	0.156	0.031	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	124	103	102	102	121	105	102	0
N.S.	1	1.00	1.14	0.94	0.94	0.94	1.11	0.96	0.94	0.00
time (sec)	N/A	0.074	0.004	0.043	1.356	0.335	0.088	0.160	0.080	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	129	106	105	105	124	108	105	0
N.S.	1	1.00	1.13	0.93	0.92	0.92	1.09	0.95	0.92	0.00
time (sec)	N/A	0.084	0.005	0.043	1.329	0.365	0.089	0.149	0.072	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	180	151	150	150	180	154	150	0
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.19	1.02	0.99	0.00
time (sec)	N/A	0.108	0.005	0.043	1.369	0.339	0.096	0.157	0.164	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	185	154	153	153	184	157	153	0
N.S.	1	1.00	1.19	0.99	0.98	0.98	1.18	1.01	0.98	0.00
time (sec)	N/A	0.113	0.018	0.043	1.382	0.345	0.099	0.164	0.161	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	236	199	198	198	241	203	198	0
N.S.	1	1.00	1.22	1.03	1.03	1.03	1.25	1.05	1.03	0.00
time (sec)	N/A	0.156	0.007	0.043	1.330	0.357	0.103	0.173	5.080	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	241	202	201	201	245	206	201	0
N.S.	1	1.00	1.22	1.02	1.02	1.02	1.24	1.04	1.02	0.00
time (sec)	N/A	0.150	0.007	0.043	1.371	0.375	0.110	0.193	0.359	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	214	177	174	0	0	280	1970	0
N.S.	1	1.00	1.61	1.33	1.31	0.00	0.00	2.11	14.81	0.00
time (sec)	N/A	0.124	0.113	0.046	3.031	0.000	0.000	0.186	5.658	0.001

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	221	208	208	0	0	328	846	0
N.S.	1	1.00	1.36	1.28	1.28	0.00	0.00	2.02	5.22	0.00
time (sec)	N/A	0.203	0.095	0.044	2.976	0.000	0.000	0.190	4.846	0.001

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	296	294	277	0	0	290	1952	0
N.S.	1	1.00	1.01	1.00	0.95	0.00	0.00	0.99	6.66	0.00
time (sec)	N/A	0.222	0.231	0.052	3.030	0.000	0.000	0.184	0.927	0.001

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	321	321	311	325	305	0	0	308	838	0
N.S.	1	1.00	0.97	1.01	0.95	0.00	0.00	0.96	2.61	0.00
time (sec)	N/A	0.334	0.239	0.053	3.015	0.000	0.000	0.185	4.854	0.001

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	318	318	315	362	305	0	517	316	478	0
N.S.	1	1.00	0.99	1.14	0.96	0.00	1.63	0.99	1.50	0.00
time (sec)	N/A	0.270	0.406	0.049	3.058	0.000	22.319	0.184	0.360	0.001

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	294	334	294	0	510	303	559	0
N.S.	1	1.00	0.95	1.08	0.95	0.00	1.65	0.98	1.80	0.00
time (sec)	N/A	0.274	0.383	0.054	3.022	0.000	44.229	0.215	5.097	0.001

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	351	351	347	432	355	0	578	354	832	0
N.S.	1	1.00	0.99	1.23	1.01	0.00	1.65	1.01	2.37	0.00
time (sec)	N/A	0.318	0.439	0.052	3.019	0.000	108.467	0.192	5.199	0.001

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	340	340	329	373	343	0	0	338	521	0
N.S.	1	1.00	0.97	1.10	1.01	0.00	0.00	0.99	1.53	0.00
time (sec)	N/A	0.328	0.391	0.060	3.084	0.000	0.000	0.294	0.396	0.001

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	382	382	379	400	402	0	0	391	879	0
N.S.	1	1.00	0.99	1.05	1.05	0.00	0.00	1.02	2.30	0.00
time (sec)	N/A	0.406	0.442	0.063	3.088	0.000	0.000	0.329	5.255	0.001

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	380	380	366	403	396	0	0	380	888	0
N.S.	1	1.00	0.96	1.06	1.04	0.00	0.00	1.00	2.34	0.00
time (sec)	N/A	0.402	0.459	0.065	3.125	0.000	0.000	0.185	0.482	0.001

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	5	7	6	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75	0.00
time (sec)	N/A	0.010	0.001	0.037	1.302	0.407	0.085	0.170	0.023	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	9	8	8	7	9	6	0
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.90	0.60	0.00
time (sec)	N/A	0.015	0.001	0.042	1.334	0.395	0.080	0.192	0.056	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	9	8	8	8	9	6	0
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.90	0.60	0.00
time (sec)	N/A	0.013	0.001	0.041	1.392	0.381	0.093	0.182	4.992	0.001

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	21	18	17	17	15	15	6	0
N.S.	1	1.00	2.10	1.80	1.70	1.70	1.50	1.50	0.60	0.00
time (sec)	N/A	0.010	0.003	0.047	1.329	0.397	0.110	0.196	0.097	0.001

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	17	16	16	24	16	16	0
N.S.	1	1.00	1.00	0.71	0.67	0.67	1.00	0.67	0.67	0.00
time (sec)	N/A	0.024	0.008	0.045	2.928	0.379	0.147	0.188	0.030	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	39	38	38	46	39	49	0
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.92	0.78	0.98	0.00
time (sec)	N/A	0.049	0.020	0.051	2.887	0.412	0.212	0.202	0.131	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	46	39	38	38	46	39	48	0
N.S.	1	1.00	0.92	0.78	0.76	0.76	0.92	0.78	0.96	0.00
time (sec)	N/A	0.048	0.016	0.049	2.954	0.408	0.242	0.183	4.987	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	56	47	46	46	56	48	52	0
N.S.	1	1.00	0.93	0.78	0.77	0.77	0.93	0.80	0.87	0.00
time (sec)	N/A	0.051	0.014	0.048	2.944	0.418	0.226	0.188	5.010	0.001

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	52	47	46	46	56	48	52	0
N.S.	1	1.00	0.87	0.78	0.77	0.77	0.93	0.80	0.87	0.00
time (sec)	N/A	0.049	0.013	0.050	3.076	0.408	0.232	0.181	4.975	0.001

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	39	38	38	48	35	46	0
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.96	0.70	0.92	0.00
time (sec)	N/A	0.026	0.007	0.049	2.926	0.406	0.156	0.173	0.086	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	39	38	38	48	39	46	0
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.96	0.78	0.92	0.00
time (sec)	N/A	0.043	0.012	0.052	2.948	0.407	0.198	0.179	0.100	0.001

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	100	85	84	115	105	86	100	0
N.S.	1	1.00	0.91	0.77	0.76	1.05	0.95	0.78	0.91	0.00
time (sec)	N/A	0.118	0.108	0.056	2.984	0.408	0.432	0.192	5.098	0.001

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	97	85	84	115	105	86	100	0
N.S.	1	1.00	0.88	0.77	0.76	1.05	0.95	0.78	0.91	0.00
time (sec)	N/A	0.117	0.093	0.049	2.943	0.405	0.468	0.208	0.189	0.001

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	81	81	122	68	61	91	70	63	52	0
N.S.	1	1.00	1.51	0.84	0.75	1.12	0.86	0.78	0.64	0.00
time (sec)	N/A	0.069	0.569	0.056	3.063	0.398	0.229	0.170	4.923	0.001

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	84	73	74	126	82	76	77	0
N.S.	1	1.00	0.91	0.79	0.80	1.37	0.89	0.83	0.84	0.00
time (sec)	N/A	0.116	0.036	0.058	2.966	0.403	0.422	0.188	0.123	0.001

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	119	115	105	256	124	111	120	0
N.S.	1	1.00	0.80	0.78	0.71	1.73	0.84	0.75	0.81	0.00
time (sec)	N/A	0.172	0.085	0.067	2.908	0.416	0.651	0.203	0.190	0.001

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	121	115	105	257	124	111	121	0
N.S.	1	1.00	0.83	0.79	0.72	1.76	0.85	0.76	0.83	0.00
time (sec)	N/A	0.173	0.090	0.066	2.896	0.413	0.647	0.177	5.089	0.001

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	111	111	95	187	116	106	110	0
N.S.	1	1.00	0.78	0.78	0.67	1.32	0.82	0.75	0.77	0.00
time (sec)	N/A	0.148	0.072	0.060	2.983	0.407	0.639	0.180	5.080	0.001

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	111	111	95	187	116	106	111	0
N.S.	1	1.00	0.78	0.78	0.67	1.32	0.82	0.75	0.78	0.00
time (sec)	N/A	0.145	0.074	0.061	2.884	0.420	0.572	0.243	0.189	0.001

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	103	102	87	131	110	89	102	0
N.S.	1	1.00	0.91	0.90	0.77	1.16	0.97	0.79	0.90	0.00
time (sec)	N/A	0.082	0.068	0.062	2.936	0.414	0.529	0.174	0.172	0.001

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	111	102	95	187	119	99	111	0
N.S.	1	1.00	0.85	0.78	0.73	1.43	0.91	0.76	0.85	0.00
time (sec)	N/A	0.148	0.080	0.060	2.986	0.413	0.695	0.177	0.190	0.001

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	91	76	75	75	102	69	91	0
N.S.	1	1.00	0.92	0.77	0.76	0.76	1.03	0.70	0.92	0.00
time (sec)	N/A	0.063	0.037	0.050	2.988	0.414	0.395	0.192	5.096	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	108	130	118	305	1251	392	115	0
N.S.	1	1.00	1.29	1.55	1.40	3.63	14.89	4.67	1.37	0.00
time (sec)	N/A	0.056	0.228	0.063	1.356	0.440	8.171	0.262	5.135	0.141

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	120	87	78	160	552	196	76	0
N.S.	1	1.00	1.97	1.43	1.28	2.62	9.05	3.21	1.25	0.00
time (sec)	N/A	0.040	0.152	0.063	1.397	0.443	4.269	0.219	5.058	0.105

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	42	45	39	56	163	65	38	0
N.S.	1	1.00	1.02	1.10	0.95	1.37	3.98	1.59	0.93	0.00
time (sec)	N/A	0.023	0.133	0.056	1.293	0.438	2.027	0.222	5.056	0.073

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	12	17	15	12	12	0
N.S.	1	1.00	1.00	1.08	1.00	1.42	1.25	1.00	1.00	0.00
time (sec)	N/A	0.003	0.002	0.043	1.319	0.432	0.067	0.167	5.011	0.021

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	0	0	66	0	0	-1	45
N.S.	1	1.00	1.00	0.00	0.00	1.47	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.395	0.305	0.691	0.000	0.443	0.000	0.000	0.000	44.616

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	21	20	20	0	0	20	0
N.S.	1	1.00	1.00	0.88	0.83	0.83	0.00	0.00	0.83	0.00
time (sec)	N/A	0.053	0.176	0.052	2.126	0.430	0.000	0.000	5.587	7.840

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-1)	F	F(-2)	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	103	103	93	0	0	0	0	0	-1	103
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.103	0.203	180.000	0.000	0.000	0.000	0.000	0.000	166.726

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	0	0	61	0	228	95	0
N.S.	1	1.00	1.00	0.00	0.00	2.18	0.00	8.14	3.39	0.00
time (sec)	N/A	0.101	0.346	1.082	0.000	0.458	0.000	0.422	5.197	0.163

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	46	138	77	119	0	237	124	0
N.S.	1	1.00	1.02	3.07	1.71	2.64	0.00	5.27	2.76	0.00
time (sec)	N/A	0.157	0.410	0.575	3.039	0.452	0.000	0.426	5.367	0.415

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	52	59	54	0	115	76	0
N.S.	1	1.00	1.00	1.68	1.90	1.74	0.00	3.71	2.45	0.00
time (sec)	N/A	0.205	0.601	0.171	2.672	0.463	0.000	0.556	5.298	0.432

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	41	136	92	88	0	155	106	0
N.S.	1	1.00	0.91	3.02	2.04	1.96	0.00	3.44	2.36	0.00
time (sec)	N/A	0.553	0.888	0.502	3.035	0.465	0.000	0.809	5.640	1.160

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [77] had the largest ratio of [.6923]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	20	0.050
2	A	2	1	1.00	22	0.045
3	A	2	1	1.00	22	0.045
4	A	2	1	1.00	25	0.040
5	A	2	1	1.00	27	0.037
6	A	2	1	1.00	27	0.037
7	A	6	6	1.00	15	0.400
8	A	7	7	1.00	15	0.467
9	A	8	7	1.00	15	0.467
10	A	9	7	1.00	15	0.467
11	A	6	6	1.00	15	0.400
12	A	6	6	1.00	16	0.375
13	A	3	3	1.00	11	0.273
14	A	3	3	1.00	15	0.200
15	A	3	3	1.00	13	0.231
16	A	3	3	1.00	13	0.231
17	A	6	6	1.00	15	0.400
18	A	3	3	1.00	19	0.158
19	A	3	3	1.00	21	0.143
20	A	3	3	1.00	31	0.097
21	A	3	3	1.00	36	0.083
22	A	12	10	1.00	35	0.286
23	A	11	9	1.00	33	0.273
24	A	10	8	1.00	36	0.222
25	A	10	10	1.00	19	0.526
26	A	9	9	1.00	18	0.500
27	A	4	4	1.00	27	0.148
28	A	4	4	1.00	28	0.143
29	A	4	4	1.00	24	0.167
30	A	4	4	1.00	24	0.167
31	A	4	4	1.00	26	0.154
32	A	4	4	1.00	26	0.154
33	A	4	4	1.00	28	0.143
34	A	4	4	1.00	30	0.133
35	A	4	4	1.00	29	0.138
36	A	4	4	1.00	29	0.138
37	A	4	4	1.00	29	0.138
38	A	4	4	1.00	32	0.125
39	A	6	6	1.00	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
40	A	4	4	1.00	49	0.082
41	A	4	4	1.00	57	0.070
42	A	2	2	1.00	31	0.065
43	A	2	2	1.00	42	0.048
44	A	4	4	1.00	42	0.095
45	A	4	4	1.00	45	0.089
46	A	4	4	1.00	45	0.089
47	A	4	4	1.00	44	0.091
48	A	3	3	1.00	20	0.150
49	A	6	6	1.00	20	0.300
50	A	2	2	1.00	16	0.125
51	A	5	5	1.00	20	0.250
52	A	3	3	1.00	18	0.167
53	A	2	1	1.00	30	0.033
54	A	2	1	1.00	30	0.033
55	A	2	1	1.00	28	0.036
56	A	2	1	1.00	30	0.033
57	A	7	7	1.00	30	0.233
58	A	8	8	1.00	30	0.267
59	A	8	8	1.00	17	0.471
60	A	10	9	1.00	17	0.529
61	A	10	9	0.99	17	0.529
62	A	10	9	0.99	22	0.409
63	A	10	9	1.00	22	0.409
64	A	10	9	1.00	22	0.409
65	A	9	8	1.00	17	0.471
66	A	9	8	1.00	19	0.421
67	A	8	7	1.00	18	0.389
68	A	7	5	1.00	16	0.312
69	A	13	9	1.00	15	0.600
70	A	8	6	1.00	16	0.375
71	A	14	10	1.00	15	0.667
72	A	9	6	1.00	16	0.375
73	A	15	10	1.00	15	0.667
74	A	10	6	1.00	16	0.375
75	A	16	10	1.00	15	0.667
76	A	7	5	1.00	15	0.333
77	A	13	9	1.00	13	0.692
78	A	7	5	1.00	21	0.238
79	A	13	9	1.00	20	0.450
80	A	8	6	1.00	21	0.286
81	A	14	10	1.00	20	0.500
82	A	9	6	1.00	21	0.286
83	A	15	10	1.00	20	0.500
84	A	10	6	1.00	21	0.286
85	A	16	10	1.00	20	0.500
86	A	3	2	1.00	11	0.182
87	A	3	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	2	1	1.00	15	0.067
89	A	3	2	1.00	14	0.143
90	A	2	1	1.00	17	0.059
91	A	3	2	1.00	19	0.105
92	A	2	1	1.00	20	0.050
93	A	2	2	1.00	14	0.143
94	A	4	3	1.00	17	0.176
95	A	4	3	1.00	19	0.158
96	A	3	2	1.00	20	0.100
97	A	4	3	1.00	21	0.143
98	A	3	2	1.00	22	0.091
99	A	4	3	1.00	24	0.125
100	A	3	2	1.00	25	0.080
101	A	3	2	1.00	25	0.080
102	A	8	6	1.00	26	0.231
103	A	9	7	1.00	26	0.269
104	A	10	7	1.00	26	0.269
105	A	10	7	1.00	11	0.636
106	A	3	3	1.00	12	0.250
107	A	13	9	1.00	15	0.600
108	A	10	7	1.00	14	0.500
109	A	9	6	1.00	17	0.353
110	A	14	10	1.00	19	0.526
111	A	13	9	1.00	20	0.450
112	A	2	2	1.00	14	0.143
113	A	12	8	1.00	17	0.471
114	A	5	5	1.00	19	0.263
115	A	15	11	1.00	20	0.550
116	A	13	9	1.00	21	0.429
117	A	12	8	1.00	22	0.364
118	A	16	12	1.00	24	0.500
119	A	15	11	1.00	25	0.440
120	A	2	2	1.00	19	0.105
121	A	11	8	1.00	17	0.471
122	A	9	7	1.00	20	0.350
123	A	15	11	1.00	19	0.579
124	A	11	8	1.00	31	0.258
125	A	8	6	1.00	31	0.194
126	A	9	7	1.00	31	0.226
127	A	10	8	1.00	31	0.258
128	A	17	12	1.00	30	0.400
129	A	14	10	1.00	30	0.333
130	A	15	11	1.00	30	0.367
131	A	16	12	1.00	30	0.400
132	A	2	2	1.00	21	0.095
133	A	2	2	1.00	21	0.095
134	A	2	1	1.00	19	0.053
135	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
136	A	2	2	1.00	21	0.095
137	A	2	2	1.00	21	0.095
138	A	2	2	1.00	21	0.095
139	A	13	9	1.00	36	0.250
140	A	13	9	1.00	41	0.220
141	A	13	9	1.00	46	0.196
142	A	19	13	1.00	35	0.371
143	A	19	13	1.00	40	0.325
144	A	19	13	1.00	45	0.289
145	A	8	6	1.00	36	0.167
146	A	8	6	1.00	41	0.146
147	A	10	8	1.00	46	0.174
148	A	14	10	1.00	35	0.286
149	A	14	10	1.00	40	0.250
150	A	16	12	1.00	45	0.267
151	A	9	7	1.00	36	0.194
152	A	9	7	1.00	41	0.171
153	A	9	7	1.00	46	0.152
154	A	15	11	1.00	35	0.314
155	A	15	11	1.00	40	0.275
156	A	15	11	1.00	45	0.244
157	A	10	8	1.00	36	0.222
158	A	10	8	1.00	41	0.195
159	A	10	8	1.00	46	0.174
160	A	16	12	1.00	35	0.343
161	A	16	12	1.00	40	0.300
162	A	16	12	1.00	45	0.267
163	A	1	1	1.00	23	0.043
164	A	1	1	1.00	26	0.038
165	A	1	1	1.00	28	0.036
166	A	1	1	1.00	31	0.032
167	A	1	1	1.00	15	0.067
168	A	3	2	1.00	11	0.182
169	A	3	2	1.00	15	0.133
170	A	3	2	1.00	30	0.067
171	A	3	2	1.00	30	0.067
172	A	3	2	1.00	30	0.067
173	A	3	2	1.00	30	0.067
174	A	3	2	1.00	30	0.067
175	A	3	2	1.00	30	0.067
176	A	3	2	1.00	30	0.067
177	A	3	2	1.00	30	0.067
178	A	3	2	1.00	30	0.067
179	A	3	2	1.00	30	0.067
180	A	9	8	1.00	30	0.267
181	A	9	8	1.00	30	0.267
182	A	9	8	1.00	30	0.267
183	A	9	8	1.00	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	9	8	1.00	30	0.267
185	A	9	8	1.00	28	0.286
186	A	8	7	1.00	27	0.259
187	A	8	7	1.00	30	0.233
188	A	8	7	1.00	30	0.233
189	A	8	7	1.00	30	0.233
190	A	8	7	1.00	30	0.233
191	A	8	7	1.00	30	0.233
192	A	8	7	1.00	30	0.233
193	A	8	7	1.00	30	0.233
194	A	8	7	1.00	30	0.233
195	A	8	7	1.00	30	0.233
196	A	8	7	1.00	30	0.233
197	A	8	7	1.00	30	0.233
198	A	3	2	1.00	30	0.067
199	A	3	2	1.00	30	0.067
200	A	3	2	1.00	30	0.067
201	A	3	2	1.00	30	0.067
202	A	3	2	1.00	30	0.067
203	A	3	2	1.00	30	0.067
204	A	3	2	1.00	30	0.067
205	A	3	2	1.00	30	0.067
206	A	3	2	1.00	30	0.067
207	A	9	8	1.00	30	0.267
208	A	12	10	1.00	30	0.333
209	A	9	8	1.00	30	0.267
210	A	11	10	1.00	30	0.333
211	A	9	8	1.00	30	0.267
212	A	10	9	1.00	28	0.321
213	A	9	9	1.00	27	0.333
214	A	9	8	1.00	30	0.267
215	A	9	8	1.00	30	0.267
216	A	9	8	1.00	30	0.267
217	A	9	8	1.00	30	0.267
218	A	9	8	1.00	30	0.267
219	A	9	8	1.00	30	0.267
220	A	9	8	1.00	30	0.267
221	A	9	8	1.00	30	0.267
222	A	9	8	1.00	30	0.267
223	A	3	2	1.00	30	0.067
224	A	3	2	1.00	30	0.067
225	A	3	2	1.00	30	0.067
226	A	3	2	1.00	30	0.067
227	A	3	2	1.00	30	0.067
228	A	3	2	1.00	30	0.067
229	A	3	2	1.00	30	0.067
230	A	3	2	1.00	30	0.067
231	A	3	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
232	A	3	2	1.00	30	0.067
233	A	10	9	1.00	30	0.300
234	A	14	10	1.00	30	0.333
235	A	10	9	1.00	30	0.300
236	A	13	10	1.00	30	0.333
237	A	10	9	1.00	30	0.300
238	A	12	10	1.00	30	0.333
239	A	10	10	1.00	30	0.333
240	A	10	10	1.00	28	0.357
241	A	9	9	1.00	27	0.333
242	A	9	9	1.00	30	0.300
243	A	9	9	1.00	30	0.300
244	A	10	9	1.00	30	0.300
245	A	10	9	1.00	30	0.300
246	A	10	8	1.00	30	0.267
247	A	10	8	1.00	30	0.267
248	A	10	8	1.00	30	0.267
249	A	10	8	1.00	30	0.267
250	A	10	8	1.00	30	0.267
251	A	8	7	1.00	16	0.438
252	A	5	4	1.00	16	0.250
253	A	8	7	1.00	16	0.438
254	A	6	6	1.00	14	0.429
255	A	6	5	1.00	16	0.312
256	A	6	5	1.00	16	0.312
257	A	3	2	1.00	16	0.125
258	A	6	6	1.00	14	0.429
259	A	6	6	1.00	16	0.375
260	A	2	1	1.00	21	0.048
261	A	2	1	1.00	19	0.053
262	A	2	1	1.00	18	0.056
263	A	2	1	1.00	21	0.048
264	A	2	1	1.00	21	0.048
265	A	2	1	1.00	21	0.048
266	A	3	2	1.00	23	0.087
267	A	3	2	1.00	21	0.095
268	A	3	2	1.00	20	0.100
269	A	2	1	1.00	23	0.043
270	A	2	1	1.00	23	0.043
271	A	2	1	1.00	23	0.043
272	A	3	2	1.00	23	0.087
273	A	3	2	1.00	21	0.095
274	A	3	2	1.00	20	0.100
275	A	2	1	1.00	23	0.043
276	A	2	1	1.00	23	0.043
277	A	2	1	1.00	23	0.043
278	A	3	2	1.00	23	0.087
279	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
280	A	3	2	1.00	20	0.100
281	A	2	1	1.00	23	0.043
282	A	2	1	1.00	23	0.043
283	A	2	1	1.00	23	0.043
284	A	10	9	1.00	23	0.391
285	A	10	9	1.00	23	0.391
286	A	10	9	1.00	21	0.429
287	A	8	8	1.00	20	0.400
288	A	10	9	1.00	23	0.391
289	A	10	9	1.00	23	0.391
290	A	10	9	1.00	23	0.391
291	A	7	7	1.00	23	0.304
292	A	7	7	1.00	21	0.333
293	A	7	7	1.00	20	0.350
294	A	11	10	1.00	23	0.435
295	A	11	10	1.00	23	0.435
296	A	11	10	1.00	23	0.435
297	A	11	10	1.00	23	0.435
298	A	8	8	1.00	23	0.348
299	A	8	8	1.00	21	0.381
300	A	8	8	1.00	20	0.400
301	A	12	10	1.00	23	0.435
302	A	12	10	1.00	23	0.435
303	A	12	10	1.00	23	0.435
304	A	12	10	1.00	23	0.435
305	A	9	8	1.00	23	0.348
306	A	9	9	1.00	21	0.429
307	A	9	8	1.00	20	0.400
308	A	13	10	1.00	23	0.435
309	A	13	10	1.00	23	0.435
310	A	13	10	1.00	23	0.435
311	A	13	10	1.00	23	0.435
312	A	5	5	1.00	20	0.250
313	A	4	4	1.00	18	0.222
314	A	5	5	1.00	20	0.250
315	A	4	4	1.00	18	0.222
316	A	4	4	1.00	27	0.148
317	A	4	4	1.00	29	0.138
318	A	4	4	1.00	28	0.143
319	A	4	4	1.00	28	0.143
320	A	2	1	1.00	36	0.028
321	A	2	1	1.00	36	0.028
322	A	2	1	1.00	36	0.028
323	A	2	1	1.00	34	0.029
324	A	2	1	1.00	33	0.030
325	A	2	1	1.00	36	0.028
326	A	2	1	1.00	36	0.028
327	A	2	1	1.00	36	0.028

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
328	A	2	1	1.00	36	0.028
329	A	2	1	1.00	36	0.028
330	A	2	1	1.00	38	0.026
331	A	2	1	1.00	38	0.026
332	A	3	2	1.00	38	0.053
333	A	3	2	1.00	36	0.056
334	A	3	2	1.00	35	0.057
335	A	3	2	1.00	38	0.053
336	A	3	2	1.00	38	0.053
337	A	3	2	1.00	38	0.053
338	A	2	1	1.00	38	0.026
339	A	2	1	1.00	38	0.026
340	A	2	1	1.00	38	0.026
341	A	2	1	1.00	38	0.026
342	A	3	2	1.00	38	0.053
343	A	3	2	1.00	36	0.056
344	A	3	2	1.00	35	0.057
345	A	3	2	1.00	38	0.053
346	A	3	2	1.00	38	0.053
347	A	3	2	1.00	38	0.053
348	A	2	1	1.00	38	0.026
349	A	2	1	1.00	38	0.026
350	A	13	10	1.00	38	0.263
351	A	13	10	1.00	38	0.263
352	A	13	10	1.00	38	0.263
353	A	13	10	1.00	36	0.278
354	A	10	9	0.99	35	0.257
355	A	10	9	0.99	38	0.237
356	A	10	9	1.00	38	0.237
357	A	10	9	0.99	38	0.237
358	A	10	9	0.99	38	0.237
359	A	11	10	1.00	38	0.263
360	A	11	10	1.00	38	0.263
361	A	11	10	0.99	38	0.263
362	A	11	10	1.00	36	0.278
363	A	9	9	1.00	35	0.257
364	A	11	10	0.99	38	0.263
365	A	11	10	1.00	38	0.263
366	A	11	10	0.99	38	0.263
367	A	11	10	0.99	38	0.263
368	A	12	11	1.00	38	0.290
369	A	10	10	1.00	38	0.263
370	A	8	8	1.00	38	0.210
371	A	8	8	1.00	36	0.222
372	A	8	8	1.00	35	0.229
373	A	12	10	0.99	38	0.263
374	A	12	10	1.00	38	0.263
375	A	12	10	0.99	38	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
376	A	12	10	0.99	38	0.263
377	A	2	1	1.00	23	0.043
378	A	2	1	1.00	26	0.038
379	A	3	2	1.00	25	0.080
380	A	3	2	1.00	28	0.071
381	A	3	2	1.00	25	0.080
382	A	3	2	1.00	28	0.071
383	A	3	2	1.00	25	0.080
384	A	3	2	1.00	28	0.071
385	A	9	7	1.00	26	0.269
386	A	12	9	1.00	29	0.310
387	A	15	11	1.00	25	0.440
388	A	18	13	1.00	28	0.464
389	A	14	10	1.00	25	0.400
390	A	14	10	1.00	28	0.357
391	A	15	11	1.00	25	0.440
392	A	15	11	1.00	28	0.393
393	A	16	11	1.00	25	0.440
394	A	16	11	1.00	28	0.393
395	A	2	2	1.00	22	0.091
396	A	2	2	1.00	35	0.057
397	A	2	2	1.00	35	0.057
398	A	2	2	1.00	22	0.091
399	A	3	3	1.00	25	0.120
400	A	6	5	1.00	15	0.333
401	A	6	5	1.00	15	0.333
402	A	7	6	1.00	20	0.300
403	A	7	6	1.00	20	0.300
404	A	7	7	1.00	17	0.412
405	A	7	6	1.00	25	0.240
406	A	11	6	1.00	35	0.171
407	A	11	6	1.00	35	0.171
408	A	8	5	1.00	22	0.227
409	A	11	7	1.00	25	0.280
410	A	17	7	1.00	15	0.467
411	A	17	7	1.00	15	0.467
412	A	14	7	1.00	20	0.350
413	A	14	7	1.00	20	0.350
414	A	15	9	1.00	17	0.529
415	A	14	7	1.00	25	0.280
416	A	13	7	1.00	18	0.389
417	A	4	3	1.00	19	0.158
418	A	4	3	1.00	19	0.158
419	A	4	2	1.00	17	0.118
420	A	1	0	1.00	9	0.000
421	A	2	2	1.00	58	0.034
422	A	1	1	1.00	46	0.022
423	A	10	8	1.00	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
424	A	1	1	1.00	48	0.021
425	A	1	1	1.00	45	0.022
426	A	1	1	1.00	69	0.014
427	A	1	1	1.00	86	0.012

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)}{b^3}+\frac{2(a+bx)^{3/2}(bd-2ae)}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {698}

$$\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)}{b^3}+\frac{2(a+bx)^{3/2}(bd-2ae)}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)*Sqrt[a + b*x])/b^3 + (2*(b*d - 2*a*e)*(a + b*x)^
(3/2))/(3*b^3) + (2*e*(a + b*x)^(5/2))/(5*b^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx &= \int \left(\frac{b^2c-abd+a^2e}{b^2\sqrt{a+bx}}+\frac{(bd-2ae)\sqrt{a+bx}}{b^2}+\frac{e(a+bx)^{3/2}}{b^2}\right)dx\\ &= \frac{2\left(b^2c-abd+a^2e\right)\sqrt{a+bx}}{b^3}+\frac{2(bd-2ae)(a+bx)^{3/2}}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.16, size = 53, normalized size = 0.74

$$\frac{2\sqrt{a+bx}\left(8a^2e-2ab(5d+2ex)+b^2(15c+x(5d+3ex))\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2*e - 2*a*b*(5*d + 2*e*x) + b^2*(15*c + x*(5*d + 3*e*
x))))/(15*b^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 62, normalized size = 0.86

$$\frac{2\sqrt{a+bx}\left(15a^2e+5bd(a+bx)-15abd-10ae(a+bx)+3e(a+bx)^2+15b^2c\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(15*b^2*c - 15*a*b*d + 15*a^2*e + 5*b*d*(a + b*x) - 10*a*e
*(a + b*x) + 3*e*(a + b*x)^2))/(15*b^3)
```

fricas [A] time = 0.41, size = 53, normalized size = 0.74

$$\frac{2\left(3\,b^2ex^2+15\,b^2c-10\,abd+8\,a^2e+\left(5\,b^2d-4\,abe\right)x\right)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*e*x^2 + 15*b^2*c - 10*a*b*d + 8*a^2*e + (5*b^2*d - 4*a*b*e)*x)*
sqrt(b*x + a)/b^3
```

giac [A] time = 0.16, size = 78, normalized size = 1.08

$$\frac{2\left(15\,\sqrt{bx+a}\,c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)d}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)e}{b^2}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

maple [A] time = 0.06, size = 53, normalized size = 0.74

$$\frac{2\sqrt{bx+a}\,\left(3e\,x^2b^2-4abex+5b^2dx+8a^2e-10abd+15b^2c\right)}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x+a)^(1/2),x)
```

```
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*e*x^2-4*a*b*e*x+5*b^2*d*x+8*a^2*e-10*a*b*d+15*b^2
*c)/b^3
```

maxima [A] time = 0.92, size = 77, normalized size = 1.07

$$\frac{2\left(15\,\sqrt{bx+a}\,c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)d}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)e}{b^2}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

mupad [B] time = 4.72, size = 58, normalized size = 0.81

$$\frac{2\,\sqrt{a+b\,x}\,\left(3\,e\,{\left(a+b\,x\right)}^2+15\,b^2\,c+15\,a^2\,e-10\,a\,e\,\left(a+b\,x\right)+5\,b\,d\,\left(a+b\,x\right)-15\,a\,b\,d\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2)*(3*e*(a + b*x)^2 + 15*b^2*c + 15*a^2*e - 10*a*e*(a + b*x
) + 5*b*d*(a + b*x) - 15*a*b*d))/(15*b^3)
```

sympy [A] time = 11.05, size = 223, normalized size = 3.10

$$\begin{cases} \frac{-\frac{2ac}{\sqrt{a+bx}} - \frac{2ad\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right)}{b} - \frac{2ae\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b^2} - 2c\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right) - \frac{2d\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b} - \frac{2e\left(-\frac{a^3}{\sqrt{a+bx}} - 3a^2\sqrt{a+bx} + a(a+bx)^{\frac{3}{2}} - \frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} & \text{for } b \neq 0 \\ \frac{cx + \frac{dx^2}{2} + \frac{ex^3}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*c/sqrt(a + b*x) - 2*a*d*(-a/sqrt(a + b*x) - sqrt(a + b*x))
/b - 2*a*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b*
*2 - 2*c*(-a/sqrt(a + b*x) - sqrt(a + b*x)) - 2*d*(a**2/sqrt(a + b*x) + 2*a
*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b - 2*e*(-a**3/sqrt(a + b*x) - 3*a**2*
sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2)/b, Ne(b, 0))
, ((c*x + d*x**2/2 + e*x**3/3)/sqrt(a), True))
```

3.2 $\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=161

$$-\frac{2(a+bx)^{5/2}\left(-6a^2e^2+6abde-\left(b^2\left(2ce+d^2\right)\right)\right)}{5b^5}+\frac{4(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)}{3b^5}+\frac{2\sqrt{a+bx}\left(a^2e-abc\right.}{b^5}$$

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {698}

$$-\frac{2(a+bx)^{5/2}\left(-6a^2e^2+6abde+b^2\left(-\left(2ce+d^2\right)\right)\right)}{5b^5}+\frac{4(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)}{3b^5}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^2}{b^5}+\frac{4e(a+bx)^{7/2}(bd-2ae)}{7b^5}+\frac{2e^2(a+bx)^{9/2}}{9b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^2*Sqrt[a + b*x])/b^5 + (4*(b*d - 2*a*e)*(b^2*c -
 a*b*d + a^2*e)*(a + b*x)^(3/2))/(3*b^5) - (2*(6*a*b*d*e - 6*a^2*e^2 - b^2*
(d^2 + 2*c*e))*(a + b*x)^(5/2))/(5*b^5) + (4*e*(b*d - 2*a*e)*(a + b*x)^(7/2
))/(7*b^5) + (2*e^2*(a + b*x)^(9/2))/(9*b^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2\right)^2}{\sqrt{a+bx}}\,dx=\int \left(\frac{\left(b^2c-abd+a^2e\right)^2}{b^4\sqrt{a+bx}}+\frac{2(bd-2ae)\left(b^2c-abd+a^2e\right)\sqrt{a+bx}}{b^4}+\frac{\left(-6abde+6a^2e^2+b^2\right.}{}\right.$$

$$=\frac{2\left(b^2c-abd+a^2e\right)^2\sqrt{a+bx}}{b^5}+\frac{4(bd-2ae)\left(b^2c-abd+a^2e\right)(a+bx)^{3/2}}{3b^5}-\frac{2\left(6abde-6\right.}{}$$

Mathematica [A] time = 0.29, size = 155, normalized size = 0.96

$$\frac{2\sqrt{a+bx}\left(128a^4e^2-32a^3be(9d+2ex)+24a^2b^2\left(2e\left(7c+ex^2\right)+7d^2+6dex\right)-4ab^3\left(21c(5d+2ex)+x\left(21d^2+27dex+10e^2x^2\right)\right)+b^4\left(315c^2+42cx(5d+3ex)+x^2\left(63d^2+90dex+35e^2x^2\right)\right)\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(128*a^4*e^2 - 32*a^3*b*e*(9*d + 2*e*x) + 24*a^2*b^2*(7*d^
2 + 6*d*e*x + 2*e*(7*c + e*x^2)) - 4*a*b^3*(21*c*(5*d + 2*e*x) + x*(21*d^2
+ 27*d*e*x + 10*e^2*x^2)) + b^4*(315*c^2 + 42*c*x*(5*d + 3*e*x) + x^2*(63*d
^2 + 90*d*e*x + 35*e^2*x^2))))/(315*b^5)
```

IntegrateAlgebraic [A] time = 0.08, size = 229, normalized size = 1.42

$$\frac{2\sqrt{a+bx}\left(315a^4e^2-630a^3bde-420a^3e^2(a+bx)+630a^2b^2ce+315a^2b^2d^2+630a^2bde(a+bx)+378a^2e^2(a+bx)^2+210b^3cd(a+bx)-630ab^3cd-420ab^2ce(a+bx)+126b^2ce(a+bx)^2-210ab^2d^2(a+bx)+63b^2d^2(a+bx)^2-378abde(a+bx)^2+90bde(a+bx)^3-180ae^2(a+bx)^3+35e^2(a+bx)^4+315b^4c^2\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(315*b^4*c^2 - 630*a*b^3*c*d + 315*a^2*b^2*d^2 + 630*a^2*b
^2*c*e - 630*a^3*b*d*e + 315*a^4*e^2 + 210*b^3*c*d*(a + b*x) - 210*a*b^2*d^
2*(a + b*x) - 420*a*b^2*c*e*(a + b*x) + 630*a^2*b*d*e*(a + b*x) - 420*a^3*e
^2*(a + b*x) + 63*b^2*d^2*(a + b*x)^2 + 126*b^2*c*e*(a + b*x)^2 - 378*a*b*d
*e*(a + b*x)^2 + 378*a^2*e^2*(a + b*x)^2 + 90*b*d*e*(a + b*x)^3 - 180*a*e^2
*(a + b*x)^3 + 35*e^2*(a + b*x)^4))/(315*b^5)
```

fricas [A]   time = 0.41, size = 192, normalized size = 1.19

$$\frac{2\left(35\,b^{4}e^{2}x^{4}+315\,b^{4}c^{2}-420\,ab^{3}cd+168\,a^{2}b^{2}d^{2}+128\,a^{4}e^{2}+10\left(9\,b^{4}de-4\,ab^{3}e^{2}\right)x^{3}+3\left(21\,b^{4}d^{2}+16\,a^{2}b^{2}e^{2}+6\left(7\,b^{4}c-6\,ab^{3}d\right)e\right)x^{2}+48\left(7\,a^{2}b^{2}c-6\,a^{3}bd\right)e+2\left(105\,b^{4}cd-42\,ab^{3}d^{2}-32\,a^{3}be^{2}-12\left(7\,ab^{3}c-6\,a^{2}b^{2}d\right)e\right)x\right)\sqrt{bx+a}}{315\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*e^2*x^4 + 315*b^4*c^2 - 420*a*b^3*c*d + 168*a^2*b^2*d^2 + 128
*a^4*e^2 + 10*(9*b^4*d*e - 4*a*b^3*e^2)*x^3 + 3*(21*b^4*d^2 + 16*a^2*b^2*e^
2 + 6*(7*b^4*c - 6*a*b^3*d)*e)*x^2 + 48*(7*a^2*b^2*c - 6*a^3*b*d)*e + 2*(10
5*b^4*c*d - 42*a*b^3*d^2 - 32*a^3*b*e^2 - 12*(7*a*b^3*c - 6*a^2*b^2*d)*e)*x
)*sqrt(b*x + a)/b^5
```

giac [A]   time = 0.17, size = 237, normalized size = 1.47

$$\frac{2\left(315\,\sqrt{bx+a}\,c^{2}+\frac{210\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)cd}{b}+\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)d^{2}}{b^{2}}+\frac{42\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)ce}{b^{2}}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)de}{b^{3}}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}\right)e^{2}}{b^{4}}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 210*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*c*
d/b + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
d^2/b^2 + 42*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a
^2)*c*e/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x +
 a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(
b*x + a)*a^4)*e^2/b^4)/b
```

maple [A]   time = 0.05, size = 194, normalized size = 1.20

$$\frac{2\sqrt{bx+a}\,\left(35e^{2}x^{4}b^{4}-40a\,b^{3}e^{2}x^{3}+90b^{4}de\,x^{3}+48a^{2}b^{2}e^{2}x^{2}-108a\,b^{3}de\,x^{2}+126b^{4}ce\,x^{2}+63b^{4}d^{2}x^{2}-64a^{3}b\,e^{2}x+144a^{2}b^{2}dex-168a\,b^{3}cex-84a\,b^{3}d^{2}x+210b^{4}cdx+128a^{4}e^{2}-288a^{3}bde+336a^{2}b^{2}ce+168a^{2}b^{2}d^{2}-420a\,b^{3}cd+315c^{2}b^{4}\right)}{315b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/315*(b*x+a)^(1/2)*(35*b^4*e^2*x^4-40*a*b^3*e^2*x^3+90*b^4*d*e*x^3+48*a^2*
b^2*e^2*x^2-108*a*b^3*d*e*x^2+126*b^4*c*e*x^2+63*b^4*d^2*x^2-64*a^3*b*e^2*x
+144*a^2*b^2*d*e*x-168*a*b^3*c*e*x-84*a*b^3*d^2*x+210*b^4*c*d*x+128*a^4*e^2
-288*a^3*b*d*e+336*a^2*b^2*c*e+168*a^2*b^2*d^2-420*a*b^3*c*d+315*b^4*c^2)/b
^5
```

maxima [A]   time = 0.90, size = 237, normalized size = 1.47

$$\frac{2\left(315\,\sqrt{bx+a}\,c^{2}+42\,c\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)d}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)e}{b^{2}}\right)+\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)d^{2}}{b^{2}}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)de}{b^{3}}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}\right)e^{2}}{b^{4}}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 42*c*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a
)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e
/b^2) + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2
)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a
)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*
x + a)*a^4)*e^2/b^4)/b
```

mupad [B] time = 4.76, size = 149, normalized size = 0.93

$$\frac{2\,e^2\,{\left(a+b\,x\right)}^{9/2}}{9\,b^5}+\frac{{\left(a+b\,x\right)}^{5/2}\,\left(12\,a^2\,e^2-12\,a\,b\,d\,e+2\,b^2\,d^2+4\,c\,b^2\,e\right)}{5\,b^5}+\frac{2\,\sqrt{a+b\,x}\,{\left(e\,a^2-d\,a\,b+c\,b^2\right)}^2}{b^5}-\frac{\left(8\,a\,e^2-4\,b\,d\,e\right)\,{\left(a+b\,x\right)}^{7/2}}{7\,b^5}-\frac{4\,\left(2\,a\,e-b\,d\right)\,{\left(a+b\,x\right)}^{3/2}\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)^2/(a + b*x)^(1/2),x)
```

```
[Out] (2*e^2*(a + b*x)^(9/2))/(9*b^5) + ((a + b*x)^(5/2)*(12*a^2*e^2 + 2*b^2*d^2
+ 4*b^2*c*e - 12*a*b*d*e))/(5*b^5) + (2*(a + b*x)^(1/2)*(b^2*c + a^2*e - a*
b*d)^2)/b^5 - ((8*a*e^2 - 4*b*d*e)*(a + b*x)^(7/2))/(7*b^5) - (4*(2*a*e - b
*d)*(a + b*x)^(3/2)*(b^2*c + a^2*e - a*b*d))/(3*b^5)
```

sympy [A] time = 85.15, size = 644, normalized size = 4.00

$$\begin{cases} \frac{- \frac{2 a c^{2}}{\sqrt{a + b x}} - \frac{4 a c d \left(- \frac{a}{\sqrt{a + b x}} - \sqrt{a + b x}\right)}{b} - \frac{4 a c e \left(\frac{a^{2}}{\sqrt{a + b x}} + 2 a \sqrt{a + b x} - \frac{\left(a + b x\right)^{\frac{3}{2}}}{3}\right)}{b^{2}} - \frac{2 a d^{2} \left(\frac{a^{2}}{\sqrt{a + b x}} + 2 a \sqrt{a + b x} - \frac{\left(a + b x\right)^{\frac{3}{2}}}{3}\right)}{b^{2}} - \frac{4 a d e \left(- \frac{a^{3}}{\sqrt{a + b x}} - 3 a^{2} \sqrt{a + b x} + a \left(a + b x\right)^{\frac{3}{2}} - \frac{\left(a + b x\right)^{\frac{5}{2}}}{5}\right)}{b^{3}} - \frac{2 a e^{2} \left(\frac{a^{4}}{\sqrt{a + b x}} + 4 a^{3} \sqrt{a + b x} - 2 a^{2} \left(a + b x\right)^{\frac{3}{2}} + \frac{4 a \left(a + b x\right)^{\frac{5}{2}}}{5} - \frac{\left(a + b x\right)^{\frac{7}{2}}}{7}\right)}{b^{4}} - 2 c^{2} \left(- \frac{a}{\sqrt{a + b x}} - \sqrt{a + b x}\right) - \frac{4 c d \left(\frac{a^{2}}{\sqrt{a + b x}} + 2 a \sqrt{a + b x} - \frac{\left(a + b x\right)^{\frac{3}{2}}}{3}\right)}{b} - \frac{4 c e \left(- \frac{a^{3}}{\sqrt{a + b x}} - 3 a^{2} \sqrt{a + b x} + a \left(a + b x\right)^{\frac{3}{2}} - \frac{\left(a + b x\right)^{\frac{5}{2}}}{5}\right)}{b^{2}} - \frac{2 d^{2} \left(- \frac{a^{3}}{\sqrt{a + b x}} - 3 a^{2} \sqrt{a + b x} + a \left(a + b x\right)^{\frac{3}{2}} - \frac{\left(a + b x\right)^{\frac{5}{2}}}{5}\right)}{b^{2}} - \frac{4 d e \left(\frac{a^{4}}{\sqrt{a + b x}} + 4 a^{3} \sqrt{a + b x} - 2 a^{2} \left(a + b x\right)^{\frac{3}{2}} + \frac{4 a \left(a + b x\right)^{\frac{5}{2}}}{5} - \frac{\left(a + b x\right)^{\frac{7}{2}}}{7}\right)}{b^{3}} - \frac{2 e^{2} \left(- \frac{a^{5}}{\sqrt{a + b x}} - 5 a^{4} \sqrt{a + b x} + \frac{10 a^{3} \left(a + b x\right)^{\frac{3}{2}}}{3} - 2 a^{2} \left(a + b x\right)^{\frac{5}{2}} + \frac{5 a \left(a + b x\right)^{\frac{7}{2}}}{7} - \frac{\left(a + b x\right)^{\frac{9}{2}}}{9}\right)}{b^{4}}}{b} & \text{for}\: b \neq 0 \\ \frac{c^{2} x + c d x^{2} + \frac{d e x^{4}}{2} + \frac{e^{2} x^{5}}{5} + \frac{x^{3} \left(2 c e + d^{2}\right)}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*c**2/sqrt(a + b*x) - 4*a*c*d*(-a/sqrt(a + b*x) - sqrt(a +
b*x))/b - 4*a*c*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2
)/3)/b**2 - 2*a*d**2*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(
3/2)/3)/b**2 - 4*a*d*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a +
 b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 - 2*a*e**2*(a**4/sqrt(a + b*x) + 4*
a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a
+ b*x)**(7/2)/7)/b**4 - 2*c**2*(-a/sqrt(a + b*x) - sqrt(a + b*x)) - 4*c*d*(
a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b - 4*c*e*(-a*
*3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(
5/2)/5)/b**2 - 2*d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a +
b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 - 4*d*e*(a**4/sqrt(a + b*x) + 4*a**3
*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*
x)**(7/2)/7)/b**3 - 2*e**2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10
*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7
 - (a + b*x)**(9/2)/9)/b**4)/b, Ne(b, 0)), ((c**2*x + c*d*x**2 + d*e*x**4/2
 + e**2*x**5/5 + x**3*(2*c*e + d**2)/3)/sqrt(a), True))
```

3.3 $\int \frac{\left(c+dx+ex^2\right)^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=274

$$-\frac{2e(a+bx)^{9/2}\left(-5a^2e^2+5abde-\left(b^2\left(ce+d^2\right)\right)\right)}{3b^7}-\frac{2(a+bx)^{7/2}(bd-2ae)\left(-10a^2e^2+10abde-\left(b^2\left(6ce+d^2\right)\right)\right)}{7b^7}$$

Rubi [A] time = 0.19, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {698}

$$-\frac{2e(a+bx)^{9/2}\left(-5a^2e^2+5abde+b^2\left(-\left(ce+d^2\right)\right)\right)}{3b^7}-\frac{2(a+bx)^{7/2}(bd-2ae)\left(-10a^2e^2+10abde+b^2\left(-\left(6ce+d^2\right)\right)\right)}{7b^7}-\frac{6(a+bx)^{5/2}\left(a^2e-abd+b^2c\right)\left(-5a^2e^2+5abde+b^2\left(-\left(ce+d^2\right)\right)\right)}{5b^7}+\frac{2(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)^2}{b^7}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^3}{b^7}+\frac{6e^2(a+bx)^{11/2}(bd-2ae)}{11b^7}+\frac{2e^3(a+bx)^{13/2}}{13b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^3*Sqrt[a + b*x])/b^7 + (2*(b*d - 2*a*e)*(b^2*c -
 a*b*d + a^2*e)^2*(a + b*x)^(3/2))/b^7 - (6*(b^2*c - a*b*d + a^2*e)*(5*a*b*
d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(5/2))/(5*b^7) - (2*(b*d - 2*a
*e)*(10*a*b*d*e - 10*a^2*e^2 - b^2*(d^2 + 6*c*e))*(a + b*x)^(7/2))/(7*b^7)
- (2*e*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(9/2))/(3*b^7) +
 (6*e^2*(b*d - 2*a*e)*(a + b*x)^(11/2))/(11*b^7) + (2*e^3*(a + b*x)^(13/2))
/(13*b^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2\right)^3}{\sqrt{a+bx}}\,dx = \int \left(\frac{\left(b^2c-abd+a^2e\right)^3}{b^6\sqrt{a+bx}}+\frac{3(bd-2ae)\left(b^2c-abd+a^2e\right)^2\sqrt{a+bx}}{b^6}+\frac{3\left(b^2c-abd+a^2\right.}{}\right.$$

$$=\frac{2\left(b^2c-abd+a^2e\right)^3\sqrt{a+bx}}{b^7}+\frac{2(bd-2ae)\left(b^2c-abd+a^2e\right)^2(a+bx)^{3/2}}{b^7}-\frac{6\left(b^2c-\right.}{}$$

Mathematica [A] time = 1.03, size = 294, normalized size = 1.07

$$\frac{2\sqrt{a+bx}(c+x(d+ex))^3}{b}-\frac{4(a+bx)^{3/2}\left(-2560a^5e^3+640a^4be^2(13d+6ex)-64a^3b^2e\left(e\left(143c+75ex^2\right)+143d^2+195dex\right)+8a^2b^3\left(78de\left(33c+25ex^2\right)+4e^2x\left(429c+175ex^2\right)+429d^3+1716d^2ex\right)-4ab^4\left(3003c^2e+429c\left(7d^2+18dex+10e^2x^2\right)+x\left(1287d^3+4290d^2ex+4550de^2x^2+1575e^3x^3\right)\right)+b^5\left(3003c^2(5d+6ex)+286cx\left(63d^2+135dex+70e^2x^2\right)+5x^2\left(1287d^3+4004d^2ex+4095de^2x^2+1386e^3x^3\right)\right)\right)}{15015b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + x*(d + e*x))^3)/b - (4*(a + b*x)^(3/2)*(-2560*a^5*e^3
 + 640*a^4*b*e^2*(13*d + 6*e*x) - 64*a^3*b^2*e*(143*d^2 + 195*d*e*x + e*(14
3*c + 75*e*x^2)) + 8*a^2*b^3*(429*d^3 + 1716*d^2*e*x + 78*d*e*(33*c + 25*e*
x^2) + 4*e^2*x*(429*c + 175*e*x^2)) + b^5*(3003*c^2*(5*d + 6*e*x) + 286*c*x
*(63*d^2 + 135*d*e*x + 70*e^2*x^2) + 5*x^2*(1287*d^3 + 4004*d^2*e*x + 4095*
d*e^2*x^2 + 1386*e^3*x^3)) - 4*a*b^4*(3003*c^2*e + 429*c*(7*d^2 + 18*d*e*x
```

```
+ 10*e^2*x^2) + x*(1287*d^3 + 4290*d^2*e*x + 4550*d*e^2*x^2 + 1575*e^3*x^3)
)))/(15015*b^7)
```

IntegrateAlgebraic [B] time = 0.17, size = 592, normalized size = 2.16

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(15015*b^6*c^3 - 45045*a*b^5*c^2*d + 45045*a^2*b^4*c*d^2 -
 15015*a^3*b^3*d^3 + 45045*a^2*b^4*c^2*e - 90090*a^3*b^3*c*d*e + 45045*a^4*
b^2*d^2*e + 45045*a^4*b^2*c*e^2 - 45045*a^5*b*d*e^2 + 15015*a^6*e^3 + 15015
*b^5*c^2*d*(a + b*x) - 30030*a*b^4*c*d^2*(a + b*x) + 15015*a^2*b^3*d^3*(a +
 b*x) - 30030*a*b^4*c^2*e*(a + b*x) + 90090*a^2*b^3*c*d*e*(a + b*x) - 60060
*a^3*b^2*d^2*e*(a + b*x) - 60060*a^3*b^2*c*e^2*(a + b*x) + 75075*a^4*b*d*e^
2*(a + b*x) - 30030*a^5*e^3*(a + b*x) + 9009*b^4*c*d^2*(a + b*x)^2 - 9009*a
*b^3*d^3*(a + b*x)^2 + 9009*b^4*c^2*e*(a + b*x)^2 - 54054*a*b^3*c*d*e*(a +
b*x)^2 + 54054*a^2*b^2*d^2*e*(a + b*x)^2 + 54054*a^2*b^2*c*e^2*(a + b*x)^2
- 90090*a^3*b*d*e^2*(a + b*x)^2 + 45045*a^4*e^3*(a + b*x)^2 + 2145*b^3*d^3*
(a + b*x)^3 + 12870*b^3*c*d*e*(a + b*x)^3 - 25740*a*b^2*d^2*e*(a + b*x)^3 -
 25740*a*b^2*c*e^2*(a + b*x)^3 + 64350*a^2*b*d*e^2*(a + b*x)^3 - 42900*a^3*
e^3*(a + b*x)^3 + 5005*b^2*d^2*e*(a + b*x)^4 + 5005*b^2*c*e^2*(a + b*x)^4 -
 25025*a*b*d*e^2*(a + b*x)^4 + 25025*a^2*e^3*(a + b*x)^4 + 4095*b*d*e^2*(a
+ b*x)^5 - 8190*a*e^3*(a + b*x)^5 + 1155*e^3*(a + b*x)^6))/(15015*b^7)
```

fricas [A] time = 0.41, size = 457, normalized size = 1.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*e^3*x^6 + 15015*b^6*c^3 - 30030*a*b^5*c^2*d + 24024*a^2*b
^4*c*d^2 - 6864*a^3*b^3*d^3 + 5120*a^6*e^3 + 315*(13*b^6*d*e^2 - 4*a*b^5*e^
3)*x^5 + 35*(143*b^6*d^2*e + 40*a^2*b^4*e^3 + 13*(11*b^6*c - 10*a*b^5*d)*e^
2)*x^4 + 5*(429*b^6*d^3 - 320*a^3*b^3*e^3 - 104*(11*a*b^5*c - 10*a^2*b^4*d)
*e^2 + 286*(9*b^6*c*d - 4*a*b^5*d^2)*e)*x^3 + 1664*(11*a^4*b^2*c - 10*a^5*b
*d)*e^2 + 3*(3003*b^6*c*d^2 - 858*a*b^5*d^3 + 640*a^4*b^2*e^3 + 208*(11*a^2
*b^4*c - 10*a^3*b^3*d)*e^2 + 143*(21*b^6*c^2 - 36*a*b^5*c*d + 16*a^2*b^4*d^
2)*e)*x^2 + 1144*(21*a^2*b^4*c^2 - 36*a^3*b^3*c*d + 16*a^4*b^2*d^2)*e + (15
015*b^6*c^2*d - 12012*a*b^5*c*d^2 + 3432*a^2*b^4*d^3 - 2560*a^5*b*e^3 - 832
*(11*a^3*b^3*c - 10*a^4*b^2*d)*e^2 - 572*(21*a*b^5*c^2 - 36*a^2*b^4*c*d + 1
6*a^3*b^3*d^2)*e)*x)*sqrt(b*x + a)/b^7
```

giac [B] time = 0.23, size = 526, normalized size = 1.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 15015*((b*x + a)^(3/2) - 3*sqrt(b*x + a)
*a)*c^2*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
+ a)*a^2)*c*d^2/b^2 + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*c^2*e/b^2 + 429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
 + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 2574*(5*(b*x +
a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)
*a^3)*c*d*e/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*
```

```
x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b
^4 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4 + 65*(63*(
b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b
*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2
/b^5 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(
9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x
+ a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

maple [A] time = 0.05, size = 495, normalized size = 1.81

$$\frac{2\sqrt{bx+a}\left(1155e^3x^6b^6-1260a b^5e^3x^5+4095b^6d e^2x^5+1400a^2b^4e^3x^4-4550a b^5d e^2x^4+5005b^6c e^2x^4+5005b^6d^2e x^4-1600a^3b^3e^3x^3+5200a^2b^4d e^2x^3-5720a b^5c e^2x^3-5720a b^5d^2e x^3+12870b^6c d e x^3+2145b^6d^3x^3+1920a^4b^2e^3x^2-6240a^3b^3d e^2x^2+6864a^2b^4c e^2x^2+6864a^2b^4d^2e x^2-15444a b^5c d e x^2-2574a b^5d^3x^2+9009b^6c^2e x^2+9009b^6c d^2x^2-2560a^5b e^3x+8320a^4b^2d e^2x-9152a^3b^3c e^2x-9152a^3b^3d^2e x+20592a^2b^4c d e x+3432a^2b^4d^3x-12012a b^5c^2e x-12012a b^5c d^2x+15015b^6c^2d x+5120a^6e^3-16640a^5b d e^2+18304a^4b^2c e^2+18304a^4b^2d^2e-41184a^3b^3c d e-6864a^3b^3d^3+24024a^2b^4c^2e+24024a^2b^4c d^2-30030a b^5c^2d+15015c^3b^6\right)}{15015b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x)
```

```
[Out] 2/15015*(b*x+a)^(1/2)*(1155*b^6*e^3*x^6-1260*a*b^5*e^3*x^5+4095*b^6*d*e^2*x
^5+1400*a^2*b^4*e^3*x^4-4550*a*b^5*d*e^2*x^4+5005*b^6*c*e^2*x^4+5005*b^6*d^
2*e*x^4-1600*a^3*b^3*e^3*x^3+5200*a^2*b^4*d*e^2*x^3-5720*a*b^5*c*e^2*x^3-57
20*a*b^5*d^2*e*x^3+12870*b^6*c*d*e*x^3+2145*b^6*d^3*x^3+1920*a^4*b^2*e^3*x^
2-6240*a^3*b^3*d*e^2*x^2+6864*a^2*b^4*c*e^2*x^2+6864*a^2*b^4*d^2*e*x^2-1544
4*a*b^5*c*d*e*x^2-2574*a*b^5*d^3*x^2+9009*b^6*c^2*e*x^2+9009*b^6*c*d^2*x^2-
2560*a^5*b*e^3*x+8320*a^4*b^2*d*e^2*x-9152*a^3*b^3*c*e^2*x-9152*a^3*b^3*d^2
*e*x+20592*a^2*b^4*c*d*e*x+3432*a^2*b^4*d^3*x-12012*a*b^5*c^2*e*x-12012*a*b
^5*c*d^2*x+15015*b^6*c^2*d*x+5120*a^6*e^3-16640*a^5*b*d*e^2+18304*a^4*b^2*c
*e^2+18304*a^4*b^2*d^2*e-41184*a^3*b^3*c*d*e-6864*a^3*b^3*d^3+24024*a^2*b^4
*c^2*e+24024*a^2*b^4*c*d^2-30030*a*b^5*c^2*d+15015*b^6*c^3)/b^7
```

maxima [B] time = 0.98, size = 525, normalized size = 1.92

$$\frac{2\left(15015\sqrt{bx+a}\,c^3+3003c^2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)d}{b}+\frac{\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)e}{b^2}\right)+143c\left(\frac{21\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)d^2}{b^2}+\frac{18\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)de}{b^3}+\frac{\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)e^2}{b^4}\right)+\frac{429\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)d^3}{b^3}+\frac{143\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)d^2e}{b^4}+\frac{65\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)de^2}{b^5}+\frac{5\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)e^3}{b^6}\right)}{15015b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 3003*c^2*(5*((b*x + a)^(3/2) - 3*sqrt(b*
x + a)*a)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a
)*a^2)*e/b^2) + 143*c*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sq
rt(b*x + a)*a^2)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 3
5*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2)
 - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^
3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4) + 429*(5*(b*x + a)^(7/2) - 21*(b*x + a)
^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 143*(35
*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b
*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b^4 + 65*(63*(b*x + a)^(11
/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2
)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2/b^5 + 5*(23
1*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8
580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a
^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

mupad [B] time = 0.10, size = 299, normalized size = 1.09

$$\frac{2e^3(a+bx)^{13/2}}{13b^7}-\frac{\left(12ae^3-6bde^2\right)(a+bx)^{11/2}}{11b^7}+\frac{(a+bx)^{9/2}\left(30a^2e^3-30abde^2+6b^2d^2e+6cb^2e^2\right)}{9b^7}+\frac{2\sqrt{a+bx}\left(ea^2-dab+cb^2\right)^3}{b^7}+\frac{(a+bx)^{5/2}\left(30a^4e^3-60a^3bde^2+36a^2b^2ce^2+36a^2b^2d^2e-36ab^3cde-6ab^3d^3+6b^4c^2e+6b^4cd^2\right)}{5b^7}-\frac{2(2ae-bd)(a+bx)^{7/2}\left(10a^2e^2-10abde+b^2d^2+6cb^2e\right)}{7b^7}-\frac{2(2ae-bd)(a+bx)^{3/2}\left(ea^2-dab+cb^2\right)^2}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)^3/(a + b*x)^(1/2),x)
```

```
[Out] (2*e^3*(a + b*x)^(13/2))/(13*b^7) - ((12*a*e^3 - 6*b*d*e^2)*(a + b*x)^(11/2
))/(11*b^7) + ((a + b*x)^(9/2)*(30*a^2*e^3 + 6*b^2*c*e^2 + 6*b^2*d^2*e - 30
```

```
*a*b*d*e^2))/(9*b^7) + (2*(a + b*x)^(1/2)*(b^2*c + a^2*e - a*b*d)^3)/b^7 +
((a + b*x)^(5/2)*(30*a^4*e^3 - 6*a*b^3*d^3 + 6*b^4*c*d^2 + 6*b^4*c^2*e + 36
*a^2*b^2*c*e^2 + 36*a^2*b^2*d^2*e - 60*a^3*b*d*e^2 - 36*a*b^3*c*d*e))/(5*b^
7) - (2*(2*a*e - b*d)*(a + b*x)^(7/2)*(10*a^2*e^2 + b^2*d^2 + 6*b^2*c*e - 1
0*a*b*d*e))/(7*b^7) - (2*(2*a*e - b*d)*(a + b*x)^(3/2)*(b^2*c + a^2*e - a*b
*d)^2)/b^7
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.4 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=114

$$\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{2(a+bx)^{5/2}(be-3af)}{5b^4}+\frac{2f(a+bx}{7b^4}$$

Rubi [A] time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1850}

$$\frac{2\sqrt{a+bx}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}+\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{2(a+bx)^{5/2}(be-3af)}{5b^4}+\frac{2f(a+bx)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x])/b^4 + (2*(b^2*d - 2*a
*b*e + 3*a^2*f)*(a + b*x)^(3/2))/(3*b^4) + (2*(b*e - 3*a*f)*(a + b*x)^(5/2)
)/(5*b^4) + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx=\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}}+\frac{\left(b^2d-2abe+3a^2f\right)\sqrt{a+bx}}{b^3}+\frac{(be-3af)(a+bx)}{b^3}\right.$$
$$=\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx}}{b^4}+\frac{2\left(b^2d-2abe+3a^2f\right)(a+bx)^{3/2}}{3b^4}+\frac{2(be-}{}$$

Mathematica [A] time = 0.18, size = 82, normalized size = 0.72

$$\frac{2\sqrt{a+bx}\left(-48a^3f+8a^2b(7e+3fx)-2ab^2(35d+x(14e+9fx))+b^3(105c+x(35d+3x(7e+5fx)))\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x) - 2*a*b^2*(35*d + x*(14
*e + 9*f*x)) + b^3*(105*c + x*(35*d + 3*x*(7*e + 5*f*x)))))/(105*b^4)
```

IntegrateAlgebraic [A] time = 0.06, size = 107, normalized size = 0.94

$$\frac{2\sqrt{a+bx}\left(-105a^3f+105a^2be+105a^2f(a+bx)+35b^2d(a+bx)-105ab^2d-70abe(a+bx)+21be(a+bx)^2-63af(a+bx)^2+15f(a+bx)^3+105b^3c\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(105*b^3*c - 105*a*b^2*d + 105*a^2*b*e - 105*a^3*f + 35*b^
2*d*(a + b*x) - 70*a*b*e*(a + b*x) + 105*a^2*f*(a + b*x) + 21*b*e*(a + b*x)
^2 - 63*a*f*(a + b*x)^2 + 15*f*(a + b*x)^3))/(105*b^4)
```

fricas [A] time = 0.41, size = 90, normalized size = 0.79

$$\frac{2\left(15\,b^3fx^3+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+3\left(7\,b^3e-6\,ab^2f\right)x^2+\left(35\,b^3d-28\,ab^2e+24\,a^2bf\right)x\right)\sqrt{bx+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*f*x^3 + 105*b^3*c - 70*a*b^2*d + 56*a^2*b*e - 48*a^3*f + 3*(7
*b^3*e - 6*a*b^2*f)*x^2 + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x)*sqrt(b*x
+ a)/b^4
```

giac [A] time = 0.17, size = 129, normalized size = 1.13

$$\frac{2\left(105\,\sqrt{bx+a}\,c+\frac{35\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)d}{b}+\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)e}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)f}{b^3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b +
 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2
+ 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35
*sqrt(b*x + a)*a^3)*f/b^3)/b
```

maple [A] time = 0.04, size = 91, normalized size = 0.80

$$-\frac{2\sqrt{bx+a}\,\left(-15f\,x^3b^3+18a\,b^2f\,x^2-21b^3e\,x^2-24a^2bfx+28a\,b^2ex-35b^3dx+48a^3f-56a^2be+70a\,b^2d-105b^3c\right)}{105b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x)
```

```
[Out] -2/105*(b*x+a)^(1/2)*(-15*b^3*f*x^3+18*a*b^2*f*x^2-21*b^3*e*x^2-24*a^2*b*f*
x+28*a*b^2*e*x-35*b^3*d*x+48*a^3*f-56*a^2*b*e+70*a*b^2*d-105*b^3*c)/b^4
```

maxima [A] time = 0.83, size = 128, normalized size = 1.12

$$\frac{2\left(105\,\sqrt{bx+a}\,c+\frac{35\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)d}{b}+\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)e}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)f}{b^3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b +
 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2
+ 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35
*sqrt(b*x + a)*a^3)*f/b^3)/b
```

mupad [B] time = 4.81, size = 103, normalized size = 0.90

$$\frac{(a+b\,x)^{3/2}\,\left(6\,f\,a^2-4\,e\,a\,b+2\,d\,b^2\right)}{3\,b^4}-\frac{\left(6\,a\,f-2\,b\,e\right)\,(a+b\,x)^{5/2}}{5\,b^4}+\frac{\sqrt{a+b\,x}\,\left(-2\,f\,a^3+2\,e\,a^2\,b-2\,d\,a\,b^2+2\,c\,b^3\right)}{b^4}+\frac{2\,f\,(a+b\,x)^{7/2}}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x)^(1/2),x)
```

```
[Out] ((a + b*x)^(3/2)*(2*b^2*d + 6*a^2*f - 4*a*b*e))/(3*b^4) - ((6*a*f - 2*b*e)*
(a + b*x)^(5/2))/(5*b^4) + ((a + b*x)^(1/2)*(2*b^3*c - 2*a^3*f - 2*a*b^2*d
+ 2*a^2*b*e))/b^4 + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

sympy [A] time = 45.83, size = 354, normalized size = 3.11

$$\begin{cases} \frac{-\frac{2ac}{\sqrt{a+bx}} - \frac{2ad\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right)}{b} - \frac{2ae\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b^2} - \frac{2af\left(-\frac{a^3}{\sqrt{a+bx}} - 3a^2\sqrt{a+bx} + a(a+bx)^{\frac{3}{2}} - \frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^3} - 2c\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right) - \frac{2d\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b} - \frac{2e\left(-\frac{a^3}{\sqrt{a+bx}} - 3a^2\sqrt{a+bx} + a(a+bx)^{\frac{3}{2}} - \frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2} - \frac{2f\left(\frac{a^4}{\sqrt{a+bx}} + 4a^3\sqrt{a+bx} - 2a^2(a+bx)^{\frac{3}{2}} + \frac{4a(a+bx)^{\frac{5}{2}}}{5} - \frac{(a+bx)^{\frac{7}{2}}}{7}\right)}{b^3}}{b} & \text{for } b \neq 0 \\ \frac{cx + \frac{dx^2}{2} + \frac{ex^3}{3} + \frac{fx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*c/sqrt(a + b*x) - 2*a*d*(-a/sqrt(a + b*x) - sqrt(a + b*x))
/b - 2*a*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b*
*2 - 2*a*f*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2)
 - (a + b*x)**(5/2)/5)/b**3 - 2*c*(-a/sqrt(a + b*x) - sqrt(a + b*x)) - 2*d*
(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b - 2*e*(-a**
3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5
/2)/5)/b**2 - 2*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a +
b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3)/b, Ne(b, 0
)), ((c*x + d*x**2/2 + e*x**3/3 + f*x**4/4)/sqrt(a), True))
```

3.5 $\int \frac{\left(c+dx+ex^2+fx^3\right)^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=320

$$-\frac{2(a+bx)^{9/2}\left(-15a^2f^2+10abef-\left(b^2\left(2df+e^2\right)\right)\right)}{9b^7}+\frac{4(a+bx)^{7/2}\left(-10a^3f^2+10a^2bef-2ab^2\left(2df+e^2\right)+b^3(cf\right.}{7b^7}$$

Rubi [A] time = 0.24, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1850}

$$\frac{2(a+bx)^{5/2}\left(6a^2b^2\left(2df+e^2\right)-20a^3bef+15a^4f^2-6ab^3(cf+de)+b^4\left(2ce+d^2\right)\right)}{5b^7}+\frac{4(a+bx)^{7/2}\left(10a^2bef-10a^3f^2-2ab^2\left(2df+e^2\right)+b^3(cf+de)\right)}{7b^7}+\frac{4(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^7}+\frac{2\sqrt{a+bx}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)^2}{b^7}-\frac{2(a+bx)^{9/2}\left(-15a^2f^2+10abef+b^2\left(-\left(2df+e^2\right)\right)\right)}{9b^7}+\frac{4f(a+bx)^{11/2}(be-3af)}{11b^7}+\frac{2f^2(a+bx)^{13/2}}{13b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x])/b^7 + (4*(b^2*d - 2
*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a + b*x)^(3/2))/(3*b
^7) + (2*(b^4*(d^2 + 2*c*e) - 20*a^3*b*e*f + 15*a^4*f^2 - 6*a*b^3*(d*e + c*
f) + 6*a^2*b^2*(e^2 + 2*d*f))*(a + b*x)^(5/2))/(5*b^7) + (4*(10*a^2*b*e*f -
 10*a^3*f^2 + b^3*(d*e + c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2))/(7*
b^7) - (2*(10*a*b*e*f - 15*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/2))/(9
*b^7) + (4*f*(b*e - 3*a*f)*(a + b*x)^(11/2))/(11*b^7) + (2*f^2*(a + b*x)^(1
3/2))/(13*b^7)
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx+ex^2+fx^3\right)^2}{\sqrt{a+bx}}\,dx &= \int \left(\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)^2}{b^6\sqrt{a+bx}}+\frac{2\left(b^2d-2abe+3a^2f\right)\left(b^3c-ab^2d+a^2be-a^3f\right.}{b^6}\right.\\ &= \frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)^2\sqrt{a+bx}}{b^7}+\frac{4\left(b^2d-2abe+3a^2f\right)\left(b^3c-ab^2d+a^2be\right.}{3b^7}\end{aligned}$$

Mathematica [A] time = 0.57, size = 303, normalized size = 0.95

$$\frac{2\left(-\frac{1}{9}(a+bx)^{9/2}\left(-15a^2f^2+10abef-\left(b^2\left(2df+e^2\right)\right)\right)+\frac{2}{7}(a+bx)^{7/2}\left(-10a^3f^2+10a^2bef-2ab^2\left(2df+e^2\right)+b^3(cf+de)\right)+\frac{2}{3}(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^2+\frac{1}{5}(a+bx)^{5/2}\left(15a^4f^2-20a^3bef+6a^2b^2\left(2df+e^2\right)-6ab^3(cf+de)+b^4\left(2ce+d^2\right)\right)+\frac{2}{11}f(a+bx)^{11/2}(be-3af)+\frac{1}{13}f^2(a+bx)^{13/2}\right)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x] + (2*(b^2*d - 2*a*b
*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a + b*x)^(3/2))/3 + ((b^
4*(d^2 + 2*c*e) - 20*a^3*b*e*f + 15*a^4*f^2 - 6*a*b^3*(d*e + c*f) + 6*a^2*b
^2*(e^2 + 2*d*f))*(a + b*x)^(5/2))/5 + (2*(10*a^2*b*e*f - 10*a^3*f^2 + b^3*
(d*e + c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2))/7 - ((10*a*b*e*f - 15
*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/2))/9 + (2*f*(b*e - 3*a*f)*(a +
b*x)^(11/2))/11 + (f^2*(a + b*x)^(13/2))/13))/b^7
```

IntegrateAlgebraic [A] time = 0.16, size = 544, normalized size = 1.70

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(45045*b^6*c^2 - 90090*a*b^5*c*d + 45045*a^2*b^4*d^2 + 900
90*a^2*b^4*c*e - 90090*a^3*b^3*d*e + 45045*a^4*b^2*e^2 - 90090*a^3*b^3*c*f
+ 90090*a^4*b^2*d*f - 90090*a^5*b*e*f + 45045*a^6*f^2 + 30030*b^5*c*d*(a +
b*x) - 30030*a*b^4*d^2*(a + b*x) - 60060*a*b^4*c*e*(a + b*x) + 90090*a^2*b^
3*d*e*(a + b*x) - 60060*a^3*b^2*e^2*(a + b*x) + 90090*a^2*b^3*c*f*(a + b*x)
 - 120120*a^3*b^2*d*f*(a + b*x) + 150150*a^4*b*e*f*(a + b*x) - 90090*a^5*f^
2*(a + b*x) + 9009*b^4*d^2*(a + b*x)^2 + 18018*b^4*c*e*(a + b*x)^2 - 54054*
a*b^3*d*e*(a + b*x)^2 + 54054*a^2*b^2*e^2*(a + b*x)^2 - 54054*a*b^3*c*f*(a
+ b*x)^2 + 108108*a^2*b^2*d*f*(a + b*x)^2 - 180180*a^3*b*e*f*(a + b*x)^2 +
135135*a^4*f^2*(a + b*x)^2 + 12870*b^3*d*e*(a + b*x)^3 - 25740*a*b^2*e^2*(a
 + b*x)^3 + 12870*b^3*c*f*(a + b*x)^3 - 51480*a*b^2*d*f*(a + b*x)^3 + 12870
0*a^2*b*e*f*(a + b*x)^3 - 128700*a^3*f^2*(a + b*x)^3 + 5005*b^2*e^2*(a + b*
x)^4 + 10010*b^2*d*f*(a + b*x)^4 - 50050*a*b*e*f*(a + b*x)^4 + 75075*a^2*f^
2*(a + b*x)^4 + 8190*b*e*f*(a + b*x)^5 - 24570*a*f^2*(a + b*x)^5 + 3465*f^2
*(a + b*x)^6))/(45045*b^7)
```

fricas [A] time = 0.42, size = 417, normalized size = 1.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*b^6*f^2*x^6 + 45045*b^6*c^2 - 60060*a*b^5*c*d + 24024*a^2*b^4
*d^2 + 18304*a^4*b^2*e^2 + 15360*a^6*f^2 + 630*(13*b^6*e*f - 6*a*b^5*f^2)*x
^5 + 35*(143*b^6*e^2 + 120*a^2*b^4*f^2 + 26*(11*b^6*d - 10*a*b^5*e)*f)*x^4
+ 10*(1287*b^6*d*e - 572*a*b^5*e^2 - 480*a^3*b^3*f^2 + 13*(99*b^6*c - 88*a*
b^5*d + 80*a^2*b^4*e)*f)*x^3 + 3*(3003*b^6*d^2 + 2288*a^2*b^4*e^2 + 1920*a^
4*b^2*f^2 + 858*(7*b^6*c - 6*a*b^5*d)*e - 52*(99*a*b^5*c - 88*a^2*b^4*d + 8
0*a^3*b^3*e)*f)*x^2 + 6864*(7*a^2*b^4*c - 6*a^3*b^3*d)*e - 416*(99*a^3*b^3*
c - 88*a^4*b^2*d + 80*a^5*b*e)*f + 2*(15015*b^6*c*d - 6006*a*b^5*d^2 - 4576
*a^3*b^3*e^2 - 3840*a^5*b*f^2 - 1716*(7*a*b^5*c - 6*a^2*b^4*d)*e + 104*(99*
a^2*b^4*c - 88*a^3*b^3*d + 80*a^4*b^2*e)*f)*x)*sqrt(b*x + a)/b^7
```

giac [A] time = 0.20, size = 516, normalized size = 1.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 30030*((b*x + a)^(3/2) - 3*sqrt(b*x + a)
*a)*c*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
a)*a^2)*d^2/b^2 + 6006*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(
b*x + a)*a^2)*c*e/b^2 + 2574*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35
*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*f/b^3 + 2574*(5*(b*x + a)^(7
/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)
*d*e/b^3 + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^
(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d*f/b^4 + 143*
(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420
*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 130*(63*(b*x + a)^(
11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5
```

```
/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*f*e/b^5 + 15*(2
31*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 -
8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*
a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

maple [A] time = 0.05, size = 447, normalized size = 1.40

$$\frac{2\sqrt{bx+a}\left(3465b^6f^2x^6-3780ab^5f^2x^5+8190b^6efx^5+4200a^2b^4f^2x^4-9100ab^5efx^4+10010b^6dfx^4+5005b^6e^2x^4-4800a^3b^3f^2x^3+10400a^2b^4efx^3-11440ab^5dfx^3-5720ab^5e^2x^3+12870b^6cfx^3+12870b^6dex^3+5760a^4b^2f^2x^2-12480a^3b^3efx^2+13728a^2b^4dfx^2+6864a^2b^4e^2x^2-15444ab^5cfx^2-15444ab^5dex^2+18018b^6cex^2+9009b^6d^2x^2-7680a^5bf^2x+16640a^4b^2efx-18304a^3b^3dfx-9152a^3b^3e^2x+20592a^2b^4cfx+20592a^2b^4dex-24024ab^5cex-12012ab^5d^2x+30030b^6cdx+15360a^6f^2-33280a^5bef+36608a^4b^2df+18304a^4b^2e^2-41184a^3b^3cf-41184a^3b^3de+48048a^2b^4ce+24024a^2b^4d^2-60060ab^5cd+45045c^2b^6\right)}{45045b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/45045*(b*x+a)^(1/2)*(3465*b^6*f^2*x^6-3780*a*b^5*f^2*x^5+8190*b^6*e*f*x^5
+4200*a^2*b^4*f^2*x^4-9100*a*b^5*e*f*x^4+10010*b^6*d*f*x^4+5005*b^6*e^2*x^4
-4800*a^3*b^3*f^2*x^3+10400*a^2*b^4*e*f*x^3-11440*a*b^5*d*f*x^3-5720*a*b^5*
e^2*x^3+12870*b^6*c*f*x^3+12870*b^6*d*e*x^3+5760*a^4*b^2*f^2*x^2-12480*a^3*
b^3*e*f*x^2+13728*a^2*b^4*d*f*x^2+6864*a^2*b^4*e^2*x^2-15444*a*b^5*c*f*x^2-
15444*a*b^5*d*e*x^2+18018*b^6*c*e*x^2+9009*b^6*d^2*x^2-7680*a^5*b*f^2*x+166
40*a^4*b^2*e*f*x-18304*a^3*b^3*d*f*x-9152*a^3*b^3*e^2*x+20592*a^2*b^4*c*f*x
+20592*a^2*b^4*d*e*x-24024*a*b^5*c*e*x-12012*a*b^5*d^2*x+30030*b^6*c*d*x+15
360*a^6*f^2-33280*a^5*b*e*f+36608*a^4*b^2*d*f+18304*a^4*b^2*e^2-41184*a^3*b
^3*c*f-41184*a^3*b^3*d*e+48048*a^2*b^4*c*e+24024*a^2*b^4*d^2-60060*a*b^5*c*
d+45045*b^6*c^2)/b^7
```

maxima [A] time = 1.00, size = 500, normalized size = 1.56

$$\frac{2\left(45045\sqrt{bx+a}\,c^2+858c\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)d}{b}+\frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)e}{b^2}+\frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)f}{b^3}\right)+\frac{3003\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)d^2}{b^2}+\frac{143\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)e^2}{b^4}+\frac{286\left(35(bx+a)^{\frac{9}{2}}f+45(be-4af)(bx+a)^{\frac{7}{2}}-189(abe-2a^2f)(bx+a)^{\frac{5}{2}}+105(3a^2be-4a^3f)(bx+a)^{\frac{3}{2}}-315(a^3be-a^4f)\sqrt{bx+a}\right)d}{b^4}+\frac{130\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)ef}{b^5}+\frac{15\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)f^2}{b^6}\right)}{45045b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 858*c*(35*((b*x + a)^(3/2) - 3*sqrt(b*x
+ a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a
)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 3003*(3*(b*x + a)^(5/2) - 10*(b*x
 + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 143*(35*(b*x + a)^(9/2) - 1
80*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 +
315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(9/2)*f + 45*(b*e - 4*a*
f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(b*x + a)^(5/2) + 105*(3*a^2*b*e
 - 4*a^3*f)*(b*x + a)^(3/2) - 315*(a^3*b*e - a^4*f)*sqrt(b*x + a))*d/b^4 +
130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2
- 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a
^5)*e*f/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*
x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 60
06*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

mupad [B] time = 4.70, size = 316, normalized size = 0.99

$$\frac{2\sqrt{a+bx}\left(-fa^3+ea^2b-dab^2+cb^3\right)^2}{b^7}+\frac{2f^2(a+bx)^{13/2}}{13b^7}-\frac{(a+bx)^{7/2}\left(40a^3f^2-40a^2bef+8ab^2e^2+16dab^2f-4db^3e-4cb^3f\right)}{7b^7}+\frac{(a+bx)^{9/2}\left(30a^2f^2-20abef+2b^2e^2+4db^2f\right)}{9b^7}+\frac{(a+bx)^{5/2}\left(30a^4f^2-40a^3bef+24a^2b^2df+12a^2b^2e^2-12ab^3de-12cab^3f+2b^4d^2+4cb^4e\right)}{5b^7}-\frac{\left(12af^2-4bef\right)(a+bx)^{11/2}}{11b^7}+\frac{4(a+bx)^{3/2}\left(3fa^2-2eab+db^2\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)^2/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)^2)/b^7 + (2*f^2*(a +
 b*x)^(13/2))/(13*b^7) - ((a + b*x)^(7/2)*(40*a^3*f^2 + 8*a*b^2*e^2 - 4*b^3
*c*f - 4*b^3*d*e + 16*a*b^2*d*f - 40*a^2*b*e*f))/(7*b^7) + ((a + b*x)^(9/2)
*(30*a^2*f^2 + 2*b^2*e^2 + 4*b^2*d*f - 20*a*b*e*f))/(9*b^7) + ((a + b*x)^(5
/2)*(2*b^4*d^2 + 30*a^4*f^2 + 12*a^2*b^2*e^2 + 4*b^4*c*e - 12*a*b^3*c*f - 1
2*a*b^3*d*e - 40*a^3*b*e*f + 24*a^2*b^2*d*f))/(5*b^7) - ((12*a*f^2 - 4*b*e*
```

```
f)*(a + b*x)^(11/2))/(11*b^7) + (4*(a + b*x)^(3/2)*(b^2*d + 3*a^2*f - 2*a*b
*e)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^7)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.6 $\int \frac{\left(c+dx+ex^2+fx^3\right)^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=708

$$-\frac{2f(a+bx)^{15/2}\left(-12a^2f^2+8abef-\left(b^2\left(df+e^2\right)\right)\right)}{5b^{10}}+\frac{2(a+bx)^{13/2}\left(-84a^3f^3+84a^2bef^2-21ab^2f\left(df+e^2\right)+b^3\right.}{13b^{10}}$$

Rubi [A] time = 0.63, antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1850}

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x])/b^10 + (2*(b^2*d -
2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^(3/2))/b
^10 + (6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b*e*
f + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)
^(5/2))/(5*b^10) - (2*(168*a^5*b*e*f^2 - 84*a^6*f^3 - b^6*(d^3 + 6*c*d*e +
3*c^2*f) - 105*a^4*b^2*f*(e^2 + d*f) + 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) -
 30*a^2*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 20*a^3*b^3*(e^3 + 6*d*e*f + 3*c*f^2
))*(a + b*x)^(7/2))/(7*b^10) + (2*(70*a^4*b*e*f^2 - 42*a^5*f^3 - 35*a^3*b^2
*f*(e^2 + d*f) + b^5*(d^2*e + c*e^2 + 2*c*d*f) - 5*a*b^4*(d*e^2 + d^2*f + 2
*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(9/2))/(3*b^10) -
(6*(56*a^3*b*e*f^2 - 42*a^4*f^3 - 21*a^2*b^2*f*(e^2 + d*f) - b^4*(d*e^2 + d
^2*f + 2*c*e*f) + 2*a*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(11/2))/(11*
b^10) + (2*(84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a*b^2*f*(e^2 + d*f) + b^3*(e^3
 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(13/2))/(13*b^10) - (2*f*(8*a*b*e*f - 12*a
^2*f^2 - b^2*(e^2 + d*f))*(a + b*x)^(15/2))/(5*b^10) + (6*f^2*(b*e - 3*a*f)
*(a + b*x)^(17/2))/(17*b^10) + (2*f^3*(a + b*x)^(19/2))/(19*b^10)
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)^3}{\sqrt{a+bx}}\,dx = \int \left(\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)^3}{b^9\sqrt{a+bx}}+\frac{3\left(b^2d-2abe+3a^2f\right)\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^9}\right.$$

$$=\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)^3\sqrt{a+bx}}{b^{10}}+\frac{2\left(b^2d-2abe+3a^2f\right)\left(b^3c-ab^2d+a^2be\right.}{b^{10}}$$

Mathematica [A] time = 2.89, size = 678, normalized size = 0.96

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x] + (b^2*d - 2*a*b*e
+ 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^(3/2) + (3*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b*e*f + 12*a^4*f^2
 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)^(5/2))/5 + ((
-168*a^5*b*e*f^2 + 84*a^6*f^3 + b^6*(d^3 + 6*c*d*e + 3*c^2*f) + 105*a^4*b^2
*f*(e^2 + d*f) - 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) + 30*a^2*b^4*(d*e^2 + d
^2*f + 2*c*e*f) - 20*a^3*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(7/2))/7
+ ((70*a^4*b*e*f^2 - 42*a^5*f^3 - 35*a^3*b^2*f*(e^2 + d*f) + b^5*(d^2*e + c
*e^2 + 2*c*d*f) - 5*a*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*
e*f + 3*c*f^2))*(a + b*x)^(9/2))/3 + (3*(-56*a^3*b*e*f^2 + 42*a^4*f^3 + 21*
a^2*b^2*f*(e^2 + d*f) + b^4*(d*e^2 + d^2*f + 2*c*e*f) - 2*a*b^3*(e^3 + 6*d*
e*f + 3*c*f^2))*(a + b*x)^(11/2))/11 + ((84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a
*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(13/2))/13 +
(f*(-8*a*b*e*f + 12*a^2*f^2 + b^2*(e^2 + d*f))*(a + b*x)^(15/2))/5 + (3*f^2
*(b*e - 3*a*f)*(a + b*x)^(17/2))/17 + (f^3*(a + b*x)^(19/2))/19))/b^10
```

IntegrateAlgebraic [B] time = 0.42, size = 2128, normalized size = 3.01

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(4849845*b^9*c^3*Sqrt[a + b*x] - 14549535*a*b^8*c^2*d*Sqrt[a + b*x] + 14
549535*a^2*b^7*c*d^2*Sqrt[a + b*x] - 4849845*a^3*b^6*d^3*Sqrt[a + b*x] + 14
549535*a^2*b^7*c^2*e*Sqrt[a + b*x] - 29099070*a^3*b^6*c*d*e*Sqrt[a + b*x] +
 14549535*a^4*b^5*d^2*e*Sqrt[a + b*x] + 14549535*a^4*b^5*c*e^2*Sqrt[a + b*x
] - 14549535*a^5*b^4*d*e^2*Sqrt[a + b*x] + 4849845*a^6*b^3*e^3*Sqrt[a + b*x
] - 14549535*a^3*b^6*c^2*f*Sqrt[a + b*x] + 29099070*a^4*b^5*c*d*f*Sqrt[a +
b*x] - 14549535*a^5*b^4*d^2*f*Sqrt[a + b*x] - 29099070*a^5*b^4*c*e*f*Sqrt[a
 + b*x] + 29099070*a^6*b^3*d*e*f*Sqrt[a + b*x] - 14549535*a^7*b^2*e^2*f*Sqr
t[a + b*x] + 14549535*a^6*b^3*c*f^2*Sqrt[a + b*x] - 14549535*a^7*b^2*d*f^2*
Sqrt[a + b*x] + 14549535*a^8*b*e*f^2*Sqrt[a + b*x] - 4849845*a^9*f^3*Sqrt[a
 + b*x] + 4849845*b^8*c^2*d*(a + b*x)^(3/2) - 9699690*a*b^7*c*d^2*(a + b*x)
^(3/2) + 4849845*a^2*b^6*d^3*(a + b*x)^(3/2) - 9699690*a*b^7*c^2*e*(a + b*x
)^(3/2) + 29099070*a^2*b^6*c*d*e*(a + b*x)^(3/2) - 19399380*a^3*b^5*d^2*e*(
a + b*x)^(3/2) - 19399380*a^3*b^5*c*e^2*(a + b*x)^(3/2) + 24249225*a^4*b^4*
d*e^2*(a + b*x)^(3/2) - 9699690*a^5*b^3*e^3*(a + b*x)^(3/2) + 14549535*a^2*
b^6*c^2*f*(a + b*x)^(3/2) - 38798760*a^3*b^5*c*d*f*(a + b*x)^(3/2) + 242492
25*a^4*b^4*d^2*f*(a + b*x)^(3/2) + 48498450*a^4*b^4*c*e*f*(a + b*x)^(3/2) -
 58198140*a^5*b^3*d*e*f*(a + b*x)^(3/2) + 33948915*a^6*b^2*e^2*f*(a + b*x)^
(3/2) - 29099070*a^5*b^3*c*f^2*(a + b*x)^(3/2) + 33948915*a^6*b^2*d*f^2*(a
+ b*x)^(3/2) - 38798760*a^7*b*e*f^2*(a + b*x)^(3/2) + 14549535*a^8*f^3*(a +
 b*x)^(3/2) + 2909907*b^7*c*d^2*(a + b*x)^(5/2) - 2909907*a*b^6*d^3*(a + b*
x)^(5/2) + 2909907*b^7*c^2*e*(a + b*x)^(5/2) - 17459442*a*b^6*c*d*e*(a + b*
x)^(5/2) + 17459442*a^2*b^5*d^2*e*(a + b*x)^(5/2) + 17459442*a^2*b^5*c*e^2*
(a + b*x)^(5/2) - 29099070*a^3*b^4*d*e^2*(a + b*x)^(5/2) + 14549535*a^4*b^3
*e^3*(a + b*x)^(5/2) - 8729721*a*b^6*c^2*f*(a + b*x)^(5/2) + 34918884*a^2*b
^5*c*d*f*(a + b*x)^(5/2) - 29099070*a^3*b^4*d^2*f*(a + b*x)^(5/2) - 5819814
0*a^3*b^4*c*e*f*(a + b*x)^(5/2) + 87297210*a^4*b^3*d*e*f*(a + b*x)^(5/2) -
61108047*a^5*b^2*e^2*f*(a + b*x)^(5/2) + 43648605*a^4*b^3*c*f^2*(a + b*x)^(
5/2) - 61108047*a^5*b^2*d*f^2*(a + b*x)^(5/2) + 81477396*a^6*b*e*f^2*(a + b
*x)^(5/2) - 34918884*a^7*f^3*(a + b*x)^(5/2) + 692835*b^6*d^3*(a + b*x)^(7/
2) + 4157010*b^6*c*d*e*(a + b*x)^(7/2) - 8314020*a*b^5*d^2*e*(a + b*x)^(7/2
) - 8314020*a*b^5*c*e^2*(a + b*x)^(7/2) + 20785050*a^2*b^4*d*e^2*(a + b*x)^
(7/2) - 13856700*a^3*b^3*e^3*(a + b*x)^(7/2) + 2078505*b^6*c^2*f*(a + b*x)^
(7/2) - 16628040*a*b^5*c*d*f*(a + b*x)^(7/2) + 20785050*a^2*b^4*d^2*f*(a +
b*x)^(7/2) + 41570100*a^2*b^4*c*e*f*(a + b*x)^(7/2) - 83140200*a^3*b^3*d*e*
f*(a + b*x)^(7/2) + 72747675*a^4*b^2*e^2*f*(a + b*x)^(7/2) - 41570100*a^3*b
^3*c*f^2*(a + b*x)^(7/2) + 72747675*a^4*b^2*d*f^2*(a + b*x)^(7/2) - 1163962
```

```
80*a^5*b*e*f^2*(a + b*x)^(7/2) + 58198140*a^6*f^3*(a + b*x)^(7/2) + 1616615
*b^5*d^2*e*(a + b*x)^(9/2) + 1616615*b^5*c*e^2*(a + b*x)^(9/2) - 8083075*a*
b^4*d*e^2*(a + b*x)^(9/2) + 8083075*a^2*b^3*e^3*(a + b*x)^(9/2) + 3233230*b
^5*c*d*f*(a + b*x)^(9/2) - 8083075*a*b^4*d^2*f*(a + b*x)^(9/2) - 16166150*a
*b^4*c*e*f*(a + b*x)^(9/2) + 48498450*a^2*b^3*d*e*f*(a + b*x)^(9/2) - 56581
525*a^3*b^2*e^2*f*(a + b*x)^(9/2) + 24249225*a^2*b^3*c*f^2*(a + b*x)^(9/2)
- 56581525*a^3*b^2*d*f^2*(a + b*x)^(9/2) + 113163050*a^4*b*e*f^2*(a + b*x)^
(9/2) - 67897830*a^5*f^3*(a + b*x)^(9/2) + 1322685*b^4*d*e^2*(a + b*x)^(11/
2) - 2645370*a*b^3*e^3*(a + b*x)^(11/2) + 1322685*b^4*d^2*f*(a + b*x)^(11/2
) + 2645370*b^4*c*e*f*(a + b*x)^(11/2) - 15872220*a*b^3*d*e*f*(a + b*x)^(11
/2) + 27776385*a^2*b^2*e^2*f*(a + b*x)^(11/2) - 7936110*a*b^3*c*f^2*(a + b*
x)^(11/2) + 27776385*a^2*b^2*d*f^2*(a + b*x)^(11/2) - 74070360*a^3*b*e*f^2*
(a + b*x)^(11/2) + 55552770*a^4*f^3*(a + b*x)^(11/2) + 373065*b^3*e^3*(a +
b*x)^(13/2) + 2238390*b^3*d*e*f*(a + b*x)^(13/2) - 7834365*a*b^2*e^2*f*(a +
 b*x)^(13/2) + 1119195*b^3*c*f^2*(a + b*x)^(13/2) - 7834365*a*b^2*d*f^2*(a
+ b*x)^(13/2) + 31337460*a^2*b*e*f^2*(a + b*x)^(13/2) - 31337460*a^3*f^3*(a
 + b*x)^(13/2) + 969969*b^2*e^2*f*(a + b*x)^(15/2) + 969969*b^2*d*f^2*(a +
b*x)^(15/2) - 7759752*a*b*e*f^2*(a + b*x)^(15/2) + 11639628*a^2*f^3*(a + b*
x)^(15/2) + 855855*b*e*f^2*(a + b*x)^(17/2) - 2567565*a*f^3*(a + b*x)^(17/2
) + 255255*f^3*(a + b*x)^(19/2)))/(4849845*b^10)
```

fricas [A] time = 0.42, size = 1221, normalized size = 1.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/4849845*(255255*b^9*f^3*x^9 + 4849845*b^9*c^3 - 9699690*a*b^8*c^2*d + 775
9752*a^2*b^7*c*d^2 - 2217072*a^3*b^6*d^3 + 1653760*a^6*b^3*e^3 - 1376256*a^
9*f^3 + 45045*(19*b^9*e*f^2 - 6*a*b^8*f^3)*x^8 + 3003*(323*b^9*e^2*f + 96*a
^2*b^7*f^3 + 19*(17*b^9*d - 16*a*b^8*e)*f^2)*x^7 + 231*(1615*b^9*e^3 - 1344
*a^3*b^6*f^3 + 19*(255*b^9*c - 238*a*b^8*d + 224*a^2*b^7*e)*f^2 + 646*(15*b
^9*d*e - 7*a*b^8*e^2)*f)*x^6 + 63*(20995*b^9*d*e^2 - 6460*a*b^8*e^3 + 5376*
a^4*b^5*f^3 - 76*(255*a*b^8*c - 238*a^2*b^7*d + 224*a^3*b^6*e)*f^2 + 323*(6
5*b^9*d^2 + 56*a^2*b^7*e^2 + 10*(13*b^9*c - 12*a*b^8*d)*e)*f)*x^5 + 35*(461
89*b^9*d^2*e + 12920*a^2*b^7*e^3 - 10752*a^5*b^4*f^3 + 4199*(11*b^9*c - 10*
a*b^8*d)*e^2 + 152*(255*a^2*b^7*c - 238*a^3*b^6*d + 224*a^4*b^5*e)*f^2 + 64
6*(143*b^9*c*d - 65*a*b^8*d^2 - 56*a^3*b^6*e^2 - 10*(13*a*b^8*c - 12*a^2*b^
7*d)*e)*f)*x^4 + 5*(138567*b^9*d^3 - 103360*a^3*b^6*e^3 + 86016*a^6*b^3*f^3
 - 33592*(11*a*b^8*c - 10*a^2*b^7*d)*e^2 - 1216*(255*a^3*b^6*c - 238*a^4*b^
5*d + 224*a^5*b^4*e)*f^2 + 92378*(9*b^9*c*d - 4*a*b^8*d^2)*e + 323*(1287*b^
9*c^2 - 2288*a*b^8*c*d + 1040*a^2*b^7*d^2 + 896*a^4*b^5*e^2 + 160*(13*a^2*b
^7*c - 12*a^3*b^6*d)*e)*f)*x^3 + 537472*(11*a^4*b^5*c - 10*a^5*b^4*d)*e^2 +
 19456*(255*a^6*b^3*c - 238*a^7*b^2*d + 224*a^8*b*e)*f^2 + 3*(969969*b^9*c*
d^2 - 277134*a*b^8*d^3 + 206720*a^4*b^5*e^3 - 172032*a^7*b^2*f^3 + 67184*(1
1*a^2*b^7*c - 10*a^3*b^6*d)*e^2 + 2432*(255*a^4*b^5*c - 238*a^5*b^4*d + 224
*a^6*b^3*e)*f^2 + 46189*(21*b^9*c^2 - 36*a*b^8*c*d + 16*a^2*b^7*d^2)*e - 64
6*(1287*a*b^8*c^2 - 2288*a^2*b^7*c*d + 1040*a^3*b^6*d^2 + 896*a^5*b^4*e^2 +
 160*(13*a^3*b^6*c - 12*a^4*b^5*d)*e)*f)*x^2 + 369512*(21*a^2*b^7*c^2 - 36*
a^3*b^6*c*d + 16*a^4*b^5*d^2)*e - 5168*(1287*a^3*b^6*c^2 - 2288*a^4*b^5*c*d
 + 1040*a^5*b^4*d^2 + 896*a^7*b^2*e^2 + 160*(13*a^5*b^4*c - 12*a^6*b^3*d)*e
)*f + (4849845*b^9*c^2*d - 3879876*a*b^8*c*d^2 + 1108536*a^2*b^7*d^3 - 8268
80*a^5*b^4*e^3 + 688128*a^8*b*f^3 - 268736*(11*a^3*b^6*c - 10*a^4*b^5*d)*e^
2 - 9728*(255*a^5*b^4*c - 238*a^6*b^3*d + 224*a^7*b^2*e)*f^2 - 184756*(21*a
*b^8*c^2 - 36*a^2*b^7*c*d + 16*a^3*b^6*d^2)*e + 2584*(1287*a^2*b^7*c^2 - 22
88*a^3*b^6*c*d + 1040*a^4*b^5*d^2 + 896*a^6*b^3*e^2 + 160*(13*a^4*b^5*c - 1
2*a^5*b^4*d)*e)*f)*x)*sqrt(b*x + a)/b^10
```

giac [B] time = 0.29, size = 1414, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 4849845*((b*x + a)^(3/2) - 3*sqrt(b*
x + a)*a)*c^2*d/b + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*c*d^2/b^2 + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2
)*a + 15*sqrt(b*x + a)*a^2)*c^2*e/b^2 + 138567*(5*(b*x + a)^(7/2) - 21*(b*x
 + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 41
5701*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)*c^2*f/b^3 + 831402*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(
5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*d*e/b^3 + 92378*(
35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*
(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*d*f/b^4 + 46189*(35*(b*x + a
)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(
3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b^4 + 20995*(63*(b*x + a)^(11/2) -
385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3
+ 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d^2*f/b^5 + 46189*(35*(
b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x
 + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4 + 41990*(63*(b*x + a)^(1
1/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/
2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*c*f*e/b^5 + 4845
*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2
 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/
2)*a^5 + 3003*sqrt(b*x + a)*a^6)*c*f^2/b^6 + 20995*(63*(b*x + a)^(11/2) - 3
85*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 +
 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2/b^5 + 9690*(231*(b
*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*
(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 +
 3003*sqrt(b*x + a)*a^6)*d*f*e/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x
 + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 3
2175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2
)*a^6 - 6435*sqrt(b*x + a)*a^7)*d*f^2/b^7 + 1615*(231*(b*x + a)^(13/2) - 16
38*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3
 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)
*a^6)*e^3/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*f*e^2/b^7 + 133*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*
a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x
 + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 -
 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*f^2*e/b^8 + 21*(121
55*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a
^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*
(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)
*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*f^3/b^9)/b
```

maple [B] time = 0.05, size = 1417, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x)
```

```
[Out] -2/4849845*(b*x+a)^(1/2)*(-255255*b^9*f^3*x^9+270270*a*b^8*f^3*x^8-855855*b
^9*e*f^2*x^8-288288*a^2*b^7*f^3*x^7+912912*a*b^8*e*f^2*x^7-969969*b^9*d*f^2
*x^7-969969*b^9*e^2*f*x^7+310464*a^3*b^6*f^3*x^6-983136*a^2*b^7*e*f^2*x^6+1
044582*a*b^8*d*f^2*x^6+1044582*a*b^8*e^2*f*x^6-1119195*b^9*c*f^2*x^6-223839
0*b^9*d*e*f*x^6-373065*b^9*e^3*x^6-338688*a^4*b^5*f^3*x^5+1072512*a^3*b^6*e
```

```
*f^2*x^5-1139544*a^2*b^7*d*f^2*x^5-1139544*a^2*b^7*e^2*f*x^5+1220940*a*b^8*
c*f^2*x^5+2441880*a*b^8*d*e*f*x^5+406980*a*b^8*e^3*x^5-2645370*b^9*c*e*f*x^
5-1322685*b^9*d^2*f*x^5-1322685*b^9*d*e^2*x^5+376320*a^5*b^4*f^3*x^4-119168
0*a^4*b^5*e*f^2*x^4+1266160*a^3*b^6*d*f^2*x^4+1266160*a^3*b^6*e^2*f*x^4-135
6600*a^2*b^7*c*f^2*x^4-2713200*a^2*b^7*d*e*f*x^4-452200*a^2*b^7*e^3*x^4+293
9300*a*b^8*c*e*f*x^4+1469650*a*b^8*d^2*f*x^4+1469650*a*b^8*d*e^2*x^4-323323
0*b^9*c*d*f*x^4-1616615*b^9*c*e^2*x^4-1616615*b^9*d^2*e*x^4-430080*a^6*b^3*
f^3*x^3+1361920*a^5*b^4*e*f^2*x^3-1447040*a^4*b^5*d*f^2*x^3-1447040*a^4*b^5
*e^2*f*x^3+1550400*a^3*b^6*c*f^2*x^3+3100800*a^3*b^6*d*e*f*x^3+516800*a^3*b
^6*e^3*x^3-3359200*a^2*b^7*c*e*f*x^3-1679600*a^2*b^7*d^2*f*x^3-1679600*a^2*
b^7*d*e^2*x^3+3695120*a*b^8*c*d*f*x^3+1847560*a*b^8*c*e^2*x^3+1847560*a*b^8
*d^2*e*x^3-2078505*b^9*c^2*f*x^3-4157010*b^9*c*d*e*x^3-692835*b^9*d^3*x^3+5
16096*a^7*b^2*f^3*x^2-1634304*a^6*b^3*e*f^2*x^2+1736448*a^5*b^4*d*f^2*x^2+1
736448*a^5*b^4*e^2*f*x^2-1860480*a^4*b^5*c*f^2*x^2-3720960*a^4*b^5*d*e*f*x^
2-620160*a^4*b^5*e^3*x^2+4031040*a^3*b^6*c*e*f*x^2+2015520*a^3*b^6*d^2*f*x^
2+2015520*a^3*b^6*d*e^2*x^2-4434144*a^2*b^7*c*d*f*x^2-2217072*a^2*b^7*c*e^2
*x^2-2217072*a^2*b^7*d^2*e*x^2+2494206*a*b^8*c^2*f*x^2+4988412*a*b^8*c*d*e*
x^2+831402*a*b^8*d^3*x^2-2909907*b^9*c^2*e*x^2-2909907*b^9*c*d^2*x^2-688128
*a^8*b*f^3*x+2179072*a^7*b^2*e*f^2*x-2315264*a^6*b^3*d*f^2*x-2315264*a^6*b^
3*e^2*f*x+2480640*a^5*b^4*c*f^2*x+4961280*a^5*b^4*d*e*f*x+826880*a^5*b^4*e^
3*x-5374720*a^4*b^5*c*e*f*x-2687360*a^4*b^5*d^2*f*x-2687360*a^4*b^5*d*e^2*x
+5912192*a^3*b^6*c*d*f*x+2956096*a^3*b^6*c*e^2*x+2956096*a^3*b^6*d^2*e*x-33
25608*a^2*b^7*c^2*f*x-6651216*a^2*b^7*c*d*e*x-1108536*a^2*b^7*d^3*x+3879876
*a*b^8*c^2*e*x+3879876*a*b^8*c*d^2*x-4849845*b^9*c^2*d*x+1376256*a^9*f^3-43
58144*a^8*b*e*f^2+4630528*a^7*b^2*d*f^2+4630528*a^7*b^2*e^2*f-4961280*a^6*b
^3*c*f^2-9922560*a^6*b^3*d*e*f-1653760*a^6*b^3*e^3+10749440*a^5*b^4*c*e*f+5
374720*a^5*b^4*d^2*f+5374720*a^5*b^4*d*e^2-11824384*a^4*b^5*c*d*f-5912192*a
^4*b^5*c*e^2-5912192*a^4*b^5*d^2*e+6651216*a^3*b^6*c^2*f+13302432*a^3*b^6*c
*d*e+2217072*a^3*b^6*d^3-7759752*a^2*b^7*c^2*e-7759752*a^2*b^7*c*d^2+969969
0*a*b^8*c^2*d-4849845*b^9*c^3)/b^10
```

maxima [B] time = 1.09, size = 1360, normalized size = 1.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 138567*c^2*(35*((b*x + a)^(3/2) - 3*
sqrt(b*x + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqr
t(b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b
*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 323*c*(3003*(3*(b*x + a)
^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 143*(35*(b*
x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
 a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(9/2)*f
+ 45*(b*e - 4*a*f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(b*x + a)^(5/2)
+ 105*(3*a^2*b*e - 4*a^3*f)*(b*x + a)^(3/2) - 315*(a^3*b*e - a^4*f)*sqrt(b*
x + a))*d/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x
 + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693
*sqrt(b*x + a)*a^5)*e*f/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11
/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a
)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6) +
 138567*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2
- 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 4199*(315*(b*x + a)^(11/2)*f + 385*(b*e -
 5*a*f)*(b*x + a)^(9/2) - 990*(2*a*b*e - 5*a^2*f)*(b*x + a)^(7/2) + 1386*(3
*a^2*b*e - 5*a^3*f)*(b*x + a)^(5/2) - 1155*(4*a^3*b*e - 5*a^4*f)*(b*x + a)^
(3/2) + 3465*(a^4*b*e - a^5*f)*sqrt(b*x + a))*d^2/b^5 + 1615*(231*(b*x + a)
^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x +
a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*s
qrt(b*x + a)*a^6)*e^3/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13
```

```
/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x
 + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6
435*sqrt(b*x + a)*a^7)*e^2*f/b^7 + 133*(6435*(b*x + a)^(17/2) - 58344*(b*x
+ a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 +
 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)
^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*e*f^2/b
^8 + 323*(3003*(b*x + a)^(15/2)*f^2 + 3465*(2*b*e*f - 7*a*f^2)*(b*x + a)^(1
3/2) + 4095*(b^2*e^2 - 12*a*b*e*f + 21*a^2*f^2)*(b*x + a)^(11/2) - 25025*(a
*b^2*e^2 - 6*a^2*b*e*f + 7*a^3*f^2)*(b*x + a)^(9/2) + 32175*(2*a^2*b^2*e^2
- 8*a^3*b*e*f + 7*a^4*f^2)*(b*x + a)^(7/2) - 9009*(10*a^3*b^2*e^2 - 30*a^4*
b*e*f + 21*a^5*f^2)*(b*x + a)^(5/2) + 15015*(5*a^4*b^2*e^2 - 12*a^5*b*e*f +
 7*a^6*f^2)*(b*x + a)^(3/2) - 45045*(a^5*b^2*e^2 - 2*a^6*b*e*f + a^7*f^2)*s
qrt(b*x + a))*d/b^7 + 21*(12155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*
a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b
*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*
a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqr
t(b*x + a)*a^9)*f^3/b^9)/b
```

mupad [B] time = 0.24, size = 896, normalized size = 1.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)^3/(a + b*x)^(1/2),x)
```

```
[Out] ((a + b*x)^(11/2)*(252*a^4*f^3 - 12*a*b^3*e^3 + 6*b^4*d*e^2 + 6*b^4*d^2*f +
 126*a^2*b^2*d*f^2 + 126*a^2*b^2*e^2*f + 12*b^4*c*e*f - 36*a*b^3*c*f^2 - 33
6*a^3*b*e*f^2 - 72*a*b^3*d*e*f))/(11*b^10) + (2*(a + b*x)^(1/2)*(b^3*c - a^
3*f - a*b^2*d + a^2*b*e)^3)/b^10 + ((a + b*x)^(9/2)*(6*b^5*c*e^2 - 252*a^5*
f^3 + 6*b^5*d^2*e + 30*a^2*b^3*e^3 + 90*a^2*b^3*c*f^2 - 210*a^3*b^2*d*f^2 -
 210*a^3*b^2*e^2*f + 12*b^5*c*d*f - 30*a*b^4*d*e^2 - 30*a*b^4*d^2*f + 420*a
^4*b*e*f^2 + 180*a^2*b^3*d*e*f - 60*a*b^4*c*e*f))/(9*b^10) + (2*f^3*(a + b*
x)^(19/2))/(19*b^10) + ((a + b*x)^(13/2)*(2*b^3*e^3 - 168*a^3*f^3 + 6*b^3*c
*f^2 + 12*b^3*d*e*f - 42*a*b^2*d*f^2 - 42*a*b^2*e^2*f + 168*a^2*b*e*f^2))/(
13*b^10) - ((18*a*f^3 - 6*b*e*f^2)*(a + b*x)^(17/2))/(17*b^10) + ((a + b*x)
^(15/2)*(72*a^2*f^3 + 6*b^2*d*f^2 + 6*b^2*e^2*f - 48*a*b*e*f^2))/(15*b^10)
- ((a + b*x)^(5/2)*(72*a^7*f^3 + 6*a*b^6*d^3 - 6*b^7*c*d^2 - 6*b^7*c^2*e -
30*a^4*b^3*e^3 - 36*a^2*b^5*c*e^2 - 36*a^2*b^5*d^2*e + 60*a^3*b^4*d*e^2 - 9
0*a^4*b^3*c*f^2 + 60*a^3*b^4*d^2*f + 126*a^5*b^2*d*f^2 + 126*a^5*b^2*e^2*f
+ 18*a*b^6*c^2*f - 168*a^6*b*e*f^2 - 72*a^2*b^5*c*d*f + 120*a^3*b^4*c*e*f -
 180*a^4*b^3*d*e*f + 36*a*b^6*c*d*e))/(5*b^10) + ((a + b*x)^(7/2)*(2*b^6*d^
3 + 168*a^6*f^3 + 6*b^6*c^2*f - 40*a^3*b^3*e^3 + 60*a^2*b^4*d*e^2 - 120*a^3
*b^3*c*f^2 + 60*a^2*b^4*d^2*f + 210*a^4*b^2*d*f^2 + 210*a^4*b^2*e^2*f + 12*
b^6*c*d*e - 24*a*b^5*c*e^2 - 24*a*b^5*d^2*e - 336*a^5*b*e*f^2 + 120*a^2*b^4
*c*e*f - 240*a^3*b^3*d*e*f - 48*a*b^5*c*d*f))/(7*b^10) + (2*(a + b*x)^(3/2)
*(b^2*d + 3*a^2*f - 2*a*b*e)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)^2)/b^10
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.7 $\int \frac{c+dx}{a+bx^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1860, 31, 634, 617, 204, 628}

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3),x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
```

```
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{a+bx^3}\,dx = \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}} + \frac{1}{2}\left(\frac{c}{\sqrt[3]{a}}+\frac{d}{\sqrt[3]{b}}\right)\int \frac{}{a^{2/3}}$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}} + \frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\mathrm{S}}{}$$

$$= -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}} + \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\right.}{6a^{2/3}b^{2/3}}$$

Mathematica [A] time = 0.08, size = 124, normalized size = 0.77

$$\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x^3), x]
```

fricas [C] time = 1.15, size = 1931, normalized size = 11.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d
^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*
d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
```

```
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)
/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^
2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a
*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3 + a*d^3)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a^2*b*d + 2*a*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^
3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c
^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^
3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^
2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3
+ a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*
a^2*b*d + 2*a*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^
2*a*b + 16*c*d)/(a*b)))
```

giac [A] time = 0.17, size = 141, normalized size = 0.88

$$-\frac{\sqrt{3}\left(bc-\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}}-\frac{\left(bc+\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}}-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(d*(-a/b)^(1/3) + c)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
```

maple [A] time = 0.05, size = 186, normalized size = 1.16

$$\frac{\sqrt{3}\, c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b}-\frac{d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b}+\frac{d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*c/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*c/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/3*c/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-1/3*d/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*d/b/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*d*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 1.91, size = 135, normalized size = 0.84

$$\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(d*(a/b)^(1/3) + c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(d*(a/b)^(1/3) - c)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(d*(a/b)^(1/3) - c)*log(x + (a/b)^(1/3
))/(b*(a/b)^(2/3))
```

mupad [B] time = 5.51, size = 127, normalized size = 0.79

$$\sum_{k=1}^{3}\ln\left(b\left(c\,d+d^{2}x+\mathrm{root}\left(27a^{2}b^{2}z^{3}+9abcdz+ad^{3}-bc^{3},z,k\right)^{2}ab9+\mathrm{root}\left(27a^{2}b^{2}z^{3}+9abcdz+ad^{3}-bc^{3},z,k\right)bcx3\right)\right)\mathrm{root}\left(27a^{2}b^{2}z^{3}+9abcdz+ad^{3}-bc^{3},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3),x)
```

```
[Out] symsum(log(b*(c*d + d^2*x + 9*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b
*c^3, z, k)^2*a*b + 3*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z,
 k)*b*c*x))*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z, k), k, 1,
 3)
```

sympy [A] time = 1.19, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^{3}a^{2}b^{2}+9tabcd+ad^{3}-bc^{3},\left(t\mapsto t\log\left(x+\frac{9t^{2}a^{2}bd+3tabc^{2}+2acd^{2}}{ad^{3}+bc^{3}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)))
)
```

3.8 $\int \frac{c+dx}{\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=189

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}$$

Rubi [A] time = 0.14, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5
/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^3\right)^2}\,dx = \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}c-\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b\right.}{18a^{5/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{(2}{}$$

Mathematica [A] time = 0.23, size = 180, normalized size = 0.95

$$\frac{\frac{\left(a^{2/3}d-2\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} - \frac{2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{6ax(c+dx)}{a+bx^3}}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^2,x]
```

```
[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b^(
1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3
)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
18*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x^3)^2, x]
```

fricas [C] time = 1.17, size = 2088, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
 a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*
(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^
5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^
5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3)
- 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/
3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2
) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/
(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*
a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3 + a^2)*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt(1/3)*(a*b*x^
3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) +
 (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^
3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a
^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2)
)^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8
*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b +
32*c*d)/(a^3*b))) + ((a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt
(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)
+ 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (
8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/
(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*
(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1
)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/
2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
```

```
 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))))/(a*b*x^3 + a^2)
```

giac [A] time = 0.18, size = 174, normalized size = 0.92

$$-\frac{\sqrt{3}\left(2\,bc-\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{\left(2\,bc+\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}}+\frac{dx^{2}+cx}{3\left(bx^{3}+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)^(1
/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c*x)/
((b*x^3 + a)*a)
```

maple [A] time = 0.05, size = 238, normalized size = 1.26

$$\frac{d\,x^{2}}{3\left(b\,x^{3}+a\right)a}+\frac{cx}{3\left(b\,x^{3}+a\right)a}+\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*c*x/a/(b*x^3+a)+2/9*c/a/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/9*c/a/b/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9*c/a/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d*x^2/a/(b*x^3+a)-1/9*d/a/b/(a/b)^(
1/3)*ln(x+(a/b)^(1/3))+1/18*d/a/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+1/9*d/a*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.00, size = 169, normalized size = 0.89

$$\frac{dx^{2}+cx}{3\left(abx^{3}+a^{2}\right)}+\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d*x^2 + c*x)/(a*b*x^3 + a^2) + 1/9*sqrt(3)*(d*(a/b)^(1/3) + 2*c)*arcta
n(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) + 1/18*(d*
(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3))
 - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

mupad [B] time = 4.87, size = 169, normalized size = 0.89

$$\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d+d^{2}\,x+\mathrm{root}\left(729\,a^{5}\,b^{2}\,z^{3}+54\,a^{2}\,b\,c\,d\,z-8\,b\,c^{3}+a\,d^{3},z,k\right)^{2}a^{3}\,b\,81+\mathrm{root}\left(729\,a^{5}\,b^{2}\,z^{3}+54\,a^{2}\,b\,c\,d\,z-8\,b\,c^{3}+a\,d^{3},z,k\right)\,a\,b\,c\,x\,18\right)}{a^{2}\,9}\right)\mathrm{root}\left(729\,a^{5}\,b^{2}\,z^{3}+54\,a^{2}\,b\,c\,d\,z-8\,b\,c^{3}+a\,d^{3},z,k\right)\right)+\frac{\frac{d\,x^{2}}{3\,a}+\frac{c\,x}{3\,a}}{b\,x^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z - 8
*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
```

```
8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*b*c
*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) + (c*x)/(3*a))/(a
+ b*x^3)
```

sympy [A] time = 2.15, size = 105, normalized size = 0.56

$$\mathrm{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{cx + dx^2}{3a^2 + 3abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

3.9 $\int \frac{c+dx}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=215

$$-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\right.}{9\sqrt{3}\,a^{8/3}b^{2/3}}$$

Rubi [A] time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] (x*(c + d*x))/(6*a*(a + b*x^3)^2) + (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3))
- ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/
3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^3\right)^3}\,dx = \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} - \frac{\int \frac{-5c-4dx}{\left(a+bx^3\right)^2}\,dx}{6a}$$

$$= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} + \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} + \frac{\int \frac{10c+4dx}{a+bx^3}\,dx}{18a^2}$$

$$= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} + \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} + \frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}c+4\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-10\sqrt[3]{b}c+4\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}\sqrt[3]{b}} + \frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \ldots}{27a^{8/3}\ldots}$$

$$= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} + \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\int \ldots}{54a^{8/3}b\ldots}$$

$$= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} + \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\ldots\right.}{54a\ldots}$$

$$= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} + \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} - \frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\ldots\right.}{27a^{8/3}b^{2/3}\ldots}$$

Mathematica [A] time = 0.25, size = 205, normalized size = 0.95

$$\frac{\frac{\left(2a^{2/3}d-5\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{9a^2x(c+dx)}{\left(a+bx^3\right)^2} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{3ax(5c+4dx)}{a+bx^3}}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] ((9*a^2*x*(c + d*x))/(a + b*x^3)^2 + (3*a*x*(5*c + 4*d*x))/(a + b*x^3) - (2
*Sqrt[3]*a^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(2/3) + (2*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)*Log[a^(1/3)
 + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*c + 2*a^(2/3)*d)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(54*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x^3)^3, x]
```

fricas [C] time = 1.20, size = 2215, normalized size = 10.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(24*b*d*x^5 + 30*b*c*x^4 + 42*a*d*x^2 + 48*a*c*x - 2*(a^2*b^2*x^6 + 2
*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*
b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a
^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3
)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(
-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*
d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125
*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(
1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (
125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 + 40*a*c*d^2 + (125*b*c^3
 + 8*a*d^3)*x) + ((a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(
1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a
^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) + 3*sqrt(1/3)*(a^2*b^2*x
^6 + 2*a^3*b*x^3 + a^4)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)
*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d
^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c
^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*
d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/
(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x + 3/2*sqrt(1/3)*
(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*
a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)
*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b))) + ((a^2*b^2*x^6
+ 2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a
^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*s
qrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)
/(a^8*b^2))^(1/3))) - 3*sqrt(1/3)*(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*sqrt(-(
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((
125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2
*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c
^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)
```

```
^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*
b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(
a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2
+ 2*(125*b*c^3 + 8*a*d^3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) -
20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2)
 + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((
125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2
*a^5*b + 160*c*d)/(a^5*b))))/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)
```

giac [A] time = 0.23, size = 194, normalized size = 0.90

$$-\frac{\sqrt{3}\left(5\,bc-2\,\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{\left(5\,bc+2\,\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{3}}+\frac{4\,bdx^{5}+5\,bcx^{4}+7\,adx^{2}+8\,acx}{18\,\left(bx^{3}+a\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b*c - 2*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*c + 2*(-a*b^2)^(1/3)
*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(2
*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/18*(
4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/((b*x^3 + a)^2*a^2)
```

maple [A] time = 0.06, size = 272, normalized size = 1.27

$$\frac{d\,x^{2}}{6\left(b\,x^{3}+a\right)^{2}a}+\frac{cx}{6\left(b\,x^{3}+a\right)^{2}a}+\frac{2d\,x^{2}}{9\left(b\,x^{3}+a\right)a^{2}}+\frac{5cx}{18\left(b\,x^{3}+a\right)a^{2}}+\frac{5\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{5c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{5c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{2\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*c/a*x/(b*x^3+a)^2+5/18*c/a^2*x/(b*x^3+a)+5/27*c/a^2/b/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))-5/54*c/a^2/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+5/2
7*c/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/6*d
/a*x^2/(b*x^3+a)^2+2/9*d/a^2*x^2/(b*x^3+a)-2/27*d/a^2/b/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))+1/27*d/a^2/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/27*
d/a^2*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 1.96, size = 203, normalized size = 0.94

$$\frac{4\,bdx^{5}+5\,bcx^{4}+7\,adx^{2}+8\,acx}{18\left(a^{2}b^{2}x^{6}+2\,a^{3}bx^{3}+a^{4}\right)}+\frac{\sqrt{3}\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/(a^2*b^2*x^6 + 2*a^3*b*x
^3 + a^4) + 1/27*sqrt(3)*(2*d*(a/b)^(1/3) + 5*c)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) + 1/54*(2*d*(a/b)^(1/3) - 5*c
```

```
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/27*(2*d*(a
/b)^(1/3) - 5*c)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

mupad [B] time = 0.27, size = 206, normalized size = 0.96

$$\frac{\frac{7\,d\,x^2}{18\,a}+\frac{4\,c\,x}{9\,a}+\frac{5\,b\,c\,x^4}{18\,a^2}+\frac{2\,b\,d\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\left(\sum_{k=1}^{3}\ln\left(\frac{b\,\left(10\,c\,d+4\,d^2\,x+\mathrm{root}\left(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k\right)^2\,a^5\,b\,729+\mathrm{root}\left(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k\right)\,a^2\,b\,c\,x\,135\right)}{a^4\,81}\right)\,\mathrm{root}\left(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3)^3,x)
```

```
[Out] ((7*d*x^2)/(18*a) + (4*c*x)/(9*a) + (5*b*c*x^4)/(18*a^2) + (2*b*d*x^5)/(9*a
^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((b*(10*c*d + 4*d^2*x + 729*ro
ot(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z, k)^2*a^5*b
 + 135*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z, k
)*a^2*b*c*x))/(81*a^4))*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^
3 + 8*a*d^3, z, k), k, 1, 3)
```

sympy [A] time = 2.47, size = 146, normalized size = 0.68

$$\mathrm{RootSum}\left(19683t^3a^8b^2+810ta^3bcd+8ad^3-125bc^3,\left(t\mapsto t\log\left(x+\frac{1458t^2a^6bd+675ta^3bc^2+40acd^2}{8ad^3+125bc^3}\right)\right)\right)+\frac{8acx+7adx^2+5bcx^4+4bdx^5}{18a^4+36a^3bx^3+18a^2b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*d*
*2)/(8*a*d**3 + 125*b*c**3)))) + (8*a*c*x + 7*a*d*x**2 + 5*b*c*x**4 + 4*b*d
*x**5)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6)
```

3.10 $\int \frac{c+dx}{(a+bx^3)^4} \, dx$

Optimal. Leaf size=240

$$-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan}{81\sqrt{3}a^{11/3}b}$$

Rubi [A] time = 0.22, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] (x*(c + d*x))/(9*a*(a + b*x^3)^3) + (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2
) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (2*(20*b^(1/3)*c + 7*a^(1/3
)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3
)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243
*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^3\right)^4}\,dx = \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} - \frac{\int \frac{-8c-7dx}{\left(a+bx^3\right)^3}\,dx}{9a}$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{\int \frac{40c+28dx}{\left(a+bx^3\right)^2}\,dx}{54a^2}$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{\int \frac{-80c-28dx}{a+bx^3}\,dx}{162a^3}$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{\int \frac{\sqrt[3]{a}\left(-160\sqrt[3]{b}c-28\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(80\sqrt[3]{b}c-28\sqrt[3]{a}\,d\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{486a^{11/3}\sqrt[3]{b}}$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{11/3}b^{2/3}}$$

Mathematica [A] time = 0.23, size = 229, normalized size = 0.95

$$\frac{\frac{2\left(7a^{2/3}d-20\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{54a^3x(c+dx)}{\left(a+bx^3\right)^3}+\frac{9a^2x(8c+7dx)}{\left(a+bx^3\right)^2}-\frac{4\sqrt{3}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{12ax(10c+7dx)}{a+bx^3}}{486a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] ((54*a^3*x*(c + d*x))/(a + b*x^3)^3 + (9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2
 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3) - (4*Sqrt[3]*a^(1/3)*(20*b^(1/3)*c +
 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(20
```

```
*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-
20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/b^(2/3))/(486*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{\left(a+bx^3\right)^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x^3)^4, x]
```

fricas [C] time = 1.19, size = 2308, normalized size = 9.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^2*d*x^8 + 240*b^2*c*x^7 + 462*a*b*d*x^5 + 624*a*b*c*x^4 + 402*
a^2*d*x^2 + 492*a^2*c*x - 2*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^
6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*
c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*
b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2
))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
- 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 + 7840*a*c*d^2
 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*
x^3 + a^6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) +
1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3))) + 3*sqrt(1/3)*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3
 + a^6)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3
) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^
3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b)))*log(-7/4*(4^(1/3)*(I*s
qrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)
/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 +
 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8
*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) +
1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 + 8*(8000*b*c^3 + 343*a*d^3)*x
+ 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqr
t(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 3
43*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b
 + 8960*c*d)/(a^7*b))) + ((a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
```

```
^(1/3))) - 3*sqrt(1/3)*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)*sq
rt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*
b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^
7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b
^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1
)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2
))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3
)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400
*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 + 8*(8000*b*c^3 + 343*a*d^3)*x - 3/4*sqrt
(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8
000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)
/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^
11*b^2))^(1/3)))*a^8*b*d + 1600*a^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/
(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*
d)/(a^7*b))))/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

giac [A] time = 0.23, size = 218, normalized size = 0.91

$$-\frac{2\sqrt{3}\left(20\,bc-7\,\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}}-\frac{\left(20\,bc+7\,\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}}-\frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{4}}+\frac{28\,b^{2}dx^{8}+40\,b^{2}cx^{7}+77\,abdx^{5}+104\,abcx^{4}+67\,a^{2}dx^{2}+82\,a^{2}cx}{162\,\left(bx^{3}+a\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*sqrt(3)*(20*b*c - 7*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*c + 7*(-a*b^2)^(
1/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 2/2
43*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 +
1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67*a^2*
d*x^2 + 82*a^2*c*x)/((b*x^3 + a)^3*a^3)
```

maple [A] time = 0.05, size = 306, normalized size = 1.28

$$\frac{d\,x^{2}}{9\left(b\,x^{3}+a\right)^{3}a}+\frac{cx}{9\left(b\,x^{3}+a\right)^{3}a}+\frac{7d\,x^{2}}{54\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{4cx}{27\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{14d\,x^{2}}{81\left(b\,x^{3}+a\right)a^{3}}+\frac{20cx}{81\left(b\,x^{3}+a\right)a^{3}}+\frac{40\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}b}+\frac{40c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}b}-\frac{20c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}b}+\frac{14\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}b}-\frac{14d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}b}+\frac{7d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] 1/9*c/a*x/(b*x^3+a)^3+4/27*c/a^2*x/(b*x^3+a)^2+20/81*c/a^3*x/(b*x^3+a)+40/2
43*c/a^3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-20/243*c/a^3/b/(a/b)^(2/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+40/243*c/a^3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))+1/9*d/a*x^2/(b*x^3+a)^3+7/54*d/a^2*x^2/(b*x^3+a)
^2+14/81*d/a^3*x^2/(b*x^3+a)-14/243*d/a^3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7
/243*d/a^3/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+14/243*d/a^3*3^(
1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.62, size = 238, normalized size = 0.99

$$\frac{28\,b^{2}dx^{8}+40\,b^{2}cx^{7}+77\,abdx^{5}+104\,abcx^{4}+67\,a^{2}dx^{2}+82\,a^{2}cx}{162\left(a^{3}b^{3}x^{9}+3\,a^{4}b^{2}x^{6}+3\,a^{5}bx^{3}+a^{6}\right)}+\frac{2\sqrt{3}\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^{3}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^{3}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^{3}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67*a^2*
d*x^2 + 82*a^2*c*x)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6) + 2/2
43*sqrt(3)*(7*d*(a/b)^(1/3) + 20*c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/243*(7*d*(a/b)^(1/3) - 20*c)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 2/243*(7*d*(a/b)^(1/3)
- 20*c)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

mupad [B] time = 4.93, size = 241, normalized size = 1.00

$$\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(560\,c\,d+196\,d^2\,x+\mathrm{root}\left(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right)^2\,a^7\,b\,59049+\mathrm{root}\left(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right)\,a^3\,b\,c\,x\,9720\right)}{a^6\,6561}\right)\mathrm{root}\left(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right)\right)+\frac{\frac{67\,d\,x^2}{162\,a}+\frac{41\,c\,x}{81\,a}+\frac{20\,b^2\,c\,x^7}{81\,a^3}+\frac{14\,b^2\,d\,x^8}{81\,a^3}+\frac{52\,b\,c\,x^4}{81\,a^2}+\frac{77\,b\,d\,x^5}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3)^4,x)
```

```
[Out] symsum(log((b*(560*c*d + 196*d^2*x + 59049*root(14348907*a^11*b^2*z^3 + 408
240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)^2*a^7*b + 9720*root(14348
907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)*a^3
*b*c*x))/(6561*a^6))*root(14348907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 6400
0*b*c^3 + 2744*a*d^3, z, k), k, 1, 3) + ((67*d*x^2)/(162*a) + (41*c*x)/(81*
a) + (20*b^2*c*x^7)/(81*a^3) + (14*b^2*d*x^8)/(81*a^3) + (52*b*c*x^4)/(81*a
^2) + (77*b*d*x^5)/(162*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6)
```

sympy [A] time = 3.64, size = 185, normalized size = 0.77

$$\operatorname{RootSum}\left(14348907t^3a^{11}b^2+408240ta^4bcd+2744ad^3-64000bc^3,\left(t\mapsto t\log\left(x+\frac{413343t^2a^8bd+194400ta^4bc^2+7840acd^2}{1372ad^3+32000bc^3}\right)\right)\right)+\frac{82a^2cx+67a^2dx^2+104abcx^4+77abdx^5+40b^2cx^7+28b^2dx^8}{162a^6+486a^5bx^3+486a^4b^2x^6+162a^3b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 64
000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4*b
*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (82*a**2*c*x + 67*
a**2*d*x**2 + 104*a*b*c*x**4 + 77*a*b*d*x**5 + 40*b**2*c*x**7 + 28*b**2*d*x
**8)/(162*a**6 + 486*a**5*b*x**3 + 486*a**4*b**2*x**6 + 162*a**3*b**3*x**9)
```

3.11 $\int \frac{a+bx}{d+ex^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1860, 31, 634, 617, 204, 628}

$$-\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(d + e*x^3),x]
```

```
[Out] -(((b*d^(1/3) + a*e^(1/3))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))
])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) - a*e^(1/3))*Log[d^(1/3) + e^(1
/3)*x])/(3*d^(2/3)*e^(2/3)) - ((a - (b*d^(1/3))/e^(1/3))*Log[d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
```

```
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx}{d+ex^3}\,dx = \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3d^{2/3}} + \frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}+2a\sqrt[3]{e}\right)+\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}\sqrt[3]{e}}$$

$$= \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{1}{2}\left(\frac{a}{\sqrt[3]{d}}+\frac{b}{\sqrt[3]{e}}\right)\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\int -}{6d^{2/}}$$

$$= \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\text{Sub}}{}$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} + \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(d^{2/3}-\sqrt[3]{d}\right.}{6d^{2/3}e^{2/3}}$$

Mathematica [A] time = 0.07, size = 125, normalized size = 0.78

$$\frac{-\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\right)-2\sqrt{3}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{6d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(d + e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt
[3]] - (b*d^(1/3) - a*e^(1/3))*(2*Log[d^(1/3) + e^(1/3)*x] - Log[d^(2/3) -
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{d+ex^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(d + e*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(d + e*x^3), x]
```

fricas [C] time = 1.19, size = 1961, normalized size = 12.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3
*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^
3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^
```

```
3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(
2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)
/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d + (b^3*d + a^3*e)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2
))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^
2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^
3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^
(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2)
- (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1
/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a
^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d + 2*(b^3*d + a^3*e)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 2*a^2*d*e
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*
e))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*
d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3
*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*
e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3
*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2
*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^
2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d + 2*(b^3*d
+ a^3*e)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(
d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3)
 + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*
b*d^2*e + 2*a^2*d*e)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(
d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3)
 + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^
2*d*e + 16*a*b)/(d*e)))
```

giac [A] time = 0.17, size = 132, normalized size = 0.82

$$-\frac{\sqrt{3}\left(ae-\left(-de^{2}\right)^{\frac{1}{3}}b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)}{3\left(-de^{2}\right)^{\frac{2}{3}}}-\frac{\left(ae+\left(-de^{2}\right)^{\frac{1}{3}}b\right)\log\left(x^{2}+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\left(-de^{2}\right)^{\frac{2}{3}}}-\frac{\left(-de^{(-1)}\right)^{\frac{1}{3}}\left(\left(-de^{(-1)}\right)^{\frac{1}{3}}b+a\right)\log\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(a*e - (-d*e^2)^(1/3)*b)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))
^(1/3))/(-d*e^(-1))^(1/3))/(-d*e^2)^(2/3) - 1/6*(a*e + (-d*e^2)^(1/3)*b)*lo
g(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3) - 1/3*(-d*e
^(-1))^(1/3)*((-d*e^(-1))^(1/3)*b + a)*log(abs(x - (-d*e^(-1))^(1/3)))/d
```

maple [A] time = 0.05, size = 186, normalized size = 1.16

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e}+\frac{a\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e}-\frac{a\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{2}{3}}e}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}e}-\frac{b\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}e}+\frac{b\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{1}{3}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(e*x^3+d),x)
```

```
[Out] 1/3*a/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))-1/6*a/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3
)*x+(d/e)^(2/3))+1/3*a/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1
/3)*x-1))-1/3*b/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6*b/e/(d/e)^(1/3)*ln(x^2-
(d/e)^(1/3)*x+(d/e)^(2/3))+1/3*b*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(
2/(d/e)^(1/3)*x-1))
```

maxima [A] time = 2.51, size = 135, normalized size = 0.84

$$\frac{\sqrt{3}\left(b\left(\frac{d}{e}\right)^{\frac{1}{3}}+a\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\left(b\left(\frac{d}{e}\right)^{\frac{1}{3}}-a\right)\log\left(x^2-x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\left(b\left(\frac{d}{e}\right)^{\frac{1}{3}}-a\right)\log\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b*(d/e)^(1/3) + a)*arctan(1/3*sqrt(3)*(2*x - (d/e)^(1/3))/(d/e
)^(1/3))/(e*(d/e)^(2/3)) + 1/6*(b*(d/e)^(1/3) - a)*log(x^2 - x*(d/e)^(1/3)
+ (d/e)^(2/3))/(e*(d/e)^(2/3)) - 1/3*(b*(d/e)^(1/3) - a)*log(x + (d/e)^(1/3
))/(e*(d/e)^(2/3))
```

mupad [B] time = 4.85, size = 127, normalized size = 0.79

$$\sum_{k=1}^{3}\ln\left(e\left(a\,b+b^2\,x+\mathrm{root}\left(27\,d^2\,e^2\,z^3+9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)^2\,d\,e\,9+\mathrm{root}\left(27\,d^2\,e^2\,z^3+9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)\,a\,e\,x\,3\right)\right)\,\mathrm{root}\left(27\,d^2\,e^2\,z^3+9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(d + e*x^3),x)
```

```
[Out] symsum(log(e*(a*b + b^2*x + 9*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a
^3*e, z, k)^2*d*e + 3*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a^3*e, z,
 k)*a*e*x))*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a^3*e, z, k), k, 1,
 3)
```

sympy [A] time = 1.43, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3d^2e^2+9tabde-a^3e+b^3d,\left(t\mapsto t\log\left(x+\frac{9t^2bd^2e+3ta^2de+2ab^2d}{a^3e+b^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x**3+d),x)
```

```
[Out] RootSum(27*_t**3*d**2*e**2 + 9*_t*a*b*d*e - a**3*e + b**3*d, Lambda(_t, _t*
log(x + (9*_t**2*b*d**2*e + 3*_t*a**2*d*e + 2*a*b**2*d)/(a**3*e + b**3*d)))
)
```

3.12 $\int \frac{a+bx}{d-ex^3}\,dx$

Optimal. Leaf size=161

$$\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

Rubi [A] time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1861, 31, 634, 617, 204, 628}

$$\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(d - e*x^3),x]
```

```
[Out] -(((b*d^(1/3) - a*e^(1/3))*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))
])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) + a*e^(1/3))*Log[d^(1/3) - e^(1
/3)*x])/(3*d^(2/3)*e^(2/3)) + ((b*d^(1/3) + a*e^(1/3))*Log[d^(2/3) + d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/
```

```
(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s
*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x]
&& NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx}{d-ex^3}\,dx = \frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}x}\,dx}{3d^{2/3}} - \frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}-2a\sqrt[3]{e}\right)-\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\sqrt[3]{e}x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}\sqrt[3]{e}}$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} - \frac{1}{2}\left(-\frac{a}{\sqrt[3]{d}}+\frac{b}{\sqrt[3]{e}}\right)\int \frac{1}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{\right.}}{}$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)}{}$$

$$= -\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} - \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\right.}{6d^{2/3}e^{2/3}}$$

Mathematica [A] time = 0.06, size = 125, normalized size = 0.78

$$\frac{-\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\left(2\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)-\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\right)-2\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{6d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(d - e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b*d^(1/3) - a*e^(1/3))*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt
[3]] - (b*d^(1/3) + a*e^(1/3))*(2*Log[d^(1/3) - e^(1/3)*x] - Log[d^(2/3) +
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{d-ex^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(d - e*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(d - e*x^3), x]
```

fricas [C] time = 1.18, size = 1905, normalized size = 11.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a
^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(
d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/36*(9*(I*sqrt(3) +
 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3
) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*
```

```
d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b^
3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sq
rt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^
2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d - (b^3*d - a^3*e)*x) + 1/36*(9*(I*sqrt(
3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^
(1/3) + 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e
^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1
/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d
*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^
2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(9*(I*sqrt(3) +
1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d
 - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b^3
*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqr
t(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2
*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d - 2*(b^3*d - a^3*e)*x + 1/12*sqrt(1/3)*(
(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/
(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^
2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 6*a^2*d*e)*sqrt(-((9
*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d
^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2)
 - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e))) + 1/36*
(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/
(d^2*e^2))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^
3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) +
1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^
(1/3)))^2*d*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d +
 a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(9*(I
*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*
e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) -
1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3) + 1)*
(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) +
a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d -
a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d - 2*(b^3*d - a^3*e)*x - 1/12*
sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*
d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3
*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 6*a^2*d*e
)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d
- a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e
)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e
)))
```

giac [A] time = 0.18, size = 115, normalized size = 0.71

$$-\frac{\sqrt{3}\left(bd^{\frac{2}{3}}e^{\frac{4}{3}}-ad^{\frac{1}{3}}e^{\frac{5}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+2x\right)e^{\frac{1}{3}}}{3d^{\frac{1}{3}}}\right)e^{(-2)}}{3d}-\frac{\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+a\right)e^{\left(-\frac{1}{3}\right)}\log\left(\left|-d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right|\right)}{3d^{\frac{2}{3}}}+\frac{\left(bd^{\frac{2}{3}}e^{\frac{4}{3}}+ad^{\frac{1}{3}}e^{\frac{5}{3}}\right)e^{(-2)}\log\left(d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^2+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*d^(2/3)*e^(4/3) - a*d^(1/3)*e^(5/3))*arctan(1/3*sqrt(3)*(d^
(1/3)*e^(-1/3) + 2*x)*e^(1/3)/d^(1/3))*e^(-2)/d - 1/3*(b*d^(1/3)*e^(-1/3) +
 a)*e^(-1/3)*log(abs(-d^(1/3)*e^(-1/3) + x))/d^(2/3) + 1/6*(b*d^(2/3)*e^(4/
3) + a*d^(1/3)*e^(5/3))*e^(-2)*log(d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2
/3))/d
```

maple [A] time = 0.05, size = 188, normalized size = 1.17

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e}-\frac{a\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e}+\frac{a\ln\left(x^{2}+\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{2}{3}}e}-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}e}-\frac{b\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}e}+\frac{b\ln\left(x^{2}+\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{1}{3}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-e*x^3+d),x)
```

```
[Out] -1/3*a/e/(d/e)^(2/3)*ln(x-(d/e)^(1/3))+1/6*a/e/(d/e)^(2/3)*ln(x^2+(d/e)^(1/
3)*x+(d/e)^(2/3))+1/3*a/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(
1/3)*x+1))-1/3*b/e/(d/e)^(1/3)*ln(x-(d/e)^(1/3))+1/6*b/e/(d/e)^(1/3)*ln(x^2
+(d/e)^(1/3)*x+(d/e)^(2/3))-1/3*b*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*
(2/(d/e)^(1/3)*x+1))
```

maxima [A] time = 2.68, size = 132, normalized size = 0.82

$$-\frac{\sqrt{3}\left(b\left(\frac{d}{e}\right)^{\frac{1}{3}}-a\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\left(b\left(\frac{d}{e}\right)^{\frac{1}{3}}+a\right)\log\left(x^{2}+x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\left(b\left(\frac{d}{e}\right)^{\frac{1}{3}}+a\right)\log\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\,e\left(\frac{d}{e}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b*(d/e)^(1/3) - a)*arctan(1/3*sqrt(3)*(2*x + (d/e)^(1/3))/(d/
e)^(1/3))/(e*(d/e)^(2/3)) + 1/6*(b*(d/e)^(1/3) + a)*log(x^2 + x*(d/e)^(1/3)
 + (d/e)^(2/3))/(e*(d/e)^(2/3)) - 1/3*(b*(d/e)^(1/3) + a)*log(x - (d/e)^(1/
3))/(e*(d/e)^(2/3))
```

mupad [B] time = 0.21, size = 124, normalized size = 0.77

$$\sum_{k=1}^{3}\ln\left(e\,\left(a\,b+b^2\,x-\mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)^2\,d\,e\,9-\mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)\,a\,e\,x\,3\right)\right)\,\mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(d - e*x^3),x)
```

```
[Out] symsum(log(e*(a*b + b^2*x - 9*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a
^3*e, z, k)^2*d*e - 3*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a^3*e, z,
 k)*a*e*x))*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a^3*e, z, k), k, 1,
 3)
```

sympy [A] time = 1.49, size = 78, normalized size = 0.48

$$-\operatorname{RootSum}\left(27t^{3}d^{2}e^{2}-9tabde-a^{3}e-b^{3}d,\left(t\mapsto t\log\left(x+\frac{9t^{2}bd^{2}e-3ta^{2}de-2ab^{2}d}{a^{3}e-b^{3}d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-e*x**3+d),x)
```

```
[Out] -RootSum(27*_t**3*d**2*e**2 - 9*_t*a*b*d*e - a**3*e - b**3*d, Lambda(_t, _t
*log(x + (9*_t**2*b*d**2*e - 3*_t*a**2*d*e - 2*a*b**2*d)/(a**3*e - b**3*d))
))
```

3.13 $\int \frac{1+x}{1+x^3} \, dx$

Optimal. Leaf size=19

$$-\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1586, 618, 204}

$$-\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(1 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x}{1+x^3} \, dx &= \int \frac{1}{1-x+x^2} \, dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, -1+2x\right)\right) \\ &= -\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(1 + x^3),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x)/(1 + x^3),x]
[Out] IntegrateAlgebraic[(1 + x)/(1 + x^3), x]
```

fricas [A] time = 0.40, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+1),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

giac [A] time = 0.19, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+1),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

maple [A] time = 0.04, size = 17, normalized size = 0.89

$$\frac{2\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^3+1),x)
[Out] 2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.48, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+1),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

mupad [B] time = 4.70, size = 16, normalized size = 0.84

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2x-1)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^3 + 1),x)
```

```
[Out] (2*3^(1/2)*atan((3^(1/2)*(2*x - 1))/3))/3
```

sympy **[A]** time = 0.37, size = 26, normalized size = 1.37

$$\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**3+1),x)
```

```
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.14 $\int \frac{1-x}{1-x^3} \, dx$

Optimal. Leaf size=19

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1586, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x}{1-x^3} \, dx &= \int \frac{1}{1+x+x^2} \, dx \\
&= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, 1+2x\right)\right) \\
&= \frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 - x^3),x]
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x}{1-x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x)/(1 - x^3),x]
[Out] IntegrateAlgebraic[(1 - x)/(1 - x^3), x]
```

fricas [A] time = 0.41, size = 16, normalized size = 0.84

$$\frac{2}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

giac [A] time = 0.17, size = 16, normalized size = 0.84

$$\frac{2}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

maple [A] time = 0.04, size = 17, normalized size = 0.89

$$\frac{2\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(-x^3+1),x)
[Out] 2/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.43, size = 16, normalized size = 0.84

$$\frac{2}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

mupad [B] time = 4.67, size = 16, normalized size = 0.84

$$\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,x+1)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^3 - 1),x)
```

```
[Out] (2*3^(1/2)*atan((3^(1/2)*(2*x + 1))/3))/3
```

sympy [**A**] time = 0.22, size = 26, normalized size = 1.37

$$\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x**3+1),x)
```

```
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

3.15 $\int \frac{1+x}{1-x^3} \, dx$

Optimal. Leaf size=22

$$\frac{1}{3} \log \left(x^2+x+1\right)-\frac{2}{3} \log (1-x)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1861, 31, 628}

$$\frac{1}{3} \log \left(x^2+x+1\right)-\frac{2}{3} \log (1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(1 - x^3),x]
```

```
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/
(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s
*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x]
&& NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{1-x^3} \, dx &= -\left(\frac{1}{3} \int \frac{-1-2x}{1+x+x^2} \, dx\right)+\frac{2}{3} \int \frac{1}{1-x} \, dx \\
&= -\frac{2}{3} \log (1-x)+\frac{1}{3} \log \left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{1}{3} \log \left(x^2+x+1\right)-\frac{2}{3} \log (1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(1 - x^3),x]
```

```
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x}{1-x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x)/(1 - x^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + x)/(1 - x^3), x]
```

fricas [A] time = 0.40, size = 16, normalized size = 0.73

$$\frac{1}{3}\,\log\left(x^2+x+1\right)-\frac{2}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

giac [A] time = 0.16, size = 17, normalized size = 0.77

$$\frac{1}{3}\,\log\left(x^2+x+1\right)-\frac{2}{3}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(abs(x - 1))
```

maple [A] time = 0.05, size = 17, normalized size = 0.77

$$-\frac{2\ln\left(x-1\right)}{3}+\frac{\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(-x^3+1),x)
```

```
[Out] -2/3*ln(x-1)+1/3*ln(x^2+x+1)
```

maxima [A] time = 2.44, size = 16, normalized size = 0.73

$$\frac{1}{3}\,\log\left(x^2+x+1\right)-\frac{2}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

mupad [B] time = 0.06, size = 16, normalized size = 0.73

$$\frac{\ln\left(x^2+x+1\right)}{3}-\frac{2\,\ln\left(x-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 1)/(x^3 - 1),x)
```

```
[Out] log(x + x^2 + 1)/3 - (2*log(x - 1))/3
```

sympy [A] time = 0.25, size = 17, normalized size = 0.77

$$-\frac{2\log\left(x-1\right)}{3}+\frac{\log\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**3+1),x)
```

```
[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3
```

3.16 $\int \frac{1-x}{1+x^3} \, dx$

Optimal. Leaf size=22

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1860, 31, 628}

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 + x^3),x]
```

```
[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{1+x^3}\,dx &= \frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx\\ &= \frac{2}{3}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 + x^3),x]
```

```
[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x)/(1 + x^3),x]
[Out] IntegrateAlgebraic[(1 - x)/(1 + x^3), x]
```

fricas [A] time = 0.40, size = 18, normalized size = 0.82

$$-\frac{1}{3}\log\left(x^2 - x + 1\right) + \frac{2}{3}\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^3+1),x, algorithm="fricas")
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

giac [A] time = 0.17, size = 19, normalized size = 0.86

$$-\frac{1}{3}\log\left(x^2 - x + 1\right) + \frac{2}{3}\log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^3+1),x, algorithm="giac")
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(abs(x + 1))
```

maple [A] time = 0.04, size = 19, normalized size = 0.86

$$\frac{2\ln\left(x + 1\right)}{3} - \frac{\ln\left(x^2 - x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x^3+1),x)
[Out] 2/3*ln(x+1)-1/3*ln(x^2-x+1)
```

maxima [A] time = 2.47, size = 18, normalized size = 0.82

$$-\frac{1}{3}\log\left(x^2 - x + 1\right) + \frac{2}{3}\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^3+1),x, algorithm="maxima")
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

mupad [B] time = 0.11, size = 18, normalized size = 0.82

$$\frac{2\ \ln\left(x + 1\right)}{3} - \frac{\ln\left(x^2 - x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x^3 + 1),x)
[Out] (2*log(x + 1))/3 - log(x^2 - x + 1)/3
```

sympy [A] time = 0.23, size = 17, normalized size = 0.77

$$\frac{2\log\left(x + 1\right)}{3} - \frac{\log\left(x^2 - x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**3+1),x)
```

```
[Out] 2*log(x + 1)/3 - log(x**2 - x + 1)/3
```

3.17 $\int \frac{3-x}{1-x^3} \, dx$

Optimal. Leaf size=41

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1861, 31, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x)/(1 - x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]
/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/
(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s
*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x]
&& NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x}{1-x^3}\,dx &= -\left(\frac{1}{3}\int \frac{-7-2x}{1+x+x^2}\,dx\right)+\frac{2}{3}\int \frac{1}{1-x}\,dx\\ &= -\frac{2}{3}\log(1-x)+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx+2\int \frac{1}{1+x+x^2}\,dx\\ &= -\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)-4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= \frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x)/(1 - x^3),x]
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]
/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-x}{1-x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x)/(1 - x^3),x]
[Out] IntegrateAlgebraic[(3 - x)/(1 - x^3), x]
```

fricas [A] time = 0.41, size = 32, normalized size = 0.78

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
x - 1)
```

giac [A] time = 0.15, size = 33, normalized size = 0.80

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="giac")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
abs(x - 1))
```

maple [A] time = 0.05, size = 33, normalized size = 0.80

$$\frac{4\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}-\frac{2\ln(x-1)}{3}+\frac{\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)/(-x^3+1),x)
```

```
[Out] -2/3*ln(x-1)+1/3*ln(x^2+x+1)+4/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.43, size = 32, normalized size = 0.78

$$\frac{4}{3}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
x - 1)
```

mupad [B] time = 0.14, size = 46, normalized size = 1.12

$$-\frac{2\ \ln(x-1)}{3}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\ 2\mathrm{i}}{3}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\ 2\mathrm{i}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3)/(x^3 - 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*2i)/3 + 1/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*2i)/3 - 1/3) - (2*log(x - 1))/3
```

sympy [A] time = 0.47, size = 44, normalized size = 1.07

$$-\frac{2\log(x-1)}{3}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{4\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x**3+1),x)
```

```
[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3 + 4*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)/3
```

3.18 $\int \frac{c+dx}{c^3+d^3x^3}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1586, 617, 204}

$$-\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(c^3 + d^3*x^3),x]
```

```
[Out] (-2*ArcTan[(c - 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{c^3+d^3x^3}\,dx &= \int \frac{1}{c^2-cdx+d^2x^2}\,dx \\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2dx}{c}\right)}{cd} \\
&= -\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.07

$$\frac{2\tan^{-1}\left(\frac{2dx-c}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(c^3 + d^3*x^3),x]
```

```
[Out] (2*ArcTan[(-c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx}{c^3 + d^3x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(c^3 + d^3*x^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(c^3 + d^3*x^3), x]
```

fricas [A] time = 0.41, size = 28, normalized size = 0.97

$$\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

giac [A] time = 0.21, size = 28, normalized size = 0.97

$$\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

maple [A] time = 0.07, size = 35, normalized size = 1.21

$$\frac{2\sqrt{3}\;\arctan\left(\frac{\left(2d^2x-cd\right)\sqrt{3}}{3cd}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(d^3*x^3+c^3),x)
```

```
[Out] 2/3*3^(1/2)/c/d*arctan(1/3*(2*d^2*x-c*d)*3^(1/2)/c/d)
```

maxima [A] time = 2.90, size = 34, normalized size = 1.17

$$\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,d^2x-cd\right)}{3\,cd}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x - c*d)/(c*d))/(c*d)
```

mupad [B] time = 0.05, size = 28, normalized size = 0.97

$$-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,d\,x}{3\,c}\right)}{3\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(c^3 + d^3*x^3),x)
```

```
[Out] -(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*d*x)/(3*c)))/(3*c*d)
```

sympy [C] time = 0.40, size = 54, normalized size = 1.86

$$\frac{-\frac{\sqrt{3}\,i\log\left(x+\frac{-c-\sqrt{3}\,ic}{2d}\right)}{3}+\frac{\sqrt{3}\,i\log\left(x+\frac{-c+\sqrt{3}\,ic}{2d}\right)}{3}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d**3*x**3+c**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (-c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (-c + s
qrt(3)*I*c)/(2*d))/3)/(c*d)
```

3.19 $\int \frac{c-dx}{c^3-d^3x^3}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1586, 617, 204}

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x)/(c^3 - d^3*x^3),x]
```

```
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c-dx}{c^3-d^3x^3}\,dx &= \int \frac{1}{c^2+cdx+d^2x^2}\,dx\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2dx}{c}\right)}{cd}\\
&= \frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x)/(c^3 - d^3*x^3),x]
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c - d*x)/(c^3 - d^3*x^3),x]
[Out] IntegrateAlgebraic[(c - d*x)/(c^3 - d^3*x^3), x]
```

fricas [A] time = 0.40, size = 26, normalized size = 0.90

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx+c)}{3\,c}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

giac [A] time = 0.17, size = 26, normalized size = 0.90

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx+c)}{3\,c}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

maple [A] time = 0.04, size = 34, normalized size = 1.17

$$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2d^2x+cd\right)\sqrt{3}}{3cd}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x+c)/(-d^3*x^3+c^3),x)
[Out] 2/3*3^(1/2)/c/d*arctan(1/3*(2*d^2*x+c*d)*3^(1/2)/c/d)
```

maxima [A] time = 2.98, size = 33, normalized size = 1.14

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,d^2x+cd\right)}{3\,cd}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x + c*d)/(c*d))/(c*d)
```

mupad [B] time = 0.04, size = 28, normalized size = 0.97

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,d\,x}{3\,c}\right)}{3\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x)/(c^3 - d^3*x^3),x)
```

```
[Out] (2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*d*x)/(3*c)))/(3*c*d)
```

sympy [C] time = 0.47, size = 53, normalized size = 1.83

$$\frac{-\frac{\sqrt{3}\,i\log\left(x+\frac{c-\sqrt{3}\,ic}{2d}\right)}{3}+\frac{\sqrt{3}\,i\log\left(x+\frac{c+\sqrt{3}\,ic}{2d}\right)}{3}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d**3*x**3+c**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (c + sqr
t(3)*I*c)/(2*d))/3)/(c*d)
```

3.20 $\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}\,B+b^{2/3}Bx}{a+bx^3}\,dx$

Optimal. Leaf size=39

$$-\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1586, 617, 204}

$$-\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (-2*B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}\,B+b^{2/3}Bx}{a+bx^3}\,dx &= \int \frac{1}{\frac{a^{2/3}}{\sqrt[3]{b}\,B}-\frac{\sqrt[3]{a}\,x}{B}+\frac{\sqrt[3]{b}\,x^2}{B}}\,dx\\
&= \frac{(2B)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}\\
&= -\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.90

$$-\frac{2B\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
[Out] (-2*B*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
[Out] IntegrateAlgebraic[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3), x]
```

fricas [A] time = 0.45, size = 107, normalized size = 2.74

$$\left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3-3\,a^{\frac{2}{3}}b^{\frac{1}{3}}x+3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}b^{\frac{2}{3}}x^2+ab^{\frac{1}{3}}x-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-a}{bx^3+a}\right),\ \frac{2\,\sqrt{\frac{1}{3}}B\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,b^{\frac{1}{3}}x-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="fricas")
[Out] [sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqrt(1
/3)*(2*a^(2/3)*b^(2/3)*x^2 + a*b^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3)) - a)/(
b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*b^(1/3)*x - a^(1/3))/a^(1/3)
)/a^(1/3)]
```

giac [A] time = 0.22, size = 48, normalized size = 1.23

$$\frac{2\,\sqrt{3}Bb^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,b^{\frac{2}{3}}x-a^{\frac{1}{3}}b^{\frac{1}{3}}\right)}{3\,\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3\,\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="giac")
[Out] 2/3*sqrt(3)*B*b^(1/3)*arctan(1/3*sqrt(3)*(2*b^(2/3)*x - a^(1/3)*b^(1/3))/sq
rt(a^(2/3)*b^(2/3)))/sqrt(a^(2/3)*b^(2/3))
```

maple [B] time = 0.06, size = 195, normalized size = 5.00

$$\frac{\sqrt{3}\,B a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{\frac{2}{3}}}+\frac{B a^{\frac{1}{3}}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{\frac{2}{3}}}-\frac{B a^{\frac{1}{3}}\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{\frac{2}{3}}}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{\frac{1}{3}}}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{\frac{1}{3}}}+\frac{B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x)
```

```
[Out] 1/3*B/b^(2/3)*a^(1/3)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*B/b^(2/3)*a^(1/3)/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*B/b^(2/3)*a^(1/3)/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*B/b^(1/3)/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))+1/6*B/b^(1/3)/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+1/3*B/b^(1/3)*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))
```

maxima [B] time = 2.98, size = 163, normalized size = 4.18

$$\frac{\sqrt{3}\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*b^(2/3)*(a/b)^(1/3) + B*a^(1/3)*b^(1/3))*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(B*b^(2/3)*(a/b)^(1/
3) - B*a^(1/3)*b^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/
3)) - 1/3*(B*b^(2/3)*(a/b)^(1/3) - B*a^(1/3)*b^(1/3))*log(x + (a/b)^(1/3))/
(b*(a/b)^(2/3))
```

mupad [B] time = 4.82, size = 49, normalized size = 1.26

$$\frac{2\sqrt{3}\,B\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{b}}{3\sqrt{-b}}-\frac{2\sqrt{3}\,b^{5/6}\,x}{3\,a^{1/3}\sqrt{-b}}\right)}{3\,a^{1/3}\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a^(1/3)*b^(1/3) + B*b^(2/3)*x)/(a + b*x^3),x)
```

```
[Out] (2*3^(1/2)*B*b^(1/2)*atanh((3^(1/2)*b^(1/2))/(3*(-b)^(1/2)) - (2*3^(1/2)*b^
(5/6)*x)/(3*a^(1/3)*(-b)^(1/2))))/(3*a^(1/3)*(-b)^(1/2))
```

sympy [C] time = 0.59, size = 88, normalized size = 2.26

$$\frac{B\left(-\frac{\sqrt{3}\,i\log\left(x+\frac{-B\sqrt[3]{a}-\sqrt{3}\,iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}+\frac{\sqrt{3}\,i\log\left(x+\frac{-B\sqrt[3]{a}+\sqrt{3}\,iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}\right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*b**(1/3)*B+b**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] B*(-sqrt(3)*I*log(x + (-B*a**(1/3) - sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))/
3 + sqrt(3)*I*log(x + (-B*a**(1/3) + sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))/
3)/a**(1/3)
```

3.21 $\int \frac{\sqrt[3]{a}\,\sqrt[3]{-b}\,B-(-b)^{2/3}Bx}{a+bx^3}\,dx$

Optimal. Leaf size=41

$$\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1586, 617, 204}

$$\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]

[Out] (2*B*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a}\,\sqrt[3]{-b}\,B-(-b)^{2/3}Bx}{a+bx^3}\,dx &= \int \frac{1}{-\frac{a^{2/3}(-b)^{2/3}}{bB}+\frac{\sqrt[3]{a}\,x}{B}+\frac{\sqrt[3]{-b}\,x^2}{B}}\,dx\\
&= -\frac{(2B)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{-b}\,x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}\\
&= \frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}}
\end{aligned}$$

Mathematica [B] time = 0.07, size = 129, normalized size = 3.15

$$\frac{\sqrt[3]{-b}B\left(\left(\sqrt[3]{-b}+\sqrt[3]{b}\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)+2\sqrt{3}\left(\sqrt[3]{-b}-\sqrt[3]{b}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] ((-b)^(1/3)*B*(2*Sqrt[3]*((-b)^(1/3) - b^(1/3))*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + ((-b)^(1/3) + b^(1/3))*(2*Log[a^(1/3) + b^(1/3)*x] - Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])))/(6*a^(1/3)*b^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3), x]
```

fricas [A] time = 0.45, size = 114, normalized size = 2.78

$$\left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^3+3a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x-3\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x^2-a(-b)^{\frac{1}{3}}x-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-a}{bx^3+a}\right),\ \frac{2\sqrt{\frac{1}{3}}B\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(-b)^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="fri
cas")
```

```
[Out] [sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x - 3*sqr
t(1/3)*(2*a^(2/3)*(-b)^(2/3)*x^2 - a*(-b)^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3
)) - a)/(b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*(-b)^(1/3)*x + a^(1
/3))/a^(1/3))/a^(1/3)]
```

giac [A] time = 0.21, size = 58, normalized size = 1.41

$$\frac{2\sqrt{3}Bb\arctan\left(-\frac{\sqrt{3}\left(2(-b)^{\frac{2}{3}}x+a^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}\,(-b)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="gia
c")
```

```
[Out] 2/3*sqrt(3)*B*b*arctan(-1/3*sqrt(3)*(2*(-b)^(2/3)*x + a^(1/3)*(-b)^(1/3))/s
qrt(a^(2/3)*(-b)^(2/3)))/(sqrt(a^(2/3)*(-b)^(2/3))*(-b)^(2/3))
```

maple [B] time = 0.06, size = 228, normalized size = 5.56

$$\frac{(-1)^{\frac{1}{3}}\sqrt{3}\,B\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{\frac{2}{3}}}+\frac{(-1)^{\frac{1}{3}}B\,a^{\frac{1}{3}}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{\frac{2}{3}}}-\frac{(-1)^{\frac{1}{3}}B\,a^{\frac{1}{3}}\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{\frac{2}{3}}}-\frac{(-1)^{\frac{1}{3}}(-b)^{\frac{1}{3}}\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{\frac{2}{3}}}+\frac{(-1)^{\frac{1}{3}}(-b)^{\frac{1}{3}}B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{\frac{2}{3}}}-\frac{(-1)^{\frac{1}{3}}(-b)^{\frac{1}{3}}B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x)
```

```
[Out] 1/3*B/b^(2/3)*(-1)^(1/3)*a^(1/3)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*B/b^(2/3
)*(-1)^(1/3)*a^(1/3)/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*B/b^
(2/3)*(-1)^(1/3)*a^(1/3)/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))+1/3*B/b^(2/3)*(-1)^(1/3)*(-b)^(1/3)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
-1/6*B/b^(2/3)*(-1)^(1/3)*(-b)^(1/3)/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))-1/3*B/b^(2/3)*(-1)^(1/3)*(-b)^(1/3)*3^(1/2)/(a/b)^(1/3)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [B] time = 2.95, size = 174, normalized size = 4.24

$$-\frac{\sqrt{3}\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="max
ima")
```

```
[Out] -1/3*sqrt(3)*(B*(-b)^(2/3)*(a/b)^(1/3) - B*a^(1/3)*(-b)^(1/3))*arctan(1/3*s
qrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) - 1/6*(B*(-b)^(2/3)
*(a/b)^(1/3) + B*a^(1/3)*(-b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(b*(a/b)^(2/3)) + 1/3*(B*(-b)^(2/3)*(a/b)^(1/3) + B*a^(1/3)*(-b)^(1/3))*lo
g(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 0.23, size = 49, normalized size = 1.20

$$-\frac{2\sqrt{3}\,B\,\sqrt{-b}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{-b}}{3\,\sqrt{b}}-\frac{2\,\sqrt{3}\,\sqrt{b}\,x}{3\,a^{1/3}\,(-b)^{1/6}}\right)}{3\,a^{1/3}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3))/(a + b*x^3),x)
```

```
[Out] -(2*3^(1/2)*B*(-b)^(1/2)*atanh((3^(1/2)*(-b)^(1/2))/(3*b^(1/2)) - (2*3^(1/2
)*b^(1/2)*x)/(3*a^(1/3)*(-b)^(1/6))))/(3*a^(1/3)*b^(1/2))
```

sympy [C] time = 0.85, size = 105, normalized size = 2.56

$$-\frac{B\left(-\frac{\sqrt{3}\,i\log\left(-\frac{\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b}-\frac{\sqrt{3}\,i\,\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(-\frac{\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b}+\frac{\sqrt{3}\,i\,\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b}+x\right)}{3}\right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*(-b)**(1/3)*B-(-b)**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] -B*(-sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) - sqrt(3)*I*a**(1/3)*(-b)**(
2/3)/(2*b) + x)/3 + sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) + sqrt(3)*I*a
**(1/3)*(-b)**(2/3)/(2*b) + x)/3)/a**(1/3)
```

3.22 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx$

Optimal. Leaf size=118

$$\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

Rubi [A] time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {260, 1593, 1871, 12, 292, 31, 634, 617, 204, 628}

$$\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]

[Out] -((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^
(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)) + (B*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx = -\left(C\int \frac{x^2}{a+bx^3}\,dx\right)+\int \frac{Bx+Cx^2}{a+bx^3}\,dx$$

$$= -\frac{C\log\left(a+bx^3\right)}{3b}+\int \frac{x(B+Cx)}{a+bx^3}\,dx$$

$$= -\frac{C\log\left(a+bx^3\right)}{3b}+C\int \frac{x^2}{a+bx^3}\,dx+\int \frac{Bx}{a+bx^3}\,dx$$

$$= B\int \frac{x}{a+bx^3}\,dx$$

$$= -\frac{B\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}+\frac{B\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}$$

$$= -\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{B\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{2/3}}+\frac{B\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}}$$

$$= -\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}+\frac{B\,\text{Subst}\left(\int \frac{1}{-3-x^2} \cdots\right)}{\sqrt[3]{a}\,b \cdots}$$

$$= -\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2 \cdots\right.}{6\sqrt[3]{a}\,b^{2/3}}$$

Mathematica [A] time = 0.03, size = 90, normalized size = 0.76

$$\frac{B\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (B*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
+ b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(1/3)*
b^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3), x]
```

fricas [A] time = 0.44, size = 310, normalized size = 2.63

$$\left[\frac{3\sqrt{\frac{1}{3}}Bab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3(-ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)+(-ab^2)^{\frac{2}{3}}B\log\left(b^2x^2+(-ab^2)^{\frac{1}{3}}bx+(-ab^2)^{\frac{2}{3}}\right)-2(-ab^2)^{\frac{2}{3}}B\log\left(bx-(-ab^2)^{\frac{1}{3}}\right)}{6ab^2},\ \frac{6\sqrt{\frac{1}{3}}Bab\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)+(-ab^2)^{\frac{2}{3}}B\log\left(b^2x^2+(-ab^2)^{\frac{1}{3}}bx+(-ab^2)^{\frac{2}{3}}\right)-2(-ab^2)^{\frac{2}{3}}B\log\left(bx-(-ab^2)^{\frac{1}{3}}\right)}{6ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*B*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqr
t(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*B*log(b^2*x^2 + (-
a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*log(b*x - (-a*b^2)^
(1/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*B*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt
(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*
B*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*l
og(b*x - (-a*b^2)^(1/3)))/(a*b^2)]
```

giac [A] time = 0.19, size = 103, normalized size = 0.87

$$\frac{\sqrt{3}B\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}}-\frac{B\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}}-\frac{B\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*B*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(-a*b^2
)^(1/3) - 1/6*B*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(1/3) - 1
/3*B*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a
```

maple [A] time = 0.05, size = 94, normalized size = 0.80

$$\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x)
```

```
[Out] -1/3*B/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*B/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/
3)*x+(a/b)^(2/3))+1/3*B*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))
```

maxima [A] time = 3.03, size = 159, normalized size = 1.35

$$-\frac{C\log\left(bx^{3}+a\right)}{3b}+\frac{\left(2C\left(\frac{a}{b}\right)^{\frac{1}{3}}+B\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-B\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\left(2Ca-\left(3B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{2Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*C*log(b*x^3 + a)/b + 1/6*(2*C*(a/b)^(1/3) + B)*log(x^2 - x*(a/b)^(1/3)
 + (a/b)^(2/3))/(b*(a/b)^(1/3)) + 1/3*(C*(a/b)^(1/3) - B)*log(x + (a/b)^(1/
3))/(b*(a/b)^(1/3)) - 1/9*sqrt(3)*(2*C*a - (3*B*(a/b)^(2/3) + 2*C*a/b)*b)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

mupad [B] time = 4.94, size = 98, normalized size = 0.83

$$-\frac{B\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(B-\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(B+\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

```
[Out] (log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^(1/3))*(B - 3^(1/2)*B*1i))/(6*a
^(1/3)*b^(2/3)) - (B*log(b^(1/3)*x + a^(1/3)))/(3*a^(1/3)*b^(2/3)) + (log(3
^(1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*(B + 3^(1/2)*B*1i))/(6*a^(1/3)
*b^(2/3))
```

sympy [A] time = 0.48, size = 26, normalized size = 0.22

$$B\operatorname{RootSum}\left(27t^{3}ab^{2}+1,\left(t\mapsto t\log\left(9t^{2}ab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x)/(b*x**3+a),x)
```

```
[Out] B*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

3.23 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx$

Optimal. Leaf size=118

$$-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {260, 1871, 12, 200, 31, 634, 617, 204, 628}

$$-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] -((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^
(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) - (A*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = -\left(C\int \frac{x^2}{a+bx^3}\,dx\right)+\int \frac{A+Cx^2}{a+bx^3}\,dx$$

$$= -\frac{C\log\left(a+bx^3\right)}{3b}+C\int \frac{x^2}{a+bx^3}\,dx+\int \frac{A}{a+bx^3}\,dx$$

$$= A\int \frac{1}{a+bx^3}\,dx$$

$$= \frac{A\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}+\frac{A\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}}$$

$$= \frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{A\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}}-\frac{A\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{b}}$$

$$= \frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx \ldots\right)}{a^{2/3}\sqrt[3]{b} \ldots}$$

$$= -\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2 \ldots\right)}{6a^{2/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.02, size = 90, normalized size = 0.76

$$-\frac{A\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] -1/6*(A*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1
/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(2/3
)*b^(1/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3), x]
```

fricas [A] time = 0.43, size = 305, normalized size = 2.58

$$\left[\frac{3\sqrt{\frac{1}{3}}Aab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(a^2b\right)^{\frac{2}{3}}A\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right)+2\left(a^2b\right)^{\frac{2}{3}}A\log\left(abx+\left(a^2b\right)^{\frac{2}{3}}\right)}{6a^2b},\ \frac{6\sqrt{\frac{1}{3}}Aab\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-\left(a^2b\right)^{\frac{2}{3}}A\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right)+2\left(a^2b\right)^{\frac{2}{3}}A\log\left(abx+\left(a^2b\right)^{\frac{2}{3}}\right)}{6a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*A*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(
1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*A*log(a*b*x^2 - (a^2
*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*A*log(a*b*x + (a^2*b)^(2/3
)))/(a^2*b), 1/6*(6*sqrt(1/3)*A*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*
(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^
(2/3)*A*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*
A*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b)]
```

giac [A] time = 0.21, size = 115, normalized size = 0.97

$$-\frac{A\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}A\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab}+\frac{\left(-ab^2\right)^{\frac{1}{3}}A\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*A*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/
3)*A*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/6*(-a*
b^2)^(1/3)*A*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

maple [A] time = 0.05, size = 94, normalized size = 0.80

$$\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x)
```

```
[Out] 1/3*A/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*A/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/3*A/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))
```

maxima [A] time = 2.99, size = 159, normalized size = 1.35

$$-\frac{C\log\left(bx^3+a\right)}{3\,b}-\frac{\sqrt{3}\left(2\,Ca-\left(3\,A\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}-A\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+A\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*C*log(b*x^3 + a)/b - 1/9*sqrt(3)*(2*C*a - (3*A*(a/b)^(1/3) + 2*C*a/b)*
b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*(a/
b)^(2/3) - A)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*
(C*(a/b)^(2/3) + A)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 5.01, size = 96, normalized size = 0.81

$$\frac{A\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(A-\sqrt{3}\,A\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(A+\sqrt{3}\,A\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

```
[Out] (A*log(b^(1/3)*x + a^(1/3)))/(3*a^(2/3)*b^(1/3)) - (log(a^(1/3) - 2*b^(1/3)
*x - 3^(1/2)*a^(1/3)*1i)*(A - 3^(1/2)*A*1i))/(6*a^(2/3)*b^(1/3)) - (log(2*b
^(1/3)*x - 3^(1/2)*a^(1/3)*1i - a^(1/3))*(A + 3^(1/2)*A*1i))/(6*a^(2/3)*b^(
1/3))
```

sympy [A] time = 0.21, size = 22, normalized size = 0.19

$$A\,\mathrm{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+A)/(b*x**3+a),x)
```

```
[Out] A*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

3.24 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right) dx$

Optimal. Leaf size=161

$$-\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Rubi [A] time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {260, 1871, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] -(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*B)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)dx = -\left(C\int \frac{x^2}{a+bx^3}\,dx\right)+\int \frac{A+Bx+Cx^2}{a+bx^3}\,dx$$

$$= -\frac{C\log\left(a+bx^3\right)}{3b}+C\int \frac{x^2}{a+bx^3}\,dx+\int \frac{A+Bx}{a+bx^3}\,dx$$

$$= \frac{\int \frac{\sqrt[3]{a}\left(2A\sqrt[3]{b}+\sqrt[3]{a}B\right)+\sqrt[3]{b}\left(-A\sqrt[3]{b}+\sqrt[3]{a}B\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$= \frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}}+$$

$$= \frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^2\right.}{6a^{2/3}b^{2/3}}$$

$$= -\frac{\left(A\sqrt[3]{b}+\sqrt[3]{a}B\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}+\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{(A}{}$$

Mathematica [A] time = 0.05, size = 124, normalized size = 0.77

$$\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(A*b^(1/3) + a^(1/3)*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (A*b^(1/3) - a^(1/3)*B)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x
]
```

```
[Out] IntegrateAlgebraic[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),
x]
```

fricas [C] time = 1.18, size = 1961, normalized size = 12.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3
*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^
3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^
3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)
/(a^2*b^2))^(1/3)))*A^2*a*b + 2*A*B^2*a + (B^3*a + A^3*b)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^
2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^
3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2)
- (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A
^3*b)/(a^2*b^2))^(1/3)))*A^2*a*b - 2*A*B^2*a + 2*(B^3*a + A^3*b)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*B*a^2*b + 2*A^2*a*b
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*
a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3
*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*
b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3
*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^
2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*A^2*a*b - 2*A*B^2*a + 2*(B^3*a
+ A^3*b)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(
a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3)
 + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*
B*a^2*b + 2*A^2*a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(
a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3)
 + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^
2*a*b + 16*A*B)/(a*b)))
```

giac [A] time = 0.20, size = 147, normalized size = 0.91

$$-\frac{\sqrt{3}\left(Ab-\left(-ab^{2}\right)^{\frac{1}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}}-\frac{\left(Ab+\left(-ab^{2}\right)^{\frac{1}{3}}B\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}}-\frac{\left(Bb\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(A*b - (-a*b^2)^(1/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(A*b + (-a*b^2)^(1/3)*B)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(B*b*(-a/b)^(1/3) + A*b)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)
```

maple [A] time = 0.04, size = 186, normalized size = 1.16

$$\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x)
```

```
[Out] 1/3/(a/b)^(2/3)*A/b*ln(x+(a/b)^(1/3))-1/6/(a/b)^(2/3)*A/b*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/3/(a/b)^(2/3)*3^(1/2)*A/b*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-1/3/(a/b)^(1/3)*B/b*ln(x+(a/b)^(1/3))+1/6/(a/b)^(1/3)*B/b*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/(a/b)^(1/3)*B/b*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.96, size = 188, normalized size = 1.17

$$-\frac{C\log\left(bx^{3}+a\right)}{3b}-\frac{\sqrt{3}\left(2Ca-\left(3B\left(\frac{a}{b}\right)^{\frac{2}{3}}+3A\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab}+\frac{\left(2C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}-A\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}+A\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*C*log(b*x^3 + a)/b - 1/9*sqrt(3)*(2*C*a - (3*B*(a/b)^(2/3) + 3*A*(a/b)
^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a
*b) + 1/6*(2*C*(a/b)^(2/3) + B*(a/b)^(1/3) - A)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(C*(a/b)^(2/3) - B*(a/b)^(1/3) + A)*log(x
 + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 4.92, size = 127, normalized size = 0.79

$$\sum_{k=1}^{3}\ln\left(b\left(B^{2}x+AB+\mathrm{root}\left(27a^{2}b^{2}z^{3}+9ABabz+B^{3}a-A^{3}b,z,k\right)^{2}ab9+A\,\mathrm{root}\left(27a^{2}b^{2}z^{3}+9ABabz+B^{3}a-A^{3}b,z,k\right)bx3\right)\right)\mathrm{root}\left(27a^{2}b^{2}z^{3}+9ABabz+B^{3}a-A^{3}b,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(b*(B^2*x + A*B + 9*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A
^3*b, z, k)^2*a*b + 3*A*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A^3*b,
z, k)*b*x))*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A^3*b, z, k), k, 1,
 3)
```

sympy [A] time = 1.28, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^{3}a^{2}b^{2} + 9tABab - A^{3}b + B^{3}a, \left(t \mapsto t\log\left(x + \frac{9t^{2}Ba^{2}b + 3tA^{2}ab + 2AB^{2}a}{A^{3}b + B^{3}a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x+A)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*A*B*a*b - A**3*b + B**3*a, Lambda(_t, _t*
log(x + (9*_t**2*B*a**2*b + 3*_t*A**2*a*b + 2*A*B**2*a)/(A**3*b + B**3*a)))
)
```

3.25 $\int \frac{bx+cx^2}{d+ex^3} dx$

Optimal. Leaf size=134

$$\frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}}-\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}}+\frac{c\log\left(d+ex^3\right)}{3e}$$

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1593, 1871, 12, 292, 31, 634, 617, 204, 628, 260}

$$\frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}}-\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}}+\frac{c\log\left(d+ex^3\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] -((b*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(1/3)*e^
(2/3))) - (b*Log[d^(1/3) + e^(1/3)*x])/(3*d^(1/3)*e^(2/3)) + (b*Log[d^(2/3)
 - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(1/3)*e^(2/3)) + (c*Log[d + e*x^3
])/(3*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{bx+cx^2}{d+ex^3}\,dx &= \int \frac{x(b+cx)}{d+ex^3}\,dx\\
&= c\int \frac{x^2}{d+ex^3}\,dx + \int \frac{bx}{d+ex^3}\,dx\\
&= \frac{c\log\left(d+ex^3\right)}{3e} + b\int \frac{x}{d+ex^3}\,dx\\
&= \frac{c\log\left(d+ex^3\right)}{3e} - \frac{b\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3\sqrt[3]{d}\sqrt[3]{e}} + \frac{b\int \frac{\sqrt[3]{d}+\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3\sqrt[3]{d}\sqrt[3]{e}}\\
&= -\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{c\log\left(d+ex^3\right)}{3e} + \frac{b\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6\sqrt[3]{d}e^{2/3}} + \frac{b\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{2\sqrt[3]{e}}\\
&= -\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} + \frac{c\log\left(d+ex^3\right)}{3e} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{\sqrt[3]{d}e^{2/3}}\\
&= -\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}} - \frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} + \frac{c\log\left(d+ex^3\right)}{3e}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 122, normalized size = 0.91

$$
\frac{b\sqrt[3]{e}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)-2b\sqrt[3]{e}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)-2\sqrt{3}b\sqrt[3]{e}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)+2c\sqrt[3]{d}\log\left(d+ex^3\right)}{6\sqrt[3]{d}e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*b*e^(1/3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*b*e^(
1/3)*Log[d^(1/3) + e^(1/3)*x] + b*e^(1/3)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x +
 e^(2/3)*x^2] + 2*c*d^(1/3)*Log[d + e*x^3])/(6*d^(1/3)*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + cx^2}{d + ex^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] IntegrateAlgebraic[(b*x + c*x^2)/(d + e*x^3), x]
```

fricas [C] time = 1.21, size = 1043, normalized size = 7.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(1/3)*e*sqrt(((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d
*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*e^2 + 4*(3*(I*sqrt(3
) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1
/3) - 2*c/e)*c*e + 4*c^2)/e^2)*arctan(1/8*sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1
/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/
e)^2*d*e^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*
(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e - 8*b^2*e*x + 4*b^2*e*sqrt(-(
(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)
/(d*e^3))^(1/3) - 2*c/e)^2*d*e^2*x - 4*b^2*e*x^2 + 4*c^2*d*x - 4*b*c*d + 2*
(2*c*d*e*x - b*d*e)*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) +
1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e))/(b^2*e)) + 4*c^2*d)*sqrt(((3*
(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d
*e^3))^(1/3) - 2*c/e)^2*e^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^
3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*e + 4*c^2)/e^2)/
b^3) + 2*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d
 - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*e*log(1/4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e
^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*d*e^
2 + (3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^
3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e + b^2*e*x + c^2*d) - ((3*(I*sqrt(3) + 1)
*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) -
2*c/e)*e + 6*c)*log(-1/4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^
2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*d*e^2*x + b^2*e*x^2 - c
^2*d*x + b*c*d - 1/2*(2*c*d*e*x - b*d*e)*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3
+ 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)))/e
```

giac [A] time = 0.18, size = 110, normalized size = 0.82

$$\frac{1}{3}\,ce^{(-1)}\log\left(\left|x^3e + d\right|\right) + \frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)}{3\left(-de^2\right)^{\frac{1}{3}}} - \frac{b\log\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}}x + \left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{1}{3}}} - \frac{\left(-de^{(-1)}\right)^{\frac{2}{3}}b\log\left(\left|x - \left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/3*c*e^(-1)*log(abs(x^3*e + d)) + 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x +
(-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))/(-d*e^2)^(1/3) - 1/6*b*log(x^2 + (-d*
```

```
e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(1/3) - 1/3*(-d*e^(-1))^(2/3)
*b*log(abs(x - (-d*e^(-1))^(1/3)))/d
```

maple [A] time = 0.05, size = 108, normalized size = 0.81

$$\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}} e}-\frac{b \ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}} e}+\frac{b \ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}} x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{1}{3}} e}+\frac{c \ln\left(e x^{3}+d\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x^3+d),x)
```

```
[Out] -1/3/(d/e)^(1/3)*b/e*ln(x+(d/e)^(1/3))+1/6/(d/e)^(1/3)*b/e*ln(x^2-(d/e)^(1/
3)*x+(d/e)^(2/3))+1/3*3^(1/2)/(d/e)^(1/3)*b/e*arctan(1/3*3^(1/2)*(2/(d/e)^(
1/3)*x-1))+1/3*c*ln(e*x^3+d)/e
```

maxima [A] time = 2.92, size = 145, normalized size = 1.08

$$\frac{\left(2c\left(\frac{d}{e}\right)^{\frac{1}{3}}+b\right)\log\left(x^{2}-x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\left(c\left(\frac{d}{e}\right)^{\frac{1}{3}}-b\right)\log\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\left(2cd-\left(3b\left(\frac{d}{e}\right)^{\frac{2}{3}}+\frac{2cd}{e}\right)e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{9de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/6*(2*c*(d/e)^(1/3) + b)*log(x^2 - x*(d/e)^(1/3) + (d/e)^(2/3))/(e*(d/e)^(
1/3)) + 1/3*(c*(d/e)^(1/3) - b)*log(x + (d/e)^(1/3))/(e*(d/e)^(1/3)) - 1/9*
sqrt(3)*(2*c*d - (3*b*(d/e)^(2/3) + 2*c*d/e)*e)*arctan(1/3*sqrt(3)*(2*x - (
d/e)^(1/3))/(d/e)^(1/3))/(d*e)
```

mupad [B] time = 0.19, size = 158, normalized size = 1.18

$$\sum_{k=1}^{3} \ln\left(-\mathrm{root}\left(27 d e^{3} z^{3}-27 c d e^{2} z^{2}+9 c^{2} d e z+b^{3} e-c^{3} d, z, k\right)\left(6 c d e-\mathrm{root}\left(27 d e^{3} z^{3}-27 c d e^{2} z^{2}+9 c^{2} d e z+b^{3} e-c^{3} d, z, k\right) d e^{2} 9\right)+c^{2} d+b^{2} e x\right) \mathrm{root}\left(27 d e^{3} z^{3}-27 c d e^{2} z^{2}+9 c^{2} d e z+b^{3} e-c^{3} d, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)/(d + e*x^3),x)
```

```
[Out] symsum(log(c^2*d - root(27*d*e^3*z^3 - 27*c*d*e^2*z^2 + 9*c^2*d*e*z + b^3*e
 - c^3*d, z, k)*(6*c*d*e - 9*root(27*d*e^3*z^3 - 27*c*d*e^2*z^2 + 9*c^2*d*e
*z + b^3*e - c^3*d, z, k)*d*e^2) + b^2*e*x)*root(27*d*e^3*z^3 - 27*c*d*e^2*
z^2 + 9*c^2*d*e*z + b^3*e - c^3*d, z, k), k, 1, 3)
```

sympy [A] time = 0.71, size = 75, normalized size = 0.56

$$\operatorname{RootSum}\left(27t^{3}de^{3}-27t^{2}cde^{2}+9tc^{2}de+b^{3}e-c^{3}d, \left(t \mapsto t\log\left(x+\frac{9t^{2}de^{2}-6tcde+c^{2}d}{b^{2}e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x**3+d),x)
```

```
[Out] RootSum(27*_t**3*d*e**3 - 27*_t**2*c*d*e**2 + 9*_t*c**2*d*e + b**3*e - c**3
*d, Lambda(_t, _t*log(x + (9*_t**2*d*e**2 - 6*_t*c*d*e + c**2*d)/(b**2*e)))
)
```

3.26 $\int \frac{a+cx^2}{d-ex^3}\,dx$

Optimal. Leaf size=134

$$\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

Rubi [A] time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1871, 12, 200, 31, 634, 617, 204, 628, 260}

$$\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d - e*x^3),x]
```

```
[Out] (a*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(1
/3)) - (a*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) + (a*Log[d^(2/3) +
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3)) - (c*Log[d - e*x^3])/
(3*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+cx^2}{d-ex^3}\,dx &= c\int \frac{x^2}{d-ex^3}\,dx+\int \frac{a}{d-ex^3}\,dx\\
&= -\frac{c\log\left(d-ex^3\right)}{3e}+a\int \frac{1}{d-ex^3}\,dx\\
&= -\frac{c\log\left(d-ex^3\right)}{3e}+\frac{a\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}x}\,dx}{3d^{2/3}}+\frac{a\int \frac{2\sqrt[3]{d}+\sqrt[3]{e}x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}}\\
&= -\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}+\frac{a\int \frac{1}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{2\sqrt[3]{d}}+\frac{a\int \frac{\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6d^{2/3}\sqrt[3]{e}}\\
&= -\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}-\frac{a\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x\right.}{d^{2/3}\sqrt[3]{e}}\\
&= \frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 123, normalized size = 0.92

$$
\frac{ae^{2/3}\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)-2ae^{2/3}\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)+2\sqrt{3}\,ae^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)-2cd^{2/3}\log\left(d-ex^3\right)}{6d^{2/3}e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d - e*x^3),x]
```

```
[Out] (2*Sqrt[3]*a*e^(2/3)*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*a*e^(2
/3)*Log[d^(1/3) - e^(1/3)*x] + a*e^(2/3)*Log[d^(2/3) + d^(1/3)*e^(1/3)*x +
e^(2/3)*x^2] - 2*c*d^(2/3)*Log[d - e*x^3])/(6*d^(2/3)*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{a+cx^2}{d-ex^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)/(d - e*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^2)/(d - e*x^3), x]
```

fricas [C] time = 1.19, size = 1267, normalized size = 9.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(1/3)*e*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2
*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*
(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/
3) + 2*c/e)*c*e + 4*c^2)/e^2)*arctan(-1/8*(2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqr
t(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2
*c/e)*a*d*e^2 - 2*a*c*d*e)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^
3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^
(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3
))^(1/3) + 2*c/e)*c*e + 4*c^2)/e^2)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3
/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*d^2*e^
2 + 4*a^2*e^2*x^2 - 4*a*c*d*e*x + 4*c^2*d^2 + 2*(a*d*e^2*x - 2*c*d^2*e)*((1
/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2
*e^3))^(1/3) + 2*c/e))/(a^2*e^2)) - sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)
*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*d
^2*e^2 - 8*a*c*d*e*x + 4*c^2*d^2 + 4*(a*d*e^2*x - c*d^2*e)*((1/2)^(1/3)*(I*
sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3)
+ 2*c/e))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*
d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)
*c*e + 4*c^2)/e^2))/(a^3*e)) - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^
3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e*log(-1/2*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^
3))^(1/3) + 2*c/e)*d*e + a*e*x + c*d) + (((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/
e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e - 6*c)*
log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^
3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*d^2*e^2 + a^2*e^2*x^2 - a*c*d*e*x + c^2*
d^2 + 1/2*(a*d*e^2*x - 2*c*d^2*e)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a
^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)))/e
```

giac [A] time = 0.17, size = 95, normalized size = 0.71

$$-\frac{1}{3}\,ce^{(-1)}\log\left(\left|x^{3}e-d\right|\right)+\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+2\,x\right)e^{\frac{1}{3}}}{3\,d^{\frac{1}{3}}}\right)e^{\left(-\frac{1}{3}\right)}}{3\,d^{\frac{2}{3}}}+\frac{ae^{\left(-\frac{1}{3}\right)}\log\left(d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^{2}+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6\,d^{\frac{2}{3}}}-\frac{ae^{\left(-\frac{1}{3}\right)}\log\left(\left|-d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right|\right)}{3\,d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*c*e^(-1)*log(abs(x^3*e - d)) + 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(d^(1/
3)*e^(-1/3) + 2*x)*e^(1/3)/d^(1/3))*e^(-1/3)/d^(2/3) + 1/6*a*e^(-1/3)*log(d
^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(2/3) - 1/3*a*e^(-1/3)*log(ab
s(-d^(1/3)*e^(-1/3) + x))/d^(2/3)
```

maple [A] time = 0.04, size = 111, normalized size = 0.83

$$\frac{\sqrt{3}\, a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}} e}-\frac{a \ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}} e}+\frac{a \ln\left(x^{2}+\left(\frac{d}{e}\right)^{\frac{1}{3}} x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{2}{3}} e}-\frac{c \ln\left(e\,x^{3}-d\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(-e*x^3+d),x)
```

```
[Out] -1/3/(d/e)^(2/3)*a/e*ln(x-(d/e)^(1/3))+1/6/(d/e)^(2/3)*a/e*ln(x^2+(d/e)^(1/
3)*x+(d/e)^(2/3))+1/3/(d/e)^(2/3)*3^(1/2)*a/e*arctan(1/3*3^(1/2)*(2/(d/e)^(
1/3)*x+1))-1/3*c/e*ln(e*x^3-d)
```

maxima [A] time = 3.05, size = 144, normalized size = 1.07

$$-\frac{\sqrt{3}\left(2\,cd-\left(3\,a\left(\frac{d}{e}\right)^{\frac{1}{3}}+\frac{2\,cd}{e}\right)e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{9\,de}-\frac{\left(2\,c\left(\frac{d}{e}\right)^{\frac{2}{3}}-a\right)\log\left(x^{2}+x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\left(c\left(\frac{d}{e}\right)^{\frac{2}{3}}+a\right)\log\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\,e\left(\frac{d}{e}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*c*d - (3*a*(d/e)^(1/3) + 2*c*d/e)*e)*arctan(1/3*sqrt(3)*(2*
x + (d/e)^(1/3))/(d/e)^(1/3))/(d*e) - 1/6*(2*c*(d/e)^(2/3) - a)*log(x^2 + x
*(d/e)^(1/3) + (d/e)^(2/3))/(e*(d/e)^(2/3)) - 1/3*(c*(d/e)^(2/3) + a)*log(x
 - (d/e)^(1/3))/(e*(d/e)^(2/3))
```

mupad [B] time = 5.01, size = 178, normalized size = 1.33

$$\sum_{k=1}^{3}\ln\left(-\left(c+\mathrm{root}\left(27\,d^{2}e^{3}z^{3}+27\,c\,d^{2}e^{2}z^{2}+9\,c^{2}d^{2}e\,z+c^{3}d^{2}+a^{3}e^{2},z,k\right)e\,3\right)\left(c\,d+\mathrm{root}\left(27\,d^{2}e^{3}z^{3}+27\,c\,d^{2}e^{2}z^{2}+9\,c^{2}d^{2}e\,z+c^{3}d^{2}+a^{3}e^{2},z,k\right)d\,e\,3+a\,e\,x\right)\right)\mathrm{root}\left(27\,d^{2}e^{3}z^{3}+27\,c\,d^{2}e^{2}z^{2}+9\,c^{2}d^{2}e\,z+c^{3}d^{2}+a^{3}e^{2},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/(d - e*x^3),x)
```

```
[Out] symsum(log(-(c + 3*root(27*d^2*e^3*z^3 + 27*c*d^2*e^2*z^2 + 9*c^2*d^2*e*z +
 c^3*d^2 + a^3*e^2, z, k)*e)*(c*d + 3*root(27*d^2*e^3*z^3 + 27*c*d^2*e^2*z^
2 + 9*c^2*d^2*e*z + c^3*d^2 + a^3*e^2, z, k)*d*e + a*e*x))*root(27*d^2*e^3*
z^3 + 27*c*d^2*e^2*z^2 + 9*c^2*d^2*e*z + c^3*d^2 + a^3*e^2, z, k), k, 1, 3)
```

sympy [A] time = 0.59, size = 70, normalized size = 0.52

$$-\operatorname{RootSum}\left(27t^{3}d^{2}e^{3}-27t^{2}cd^{2}e^{2}+9tc^{2}d^{2}e-a^{3}e^{2}-c^{3}d^{2},\left(t\mapsto t\log\left(x+\frac{-3tde+cd}{ae}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(-e*x**3+d),x)
```

```
[Out] -RootSum(27*_t**3*d**2*e**3 - 27*_t**2*c*d**2*e**2 + 9*_t*c**2*d**2*e - a**
3*e**2 - c**3*d**2, Lambda(_t, _t*log(x + (-3*_t*d*e + c*d)/(a*e))))
```

3.27 $\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx$

Optimal. Leaf size=37

$$\frac{\log(a+bx)}{b}-\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}\,a}\right)}{\sqrt{3}\,b}$$

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1868, 31, 617, 204}

$$\frac{\log(a+bx)}{b}-\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}\,a}\right)}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) + Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx &= \frac{a\int \frac{1}{\frac{a^2}{b^2}-\frac{ax}{b}+x^2}\,dx}{b^2}+\frac{\int \frac{1}{\frac{a}{b}+x}\,dx}{b}\\
&= \frac{\log(a+bx)}{b}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2bx}{a}\right)}{b}\\
&= -\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}\,a}\right)}{\sqrt{3}\,b}+\frac{\log(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.95

$$\frac{\log\left(a^3+b^3x^3\right)-\log\left(a^2-abx+b^2x^2\right)+2\log(a+bx)+2\sqrt{3}\,\tan^{-1}\left(\frac{2bx-a}{\sqrt{3}a}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*b*x)/(Sqrt[3]*a)] + 2*Log[a + b*x] - Log[a^2 - a*
b*x + b^2*x^2] + Log[a^3 + b^3*x^3])/(3*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
[Out] IntegrateAlgebraic[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3), x]
```

fricas [A] time = 0.41, size = 36, normalized size = 0.97

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx-a)}{3a}\right)+3\log\left(bx+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a) + 3*log(b*x + a))/b
```

giac [A] time = 0.17, size = 37, normalized size = 1.00

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx-a)}{3a}\right)}{3b}+\frac{\log\left(|bx+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a)/b + log(abs(b*x + a))/b
```

maple [A] time = 0.05, size = 43, normalized size = 1.16

$$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2b^2x-ab\right)\sqrt{3}}{3ab}\right)}{3b}+\frac{\ln\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x)
[Out] 2/3*3^(1/2)/b*arctan(1/3*(2*b^2*x-a*b)*3^(1/2)/a/b)+ln(b*x+a)/b
```

maxima [A] time = 2.99, size = 42, normalized size = 1.14

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2b^2x-ab\right)}{3ab}\right)}{3b}+\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x - a*b)/(a*b))/b + log(b*x + a)/b
```

mupad [B] time = 4.81, size = 84, normalized size = 2.27

$$\frac{\ln(a+b\,x)}{b}-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{4\,\sqrt{3}\,a^3\,b^4}{4\,a^3\,b^4+4\,x\,a^2\,b^5}-\frac{4\,\sqrt{3}\,a^2\,b^5\,x}{4\,a^3\,b^4+4\,x\,a^2\,b^5}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x)
```

```
[Out] log(a + b*x)/b - (2*3^(1/2)*atan((4*3^(1/2)*a^3*b^4)/(4*a^3*b^4 + 4*a^2*b^5
*x) - (4*3^(1/2)*a^2*b^5*x)/(4*a^3*b^4 + 4*a^2*b^5*x)))/(3*b)
```

sympy [C] time = 0.50, size = 60, normalized size = 1.62

$$\frac{-\frac{\sqrt{3}\,i\log\left(x+\frac{-a-\sqrt{3}\,ia}{2b}\right)}{3}+\frac{\sqrt{3}\,i\log\left(x+\frac{-a+\sqrt{3}\,ia}{2b}\right)}{3}+\log\left(\frac{a}{b}+x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a**2)/(b**3*x**3+a**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (-a - sqrt(3)*I*a)/(2*b))/3 + sqrt(3)*I*log(x + (-a + s
qrt(3)*I*a)/(2*b))/3 + log(a/b + x))/b
```

3.28 $\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx$

Optimal. Leaf size=39

$$\frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1868, 31, 617, 204}

$$\frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

```
[Out] (2*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) - Log[a - b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx &= \frac{a\int \frac{1}{\frac{a^2}{b^2}+\frac{ax}{b}+x^2}\,dx}{b^2}-\frac{\int \frac{1}{-\frac{a}{b}+x}\,dx}{b}\\ &= -\frac{\log(a-bx)}{b}-\frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2bx}{a}\right)}{b}\\ &= \frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.82

$$\frac{-\log\left(a^3-b^3x^3\right)+\log\left(a^2+abx+b^2x^2\right)-2\log(a-bx)+2\sqrt{3}\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
[Out] (2*Sqrt[3]*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)] - 2*Log[a - b*x] + Log[a^2 + a*b
*x + b^2*x^2] - Log[a^3 - b^3*x^3])/(3*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
[Out] IntegrateAlgebraic[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3), x]
```

fricas [A] time = 0.41, size = 36, normalized size = 0.92

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx+a)}{3a}\right)-3\log(bx-a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a) - 3*log(b*x - a))/b
```

giac [A] time = 0.15, size = 38, normalized size = 0.97

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx+a)}{3a}\right)}{3b}-\frac{\log(|bx-a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a)/b - log(abs(b*x - a))/b
```

maple [A] time = 0.06, size = 45, normalized size = 1.15

$$\frac{2\sqrt{3}\arctan\left(\frac{\left(2b^2x+ab\right)\sqrt{3}}{3ab}\right)}{3b}-\frac{\ln(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x)
[Out] -1/b*ln(b*x-a)+2/3*3^(1/2)/b*arctan(1/3*(2*b^2*x+a*b)*3^(1/2)/a/b)
```

maxima [A] time = 2.97, size = 44, normalized size = 1.13

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2b^2x+ab\right)}{3ab}\right)}{3b}-\frac{\log(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x + a*b)/(a*b))/b - log(b*x - a)/b
```

mupad [B] time = 0.09, size = 86, normalized size = 2.21

$$\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,a^3\,b^4}{4\,a^3\,b^4-4\,a^2\,b^5\,x}+\frac{4\sqrt{3}\,a^2\,b^5\,x}{4\,a^3\,b^4-4\,a^2\,b^5\,x}\right)}{3\,b}-\frac{\ln\left(a-b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x)
```

```
[Out] (2*3^(1/2)*atan((4*3^(1/2)*a^3*b^4)/(4*a^3*b^4 - 4*a^2*b^5*x) + (4*3^(1/2)*
a^2*b^5*x)/(4*a^3*b^4 - 4*a^2*b^5*x)))/(3*b) - log(a - b*x)/b
```

sympy [C] time = 0.70, size = 60, normalized size = 1.54

$$-\frac{\frac{\sqrt{3}\,i\log\left(x+\frac{a-\sqrt{3}\,ia}{2b}\right)}{3}-\frac{\sqrt{3}\,i\log\left(x+\frac{a+\sqrt{3}\,ia}{2b}\right)}{3}+\log\left(-\frac{a}{b}+x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a**2)/(-b**3*x**3+a**3),x)
```

```
[Out] -(sqrt(3)*I*log(x + (a - sqrt(3)*I*a)/(2*b))/3 - sqrt(3)*I*log(x + (a + sqr
t(3)*I*a)/(2*b))/3 + log(-a/b + x))/b
```

3.29 $\int \frac{8C+b^{2/3}Cx^2}{8+bx^3} \, dx$

Optimal. Leaf size=48

$$\frac{C \log \left(\sqrt[3]{b} x+2\right)}{\sqrt[3]{b}}-\frac{2 C \tan ^{-1}\left(\frac{1-\sqrt[3]{b} x}{\sqrt{3}}\right)}{\sqrt{3} \sqrt[3]{b}}$$

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1863, 31, 617, 204}

$$\frac{C \log \left(\sqrt[3]{b} x+2\right)}{\sqrt[3]{b}}-\frac{2 C \tan ^{-1}\left(\frac{1-\sqrt[3]{b} x}{\sqrt{3}}\right)}{\sqrt{3} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - b^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (C*Log[2 + b^(1/
3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx = \frac{(2C)\int \frac{1}{\frac{4}{b^{2/3}}-\frac{2x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}} + \frac{C\int \frac{1}{\frac{2}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}}$$
$$= \frac{C\log\left(2+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} + \frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$
$$= -\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{b}\,x}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} + \frac{C\log\left(2+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}$$

Mathematica [A] time = 0.02, size = 76, normalized size = 1.58

$$\frac{C\left(-\log\left(b^{2/3}x^2-2\sqrt[3]{b}\,x+4\right)+\log\left(bx^3+8\right)+2\log\left(\sqrt[3]{b}\,x+2\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{b}\,x-1}{\sqrt{3}}\right)\right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*ArcTan[(-1 + b^(1/3)*x)/Sqrt[3]] + 2*Log[2 + b^(1/3)*x] - Log
[4 - 2*b^(1/3)*x + b^(2/3)*x^2] + Log[8 + b*x^3]))/(3*b^(1/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3), x]
```

fricas [A] time = 0.45, size = 134, normalized size = 2.79

$$\left[\frac{\sqrt{\frac{1}{3}}\,Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{bx^3+6\sqrt{\frac{1}{3}}\left(bx^2+b^{\frac{2}{3}}x-2\,b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-6\,b^{\frac{1}{3}}x-4}{bx^3+8}\right)+Cb^{\frac{2}{3}}\log\left(bx+2\,b^{\frac{2}{3}}\right)}{b}, \frac{2\sqrt{\frac{1}{3}}\,Cb^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{2}{3}}x-b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right)+Cb^{\frac{2}{3}}\log\left(bx+2\,b^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((b*x^3 + 6*sqrt(1/3)*(b*x^2 + b^(2/3)*
x - 2*b^(1/3))*sqrt(-1/b^(2/3)) - 6*b^(1/3)*x - 4)/(b*x^3 + 8)) + C*b^(2/3)
*log(b*x + 2*b^(2/3)))/b, (2*sqrt(1/3)*C*b^(2/3)*arctan(sqrt(1/3)*(b^(2/3)*
x - b^(1/3))/b^(1/3)) + C*b^(2/3)*log(b*x + 2*b^(2/3)))/b]
```

giac [B] time = 0.42, size = 115, normalized size = 2.40

$$\frac{2}{3}\sqrt{3}\,C\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(x+\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{b}\right)^{\frac{1}{3}}}\right)-\frac{1}{3}\left(Cb^{\frac{2}{3}}\left(-\frac{1}{b}\right)^{\frac{2}{3}}+2\,C\right)\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(\left|x-2\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right|\right)+\frac{1}{3}\left(C\left(-\frac{1}{b}\right)^{\frac{1}{3}}+\frac{C}{b^{\frac{1}{3}}}\right)\log\left(x^2+2\,x\left(-\frac{1}{b}\right)^{\frac{1}{3}}+4\left(-\frac{1}{b}\right)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*(-1/b)^(1/3)*arctan(1/3*sqrt(3)*(x + (-1/b)^(1/3))/(-1/b)^(1/
3)) - 1/3*(C*b^(2/3)*(-1/b)^(2/3) + 2*C)*(-1/b)^(1/3)*log(abs(x - 2*(-1/b)^
(1/3))) + 1/3*(C*(-1/b)^(1/3) + C/b^(1/3))*log(x^2 + 2*x*(-1/b)^(1/3) + 4*(
-1/b)^(2/3))
```

maple [B] time = 0.06, size = 117, normalized size = 2.44

$$\frac{C\ln\left(b\,x^{3}+8\right)}{3b^{\frac{1}{3}}}+\frac{8^{\frac{1}{3}}\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(\frac{8^{\frac{2}{3}}x}{4\left(\frac{1}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{1}{b}\right)^{\frac{2}{3}}b}+\frac{8^{\frac{1}{3}}C\ln\left(x+8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{1}{b}\right)^{\frac{2}{3}}b}-\frac{8^{\frac{1}{3}}C\ln\left(x^{2}-8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}x+8^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{1}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x)
```

```
[Out] 1/3*C/b*8^(1/3)/(1/b)^(2/3)*ln(x+8^(1/3)*(1/b)^(1/3))-1/6*C/b*8^(1/3)/(1/b)
^(2/3)*ln(x^2-8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))+1/3*C/b*8^(1/3)/(1
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x-1))+1/3*C/b
^(1/3)*ln(b*x^3+8)
```

maxima [A] time = 2.99, size = 47, normalized size = 0.98

$$\frac{2\,\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(b^{\frac{2}{3}}x-b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{1}{3}}}+\frac{C\log\left(\frac{b^{\frac{1}{3}}x+2}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(b^(2/3)*x - b^(1/3))/b^(1/3))/b^(1/3) + C
*log((b^(1/3)*x + 2)/b^(1/3))/b^(1/3)
```

mupad [B] time = 5.14, size = 147, normalized size = 3.06

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C-\mathrm{root}\left(27\,b^{3}\,z^{3}-27\,C\,b^{8/3}\,z^{2}+9\,C^{2}\,b^{7/3}\,z-9\,C^{3}\,b^{2},z,k\right)\,b^{1/3}\,3\right)\,\left(-C+\mathrm{root}\left(27\,b^{3}\,z^{3}-27\,C\,b^{8/3}\,z^{2}+9\,C^{2}\,b^{7/3}\,z-9\,C^{3}\,b^{2},z,k\right)\,b^{1/3}\,3+C\,b^{1/3}\,x\right)\,8}{b^{5/3}}\right)\,\mathrm{root}\left(27\,b^{3}\,z^{3}-27\,C\,b^{8/3}\,z^{2}+9\,C^{2}\,b^{7/3}\,z-9\,C^{3}\,b^{2},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*C + C*b^(2/3)*x^2)/(b*x^3 + 8),x)
```

```
[Out] symsum(log(-(8*(C - 3*root(27*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b^(7/3)*z
- 9*C^3*b^2, z, k)*b^(1/3))*(3*root(27*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b
^(7/3)*z - 9*C^3*b^2, z, k)*b^(1/3) - C + C*b^(1/3)*x))/b^(5/3))*root(27*b^
3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b^(7/3)*z - 9*C^3*b^2, z, k), k, 1, 3)
```

sympy [A] time = 0.63, size = 58, normalized size = 1.21

$$\operatorname{RootSum}\left(3t^{3}b^{\frac{5}{3}}-3t^{2}Cb^{\frac{4}{3}}+tC^{2}b-C^{3}b^{\frac{2}{3}},\left(t\mapsto t\log\left(x+\frac{3t\sqrt[3]{b}-C}{C\sqrt[3]{b}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b**(2/3)*C*x**2)/(b*x**3+8),x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
Lambda(_t, _t*log(x + (3*_t*b**(1/3) - C)/(C*b**(1/3)))))
```

3.30 $\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx$

Optimal. Leaf size=47

$$\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)-\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1863, 31, 617, 204}

$$\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)-\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

```
[Out] -(C*ArcTan[(a^(1/3) - 4*x)/(Sqrt[3]*a^(1/3))])/(2*Sqrt[3]) + (C*Log[a^(1/3)
 + 2*x])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3}\,dx = \frac{1}{4}C\int \frac{1}{\frac{\sqrt[3]{a}}{2} + x}\,dx + \frac{1}{8}\left(\sqrt[3]{a}\,C\right)\int \frac{1}{\frac{a^{2/3}}{4} - \frac{\sqrt[3]{a}\,x}{2} + x^2}\,dx$$
$$= \frac{1}{4}C\log\left(\sqrt[3]{a} + 2x\right) + \frac{1}{2}C\,\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, 1 - \frac{4x}{\sqrt[3]{a}}\right)$$
$$= -\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a} - 4x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}} + \frac{1}{4}C\log\left(\sqrt[3]{a} + 2x\right)$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.53

$$\frac{1}{12}C\left(-\log\left(a^{2/3} - 2\sqrt[3]{a}\,x + 4x^2\right) + \log\left(a + 8x^3\right) + 2\log\left(\sqrt[3]{a} + 2x\right) - 2\sqrt{3}\tan^{-1}\left(\frac{1 - \frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (4*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) + 2*x] -
 Log[a^(2/3) - 2*a^(1/3)*x + 4*x^2] + Log[a + 8*x^3]))/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3), x]
```

fricas [A] time = 0.43, size = 40, normalized size = 0.85

$$\frac{1}{6}\sqrt{3}\,C\arctan\left(\frac{4\sqrt{3}\,a^{\frac{2}{3}}x - \sqrt{3}\,a}{3\,a}\right) + \frac{1}{4}\,C\log\left(2\,x + a^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*a^(2/3)*x - sqrt(3)*a)/a) + 1/4*C*log(2
*x + a^(1/3))
```

giac [B] time = 0.20, size = 111, normalized size = 2.36

$$\frac{\sqrt{3}\left(\sqrt{3}\,i|a| + a\right)C\arctan\left(\frac{\sqrt{3}\left(4\,x + (-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{12\,a} + \frac{\left(\sqrt{3}\,i|a| + 3\,a\right)C\log\left(x^2 + \frac{1}{2}\,(-a)^{\frac{1}{3}}\,x + \frac{1}{4}\,(-a)^{\frac{2}{3}}\right)}{24\,a} - \frac{\left(C\,(-a)^{\frac{2}{3}} + 2\,Ca^{\frac{2}{3}}\right)(-a)^{\frac{1}{3}}\log\left(\left|x - \frac{1}{2}\,(-a)^{\frac{1}{3}}\right|\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*i*abs(a) + a)*C*arctan(1/3*sqrt(3)*(4*x + (-a)^(1/3))
/(-a)^(1/3))/a + 1/24*(sqrt(3)*i*abs(a) + 3*a)*C*log(x^2 + 1/2*(-a)^(1/3)*x
```

```
 + 1/4*(-a)^(2/3))/a - 1/12*(C*(-a)^(2/3) + 2*C*a^(2/3))*(-a)^(1/3)*log(abs
(x - 1/2*(-a)^(1/3)))/a
```

maple [B] time = 0.04, size = 84, normalized size = 1.79

$$\frac{8^{\frac{2}{3}}\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(\frac{2\,8^{\frac{1}{3}}x}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{24}+\frac{8^{\frac{2}{3}}C\ln\left(x+\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}}{8}\right)}{24}-\frac{8^{\frac{2}{3}}C\ln\left(x^{2}-\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}x}{8}+\frac{8^{\frac{1}{3}}a^{\frac{2}{3}}}{8}\right)}{48}+\frac{C\ln\left(8x^{3}+a\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x)
```

```
[Out] 1/24*C*8^(2/3)*ln(x+1/8*8^(2/3)*a^(1/3))-1/48*C*8^(2/3)*ln(x^2-1/8*8^(2/3)*
a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*C*8^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
*8^(1/3)/a^(1/3)*x-1))+1/12*C*ln(8*x^3+a)
```

maxima [A] time = 3.00, size = 36, normalized size = 0.77

$$\frac{1}{6}\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(4\,x-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+\frac{1}{4}\,C\log\left(x+\frac{1}{2}\,a^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*sqrt(3)*(4*x - a^(1/3))/a^(1/3)) + 1/4*C*log(x + 1
/2*a^(1/3))
```

mupad [B] time = 5.02, size = 145, normalized size = 3.09

$$\sum_{k=1}^{3}\ln\left(-\frac{a^{2/3}\left(C-12\,\mathrm{root}\left(1728\,a^2\,z^3-432\,C\,a^2\,z^2+36\,C^2\,a^2\,z-9\,C^3\,a^2,z,k\right)\right)\left(4\,C\,x-C\,a^{1/3}+\mathrm{root}\left(1728\,a^2\,z^3-432\,C\,a^2\,z^2+36\,C^2\,a^2\,z-9\,C^3\,a^2,z,k\right)\,a^{1/3}\,12\right)}{128}\right)\mathrm{root}\left(1728\,a^2\,z^3-432\,C\,a^2\,z^2+36\,C^2\,a^2\,z-9\,C^3\,a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*a^(2/3) + 2*C*x^2)/(a + 8*x^3),x)
```

```
[Out] symsum(log(-(a^(2/3)*(C - 12*root(1728*a^2*z^3 - 432*C*a^2*z^2 + 36*C^2*a^2
*z - 9*C^3*a^2, z, k))*(4*C*x - C*a^(1/3) + 12*root(1728*a^2*z^3 - 432*C*a^
2*z^2 + 36*C^2*a^2*z - 9*C^3*a^2, z, k)*a^(1/3)))/128)*root(1728*a^2*z^3 -
432*C*a^2*z^2 + 36*C^2*a^2*z - 9*C^3*a^2, z, k), k, 1, 3)
```

sympy [C] time = 0.74, size = 85, normalized size = 1.81

$$C\left(\frac{\log\left(\frac{\sqrt[3]{a}}{2}+x\right)}{4}-\frac{\sqrt{3}\,i\log\left(x+\frac{-C\sqrt[3]{a}-\sqrt{3}\,iC\sqrt[3]{a}}{4C}\right)}{12}+\frac{\sqrt{3}\,i\log\left(x+\frac{-C\sqrt[3]{a}+\sqrt{3}\,iC\sqrt[3]{a}}{4C}\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(2/3)*C+2*C*x**2)/(8*x**3+a),x)
```

```
[Out] C*(log(a**(1/3)/2 + x)/4 - sqrt(3)*I*log(x + (-C*a**(1/3) - sqrt(3)*I*C*a**
(1/3))/(4*C))/12 + sqrt(3)*I*log(x + (-C*a**(1/3) + sqrt(3)*I*C*a**(1/3))/(
4*C))/12)
```

3.31 $\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx$

Optimal. Leaf size=57

$$\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}\,x}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{-b}}-\frac{C\log\left(\sqrt[3]{-b}\,x+2\right)}{\sqrt[3]{-b}}$$

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1864, 31, 617, 204}

$$\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}\,x}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{-b}}-\frac{C\log\left(\sqrt[3]{-b}\,x+2\right)}{\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (-b)^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3)) - (C*Log[2 +
(-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1864

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)},
Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2)
, x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C, 0
]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx = -\frac{(2C)\int \frac{1}{\frac{4}{(-b)^{2/3}}-\frac{2x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}} - \frac{C\int \frac{1}{\frac{2}{\sqrt[3]{-b}}+x}\,dx}{\sqrt[3]{-b}}$$
$$= -\frac{C\log\left(2+\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}} - \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}$$
$$= \frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}\,x}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{-b}} - \frac{C\log\left(2+\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}$$

Mathematica [A] time = 0.03, size = 99, normalized size = 1.74

$$\frac{C\left(-b^{2/3}\log\left(b^{2/3}x^2+2\sqrt[3]{b}\,x+4\right)+2b^{2/3}\log\left(2-\sqrt[3]{b}\,x\right)-2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}\,x+1}{\sqrt{3}}\right)+(-b)^{2/3}\log\left(8-bx^3\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 + b^(1/3)*x)/Sqrt[3]] + 2*b^(2/3)*Log[2 -
b^(1/3)*x] - b^(2/3)*Log[4 + 2*b^(1/3)*x + b^(2/3)*x^2] + (-b)^(2/3)*Log[8
- b*x^3]))/(3*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3), x]
```

fricas [A] time = 0.45, size = 182, normalized size = 3.19

$$\left[\frac{\sqrt{\frac{1}{3}}\,Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{bx^3-6\sqrt{\frac{1}{3}}\left(bx^2-(-b)^{\frac{2}{3}}x+2(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+6(-b)^{\frac{1}{3}}x+4}{bx^3-8}\right)+C(-b)^{\frac{2}{3}}\log\left(bx-2(-b)^{\frac{2}{3}}\right)}{b}, -\frac{2\sqrt{\frac{1}{3}}\,Cb\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(\sqrt{\frac{1}{3}}\left((-b)^{\frac{2}{3}}x-(-b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\right)-C(-b)^{\frac{2}{3}}\log\left(bx-2(-b)^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((b*x^3 - 6*sqrt(1/3)*(b*x^2 - (-b)^(
2/3)*x + 2*(-b)^(1/3))*sqrt((-b)^(1/3)/b) + 6*(-b)^(1/3)*x + 4)/(b*x^3 - 8)
) + C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(
1/3)/b)*arctan(sqrt(1/3)*((-b)^(2/3)*x - (-b)^(1/3))*sqrt(-(-b)^(1/3)/b)) -
 C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b]
```

giac [B] time = 0.31, size = 91, normalized size = 1.60

$$-\frac{2\sqrt{3}\,C|b|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,b^{\frac{1}{3}}\left(x+\frac{1}{b^{\frac{1}{3}}}\right)\right)}{3b}+\frac{1}{3}\left(\frac{C(-b)^{\frac{2}{3}}}{b}-\frac{C}{b^{\frac{1}{3}}}\right)\log\left(x^2+\frac{2x}{b^{\frac{1}{3}}}+\frac{4}{b^{\frac{2}{3}}}\right)+\frac{\left(2C+\frac{C(-b)^{\frac{2}{3}}}{b^{\frac{2}{3}}}\right)\log\left(\left|x-\frac{2}{b^{\frac{1}{3}}}\right|\right)}{3b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*abs(b)^(2/3)*arctan(1/3*sqrt(3)*b^(1/3)*(x + 1/b^(1/3)))/b +
 1/3*(C*(-b)^(2/3)/b - C/b^(1/3))*log(x^2 + 2*x/b^(1/3) + 4/b^(2/3)) + 1/3*
(2*C + C*(-b)^(2/3)/b^(2/3))*log(abs(x - 2/b^(1/3)))/b^(1/3)
```

maple [B] time = 0.05, size = 122, normalized size = 2.14

$$-\frac{8^{\frac{1}{3}}\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(\frac{8^{\frac{2}{3}}x}{4\left(\frac{1}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{1}{b}\right)^{\frac{2}{3}}b}+\frac{8^{\frac{1}{3}}C\ln\left(x-8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{1}{b}\right)^{\frac{2}{3}}b}-\frac{8^{\frac{1}{3}}C\ln\left(x^{2}+8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}x+8^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{1}{b}\right)^{\frac{2}{3}}b}+\frac{(-b)^{\frac{2}{3}}C\ln\left(b\,x^{3}-8\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x)
```

```
[Out] 1/3*C/b*8^(1/3)/(1/b)^(2/3)*ln(x-8^(1/3)*(1/b)^(1/3))-1/6*C/b*8^(1/3)/(1/b)
^(2/3)*ln(x^2+8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))-1/3*C/b*8^(1/3)/(1
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x+1))+1/3*C*(
-b)^(2/3)/b*ln(b*x^3-8)
```

maxima [B] time = 2.94, size = 122, normalized size = 2.14

$$\frac{\left(C\,(-b)^{\frac{2}{3}}-Cb^{\frac{2}{3}}\right)\log\left(b^{\frac{2}{3}}x^{2}+2\,b^{\frac{1}{3}}x+4\right)}{3\,b}+\frac{\left(C\,(-b)^{\frac{2}{3}}+2\,Cb^{\frac{2}{3}}\right)\log\left(\frac{b^{\frac{1}{3}}x-2}{b^{\frac{1}{3}}}\right)}{3\,b}+\frac{2\,\sqrt{3}\left(C\,(-b)^{\frac{2}{3}}\,b^{\frac{4}{3}}-\left(C\,(-b)^{\frac{2}{3}}\,b^{\frac{1}{3}}+3\,Cb\right)b\right)\arctan\left(\frac{\sqrt{3}\left(b^{\frac{2}{3}}x+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{9\,b^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="maxima")
```

```
[Out] 1/3*(C*(-b)^(2/3) - C*b^(2/3))*log(b^(2/3)*x^2 + 2*b^(1/3)*x + 4)/b + 1/3*(
C*(-b)^(2/3) + 2*C*b^(2/3))*log((b^(1/3)*x - 2)/b^(1/3))/b + 2/9*sqrt(3)*(C
*(-b)^(2/3)*b^(4/3) - (C*(-b)^(2/3)*b^(1/3) + 3*C*b)*b)*arctan(1/3*sqrt(3)*
(b^(2/3)*x + b^(1/3))/b^(1/3))/b^(7/3)
```

mupad [B] time = 5.27, size = 176, normalized size = 3.09

$$\sum_{k=1}^{3}\ln\left(\frac{8\,C^{2}}{(-b)^{5/3}}+\mathrm{root}\left(27\,b^{3}\,z^{3}-27\,C\,(-b)^{8/3}\,z^{2}-9\,C^{2}\,(-b)^{7/3}\,z-9\,C^{3}\,b^{2},z,k\right)\left(-\frac{\mathrm{root}\left(27\,b^{3}\,z^{3}-27\,C\,(-b)^{8/3}\,z^{2}-9\,C^{2}\,(-b)^{7/3}\,z-9\,C^{3}\,b^{2},z,k\right)\,72}{b}+\frac{48\,C}{(-b)^{4/3}}+\frac{24\,C\,x}{b}\right)-\frac{8\,C^{2}\,x}{(-b)^{4/3}}\right)\mathrm{root}\left(27\,b^{3}\,z^{3}-27\,C\,(-b)^{8/3}\,z^{2}-9\,C^{2}\,(-b)^{7/3}\,z-9\,C^{3}\,b^{2},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*C + C*(-b)^(2/3)*x^2)/(b*x^3 - 8),x)
```

```
[Out] symsum(log((8*C^2)/(-b)^(5/3) + root(27*b^3*z^3 - 27*C*(-b)^(8/3)*z^2 - 9*C
^2*(-b)^(7/3)*z - 9*C^3*b^2, z, k)*((48*C)/(-b)^(4/3) - (72*root(27*b^3*z^3
 - 27*C*(-b)^(8/3)*z^2 - 9*C^2*(-b)^(7/3)*z - 9*C^3*b^2, z, k))/b + (24*C*x
)/b) - (8*C^2*x)/(-b)^(4/3))*root(27*b^3*z^3 - 27*C*(-b)^(8/3)*z^2 - 9*C^2*
(-b)^(7/3)*z - 9*C^3*b^2, z, k), k, 1, 3)
```

sympy [A] time = 0.99, size = 58, normalized size = 1.02

$$\operatorname{RootSum}\left(3t^{3}b^{2}-3t^{2}Cb\,(-b)^{\frac{2}{3}}+tC^{2}\,(-b)^{\frac{4}{3}}-C^{3}b,\left(t\mapsto t\log\left(-\frac{3t}{C}+x+\frac{(-b)^{\frac{2}{3}}}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)**(2/3)*C*x**2)/(b*x**3-8),x)
```

```
[Out] RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C**3
*b, Lambda(_t, _t*log(-3*_t/C + x + (-b)**(2/3)/b)))
```

3.32 $\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx$

Optimal. Leaf size=47

$$\frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1864, 31, 617, 204}

$$\frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

```
[Out] (C*ArcTan[(1 - (4*x)/(-a)^(1/3))/Sqrt[3]])/(2*Sqrt[3]) - (C*Log[(-a)^(1/3)
+ 2*x])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1864

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)},
Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2)
, x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C, 0
]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx = -\left(\frac{1}{4}C\int \frac{1}{\frac{\sqrt[3]{-a}}{2}+x}\,dx\right) - \frac{1}{8}\left(\sqrt[3]{-a}\,C\right)\int \frac{1}{\frac{1}{4}(-a)^{2/3}-\frac{1}{2}\sqrt[3]{-a}\,x+x^2}\,dx$$
$$= -\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right) - \frac{1}{2}C\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{4x}{\sqrt[3]{-a}}\right)$$
$$= \frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

Mathematica [B] time = 0.04, size = 106, normalized size = 2.26

$$\frac{C\left(-a^{2/3}\log\left(8x^3-a\right)+(-a)^{2/3}\log\left(a^{2/3}+2\sqrt[3]{a}\,x+4x^2\right)-2(-a)^{2/3}\log\left(\sqrt[3]{a}-2x\right)+2\sqrt{3}\,(-a)^{2/3}\tan^{-1}\left(\frac{\frac{4x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{12a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(-a)^(2/3)*ArcTan[(1 + (4*x)/a^(1/3))/Sqrt[3]] - 2*(-a)^(2/3)
*Log[a^(1/3) - 2*x] + (-a)^(2/3)*Log[a^(2/3) + 2*a^(1/3)*x + 4*x^2] - a^(2/
3)*Log[-a + 8*x^3]))/(12*a^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

```
[Out] IntegrateAlgebraic[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3), x]
```

fricas [A] time = 0.43, size = 43, normalized size = 0.91

$$\frac{1}{6}\,\sqrt{3}\,C\arctan\left(\frac{4\,\sqrt{3}\,(-a)^{\frac{2}{3}}\,x+\sqrt{3}\,a}{3\,a}\right) - \frac{1}{4}\,C\log\left(2\,x+(-a)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*(-a)^(2/3)*x + sqrt(3)*a)/a) - 1/4*C*lo
g(2*x + (-a)^(1/3))
```

giac [B] time = 0.21, size = 98, normalized size = 2.09

$$\frac{\sqrt{3}\left(\sqrt{3}\,i|a|-a\right)C\arctan\left(\frac{\sqrt{3}\left(4\,x+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{12\,a} + \frac{\left(\sqrt{3}\,i|a|-3\,a\right)C\log\left(x^2+\frac{1}{2}\,a^{\frac{1}{3}}x+\frac{1}{4}\,a^{\frac{2}{3}}\right)}{24\,a} - \frac{\left(2\,C\,(-a)^{\frac{2}{3}}+Ca^{\frac{2}{3}}\right)\log\left(\left|x-\frac{1}{2}\,a^{\frac{1}{3}}\right|\right)}{12\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*i*abs(a) - a)*C*arctan(1/3*sqrt(3)*(4*x + a^(1/3))/a^
(1/3))/a + 1/24*(sqrt(3)*i*abs(a) - 3*a)*C*log(x^2 + 1/2*a^(1/3)*x + 1/4*a^
(2/3))/a - 1/12*(2*C*(-a)^(2/3) + C*a^(2/3))*log(abs(x - 1/2*a^(1/3)))/a^(2
/3)
```

maple [B] time = 0.05, size = 110, normalized size = 2.34

$$-\frac{C\ln\left(8x^{3}-a\right)}{12}+\frac{(-a)^{\frac{2}{3}}\,8^{\frac{2}{3}}\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(\frac{2\,8^{\frac{1}{3}}x}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{24a^{\frac{2}{3}}}-\frac{(-a)^{\frac{2}{3}}\,8^{\frac{2}{3}}C\ln\left(x-\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}}{8}\right)}{24a^{\frac{2}{3}}}+\frac{(-a)^{\frac{2}{3}}\,8^{\frac{2}{3}}C\ln\left(x^{2}+\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}x}{8}+\frac{8^{\frac{1}{3}}a^{\frac{2}{3}}}{8}\right)}{48a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x)
```

```
[Out] -1/24*C*(-a)^(2/3)*8^(2/3)/a^(2/3)*ln(x-1/8*8^(2/3)*a^(1/3))+1/48*C*(-a)^(2
/3)*8^(2/3)/a^(2/3)*ln(x^2+1/8*8^(2/3)*a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*
C*(-a)^(2/3)*8^(2/3)/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*
x+1))-1/12*C*ln(8*x^3-a)
```

maxima [B] time = 2.99, size = 93, normalized size = 1.98

$$\frac{\sqrt{3}\,C\,(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(4\,x+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{6\,a^{\frac{2}{3}}}+\frac{\left(C\,(-a)^{\frac{2}{3}}-Ca^{\frac{2}{3}}\right)\log\left(4\,x^{2}+2\,a^{\frac{1}{3}}x+a^{\frac{2}{3}}\right)}{12\,a^{\frac{2}{3}}}-\frac{\left(2\,C\,(-a)^{\frac{2}{3}}+Ca^{\frac{2}{3}}\right)\log\left(x-\frac{1}{2}\,a^{\frac{1}{3}}\right)}{12\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*C*(-a)^(2/3)*arctan(1/3*sqrt(3)*(4*x + a^(1/3))/a^(1/3))/a^(2/3
) + 1/12*(C*(-a)^(2/3) - C*a^(2/3))*log(4*x^2 + 2*a^(1/3)*x + a^(2/3))/a^(2
/3) - 1/12*(2*C*(-a)^(2/3) + C*a^(2/3))*log(x - 1/2*a^(1/3))/a^(2/3)
```

mupad [B] time = 0.33, size = 142, normalized size = 3.02

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C+12\,\mathrm{root}\left(1728\,a^{2}\,z^{3}+432\,C\,a^{2}\,z^{2}+36\,C^{2}\,a^{2}\,z+9\,C^{3}\,a^{2},z,k\right)\right)\left(C\,a+\mathrm{root}\left(1728\,a^{2}\,z^{3}+432\,C\,a^{2}\,z^{2}+36\,C^{2}\,a^{2}\,z+9\,C^{3}\,a^{2},z,k\right)\,a\,12+4\,C\,(-a)^{2/3}\,x\right)}{128}\right)\mathrm{root}\left(1728\,a^{2}\,z^{3}+432\,C\,a^{2}\,z^{2}+36\,C^{2}\,a^{2}\,z+9\,C^{3}\,a^{2},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*C*x^2 + C*(-a)^(2/3))/(a - 8*x^3),x)
```

```
[Out] symsum(log(-((C + 12*root(1728*a^2*z^3 + 432*C*a^2*z^2 + 36*C^2*a^2*z + 9*C
^3*a^2, z, k))*(C*a + 12*root(1728*a^2*z^3 + 432*C*a^2*z^2 + 36*C^2*a^2*z +
 9*C^3*a^2, z, k)*a + 4*C*(-a)^(2/3)*x))/128)*root(1728*a^2*z^3 + 432*C*a^2
*z^2 + 36*C^2*a^2*z + 9*C^3*a^2, z, k), k, 1, 3)
```

sympy [C] time = 0.93, size = 95, normalized size = 2.02

$$-C\left(\frac{\log\left(-\frac{a}{2(-a)^{\frac{2}{3}}}+x\right)}{4}+\frac{\sqrt{3}\,i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{4(-a)^{\frac{2}{3}}}+x\right)}{12}-\frac{\sqrt{3}\,i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{4(-a)^{\frac{2}{3}}}+x\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)**(2/3)*C+2*C*x**2)/(-8*x**3+a),x)
```

```
[Out] -C*(log(-a/(2*(-a)**(2/3)) + x)/4 + sqrt(3)*I*log(a/(4*(-a)**(2/3)) - sqrt(
3)*I*a/(4*(-a)**(2/3)) + x)/12 - sqrt(3)*I*log(a/(4*(-a)**(2/3)) + sqrt(3)*
I*a/(4*(-a)**(2/3)) + x)/12)
```

3.33 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=50

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1867, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(
1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b}+\frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$

$$=\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$

$$=-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] time = 0.05, size = 146, normalized size = 2.92

$$\frac{C\left(-b^{2/3}\left(\frac{a}{b}\right)^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^{2/3}\log\left(a+bx^3\right)+2b^{2/3}\left(\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\,b^{2/3}\left(\frac{a}{b}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(2/3)*b^(2/3)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b*x^3]))/(
3*a^(2/3)*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3), x]
```

fricas [A] time = 0.43, size = 52, normalized size = 1.04

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+3\,C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 3*
C*log(x + (a/b)^(1/3)))/b
```

giac [B] time = 0.22, size = 166, normalized size = 3.32

$$\frac{\sqrt{3}\left(ab^2+\sqrt{3}\sqrt{a^2b^4}\,i\right)C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}+\frac{\left(3\,ab^2+\sqrt{3}\sqrt{a^2b^4}\,i\right)C\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(a*b^2 + sqrt(3)*sqrt(a^2*b^4)*i)*C*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) - 1/3*(C*b^2*(-a/b)^(2/3) + 2*(a*b^2)^(2
/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/6*(3*a*b^2 + sqr
t(3)*sqrt(a^2*b^4)*i)*C*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

maple [A] time = 0.05, size = 87, normalized size = 1.74

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b}+\frac{2C\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}-\frac{C\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b}+\frac{C\ln\left(b\,x^{3}+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*ln(x+(a/b)^(1/3))/b-1/3*C/b*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/3*C/b
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)
```

maxima [A] time = 3.03, size = 51, normalized size = 1.02

$$\frac{2\,\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}+\frac{C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/b + C*log
(x + (a/b)^(1/3))/b
```

mupad [B] time = 5.10, size = 172, normalized size = 3.44

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C-\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z-9\,C^3\,a^2,z,k\right)\,b\,3\right)\,\left(-C\,a+\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z-9\,C^3\,a^2,z,k\right)\,a\,b\,3+2\,C\,b\,x\,{\left(\frac{a}{b}\right)}^{2/3}\right)}{b^3}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z-9\,C^3\,a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2 + 2*C*(a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log(-((C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
- 9*C^3*a^2, z, k)*b)*(3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2
*b*z - 9*C^3*a^2, z, k)*a*b - C*a + 2*C*b*x*(a/b)^(2/3)))/b^3)*root(27*a^2*
b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z - 9*C^3*a^2, z, k), k, 1, 3)
```

sympy [C] time = 0.74, size = 100, normalized size = 2.00

$$\frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) - sqrt
(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

3.34 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1867, 31, 617, 204}

$$\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a/
b))^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3}\, dx = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\, dx}{b} - \frac{\left(\sqrt[3]{-\frac{a}{b}}\, C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3} - \sqrt[3]{-\frac{a}{b}}\, x + x^2}\, dx}{b}$$
$$= -\frac{C\log\left(\sqrt[3]{-\frac{a}{b}} + x\right)}{b} - \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\, dx, x, 1 - \frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$
$$= \frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\, b} - \frac{C\log\left(\sqrt[3]{-\frac{a}{b}} + x\right)}{b}$$

Mathematica [B] time = 0.10, size = 150, normalized size = 2.83

$$\frac{C\left(b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{b}\, x + b^{2/3}x^2\right) - a^{2/3}\log\left(a - bx^3\right) - 2b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a} - \sqrt[3]{b}\, x\right) + 2\sqrt{3}\, b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\, x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] - 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^(2/3)*
b^(2/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*
x^3]))/(3*a^(2/3)*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3), x]
```

fricas [A] time = 0.43, size = 53, normalized size = 1.00

$$\frac{2\sqrt{3}\, C\arctan\left(\frac{2\sqrt{3}\, bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} + \sqrt{3}\, a}{3\, a}\right) - 3\, C\log\left(x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a) - 3
*C*log(x + (-a/b)^(1/3)))/b
```

giac [B] time = 0.22, size = 162, normalized size = 3.06

$$-\frac{\sqrt{3}\left(ab^2 - \sqrt{3}\sqrt{a^2b^4}\, i\right)C\arctan\left(\frac{\sqrt{3}\left(2\, x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\, ab^3} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\left(-ab^2\right)^{\frac{2}{3}} C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\, ab^2} - \frac{\left(3\, ab^2 - \sqrt{3}\sqrt{a^2b^4}\, i\right)C\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\, ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(a*b^2 - sqrt(3)*sqrt(a^2*b^4)*i)*C*arctan(1/3*sqrt(3)*(2*x +
(a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/3*(C*b^2*(a/b)^(2/3) + 2*(-a*b^2)^(2/
3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*b^2) - 1/6*(3*a*b^2 - sqrt(3
)*sqrt(a^2*b^4)*i)*C*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3)
```

maple [B] time = 0.05, size = 135, normalized size = 2.55

$$\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\sqrt{3}\,C\arctan\left(\frac{\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)\sqrt{3}}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}C\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}C\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{C\ln\left(b\,x^{3}-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C*(-a/b)^(2/3)/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/3*C*(-a/b)^(2/3)/b/(a
/b)^(2/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+2/3*C*(-a/b)^(2/3)/b/(a/b)^(2/3
)*3^(1/2)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2))-1/3*C/b*ln(b*x^3-a)
```

maxima [B] time = 3.03, size = 167, normalized size = 3.15

$$-\frac{2\sqrt{3}\left(Ca-\left(3C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}-\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x^{2}+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*(C*a - (3*C*(a/b)^(1/3)*(-a/b)^(2/3) + C*a/b)*b)*arctan(1/3*sq
rt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/3*(C*(a/b)^(2/3) - C*(-a/b
)^(2/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(C*(a
/b)^(2/3) + 2*C*(-a/b)^(2/3))*log(x - (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 5.40, size = 172, normalized size = 3.25

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C+\mathrm{root}\left(27\,a^{2}\,b^{3}\,z^{3}+27\,C\,a^{2}\,b^{2}\,z^{2}+9\,C^{2}\,a^{2}\,b\,z+9\,C^{3}\,a^{2},z,k\right)\,b\,3\right)\left(C\,a+\mathrm{root}\left(27\,a^{2}\,b^{3}\,z^{3}+27\,C\,a^{2}\,b^{2}\,z^{2}+9\,C^{2}\,a^{2}\,b\,z+9\,C^{3}\,a^{2},z,k\right)\,a\,b\,3+2\,C\,b\,x\left(-\frac{a}{b}\right)^{2/3}\right)}{b^{3}}\right)\mathrm{root}\left(27\,a^{2}\,b^{3}\,z^{3}+27\,C\,a^{2}\,b^{2}\,z^{2}+9\,C^{2}\,a^{2}\,b\,z+9\,C^{3}\,a^{2},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2 + 2*C*(-a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log(-((C + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
+ 9*C^3*a^2, z, k)*b)*(C*a + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C
^2*a^2*b*z + 9*C^3*a^2, z, k)*a*b + 2*C*b*x*(-a/b)^(2/3)))/b^3)*root(27*a^2
*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z + 9*C^3*a^2, z, k), k, 1, 3)
```

sympy [C] time = 0.84, size = 110, normalized size = 2.08

$$-\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3)) - s
qrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

3.35 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=54

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1869, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(a
/b))^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=-\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b}-\frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}\,x+x^2}\,dx}{b}$$

$$=\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}+\frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$=-\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] time = 0.07, size = 149, normalized size = 2.76

$$\frac{C\left(-b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+a^{2/3}\log\left(a+bx^3\right)+2b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt{3}\,b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] + 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (-(a/b))^(2/3)
*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b
*x^3]))/(3*a^(2/3)*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3), x]
```

fricas [A] time = 0.43, size = 56, normalized size = 1.04

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+3\,C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 3
*C*log(x - (-a/b)^(1/3)))/b
```

giac [A] time = 0.18, size = 91, normalized size = 1.69

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/
3*(C*b^2*(-a/b)^(2/3) + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b^2)
```

maple [B] time = 0.04, size = 132, normalized size = 2.44

$$\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}C\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}C\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{C\ln\left(b\,x^{3}+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*(-a/b)^(2/3)/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/3*C*(-a/b)^(2/3)/b/(a/
b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/3*C*(-a/b)^(2/3)/b/(a/b)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)
```

maxima [B] time = 3.15, size = 168, normalized size = 3.11

$$-\frac{2\sqrt{3}\left(Ca-\left(3C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*(C*a - (3*C*(a/b)^(1/3)*(-a/b)^(2/3) + C*a/b)*b)*arctan(1/3*sq
rt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/3*(C*(a/b)^(2/3) - C*(-a/b
)^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(C*(a
/b)^(2/3) + 2*C*(-a/b)^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 5.27, size = 173, normalized size = 3.20

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C-\mathrm{root}\left(27\,a^{2}\,b^{3}\,z^{3}-27\,C\,a^{2}\,b^{2}\,z^{2}+9\,C^{2}\,a^{2}\,b\,z-9\,C^{3}\,a^{2},z,k\right)\,b\,3\right)\left(-C\,a+\mathrm{root}\left(27\,a^{2}\,b^{3}\,z^{3}-27\,C\,a^{2}\,b^{2}\,z^{2}+9\,C^{2}\,a^{2}\,b\,z-9\,C^{3}\,a^{2},z,k\right)\,a\,b\,3+2\,C\,b\,x\left(-\frac{a}{b}\right)^{2/3}\right)}{b^{3}}\right)\mathrm{root}\left(27\,a^{2}\,b^{3}\,z^{3}-27\,C\,a^{2}\,b^{2}\,z^{2}+9\,C^{2}\,a^{2}\,b\,z-9\,C^{3}\,a^{2},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2 + 2*C*(-a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log(-((C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
- 9*C^3*a^2, z, k)*b)*(3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2
*b*z - 9*C^3*a^2, z, k)*a*b - C*a + 2*C*b*x*(-a/b)^(2/3)))/b^3)*root(27*a^2
*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z - 9*C^3*a^2, z, k), k, 1, 3)
```

sympy [C] time = 0.77, size = 109, normalized size = 2.02

$$\frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3)) - sq
rt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

3.36 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1869, 31, 617, 204}

$$\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(1
/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3}\, dx = \frac{C \int \frac{1}{\sqrt[3]{\frac{a}{b}} - x}\, dx}{b} + \frac{\left(\sqrt[3]{\frac{a}{b}}\, C\right) \int \frac{1}{\left(\frac{a}{b}\right)^{2/3} + \sqrt[3]{\frac{a}{b}}\, x + x^2}\, dx}{b}$$

$$= -\frac{C \log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b} - \frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\, dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$

$$= \frac{2C \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\, b} - \frac{C \log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b}$$

Mathematica [B] time = 0.05, size = 147, normalized size = 2.77

$$\frac{C\left(b^{2/3}\left(\frac{a}{b}\right)^{2/3} \log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{b}\, x + b^{2/3}x^2\right) - a^{2/3} \log\left(a - bx^3\right) - 2b^{2/3}\left(\frac{a}{b}\right)^{2/3} \log\left(\sqrt[3]{a} - \sqrt[3]{b}\, x\right) + 2\sqrt{3}\, b^{2/3}\left(\frac{a}{b}\right)^{2/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\, x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] - 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (a/b)^(2/3)*b^(2/3)*L
og[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*x^3]))/(3
*a^(2/3)*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3), x]
```

fricas [A] time = 0.43, size = 53, normalized size = 1.00

$$\frac{2\sqrt{3}\, C \arctan\left(\frac{2\sqrt{3}\, bx\left(\frac{a}{b}\right)^{\frac{2}{3}} + \sqrt{3}\, a}{3\, a}\right) - 3\, C \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a) - 3*
C*log(x - (a/b)^(1/3)))/b
```

giac [A] time = 0.21, size = 85, normalized size = 1.60

$$\frac{2\sqrt{3}\, C \arctan\left(\frac{\sqrt{3}\left(2\, x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\, b} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\left(ab^2\right)^{\frac{2}{3}} C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\, ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - 1/3*(
C*b^2*(a/b)^(2/3) + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3))
)/(a*b^2)
```

maple [A] time = 0.05, size = 90, normalized size = 1.70

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)\sqrt{3}}{3}\right)}{3b}-\frac{2C\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}+\frac{C\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b}-\frac{C\ln\left(b\,x^{3}-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C/b*ln(x-(a/b)^(1/3))+1/3*C/b*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+2/3*C*
arctan(1/3*(2/(a/b)^(1/3)*x+1)*3^(1/2))/b*3^(1/2)-1/3*C/b*ln(b*x^3-a)
```

maxima [A] time = 3.00, size = 52, normalized size = 0.98

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - C*log
(x - (a/b)^(1/3))/b
```

mupad [B] time = 5.19, size = 171, normalized size = 3.23

$$\sum_{k=1}^{3}\ln\left(-\frac{\left(C+\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z+9\,C^3\,a^2,z,k\right)\,b\,3\right)\,\left(C\,a+\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z+9\,C^3\,a^2,z,k\right)\,a\,b\,3+2\,C\,b\,x\,{\left(\frac{a}{b}\right)}^{2/3}\right)}{b^3}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z+9\,C^3\,a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2 + 2*C*(a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log(-((C + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
+ 9*C^3*a^2, z, k)*b)*(C*a + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C
^2*a^2*b*z + 9*C^3*a^2, z, k)*a*b + 2*C*b*x*(a/b)^(2/3)))/b^3)*root(27*a^2*
b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z + 9*C^3*a^2, z, k), k, 1, 3)
```

sympy [C] time = 0.79, size = 102, normalized size = 1.92

$$-\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) - sqr
t(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

3.37 $\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=61

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1863, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3))
+ (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx = \frac{\left(\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}} + \frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}}$$

$$= \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Mathematica [A] time = 0.02, size = 95, normalized size = 1.56

$$\frac{C\left(-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\log\left(a+bx^3\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3)
+ b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + Log[a + b*x
^3]))/(3*b^(1/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3), x]
```

fricas [A] time = 0.46, size = 160, normalized size = 2.62

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2bx^3-3a^{\frac{2}{3}}b^{\frac{1}{3}}x+3\sqrt{\frac{1}{3}}\left(2a^{\frac{1}{3}}bx^2+a^{\frac{2}{3}}b^{\frac{2}{3}}x-ab^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-a}{bx^3+a}\right)+Cb^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b}, \frac{2\sqrt{\frac{1}{3}}Cb^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}b^{\frac{2}{3}}x-ab^{\frac{1}{3}}\right)}{ab^{\frac{1}{3}}}\right)+Cb^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqr
t(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*b^(2/3)*x - a*b^(1/3))*sqrt(-1/b^(2/3)) -
 a)/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b, (2*sqrt(1/3)*C*
b^(2/3)*arctan(sqrt(1/3)*(2*a^(2/3)*b^(2/3)*x - a*b^(1/3))/(a*b^(1/3))) + C
*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.06, size = 117, normalized size = 1.92

$$\frac{2\sqrt{3}\, C\, a^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{2C\, a^{\frac{2}{3}} \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{C\, a^{\frac{2}{3}} \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{C \ln\left(b\, x^3+a\right)}{3b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*a^(2/3)/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/3*C*a^(2/3)/b/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/3*C*a^(2/3)/b/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^3+a)
```

maxima [B] time = 2.87, size = 162, normalized size = 2.66

$$-\frac{2\sqrt{3}\left(Cab^{\frac{2}{3}}-\left(3\, Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca}{b^{\frac{1}{3}}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\, x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\, ab}+\frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-Ca^{\frac{2}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\, b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\, Ca^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\, b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*(C*a*b^(2/3) - (3*C*a^(2/3)*(a/b)^(1/3) + C*a/b^(1/3))*b)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/3*(C*b^(2/3)*(a/b
)^(2/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3))
 + 1/3*(C*b^(2/3)*(a/b)^(2/3) + 2*C*a^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^
(2/3))
```

mupad [B] time = 5.31, size = 193, normalized size = 3.16

$$\sum_{k=1}^{3}\ln\left(-\frac{a^{2/3}\left(C-\mathrm{root}\left(27\, a^2\, b^3\, z^3-27\, C\, a^2\, b^{8/3}\, z^2+9\, C^2\, a^2\, b^{7/3}\, z-9\, C^3\, a^2\, b^2, z, k\right) b^{1/3}\, 3\right)\left(-C\, a^{1/3}+\mathrm{root}\left(27\, a^2\, b^3\, z^3-27\, C\, a^2\, b^{8/3}\, z^2+9\, C^2\, a^2\, b^{7/3}\, z-9\, C^3\, a^2\, b^2, z, k\right) a^{1/3}\, b^{1/3}\, 3+2\, C\, b^{1/3}\, x\right)}{b^{5/3}}\right)\mathrm{root}\left(27\, a^2\, b^3\, z^3-27\, C\, a^2\, b^{8/3}\, z^2+9\, C^2\, a^2\, b^{7/3}\, z-9\, C^3\, a^2\, b^2, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*C*a^(2/3) + C*b^(2/3)*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(-(a^(2/3)*(C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^(8/3)*z^2 + 9*
C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k)*b^(1/3))*(3*root(27*a^2*b^3*z^3 -
27*C*a^2*b^(8/3)*z^2 + 9*C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k)*a^(1/3)*b
^(1/3) - C*a^(1/3) + 2*C*b^(1/3)*x))/b^(5/3))*root(27*a^2*b^3*z^3 - 27*C*a^
2*b^(8/3)*z^2 + 9*C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

sympy [A] time = 0.73, size = 70, normalized size = 1.15

$$\operatorname{RootSum}\left(3t^3b^{\frac{5}{3}}-3t^2Cb^{\frac{4}{3}}+tC^2b-C^3b^{\frac{2}{3}},\left(t\mapsto t\log\left(x+\frac{3t\sqrt[3]{a}\,\sqrt[3]{b}-C\sqrt[3]{a}}{2C\sqrt[3]{b}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a**(2/3)*C+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
Lambda(_t, _t*log(x + (3*_t*a**(1/3)*b**(1/3) - C*a**(1/3))/(2*C*b**(1/3)))
))
```

3.38 $\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1866, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(-b)^(
1/3)) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1866

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, -Di
st[C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2),
x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /; Fr
eeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = -\frac{\left(\sqrt[3]{a}\,C\right)\int \frac{1}{\frac{a^{2/3}}{(-b)^{2/3}}+\frac{\sqrt[3]{a}\,x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}} - \frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}}-x}\,dx}{\sqrt[3]{-b}}$$

$$= \frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}} + \frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{-b}\,x}{\sqrt[3]{a}}\right)}{\sqrt[3]{-b}}$$

$$= -\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}} + \frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\,x\right)}{\sqrt[3]{-b}}$$

Mathematica [A] time = 0.03, size = 116, normalized size = 1.66

$$-\frac{C\left(-b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+(-b)^{2/3}\log\left(a+bx^3\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] -1/3*(C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2
*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2] + (-b)^(2/3)*Log[a + b*x^3]))/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3), x]
```

fricas [A] time = 0.45, size = 205, normalized size = 2.93

$$\left[\frac{\sqrt{\frac{1}{3}}\,Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{2bx^3+3a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x-3\sqrt{\frac{1}{3}}\left(2a^{\frac{1}{3}}bx^2+a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x+a(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}-a}{bx^3+a}\right)-C(-b)^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}(-b)^{\frac{2}{3}}\right)}{b},\ -\frac{2\sqrt{\frac{1}{3}}\,Cb\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x+a(-b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{a}\right)+C(-b)^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}(-b)^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x -
3*sqrt(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*(-b)^(2/3)*x + a*(-b)^(1/3))*sqrt((-
b)^(1/3)/b) - a)/(b*x^3 + a)) - C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/3)))
/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(1/3)/b)*arctan(sqrt(1/3)*(2*a^(2/3)*(-b)^
(2/3)*x + a*(-b)^(1/3))*sqrt(-(-b)^(1/3)/b)/a) + C*(-b)^(2/3)*log(b*x + a^(
1/3)*(-b)^(2/3)))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.05, size = 122, normalized size = 1.74

$$-\frac{2\sqrt{3}\,C\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{2C\,a^{\frac{2}{3}}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{C\,a^{\frac{2}{3}}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{(-b)^{\frac{2}{3}}\,C\ln\left(b\,x^{3}+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] -2/3/(a/b)^(2/3)*C*a^(2/3)/b*ln(x+(a/b)^(1/3))+1/3/(a/b)^(2/3)*C*a^(2/3)/b*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-2/3/(a/b)^(2/3)*3^(1/2)*C*a^(2/3)/b*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*C*(-b)^(2/3)/b*ln(b*x^3+a)
```

maxima [B] time = 3.02, size = 173, normalized size = 2.47

$$\frac{2\sqrt{3}\left(Ca\,(-b)^{\frac{2}{3}}-\left(3\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca(-b)^{\frac{2}{3}}}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}-\frac{\left(C\,(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-Ca^{\frac{2}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(C\,(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,Ca^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*(C*a*(-b)^(2/3) - (3*C*a^(2/3)*(a/b)^(1/3) + C*a*(-b)^(2/3)/b)*
b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/3*(C*(-b)^
(2/3)*(a/b)^(2/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a
/b)^(2/3)) - 1/3*(C*(-b)^(2/3)*(a/b)^(2/3) + 2*C*a^(2/3))*log(x + (a/b)^(1/
3))/(b*(a/b)^(2/3))
```

mupad [B] time = 5.24, size = 221, normalized size = 3.16

$$\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,a^{2}\,b^{3}\,z^{3}+27\,C\,a^{2}\,(-b)^{8/3}\,z^{2}-9\,C^{2}\,a^{2}\,(-b)^{7/3}\,z+9\,C^{3}\,a^{2}\,b^{2},z,k\right)\left(\frac{6\,C\,a}{(-b)^{4/3}}+\frac{\mathrm{root}\left(27\,a^{2}\,b^{3}\,z^{3}+27\,C\,a^{2}\,(-b)^{8/3}\,z^{2}-9\,C^{2}\,a^{2}\,(-b)^{7/3}\,z+9\,C^{3}\,a^{2}\,b^{2},z,k\right)\,a\,9}{b}-\frac{6\,C\,a^{2/3}\,x}{b}\right)-\frac{C^{2}\,a}{(-b)^{5/3}}-\frac{2\,C^{2}\,a^{2/3}\,x}{(-b)^{4/3}}\right)\mathrm{root}\left(27\,a^{2}\,b^{3}\,z^{3}+27\,C\,a^{2}\,(-b)^{8/3}\,z^{2}-9\,C^{2}\,a^{2}\,(-b)^{7/3}\,z+9\,C^{3}\,a^{2}\,b^{2},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*C*a^(2/3) + C*(-b)^(2/3)*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(
7/3)*z + 9*C^3*a^2*b^2, z, k)*((6*C*a)/(-b)^(4/3) + (9*root(27*a^2*b^3*z^3
+ 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(7/3)*z + 9*C^3*a^2*b^2, z, k)*a
)/b - (6*C*a^(2/3)*x)/b) - (C^2*a)/(-b)^(5/3) - (2*C^2*a^(2/3)*x)/(-b)^(4/3
))*root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(7/3)*z +
 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

sympy [A] time = 1.24, size = 73, normalized size = 1.04

$$-\operatorname{RootSum}\left(3t^{3}b^{2}-3t^{2}Cb\,(-b)^{\frac{2}{3}}+tC^{2}\,(-b)^{\frac{4}{3}}-C^{3}b,\left(t\mapsto t\log\left(\frac{3t\sqrt[3]{a}}{2C}-\frac{\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a**(2/3)*C-(-b)**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] -RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C**
3*b, Lambda(_t, _t*log(3*_t*a**(1/3)/(2*C) - a**(1/3)*(-b)**(2/3)/(2*b) + x
)))
```

3.39 $\int \frac{-3+x^2}{-1+x^3} \, dx$

Optimal. Leaf size=40

$$\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1875, 31, 634, 618, 204, 628}

$$\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x^2)/(-1 + x^3),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - (2*Log[1 - x])/3 + (5*Log[1 + x + x^2])
/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-3+x^2}{-1+x^3}\,dx &= -\left(\frac{1}{3}\int \frac{-7-5x}{1+x+x^2}\,dx\right)+\frac{2}{3}\int \frac{1}{1-x}\,dx\\
&= -\frac{2}{3}\log(1-x)+\frac{5}{6}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{3}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= -\frac{2}{3}\log(1-x)+\frac{5}{6}\log\left(1+x+x^2\right)-3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{2}{3}\log(1-x)+\frac{5}{6}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.25

$$\frac{1}{3}\log\left(1-x^3\right)+\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x^2)/(-1 + x^3),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[1 - x] + Log[1 + x + x^2]/2 + Log[1
 - x^3]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3+x^2}{-1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + x^2)/(-1 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(-3 + x^2)/(-1 + x^3), x]
```

fricas [A] time = 0.43, size = 31, normalized size = 0.78

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="fricas")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
1)
```

giac [A] time = 0.32, size = 32, normalized size = 0.80

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(abs(
x - 1))
```

maple [A] time = 0.05, size = 32, normalized size = 0.80

$$\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)-\frac{2\ln(x-1)}{3}+\frac{5\ln\left(x^2+x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3)/(x^3-1),x)
```

```
[Out] -2/3*ln(x-1)+5/6*ln(x^2+x+1)+3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.99, size = 31, normalized size = 0.78

$$\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
1)
```

mupad [B] time = 0.16, size = 46, normalized size = 1.15

$$-\frac{2\ln(x-1)}{3}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{5}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 3)/(x^3 - 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/2 + 5/6) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*1i)/2 - 5/6) - (2*log(x - 1))/3
```

sympy [A] time = 0.29, size = 42, normalized size = 1.05

$$-\frac{2\log(x-1)}{3}+\frac{5\log\left(x^2+x+1\right)}{6}+\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3)/(x**3-1),x)
```

```
[Out] -2*log(x - 1)/3 + 5*log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)
```

3.40 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {1863, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(a + b*
x^3),x]
```

```
[Out] (-2*(B/a^(1/3) + C/b^(1/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/Sqrt[3] + (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx = \frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}} + \frac{\left(\sqrt[3]{b}B+\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}}$$

$$= \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \left(2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x$$

$$= -\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} + \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Mathematica [A] time = 0.05, size = 122, normalized size = 1.74

$$\frac{\sqrt[3]{a}C\left(-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\log\left(a+bx^3\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}C+\sqrt[3]{b}B\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(
a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*B + a^(1/3)*C)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + a^(1/3)*C*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2] + Log[a + b*x^3]))/(3*a^(1/3)*b^(1/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)
*C*x^2)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)
*C*x^2)/(a + b*x^3), x]
```

fricas [B] time = 3.30, size = 430, normalized size = 6.14

$$\left[\frac{\sqrt{\frac{1}{3}}\,b\sqrt{-\frac{C^2ab^{\frac{1}{3}}+2BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^2a^{\frac{1}{3}}b}{ab}}\log\left(-\frac{C^3a^2+B^3ab-2\left(C^3ab+B^3b^2\right)x^3+3\left(C^3a+B^3b\right)a^{\frac{2}{3}}b^{\frac{1}{3}}x-3\sqrt{\frac{1}{3}}\left(\left(2B^2bx^2+C^2ax+BCa\right)a^{\frac{2}{3}}b^{\frac{2}{3}}+\left(2C^2abx^2-BCabx-B^2ab\right)a^{\frac{1}{3}}-\left(2BCabx^2-B^2abx+C^2a^2\right)b^{\frac{1}{3}}\right)\sqrt{-\frac{C^2ab^{\frac{1}{3}}+2BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^2a^{\frac{1}{3}}b}{ab}}}{bx^3+a}\right)+Cb^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b}, \frac{2\sqrt{\frac{1}{3}}\,b\sqrt{\frac{C^2ab^{\frac{1}{3}}+2BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^2a^{\frac{1}{3}}b}{ab}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(\left(2C^2x+BC\right)a^{\frac{2}{3}}b^{\frac{2}{3}}-\left(2BCbx+B^2b\right)a^{\frac{1}{3}}+\left(2B^2bx-C^2a\right)b^{\frac{1}{3}}\right)\sqrt{\frac{C^2ab^{\frac{1}{3}}+2BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^2a^{\frac{1}{3}}b}{ab}}}{C^3a+B^3b}\right)+Cb^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*b*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)
/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 + 3*(C^3*a + B^
3*b)*a^(2/3)*b^(1/3)*x - 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*a)*a^(2/
3)*b^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) - (2*B*C*a*b*x^2
 - B^2*a*b*x + C^2*a^2)*b^(1/3))*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/
3) + B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2
/3)))/b, (2*sqrt(1/3)*b*sqrt((C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a
^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*x + B*C)*a^(2/3)*b^(2/3) - (2*B*C
```

```
*b*x + B^2*b)*a^(1/3) + (2*B^2*b*x - C^2*a)*b^(1/3))*sqrt((C^2*a*b^(1/3) +
2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b))/(C^3*a + B^3*b)) + C*b^(2/3)*
log(b*x + a^(1/3)*b^(2/3)))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.06, size = 310, normalized size = 4.43

$$\frac{\sqrt{3}\, B\, a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{\frac{2}{3}}}+\frac{B\, a^{\frac{1}{3}} \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{\frac{2}{3}}}-\frac{B\, a^{\frac{1}{3}} \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{\frac{2}{3}}}+\frac{\sqrt{3}\, B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{\frac{1}{3}}}-\frac{B \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{\frac{1}{3}}}+\frac{B \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{\frac{1}{3}}}+\frac{2\sqrt{3}\, C\, a^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{2C\, a^{\frac{2}{3}} \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{C\, a^{\frac{2}{3}} \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{C \ln\left(b x^{3}+a\right)}{3 b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] 1/3/(a/b)^(2/3)*B*a^(1/3)/b^(2/3)*ln(x+(a/b)^(1/3))+2/3/(a/b)^(2/3)*C*a^(2/
3)/b*ln(x+(a/b)^(1/3))-1/6/(a/b)^(2/3)*B*a^(1/3)/b^(2/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))-1/3/(a/b)^(2/3)*C*a^(2/3)/b*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+1/3/(a/b)^(2/3)*3^(1/2)*B*a^(1/3)/b^(2/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))+2/3/(a/b)^(2/3)*3^(1/2)*C*a^(2/3)/b*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))-1/3/(a/b)^(1/3)*B/b^(1/3)*ln(x+(a/b)^(1/3))+1/6/(a/b)^(1/3)*B/b^
(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/(a/b)^(1/3)*B/b^(1/3)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^3+a)
```

maxima [B] time = 3.12, size = 236, normalized size = 3.37

$$-\frac{\sqrt{3}\left(2\, C a b^{\frac{2}{3}}-\left(6\, C a^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+3\, B a^{\frac{1}{3}} b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(3\, B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{2\, C a}{b}\right) b^{\frac{2}{3}}\right) b\right) \arctan\left(\frac{\sqrt{3}\left(2\, x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\, a b}-\frac{\left(2\, C a^{\frac{2}{3}}+B a^{\frac{1}{3}} b^{\frac{1}{3}}-\left(2\, C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) b^{\frac{2}{3}}\right) \log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\, b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\, C a^{\frac{2}{3}}+B a^{\frac{1}{3}} b^{\frac{1}{3}}+\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) b^{\frac{2}{3}}\right) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\, b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*C*a*b^(2/3) - (6*C*a^(2/3)*(a/b)^(1/3) + 3*B*a^(1/3)*b^(1/3
)*(a/b)^(1/3) + (3*B*(a/b)^(2/3) + 2*C*a/b)*b^(2/3))*b)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/6*(2*C*a^(2/3) + B*a^(1/3)*b^(1/
3) - (2*C*(a/b)^(2/3) + B*(a/b)^(1/3))*b^(2/3))*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(2*C*a^(2/3) + B*a^(1/3)*b^(1/3) + (C*(a/
b)^(2/3) - B*(a/b)^(1/3))*b^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 6.23, size = 386, normalized size = 5.51

$$\sum_{k=1}^{3} \ln\left(-\frac{x\left(2\, C^{2} a^{2/3} b^{2/3}-B^{2} b^{4/3}+B\, C\, a^{1/3} b\right)}{b^{2}}+\frac{a^{1/3}\left(B\, b^{1/3}+C\, a^{1/3}\right)^{2}}{b^{5/3}}+\frac{\mathrm{root}\left(27\, a^{2} b^{3} z^{3}-27\, C\, a^{2} b^{8/3} z^{2}+18\, B\, C\, a^{5/3} b^{8/3} z+9\, C^{2} a^{2} b^{7/3} z+9\, B^{2} a^{4/3} b^{3} z-18\, B\, C^{2} a^{5/3} b^{7/3}-9\, B^{2} C\, a^{4/3} b^{8/3}-9\, C^{3} a^{2} b^{2}, z, k\right)\left(-6\, C\, a+\mathrm{root}\left(27\, a^{2} b^{3} z^{3}-27\, C\, a^{2} b^{8/3} z^{2}+18\, B\, C\, a^{5/3} b^{8/3} z+9\, C^{2} a^{2} b^{7/3} z+9\, B^{2} a^{4/3} b^{3} z-18\, B\, C^{2} a^{5/3} b^{7/3}-9\, B^{2} C\, a^{4/3} b^{8/3}-9\, C^{3} a^{2} b^{2}, z, k\right) a\, b^{1/3}\, 9+3\, B\, a^{1/3} b^{2/3} x+6\, C\, a^{2/3} b^{1/3} x\right)}{b^{4/3}}\right) \mathrm{root}\left(27\, a^{2} b^{3} z^{3}-27\, C\, a^{2} b^{8/3} z^{2}+18\, B\, C\, a^{5/3} b^{8/3} z+9\, C^{2} a^{2} b^{7/3} z+9\, B^{2} a^{4/3} b^{3} z-18\, B\, C^{2} a^{5/3} b^{7/3}-9\, B^{2} C\, a^{4/3} b^{8/3}-9\, C^{3} a^{2} b^{2}, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*C*a^(2/3) + B*a^(1/3)*b^(1/3) + C*b^(2/3)*x^2 + B*b^(2/3)*x)/(a + b*
x^3),x)
```

```
[Out] symsum(log((a^(1/3)*(B*b^(1/3) + C*a^(1/3))^2)/b^(5/3) - (x*(2*C^2*a^(2/3)*
b^(2/3) - B^2*b^(4/3) + B*C*a^(1/3)*b))/b^2 + (root(27*a^2*b^3*z^3 - 27*C*a
```

```
^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z + 9*B^2*a^(
4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) - 9*C^3*a^2
*b^2, z, k)*(9*root(27*a^2*b^3*z^3 - 27*C*a^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*
b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z + 9*B^2*a^(4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^
(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) - 9*C^3*a^2*b^2, z, k)*a*b^(1/3) - 6*C*a +
3*B*a^(1/3)*b^(2/3)*x + 6*C*a^(2/3)*b^(1/3)*x))/b^(4/3))*root(27*a^2*b^3*z^
3 - 27*C*a^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z +
 9*B^2*a^(4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) -
 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*b**(1/3)*B+2*a**(2/3)*C+b**(2/3)*B*x+b**(2/3)*C*x**2)/(
b*x**3+a),x)
```

```
[Out] Timed out
```

3.41 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=88

$$\frac{2\left(\sqrt[3]{a}(-b)^{2/3}C+bB\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {1866, 31, 617, 204}

$$\frac{2\left(\sqrt[3]{a}(-b)^{2/3}C+bB\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)*C*x^2
)/(a + b*x^3),x]
```

```
[Out] (2*(b*B + a^(1/3)*(-b)^(2/3)*C)*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(1/3)*b) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1866

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, -Di
st[C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2),
x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /; Fr
eeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = -\frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}}-x}\,dx}{\sqrt[3]{-b}} + \frac{\left(\sqrt[3]{-b}B-\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{(-b)^{2/3}}+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}}$$

$$= \frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}} - \left(2\left(\frac{B}{\sqrt[3]{a}}+\frac{bC}{(-b)^{4/3}}\right)\right)\text{Subst}\left(\int \right.$$

$$= \frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{bC}{(-b)^{4/3}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} + \frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}\right.}{\sqrt[3]{-b}}$$

Mathematica [B] time = 0.66, size = 238, normalized size = 2.70

$$\frac{\frac{\left(2\sqrt[3]{a}b\sqrt[3]{-b}C+b^{5/3}B+(-b)^{5/3}B\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2b\left(2\sqrt[3]{a}\sqrt[3]{-b}C+\left(b^{2/3}-(-b)^{2/3}\right)B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt[3]{a}(-b)^{2/3}\sqrt[3]{-b^2}C\log\left(a+bx^3\right)}{\sqrt[3]{-b^2}}+2\sqrt{3}\sqrt[3]{b}\left(2\sqrt[3]{a}\sqrt[3]{b}C+\left((-b)^{2/3}-\sqrt[3]{-b^2}\right)B\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt[3]{a}b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)
*C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*b^(1/3)*(((-b)^(2/3) - (-b^2)^(1/3))*B + 2*a^(1/3)*b^(1/3)*C)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (-2*b*((-(-b)^(2/3) + b^(2/3))*
B + 2*a^(1/3)*(-b)^(1/3)*C)*Log[a^(1/3) + b^(1/3)*x] + ((-b)^(5/3)*B + b^(5
/3)*B + 2*a^(1/3)*(-b)^(1/3)*b*C)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2] - 2*a^(1/3)*(-b)^(2/3)*(-b^2)^(1/3)*C*Log[a + b*x^3])/(-b^2)^(1/3))/(
6*a^(1/3)*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (
-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (
-b)^(2/3)*C*x^2)/(a + b*x^3), x]
```

fricas [B] time = 2.72, size = 470, normalized size = 5.34

$$\left[\frac{\sqrt{\frac{1}{3}}b\sqrt{\frac{C^2a(-b)^{\frac{1}{3}}-2BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}}-B^2a^{\frac{1}{3}}b}{ab}}\log\left(-\frac{C^3a^2+B^3ab-2\left(C^3ab+B^3b^2\right)x^3-3\left(C^3a+B^3b\right)a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x+3\sqrt{\frac{1}{3}}\left(\left(2B^2bx^2+C^2ax+BCa\right)a^{\frac{2}{3}}(-b)^{\frac{2}{3}}+\left(2C^2abx^2-BCabx-B^2ab\right)a^{\frac{1}{3}}+\left(2BCabx^2-B^2abx+C^2a^2\right)(-b)^{\frac{1}{3}}\right)\sqrt{\frac{C^2a(-b)^{\frac{1}{3}}-2BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}}-B^2a^{\frac{1}{3}}b}{ab}}}{bx^3+a}\right)-C(-b)^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}(-b)^{\frac{2}{3}}\right)}{b}, -\frac{2\sqrt{\frac{1}{3}}b\sqrt{-\frac{C^2a(-b)^{\frac{1}{3}}-2BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}}-B^2a^{\frac{1}{3}}b}{ab}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(\left(2C^2x+BC\right)a^{\frac{2}{3}}(-b)^{\frac{2}{3}}-\left(2BCbx+B^2b\right)a^{\frac{1}{3}}-\left(2B^2bx-C^2a\right)(-b)^{\frac{1}{3}}\right)\sqrt{-\frac{C^2a(-b)^{\frac{1}{3}}-2BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}}-B^2a^{\frac{1}{3}}b}{ab}}}{C^3a+B^3b}\right)+C(-b)^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}(-b)^{\frac{2}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*b*sqrt((C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/
3)*b)/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 - 3*(C^3*a
 + B^3*b)*a^(2/3)*(-b)^(1/3)*x + 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*
a)*a^(2/3)*(-b)^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) + (2*
B*C*a*b*x^2 - B^2*a*b*x + C^2*a^2)*(-b)^(1/3))*sqrt((C^2*a*(-b)^(1/3) - 2*B
*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) - C*(-b)^(2/3)*
log(b*x + a^(1/3)*(-b)^(2/3)))/b, -(2*sqrt(1/3)*b*sqrt(-(C^2*a*(-b)^(1/3) -
```

```
 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*
x + B*C)*a^(2/3)*(-b)^(2/3) - (2*B*C*b*x + B^2*b)*a^(1/3) - (2*B^2*b*x - C^
2*a)*(-b)^(1/3))*sqrt(-(C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a
^(1/3)*b)/(a*b))/(C^3*a + B^3*b)) + C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/
3)))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.05, size = 345, normalized size = 3.92

$$-\frac{2\sqrt{3}\,C a^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{2 C a^{\frac{2}{3}} \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{C a^{\frac{2}{3}} \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{\sqrt{3}\,\left(-b\right)^{\frac{1}{3}} B a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{\left(-b\right)^{\frac{1}{3}} B a^{\frac{1}{3}} \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{\left(-b\right)^{\frac{1}{3}} B a^{\frac{1}{3}} \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{\left(-b\right)^{\frac{2}{3}}\sqrt{3}\,B \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b}+\frac{\left(-b\right)^{\frac{2}{3}} B \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b}-\frac{\left(-b\right)^{\frac{2}{3}} B \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b}-\frac{\left(-b\right)^{\frac{2}{3}} C \ln\left(b x^{3}+a\right)}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2)/(b*x
^3+a),x)
```

```
[Out] 1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^(1/3)*(-b)^(1/3)*B-2/3/(a/b)^(2/3)*C*
a^(2/3)/b*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))*a^(1/3)*(-b)^(1/3)*B+1/3/(a/b)^(2/3)*C*a^(2/3)/b*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))*a^(1/3)*(-b)^(1/3)*B-2/3/(a/b)^(2/3)*3^(1/2)*C*a^(2/3)/b*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))+1/3*B*(-b)^(2/3)/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-
1/6*B*(-b)^(2/3)/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*B*(-b)
^(2/3)*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*(-
b)^(2/3)*C/b*ln(b*x^3+a)
```

maxima [B] time = 3.04, size = 252, normalized size = 2.86

$$\frac{\sqrt{3}\left(2\,C a\left(-b\right)^{\frac{2}{3}}-\left(6\,C a^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-3\,B a^{\frac{1}{3}}\left(-b\right)^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{2\,C a}{b}\right)\left(-b\right)^{\frac{2}{3}}\right) b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a b}+\frac{\left(2\,C a^{\frac{2}{3}}-B a^{\frac{1}{3}}\left(-b\right)^{\frac{1}{3}}-\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-b\right)^{\frac{2}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,C a^{\frac{2}{3}}-B a^{\frac{1}{3}}\left(-b\right)^{\frac{1}{3}}+\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-b\right)^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(2*C*a*(-b)^(2/3) - (6*C*a^(2/3)*(a/b)^(1/3) - 3*B*a^(1/3)*(-b)
^(1/3)*(a/b)^(1/3) + (3*B*(a/b)^(2/3) + 2*C*a/b)*(-b)^(2/3))*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*a^(2/3) - B*a^(1/
3)*(-b)^(1/3) - (2*C*(a/b)^(2/3) + B*(a/b)^(1/3))*(-b)^(2/3))*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(2*C*a^(2/3) - B*a^(1/3)*(-
b)^(1/3) + (C*(a/b)^(2/3) - B*(a/b)^(1/3))*(-b)^(2/3))*log(x + (a/b)^(1/3))
/(b*(a/b)^(2/3))
```

mupad [B] time = 6.32, size = 444, normalized size = 5.05

$$\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,a^{2} b^{3} z^{3}+27\,C a^{2}\left(-b\right)^{8/3} z^{2}+18\,B C a^{5/3}\left(-b\right)^{8/3} z+9\,B^{2} a^{4/3} b^{3} z-9\,C^{2} a^{2}\left(-b\right)^{7/3} z-18\,B C^{2} a^{5/3}\left(-b\right)^{7/3}+9\,B^{2} C a^{4/3}\left(-b\right)^{8/3}+9\,C^{3} a^{2} b^{2},z,k\right)\left(\frac{6\,C a}{\left(-b\right)^{4/3}}-\frac{x\left(3\,B a^{1/3}\left(-b\right)^{4/3}+6\,C a^{2/3} b\right)}{b^{2}}+\frac{\mathrm{root}\left(27\,a^{2} b^{3} z^{3}+27\,C a^{2}\left(-b\right)^{8/3} z^{2}+18\,B C a^{5/3}\left(-b\right)^{8/3} z+9\,B^{2} a^{4/3} b^{3} z-9\,C^{2} a^{2}\left(-b\right)^{7/3} z-18\,B C^{2} a^{5/3}\left(-b\right)^{7/3}+9\,B^{2} C a^{4/3}\left(-b\right)^{8/3}+9\,C^{3} a^{2} b^{2},z,k\right) a\,9}{b}\right)+\frac{B^{2} a^{1/3} b^{2}+C^{2} a\left(-b\right)^{4/3}-2\,B C a^{2/3}\left(-b\right)^{5/3}}{b^{3}}-\frac{x\left(2\,C^{2} a^{2/3}\left(-b\right)^{2/3}-B^{2}\left(-b\right)^{4/3}+B C a^{1/3} b\right)}{b^{2}}\right)\mathrm{root}\left(27\,a^{2} b^{3} z^{3}+27\,C a^{2}\left(-b\right)^{8/3} z^{2}+18\,B C a^{5/3}\left(-b\right)^{8/3} z+9\,B^{2} a^{4/3} b^{3} z-9\,C^{2} a^{2}\left(-b\right)^{7/3} z-18\,B C^{2} a^{5/3}\left(-b\right)^{7/3}+9\,B^{2} C a^{4/3}\left(-b\right)^{8/3}+9\,C^{3} a^{2} b^{2},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*C*a^(2/3) + B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3) + C*(-b)^(2/3)*x^
2)/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(
-b)^(8/3)*z + 9*B^2*a^(4/3)*b^3*z - 9*C^2*a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/
3)*(-b)^(7/3) + 9*B^2*C*a^(4/3)*(-b)^(8/3) + 9*C^3*a^2*b^2, z, k)*((6*C*a)/
(-b)^(4/3) - (x*(3*B*a^(1/3)*(-b)^(4/3) + 6*C*a^(2/3)*b))/b^2 + (9*root(27*
a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(-b)^(8/3)*z + 9*B^2
*a^(4/3)*b^3*z - 9*C^2*a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/3)*(-b)^(7/3) + 9*B
^2*C*a^(4/3)*(-b)^(8/3) + 9*C^3*a^2*b^2, z, k)*a)/b) + (B^2*a^(1/3)*b^2 + C
^2*a*(-b)^(4/3) - 2*B*C*a^(2/3)*(-b)^(5/3))/b^3 - (x*(2*C^2*a^(2/3)*(-b)^(2
/3) - B^2*(-b)^(4/3) + B*C*a^(1/3)*b))/b^2)*root(27*a^2*b^3*z^3 + 27*C*a^2*
(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(-b)^(8/3)*z + 9*B^2*a^(4/3)*b^3*z - 9*C^2*
a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/3)*(-b)^(7/3) + 9*B^2*C*a^(4/3)*(-b)^(8/3)
 + 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*(-b)**(1/3)*B-2*a**(2/3)*C-(-b)**(2/3)*B*x-(-b)**(2/3)*
C*x**2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.42 $\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3} \, dx$

Optimal. Leaf size=11

$$\frac{\log(B-Cx)}{C}$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1586, 31}

$$\frac{\log(B-Cx)}{C}$$

Antiderivative was successfully verified.

```
[In] Int[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
[Out] Log[B - C*x]/C
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3} \, dx &= \int \frac{1}{-B+Cx} \, dx \\ &= \frac{\log(B-Cx)}{C} \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.09

$$\frac{\log(Cx-B)}{C}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
[Out] Log[-B + C*x]/C
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
[Out] IntegrateAlgebraic[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3), x]
```

fricas [**A**] time = 0.39, size = 12, normalized size = 1.09

$$\frac{\log\left(Cx - B\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="fricas")
[Out] log(C*x - B)/C
```

giac [**A**] time = 0.36, size = 13, normalized size = 1.18

$$\frac{\log\left(|Cx - B|\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="giac")
[Out] log(abs(C*x - B))/C
```

maple [**A**] time = 0.04, size = 12, normalized size = 1.09

$$\frac{\ln\left(-Cx + B\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x)
[Out] ln(-C*x+B)/C
```

maxima [**A**] time = 1.36, size = 12, normalized size = 1.09

$$\frac{\log\left(Cx - B\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="maxima")
[Out] log(C*x - B)/C
```

mupad [**B**] time = 0.04, size = 12, normalized size = 1.09

$$\frac{\ln\left(C\,x - B\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B^2 + C^2*x^2 + B*C*x)/(B^3 - C^3*x^3),x)
[Out] log(C*x - B)/C
```

sympy [**A**] time = 0.24, size = 7, normalized size = 0.64

$$\frac{\log\left(-B + Cx\right)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C**2*x**2+B*C*x+B**2)/(C**3*x**3-B**3),x)
[Out] log(-B + C*x)/C
```

3.43 $\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=21

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1586, 31}

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx &= \int \frac{1}{\frac{\sqrt[3]{a}}{C}+\frac{\sqrt[3]{b}x}{C}}\,dx\\ &= \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b
*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b
*x^3), x]
```

fricas [A] time = 0.42, size = 17, normalized size = 0.81

$$\frac{C\log\left(bx + a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="fricas")
```

```
[Out] C*log(b*x + a^(1/3)*b^(2/3))/b^(1/3)
```

giac [A] time = 0.31, size = 16, normalized size = 0.76

$$\frac{C\log\left(\left|b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right|\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="giac")
```

```
[Out] C*log(abs(b^(1/3)*x + a^(1/3)))/b^(1/3)
```

maple [B] time = 0.05, size = 218, normalized size = 10.38

$$\frac{\sqrt{3}\, C a^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} + \frac{C a^{\frac{2}{3}} \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{C a^{\frac{2}{3}} \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{\sqrt{3}\, C a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{\frac{2}{3}}} + \frac{C a^{\frac{1}{3}} \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{\frac{2}{3}}} - \frac{C a^{\frac{1}{3}} \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{\frac{2}{3}}} + \frac{C\ln\left(b x^3+a\right)}{3 b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] 1/3/(a/b)^(2/3)*C*a^(2/3)/b*ln(x+(a/b)^(1/3))-1/6/(a/b)^(2/3)*C*a^(2/3)/b*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/(a/b)^(2/3)*3^(1/2)*C*a^(2/3)/b*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b^(2/3)*a^(1/3)/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))-1/6*C/b^(2/3)*a^(1/3)/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))-1/3*C/b^(2/3)*a^(1/3)*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^3+a)
```

maxima [B] time = 2.99, size = 210, normalized size = 10.00

$$-\frac{\sqrt{3}\left(2 C a b^{\frac{2}{3}} + \left(3 C a^{\frac{1}{3}} b^{\frac{1}{3}} \left(\frac{a}{b}\right)^{\frac{2}{3}} - 3 C a^{\frac{2}{3}} \left(\frac{a}{b}\right)^{\frac{1}{3}} - \frac{2 C a}{b^{\frac{1}{3}}}\right) b\right) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab} + \frac{\left(2 C b^{\frac{2}{3}} \left(\frac{a}{b}\right)^{\frac{2}{3}} - C a^{\frac{1}{3}} b^{\frac{1}{3}} \left(\frac{a}{b}\right)^{\frac{1}{3}} - C a^{\frac{2}{3}}\right) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(C b^{\frac{2}{3}} \left(\frac{a}{b}\right)^{\frac{2}{3}} + C a^{\frac{1}{3}} b^{\frac{1}{3}} \left(\frac{a}{b}\right)^{\frac{1}{3}} + C a^{\frac{2}{3}}\right) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*C*a*b^(2/3) + (3*C*a^(1/3)*b^(1/3)*(a/b)^(2/3) - 3*C*a^(2/3
)*(a/b)^(1/3) - 2*C*a/b^(1/3))*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a
```

```
/b)^(1/3))/(a*b) + 1/6*(2*C*b^(2/3)*(a/b)^(2/3) - C*a^(1/3)*b^(1/3)*(a/b)^(
1/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) +
1/3*(C*b^(2/3)*(a/b)^(2/3) + C*a^(1/3)*b^(1/3)*(a/b)^(1/3) + C*a^(2/3))*log
(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 4.90, size = 15, normalized size = 0.71

$$\frac{C\ \ln\left(x+\frac{a^{1/3}}{b^{1/3}}\right)}{b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*a^(2/3) + C*b^(2/3)*x^2 - C*a^(1/3)*b^(1/3)*x)/(a + b*x^3),x)
[Out] (C*log(x + a^(1/3)/b^(1/3)))/b^(1/3)
```

sympy [A] time = 0.26, size = 20, normalized size = 0.95

$$\frac{C\log\left(\sqrt[3]{a}\,b^{\frac{2}{3}}+bx\right)}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(2/3)*C-a**(1/3)*b**(1/3)*C*x+b**(2/3)*C*x**2)/(b*x**3+a),x)
[Out] C*log(a**(1/3)*b**(2/3) + b*x)/b**(1/3)
```

3.44 $\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=71

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(C\sqrt[3]{\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}$$

Rubi [A] time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1867, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(C\sqrt[3]{\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}$$

Antiderivative was successfully verified.

```
[In] Int[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*(a/b)^(2/3)*(B + (a/b)^(1/3)*C)*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]]
)/(Sqrt[3]*a) + (C*Log[(a/b)^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3}\,dx = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b} + \frac{\left(B + \sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3} - \sqrt[3]{\frac{a}{b}}x + x^2}\,dx}{b}$$

$$= \frac{C\log\left(\sqrt[3]{\frac{a}{b}} + x\right)}{b} + \left(2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a} + \frac{C}{b}\right)\right)\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, 1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)$$

$$= -\frac{2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a} + \frac{C}{b}\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{C\log\left(\sqrt[3]{\frac{a}{b}} + x\right)}{b}$$

Mathematica [B] time = 0.33, size = 247, normalized size = 3.48

$$\frac{\sqrt[3]{b}\left(a^{2/3}B - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}} + B\right)\right)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) + 2\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}} + B\right) - a^{2/3}B\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right) + 2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}} + B\right) + \sqrt[3]{a}B\right)\tan^{-1}\left(\frac{2\sqrt[3]{b}x - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right) + 2aC\log\left(a + bx^3\right)}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1
/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 2*b^(1/3)*(-(a
^(2/3)*B) + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1/3)*C))*Log[a^(1/3)
+ b^(1/3)*x] + b^(1/3)*(a^(2/3)*B - a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b
)^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*C*Log[a +
b*x^3])/(6*a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x
^3),x]
```

```
[Out] IntegrateAlgebraic[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x
^3), x]
```

fricas [B] time = 1.86, size = 429, normalized size = 6.04

$$\left[\frac{C\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + \sqrt{\frac{1}{3}}\sqrt{-\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}} + B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}} + C^2a}{a}}\log\left(-\frac{C^3a^2 + B^3ab - 2\left(C^3ab + B^3b^2\right)x^3 + 3\left(C^3ab + B^3b^2\right)x\left(\frac{a}{b}\right)^{\frac{2}{3}} + 3\sqrt{\frac{1}{3}}\left(2BCabx^2 - B^2abx + C^2a^2 - \left(2B^2b^2x^2 + C^2abx + BCab\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} - \left(2C^2abx^2 - BCabx - B^2ab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}} + B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}} + C^2a}{a}}}{bx^3 + a}\right)}{b}, \frac{2\sqrt{\frac{1}{3}}\sqrt{\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}} + B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}} + C^2a}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2B^2bx - C^2a + \left(2C^2bx + BCb\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} - \left(2BCbx + B^2b\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}} + B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}} + C^2a}{a}}}{C^3a + B^3b}\right) + C\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"fricas")
```

```
[Out] [(C*log(x + (a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/
b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 +
3*(C^3*a*b + B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*
x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(a/b)^(2/3) - (2*C^2*a*
b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*
b*(a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a)))/b, (2*sqrt(1/3)*sqrt((2*B*C*b*(a/b
```

```
)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b*x - C^2*a
 + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqrt(
(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)/(C^3*a + B^3*b)) + C*l
og(x + (a/b)^(1/3)))/b]
```

giac [B] time = 0.20, size = 242, normalized size = 3.41

$$\frac{\left(2\,Cab + \left(-a^2b^4\right)^{\frac{1}{3}} B\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab^2 - \sqrt{3}\sqrt{a^2b^4}\,i} - \frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}} + Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(ab^2\right)^{\frac{1}{3}} Bb + 2\left(ab^2\right)^{\frac{2}{3}} C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2} + \frac{\sqrt{3}\left(\left(9\left(-a^2b^4\right)^{\frac{1}{3}} ab^2 - 27^{\frac{5}{6}}\left(-a^2b^4\right)^{\frac{5}{6}}\right)B + 18\left(a^2b^3 - \sqrt{3}\sqrt{a^4b^6}\,i\right)C\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{54\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"giac")
```

```
[Out] (2*C*a*b + (-a^2*b^4)^(1/3)*B)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(3*
a*b^2 - sqrt(3)*sqrt(a^2*b^4)*i) - 1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(
1/3) + (a*b^2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a*b^2) + 1/54*sqrt(3)*((9*(-a^2*b^4)^(1/3)*a*b^2 - 27^(5/6)*(-a
^2*b^4)^(5/6))*B + 18*(a^2*b^3 - sqrt(3)*sqrt(a^4*b^6)*i)*C)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4)
```

maple [A] time = 0.05, size = 121, normalized size = 1.70

$$\frac{2\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b} + \frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b} + \frac{2C\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b} - \frac{C\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b} + \frac{C\ln\left(b\,x^3 + a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C/b*ln(x+(a/b)^(1/3))-1/3*C/b*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/3*3^(
1/2)*C/b*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/3*3^(1/2)/(a/b)^(1/3)*B/
b*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)
```

maxima [A] time = 2.95, size = 78, normalized size = 1.10

$$\frac{C\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b} - \frac{2\sqrt{3}\left(Ca - \left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}} + \frac{4\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"maxima")
```

```
[Out] C*log(x + (a/b)^(1/3))/b - 2/9*sqrt(3)*(C*a - (3*B*(a/b)^(2/3) + 4*C*a/b)*b
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

mupad [B] time = 6.08, size = 436, normalized size = 6.14

$$\sum_{k=1}^{3}\ln\left(\frac{C^2a + B^2b\left(\frac{a}{b}\right)^{1/3} + 2BCb\left(\frac{a}{b}\right)^{2/3}}{b^3} + \frac{\mathrm{root}\left(27a^2b^3z^3 - 27Ca^2b^2z^2 + 18BCab^2z\left(\frac{a}{b}\right)^{2/3} + 9B^2ab^2z\left(\frac{a}{b}\right)^{1/3} + 9C^2a^2bz - 18BC^2ab\left(\frac{a}{b}\right)^{2/3} - 9B^2Cab\left(\frac{a}{b}\right)^{1/3} - 9C^3a^2, z, k\right)\left(-6Ca + \mathrm{root}\left(27a^2b^3z^3 - 27Ca^2b^2z^2 + 18BCab^2z\left(\frac{a}{b}\right)^{2/3} + 9B^2ab^2z\left(\frac{a}{b}\right)^{1/3} + 9C^2a^2bz - 18BC^2ab\left(\frac{a}{b}\right)^{2/3} - 9B^2Cab\left(\frac{a}{b}\right)^{1/3} - 9C^3a^2, z, k\right)ab9 + 3Bbx\left(\frac{a}{b}\right)^{1/3} + 6Cbx\left(\frac{a}{b}\right)^{2/3}\right)}{b^2} - \frac{x\left(2C^2\left(\frac{a}{b}\right)^{2/3} - B^2 + BC\left(\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\mathrm{root}\left(27a^2b^3z^3 - 27Ca^2b^2z^2 + 18BCab^2z\left(\frac{a}{b}\right)^{2/3} + 9B^2ab^2z\left(\frac{a}{b}\right)^{1/3} + 9C^2a^2bz - 18BC^2ab\left(\frac{a}{b}\right)^{2/3} - 9B^2Cab\left(\frac{a}{b}\right)^{1/3} - 9C^3a^2, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x + C*x^2 + B*(a/b)^(1/3) + 2*C*(a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a + B^2*b*(a/b)^(1/3) + 2*B*C*b*(a/b)^(2/3))/b^3 + (root(27
*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*
z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b
)^(1/3) - 9*C^3*a^2, z, k)*(9*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B
*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C
^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b)^(1/3) - 9*C^3*a^2, z, k)*a*b - 6*C*a
 + 3*B*b*x*(a/b)^(1/3) + 6*C*b*x*(a/b)^(2/3)))/b^2 - (x*(2*C^2*(a/b)^(2/3)
- B^2 + B*C*(a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*
B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*
C^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b)^(1/3) - 9*C^3*a^2, z, k), k, 1, 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.45 $\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=76

$$\frac{2\left(C\sqrt[3]{-\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b\sqrt[3]{-\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {1867, 31, 617, 204}

$$\frac{2\left(C\sqrt[3]{-\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b\sqrt[3]{-\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*(B + (-(a/b))^(1/3)*C)*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt
[3]*(-(a/b))^(1/3)*b) - (C*Log[(-(a/b))^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b}-\frac{\left(B+\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}\,x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}-\frac{\left(2\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}}+C\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= \frac{2\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}}+C\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] time = 0.25, size = 288, normalized size = 3.79

$$-\frac{\left(-a^{2/3}B-\sqrt[3]{a}\sqrt[3]{b}B\sqrt[3]{-\frac{a}{b}}-2\sqrt[3]{a}\sqrt[3]{b}C\left(-\frac{a}{b}\right)^{2/3}\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6ab^{2/3}}-\frac{\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}B\sqrt[3]{-\frac{a}{b}}+2\sqrt[3]{a}\sqrt[3]{b}C\left(-\frac{a}{b}\right)^{2/3}\right)\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3ab^{2/3}}-\frac{\left(a^{2/3}B-\sqrt[3]{a}\sqrt[3]{b}B\sqrt[3]{-\frac{a}{b}}-2\sqrt[3]{a}\sqrt[3]{b}C\left(-\frac{a}{b}\right)^{2/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,ab^{2/3}}-\frac{C\log\left(a-bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3)
,x]
```

```
[Out] -(((a^(2/3)*B - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3)
*b^(1/3)*C)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a*b
^(2/3))) - ((a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B + 2*a^(1/3)*(-(a/
b))^(2/3)*b^(1/3)*C)*Log[a^(1/3) - b^(1/3)*x])/(3*a*b^(2/3)) - ((-(a^(2/3)*
B) - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3)*b^(1/3)*C)
*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(2/3)) - (C*Log[a -
 b*x^3])/(3*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a
 - b*x^3),x]
```

```
[Out] IntegrateAlgebraic[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a
 - b*x^3), x]
```

fricas [B] time = 1.78, size = 459, normalized size = 6.04

$$\left[-\frac{C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)-\sqrt{\frac{1}{3}}\sqrt{\frac{2BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\log\left(-\frac{C^3a^2-B^3ab+2\left(C^3ab-B^3b^2\right)x^3-3\left(C^3ab-B^3b^2\right)x\left(-\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2BCabx^2-B^2abx-C^2a^2+\left(2B^2b^2x^2-C^2abx-BCab\right)\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\left(2C^2abx^2-BCabx-B^2ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{\frac{2BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}}{bx^3-a}\right)}{b},\ -\frac{2\sqrt{\frac{1}{3}}\sqrt{-\frac{2BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2B^2bx+C^2a+\left(2C^2bx+BCb\right)\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\left(2BCbx+B^2b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}}{C^3a-B^3b}\right)+C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="fricas")
```

```
[Out] [-(C*log(x + (-a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(-a/b)^(2/3) + B^2*b*(
-a/b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3
 - 3*(C^3*a*b - B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*
a*b*x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(-a/b)^(2/3) - (2*C
```

```
^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt((2*B*C*b*(-a/b)^(2/3)
+ B^2*b*(-a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a)))/b, -(2*sqrt(1/3)*sqrt(-(2*B
*C*b*(-a/b)^(2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2
*b*x + C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(-a/b
)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^(2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)/a)/(C^3*
a - B^3*b)) + C*log(x + (-a/b)^(1/3)))/b]
```

giac [B] time = 0.21, size = 235, normalized size = 3.09

$$-\frac{\left(2\,Cab - \left(-a^2b^4\right)^{\frac{1}{3}} B\right)\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab^2 + \sqrt{3}\,\sqrt{a^2b^4}\,i} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{1}{3}} Bb + 2\left(-ab^2\right)^{\frac{2}{3}} C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2} + \frac{\sqrt{3}\left(\left(9\left(-a^2b^4\right)^{\frac{1}{3}} ab^2 + 27^{\frac{5}{6}}\left(-a^2b^4\right)^{\frac{5}{6}}\right)B - 18\left(a^2b^3 + \sqrt{3}\,\sqrt{a^4b^6}\,i\right)C\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{54\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="giac")
```

```
[Out] -(2*C*a*b - (-a^2*b^4)^(1/3)*B)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(3*a
*b^2 + sqrt(3)*sqrt(a^2*b^4)*i) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3
) + (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^
(1/3)))/(a*b^2) + 1/54*sqrt(3)*((9*(-a^2*b^4)^(1/3)*a*b^2 + 27^(5/6)*(-a^2*
b^4)^(5/6))*B - 18*(a^2*b^3 + sqrt(3)*sqrt(a^4*b^6)*i)*C)*arctan(1/3*sqrt(3
)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4)
```

maple [B] time = 0.05, size = 345, normalized size = 4.54

$$\frac{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}} B\arctan\left(\frac{\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)\sqrt{3}}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{\sqrt{3}\,B\arctan\left(\frac{\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)\sqrt{3}}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b} - \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}} B\ln\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{B\ln\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}} B\ln\left(x^2 + \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b} + \frac{B\ln\left(x^2 + \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b} + \frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\sqrt{3}\,C\arctan\left(\frac{\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)\sqrt{3}}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}} C\ln\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} + \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}} C\ln\left(x^2 + \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{C\ln\left(bx^3 - a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*(-a/b)^(2/3)/(a/b)^(2/3)*C/b*ln(x-(a/b)^(1/3))-1/3/b/(a/b)^(2/3)*ln(x-
(a/b)^(1/3))*(-a/b)^(1/3)*B+1/3*(-a/b)^(2/3)/(a/b)^(2/3)*C/b*ln(x^2+(a/b)^(
1/3)*x+(a/b)^(2/3))+1/6/b/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))*(-a
/b)^(1/3)*B+2/3*(-a/b)^(2/3)/(a/b)^(2/3)*3^(1/2)*C/b*arctan(1/3*(2/(a/b)^(1
/3)*x+1)*3^(1/2))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*(2/(a/b)^(1/3)*x+1)*
3^(1/2))*(-a/b)^(1/3)*B-1/3*B/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6*B/b/(a/b)
^(1/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*B*3^(1/2)/b/(a/b)^(1/3)*arctan
(1/3*(2/(a/b)^(1/3)*x+1)*3^(1/2))-1/3*C/b*ln(b*x^3-a)
```

maxima [B] time = 3.01, size = 238, normalized size = 3.13

$$-\frac{\sqrt{3}\left(2\,Ca - \left(6\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}} - 3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}} + 3\,B\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} - \frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}} - B\left(\frac{a}{b}\right)^{\frac{1}{3}} - B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}} + B\left(\frac{a}{b}\right)^{\frac{1}{3}} + B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*C*a - (6*C*(a/b)^(1/3)*(-a/b)^(2/3) - 3*B*(a/b)^(2/3) + 3*B
*(a/b)^(1/3)*(-a/b)^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/
3))/(a/b)^(1/3))/(a*b) - 1/6*(2*C*(a/b)^(2/3) - 2*C*(-a/b)^(2/3) - B*(a/b)^
(1/3) - B*(-a/b)^(1/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/
3)) - 1/3*(C*(a/b)^(2/3) + 2*C*(-a/b)^(2/3) + B*(a/b)^(1/3) + B*(-a/b)^(1/3
))*log(x - (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 6.48, size = 456, normalized size = 6.00

$$\sum_{k=1}^{3}\ln\left(\frac{B^2\,b\left(-\frac{a}{b}\right)^{1/3} - C^2\,a + 2\,B\,C\,b\left(-\frac{a}{b}\right)^{2/3}}{b^3} - \frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,b^2\,z^2 - 18\,B\,C\,a\,b^2\,z\left(-\frac{a}{b}\right)^{2/3} - 9\,B^2\,a\,b^2\,z\left(-\frac{a}{b}\right)^{1/3} + 9\,C^2\,a^2\,b\,z - 18\,B\,C^2\,a\,b\left(-\frac{a}{b}\right)^{2/3} - 9\,B^2\,C\,a\,b\left(-\frac{a}{b}\right)^{1/3} + 9\,C^3\,a^2, z, k\right)\left(6\,C\,a + \mathrm{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,b^2\,z^2 - 18\,B\,C\,a\,b^2\,z\left(-\frac{a}{b}\right)^{2/3} - 9\,B^2\,a\,b^2\,z\left(-\frac{a}{b}\right)^{1/3} + 9\,C^2\,a^2\,b\,z - 18\,B\,C^2\,a\,b\left(-\frac{a}{b}\right)^{2/3} - 9\,B^2\,C\,a\,b\left(-\frac{a}{b}\right)^{1/3} + 9\,C^3\,a^2, z, k\right)a\,b\,9 + 3\,B\,b\,x\left(-\frac{a}{b}\right)^{1/3} + 6\,C\,b\,x\left(-\frac{a}{b}\right)^{2/3}\right)}{b^2} - \frac{x\left(2\,C^2\left(-\frac{a}{b}\right)^{2/3} - B^2 + B\,C\left(-\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\mathrm{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,b^2\,z^2 - 18\,B\,C\,a\,b^2\,z\left(-\frac{a}{b}\right)^{2/3} - 9\,B^2\,a\,b^2\,z\left(-\frac{a}{b}\right)^{1/3} + 9\,C^2\,a^2\,b\,z - 18\,B\,C^2\,a\,b\left(-\frac{a}{b}\right)^{2/3} - 9\,B^2\,C\,a\,b\left(-\frac{a}{b}\right)^{1/3} + 9\,C^3\,a^2, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x + C*x^2 + B*(-a/b)^(1/3) + 2*C*(-a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log((B^2*b*(-a/b)^(1/3) - C^2*a + 2*B*C*b*(-a/b)^(2/3))/b^3 - (root(
27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b
^2*z*(-a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b
*(-a/b)^(1/3) + 9*C^3*a^2, z, k)*(6*C*a + 9*root(27*a^2*b^3*z^3 + 27*C*a^2*
b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2*
a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b*(-a/b)^(1/3) + 9*C^3*a^2,
 z, k)*a*b + 3*B*b*x*(-a/b)^(1/3) + 6*C*b*x*(-a/b)^(2/3)))/b^2 - (x*(2*C^2*
(-a/b)^(2/3) - B^2 + B*C*(-a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 + 27*C*a^2
*b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2
*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b*(-a/b)^(1/3) + 9*C^3*a^2
, z, k), k, 1, 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

```
[Out] Timed out
```

3.46 $\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=78

$$\frac{2\left(B-C\sqrt[3]{-\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b\sqrt[3]{-\frac{a}{b}}}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {1869, 31, 617, 204}

$$\frac{2\left(B-C\sqrt[3]{-\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b\sqrt[3]{-\frac{a}{b}}}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*(B - (-(a/b))^(1/3)*C)*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt
[3]*(-(a/b))^(1/3)*b) + (C*Log[(-(a/b))^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b}+\frac{\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$

$$=\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{\left(2\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right.}{\sqrt[3]{-\frac{a}{b}}\,b}$$

$$=\frac{2\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-\frac{a}{b}}\,b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] time = 0.36, size = 253, normalized size = 3.24

$$\frac{\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\left(B-2C\sqrt[3]{-\frac{a}{b}}\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\left(B-2C\sqrt[3]{-\frac{a}{b}}\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\left(2C\sqrt[3]{-\frac{a}{b}}-B\right)+\sqrt[3]{a}B\right)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)+2aC\log\left(a+bx^3\right)}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x
^3),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (-(a/b))^(1/3)*b^(1/3)*(-B + 2*(-(a
/b))^(1/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] - 2*b^(1/
3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*(B - 2*(-(a/b))^(1/3)*C))*Lo
g[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3
)*(B - 2*(-(a/b))^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
+ 2*a*C*Log[a + b*x^3])/(6*a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)
/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)
/(a + b*x^3), x]
```

fricas [B] time = 1.78, size = 450, normalized size = 5.77

$$\left[\frac{C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+\sqrt{\frac{1}{3}}\sqrt{-\frac{2BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}\log\left(-\frac{C^3a^2+B^3ab-2\left(C^3ab+B^3b^2\right)x^3+3\left(C^3ab+B^3b^2\right)x\left(-\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2BCabx^2-B^2abx+C^2a^2-\left(2B^2b^2x^2+C^2abx+BCab\right)\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\left(2C^2abx^2-BCabx-B^2ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}}{bx^3+a}\right)}{b},\ \frac{2\sqrt{\frac{1}{3}}\sqrt{\frac{2BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2B^2bx-C^2a+\left(2C^2bx+BCb\right)\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\left(2BCbx+B^2b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{\frac{2BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+C^2a}{a}}}{C^3a+B^3b}\right)+C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="fricas")
```

```
[Out] [(C*log(x - (-a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(-a/b)^(2/3) - B^2*b*(
-a/b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3
 + 3*(C^3*a*b + B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*
```

```
a*b*x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(-a/b)^(2/3) + (2*C
^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^(2/3)
 - B^2*b*(-a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a)))/b, (2*sqrt(1/3)*sqrt((2*B*
C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b
*x - C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(-a/b)^
(1/3))*sqrt((2*B*C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)/(C^3*a +
 B^3*b)) + C*log(x - (-a/b)^(1/3)))/b]
```

giac [A] time = 0.19, size = 133, normalized size = 1.71

$$-\frac{2\sqrt{3}\left(Cab+\left(-ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}Bb+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="giac")
```

```
[Out] -2/3*sqrt(3)*(C*a*b + (-a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3) -
 (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(
1/3)))/(a*b^2)
```

maple [B] time = 0.05, size = 340, normalized size = 4.36

$$-\frac{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{\sqrt{3}B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}C\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}C\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{C\ln\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*(-a/b)^(2/3)/(a/b)^(2/3)*C/b*ln(x+(a/b)^(1/3))-1/3/b/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*(-a/b)^(1/3)*B-1/3*(-a/b)^(2/3)/(a/b)^(2/3)*C/b*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))+1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*(-a/
b)^(1/3)*B+2/3*(-a/b)^(2/3)/(a/b)^(2/3)*3^(1/2)*C/b*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))-1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x-1))*(-a/b)^(1/3)*B-1/3/(a/b)^(1/3)*B/b*ln(x+(a/b)^(1/3))+1/6/(a/b)^(1/3
)*B/b*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/(a/b)^(1/3)*B/b*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)
```

maxima [B] time = 3.03, size = 239, normalized size = 3.06

$$-\frac{\sqrt{3}\left(2Ca-\left(6C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+3B\left(\frac{a}{b}\right)^{\frac{2}{3}}-3B\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab}+\frac{\left(2C\left(\frac{a}{b}\right)^{\frac{2}{3}}-2C\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}+B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2C\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}-B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*C*a - (6*C*(a/b)^(1/3)*(-a/b)^(2/3) + 3*B*(a/b)^(2/3) - 3*B
*(a/b)^(1/3)*(-a/b)^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/
3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*(a/b)^(2/3) - 2*C*(-a/b)^(2/3) + B*(a/b)^
(1/3) + B*(-a/b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/
3)) + 1/3*(C*(a/b)^(2/3) + 2*C*(-a/b)^(2/3) - B*(a/b)^(1/3) - B*(-a/b)^(1/3
))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 6.05, size = 453, normalized size = 5.81

$$\sum_{k=1}^{3}\ln\left(\frac{C^2a-B^2b\left(-\frac{a}{b}\right)^{1/3}+2BCb\left(-\frac{a}{b}\right)^{2/3}}{b^3}-\frac{\mathrm{root}\left(27a^2b^3z^3-27Ca^2b^2z^2+18BCab^2z\left(-\frac{a}{b}\right)^{2/3}-9B^2ab^2z\left(-\frac{a}{b}\right)^{1/3}+9C^2a^2bz-18BC^2ab\left(-\frac{a}{b}\right)^{2/3}+9B^2Cab\left(-\frac{a}{b}\right)^{1/3}-9C^3a^2,z,k\right)\left(6Ca-\mathrm{root}\left(27a^2b^3z^3-27Ca^2b^2z^2+18BCab^2z\left(-\frac{a}{b}\right)^{2/3}-9B^2ab^2z\left(-\frac{a}{b}\right)^{1/3}+9C^2a^2bz-18BC^2ab\left(-\frac{a}{b}\right)^{2/3}+9B^2Cab\left(-\frac{a}{b}\right)^{1/3}-9C^3a^2,z,k\right)ab9+3Bbx\left(-\frac{a}{b}\right)^{1/3}-6Cbx\left(-\frac{a}{b}\right)^{2/3}\right)}{b^2}+\frac{x\left(B^2-2C^2\left(-\frac{a}{b}\right)^{2/3}+BC\left(-\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\mathrm{root}\left(27a^2b^3z^3-27Ca^2b^2z^2+18BCab^2z\left(-\frac{a}{b}\right)^{2/3}-9B^2ab^2z\left(-\frac{a}{b}\right)^{1/3}+9C^2a^2bz-18BC^2ab\left(-\frac{a}{b}\right)^{2/3}+9B^2Cab\left(-\frac{a}{b}\right)^{1/3}-9C^3a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x + C*x^2 - B*(-a/b)^(1/3) + 2*C*(-a/b)^(2/3))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a - B^2*b*(-a/b)^(1/3) + 2*B*C*b*(-a/b)^(2/3))/b^3 - (root(
27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b
^2*z*(-a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b
*(-a/b)^(1/3) - 9*C^3*a^2, z, k)*(6*C*a - 9*root(27*a^2*b^3*z^3 - 27*C*a^2*
b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2*
a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b*(-a/b)^(1/3) - 9*C^3*a^2,
 z, k)*a*b + 3*B*b*x*(-a/b)^(1/3) - 6*C*b*x*(-a/b)^(2/3)))/b^2 + (x*(B^2 -
2*C^2*(-a/b)^(2/3) + B*C*(-a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 - 27*C*a^2
*b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9*C^2
*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b*(-a/b)^(1/3) - 9*C^3*a^2
, z, k), k, 1, 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.47 $\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=75

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B-C\sqrt[3]{\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1869, 31, 617, 204}

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B-C\sqrt[3]{\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (-2*(a/b)^(2/3)*(B - (a/b)^(1/3)*C)*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]]
)/(Sqrt[3]*a) - (C*Log[(a/b)^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b} - \frac{\left(B-\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b} + \left(2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}-\frac{C}{b}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\cdots}}\right.$$

$$= -\frac{2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}-\frac{C}{b}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] time = 0.32, size = 244, normalized size = 3.25

$$\frac{\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}}-B\right)\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}}-B\right)\right)\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(B-2C\sqrt[3]{\frac{a}{b}}\right)+\sqrt[3]{a}B\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2aC\log\left(a-bx^3\right)}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B - 2*(a/b)^(
1/3)*C))*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*(a^(2/3)*B
 + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)*C))*Log[a^(1/3) - b^(1/3
)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)
*C))*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*C*Log[a - b*x^3])
/(6*a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a -
b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a -
b*x^3), x]
```

fricas [B] time = 1.72, size = 450, normalized size = 6.00

$$\left[-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\sqrt{\frac{1}{3}}\sqrt{\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\log\left(-\frac{C^3a^2-B^3ab+2\left(C^3ab-B^3b^2\right)x^3-3\left(C^3ab-B^3b^2\right)x\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2BCabx^2-B^2abx-C^2a^2+\left(2B^2b^2x^2-C^2abx-BCab\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}+\left(2C^2abx^2-BCabx-B^2ab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}}{bx^3-a}\right)}{b}, -\frac{2\sqrt{\frac{1}{3}}\sqrt{-\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2B^2bx+C^2a+\left(2C^2bx+BCb\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}+\left(2BCbx+B^2b\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}}{C^3a-B^3b}\right)+C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="fricas")
```

```
[Out] [-(C*log(x - (a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b*(a/
b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3 -
3*(C^3*a*b - B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*
x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(a/b)^(2/3) + (2*C^2*a*
b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b
```

```
*(a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a)))/b, -(2*sqrt(1/3)*sqrt(-(2*B*C*b*(a/
b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2*b*x + C^2
*a + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqr
t(-(2*B*C*b*(a/b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)/(C^3*a - B^3*b)) +
C*log(x - (a/b)^(1/3)))/b]
```

giac [A] time = 0.18, size = 125, normalized size = 1.67

$$\frac{2\sqrt{3}\left(Cab-\left(ab^{2}\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^{2}}-\frac{\left(Cb^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}+Bb^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}-\left(ab^{2}\right)^{\frac{1}{3}}Bb+2\left(ab^{2}\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="giac")
```

```
[Out] 2/3*sqrt(3)*(C*a*b - (a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3)
)/(a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3) - (a*b^
2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*
b^2)
```

maple [A] time = 0.05, size = 124, normalized size = 1.65

$$-\frac{2\sqrt{3}\,B\arctan\left(\frac{\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)\sqrt{3}}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{2\sqrt{3}\,C\arctan\left(\frac{\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)\sqrt{3}}{3}\right)}{3b}-\frac{2C\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}+\frac{C\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b}-\frac{C\ln\left(bx^{3}-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C/b*ln(x-(a/b)^(1/3))+1/3*C/b*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+2/3*3^
(1/2)*C/b*arctan(1/3*(2/(a/b)^(1/3)*x+1)*3^(1/2))-2/3*3^(1/2)/(a/b)^(1/3)*B
/b*arctan(1/3*(2/(a/b)^(1/3)*x+1)*3^(1/2))-1/3*C/b*ln(b*x^3-a)
```

maxima [A] time = 3.14, size = 78, normalized size = 1.04

$$-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}-\frac{2\sqrt{3}\left(Ca+\left(3B\left(\frac{a}{b}\right)^{\frac{2}{3}}-\frac{4Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="maxima")
```

```
[Out] -C*log(x - (a/b)^(1/3))/b - 2/9*sqrt(3)*(C*a + (3*B*(a/b)^(2/3) - 4*C*a/b)*
b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

mupad [B] time = 6.36, size = 435, normalized size = 5.80

$$\sum_{k=1}^{3}\ln\left(-\frac{C^{2}a+B^{2}b\left(\frac{a}{b}\right)^{1/3}-2BCb\left(\frac{a}{b}\right)^{2/3}}{b^{3}}-\frac{\mathrm{root}\left(27a^{2}b^{3}z^{3}+27Ca^{2}b^{2}z^{2}-18BCab^{2}z\left(\frac{a}{b}\right)^{2/3}+9B^{2}ab^{2}z\left(\frac{a}{b}\right)^{1/3}+9C^{2}a^{2}bz-18BC^{2}ab\left(\frac{a}{b}\right)^{2/3}+9B^{2}Cab\left(\frac{a}{b}\right)^{1/3}+9C^{3}a^{2},z,k\right)\left(6Ca+\mathrm{root}\left(27a^{2}b^{3}z^{3}+27Ca^{2}b^{2}z^{2}-18BCab^{2}z\left(\frac{a}{b}\right)^{2/3}+9B^{2}ab^{2}z\left(\frac{a}{b}\right)^{1/3}+9C^{2}a^{2}bz-18BC^{2}ab\left(\frac{a}{b}\right)^{2/3}+9B^{2}Cab\left(\frac{a}{b}\right)^{1/3}+9C^{3}a^{2},z,k\right)ab9-3Bbx\left(\frac{a}{b}\right)^{1/3}+6Cbx\left(\frac{a}{b}\right)^{2/3}\right)}{b^{2}}+\frac{x\left(B^{2}-2C^{2}\left(\frac{a}{b}\right)^{2/3}+BC\left(\frac{a}{b}\right)^{1/3}\right)}{b^{2}}\right)\mathrm{root}\left(27a^{2}b^{3}z^{3}+27Ca^{2}b^{2}z^{2}-18BCab^{2}z\left(\frac{a}{b}\right)^{2/3}+9B^{2}ab^{2}z\left(\frac{a}{b}\right)^{1/3}+9C^{2}a^{2}bz-18BC^{2}ab\left(\frac{a}{b}\right)^{2/3}+9B^{2}Cab\left(\frac{a}{b}\right)^{1/3}+9C^{3}a^{2},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x + C*x^2 - B*(a/b)^(1/3) + 2*C*(a/b)^(2/3))/(a - b*x^3),x)
```

```
[Out] symsum(log((x*(B^2 - 2*C^2*(a/b)^(2/3) + B*C*(a/b)^(1/3)))/b^2 - (root(27*a
^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*
(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^
(1/3) + 9*C^3*a^2, z, k)*(6*C*a + 9*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2
- 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z -
18*B*C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^(1/3) + 9*C^3*a^2, z, k)*a*b -
 3*B*b*x*(a/b)^(1/3) + 6*C*b*x*(a/b)^(2/3)))/b^2 - (C^2*a + B^2*b*(a/b)^(1/
3) - 2*B*C*b*(a/b)^(2/3))/b^3)*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*
B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*
C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^(1/3) + 9*C^3*a^2, z, k), k, 1, 3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

```
[Out] Timed out
```

3.48 $\int \frac{a+ax+cx^2}{1-x^3} \, dx$

Optimal. Leaf size=32

$$\frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(1-x)$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1875, 31, 628}

$$\frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x + c*x^2)/(1 - x^3),x]
[Out] -((2*a + c)*Log[1 - x])/3 + ((a - c)*Log[1 + x + x^2])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+ax+cx^2}{1-x^3}\,dx &= \frac{1}{3}\int \frac{a-c+(2a-2c)x}{1+x+x^2}\,dx+\frac{1}{3}(2a+c)\int \frac{1}{1-x}\,dx\\ &= -\frac{1}{3}(2a+c)\log(1-x)+\frac{1}{3}(a-c)\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.97

$$\frac{1}{3}\left((a-c)\log\left(x^2+x+1\right)-(2a+c)\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x + c*x^2)/(1 - x^3),x]
[Out] (-((2*a + c)*Log[1 - x]) + (a - c)*Log[1 + x + x^2])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + a*x + c*x^2)/(1 - x^3),x]
[Out] IntegrateAlgebraic[(a + a*x + c*x^2)/(1 - x^3), x]
```

fricas [A] time = 0.40, size = 26, normalized size = 0.81

$$\frac{1}{3}\,(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(2\,a+c)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="fricas")
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

giac [A] time = 0.15, size = 27, normalized size = 0.84

$$\frac{1}{3}\,(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(2\,a+c)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="giac")
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(abs(x - 1))
```

maple [A] time = 0.05, size = 36, normalized size = 1.12

$$-\frac{2a\ln\left(x-1\right)}{3}+\frac{a\ln\left(x^2+x+1\right)}{3}-\frac{c\ln\left(x-1\right)}{3}-\frac{c\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a*x+a)/(-x^3+1),x)
[Out] -1/3*ln(x-1)*c-2/3*ln(x-1)*a+1/3*ln(x^2+x+1)*a-1/3*ln(x^2+x+1)*c
```

maxima [A] time = 2.97, size = 26, normalized size = 0.81

$$\frac{1}{3}\,(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(2\,a+c)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="maxima")
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

mupad [B] time = 4.78, size = 35, normalized size = 1.09

$$\frac{a\,\ln\left(x^2+x+1\right)}{3}-\frac{c\,\ln\left(x-1\right)}{3}-\frac{2\,a\,\ln\left(x-1\right)}{3}-\frac{c\,\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + a*x + c*x^2)/(x^3 - 1),x)
```

```
[Out] (a*log(x + x^2 + 1))/3 - (c*log(x - 1))/3 - (2*a*log(x - 1))/3 - (c*log(x +
 x^2 + 1))/3
```

sympy [A] time = 0.87, size = 24, normalized size = 0.75

$$\frac{(a-c)\log\left(x^2+x+1\right)}{3}-\frac{(2a+c)\log(x-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a*x+a)/(-x**3+1),x)
```

```
[Out] (a - c)*log(x**2 + x + 1)/3 - (2*a + c)*log(x - 1)/3
```

3.49 $\int \frac{a+bx+cx^2}{1-x^3}\,dx$

Optimal. Leaf size=55

$$\frac{1}{6}\log\left(x^2+x+1\right)(a+b-2c)-\frac{1}{3}\log(1-x)(a+b+c)+\frac{(a-b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1875, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2+x+1\right)(a+b-2c)-\frac{1}{3}\log(1-x)(a+b+c)+\frac{(a-b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(1 - x^3),x]
```

```
[Out] ((a - b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - ((a + b + c)*Log[1 - x])/3 +
((a + b - 2*c)*Log[1 + x + x^2])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
```

```
2] && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{1-x^3}\,dx &= \frac{1}{3}\int \frac{2a-b-c+(a+b-2c)x}{1+x+x^2}\,dx+\frac{1}{3}(a+b+c)\int \frac{1}{1-x}\,dx\\ &= -\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{2}(a-b)\int \frac{1}{1+x+x^2}\,dx+\frac{1}{6}(a+b-2c)\int \frac{1+2x}{1+x+x^2}\,dx\\ &= -\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)+(-a+b)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\right.\\ &= \frac{(a-b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 1.13

$$\frac{1}{6}\left((a+b)\log\left(x^2+x+1\right)-2(a+b)\log(1-x)+2\sqrt{3}(a-b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2c\log\left(1-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(1 - x^3),x]
[Out] (2*Sqrt[3]*(a - b)*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*(a + b)*Log[1 - x] + (a +
b)*Log[1 + x + x^2] - 2*c*Log[1 - x^3])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(1 - x^3),x]
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)/(1 - x^3), x]
```

fricas [A] time = 0.42, size = 47, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right)-\frac{1}{3}(a+b+c)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(x
^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

giac [A] time = 0.17, size = 52, normalized size = 0.95

$$\frac{1}{3}\left(\sqrt{3}a-\sqrt{3}b\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right)-\frac{1}{3}(a+b+c)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(3)*a - sqrt(3)*b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*
c)*log(x^2 + x + 1) - 1/3*(a + b + c)*log(abs(x - 1))
```

maple [A] time = 0.05, size = 87, normalized size = 1.58

$$\frac{\sqrt{3}\,a\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}-\frac{a\ln(x-1)}{3}+\frac{a\ln\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,b\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}-\frac{b\ln(x-1)}{3}+\frac{b\ln\left(x^2+x+1\right)}{6}-\frac{c\ln(x-1)}{3}-\frac{c\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-x^3+1),x)
```

```
[Out] -1/3*c*ln(x-1)-1/3*ln(x-1)*b-1/3*a*ln(x-1)+1/6*a*ln(x^2+x+1)+1/6*ln(x^2+x+1
)*b-1/3*c*ln(x^2+x+1)+1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))*a-1/3*3^(1/2)
*arctan(1/3*(2*x+1)*3^(1/2))*b
```

maxima [A] time = 2.99, size = 47, normalized size = 0.85

$$\frac{1}{3}\,\sqrt{3}\,(a-b)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\,(a+b-2\,c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(a+b+c)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(x
^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

mupad [B] time = 4.95, size = 87, normalized size = 1.58

$$\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}-\frac{\sqrt{3}\,a\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,b\,1\mathrm{i}}{6}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}+\frac{\sqrt{3}\,a\,1\mathrm{i}}{6}-\frac{\sqrt{3}\,b\,1\mathrm{i}}{6}\right)-\ln\left(x-1\right)\left(\frac{a}{3}+\frac{b}{3}+\frac{c}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + c*x^2)/(x^3 - 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*(a/6 + b/6 - c/3 - (3^(1/2)*a*1i)/6 + (3^(1/2
)*b*1i)/6) + log(x + (3^(1/2)*1i)/2 + 1/2)*(a/6 + b/6 - c/3 + (3^(1/2)*a*1i
)/6 - (3^(1/2)*b*1i)/6) - log(x - 1)*(a/3 + b/3 + c/3)
```

sympy [C] time = 1.89, size = 323, normalized size = 5.87

$$-\frac{(a+b+c)\log\left(x+\frac{a^2c-a^2(a+b+c)-2ab^2+bc^2-2bc(a+b+c)+b(a+b+c)^2}{a^3-b^3}\right)}{3}-\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}\,i\,(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}\,i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}\,i(a-b)}{6}\right)+9b\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}\,i(a-b)}{6}\right)^2}{a^3-b^3}\right)-\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}\,i\,(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}\,i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}\,i(a-b)}{6}\right)+9b\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}\,i(a-b)}{6}\right)^2}{a^3-b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-x**3+1),x)
```

```
[Out] -(a + b + c)*log(x + (a**2*c - a**2*(a + b + c) - 2*a*b**2 + b*c**2 - 2*b*c
*(a + b + c) + b*(a + b + c)**2)/(a**3 - b**3))/3 - (-a/6 - b/6 + c/3 - sqr
t(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a
 - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/
6) + 9*b*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/6)**2)/(a**3 - b**3)) - (-a/
6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 +
 c/3 + sqrt(3)*I*(a - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 +
 sqrt(3)*I*(a - b)/6) + 9*b*(-a/6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6)**2)/(a
**3 - b**3))
```

3.50 $\int \frac{1+x+x^2}{1-x^3}\,dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(1 - x^3),x]
```

```
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2}{1-x^3}\,dx &= \int \frac{1}{1-x}\,dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(1 - x^3),x]
```

```
[Out] -Log[1 - x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{1-x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2)/(1 - x^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + x + x^2)/(1 - x^3), x]
```

fricas [A] time = 0.38, size = 6, normalized size = 0.75

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="fricas")
[Out] -log(x - 1)
```

giac [A] time = 0.15, size = 7, normalized size = 0.88

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="giac")
[Out] -log(abs(x - 1))
```

maple [A] time = 0.05, size = 7, normalized size = 0.88

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(-x^3+1),x)
[Out] -ln(x-1)
```

maxima [A] time = 1.27, size = 6, normalized size = 0.75

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="maxima")
[Out] -log(x - 1)
```

mupad [B] time = 0.02, size = 6, normalized size = 0.75

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2 + 1)/(x^3 - 1),x)
[Out] -log(x - 1)
```

sympy [A] time = 0.13, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(-x**3+1),x)
[Out] -log(x - 1)
```

3.51 $\int \frac{1-x+3x^2}{1-x^3} \, dx$

Optimal. Leaf size=30

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1871, 1586, 618, 204, 260}

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 3*x^2)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x+3x^2}{1-x^3}\,dx &= 3\int \frac{x^2}{1-x^3}\,dx + \int \frac{1-x}{1-x^3}\,dx \\
&= -\log\left(1-x^3\right) + \int \frac{1}{1+x+x^2}\,dx \\
&= -\log\left(1-x^3\right) - 2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= \frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(1 - x^3),x]
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x+3x^2}{1-x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x + 3*x^2)/(1 - x^3),x]
[Out] IntegrateAlgebraic[(1 - x + 3*x^2)/(1 - x^3), x]
```

fricas [A] time = 0.42, size = 32, normalized size = 1.07

$$\frac{2}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right) - \log\left(x^2+x+1\right) - \log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

giac [A] time = 0.16, size = 33, normalized size = 1.10

$$\frac{2}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right) - \log\left(x^2+x+1\right) - \log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(abs(x -
1))
```

maple [A] time = 0.05, size = 33, normalized size = 1.10

$$\frac{2\sqrt{3}\;\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} - \ln\left(x-1\right) - \ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(-x^3+1),x)
[Out] -ln(x-1)-ln(x^2+x+1)+2/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.96, size = 32, normalized size = 1.07

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

mupad [B] time = 4.93, size = 63, normalized size = 2.10

$$-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln(x-1)-\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3}+\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 - x + 1)/(x^3 - 1),x)
[Out] (3^(1/2)*log(x + (3^(1/2)*1i)/2 + 1/2)*1i)/3 - log(x + (3^(1/2)*1i)/2 + 1/2
) - log(x - 1) - (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - log(x - (3^
(1/2)*1i)/2 + 1/2)
```

sympy [A] time = 0.34, size = 5, normalized size = 0.17

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(-x**3+1),x)
[Out] -log(x - 1)
```

3.52 $\int \frac{1+x+4x^2}{1-x^3} \, dx$

Optimal. Leaf size=18

$$-\log\left(x^2+x+1\right)-2\log(1-x)$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1875, 31, 628}

$$-\log\left(x^2+x+1\right)-2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(1 - x^3),x]
```

```
[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+4x^2}{1-x^3} \, dx &= \frac{1}{3}\int \frac{-3-6x}{1+x+x^2} \, dx+2\int \frac{1}{1-x} \, dx \\ &= -2\log(1-x)-\log\left(1+x+x^2\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\log\left(x^2+x+1\right)-2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(1 - x^3),x]
```

```
[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+4x^2}{1-x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + 4*x^2)/(1 - x^3),x]
[Out] IntegrateAlgebraic[(1 + x + 4*x^2)/(1 - x^3), x]
```

fricas [A] time = 0.39, size = 16, normalized size = 0.89

$$-\log\left(x^2+x+1\right)-2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="fricas")
[Out] -log(x^2 + x + 1) - 2*log(x - 1)
```

giac [A] time = 0.15, size = 17, normalized size = 0.94

$$-\log\left(x^2+x+1\right)-2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="giac")
[Out] -log(x^2 + x + 1) - 2*log(abs(x - 1))
```

maple [A] time = 0.05, size = 17, normalized size = 0.94

$$-2\ln\left(x-1\right)-\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(-x^3+1),x)
[Out] -2*ln(x-1)-ln(x^2+x+1)
```

maxima [A] time = 2.93, size = 16, normalized size = 0.89

$$-\log\left(x^2+x+1\right)-2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="maxima")
[Out] -log(x^2 + x + 1) - 2*log(x - 1)
```

mupad [B] time = 0.04, size = 16, normalized size = 0.89

$$-\ln\left(x^2+x+1\right)-2\,\ln\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 4*x^2 + 1)/(x^3 - 1),x)
[Out] - log(x + x^2 + 1) - 2*log(x - 1)
```

sympy [A] time = 0.16, size = 15, normalized size = 0.83

$$-2\log\left(x-1\right)-\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+x+1)/(-x**3+1),x)
[Out] -2*log(x - 1) - log(x**2 + x + 1)
```

3.53 $\int \left(a+bx^3\right)^3 \left(ac+adx+bcx^3+bdx^4\right) dx$

Optimal. Leaf size=113

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1850}

$$\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+a^3bcx^4+\frac{4}{5}a^3bdx^5+a^4cx+\frac{1}{2}a^4dx^2+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(a+bx^3\right)^3 \left(ac+adx+bcx^3+bdx^4\right) dx = \int \left(a^4c+a^4dx+4a^3bcx^3+4a^3bdx^4+6a^2b^2cx^6+6a^2b^2dx^7+4ab^3\right.$$
$$= a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3$$

Mathematica [A] time = 0.00, size = 113, normalized size = 1.00

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^3 \left(ac+adx+bcx^3+bdx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x]
```

fricas [A] time = 0.35, size = 97, normalized size = 0.86

$$\frac{1}{14}x^{14}db^4+\frac{1}{13}x^{13}cb^4+\frac{4}{11}x^{11}db^3a+\frac{2}{5}x^{10}cb^3a+\frac{3}{4}x^8db^2a^2+\frac{6}{7}x^7cb^2a^2+\frac{4}{5}x^5dba^3+x^4cba^3+\frac{1}{2}x^2da^4+xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d*b^4 + 1/13*x^13*c*b^4 + 4/11*x^11*d*b^3*a + 2/5*x^10*c*b^3*a +
3/4*x^8*d*b^2*a^2 + 6/7*x^7*c*b^2*a^2 + 4/5*x^5*d*b*a^3 + x^4*c*b*a^3 + 1/2
*x^2*d*a^4 + x*c*a^4
```

giac [A] time = 0.16, size = 97, normalized size = 0.86

$$\frac{1}{14}b^4dx^{14}+\frac{1}{13}b^4cx^{13}+\frac{4}{11}ab^3dx^{11}+\frac{2}{5}ab^3cx^{10}+\frac{3}{4}a^2b^2dx^8+\frac{6}{7}a^2b^2cx^7+\frac{4}{5}a^3bdx^5+a^3bcx^4+\frac{1}{2}a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2
*a^4*d*x^2 + a^4*c*x
```

maple [A] time = 0.04, size = 98, normalized size = 0.87

$$\frac{1}{14}b^4d\,x^{14}+\frac{1}{13}b^4c\,x^{13}+\frac{4}{11}a\,b^3d\,x^{11}+\frac{2}{5}a\,b^3c\,x^{10}+\frac{3}{4}a^2b^2d\,x^8+\frac{6}{7}a^2b^2c\,x^7+\frac{4}{5}a^3bd\,x^5+a^3bc\,x^4+\frac{1}{2}a^4d\,x^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^2
*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*x^
14
```

maxima [A] time = 1.39, size = 97, normalized size = 0.86

$$\frac{1}{14}b^4dx^{14}+\frac{1}{13}b^4cx^{13}+\frac{4}{11}ab^3dx^{11}+\frac{2}{5}ab^3cx^{10}+\frac{3}{4}a^2b^2dx^8+\frac{6}{7}a^2b^2cx^7+\frac{4}{5}a^3bdx^5+a^3bcx^4+\frac{1}{2}a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2
*a^4*d*x^2 + a^4*c*x
```

mupad [B] time = 0.06, size = 97, normalized size = 0.86

$$\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{4\,d\,a^3\,b\,x^5}{5}+c\,a^3\,b\,x^4+\frac{3\,d\,a^2\,b^2\,x^8}{4}+\frac{6\,c\,a^2\,b^2\,x^7}{7}+\frac{4\,d\,a\,b^3\,x^{11}}{11}+\frac{2\,c\,a\,b^3\,x^{10}}{5}+\frac{d\,b^4\,x^{14}}{14}+\frac{c\,b^4\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

```
[Out] (a^4*d*x^2)/2 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14 + a^4*c*x + (6*a^2*b^2*c*
x^7)/7 + (3*a^2*b^2*d*x^8)/4 + a^3*b*c*x^4 + (2*a*b^3*c*x^10)/5 + (4*a^3*b*
d*x^5)/5 + (4*a*b^3*d*x^11)/11
```

sympy [A] time = 0.73, size = 117, normalized size = 1.04

$$a^4cx + \frac{a^4dx^2}{2} + a^3bcx^4 + \frac{4a^3bdx^5}{5} + \frac{6a^2b^2cx^7}{7} + \frac{3a^2b^2dx^8}{4} + \frac{2ab^3cx^{10}}{5} + \frac{4ab^3dx^{11}}{11} + \frac{b^4cx^{13}}{13} + \frac{b^4dx^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/5 + 6*a**2*b**2*
c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11
+ b**4*c*x**13/13 + b**4*d*x**14/14
```

3.54 $\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right) \, dx$

Optimal. Leaf size=88

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1850}

$$\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right) \, dx = \int \left(a^3c+a^3dx+3a^2bcx^3+3a^2bdx^4+3ab^2cx^6+3ab^2dx^7+b^3cx\right.$$
$$=a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}$$

Mathematica [A] time = 0.00, size = 88, normalized size = 1.00

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x]
```

fricas [A] time = 0.36, size = 74, normalized size = 0.84

$$\frac{1}{11}x^{11}db^3+\frac{1}{10}x^{10}cb^3+\frac{3}{8}x^8db^2a+\frac{3}{7}x^7cb^2a+\frac{3}{5}x^5dba^2+\frac{3}{4}x^4cba^2+\frac{1}{2}x^2da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d*b^3 + 1/10*x^10*c*b^3 + 3/8*x^8*d*b^2*a + 3/7*x^7*c*b^2*a + 3/5
*x^5*d*b*a^2 + 3/4*x^4*c*b*a^2 + 1/2*x^2*d*a^3 + x*c*a^3
```

giac [A] time = 0.16, size = 74, normalized size = 0.84

$$\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5
*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

maple [A] time = 0.05, size = 75, normalized size = 0.85

$$\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{3}{8}a\,b^2d\,x^8+\frac{3}{7}a\,b^2c\,x^7+\frac{3}{5}a^2bd\,x^5+\frac{3}{4}a^2bc\,x^4+\frac{1}{2}a^3d\,x^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*a
*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

maxima [A] time = 1.39, size = 74, normalized size = 0.84

$$\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5
*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

mupad [B] time = 0.04, size = 74, normalized size = 0.84

$$\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,d\,a^2\,b\,x^5}{5}+\frac{3\,c\,a^2\,b\,x^4}{4}+\frac{3\,d\,a\,b^2\,x^8}{8}+\frac{3\,c\,a\,b^2\,x^7}{7}+\frac{d\,b^3\,x^{11}}{11}+\frac{c\,b^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + a^3*c*x + (3*a^2*b*c*x^
4)/4 + (3*a*b^2*c*x^7)/7 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*d*x^8)/8
```

sympy [A] time = 0.16, size = 90, normalized size = 1.02

$$a^3cx+\frac{a^3dx^2}{2}+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x**5/5 + 3*a*b**2
*c*x**7/7 + 3*a*b**2*d*x**8/8 + b**3*c*x**10/10 + b**3*d*x**11/11
```

3.55 $\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx$

Optimal. Leaf size=60

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1850}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx &= \int \left(a^2c+a^2dx+2abcx^3+2abdx^4+b^2cx^6+b^2dx^7\right)\,dx\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8\end{aligned}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.00

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x]
```

fricas [A] time = 0.35, size = 50, normalized size = 0.83

$$\frac{1}{8}x^8db^2+\frac{1}{7}x^7cb^2+\frac{2}{5}x^5dba+\frac{1}{2}x^4cba+\frac{1}{2}x^2da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d*b^2 + 1/7*x^7*c*b^2 + 2/5*x^5*d*b*a + 1/2*x^4*c*b*a + 1/2*x^2*d*a
^2 + x*c*a^2
```

giac [A] time = 0.17, size = 50, normalized size = 0.83

$$\frac{1}{8}\,b^2dx^8 + \frac{1}{7}\,b^2cx^7 + \frac{2}{5}\,abdx^5 + \frac{1}{2}\,abcx^4 + \frac{1}{2}\,a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x
^2 + a^2*c*x
```

maple [A] time = 0.04, size = 51, normalized size = 0.85

$$\frac{1}{8}b^2d\,x^8 + \frac{1}{7}b^2c\,x^7 + \frac{2}{5}abd\,x^5 + \frac{1}{2}abc\,x^4 + \frac{1}{2}a^2d\,x^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*x
^8
```

maxima [A] time = 1.40, size = 50, normalized size = 0.83

$$\frac{1}{8}\,b^2dx^8 + \frac{1}{7}\,b^2cx^7 + \frac{2}{5}\,abdx^5 + \frac{1}{2}\,abcx^4 + \frac{1}{2}\,a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x
^2 + a^2*c*x
```

mupad [B] time = 0.03, size = 50, normalized size = 0.83

$$\frac{d\,a^2\,x^2}{2} + c\,a^2\,x + \frac{2\,d\,a\,b\,x^5}{5} + \frac{c\,a\,b\,x^4}{2} + \frac{d\,b^2\,x^8}{8} + \frac{c\,b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + a^2*c*x + (a*b*c*x^4)/2 + (
2*a*b*d*x^5)/5
```

sympy [A] time = 0.10, size = 58, normalized size = 0.97

$$a^2cx + \frac{a^2dx^2}{2} + \frac{abcx^4}{2} + \frac{2abdx^5}{5} + \frac{b^2cx^7}{7} + \frac{b^2dx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 + b**2*c*x**7/7 +
b**2*d*x**8/8
```

3.56 $\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1586}

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
[Out] c*x + (d*x^2)/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx &= \int (c+dx)\,dx\\ &= cx+\frac{dx^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
[Out] c*x + (d*x^2)/2
```

IntegrateAlgebraic [A] time = 0.03, size = 14, normalized size = 1.17

$$\frac{(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
[Out] (c + d*x)^2/(2*d)
```

fricas [A] time = 0.39, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="fricas")
[Out] 1/2*d*x^2 + c*x
```

giac [A] time = 0.20, size = 10, normalized size = 0.83

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="giac")
[Out] 1/2*d*x^2 + c*x
```

maple [A] time = 0.04, size = 11, normalized size = 0.92

$$\frac{1}{2}d\,x^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x)
[Out] c*x+1/2*d*x^2
```

maxima [A] time = 1.32, size = 10, normalized size = 0.83

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="maxima")
[Out] 1/2*d*x^2 + c*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{d\,x^2}{2} + c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x)
[Out] c*x + (d*x^2)/2
```

sympy [A] time = 0.13, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a),x)
[Out] c*x + d*x**2/2
```

3.57 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Rubi [A] time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1586, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^2}\,dx=\int \frac{c+dx}{a+bx^3}\,dx$$

$$=\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$=\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}}+\frac{1}{2}\left(\frac{c}{\sqrt[3]{a}}+\right.$$

$$=\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}}+\frac{(\sqrt[}{}$$

$$=-\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right.}{}$$

Mathematica [A] time = 0.05, size = 124, normalized size = 0.77

$$\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2, x]
```

fricas [C] time = 1.18, size = 1931, normalized size = 11.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d
^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*
d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)
/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^
2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a
*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3 + a*d^3)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a^2*b*d + 2*a*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^
3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c
^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^
3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^
2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3
+ a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*
a^2*b*d + 2*a*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^
2*a*b + 16*c*d)/(a*b)))
```

giac [A] time = 0.18, size = 141, normalized size = 0.88

$$-\frac{\sqrt{3}\left(bc-\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}}-\frac{\left(bc+\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}}-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(d*(-a/b)^(1/3) + c)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
```

maple [A] time = 0.05, size = 186, normalized size = 1.16

$$\frac{\sqrt{3}\, c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b}-\frac{d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b}+\frac{d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3/(a/b)^(2/3)/b*c*ln(x+(a/b)^(1/3))-1/6*c/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/3*c/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-1/3*d/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*d/b/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*d*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.98, size = 135, normalized size = 0.84

$$\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(d*(a/b)^(1/3) + c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(d*(a/b)^(1/3) - c)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(d*(a/b)^(1/3) - c)*log(x + (a/b)^(1/3
))/(b*(a/b)^(2/3))
```

mupad [B] time = 5.09, size = 127, normalized size = 0.79

$$\sum_{k=1}^{3}\ln\left(b\left(c\,d+d^{2}x+\mathrm{root}\left(27a^{2}b^{2}z^{3}+9\,a\,b\,c\,d\,z+a\,d^{3}-b\,c^{3},z,k\right)^{2}a\,b\,9+\mathrm{root}\left(27a^{2}b^{2}z^{3}+9\,a\,b\,c\,d\,z+a\,d^{3}-b\,c^{3},z,k\right)\,b\,c\,x\,3\right)\right)\mathrm{root}\left(27a^{2}b^{2}z^{3}+9\,a\,b\,c\,d\,z+a\,d^{3}-b\,c^{3},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log(b*(c*d + d^2*x + 9*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b
*c^3, z, k)^2*a*b + 3*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z,
 k)*b*c*x))*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z, k), k, 1,
 3)
```

sympy [A] time = 1.34, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^{3}a^{2}b^{2}+9tabcd+ad^{3}-bc^{3},\left(t\mapsto t\log\left(x+\frac{9t^{2}a^{2}bd+3tabc^{2}+2acd^{2}}{ad^{3}+bc^{3}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)))
)
```

3.58 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}\,d+2\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\sqrt[3]{}}\right.}{3\sqrt{3}\,a^{5/3}b^{2/3}}$$

Rubi [A] time = 0.13, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1586, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}\,d+2\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5
/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^3}\,dx = \int \frac{c+dx}{\left(a+bx^3\right)^2}\,dx$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}c-\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x} \ldots}{9a^{5/3}}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\ldots}{a^{2/3}-\sqrt[3]{a}\ldots}}{18a^{5/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\ldots\right.}{18a^{5/3}b^{2\ldots}}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}-\ldots\right.}{9a^{5/3}b^{2/3}}$$

Mathematica [A] time = 0.17, size = 180, normalized size = 0.95

$$\frac{\frac{\left(a^{2/3}d-2\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6ax(c+dx)}{a+bx^3}}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

```
[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b^(
```

```
1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3
)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
18*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\left(a + bx^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3, x]
```

fricas [C] time = 1.19, size = 2088, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
 a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*
(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^
5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^
5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3)
- 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/
3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2
) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/
(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*
a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3 + a^2)*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt(1/3)*(a*b*x^
3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) +
 (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^
3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a
^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2)
)^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8
*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b +
32*c*d)/(a^3*b))) + ((a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt
(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)
+ 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (
8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^
```

```
(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/
(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*
(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1
)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/
2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))))/(a*b*x^3 + a^2)
```

giac [A] time = 0.21, size = 174, normalized size = 0.92

$$-\frac{\sqrt{3}\left(2\,bc-\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{\left(2\,bc+\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}}+\frac{dx^{2}+cx}{3\left(bx^{3}+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)^(1
/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c*x)/
((b*x^3 + a)*a)
```

maple [A] time = 0.05, size = 238, normalized size = 1.26

$$\frac{d\,x^{2}}{3\left(b\,x^{3}+a\right)a}+\frac{cx}{3\left(b\,x^{3}+a\right)a}+\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x)
```

```
[Out] 1/3*c*x/a/(b*x^3+a)+2/9/(a/b)^(2/3)/a/b*c*ln(x+(a/b)^(1/3))-1/9*c/a/b/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9*c/a/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/(b*x^3+a)/a*d*x^2-1/9*d/a/b/(a/b)^(
1/3)*ln(x+(a/b)^(1/3))+1/18*d/a/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+1/9*d/a*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 3.02, size = 169, normalized size = 0.89

$$\frac{dx^{2}+cx}{3\left(abx^{3}+a^{2}\right)}+\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(d*x^2 + c*x)/(a*b*x^3 + a^2) + 1/9*sqrt(3)*(d*(a/b)^(1/3) + 2*c)*arcta
n(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) + 1/18*(d*
(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3))
 - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

mupad [B] time = 5.08, size = 169, normalized size = 0.89

$$\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d+d^2\,x+\mathrm{root}\left(729\,a^5\,b^2\,z^3+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k\right)^2\,a^3\,b\,81+\mathrm{root}\left(729\,a^5\,b^2\,z^3+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k\right)\,a\,b\,c\,x\,18\right)}{a^2\,9}\right)\,\mathrm{root}\left(729\,a^5\,b^2\,z^3+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k\right)\right)+\frac{\frac{d\,x^2}{3\,a}+\frac{c\,x}{3\,a}}{b\,x^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x)
```

```
[Out] symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z - 8
*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*b*c
*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) + (c*x)/(3*a))/(a
+ b*x^3)
```

sympy [A] time = 1.85, size = 105, normalized size = 0.56

$$\mathrm{RootSum}\left(729t^3a^5b^2+54ta^2bcd+ad^3-8bc^3,\left(t\mapsto t\log\left(x+\frac{81t^2a^4bd+36ta^2bc^2+4acd^2}{ad^3+8bc^3}\right)\right)\right)+\frac{cx+dx^2}{3a^2+3abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

3.59 $\int \frac{(a+bx)^2}{c+dx^3}\,dx$

Optimal. Leaf size=186

$$\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}-\frac{a\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\tan^{-1}\left(\cdots\right.}{\sqrt{3}\,c^{2/3}d^{2/3}}$$

Rubi [A] time = 0.18, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}-\frac{a\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x^3),x]
```

```
[Out] -((a*(2*b*c^(1/3) + a*d^(1/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1
/3))])/(Sqrt[3]*c^(2/3)*d^(2/3))) - (a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(1/3
) + d^(1/3)*x])/(3*c^(2/3)*d^(2/3)) + (a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(2
/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(2/3)) + (b^2*Log[c +
d*x^3])/(3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = b^2\int \frac{x^2}{c+dx^3}\,dx + \int \frac{a^2+2abx}{c+dx^3}\,dx$$

$$= \frac{b^2\log\left(c+dx^3\right)}{3d} + \frac{\int \frac{\sqrt[3]{c}\left(2ab\sqrt[3]{c}+2a^2\sqrt[3]{d}\right)+\left(2ab\sqrt[3]{c}-a^2\sqrt[3]{d}\right)\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}\sqrt[3]{d}} - \frac{\left(2ab\sqrt[3]{c}-a^2\sqrt[3]{d}\right)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}\sqrt[3]{d}}$$

$$= -\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d} + \frac{1}{2}\left(a\left(\frac{a}{\sqrt[3]{c}}+\frac{2b}{\sqrt[3]{d}}\right)\right)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x$$

$$= -\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}} + \frac{b^2\log$$

$$= -\frac{a\left(2b\sqrt[3]{c}+a\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}} - \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)$$

Mathematica [A] time = 0.09, size = 200, normalized size = 1.08

$$-\frac{\left(a^2\sqrt[3]{c}\sqrt[3]{d}-2abc^{2/3}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6cd^{2/3}} + \frac{\left(a^2\sqrt[3]{c}\sqrt[3]{d}-2abc^{2/3}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3cd^{2/3}} + \frac{\left(a^2\sqrt[3]{c}\sqrt[3]{d}+2abc^{2/3}\right)\tan^{-1}\left(\frac{2\sqrt[3]{d}x-\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}cd^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x^3),x]
```

```
[Out] ((2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3))*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c*d^(2/3)) + ((-2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3
))*Log[c^(1/3) + d^(1/3)*x])/(3*c*d^(2/3)) - ((-2*a*b*c^(2/3) + a^2*c^(1/3)
*d^(1/3))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c*d^(2/3)) + (
b^2*Log[c + d*x^3])/(3*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(c + d*x^3), x]
```

fricas [C] time = 1.20, size = 5014, normalized size = 26.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3)
 + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^
(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(
3) + 1) - 2*b^2/d)*d*log(2*b^5*c^2 + 7*a^3*b^2*c*d + 1/2*(2*(1/2)^(2/3)*(b^
4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c
 + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*
a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c
 + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*
a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)^2*b*c^2*
d^2 + 1/2*(4*b^3*c^2*d - a^3*c*d^2)*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^
3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d) + (8*a^2*b^3*c*d + a^5*d^2)*x
) - (6*b^2 + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqr
t(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b
*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1
/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b
*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*s
qrt(3) + 1) - 2*b^2/d)*d + 3*sqrt(1/3)*d*sqrt(-(4*b^4*c + 32*a^3*b*d + 4*(2
*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2
*b^2/d)*b^2*c*d + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-
I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*
a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)
 + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*
a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)
*(I*sqrt(3) + 1) - 2*b^2/d)^2*c*d^2)/(c*d^2)))*log(-2*b^5*c^2 - 7*a^3*b^2*c
*d - 1/2*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3)
 + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^
(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(
3) + 1) - 2*b^2/d)^2*b*c^2*d^2 - 1/2*(4*b^3*c^2*d - a^3*c*d^2)*(2*(1/2)^(2/
3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8
*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^
2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8
*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^
2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d) +
2*(8*a^2*b^3*c*d + a^5*d^2)*x + 3/2*sqrt(1/3)*(2*b^3*c^2*d + a^3*c*d^2 + (2
*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
```

```
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2
*b^2/d)*b*c^2*d^2)*sqrt(-(4*b^4*c + 32*a^3*b*d + 4*(2*(1/2)^(2/3)*(b^4/d^2
 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3
*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^
3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3
*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^
3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)*b^2*c*d + (2*(
1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/
d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) +
 (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/
d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) +
 (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b
^2/d)^2*c*d^2)/(c*d^2))) - (6*b^2 + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^
3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)*d - 3*sqrt(1/3)*d*sqrt(-(4*b^
4*c + 32*a^3*b*d + 4*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))
*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c +
 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1
/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c +
 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1
/3)*(I*sqrt(3) + 1) - 2*b^2/d)*b^2*c*d + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c +
 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c
^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^
6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c
^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^
6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)^2*c*d^2)/(c*d^2)))*log(-
2*b^5*c^2 - 7*a^3*b^2*c*d - 1/2*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*
d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) -
 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c
^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) -
 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c
^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)^2*b*c^2*d^2 - 1/2*(4*b^3*c^2*d -
a^3*c*d^2)*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(
3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d
)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2
)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d
)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqr
t(3) + 1) - 2*b^2/d) + 2*(8*a^2*b^3*c*d + a^5*d^2)*x - 3/2*sqrt(1/3)*(2*b^3
*c^2*d + a^3*c*d^2 + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))
*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c +
 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1
/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c +
 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1
/3)*(I*sqrt(3) + 1) - 2*b^2/d)*b*c^2*d^2)*sqrt(-(4*b^4*c + 32*a^3*b*d + 4*(
2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b
^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3
) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b
^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3
) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) -
2*b^2/d)*b^2*c*d + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(
-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2
*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3
) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2
*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3
)*(I*sqrt(3) + 1) - 2*b^2/d)^2*c*d^2)/(c*d^2))))/d
```

giac [A] time = 0.18, size = 175, normalized size = 0.94

$$\frac{b^2\log\left(\left|dx^3+c\right|\right)}{3\,d}-\frac{\sqrt{3}\left(a^2d-2\left(-cd^2\right)^{\frac{1}{3}}ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}}-\frac{\left(a^2d+2\left(-cd^2\right)^{\frac{1}{3}}ab\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}}-\frac{\left(2\,abd\left(-\frac{c}{d}\right)^{\frac{1}{3}}+a^2d\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*log(abs(d*x^3 + c))/d - 1/3*sqrt(3)*(a^2*d - 2*(-c*d^2)^(1/3)*a*b)*
arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(-c*d^2)^(2/3) - 1/6*
(a^2*d + 2*(-c*d^2)^(1/3)*a*b)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(-c
*d^2)^(2/3) - 1/3*(2*a*b*d*(-c/d)^(1/3) + a^2*d)*(-c/d)^(1/3)*log(abs(x - (
-c/d)^(1/3)))/(c*d)
```

maple [A] time = 0.05, size = 211, normalized size = 1.13

$$\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{a^2\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{a^2\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{2\sqrt{3}\,ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}d}-\frac{2ab\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}d}+\frac{ab\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}d}+\frac{b^2\ln\left(dx^3+c\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x^3+c),x)
```

```
[Out] 1/3*a^2/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6*a^2/d/(c/d)^(2/3)*ln(x^2-(c/d)^
(1/3)*x+(c/d)^(2/3))+1/3*a^2/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c
/d)^(1/3)*x-1))-2/3*a*b/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/3*a*b/d/(c/d)^(1/
3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+2/3*a*b*3^(1/2)/d/(c/d)^(1/3)*arctan(1
/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))+1/3*b^2*ln(d*x^3+c)/d
```

maxima [A] time = 2.92, size = 192, normalized size = 1.03

$$-\frac{\sqrt{3}\left(2\,b^2c-\left(6\,ab\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2\left(\frac{c}{d}\right)^{\frac{1}{3}}+\frac{2\,b^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd}+\frac{\left(2\,b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}+2\,ab\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^2\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\left(b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,ab\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*(2*b^2*c - (6*a*b*(c/d)^(2/3) + 3*a^2*(c/d)^(1/3) + 2*b^2*c/d)
*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d) + 1/6*(2*b^2*
(c/d)^(2/3) + 2*a*b*(c/d)^(1/3) - a^2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3
))/(d*(c/d)^(2/3)) + 1/3*(b^2*(c/d)^(2/3) - 2*a*b*(c/d)^(1/3) + a^2)*log(x
+ (c/d)^(1/3))/(d*(c/d)^(2/3))
```

mupad [B] time = 0.26, size = 357, normalized size = 1.92

$$\sum_{k=1}^{3}\ln\left(b^4c+\mathrm{root}\left(27\,c^2d^3z^3-27\,b^2c^2d^2z^2+18\,a^3bcd^2z+9\,b^4c^2dz+2\,a^3b^3cd-b^6c^2-a^6d^2,z,k\right)^2cd^2\,9+2\,a^3bd-\mathrm{root}\left(27\,c^2d^3z^3-27\,b^2c^2d^2z^2+18\,a^3bcd^2z+9\,b^4c^2dz+2\,a^3b^3cd-b^6c^2-a^6d^2,z,k\right)b^2cd\,6+\mathrm{root}\left(27\,c^2d^3z^3-27\,b^2c^2d^2z^2+18\,a^3bcd^2z+9\,b^4c^2dz+2\,a^3b^3cd-b^6c^2-a^6d^2,z,k\right)a^2d^2x\,3+3\,a^2b^2dx\right)\mathrm{root}\left(27\,c^2d^3z^3-27\,b^2c^2d^2z^2+18\,a^3bcd^2z+9\,b^4c^2dz+2\,a^3b^3cd-b^6c^2-a^6d^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x^3),x)
```

```
[Out] symsum(log(b^4*c + 9*root(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*
d^2*z + 9*b^4*c^2*d*z + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)^2*c*d^2 +
2*a^3*b*d - 6*root(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z +
 9*b^4*c^2*d*z + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)*b^2*c*d + 3*root(
```

```
27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z + 9*b^4*c^2*d*z + 2*
a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)*a^2*d^2*x + 3*a^2*b^2*d*x)*root(27*c
^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z + 9*b^4*c^2*d*z + 2*a^3*
b^3*c*d - b^6*c^2 - a^6*d^2, z, k), k, 1, 3)
```

sympy [A]   time = 1.38, size = 156, normalized size = 0.84

$$\operatorname{RootSum}\left(27t^3c^2d^3 - 27t^2b^2c^2d^2 + t\left(18a^3bcd^2 + 9b^4c^2d\right) - a^6d^2 + 2a^3b^3cd - b^6c^2, \left(t \mapsto t\log\left(x + \frac{18t^2bc^2d^2 + 3ta^3cd^2 - 12tb^3c^2d + 7a^3b^2cd + 2b^5c^2}{a^5d^2 + 8a^2b^3cd}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x**3+c),x)
```

```
[Out] RootSum(27*_t**3*c**2*d**3 - 27*_t**2*b**2*c**2*d**2 + _t*(18*a**3*b*c*d**2
 + 9*b**4*c**2*d) - a**6*d**2 + 2*a**3*b**3*c*d - b**6*c**2, Lambda(_t, _t*
log(x + (18*_t**2*b*c**2*d**2 + 3*_t*a**3*c*d**2 - 12*_t*b**3*c**2*d + 7*a*
*3*b**2*c*d + 2*b**5*c**2)/(a**5*d**2 + 8*a**2*b**3*c*d))))
```

3.60 $\int \frac{(a+bx)^3}{c+dx^3} \, dx$

Optimal. Leaf size=222

$$\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}\,d^{2/3}+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}\,d^{2/3}+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\cdots$$

Rubi [A] time = 0.32, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\left(3a^2b\sqrt[3]{c}\,d^{2/3}+a^3(-d)+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{\left(3a^2b\sqrt[3]{c}\,d^{2/3}+a^3(-d)+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{\left(-3a^2b\sqrt[3]{c}\,d^{2/3}+a^3(-d)+b^3c\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}}+\frac{ab^2\log\left(c+dx^3\right)}{d}+\frac{b^3x}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] (b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^(1/3) - 2*
d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b^3*c + 3*a^2*
b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(4/3)) +
((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \int \left(\frac{b^3}{d} - \frac{b^3c-a^3d-3a^2bdx-3ab^2dx^2}{d\left(c+dx^3\right)}\right)dx$$

$$= \frac{b^3x}{d} - \frac{\int \frac{b^3c-a^3d-3a^2bdx-3ab^2dx^2}{c+dx^3}\,dx}{d}$$

$$= \frac{b^3x}{d} + \left(3ab^2\right)\int \frac{x^2}{c+dx^3}\,dx - \frac{\int \frac{b^3c-a^3d-3a^2bdx}{c+dx^3}\,dx}{d}$$

$$= \frac{b^3x}{d} + \frac{ab^2\log\left(c+dx^3\right)}{d} - \frac{\int \frac{\sqrt[3]{c}\left(-3a^2b\sqrt[3]{c}\,d+2\sqrt[3]{d}\left(b^3c-a^3d\right)\right)+\sqrt[3]{d}\left(-3a^2b\sqrt[3]{c}\,d-\sqrt[3]{d}\left(b^3c-a^3d\right)\right)x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3c^{2/3}d^{4/3}} - \frac{\left(b^3c+3\cdots\right.}{\cdots}$$

$$= \frac{b^3x}{d} - \frac{\left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}} + \frac{ab^2\log\left(c+dx^3\right)}{d} - \frac{\left(b^3c-3a^2b\sqrt[3]{c}\,d^{2/3}-\cdots\right.}{2\cdots}$$

$$= \frac{b^3x}{d} - \frac{\left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}} + \frac{\left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(c^{2/3}-\sqrt[3]{c}\cdots\right.}{6c^{2/3}d^{4/3}}$$

$$= \frac{b^3x}{d} + \frac{\left(b^3c-3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}} - \frac{\left(b^3c+3a^2b\sqrt[3]{c}\,d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{3c^{2/3}d^{4/3}}$$

Mathematica [A] time = 0.24, size = 214, normalized size = 0.96

$$\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}\,d^{2/3}+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)-2\left(a^3(-d)+3a^2b\sqrt[3]{c}\,d^{2/3}+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)+2\sqrt{3}\left(a^3(-d)-3a^2b\sqrt[3]{c}\,d^{2/3}+b^3c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)+6ab^2c^{2/3}\sqrt[3]{d}\log\left(c+dx^3\right)+6b^3c^{2/3}\sqrt[3]{d}\,x}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] (6*b^3*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3
*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] - 2*(b^3*c + 3*a^2*b*c^(1/3
```

```
)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x] + (b^3*c + 3*a^2*b*c^(1/3)*d^(2
/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2] + 6*a*b^2*c^(2/
3)*d^(1/3)*Log[c + d*x^3]])/(6*c^(2/3)*d^(4/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/(c + d*x^3), x]
```

fricas [C] time = 1.82, size = 7245, normalized size = 32.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^3*x - 2*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/
(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^
2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^
4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1
/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*
a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^
2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4)
)^(1/3)*(I*sqrt(3) + 1))*d*log(-3*a*b^8*c^3 + 15*a^4*b^5*c^2*d + 15*a^7*b^2
*c*d^2 + 3/4*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2
))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d
^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (
b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) -
6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/
(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4)
 - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3
)*(I*sqrt(3) + 1))^2*a^2*b*c^2*d^3 - 1/2*(b^6*c^3*d - 20*a^3*b^3*c^2*d^2 +
a^6*c*d^3)*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))
*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3
) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^
9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*
a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c
*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) -
 (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*
(I*sqrt(3) + 1)) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)
*x) + (18*a*b^2 + (6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(
c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2
/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4
) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/
3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a
*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2
*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))
^(1/3)*(I*sqrt(3) + 1))*d + 3*sqrt(1/3)*d*sqrt((12*a^2*b^4*c - 48*a^5*b*d -
 12*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sq
rt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^
9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 -
 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d
 + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) -
 (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c
^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt
(3) + 1))*a*b^2*c*d - (6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*
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d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a
*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2
*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))
^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*
d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/
(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d
^4))^(1/3)*(I*sqrt(3) + 1))^2*c*d^2)/(c*d^2)))*log(3*a*b^8*c^3 - 15*a^4*b^5
*c^2*d - 15*a^7*b^2*c*d^2 - 3/4*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*
c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c +
a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9
*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/
(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*
c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 -
 a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d
^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))^2*a^2*b*c^2*d^3 + 1/2*(b^6*c^3*d - 20
*a^3*b^3*c^2*d^2 + a^6*c*d^3)*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c
+ a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^
5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d
^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c
^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c
+ a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a
^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3
)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1)) - 2*(b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6
*b^3*c*d^2 - a^9*d^3)*x + 3/4*sqrt(1/3)*(2*b^6*c^3*d + 14*a^3*b^3*c^2*d^2 +
 2*a^6*c*d^3 + 3*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c
*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/
(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4)
 - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3
) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*
b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*
d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^
(1/3)*(I*sqrt(3) + 1))*a^2*b*c^2*d^3)*sqrt((12*a^2*b^4*c - 48*a^5*b*d - 12*
(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3
) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^
3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a
^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (
1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^
9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 -
 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3)
+ 1))*a*b^2*c*d - (6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(
c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2
/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4
) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/
3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a
*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2
*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))
^(1/3)*(I*sqrt(3) + 1))^2*c*d^2)/(c*d^2))) + (18*a*b^2 + (6*(1/2)^(2/3)*(3*
a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6
/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*
d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*
a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3
*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*
c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d -
 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))*d - 3*sqrt(1
/3)*d*sqrt((12*a^2*b^4*c - 48*a^5*b*d - 12*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 -
(2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a
^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*
c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2
- a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*
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(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*
b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*
d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))*a*b^2*c*d - (6*(1/2)^(2/3)
*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3
*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*
c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d -
 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54
*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*
b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2
*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))^2*c*d^2)
/(c*d^2)))*log(3*a*b^8*c^3 - 15*a^4*b^5*c^2*d - 15*a^7*b^2*c*d^2 - 3/4*(6*(
1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) +
1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 -
3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b
^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)
^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^
3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a
^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1)
)^2*a^2*b*c^2*d^3 + 1/2*(b^6*c^3*d - 20*a^3*b^3*c^2*d^2 + a^6*c*d^3)*(6*(1/
2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)
/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*
a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6
*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(
1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3
- 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3
*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))
- 2*(b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)*x - 3/4*sqrt(1
/3)*(2*b^6*c^3*d + 14*a^3*b^3*c^2*d^2 + 2*a^6*c*d^3 + 3*(6*(1/2)^(2/3)*(3*a
^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/
d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d
 + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a
^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*
b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c
^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d -
24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))*a^2*b*c^2*d^3
)*sqrt((12*a^2*b^4*c - 48*a^5*b*d - 12*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a
^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b
^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^
2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^
9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a
^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*
c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2
- a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))*a*b^2*c*d - (6*(1/2)^(2/3)*(3*
a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6
/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*
d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*
a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3
*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*
c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d -
 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))^2*c*d^2)/(c*
d^2))))/d
```

giac [A] time = 0.19, size = 214, normalized size = 0.96

$$\frac{b^{3} x}{d} + \frac{a b^{2} \log\left(\left|d x^{3} + c\right|\right)}{d} + \frac{\sqrt{3} \left(b^{3} c - a^{3} d + 3 \left(-c d^{2}\right)^{\frac{1}{3}} a^{2} b\right) \arctan\left(\frac{\sqrt{3} \left(2 x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3 \left(-c d^{2}\right)^{\frac{2}{3}}} + \frac{\left(b^{3} c - a^{3} d - 3 \left(-c d^{2}\right)^{\frac{1}{3}} a^{2} b\right) \log\left(x^{2} + x \left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6 \left(-c d^{2}\right)^{\frac{2}{3}}} - \frac{\left(3 a^{2} b d^{3} \left(-\frac{c}{d}\right)^{\frac{1}{3}} - b^{3} c d^{2} + a^{3} d^{3}\right) \left(-\frac{c}{d}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3 c d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] b^3*x/d + a*b^2*log(abs(d*x^3 + c))/d + 1/3*sqrt(3)*(b^3*c - a^3*d + 3*(-c*
d^2)^(1/3)*a^2*b)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(-c
*d^2)^(2/3) + 1/6*(b^3*c - a^3*d - 3*(-c*d^2)^(1/3)*a^2*b)*log(x^2 + x*(-c/
d)^(1/3) + (-c/d)^(2/3))/(-c*d^2)^(2/3) - 1/3*(3*a^2*b*d^3*(-c/d)^(1/3) - b
^3*c*d^2 + a^3*d^3)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^3)
```

maple [A] time = 0.05, size = 325, normalized size = 1.46

$$\frac{\sqrt{3}\,a^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{a^3\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{a^3\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{\sqrt{3}\,a^2b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{c}{d}\right)^{\frac{1}{3}}d}-\frac{a^2b\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{\left(\frac{c}{d}\right)^{\frac{1}{3}}d}+\frac{a^2b\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{3}}d}+\frac{a\,b^2\ln\left(d\,x^3+c\right)}{d}-\frac{\sqrt{3}\,b^3c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}-\frac{b^3c\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{b^3c\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{b^3x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x^3+c),x)
```

```
[Out] b^3*x/d+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^3-1/3/d^2/(c/d)^(2/3)*ln(x+(c
/d)^(1/3))*b^3*c-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^3+1/
6/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b^3*c+1/3/d/(c/d)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^3-1/3/d^2/(c/d)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^3*c-1/d*a^2*b/(c/d)^(1/3)*l
n(x+(c/d)^(1/3))+1/2/d*a^2*b/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+
1/d*a^2*b*3^(1/2)/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))+a*b^2
*ln(d*x^3+c)/d
```

maxima [A] time = 2.96, size = 240, normalized size = 1.08

$$\frac{b^3x}{d}-\frac{\sqrt{3}\left(\left(b^3\left(\frac{c}{d}\right)^{\frac{1}{3}}+2\,ab^2\right)c-\left(3\,a^2b\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^3\left(\frac{c}{d}\right)^{\frac{1}{3}}+\frac{2\,ab^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd}+\frac{\left(b^3c+\left(6\,ab^2\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2b\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^3\right)d\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\left(b^3c-\left(3\,ab^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-3\,a^2b\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^3\right)d\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] b^3*x/d - 1/3*sqrt(3)*((b^3*(c/d)^(1/3) + 2*a*b^2)*c - (3*a^2*b*(c/d)^(2/3)
 + a^3*(c/d)^(1/3) + 2*a*b^2*c/d)*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))
/(c/d)^(1/3))/(c*d) + 1/6*(b^3*c + (6*a*b^2*(c/d)^(2/3) + 3*a^2*b*(c/d)^(1/
3) - a^3)*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^2*(c/d)^(2/3)) - 1/3
*(b^3*c - (3*a*b^2*(c/d)^(2/3) - 3*a^2*b*(c/d)^(1/3) + a^3)*d)*log(x + (c/d
)^(1/3))/(d^2*(c/d)^(2/3))
```

mupad [B] time = 5.14, size = 370, normalized size = 1.67

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,c^2\,d^4\,z^3-81\,a\,b^2\,c^2\,d^3\,z^2+54\,a^2\,b^4\,c^2\,d^2\,z+27\,a^5\,b\,c\,d^3\,z+3\,a^6\,b^3\,c\,d^2-3\,a^3\,b^6\,c^2\,d+b^9\,c^3-a^9\,d^3,z,k\right)\,\left(x\,\left(3\,a^3\,d^2-3\,b^3\,c\,d\right)+\mathrm{root}\left(27\,c^2\,d^4\,z^3-81\,a\,b^2\,c^2\,d^3\,z^2+54\,a^2\,b^4\,c^2\,d^2\,z+27\,a^5\,b\,c\,d^3\,z+3\,a^6\,b^3\,c\,d^2-3\,a^3\,b^6\,c^2\,d+b^9\,c^3-a^9\,d^3,z,k\right)\,c\,d^2\,9-18\,a\,b^2\,c\,d\right)+x\,\left(6\,d\,a^4\,b^2+3\,c\,a\,b^5\right)+6\,a^2\,b^4\,c+3\,a^5\,b\,d\right)\,\mathrm{root}\left(27\,c^2\,d^4\,z^3-81\,a\,b^2\,c^2\,d^3\,z^2+54\,a^2\,b^4\,c^2\,d^2\,z+27\,a^5\,b\,c\,d^3\,z+3\,a^6\,b^3\,c\,d^2-3\,a^3\,b^6\,c^2\,d+b^9\,c^3-a^9\,d^3,z,k\right)\right)+\frac{b^3\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x^3),x)
```

```
[Out] symsum(log(root(27*c^2*d^4*z^3 - 81*a*b^2*c^2*d^3*z^2 + 54*a^2*b^4*c^2*d^2*
z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a^3*b^6*c^2*d + b^9*c^3 - a^9*d^
3, z, k)*(x*(3*a^3*d^2 - 3*b^3*c*d) + 9*root(27*c^2*d^4*z^3 - 81*a*b^2*c^2*
d^3*z^2 + 54*a^2*b^4*c^2*d^2*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a^3
*b^6*c^2*d + b^9*c^3 - a^9*d^3, z, k)*c*d^2 - 18*a*b^2*c*d) + x*(6*a^4*b^2*
d + 3*a*b^5*c) + 6*a^2*b^4*c + 3*a^5*b*d)*root(27*c^2*d^4*z^3 - 81*a*b^2*c^
2*d^3*z^2 + 54*a^2*b^4*c^2*d^2*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a
^3*b^6*c^2*d + b^9*c^3 - a^9*d^3, z, k), k, 1, 3) + (b^3*x)/d
```

sympy [A] time = 5.71, size = 245, normalized size = 1.10

$$\frac{b^3x}{d}+\operatorname{RootSum}\left(27t^3c^2d^4-81t^2ab^2c^2d^3+t\left(27a^5bcd^3+54a^2b^4c^2d^2\right)-a^9d^3+3a^6b^3cd^2-3a^3b^6c^2d+b^9c^3,\left(t\mapsto t\log\left(x+\frac{27t^2a^2bc^2d^3+3ta^6cd^3-60ta^3b^3c^2d^2+3tb^6c^3d+15a^7b^2cd^2+15a^4b^5c^2d-3ab^8c^3}{a^9d^3+24a^6b^3cd^2+3a^3b^6c^2d-b^9c^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x**3+c),x)
```

```
[Out] b**3*x/d + RootSum(27*_t**3*c**2*d**4 - 81*_t**2*a*b**2*c**2*d**3 + _t*(27*
a**5*b*c*d**3 + 54*a**2*b**4*c**2*d**2) - a**9*d**3 + 3*a**6*b**3*c*d**2 -
3*a**3*b**6*c**2*d + b**9*c**3, Lambda(_t, _t*log(x + (27*_t**2*a**2*b*c**2
*d**3 + 3*_t*a**6*c*d**3 - 60*_t*a**3*b**3*c**2*d**2 + 3*_t*b**6*c**3*d + 1
5*a**7*b**2*c*d**2 + 15*a**4*b**5*c**2*d - 3*a*b**8*c**3)/(a**9*d**3 + 24*a
**6*b**3*c*d**2 + 3*a**3*b**6*c**2*d - b**9*c**3))))
```

3.61 $\int \frac{(a+bx)^4}{c+dx^3} \, dx$

Optimal. Leaf size=282

$$\frac{2a^2b^2 \log\left(c + dx^3\right)}{d} - \frac{\left(b\sqrt[3]{c}\left(b^3c - 4a^3d\right) - \sqrt[3]{d}\left(4ab^3c - a^4d\right)\right)\log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}\,x + d^{2/3}x^2\right)}{6c^{2/3}d^{5/3}} + \frac{\left(b\sqrt[3]{c}\left(b^3c - 4a^3d\right) -\right.}{}$$

Rubi [A] time = 0.44, antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\left(-\frac{b\sqrt[3]{c}\left(b^3c-4a^3d\right)}{\sqrt[3]{d}} + a^4(-d) + 4ab^3c\right)\log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}\,x + d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} + \frac{\left(b\sqrt[3]{c}\left(b^3c - 4a^3d\right) - \sqrt[3]{d}\left(4ab^3c - a^4d\right)\right)\log\left(\sqrt[3]{c} + \sqrt[3]{d}\,x\right)}{3c^{2/3}d^{5/3}} + \frac{\left(-4a^3b\sqrt[3]{c}\,d + a^4\left(-d^{4/3}\right) + 4ab^3c\sqrt[3]{d} + b^4c^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{5/3}} + \frac{2a^2b^2\log\left(c + dx^3\right)}{d} + \frac{4ab^3x}{d} + \frac{b^4x^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] (4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^(4/3) + 4*a*b^3*c*d^(1/3) - 4*a^3
*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3)
)])/(Sqrt[3]*c^(2/3)*d^(5/3)) + ((b*c^(1/3)*(b^3*c - 4*a^3*d) - d^(1/3)*(4*
a*b^3*c - a^4*d))*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(5/3)) + ((4*a*b^3
*c - a^4*d - (b*c^(1/3)*(b^3*c - 4*a^3*d))/d^(1/3))*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (2*a^2*b^2*Log[c + d*x^3])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \int \left(\frac{4ab^3}{d}+\frac{b^4x}{d}-\frac{4ab^3c-a^4d+b\left(b^3c-4a^3d\right)x-6a^2b^2dx^2}{d\left(c+dx^3\right)}\right)dx$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}-\frac{\int \frac{4ab^3c-a^4d+b\left(b^3c-4a^3d\right)x-6a^2b^2dx^2}{c+dx^3}\,dx}{d}$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}+\left(6a^2b^2\right)\int \frac{x^2}{c+dx^3}\,dx-\frac{\int \frac{4ab^3c-a^4d+b\left(b^3c-4a^3d\right)x}{c+dx^3}\,dx}{d}$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}+\frac{2a^2b^2\log\left(c+dx^3\right)}{d}-\frac{\int \frac{\sqrt[3]{c}\left(b\sqrt[3]{c}\left(b^3c-4a^3d\right)+2\sqrt[3]{d}\left(4ab^3c-a^4d\right)\right)+\sqrt[3]{d}\left(b\sqrt[3]{c}\left(b^3c-4a^3d\right)-\cdots\right.}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}}{3c^{2/3}d^{4/3}}\cdots$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}-\frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}\left(b^3c-4a^3d\right)}{\sqrt[3]{d}}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{2a^2b^2\log\left(c+dx^3\right)}{d}-\frac{\left(b^4\cdots\right.}{\cdots}$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}-\frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}\left(b^3c-4a^3d\right)}{\sqrt[3]{d}}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}\left(b^3c-4a\cdots\right.}{\sqrt[3]{d}}\cdots\right.}{\cdots}$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}+\frac{\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}\,d-a^4d^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{5/3}}-\frac{\left(4ab^3c-a^4d\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.37, size = 277, normalized size = 0.98

$$\frac{12a^2b^2d^{2/3}\log\left(c+dx^3\right)-\frac{\left(a^4d^{4/3}-4a^3b\sqrt[3]{c}d-4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{2/3}}+\frac{2\left(a^4d^{4/3}-4a^3b\sqrt[3]{c}d-4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}}+\frac{2\sqrt{3}\left(a^4\left(-d^{4/3}\right)-4a^3b\sqrt[3]{c}d+4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}}+24ab^3d^{2/3}x+3b^4d^{2/3}x^2}{6d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] (24*a*b^3*d^(2/3)*x + 3*b^4*d^(2/3)*x^2 + (2*Sqrt[3]*(b^4*c^(4/3) + 4*a*b^3
*c*d^(1/3) - 4*a^3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(1 - (2*d^(1/3)*x)/c^(
1/3))/Sqrt[3]])/c^(2/3) + (2*(b^4*c^(4/3) - 4*a*b^3*c*d^(1/3) - 4*a^3*b*c^(
1/3)*d + a^4*d^(4/3))*Log[c^(1/3) + d^(1/3)*x])/c^(2/3) - ((b^4*c^(4/3) - 4
*a*b^3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d + a^4*d^(4/3))*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/c^(2/3) + 12*a^2*b^2*d^(2/3)*Log[c + d*x^3])/(6*d^
(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4/(c + d*x^3), x]
```

fricas [C] time = 5.15, size = 8787, normalized size = 31.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b^4*x^2 + 48*a*b^3*x + 2*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/
d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1
)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^
2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^
2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3
+ a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c
^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9
*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3
*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sq
rt(3) + 1))*d*log(-8*a*b^11*c^4 - 66*a^4*b^8*c^3*d + 48*a^7*b^5*c^2*d^2 + 2
6*a^10*b^2*c*d^3 - 1/4*(b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1
/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c
*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d
 + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3
*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*
d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b
^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) -
(b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^
12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/
(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2 + 1/2*(28*a^2*b^6*c^3*d^2 - 56*a^5*b^3*
c^2*d^3 + a^8*c*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b
^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b
^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) -
(b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^
12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/
(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*
b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*
a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b
^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))
- (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)*x) + (36*a^2*
b^2 - (12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4
*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*
a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*
a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*
b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3
) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7
```

```
*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 -
 a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*
a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))*d + 3*sqrt(1/3)
*d*sqrt(-(64*a*b^7*c^2 - 128*a^4*b^4*c*d + 64*a^7*b*d^2 - 24*(12*a^2*b^2/d
- 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d
^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b
^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a
^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^
6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432
*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d
^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5)
 + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12
*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))*a^2*b^2*c*d^2 + (12*a^2*b^2/d - 2*(
1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(
c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*
d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^
3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2
*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*
b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) -
 (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b
^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)
/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2*c*d^3)/(c*d^3)))*log(8*a*b^11*c^4 + 66
*a^4*b^8*c^3*d - 48*a^7*b^5*c^2*d^2 - 26*a^10*b^2*c*d^3 + 1/4*(b^4*c^3*d^3
- 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7
*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6
/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b
^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12
*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c
^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^
4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^
9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6
*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2
- 1/2*(28*a^2*b^6*c^3*d^2 - 56*a^5*b^3*c^2*d^3 + a^8*c*d^4)*(12*a^2*b^2/d -
 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^
2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^
4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^
9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6
*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*
a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^
4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5)
+ (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*
d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)) - 2*(b^12*c^4 + 52*a^3*b^9*c^3*d - 5
2*a^9*b^3*c*d^3 - a^12*d^4)*x + 3/4*sqrt(1/3)*(20*a^2*b^6*c^3*d^2 + 32*a^5*
b^3*c^2*d^3 + 2*a^8*c*d^4 + (b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d -
 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^
2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^
4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^
9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6
*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*
a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^
4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5)
+ (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*
d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)))*sqrt(-(64*a*b^7*c^2 - 128*a^4*b^4*c
*d + 64*a^7*b*d^2 - 24*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a
*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6
*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4)
- (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (
b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4
)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^
```

```
4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 5
2*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6
*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)
)*a^2*b^2*c*d^2 + (12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*
c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/
d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^
12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*
c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^
2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4
*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9
*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*
c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2*c
*d^3)/(c*d^3))) + (36*a^2*b^2 - (12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d
^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)
/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2
/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2
*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 +
 a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^
2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*
c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*
d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqr
t(3) + 1))*d - 3*sqrt(1/3)*d*sqrt(-(64*a*b^7*c^2 - 128*a^4*b^4*c*d + 64*a^7
*b*d^2 - 24*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 +
19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 -
18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4
 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 -
4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5)
)^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d +
 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c
*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^
2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))*a^2*b^2*c
*d^2 + (12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^
4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4
*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52
*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3
*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/
3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^
7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3
 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4
*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2*c*d^3)/(c*d^
3)))*log(8*a*b^11*c^4 + 66*a^4*b^8*c^3*d - 48*a^7*b^5*c^2*d^2 - 26*a^10*b^2
*c*d^3 + 1/4*(b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*
(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I
*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b
*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a
^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^
9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 1
8*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4
+ 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4
*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))
^(1/3)*(I*sqrt(3) + 1))^2 - 1/2*(28*a^2*b^6*c^3*d^2 - 56*a^5*b^3*c^2*d^3 +
a^8*c*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 1
9*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 1
8*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4
+ 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4
*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))
^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d +
4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*
d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2
```

```
 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)) - 2*(b^12*
c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)*x - 3/4*sqrt(1/3)*(20
*a^2*b^6*c^3*d^2 + 32*a^5*b^3*c^2*d^3 + 2*a^8*c*d^4 + (b^4*c^3*d^3 - 4*a^3*
b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 1
9*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 1
8*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4
+ 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4
*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))
^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d +
4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*
d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2
 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)))*sqrt(-(64
*a*b^7*c^2 - 128*a^4*b^4*c*d + 64*a^7*b*d^2 - 24*(12*a^2*b^2/d - 2*(1/2)^(2
/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))
*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a
^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3
 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 -
4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3
 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*
c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4
 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d
^5))^(1/3)*(I*sqrt(3) + 1))*a^2*b^2*c*d^2 + (12*a^2*b^2/d - 2*(1/2)^(2/3)*(
36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*
sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*
d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^
12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9
*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18
*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 +
 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*
a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^
(1/3)*(I*sqrt(3) + 1))^2*c*d^3)/(c*d^3))))/d
```

giac [A] time = 0.19, size = 294, normalized size = 1.04

$$\frac{2\,a^2b^2\log\left(\left|dx^3+c\right|\right)}{d}+\frac{\sqrt{3}\left(4\,ab^3cd-a^4d^2-\left(-cd^2\right)^{\frac{1}{3}}b^4c+4\,\left(-cd^2\right)^{\frac{1}{3}}a^3bd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,\left(-cd^2\right)^{\frac{2}{3}}d}+\frac{\left(4\,ab^3cd-a^4d^2+\left(-cd^2\right)^{\frac{1}{3}}b^4c-4\,\left(-cd^2\right)^{\frac{1}{3}}a^3bd\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,\left(-cd^2\right)^{\frac{2}{3}}d}+\frac{b^4dx^2+8\,ab^3dx}{2\,d^2}+\frac{\left(b^4cd^4\left(-\frac{c}{d}\right)^{\frac{1}{3}}-4\,a^3bd^5\left(-\frac{c}{d}\right)^{\frac{1}{3}}+4\,ab^3cd^4-a^4d^5\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 2*a^2*b^2*log(abs(d*x^3 + c))/d + 1/3*sqrt(3)*(4*a*b^3*c*d - a^4*d^2 - (-c*
d^2)^(1/3)*b^4*c + 4*(-c*d^2)^(1/3)*a^3*b*d)*arctan(1/3*sqrt(3)*(2*x + (-c/
d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) + 1/6*(4*a*b^3*c*d - a^4*d^2 + (
-c*d^2)^(1/3)*b^4*c - 4*(-c*d^2)^(1/3)*a^3*b*d)*log(x^2 + x*(-c/d)^(1/3) +
(-c/d)^(2/3))/((-c*d^2)^(2/3)*d) + 1/2*(b^4*d*x^2 + 8*a*b^3*d*x)/d^2 + 1/3*
(b^4*c*d^4*(-c/d)^(1/3) - 4*a^3*b*d^5*(-c/d)^(1/3) + 4*a*b^3*c*d^4 - a^4*d^
5)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^5)
```

maple [A] time = 0.05, size = 446, normalized size = 1.58

$$\frac{b^4x^2}{2d}+\frac{\sqrt{3}\,a^4\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{a^4\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{a^4\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{4\sqrt{3}\,a^3b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}d}-\frac{4a^3b\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}d}+\frac{2a^3b\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}d}+\frac{2a^2b^2\ln\left(d\,x^3+c\right)}{d}-\frac{4\sqrt{3}\,a\,b^3c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}-\frac{4a\,b^3c\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{2a\,b^3c\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{4a\,b^3x}{d}-\frac{\sqrt{3}\,b^4c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}d^2}+\frac{b^4c\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}d^2}-\frac{b^4c\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{1}{3}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x^3+c),x)
```

```
[Out] 1/2*b^4*x^2/d+4*a*b^3*x/d+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^4-4/3/d^2/(
c/d)^(2/3)*ln(x+(c/d)^(1/3))*a*b^3*c-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x
+(c/d)^(2/3))*a^4+2/3/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a*b
```

```
^3*c+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^4-
4/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a*b^3*c
-4/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))*a^3*b+1/3/d^2/(c/d)^(1/3)*ln(x+(c/d)^(
1/3))*b^4*c+2/3/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^3*b-1/6/d
^2/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b^4*c+4/3/d*3^(1/2)/(c/d)^
(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^3*b-1/3/d^2*3^(1/2)/(c/d)^(
1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^4*c+2*a^2*b^2*ln(d*x^3+c)/d
```

maxima [A] time = 3.04, size = 303, normalized size = 1.07

$$-\frac{\sqrt{3}\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{2}{3}} + 4\,ab^3\left(\frac{c}{d}\right)^{\frac{1}{3}} + 4\,a^2b^2\right)c - \left(4\,a^3b\left(\frac{c}{d}\right)^{\frac{2}{3}} + a^4\left(\frac{c}{d}\right)^{\frac{1}{3}} + \frac{4\,a^2b^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd} + \frac{b^4x^2 + 8\,ab^3x}{2\,d} - \frac{\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{1}{3}} - 4\,ab^3\right)c - \left(12\,a^2b^2\left(\frac{c}{d}\right)^{\frac{2}{3}} + 4\,a^3b\left(\frac{c}{d}\right)^{\frac{1}{3}} - a^4\right)d\right)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{1}{3}} - 4\,ab^3\right)c + \left(6\,a^2b^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - 4\,a^3b\left(\frac{c}{d}\right)^{\frac{1}{3}} + a^4\right)d\right)\log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*((b^4*(c/d)^(2/3) + 4*a*b^3*(c/d)^(1/3) + 4*a^2*b^2)*c - (4*a^
3*b*(c/d)^(2/3) + a^4*(c/d)^(1/3) + 4*a^2*b^2*c/d)*d)*arctan(1/3*sqrt(3)*(2
*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d) + 1/2*(b^4*x^2 + 8*a*b^3*x)/d - 1/6*((
b^4*(c/d)^(1/3) - 4*a*b^3)*c - (12*a^2*b^2*(c/d)^(2/3) + 4*a^3*b*(c/d)^(1/3
) - a^4)*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^2*(c/d)^(2/3)) + 1/3*
((b^4*(c/d)^(1/3) - 4*a*b^3)*c + (6*a^2*b^2*(c/d)^(2/3) - 4*a^3*b*(c/d)^(1/
3) + a^4)*d)*log(x + (c/d)^(1/3))/(d^2*(c/d)^(2/3))
```

mupad [B] time = 4.97, size = 513, normalized size = 1.82

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,c^2\,d^5\,z^3 - 162\,a^2\,b^2\,c^2\,d^4\,z^2 + 171\,a^4\,b^4\,c^2\,d^3\,z + 36\,a\,b^7\,c^3\,d^2\,z + 36\,a^7\,b\,c\,d^4\,z - 6\,a^6\,b^6\,c^2\,d^2 + 4\,a^9\,b^3\,c\,d^3 + 4\,a^3\,b^9\,c^3\,d - b^{12}\,c^4 - a^{12}\,d^4, z, k\right)\left(\frac{x\left(3\,a^4\,d^3 - 12\,a\,b^3\,c\,d^2\right)}{d} + \mathrm{root}\left(27\,c^2\,d^5\,z^3 - 162\,a^2\,b^2\,c^2\,d^4\,z^2 + 171\,a^4\,b^4\,c^2\,d^3\,z + 36\,a\,b^7\,c^3\,d^2\,z + 36\,a^7\,b\,c\,d^4\,z - 6\,a^6\,b^6\,c^2\,d^2 + 4\,a^9\,b^3\,c\,d^3 + 4\,a^3\,b^9\,c^3\,d - b^{12}\,c^4 - a^{12}\,d^4, z, k\right)\,c\,d^2\,9 - 36\,a^2\,b^2\,c\,d\right) + \frac{4\,a^7\,b\,d^2 + 19\,a^4\,b^4\,c\,d + 4\,a\,b^7\,c^2}{d} + \frac{x\left(10\,a^6\,b^2\,d^2 + 16\,a^3\,b^5\,c\,d + b^8\,c^2\right)}{d}\right)\mathrm{root}\left(27\,c^2\,d^5\,z^3 - 162\,a^2\,b^2\,c^2\,d^4\,z^2 + 171\,a^4\,b^4\,c^2\,d^3\,z + 36\,a\,b^7\,c^3\,d^2\,z + 36\,a^7\,b\,c\,d^4\,z - 6\,a^6\,b^6\,c^2\,d^2 + 4\,a^9\,b^3\,c\,d^3 + 4\,a^3\,b^9\,c^3\,d - b^{12}\,c^4 - a^{12}\,d^4, z, k\right)\right) + \frac{b^4\,x^2}{2\,d} + \frac{4\,a\,b^3\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x^3),x)
```

```
[Out] symsum(log(root(27*c^2*d^5*z^3 - 162*a^2*b^2*c^2*d^4*z^2 + 171*a^4*b^4*c^2*
d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*z - 6*a^6*b^6*c^2*d^2 + 4*a^9*b
^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12*d^4, z, k)*((x*(3*a^4*d^3 - 12
*a*b^3*c*d^2))/d + 9*root(27*c^2*d^5*z^3 - 162*a^2*b^2*c^2*d^4*z^2 + 171*a^
4*b^4*c^2*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*z - 6*a^6*b^6*c^2*d^2
 + 4*a^9*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12*d^4, z, k)*c*d^2 - 3
6*a^2*b^2*c*d) + (4*a*b^7*c^2 + 4*a^7*b*d^2 + 19*a^4*b^4*c*d)/d + (x*(b^8*c
^2 + 10*a^6*b^2*d^2 + 16*a^3*b^5*c*d))/d)*root(27*c^2*d^5*z^3 - 162*a^2*b^2
*c^2*d^4*z^2 + 171*a^4*b^4*c^2*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*
z - 6*a^6*b^6*c^2*d^2 + 4*a^9*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12
*d^4, z, k), k, 1, 3) + (b^4*x^2)/(2*d) + (4*a*b^3*x)/d
```

sympy [A] time = 60.25, size = 325, normalized size = 1.15

$$\frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + \operatorname{RootSum}\left(27t^3c^2d^5 - 162t^2a^2b^2c^2d^4 + t\left(36a^7bcd^4 + 171a^4b^4c^2d^3 + 36ab^7c^3d^2\right) - a^{12}d^4 + 4a^9b^3cd^3 - 6a^6b^6c^2d^2 + 4a^3b^9c^3d - b^{12}c^4, \left(t \mapsto t\log\left(x + \frac{36t^2a^3bc^2d^4 - 9t^2b^4c^3d^3 + 3ta^8cd^4 - 168ta^5b^3c^2d^3 + 84ta^2b^6c^3d^2 + 26a^{10}b^2cd^3 + 48a^7b^5c^2d^2 - 66a^4b^8c^3d - 8ab^{11}c^4}{a^{12}d^4 + 52a^9b^3cd^3 - 52a^3b^9c^3d - b^{12}c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x**3+c),x)
```

```
[Out] 4*a*b**3*x/d + b**4*x**2/(2*d) + RootSum(27*_t**3*c**2*d**5 - 162*_t**2*a**
2*b**2*c**2*d**4 + _t*(36*a**7*b*c*d**4 + 171*a**4*b**4*c**2*d**3 + 36*a*b*
*7*c**3*d**2) - a**12*d**4 + 4*a**9*b**3*c*d**3 - 6*a**6*b**6*c**2*d**2 + 4
*a**3*b**9*c**3*d - b**12*c**4, Lambda(_t, _t*log(x + (36*_t**2*a**3*b*c**2
*d**4 - 9*_t**2*b**4*c**3*d**3 + 3*_t*a**8*c*d**4 - 168*_t*a**5*b**3*c**2*d
**3 + 84*_t*a**2*b**6*c**3*d**2 + 26*a**10*b**2*c*d**3 + 48*a**7*b**5*c**2*
d**2 - 66*a**4*b**8*c**3*d - 8*a*b**11*c**4)/(a**12*d**4 + 52*a**9*b**3*c*d
**3 - 52*a**3*b**9*c**3*d - b**12*c**4))))
```

3.62 $\int \frac{(a+bx+cx^2)^2}{d+ex^3} dx$

Optimal. Leaf size=272

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(\sqrt[3]{e}\left(2bcd-a^2e\right)-\sqrt[3]{d}\left(c^2d-2abe\right)\right)}{6d^{2/3}e^{5/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}\left(2bcd-a^2e\right)-\sqrt[3]{d}\right.}{3d^{2/3}e^{5/3}}$$

Rubi [A] time = 0.49, antiderivative size = 270, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^2(-e)-\frac{\sqrt[3]{d}\left(c^2d-2abe\right)}{\sqrt[3]{e}}+2bcd\right)}{6d^{2/3}e^{4/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}\left(2bcd-a^2e\right)-\sqrt[3]{d}\left(c^2d-2abe\right)\right)}{3d^{2/3}e^{5/3}}+\frac{\left(2ac+b^2\right)\log\left(d+ex^3\right)}{3e}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-ae\left(a\sqrt[3]{e}+2b\sqrt[3]{d}\right)+2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{\sqrt{3}d^{2/3}e^{5/3}}+\frac{2bcx}{e}+\frac{c^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

```
[Out] (2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^(4/3) + 2*b*c*d*e^(1/3) - a*(2*b*d^
(1/3) + a*e^(1/3))*e)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(S
qrt[3]*d^(2/3)*e^(5/3)) - ((e^(1/3)*(2*b*c*d - a^2*e) - d^(1/3)*(c^2*d - 2*
a*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(5/3)) + ((2*b*c*d - a^2*e -
 (d^(1/3)*(c^2*d - 2*a*b*e))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(
2/3)*x^2])/(6*d^(2/3)*e^(4/3)) + ((b^2 + 2*a*c)*Log[d + e*x^3])/(3*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{d+ex^3}\,dx = \int \left(\frac{2bc}{e}+\frac{c^2x}{e}-\frac{2bcd-a^2e+\left(c^2d-2abe\right)x-\left(b^2+2ac\right)ex^2}{e\left(d+ex^3\right)}\right)dx$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}-\frac{\int \frac{2bcd-a^2e+\left(c^2d-2abe\right)x-\left(b^2+2ac\right)ex^2}{d+ex^3}\,dx}{e}$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}-\left(-b^2-2ac\right)\int \frac{x^2}{d+ex^3}\,dx-\frac{\int \frac{2bcd-a^2e+\left(c^2d-2abe\right)x}{d+ex^3}\,dx}{e}$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}+\frac{\left(b^2+2ac\right)\log\left(d+ex^3\right)}{3e}-\frac{\int \frac{\sqrt[3]{d}\left(2\sqrt[3]{e}\left(2bcd-a^2e\right)+\sqrt[3]{d}\left(c^2d-2abe\right)\right)+\sqrt[3]{e}\left(-\sqrt[3]{e}\left(2bcd-\right.\right.}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}}{3d^{2/3}e^{4/3}}$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}-\frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}\left(c^2d-2abe\right)}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{4/3}}+\frac{\left(b^2+2ac\right)\log\left(d+ex^3\right)}{3e}$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}-\frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}\left(c^2d-2abe\right)}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{4/3}}+\frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}\left(c^2d-2abe\right.}{\sqrt[3]{e}}\right.}{6}$$

$$= \frac{2bcx}{e}+\frac{c^2x^2}{2e}+\frac{\left(c^2d^{4/3}+2bcd\sqrt[3]{e}-a\left(2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{5/3}}-\frac{\left(2bcd-a^2e\right.}{}$$

Mathematica [A] time = 0.42, size = 269, normalized size = 0.99

$$\frac{2e^{2/3}\left(2ac+b^2\right)\log\left(d+ex^3\right)-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae\left(a\sqrt[3]{e}-2b\sqrt[3]{d}\right)-2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{d^{2/3}}+\frac{2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae\left(a\sqrt[3]{e}-2b\sqrt[3]{d}\right)-2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{d^{2/3}}+\frac{2\sqrt{3}\left(cd^{2/3}-ae^{2/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(ae^{2/3}+2b\sqrt[3]{d}\sqrt[3]{e}+cd^{2/3}\right)}{d^{2/3}}+12bce^{2/3}x+3c^2e^{2/3}x^2}{6e^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

```
[Out] (12*b*c*e^(2/3)*x + 3*c^2*e^(2/3)*x^2 + (2*Sqrt[3]*(c*d^(2/3) - a*e^(2/3))*
(c*d^(2/3) + 2*b*d^(1/3)*e^(1/3) + a*e^(2/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(
1/3))/Sqrt[3]])/d^(2/3) + (2*(c^2*d^(4/3) - 2*b*c*d*e^(1/3) + a*(-2*b*d^(1/
3) + a*e^(1/3))*e)*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - ((c^2*d^(4/3) - 2*b*
c*d*e^(1/3) + a*(-2*b*d^(1/3) + a*e^(1/3))*e)*Log[d^(2/3) - d^(1/3)*e^(1/3)
*x + e^(2/3)*x^2])/d^(2/3) + 2*(b^2 + 2*a*c)*e^(2/3)*Log[d + e*x^3])/(6*e^(
5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^2}{d+ex^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)^2/(d + e*x^3), x]
```

fricas [C] time = 1.76, size = 12827, normalized size = 47.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] 1/12*(6*c^2*x^2 + 24*b*c*x - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*
c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(
2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*
e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a
^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3
)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 +
 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^
5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3
*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*
d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e
^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2
*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e -
 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*e*l
og(-4*b*c^5*d^4 - (5*b^4*c^2 - 4*a*b^2*c^3 + 2*a^2*c^4)*d^3*e + 2*(a*b^5 -
2*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^2 + (7*a^4*b^2 - 2*a^5*c)*d*e^3 - 1/4*(c^2
*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/
e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^
2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*
(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^
2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3
 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4
*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(
1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2
+ b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 -
 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 +
a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)
*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4
*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)^2 - 1/2*
(a^4*d*e^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e^2 - 4*(a*b^3 + 3*a^2*b*c)*d^2*e^
3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^
4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2
*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) +
 (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b
*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4
*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d
```

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^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d
*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*
d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^
2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^
4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d
*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)*x) + (6*b^2 + 12
*a*c + (2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2
+ b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 -
3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^
4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a
^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e -
 b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^
4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c
^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e +
b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^
2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^
6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*
c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*e + 3*sqrt(1/3)*e*sqrt(-(32
*b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^
2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(
b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2
)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*
b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a
^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*
(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^
2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4
 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3
+ a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^
2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*
(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)^2*d*e^
3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 +
 b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3
*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4
) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^
4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e -
b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4
)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(
I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^
2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b
^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2
*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6
*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c
)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*(b^2 + 2*a*c)*d*e^2 + 4*(b^4
 - 12*a*b^2*c)*d*e)/(d*e^3)))*log(4*b*c^5*d^4 + (5*b^4*c^2 - 4*a*b^2*c^3 +
2*a^2*c^4)*d^3*e - 2*(a*b^5 - 2*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^2 - (7*a^4*b
^2 - 2*a^5*c)*d*e^3 + 1/4*(c^2*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e
+ 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e +
 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*
d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2
*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^
5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 -
3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 +
2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2
 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^
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2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6
*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3)
 - 2*(b^2 + 2*a*c)/e)^2 + 1/2*(a^4*d*e^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e^2
- 4*(a*b^3 + 3*a^2*b*c)*d^2*e^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*
a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))
/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*
b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9
*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e
^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4
 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*
e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c
^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^
6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d
*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d
^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e
 - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e) -
 2*(c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*
a^4*b*c)*d*e^3)*x + 3/4*sqrt(1/3)*(4*a*b^3*d^2*e^3 + 2*a^4*d*e^4 + 2*(3*b^2
*c^2 - 2*a*c^3)*d^3*e^2 - (c^2*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e
+ 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e +
 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*
d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2
*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^
5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 -
3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 +
2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2
 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^
2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6
*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3)
 - 2*(b^2 + 2*a*c)/e))*sqrt(-(32*b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*
d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d
*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*
c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4
 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^
2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^
3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^
2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*
(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^
2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3
 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4
*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(
1/3) - 2*(b^2 + 2*a*c)/e)^2*d*e^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2
 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*
e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2
*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^
2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^
3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^
6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/
(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(
2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4)
+ (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*
b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^
4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*
d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)
/e)*(b^2 + 2*a*c)*d*e^2 + 4*(b^4 - 12*a*b^2*c)*d*e)/(d*e^3))) + (6*b^2 + 12
```

```
*a*c + (2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2
+ b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 -
3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^
4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a
^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e -
 b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^
4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c
^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e +
b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^
2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^
6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*
c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*e - 3*sqrt(1/3)*e*sqrt(-(32
*b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^
2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(
b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2
)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*
b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a
^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*
(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^
2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4
 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3
+ a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^
2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*
(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)^2*d*e^
3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 +
 b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3
*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4
) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^
4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e -
b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4
)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(
I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^
2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b
^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2
*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6
*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c
)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*(b^2 + 2*a*c)*d*e^2 + 4*(b^4
 - 12*a*b^2*c)*d*e)/(d*e^3)))*log(4*b*c^5*d^4 + (5*b^4*c^2 - 4*a*b^2*c^3 +
2*a^2*c^4)*d^3*e - 2*(a*b^5 - 2*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^2 - (7*a^4*b
^2 - 2*a^5*c)*d*e^3 + 1/4*(c^2*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e
+ 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e +
 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*
d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2
*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^
5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 -
3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 +
2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2
 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^
2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6
*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3)
 - 2*(b^2 + 2*a*c)/e)^2 + 1/2*(a^4*d*e^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e^2
- 4*(a*b^3 + 3*a^2*b*c)*d^2*e^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*
a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))
/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*
b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9
```

```
*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e
^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4
 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*
e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c
^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^
6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d
*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d
^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e
 - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e) -
 2*(c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*
a^4*b*c)*d*e^3)*x - 3/4*sqrt(1/3)*(4*a*b^3*d^2*e^3 + 2*a^4*d*e^4 + 2*(3*b^2
*c^2 - 2*a*c^3)*d^3*e^2 - (c^2*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e
+ 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e +
 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*
d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2
*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^
5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 -
3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 +
2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2
 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^
2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6
*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3)
 - 2*(b^2 + 2*a*c)/e))*sqrt(-(32*b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*
d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d
*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*
c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4
 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^
2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^
3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^
2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*
(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^
2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3
 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4
*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(
1/3) - 2*(b^2 + 2*a*c)/e)^2*d*e^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2
 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*
e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2
*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^
2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^
3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^
6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/
(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(
2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4)
+ (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*
b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^
4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*
d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)
/e)*(b^2 + 2*a*c)*d*e^2 + 4*(b^4 - 12*a*b^2*c)*d*e)/(d*e^3))))/e
```

giac [A] time = 0.21, size = 264, normalized size = 0.97

$$\frac{1}{3}\left(b^{2}+2\,ac\right)e^{(-1)}\log\left(\left|x^{3}e+d\right|\right)+\frac{\sqrt{3}\left(2\,bcde-\left(-de^{2}\right)^{\frac{1}{3}}c^{2}d+2\,\left(-de^{2}\right)^{\frac{1}{3}}abe-a^{2}e^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{3\,\left(-de^{2}\right)^{\frac{2}{3}}}+\frac{\left(2\,bcde+\left(-de^{2}\right)^{\frac{1}{3}}c^{2}d-2\,\left(-de^{2}\right)^{\frac{1}{3}}abe-a^{2}e^{2}\right)e^{(-1)}\log\left(x^{2}+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\,\left(-de^{2}\right)^{\frac{2}{3}}}+\frac{\left(\left(-de^{(-1)}\right)^{\frac{1}{3}}c^{2}de^{4}+2\,bcde^{4}-2\,\left(-de^{(-1)}\right)^{\frac{1}{3}}abe^{5}-a^{2}e^{5}\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-5)}\log\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3\,d}+\frac{1}{2}\left(c^{2}x^{2}e+4\,bcxe\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/3*(b^2 + 2*a*c)*e^(-1)*log(abs(x^3*e + d)) + 1/3*sqrt(3)*(2*b*c*d*e - (-d
*e^2)^(1/3)*c^2*d + 2*(-d*e^2)^(1/3)*a*b*e - a^2*e^2)*arctan(1/3*sqrt(3)*(2
*x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/(-d*e^2)^(2/3) + 1/6*(2*b
*c*d*e + (-d*e^2)^(1/3)*c^2*d - 2*(-d*e^2)^(1/3)*a*b*e - a^2*e^2)*e^(-1)*lo
g(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3) + 1/3*((-d*
e^(-1))^(1/3)*c^2*d*e^4 + 2*b*c*d*e^4 - 2*(-d*e^(-1))^(1/3)*a*b*e^5 - a^2*e
^5)*(-d*e^(-1))^(1/3)*e^(-5)*log(abs(x - (-d*e^(-1))^(1/3)))/d + 1/2*(c^2*x
^2*e + 4*b*c*x*e)*e^(-2)
```

maple [B] time = 0.07, size = 444, normalized size = 1.63

$$\frac{c^2x^2}{2e}+\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e}+\frac{a^2\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e}-\frac{a^2\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{2}{3}}e}+\frac{2\sqrt{3}\,ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}e}-\frac{2ab\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}e}+\frac{ab\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}e}+\frac{2ac\ln\left(ex^3+d\right)}{3e}+\frac{b^2\ln\left(ex^3+d\right)}{3e}-\frac{2\sqrt{3}\,bcd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2}-\frac{2bcd\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2}+\frac{bcd\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2}+\frac{2bcx}{e}-\frac{\sqrt{3}\,c^2d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}e^2}+\frac{c^2d\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}e^2}-\frac{c^2d\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{1}{3}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x^3+d),x)
```

```
[Out] 1/2*c^2*x^2/e+2*b*c*x/e+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^2-2/3/e^2/(d/
e)^(2/3)*ln(x+(d/e)^(1/3))*b*c*d-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/
e)^(2/3))*a^2+1/3/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b*c*d+1
/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^2-2/3/e^
2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b*c*d-2/3/e/(
d/e)^(1/3)*ln(x+(d/e)^(1/3))*a*b+1/3/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*c^2*
d+1/3/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a*b-1/6/e^2/(d/e)^(1/
3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*c^2*d+2/3/e*3^(1/2)/(d/e)^(1/3)*arctan
(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*b-1/3/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/
3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*c^2*d+2/3/e*ln(e*x^3+d)*a*c+1/3/e*ln(e*x^3+d
)*b^2
```

maxima [A] time = 3.03, size = 314, normalized size = 1.15

$$-\frac{\sqrt{3}\left(\left(3\,c^2\left(\frac{d}{e}\right)^{\frac{2}{3}}+2\,b^2+2\left(3\,b\left(\frac{d}{e}\right)^{\frac{1}{3}}+2\,a\right)c\right)d-\left(6\,ab\left(\frac{d}{e}\right)^{\frac{2}{3}}+3\,a^2\left(\frac{d}{e}\right)^{\frac{1}{3}}+\frac{2\,b^2d}{e}+\frac{4\,acd}{e}\right)e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{9\,de}+\frac{c^2x^2+4\,bcx}{2\,e}-\frac{\left(\left(c^2\left(\frac{d}{e}\right)^{\frac{1}{3}}-2\,bc\right)d-\left(2\,b^2\left(\frac{d}{e}\right)^{\frac{2}{3}}+4\,ac\left(\frac{d}{e}\right)^{\frac{2}{3}}+2\,ab\left(\frac{d}{e}\right)^{\frac{1}{3}}-a^2\right)e\right)\log\left(x^2-x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,e^2\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\left(\left(c^2\left(\frac{d}{e}\right)^{\frac{1}{3}}-2\,bc\right)d+\left(b^2\left(\frac{d}{e}\right)^{\frac{2}{3}}+2\,ac\left(\frac{d}{e}\right)^{\frac{2}{3}}-2\,ab\left(\frac{d}{e}\right)^{\frac{1}{3}}+a^2\right)e\right)\log\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\,e^2\left(\frac{d}{e}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*((3*c^2*(d/e)^(2/3) + 2*b^2 + 2*(3*b*(d/e)^(1/3) + 2*a)*c)*d -
 (6*a*b*(d/e)^(2/3) + 3*a^2*(d/e)^(1/3) + 2*b^2*d/e + 4*a*c*d/e)*e)*arctan(
1/3*sqrt(3)*(2*x - (d/e)^(1/3))/(d/e)^(1/3))/(d*e) + 1/2*(c^2*x^2 + 4*b*c*x
)/e - 1/6*((c^2*(d/e)^(1/3) - 2*b*c)*d - (2*b^2*(d/e)^(2/3) + 4*a*c*(d/e)^(
2/3) + 2*a*b*(d/e)^(1/3) - a^2)*e)*log(x^2 - x*(d/e)^(1/3) + (d/e)^(2/3))/(
e^2*(d/e)^(2/3)) + 1/3*((c^2*(d/e)^(1/3) - 2*b*c)*d + (b^2*(d/e)^(2/3) + 2*
a*c*(d/e)^(2/3) - 2*a*b*(d/e)^(1/3) + a^2)*e)*log(x + (d/e)^(1/3))/(e^2*(d/
e)^(2/3))
```

mupad [B] time = 5.13, size = 769, normalized size = 2.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^2/(d + e*x^3),x)
```

```
[Out] symsum(log((2*a^3*b*e^2 + 2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e)/e + (x*(c^
4*d^2 - 2*a^3*c*e^2 + 3*a^2*b^2*e^2 + 2*b^3*c*d*e))/e - 3*root(27*d^2*e^5*z
^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4*z^2 + 27*a^2*c^2*d^2*e^3*z + 18*b*
c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4*d^2*e^3*z + 6*a*b^4*c*d^2*e^2 - 9*
a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 - 6*a*b*c^4*d^3*e - 2*a^3*c^3*d^2*e^2
 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b^6*d^2*e^2 - c^6*d^4 - a^6*e^4, z,
```

```
k)*e*(2*b^2*d - 3*root(27*d^2*e^5*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4
*z^2 + 27*a^2*c^2*d^2*e^3*z + 18*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4
*d^2*e^3*z + 6*a*b^4*c*d^2*e^2 - 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 -
6*a*b*c^4*d^3*e - 2*a^3*c^3*d^2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b
^6*d^2*e^2 - c^6*d^4 - a^6*e^4, z, k)*d*e + 4*a*c*d - a^2*e*x + 2*b*c*d*x))
*root(27*d^2*e^5*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4*z^2 + 27*a^2*c^2
*d^2*e^3*z + 18*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4*d^2*e^3*z + 6*a*
b^4*c*d^2*e^2 - 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 - 6*a*b*c^4*d^3*e -
 2*a^3*c^3*d^2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b^6*d^2*e^2 - c^6*
d^4 - a^6*e^4, z, k), k, 1, 3) + (c^2*x^2)/(2*e) + (2*b*c*x)/e
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x**3+d),x)
```

```
[Out] Timed out
```

3.63 $\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx$

Optimal. Leaf size=416

$$-\frac{\log\left(d+ex^3\right)\left(a^2(-c)e-ab^2e+bc^2d\right)}{e^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(-e\left(b^3d-a^3e\right)+3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d\right.\right.}{6d^{2/3}e^{7/3}}$$

Rubi [A] time = 0.70, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d+b^2cd\right)-e\left(b^3d-a^3e\right)-6abcde+c^3d^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d+b^2cd\right)-e\left(b^3d-a^3e\right)-6abcde+c^3d^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(3a^2b\sqrt[3]{d}e^{5/3}+a^3e^2-6abcde-3ac^2d^{4/3}e^{2/3}-3b^2cd^{4/3}e^{2/3}-b^3de+c^3d^2\right)}{\sqrt{3}d^{2/3}e^{7/3}}-\frac{\log\left(d+ex^3\right)\left(a^2(-c)e-ab^2e+bc^2d\right)}{e^2}-\frac{x\left(-6abce+b^3(-e)+c^3d\right)}{e^2}+\frac{3cx^2\left(ac+b^2\right)}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

```
[Out] -(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2*e) + (b*c
^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/3)*e^(2/3) - 3*a*c^2
*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^
2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7
/3)) + ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2
*c*d + a*c^2*d - a^2*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) -
((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d +
a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2
/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[d + e*x^3])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{d+ex^3}\,dx = \int \left(-\frac{c^3d-b^3e-6abce}{e^2}+\frac{3c\left(b^2+ac\right)x}{e}+\frac{3bc^2x^2}{e}+\frac{c^3x^3}{e}+\frac{c^3d^2-6abcde-e\left(b^3d-\ldots\right.}{\ldots}\right.$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\int \frac{c^3d^2-6abcde-e\left(b^3d-a^3e\right)-3e\left(\ldots\right.}{\ldots}$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\int \frac{c^3d^2-6abcde-e\left(b^3d-a^3e\right)-3e\left(\ldots\right.}{d+ex^3}}{e^2}$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}-\frac{\left(bc^2d-ab^2e-a^2ce\right)\log\left(\ldots\right.}{e^2}$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\left(c^3d^2-6abcde-e\left(b^3d-\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\left(c^3d^2-6abcde-e\left(b^3d-\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}-\frac{\left(c^3d^2-3b^2cd^{4/3}e^{2/3}-3ac\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.58, size = 439, normalized size = 1.06

$$\frac{12\sqrt[3]{e}\log\left(d+ex^3\right)\left(a^2ce+ab^2e-bc^2d\right)-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(e\left(a^3e+3a^2b\sqrt[3]{d}e^{2/3}-b^3d\right)-3c\left(2abde+b^2d^{4/3}e^{2/3}\right)-3ac^2d^{4/3}e^{2/3}+c^3d^2\right)}{d^{2/3}}-\frac{2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^3e^2-3a^2b\sqrt[3]{d}e^{5/3}-6abcde+3ac^2d^{4/3}e^{2/3}-b^3de+3b^2cd^{4/3}e^{2/3}+c^3d^2\right)}{d^{2/3}}+\frac{4\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(a^3e^2-3a^2b\sqrt[3]{d}e^{5/3}-6abcde+3ac^2d^{4/3}e^{2/3}-b^3de+3b^2cd^{4/3}e^{2/3}+c^3d^2\right)}{d^{2/3}}+12\sqrt[3]{e}x\left(6abce+b^3e-c^3d\right)+18ce^{4/3}x^2\left(ac+b^2\right)+12bc^2e^{4/3}x^3+3c^3e^{4/3}x^4}{12e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c^3*d) + b^3*e + 6*a*b*c*e)*x + 18*c*(b^2 + a*c)*e^(4/3)*x^2
 + 12*b*c^2*e^(4/3)*x^3 + 3*c^3*e^(4/3)*x^4 - (4*Sqrt[3]*(c^3*d^2 - 3*a*c^2
*d^(4/3)*e^(2/3) + e*(-(b^3*d) + 3*a^2*b*d^(1/3)*e^(2/3) + a^3*e) - 3*c*(b^
2*d^(4/3)*e^(2/3) + 2*a*b*d*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]]
)/d^(2/3) + (4*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4/3)*e^(2/3)
 - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Log[d^(1/3) +
 e^(1/3)*x])/d^(2/3) - (2*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4
/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Lo
g[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3) + 12*e^(1/3)*(-(b*c^2
*d) + a*b^2*e + a^2*c*e)*Log[d + e*x^3])/(12*e^(7/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^2\right)^3}{d + ex^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)^3/(d + e*x^3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.23, size = 432, normalized size = 1.04

$$-\left(bc^2d - ab^2e - a^2ce\right)e^{(-2)}\log\left(\left|x^3e + d\right|\right) - \frac{\sqrt{3}\left(c^3d^2 - b^3de - 6\,abcde + 3\left(-de^2\right)^{\frac{1}{3}}b^2cd + 3\left(-de^2\right)^{\frac{1}{3}}ac^2d - 3\left(-de^2\right)^{\frac{1}{3}}a^2be + a^3e^2\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{3\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(c^3d^2 - b^3de - 6\,abcde - 3\left(-de^2\right)^{\frac{1}{3}}b^2cd - 3\left(-de^2\right)^{\frac{1}{3}}ac^2d + 3\left(-de^2\right)^{\frac{1}{3}}a^2be + a^3e^2\right)e^{(-1)}\log\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}}x + \left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(c^3d^2e^7 - 3\left(-de^{(-1)}\right)^{\frac{1}{3}}b^2cde^8 - 3\left(-de^{(-1)}\right)^{\frac{1}{3}}ac^2de^8 - b^3de^8 - 6\,abcde^8 + 3\left(-de^{(-1)}\right)^{\frac{1}{3}}a^2be^9 + a^3e^9\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-9)}\log\left(\left|x - \left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3\,d} + \frac{1}{4}\left(c^3x^4e^3 + 4\,bc^2x^3e^3 + 6\,b^2cx^2e^3 + 6\,ac^2x^2e^3 - 4\,c^3dxe^2 + 4\,b^3xe^3 + 24\,abcxe^3\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -(b*c^2*d - a*b^2*e - a^2*c*e)*e^(-2)*log(abs(x^3*e + d)) - 1/3*sqrt(3)*(c^
3*d^2 - b^3*d*e - 6*a*b*c*d*e + 3*(-d*e^2)^(1/3)*b^2*c*d + 3*(-d*e^2)^(1/3)
*a*c^2*d - 3*(-d*e^2)^(1/3)*a^2*b*e + a^3*e^2)*arctan(1/3*sqrt(3)*(2*x + (-
d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/(-d*e^2)^(2/3) - 1/6*(c^3*d^2 -
b^3*d*e - 6*a*b*c*d*e - 3*(-d*e^2)^(1/3)*b^2*c*d - 3*(-d*e^2)^(1/3)*a*c^2*d
 + 3*(-d*e^2)^(1/3)*a^2*b*e + a^3*e^2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x
 + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3) - 1/3*(c^3*d^2*e^7 - 3*(-d*e^(-1))^(1/
3)*b^2*c*d*e^8 - 3*(-d*e^(-1))^(1/3)*a*c^2*d*e^8 - b^3*d*e^8 - 6*a*b*c*d*e^
8 + 3*(-d*e^(-1))^(1/3)*a^2*b*e^9 + a^3*e^9)*(-d*e^(-1))^(1/3)*e^(-9)*log(a
bs(x - (-d*e^(-1))^(1/3)))/d + 1/4*(c^3*x^4*e^3 + 4*b*c^2*x^3*e^3 + 6*b^2*c
*x^2*e^3 + 6*a*c^2*x^2*e^3 - 4*c^3*d*x*e^2 + 4*b^3*x*e^3 + 24*a*b*c*x*e^3)*
e^(-4)
```

maple [B] time = 0.05, size = 837, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x^3+d),x)
```

```
[Out] -2/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*b*c*d-
1/2/e^2/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b^2*c*d+1/e*3^(1/2)/(
```

```
d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^2*b+1/e^2/(d/e)^(1/3)*
ln(x+(d/e)^(1/3))*a*c^2*d+1/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*b^2*c*d-1/2/e
^2/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a*c^2*d-1/3/e^2/(d/e)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b^3*d+1/3/e^3/(d/e)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*c^3*d^2-2/e^2/(d/e)^(2/3)*
ln(x+(d/e)^(1/3))*a*b*c*d+1/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3
))*a*b*c*d-1/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)
)*a*c^2*d-1/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))
*b^2*c*d-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a^3+1/e*ln(e*x
^3+d)*a^2*c+1/e*ln(e*x^3+d)*a*b^2+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^3+3
/2/e*x^2*a*c^2+3/2/e*x^2*b^2*c-1/e^2*c^3*d*x+6/e*a*b*c*x-1/e/(d/e)^(1/3)*ln
(x+(d/e)^(1/3))*a^2*b+1/2/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a
^2*b-1/e^2*ln(e*x^3+d)*b*c^2*d+1/3/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*c^3*d^
2+1/6/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b^3*d-1/6/e^3/(d/e)
^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*c^3*d^2+1/3/e/(d/e)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^3-1/3/e^2/(d/e)^(2/3)*ln(x+(d/e)^
(1/3))*b^3*d+1/e*b^3*x+1/4*c^3*x^4/e+b*c^2*x^3/e
```

maxima [A] time = 3.05, size = 520, normalized size = 1.25

$$\frac{\sqrt{3}\left(\left(c^3\left(\frac{d}{e}\right)^{\frac{1}{3}}+2bc^2\right)d^2-\left(b^3\left(\frac{d}{e}\right)^{\frac{1}{3}}+2ab^2+\left(3a\left(\frac{d}{e}\right)^{\frac{2}{3}}+\frac{2bd}{e}\right)c^2+\left(3b^2\left(\frac{d}{e}\right)^{\frac{2}{3}}+6ab\left(\frac{d}{e}\right)^{\frac{1}{3}}+2a^2\right)c\right)de+\left(3a^2b\left(\frac{d}{e}\right)^{\frac{2}{3}}+a^3\left(\frac{d}{e}\right)^{\frac{1}{3}}+\frac{2ab^2d}{e}+\frac{2a^2cd}{e}\right)e^2\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3de^2}+\frac{c^3ex^4+4bc^2ex^3+6\left(b^2c+ac^2\right)ex^2-4\left(c^3d-\left(b^3+6abc\right)e\right)x}{4e^2}-\frac{\left(c^3d^2-\left(b^3-3\left(2b\left(\frac{d}{e}\right)^{\frac{2}{3}}+a\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)c^2-3\left(b^2\left(\frac{d}{e}\right)^{\frac{1}{3}}-2ab\right)c\right)de-\left(6ab^2\left(\frac{d}{e}\right)^{\frac{2}{3}}+6a^2c\left(\frac{d}{e}\right)^{\frac{2}{3}}+3a^2b\left(\frac{d}{e}\right)^{\frac{1}{3}}-a^3\right)e^2\right)\log\left(x^2-x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\left(c^3d^2-\left(b^3+3\left(b\left(\frac{d}{e}\right)^{\frac{2}{3}}-a\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)c^2-3\left(b^2\left(\frac{d}{e}\right)^{\frac{1}{3}}-2ab\right)c\right)de+\left(3ab^2\left(\frac{d}{e}\right)^{\frac{2}{3}}+3a^2c\left(\frac{d}{e}\right)^{\frac{2}{3}}-3a^2b\left(\frac{d}{e}\right)^{\frac{1}{3}}+a^3\right)e^2\right)\log\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*((c^3*(d/e)^(1/3) + 2*b*c^2)*d^2 - (b^3*(d/e)^(1/3) + 2*a*b^2 +
 (3*a*(d/e)^(2/3) + 2*b*d/e)*c^2 + (3*b^2*(d/e)^(2/3) + 6*a*b*(d/e)^(1/3) +
 2*a^2)*c)*d*e + (3*a^2*b*(d/e)^(2/3) + a^3*(d/e)^(1/3) + 2*a*b^2*d/e + 2*a
^2*c*d/e)*e^2)*arctan(1/3*sqrt(3)*(2*x - (d/e)^(1/3))/(d/e)^(1/3))/(d*e^2)
+ 1/4*(c^3*e*x^4 + 4*b*c^2*e*x^3 + 6*(b^2*c + a*c^2)*e*x^2 - 4*(c^3*d - (b^
3 + 6*a*b*c)*e)*x)/e^2 - 1/6*(c^3*d^2 - (b^3 - 3*(2*b*(d/e)^(2/3) + a*(d/e)
^(1/3))*c^2 - 3*(b^2*(d/e)^(1/3) - 2*a*b)*c)*d*e - (6*a*b^2*(d/e)^(2/3) + 6
*a^2*c*(d/e)^(2/3) + 3*a^2*b*(d/e)^(1/3) - a^3)*e^2)*log(x^2 - x*(d/e)^(1/3
) + (d/e)^(2/3))/(e^3*(d/e)^(2/3)) + 1/3*(c^3*d^2 - (b^3 + 3*(b*(d/e)^(2/3)
 - a*(d/e)^(1/3))*c^2 - 3*(b^2*(d/e)^(1/3) - 2*a*b)*c)*d*e + (3*a*b^2*(d/e)
^(2/3) + 3*a^2*c*(d/e)^(2/3) - 3*a^2*b*(d/e)^(1/3) + a^3)*e^2)*log(x + (d/e
)^(1/3))/(e^3*(d/e)^(2/3))
```

mupad [B] time = 4.91, size = 1700, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/(d + e*x^3),x)
```

```
[Out] x*((b^3 + 6*a*b*c)/e - (c^3*d)/e^2) + symsum(log(root(27*d^2*e^7*z^3 + 81*b
*c^2*d^3*e^5*z^2 - 81*a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2
*c*d^2*e^5*z + 27*a^2*b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4
*d^4*e^3*z + 54*a^4*c^2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4
*z - 27*a*c^5*d^4*e^3*z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*
b*c^4*d^3*e^3 + 18*a*b^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d
^2*e^4 + 27*a^2*b^5*c^2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d
^3*e^3 - 9*a^7*b*c*d*e^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*
d^2*e^4 - 3*a^3*c^6*d^4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*b
^3*d*e^5 + b^9*d^3*e^3 - c^9*d^6 - a^9*e^6, z, k)*((3*x*(a^3*e^4 - b^3*d*e^
3 + c^3*d^2*e^2 - 6*a*b*c*d*e^3))/e^2 - (3*(6*a*b^2*d*e^3 - 6*b*c^2*d^2*e^2
 + 6*a^2*c*d*e^3))/e^2 + 9*root(27*d^2*e^7*z^3 + 81*b*c^2*d^3*e^5*z^2 - 81*
a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2*c*d^2*e^5*z + 27*a^2*
b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4*d^4*e^3*z + 54*a^4*c^
2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4*z - 27*a*c^5*d^4*e^3*
```

```
z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*b*c^4*d^3*e^3 + 18*a*b
^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d^2*e^4 + 27*a^2*b^5*c^
2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d^3*e^3 - 9*a^7*b*c*d*e
^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*d^2*e^4 - 3*a^3*c^6*d^
4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*b^3*d*e^5 + b^9*d^3*e^3
 - c^9*d^6 - a^9*e^6, z, k)*d*e^2) + (3*(a^5*b*e^3 - a*c^5*d^3 + 2*b^2*c^4*
d^3 + 2*a^2*b^4*d*e^2 + 2*a^4*c^2*d*e^2 + b^5*c*d^2*e + a*b^3*c^2*d^2*e + a
^2*b*c^3*d^2*e - a^3*b^2*c*d*e^2))/e^2 + (3*x*(b*c^5*d^3 - a^5*c*e^3 + 2*a^
4*b^2*e^3 + 2*a^2*c^4*d^2*e + 2*b^4*c^2*d^2*e + a*b^5*d*e^2 - a*b^2*c^3*d^2
*e + a^2*b^3*c*d*e^2 + a^3*b*c^2*d*e^2))/e^2)*root(27*d^2*e^7*z^3 + 81*b*c^
2*d^3*e^5*z^2 - 81*a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2*c*
d^2*e^5*z + 27*a^2*b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4*d^
4*e^3*z + 54*a^4*c^2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4*z
- 27*a*c^5*d^4*e^3*z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*b*c
^4*d^3*e^3 + 18*a*b^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d^2*
e^4 + 27*a^2*b^5*c^2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d^3*
e^3 - 9*a^7*b*c*d*e^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*d^2
*e^4 - 3*a^3*c^6*d^4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*b^3*
d*e^5 + b^9*d^3*e^3 - c^9*d^6 - a^9*e^6, z, k), k, 1, 3) + (c^3*x^4)/(4*e)
+ (b*c^2*x^3)/e + (3*c*x^2*(a*c + b^2))/(2*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x**3+d),x)
```

```
[Out] Timed out
```

3.64 $\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$

Optimal. Leaf size=645

$$-\frac{x^2\left(-6a^2c^2e-12ab^2ce+b^4(-e)+4bc^3d\right)}{2e^2}-\frac{2x\left(-6a^2bce-2ab^3e+2ac^3d+3b^2c^2d\right)}{e^2}+\frac{\log\left(d+ex^3\right)\left(-4ce\left(b^3d\right.\right.}{}$$

Rubi [A] time = 1.10, antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] (-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 - ((4*b*c^3*
d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) - (c*(c^3*d - 4*b^3*e
- 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*a*c)*x^4)/(2*e) + (4*b*c^3*x^5
)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^(1/3) + a*e^(1/3))*(4*c^3*d^2 + 6*c^2*(b*
d^(5/3)*e^(1/3) - a*d^(4/3)*e^(2/3)) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b^2*d^
(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))*ArcTan[(d^(1/3) - 2*e^(1/
3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(8/3)) + ((e^(1/3)*(6*b^2*c^2*
d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^(1/3)*(b^4*
d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^(1/3
) + e^(1/3)*x])/(3*d^(2/3)*e^(8/3)) - ((6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b
^3*d*e - 12*a^2*b*c*d*e + a^4*e^2 + (d^(1/3)*(b^4*d*e + 12*a*b^2*c*d*e + 6*
a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1
/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(7/3)) + ((c^4*d^2 - 12*a*b*c^2*d*e + 6*
a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3])/(3*e^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^4}{d+ex^3}\,dx = \int \left(-\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x}{e^2}-\frac{c\left(c^3d\right.}{}\right.$$

$$= -\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d\right.}{}$$

$$= -\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d\right.}{}$$

$$= -\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d\right.}{}$$

$$= -\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d\right.}{}$$

$$= -\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d\right.}{}$$

$$= -\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d\right.}{}$$

Mathematica [A] time = 0.48, size = 678, normalized size = 1.05

$$\frac{15e^{2/3}\left(-4bc^3d+b^4e+12ab^2ce+6a^2c^2e\right)x^2+60e^{2/3}\left(-3b^2c^2d-2ac^3d+2ab^3e+6a^2bce\right)x+\frac{10\sqrt{3}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\left(c^2\left(6ad^{4/3}e^{2/3}-6bd^{5/3}\sqrt[3]{e}\right)-4c^3d^2+e\left(-a^3e-3a^2b\sqrt[3]{d}e^{2/3}+3ab^2d^{2/3}\sqrt[3]{e}+b^3d\right)+12abcde\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{2/3}}-\frac{5\left(4ac^3d^2\sqrt[3]{e}+6a^2c^2d^{4/3}e+a^4e^{7/3}-4ab^3de^{4/3}+6b^2\left(c^2d^2\sqrt[3]{e}+2acd^{4/3}e\right)-4b\left(c^3d^{7/3}+3a^2cde^{4/3}+a^3\sqrt[3]{d}e^2\right)+b^4d^{4/3}e\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{d^{2/3}}+\frac{10\left(4ac^3d^2\sqrt[3]{e}+6a^2c^2d^{4/3}e+a^4e^{7/3}-4ab^3de^{4/3}+6b^2\left(c^2d^2\sqrt[3]{e}+2acd^{4/3}e\right)-4b\left(c^3d^{7/3}+3a^2cde^{4/3}+a^3\sqrt[3]{d}e^2\right)+b^4d^{4/3}e\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{2/3}}+\frac{10\left(6a^2b^2e^2-12abc^2de+4ce\left(a^3e-b^3d\right)+c^4d^2\right)\log\left(d+ex^3\right)}{\sqrt[3]{e}}+10ce^{2/3}x^3\left(12abce+4b^3e-c^3d\right)+15c^2e^{5/3}x^4\left(2ac+3b^2\right)+24bc^3e^{5/3}x^5+5c^4e^{5/3}x^6}{30e^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] (60*e^(2/3)*(-3*b^2*c^2*d - 2*a*c^3*d + 2*a*b^3*e + 6*a^2*b*c*e)*x + 15*e^(
2/3)*(-4*b*c^3*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*x^2 + 10*c*e^(2/3)*(
-(c^3*d) + 4*b^3*e + 12*a*b*c*e)*x^3 + 15*c^2*(3*b^2 + 2*a*c)*e^(5/3)*x^4 +
 24*b*c^3*e^(5/3)*x^5 + 5*c^4*e^(5/3)*x^6 + (10*Sqrt[3]*(b*d^(1/3) + a*e^(1
/3))*(-4*c^3*d^2 + c^2*(-6*b*d^(5/3)*e^(1/3) + 6*a*d^(4/3)*e^(2/3)) + 12*a*
b*c*d*e + e*(b^3*d + 3*a*b^2*d^(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^
3*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (10*(4*a*c^3*d
^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4/3) + a^4*
e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^(7/3) + 3*
a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (5*
(4*a*c^3*d^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4
/3) + a^4*e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^
(7/3) + 3*a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(2/3) - d^(1/3)*e^(1/3)
*x + e^(2/3)*x^2])/d^(2/3) + (10*(c^4*d^2 - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2
+ 4*c*e*(-(b^3*d) + a^3*e))*Log[d + e*x^3])/e^(1/3))/(30*e^(8/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^4}{d+ex^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)^4/(d + e*x^3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 723, normalized size = 1.12

$$\frac{1}{3}\left(c^4d^2-4b^3cde-12abc^2de+6a^2b^2e^2+4a^3ce^2\right)e^{(-3)}\log\left(\left|x^3e+d\right|\right)-\frac{\sqrt{3}\left(6b^2c^2d^2e+4ac^3d^2e-4\left(-de^2\right)^{\frac{1}{3}}bc^3d^2+\left(-de^2\right)^{\frac{1}{3}}b^4de+12\left(-de^2\right)^{\frac{1}{3}}ab^2cde+6\left(-de^2\right)^{\frac{1}{3}}a^2c^2de-4ab^3de^2-12a^2bcde^2-4\left(-de^2\right)^{\frac{1}{3}}a^3be^2+a^4e^3\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-2)}}{3\left(-de^2\right)^{\frac{2}{3}}}-\frac{\left(6b^2c^2d^2e+4ac^3d^2e+4\left(-de^2\right)^{\frac{1}{3}}bc^3d^2-\left(-de^2\right)^{\frac{1}{3}}b^4de-12\left(-de^2\right)^{\frac{1}{3}}ab^2cde-6\left(-de^2\right)^{\frac{1}{3}}a^2c^2de-4ab^3de^2-12a^2bcde^2+4\left(-de^2\right)^{\frac{1}{3}}a^3be^2+a^4e^3\right)e^{(-2)}\log\left(x^2+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}}-\frac{\left(4\left(-de^{(-1)}\right)^{\frac{1}{3}}bc^3d^2e^{11}+6b^2c^2d^2e^{11}+4ac^3d^2e^{11}-\left(-de^{(-1)}\right)^{\frac{1}{3}}b^4de^{12}-12\left(-de^{(-1)}\right)^{\frac{1}{3}}ab^2cde^{12}-6\left(-de^{(-1)}\right)^{\frac{1}{3}}a^2c^2de^{12}-4ab^3de^{12}-12a^2bcde^{12}+4\left(-de^{(-1)}\right)^{\frac{1}{3}}a^3be^{13}+a^4e^{13}\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-13)}\log\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3d}+\frac{1}{30}\left(5c^4x^6e^5+24bc^3x^5e^5+45b^2c^2x^4e^5+30ac^3x^4e^5-10c^4dx^3e^4+40b^3cx^3e^5+120abc^2x^3e^5-60bc^3dx^2e^4+15b^4x^2e^5+180ab^2cx^2e^5+90a^2c^2x^2e^5-180b^2c^2dxe^4-120ac^3dxe^4+120ab^3xe^5+360a^2bcxe^5\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/3*(c^4*d^2 - 4*b^3*c*d*e - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2 + 4*a^3*c*e^2)*
e^(-3)*log(abs(x^3*e + d)) - 1/3*sqrt(3)*(6*b^2*c^2*d^2*e + 4*a*c^3*d^2*e -
 4*(-d*e^2)^(1/3)*b*c^3*d^2 + (-d*e^2)^(1/3)*b^4*d*e + 12*(-d*e^2)^(1/3)*a*
b^2*c*d*e + 6*(-d*e^2)^(1/3)*a^2*c^2*d*e - 4*a*b^3*d*e^2 - 12*a^2*b*c*d*e^2
 - 4*(-d*e^2)^(1/3)*a^3*b*e^2 + a^4*e^3)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-
1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-2)/(-d*e^2)^(2/3) - 1/6*(6*b^2*c^2*d^2*e
+ 4*a*c^3*d^2*e + 4*(-d*e^2)^(1/3)*b*c^3*d^2 - (-d*e^2)^(1/3)*b^4*d*e - 12*
(-d*e^2)^(1/3)*a*b^2*c*d*e - 6*(-d*e^2)^(1/3)*a^2*c^2*d*e - 4*a*b^3*d*e^2 -
 12*a^2*b*c*d*e^2 + 4*(-d*e^2)^(1/3)*a^3*b*e^2 + a^4*e^3)*e^(-2)*log(x^2 +
```

```
(-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3) - 1/3*(4*(-d*e^(-1)
)^(1/3)*b*c^3*d^2*e^11 + 6*b^2*c^2*d^2*e^11 + 4*a*c^3*d^2*e^11 - (-d*e^(-1)
)^(1/3)*b^4*d*e^12 - 12*(-d*e^(-1))^(1/3)*a*b^2*c*d*e^12 - 6*(-d*e^(-1))^(1
/3)*a^2*c^2*d*e^12 - 4*a*b^3*d*e^12 - 12*a^2*b*c*d*e^12 + 4*(-d*e^(-1))^(1/
3)*a^3*b*e^13 + a^4*e^13)*(-d*e^(-1))^(1/3)*e^(-13)*log(abs(x - (-d*e^(-1))
^(1/3)))/d + 1/30*(5*c^4*x^6*e^5 + 24*b*c^3*x^5*e^5 + 45*b^2*c^2*x^4*e^5 +
30*a*c^3*x^4*e^5 - 10*c^4*d*x^3*e^4 + 40*b^3*c*x^3*e^5 + 120*a*b*c^2*x^3*e^
5 - 60*b*c^3*d*x^2*e^4 + 15*b^4*x^2*e^5 + 180*a*b^2*c*x^2*e^5 + 90*a^2*c^2*
x^2*e^5 - 180*b^2*c^2*d*x*e^4 - 120*a*c^3*d*x*e^4 + 120*a*b^3*x*e^5 + 360*a
^2*b*c*x*e^5)*e^(-6)
```

maple [B] time = 0.06, size = 1339, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x^3+d),x)
```

```
[Out] -4/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^2*b*c*
d-4/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*b^2*c
*d-1/e^2/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a^2*c^2*d-1/3/e^2*3^
(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b^4*d-4/e^2*ln(e*
x^3+d)*a*b*c^2*d+2/3/e^3/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b*c^
3*d^2+4/3/e*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^3
*b+2/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*a^2*c^2*d-4/3/e^3/(d/e)^(1/3)*ln(x+(
d/e)^(1/3))*b*c^3*d^2+2/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*b^2*c^2*d^2+2/3/e
^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a*b^3*d-2/3/e^3/(d/e)^(2/3
)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a*c^3*d^2-1/e^3/(d/e)^(2/3)*ln(x^2-(d/e
)^(1/3)*x+(d/e)^(2/3))*b^2*c^2*d^2-4/3/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a*
b^3*d+4/3/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a*c^3*d^2+1/2/e*x^2*b^4-4/3/e^2
/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*b^3*d+4/3/e^
3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*c^3*d^2+2/e
^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b^2*c^2*d^2+
4/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*a*b^2*c*d-2/e^2/(d/e)^(1/3)*ln(x^2-(d/e
)^(1/3)*x+(d/e)^(2/3))*a*b^2*c*d-2/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/
2)*(2/(d/e)^(1/3)*x-1))*a^2*c^2*d+4/3/e^3*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^
(1/2)*(2/(d/e)^(1/3)*x-1))*b*c^3*d^2-4/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^
2*b*c*d+2/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a^2*b*c*d+4/3/e
*x^3*b^3*c+3/e*x^2*a^2*c^2+4/e*a*b^3*x+1/e*x^4*a*c^3+3/2/e*x^4*b^2*c^2+4/3/
e*ln(e*x^3+d)*a^3*c+2/e*ln(e*x^3+d)*a^2*b^2+1/3/e^3*ln(e*x^3+d)*c^4*d^2+1/3
/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^4-1/3/e^2*x^3*c^4*d-1/6/e/(d/e)^(2/3)*ln
(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a^4+1/6*c^4*x^6/e+2/3/e/(d/e)^(1/3)*ln(x^2-
(d/e)^(1/3)*x+(d/e)^(2/3))*a^3*b-1/6/e^2/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(
d/e)^(2/3))*b^4*d-4/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*a^3*b-4/3/e^2*ln(e*x^
3+d)*b^3*c*d+1/3/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*b^4*d-2/e^2*x^2*b*c^3*d+
12/e*a^2*b*c*x-4/e^2*a*c^3*d*x-6/e^2*b^2*c^2*d*x+4/e*x^3*a*b*c^2+6/e*x^2*a*
b^2*c+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^4
+4/5*b*c^3*x^5/e
```

maxima [A] time = 3.13, size = 833, normalized size = 1.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/30*(5*c^4*e*x^6 + 24*b*c^3*e*x^5 + 15*(3*b^2*c^2 + 2*a*c^3)*e*x^4 - 10*(c
^4*d - 4*(b^3*c + 3*a*b*c^2)*e)*x^3 - 15*(4*b*c^3*d - (b^4 + 12*a*b^2*c + 6
*a^2*c^2)*e)*x^2 - 60*((3*b^2*c^2 + 2*a*c^3)*d - 2*(a*b^3 + 3*a^2*b*c)*e)*x
)/e^2 - 1/9*sqrt(3)*(2*c^4*d^3 - 2*(4*b^3*c + 6*(b*(d/e)^(2/3) + a*(d/e)^(1
/3))*c^3 + c^4*d/e + 3*(3*b^2*(d/e)^(1/3) + 4*a*b)*c^2)*d^2*e + (3*b^4*(d/e
```

```
)^(2/3) + 12*a*b^3*(d/e)^(1/3) + 12*a^2*b^2 + 6*(3*a^2*(d/e)^(2/3) + 4*a*b*
d/e)*c^2 + 4*(9*a*b^2*(d/e)^(2/3) + 9*a^2*b*(d/e)^(1/3) + 2*a^3 + 2*b^3*d/e
)*c)*d*e^2 - (12*a^3*b*(d/e)^(2/3) + 3*a^4*(d/e)^(1/3) + 12*a^2*b^2*d/e + 8
*a^3*c*d/e)*e^3)*arctan(1/3*sqrt(3)*(2*x - (d/e)^(1/3))/(d/e)^(1/3))/(d*e^3
) + 1/6*(2*(c^4*(d/e)^(2/3) - 3*b^2*c^2 + 2*(b*(d/e)^(1/3) - a)*c^3)*d^2 -
(b^4*(d/e)^(1/3) - 4*a*b^3 + 6*(4*a*b*(d/e)^(2/3) + a^2*(d/e)^(1/3))*c^2 +
4*(2*b^3*(d/e)^(2/3) + 3*a*b^2*(d/e)^(1/3) - 3*a^2*b)*c)*d*e + (12*a^2*b^2*
(d/e)^(2/3) + 8*a^3*c*(d/e)^(2/3) + 4*a^3*b*(d/e)^(1/3) - a^4)*e^2)*log(x^2
 - x*(d/e)^(1/3) + (d/e)^(2/3))/(e^3*(d/e)^(2/3)) + 1/3*((c^4*(d/e)^(2/3) +
 6*b^2*c^2 - 4*(b*(d/e)^(1/3) - a)*c^3)*d^2 + (b^4*(d/e)^(1/3) - 4*a*b^3 -
6*(2*a*b*(d/e)^(2/3) - a^2*(d/e)^(1/3))*c^2 - 4*(b^3*(d/e)^(2/3) - 3*a*b^2*
(d/e)^(1/3) + 3*a^2*b)*c)*d*e + (6*a^2*b^2*(d/e)^(2/3) + 4*a^3*c*(d/e)^(2/3
) - 4*a^3*b*(d/e)^(1/3) + a^4)*e^2)*log(x + (d/e)^(1/3))/(e^3*(d/e)^(2/3))
```

mupad [B] time = 5.05, size = 2971, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^4/(d + e*x^3),x)
```

```
[Out] x^2*((b^4 + 6*a^2*c^2 + 12*a*b^2*c)/(2*e) - (2*b*c^3*d)/e^2) - x^3*((c^4*d)
/(3*e^2) - (4*b*c*(3*a*c + b^2))/(3*e)) + symsum(log(root(27*d^2*e^9*z^3 +
324*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^3*c*d^2*e^8*z^2 - 1
62*a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6*d^5*e^4*z + 216*a^
2*b^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^5*b^2*c*d^2*e^7*z +
 108*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a*b^4*c^3*d^4*e^5*z
+ 144*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3*c^5*d^4*e^5*z + 90
*a^6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^3*e^6*z + 36*a^7*b*
d*e^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^7*b*c^4*d^3*e^5 - 3
6*a^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^4*d^5*e^3 + 36*a*b^
4*c^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^3*e^5 - 108*a^5*b^2
*c^5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*d^3*e^5 + 96*a^3*b^
6*c^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*d^2*e^6 - 54*a^2*b^
8*c^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d^4*e^4 - 12*a^10*b*
c*d*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*c^3*d^5*e^3 - 6*a^6
*c^6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6*a^6*b^6*d^2*e^6 +
4*a^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 - b^12*d^4*e^4 - c^12
*d^8 - a^12*e^8, z, k)*((x*(3*a^4*e^5 + 12*a*c^3*d^2*e^3 + 18*b^2*c^2*d^2*e
^3 - 12*a*b^3*d*e^4 - 36*a^2*b*c*d*e^4))/e^3 - (6*c^4*d^3*e^3 + 36*a^2*b^2*
d*e^5 - 24*b^3*c*d^2*e^4 + 24*a^3*c*d*e^5 - 72*a*b*c^2*d^2*e^4)/e^4 + 9*roo
t(27*d^2*e^9*z^3 + 324*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^
3*c*d^2*e^8*z^2 - 162*a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6
*d^5*e^4*z + 216*a^2*b^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^
5*b^2*c*d^2*e^7*z + 108*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a
*b^4*c^3*d^4*e^5*z + 144*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3
*c^5*d^4*e^5*z + 90*a^6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^
3*e^6*z + 36*a^7*b*d*e^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^
7*b*c^4*d^3*e^5 - 36*a^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^
4*d^5*e^3 + 36*a*b^4*c^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^
3*e^5 - 108*a^5*b^2*c^5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*
d^3*e^5 + 96*a^3*b^6*c^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*
d^2*e^6 - 54*a^2*b^8*c^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d
^4*e^4 - 12*a^10*b*c*d*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*
c^3*d^5*e^3 - 6*a^6*c^6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6
*a^6*b^6*d^2*e^6 + 4*a^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 -
b^12*d^4*e^4 - c^12*d^8 - a^12*e^8, z, k)*d*e^2) + (c^8*d^5 + 4*a^7*b*e^5 +
 4*a*b^7*d^2*e^3 + 19*a^4*b^4*d*e^4 + 10*a^6*c^2*d*e^4 + 16*b^3*c^5*d^4*e -
 16*a^3*c^5*d^3*e^2 + 10*b^6*c^2*d^3*e^2 - 8*a*b*c^6*d^4*e + 24*a^2*b^2*c^4
*d^3*e^2 + 16*a^3*b^3*c^2*d^2*e^3 - 12*a^5*b^2*c*d*e^4 + 4*a*b^4*c^3*d^3*e^
```

```
2 + 12*a^2*b^5*c*d^2*e^3 - 4*a^4*b*c^3*d^2*e^3)/e^4 + (x*(10*a^6*b^2*e^4 -
4*a^7*c*e^4 - 4*a*c^7*d^4 + 10*b^2*c^6*d^4 + b^8*d^2*e^2 + 16*a^3*b^5*d*e^3
 + 16*b^5*c^3*d^3*e + 19*a^4*c^4*d^2*e^2 + 24*a^2*b^4*c^2*d^2*e^2 - 16*a^3*
b^2*c^3*d^2*e^2 - 4*a*b^3*c^4*d^3*e + 8*a*b^6*c*d^2*e^2 + 12*a^2*b*c^5*d^3*
e - 4*a^4*b^3*c*d*e^3 + 12*a^5*b*c^2*d*e^3))/e^3)*root(27*d^2*e^9*z^3 + 324
*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^3*c*d^2*e^8*z^2 - 162*
a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6*d^5*e^4*z + 216*a^2*b
^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^5*b^2*c*d^2*e^7*z + 10
8*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a*b^4*c^3*d^4*e^5*z + 1
44*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3*c^5*d^4*e^5*z + 90*a^
6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^3*e^6*z + 36*a^7*b*d*e
^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^7*b*c^4*d^3*e^5 - 36*a
^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^4*d^5*e^3 + 36*a*b^4*c
^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^3*e^5 - 108*a^5*b^2*c^
5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*d^3*e^5 + 96*a^3*b^6*c
^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*d^2*e^6 - 54*a^2*b^8*c
^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d^4*e^4 - 12*a^10*b*c*d
*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*c^3*d^5*e^3 - 6*a^6*c^
6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6*a^6*b^6*d^2*e^6 + 4*a
^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 - b^12*d^4*e^4 - c^12*d^
8 - a^12*e^8, z, k), k, 1, 3) - x*((d*(4*a*c^3 + 6*b^2*c^2))/e^2 - (4*a*b*(
3*a*c + b^2))/e) + (c^4*x^6)/(6*e) + (x^4*(4*a*c^3 + 6*b^2*c^2))/(4*e) + (4
*b*c^3*x^5)/(5*e)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x**3+d),x)
```

```
[Out] Timed out
```

3.65 $\int \frac{2x^2+x^4}{1+x^3} \, dx$

Optimal. Leaf size=43

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1593, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x^2 + x^4)/(1 + x^3),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x] + Log[1 - x + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2x^2+x^4}{1+x^3}\,dx &= \int \frac{x^2\left(2+x^2\right)}{1+x^3}\,dx\\
&= \int \left(x+\frac{x(-1+2x)}{1+x^3}\right)dx\\
&= \frac{x^2}{2}+\int \frac{x(-1+2x)}{1+x^3}\,dx\\
&= \frac{x^2}{2}+\frac{1}{3}\int \frac{-3+3x}{1-x+x^2}\,dx+\int \frac{1}{1+x}\,dx\\
&= \frac{x^2}{2}+\log(1+x)-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= \frac{x^2}{2}+\log(1+x)+\frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.26

$$\frac{1}{6}\left(4\log\left(x^3+1\right)+3x^2-\log\left(x^2-x+1\right)+2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x^2 + x^4)/(1 + x^3),x]
```

```
[Out] (3*x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[1 - x +
x^2] + 4*Log[1 + x^3])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2+x^4}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*x^2 + x^4)/(1 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(2*x^2 + x^4)/(1 + x^3), x]
```

fricas [A] time = 0.41, size = 37, normalized size = 0.86

$$\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(x + 1)
```

giac [A] time = 0.15, size = 38, normalized size = 0.88

$$\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log\left(x^2-x+1\right) + \log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(abs(x + 1))
```

maple [A] time = 0.05, size = 38, normalized size = 0.88

$$\frac{x^2}{2} - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} + \ln(x+1) + \frac{\ln\left(x^2-x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x^2)/(x^3+1),x)
```

```
[Out] 1/2*x^2+ln(x+1)+1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.86, size = 37, normalized size = 0.86

$$\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log\left(x^2-x+1\right) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(x + 1)
```

mupad [B] time = 0.10, size = 49, normalized size = 1.14

$$\ln(x+1) + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + x^4)/(x^3 + 1),x)
```

```
[Out] log(x + 1) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/2) - log(x +
 (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/2) + x^2/2
```

sympy [A] time = 0.25, size = 44, normalized size = 1.02

$$\frac{x^2}{2} + \log(x+1) + \frac{\log\left(x^2-x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x**2)/(x**3+1),x)
```

```
[Out] x**2/2 + log(x + 1) + log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sq
rt(3)/3)/3
```

3.66 $\int \frac{2x^2+x^4}{1-x^3} \, dx$

Optimal. Leaf size=46

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1593, 1887, 1875, 31, 634, 618, 204, 628}

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x^2 + x^4)/(1 - x^3),x]
```

```
[Out] -x^2/2 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x] - Log[1 + x + x^2]/
2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{2x^2+x^4}{1-x^3}\,dx &= \int \frac{x^2\left(2+x^2\right)}{1-x^3}\,dx \\
&= \int \left(-x+\frac{x(1+2x)}{1-x^3}\right)dx \\
&= -\frac{x^2}{2}+\int \frac{x(1+2x)}{1-x^3}\,dx \\
&= -\frac{x^2}{2}+\frac{1}{3}\int \frac{-3-3x}{1+x+x^2}\,dx+\int \frac{1}{1-x}\,dx \\
&= -\frac{x^2}{2}-\log(1-x)-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx \\
&= -\frac{x^2}{2}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= -\frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica **[A]** time = 0.03, size = 54, normalized size = 1.17

$$\frac{1}{6}\left(-4\log\left(1-x^3\right)-3x^2+\log\left(x^2+x+1\right)-2\log(1-x)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x^2 + x^4)/(1 - x^3),x]
[Out] (-3*x^2 - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + Log[1 + x +
x^2] - 4*Log[1 - x^3])/6
```

IntegrateAlgebraic **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2+x^4}{1-x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*x^2 + x^4)/(1 - x^3),x]
[Out] IntegrateAlgebraic[(2*x^2 + x^4)/(1 - x^3), x]
```

fricas **[A]** time = 0.41, size = 37, normalized size = 0.80

$$-\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(x - 1)
```

giac [A] time = 0.16, size = 38, normalized size = 0.83

$$-\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-\frac{1}{2}\,\log\left(x^2+x+1\right)-\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(abs(x - 1))
```

maple [A] time = 0.05, size = 38, normalized size = 0.83

$$-\frac{x^2}{2}-\frac{\sqrt{3}\;\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}-\ln\left(x-1\right)-\frac{\ln\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x^2)/(-x^3+1),x)
```

```
[Out] -1/2*x^2-ln(x-1)-1/2*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.90, size = 37, normalized size = 0.80

$$-\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-\frac{1}{2}\,\log\left(x^2+x+1\right)-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(x - 1)
```

mupad [B] time = 0.09, size = 51, normalized size = 1.11

$$-\ln\left(x-1\right)+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + x^4)/(x^3 - 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x - 1) - log(x +
 (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) - x^2/2
```

sympy [A] time = 0.31, size = 46, normalized size = 1.00

$$-\frac{x^2}{2}-\log\left(x-1\right)-\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x**2)/(-x**3+1),x)
```

```
[Out] -x**2/2 - log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/3
```

3.67 $\int \frac{1-x+4x^3}{1+x^3} \, dx$

Optimal. Leaf size=44

$$\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)+\frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1887, 1860, 31, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)+\frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 4*x^3)/(1 + x^3),x]
```

```
[Out] 4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
+ x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x+4x^3}{1+x^3}\,dx &= \int \left(4-\frac{3+x}{1+x^3}\right) dx \\
&= 4x-\int \frac{3+x}{1+x^3}\,dx \\
&= 4x-\frac{1}{3}\int \frac{7-2x}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1+x}\,dx \\
&= 4x-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1+2x}{1-x+x^2}\,dx-2\int \frac{1}{1-x+x^2}\,dx \\
&= 4x-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)+4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= 4x+\frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)-\frac{4\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 4*x^3)/(1 + x^3),x]
```

```
[Out] 4*x - (4*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
 + x^2]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x+4x^3}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x + 4*x^3)/(1 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(1 - x + 4*x^3)/(1 + x^3), x]
```

fricas [A] time = 0.40, size = 37, normalized size = 0.84

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+4\,x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(x + 1)
```

giac [A] time = 0.15, size = 38, normalized size = 0.86

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(abs(x + 1))
```

maple [A] time = 0.05, size = 38, normalized size = 0.86

$$4x-\frac{4\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{2\ln(x+1)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-x+1)/(x^3+1),x)
```

```
[Out] 4*x-2/3*ln(x+1)+1/3*ln(x^2-x+1)-4/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.81, size = 37, normalized size = 0.84

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(x + 1)
```

mupad [B] time = 4.70, size = 49, normalized size = 1.11

$$4x-\frac{2\ln(x+1)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,2i}{3}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,2i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3 - x + 1)/(x^3 + 1),x)
```

```
[Out] 4*x - (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*2i)/3 + 1/
3) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*2i)/3 - 1/3)
```

sympy [A] time = 0.33, size = 48, normalized size = 1.09

$$4x-\frac{2\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{3}-\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-x+1)/(x**3+1),x)
```

```
[Out] 4*x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3 - 4*sqrt(3)*atan(2*sqrt(3)*x/3 -
 sqrt(3)/3)/3
```

3.68 $\int \frac{c+dx}{a-bx^4}\,dx$

Optimal. Leaf size=87

$$\frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1876, 212, 208, 205, 275}

$$\frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4),x]
```

```
[Out] (c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[(b^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqr
t[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{a-bx^4}\,dx &= \int \left(\frac{c}{a-bx^4}+\frac{dx}{a-bx^4}\right)dx \\
&= c\int \frac{1}{a-bx^4}\,dx + d\int \frac{x}{a-bx^4}\,dx \\
&= \frac{c\int \frac{1}{\sqrt{a}-\sqrt{b}\,x^2}\,dx}{2\sqrt{a}} + \frac{c\int \frac{1}{\sqrt{a}+\sqrt{b}\,x^2}\,dx}{2\sqrt{a}} + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right) \\
&= \frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 134, normalized size = 1.54

$$
\frac{-\left(\sqrt[4]{a}\,d+\sqrt[4]{b}\,c\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)+\sqrt[4]{b}\,c\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+2\sqrt[4]{b}\,c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt[4]{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)-\sqrt[4]{a}\,d\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{4a^{3/4}\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4),x]
```

```
[Out] (2*b^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(1/4)*c + a^(1/4)*d)*Log[a^(1
/4) - b^(1/4)*x] + b^(1/4)*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*d*Log[a^(1/
4) + b^(1/4)*x] + a^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3/4)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{c+dx}{a-bx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a - b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a - b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.18, size = 225, normalized size = 2.59

$$
\frac{\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}\,bd+\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}\,bd+\left(-ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))
/(a*b) - 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/(a*b) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d + (-a*b^3)^(1/4)*b*c)
*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2) + 1/
```

```
4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d + (-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2)
```

maple [A] time = 0.05, size = 101, normalized size = 1.16

$$-\frac{d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*c*(a/b)^(1/4)/a
*arctan(x/(a/b)^(1/4))-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a
*b)^(1/2)))
```

maxima [B] time = 2.88, size = 126, normalized size = 1.45

$$\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}}-\frac{d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}}-\frac{c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + 1/4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*d*log(sqrt(b)
*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*c*log((sqrt(b)*x - sqrt(sqrt(a)*sqr
t(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
```

mupad [B] time = 5.01, size = 182, normalized size = 2.09

$$\begin{cases} \frac{2c+3dx}{6bx^3} & \text{if } a=0 \\ \frac{\operatorname{atan}\left(\frac{\sqrt{2}(-b)^{1/4}x}{a^{1/4}}-1\right)\left(2a^{1/4}d+\sqrt{2}(-b)^{1/4}c\right)}{4a^{3/4}\sqrt{-b}}-\frac{\operatorname{atan}\left(\frac{\sqrt{2}(-b)^{1/4}x}{a^{1/4}}+1\right)\left(4a^{1/4}d-2\sqrt{2}(-b)^{1/4}c\right)}{8a^{3/4}\sqrt{-b}}+\frac{\sqrt{2}c\ln\left(\frac{\sqrt{-b}x^2+\sqrt{a}+\sqrt{2}a^{1/4}(-b)^{1/4}x}{\sqrt{-b}x^2+\sqrt{a}-\sqrt{2}a^{1/4}(-b)^{1/4}x}\right)}{8a^{3/4}(-b)^{1/4}} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - b*x^4),x)
```

```
[Out] piecewise(a == 0, (2*c + 3*d*x)/(6*b*x^3), a ~= 0, (atan((2^(1/2)*(-b)^(1/4
)*x)/a^(1/4) - 1)*(2*a^(1/4)*d + 2^(1/2)*(-b)^(1/4)*c))/(4*a^(3/4)*(-b)^(1/
2)) - (atan((2^(1/2)*(-b)^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*d - 2*2^(1/2)*(-
b)^(1/4)*c))/(8*a^(3/4)*(-b)^(1/2)) + (2^(1/2)*c*log(((-b)^(1/2)*x^2 + a^(1
/2) + 2^(1/2)*a^(1/4)*(-b)^(1/4)*x)/((-b)^(1/2)*x^2 + a^(1/2) - 2^(1/2)*a^(
1/4)*(-b)^(1/4)*x)))/(8*a^(3/4)*(-b)^(1/4)))
```

sympy [A] time = 1.22, size = 126, normalized size = 1.45

$$-\operatorname{RootSum}\left(256t^4a^3b^2-32t^2a^2bd^2-16tabc^2d+ad^4-bc^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3bd^2+16t^2a^2bc^2d+8ta^2d^4-4tabc^4+5ac^2d^3}{4acd^4+bc^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*
d**4 - b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 + 16*_t**2*a**
2*b*c**2*d + 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4 +
b*c**5))))
```

3.69 $\int \frac{c+dx}{a+bx^4}\,dx$

Optimal. Leaf size=219

$$-\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\tan^{-1}\left(\frac{\sqrt{2}}{\ldots}\right.}{2\sqrt{2}a^{3/\ldots}}$$

Rubi [A] time = 0.17, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*ArcTan[1 + (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1
/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx}{a+bx^4}\,dx = \int \left(\frac{c}{a+bx^4}+\frac{dx}{a+bx^4}\right) dx$$

$$= c\int \frac{1}{a+bx^4}\,dx + d\int \frac{x}{a+bx^4}\,dx$$

$$= \frac{c\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}} + \frac{c\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}} + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{c\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\sqrt{b}} + \frac{c\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\sqrt{b}} - \frac{c\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c\int \ldots}{4\ldots}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\,\text{Su}\ldots}{\ldots}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}\ldots\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Mathematica [A] time = 0.09, size = 184, normalized size = 0.84

$$\frac{-2\left(2\sqrt[4]{a}d+\sqrt{2}\sqrt[4]{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{b}c-2\sqrt[4]{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)+\sqrt{2}\sqrt[4]{b}c\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\right)}{8a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*b^(1/4)*c + 2*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[2]*b^(1/4)*c - 2*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)] + Sqrt[2]*b^(1/4)*c*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/
4)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 213, normalized size = 0.97

$$\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}} c\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab}-\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}} c\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,bd-\left(ab^{3}\right)^{\frac{1}{4}} bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^{2}}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,bd-\left(ab^{3}\right)^{\frac{1}{4}} bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b) - 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*
(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2)
```

maple [A] time = 0.04, size = 151, normalized size = 0.69

$$\frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{2\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2
-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x-1)+1/2*d/(a*b)^(1/2)*arctan(x^2*(1/a*b)^(1/2))
```

maxima [A] time = 3.04, size = 207, normalized size = 0.95

$$\frac{\sqrt{2}\,c\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,c\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}c-2\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}c+2\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - 1/8*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + s
qrt(a))/(a^(3/4)*b^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(1/4)*c - 2*sqrt(a)*d)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(1/4
)*c + 2*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4))
```

mupad [B] time = 4.80, size = 160, normalized size = 0.73

$$\begin{cases} -\frac{2\,c+3\,d\,x}{6\,b\,x^3} & \text{if } a=0 \\ \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,d+\sqrt{2}\,b^{1/4}\,c\right)}{4\,a^{3/4}\,\sqrt{b}}-\frac{\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,d-2\,\sqrt{2}\,b^{1/4}\,c\right)}{8\,a^{3/4}\,\sqrt{b}}+\frac{\sqrt{2}\,c\,\ln\left(\frac{\sqrt{a}+\sqrt{b}\,x^2+\sqrt{2}\,a^{1/4}\,b^{1/4}\,x}{\sqrt{a}+\sqrt{b}\,x^2-\sqrt{2}\,a^{1/4}\,b^{1/4}\,x}\right)}{8\,a^{3/4}\,b^{1/4}} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^4),x)
```

```
[Out] piecewise(a == 0, -(2*c + 3*d*x)/(6*b*x^3), a ~= 0, (atan((2^(1/2)*b^(1/4)*
x)/a^(1/4) - 1)*(2*a^(1/4)*d + 2^(1/2)*b^(1/4)*c))/(4*a^(3/4)*b^(1/2)) - (a
tan((2^(1/2)*b^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*d - 2*2^(1/2)*b^(1/4)*c))/(
8*a^(3/4)*b^(1/2)) + (2^(1/2)*c*log((a^(1/2) + b^(1/2)*x^2 + 2^(1/2)*a^(1/4
)*b^(1/4)*x)/(a^(1/2) + b^(1/2)*x^2 - 2^(1/2)*a^(1/4)*b^(1/4)*x)))/(8*a^(3/
4)*b^(1/4)))
```

sympy [A] time = 1.03, size = 124, normalized size = 0.57

$$\mathrm{RootSum}\left(256t^4a^3b^2+32t^2a^2bd^2-16tabc^2d+ad^4+bc^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3bd^2-16t^2a^2bc^2d-8ta^2d^4-4tabc^4+5ac^2d^3}{4acd^4-bc^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*d
**4 + b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 - 16*_t**2*a**2
*b*c**2*d - 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4 - b
*c**5))))
```

3.70 $\int \frac{c+dx}{\left(a-bx^4\right)^2} \, dx$

Optimal. Leaf size=110

$$\frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx)}{4a\left(a-bx^4\right)}$$

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1855, 1876, 212, 208, 205, 275}

$$\frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx)}{4a\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] (x*(c + d*x))/(4*a*(a - b*x^4)) + (3*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7
/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(1/4)) + (d*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
```

```
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\left(a-bx^4\right)^2}\,dx &= \frac{x(c+dx)}{4a\left(a-bx^4\right)} - \frac{\int \frac{-3c-2dx}{a-bx^4}\,dx}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)} - \frac{\int \left(-\frac{3c}{a-bx^4} - \frac{2dx}{a-bx^4}\right)dx}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)} + \frac{(3c)\int \frac{1}{a-bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a-bx^4}\,dx}{2a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)} + \frac{(3c)\int \frac{1}{\sqrt{a}-\sqrt{b}\,x^2}\,dx}{8a^{3/2}} + \frac{(3c)\int \frac{1}{\sqrt{a}+\sqrt{b}\,x^2}\,dx}{8a^{3/2}} + \frac{d\,\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)} + \frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 168, normalized size = 1.53

$$\frac{\frac{4ax(c+dx)}{a-bx^4} - \frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c+2\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c-2\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{6\sqrt[4]{a}c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{2\sqrt{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{b}}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*x*(c + d*x))/(a - b*x^4) + (6*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/
b^(1/4) - ((3*a^(1/4)*b^(1/4)*c + 2*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sq
rt[b] + ((3*a^(1/4)*b^(1/4)*c - 2*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt
[b] + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{\left(a-bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a - b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.17, size = 254, normalized size = 2.31

$$\frac{3 \sqrt{2} \left(-a b^{3}\right)^{\frac{1}{4}} c \log\left(x^{2} + \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32 a^{2} b} - \frac{3 \sqrt{2} \left(-a b^{3}\right)^{\frac{1}{4}} c \log\left(x^{2} - \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32 a^{2} b} - \frac{d x^{2} + c x}{4 \left(b x^{4} - a\right) a} - \frac{\sqrt{2} \left(2 \sqrt{2} \sqrt{-a b} b d - 3 \left(-a b^{3}\right)^{\frac{1}{4}} b c\right) \arctan\left(\frac{\sqrt{2} \left(2 x + \sqrt{2} \left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 a^{2} b^{2}} - \frac{\sqrt{2} \left(2 \sqrt{2} \sqrt{-a b} b d - 3 \left(-a b^{3}\right)^{\frac{1}{4}} b c\right) \arctan\left(\frac{\sqrt{2} \left(2 x - \sqrt{2} \left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 a^{2} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a^2*b) - 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4)
+ sqrt(-a/b))/(a^2*b) - 1/4*(d*x^2 + c*x)/((b*x^4 - a)*a) - 1/16*sqrt(2)*(2
*sqrt(2)*sqrt(-a*b)*b*d - 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + s
qrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^2) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt
(-a*b)*b*d - 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/(a^2*b^2)
```

maple [A] time = 0.05, size = 142, normalized size = 1.29

$$-\frac{d x^{2}}{4 \left(b x^{4}-a\right) a}-\frac{c x}{4 \left(b x^{4}-a\right) a}-\frac{d \ln\left(\frac{\sqrt{a b}\, x^{2}-a}{-\sqrt{a b}\, x^{2}-a}\right)}{8 \sqrt{a b}\, a}+\frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} c \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 a^{2}}+\frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} c \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] -1/4*c*x/a/(b*x^4-a)+3/16*c/a^2*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))+3/8*c/a^2*(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)-1/4*d*x^2/a/(b*x^4-a)-1/
8*d/a/(a*b)^(1/2)*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))
```

maxima [A] time = 3.04, size = 157, normalized size = 1.43

$$-\frac{d x^{2} + c x}{4 \left(a b x^{4} - a^{2}\right)} + \frac{\frac{6 c \arctan\left(\frac{\sqrt{b} x}{\sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}} + \frac{2 d \log\left(\sqrt{b} x^{2} + \sqrt{a}\right)}{\sqrt{a} \sqrt{b}} - \frac{2 d \log\left(\sqrt{b} x^{2} - \sqrt{a}\right)}{\sqrt{a} \sqrt{b}} - \frac{3 c \log\left(\frac{\sqrt{b} x - \sqrt{\sqrt{a} \sqrt{b}}}{\sqrt{b} x + \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}}}{16 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(d*x^2 + c*x)/(a*b*x^4 - a^2) + 1/16*(6*c*arctan(sqrt(b)*x/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*d*log(sqrt(b)*x^2 + sqrt(a)
)/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 3*
c*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b)
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))))/a
```

mupad [B] time = 4.92, size = 283, normalized size = 2.57

$$\left(\sum_{k=1}^{4} \ln\left(-\frac{b^{2} \left(3 c d^{2} + 2 d^{3} x + \mathrm{root}\left(65536 a^{7} b^{2} z^{4} - 2048 a^{4} b d^{2} z^{2} + 1152 a^{2} b c^{2} d z - 81 b c^{4} + 16 a d^{4}, z, k\right)^{2} a^{3} b c 192 - \mathrm{root}\left(65536 a^{7} b^{2} z^{4} - 2048 a^{4} b d^{2} z^{2} + 1152 a^{2} b c^{2} d z - 81 b c^{4} + 16 a d^{4}, z, k\right)^{2} a^{3} b d x 128 + \mathrm{root}\left(65536 a^{7} b^{2} z^{4} - 2048 a^{4} b d^{2} z^{2} + 1152 a^{2} b c^{2} d z - 81 b c^{4} + 16 a d^{4}, z, k\right) a b c^{2} x 36\right)}{a^{3} 16}\right) \mathrm{root}\left(65536 a^{7} b^{2} z^{4} - 2048 a^{4} b d^{2} z^{2} + 1152 a^{2} b c^{2} d z - 81 b c^{4} + 16 a d^{4}, z, k\right)\right) + \frac{\frac{d x^{2}}{4 a} + \frac{c x}{4 a}}{a - b x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(-(b^2*(3*c*d^2 + 2*d^3*x + 192*root(65536*a^7*b^2*z^4 - 2048*a^4
*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c^4 + 16*a*d^4, z, k)^2*a^3*b*c - 12
8*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c
^4 + 16*a*d^4, z, k)^2*a^3*b*d*x + 36*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d
```

```
^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c^4 + 16*a*d^4, z, k)*a*b*c^2*x))/(16*a^
3))*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b
*c^4 + 16*a*d^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) + (c*x)/(4*a))/(a - b*x^4
)
```

sympy [A] time = 1.80, size = 156, normalized size = 1.42

$$\operatorname{RootSum}\left(65536t^4a^7b^2 - 2048t^2a^4bd^2 + 1152ta^2bc^2d + 16ad^4 - 81bc^4, \left(t \mapsto t\log\left(x + \frac{32768t^3a^6bd^2 + 4608t^2a^4bc^2d - 512ta^3d^4 + 1296ta^2bc^4 + 360ac^2d^3}{192acd^4 + 243bc^5}\right)\right)\right) + \frac{-cx - dx^2}{-4a^2 + 4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**2 - 2048*_t**2*a**4*b*d**2 + 1152*_t*a**2*b*c**
2*d + 16*a*d**4 - 81*b*c**4, Lambda(_t, _t*log(x + (32768*_t**3*a**6*b*d**2
 + 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 + 1296*_t*a**2*b*c**4 + 360*
a*c**2*d**3)/(192*a*c*d**4 + 243*b*c**5)))) + (-c*x - d*x**2)/(-4*a**2 + 4*
a*b*x**4)
```

3.71 $\int \frac{c+dx}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=241

$$-\frac{3c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3c\tan^{-1}}{8\sqrt{}}$$

Rubi [A] time = 0.20, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{3c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx)}{4a\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*b^(1/4)) - (3*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(16*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^4\right)^2}\,dx = \frac{x(c+dx)}{4a\left(a+bx^4\right)} - \frac{\int \frac{-3c-2dx}{a+bx^4}\,dx}{4a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} - \frac{\int \left(-\frac{3c}{a+bx^4} - \frac{2dx}{a+bx^4}\right)dx}{4a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{(3c)\int \frac{1}{a+bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{(3c)\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}} + \frac{(3c)\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}} + \frac{d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{(3c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx}{16a^{3/2}\sqrt{b}} + \frac{(3c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx}{16a^{3/2}\sqrt{b}} - \frac{(3c)}{}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x + \sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{3c\log\left(\sqrt{a} + \sqrt{2}\right.}{16\sqrt{2}\,a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{3c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}} - \frac{3c\log}{}$$

Mathematica [A] time = 0.28, size = 224, normalized size = 0.93

$$\frac{\frac{8a^{3/4}x(c+dx)}{a+bx^4} - \frac{2\left(4\sqrt[4]{a}\,d+3\sqrt{2}\sqrt[4]{b}\,c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{2\left(3\sqrt{2}\sqrt[4]{b}\,c-4\sqrt[4]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{\sqrt{b}} - \frac{3\sqrt{2}\,c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}\,c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt[4]{b}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(c + d*x))/(a + b*x^4) - (2*(3*Sqrt[2]*b^(1/4)*c + 4*a^(1/4)*
d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] + (2*(3*Sqrt[2]*b^(1/4)
*c - 4*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (3*Sqr
t[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (3
*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))
/(32*a^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.17, size = 238, normalized size = 0.99

$$\frac{3\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^{2}b}-\frac{3\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^{2}b}+\frac{dx^{2}+cx}{4\left(bx^{4}+a\right)a}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,bd+3\left(ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^{2}b^{2}}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,bd+3\left(ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a^2*b) - 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(a^2*b) + 1/4*(d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(
2)*sqrt(a*b)*b*d + 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b*d
+ 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^2*b^2)
```

maple [A] time = 0.05, size = 188, normalized size = 0.78

$$\frac{d\,x^{2}}{4\left(b\,x^{4}+a\right)a}+\frac{cx}{4\left(b\,x^{4}+a\right)a}+\frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{4\sqrt{ab}\;a}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;c\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] 1/4*c*x/a/(b*x^4+a)+3/32*c/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1
/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/16*c/a^2*(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/16*c/a^2*(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*d*x^2/a/(b*x^4+a)+1/4*d/a/(a*b)^(1/
2)*arctan((1/a*b)^(1/2)*x^2)
```

maxima [A] time = 2.93, size = 238, normalized size = 0.99

$$\frac{dx^{2}+cx}{4\left(abx^{4}+a^{2}\right)}+\frac{\frac{3\sqrt{2}c\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{3\sqrt{2}c\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-4\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c+4\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(d*x^2 + c*x)/(a*b*x^4 + a^2) + 1/32*(3*sqrt(2)*c*log(sqrt(b)*x^2 + sqr
t(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 3*sqrt(2)*c*log(sqrt(
b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(3*sqrt
(2)*a^(1/4)*b^(1/4)*c - 4*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt
(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*
b^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(1/4)*c + 4*sqrt(a)*d)*arctan(1/2*sqrt(2)
*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(b))*b^(1/4)))/a
```

mupad [B] time = 4.94, size = 282, normalized size = 1.17

$$\left(\sum_{k=1}^{4}\ln\left(\frac{b^{2}\left(3\,c\,d^{2}+2\,d^{3}\,x-\mathrm{root}\left(65536\,a^{7}\,b^{2}\,z^{4}+2048\,a^{4}\,b\,d^{2}\,z^{2}-1152\,a^{2}\,b\,c^{2}\,d\,z+81\,b\,c^{4}+16\,a\,d^{4},z,k\right)^{2}a^{3}\,b\,c\,192+\mathrm{root}\left(65536\,a^{7}\,b^{2}\,z^{4}+2048\,a^{4}\,b\,d^{2}\,z^{2}-1152\,a^{2}\,b\,c^{2}\,d\,z+81\,b\,c^{4}+16\,a\,d^{4},z,k\right)^{2}a^{3}\,b\,d\,x\,128-\mathrm{root}\left(65536\,a^{7}\,b^{2}\,z^{4}+2048\,a^{4}\,b\,d^{2}\,z^{2}-1152\,a^{2}\,b\,c^{2}\,d\,z+81\,b\,c^{4}+16\,a\,d^{4},z,k\right)\,a\,b\,c^{2}\,x\,36\right)}{a^{3}\,16}\right)\mathrm{root}\left(65536\,a^{7}\,b^{2}\,z^{4}+2048\,a^{4}\,b\,d^{2}\,z^{2}-1152\,a^{2}\,b\,c^{2}\,d\,z+81\,b\,c^{4}+16\,a\,d^{4},z,k\right)\right)+\frac{\frac{d\,x^{2}}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^{4}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((b^2*(3*c*d^2 + 2*d^3*x - 192*root(65536*a^7*b^2*z^4 + 2048*a^4*
b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^4 + 16*a*d^4, z, k)^2*a^3*b*c + 128
*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^
4 + 16*a*d^4, z, k)^2*a^3*b*d*x - 36*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^
2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^4 + 16*a*d^4, z, k)*a*b*c^2*x))/(16*a^3
))*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*
c^4 + 16*a*d^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) + (c*x)/(4*a))/(a + b*x^4)
```

sympy [A] time = 1.51, size = 155, normalized size = 0.64

$$\operatorname{RootSum}\left(65536t^4a^7b^2 + 2048t^2a^4bd^2 - 1152ta^2bc^2d + 16ad^4 + 81bc^4, \left(t \mapsto t\log\left(x + \frac{-32768t^3a^6bd^2 - 4608t^2a^4bc^2d - 512ta^3d^4 - 1296ta^2bc^4 + 360ac^2d^3}{192acd^4 - 243bc^5}\right)\right)\right) + \frac{cx + dx^2}{4a^2 + 4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**2 + 2048*_t**2*a**4*b*d**2 - 1152*_t*a**2*b*c**
2*d + 16*a*d**4 + 81*b*c**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*b*d**
2 - 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 - 1296*_t*a**2*b*c**4 + 360
*a*c**2*d**3)/(192*a*c*d**4 - 243*b*c**5)))) + (c*x + d*x**2)/(4*a**2 + 4*a
*b*x**4)
```

3.72 $\int \frac{c+dx}{\left(a-bx^4\right)^3}\,dx$

Optimal. Leaf size=136

$$\frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1855, 1876, 212, 208, 205, 275}

$$\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] (x*(c + d*x))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a - b*x^4))
+ (21*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (21*c*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \frac{x(c+dx)}{8a(a-bx^4)^2} - \frac{\int \frac{-7c-6dx}{(a-bx^4)^2}\,dx}{8a}$$

$$= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{\int \frac{21c+12dx}{a-bx^4}\,dx}{32a^2}$$

$$= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{\int \left(\frac{21c}{a-bx^4} + \frac{12dx}{a-bx^4}\right)dx}{32a^2}$$

$$= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{(21c)\int \frac{1}{a-bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}$$

$$= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{(21c)\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx}{64a^{5/2}} + \frac{(21c)\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx}{64a^{5/2}} + \frac{(3d)\,\mathrm{Subst}}{}$$

$$= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

Mathematica [A] time = 0.21, size = 193, normalized size = 1.42

$$\frac{\frac{16a^2x(c+dx)}{(a-bx^4)^2} + \frac{4ax(7c+6dx)}{a-bx^4} - \frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c+4\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c-4\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{42\sqrt[4]{a}c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{12\sqrt{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{b}}}{128a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] ((16*a^2*x*(c + d*x))/(a - b*x^4)^2 + (4*a*x*(7*c + 6*d*x))/(a - b*x^4) + (
42*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(1/4) - (3*(7*a^(1/4)*b^(1/4)*c
 + 4*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sqrt[b] + (3*(7*a^(1/4)*b^(1/4)*c
 - 4*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt[b] + (12*Sqrt[a]*d*Log[Sqrt[
a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a - b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.19, size = 272, normalized size = 2.00

$$\frac{21\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256a^{3}b}-\frac{21\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256a^{3}b}+\frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-ab}bd+7\left(-ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^{3}b^{2}}+\frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-ab}bd+7\left(-ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^{3}b^{2}}-\frac{6bdx^{6}+7bcx^{5}-10adx^{2}-11acx}{32\left(bx^{4}-a\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/
b))/(a^3*b) - 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1
/4) + sqrt(-a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d + 7*(-a
*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/(a^3*b^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d + 7*(-a*b^3)^(1/4)*
b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^2
) - 1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2
)
```

maple [A] time = 0.05, size = 180, normalized size = 1.32

$$\frac{d\,x^{2}}{8\left(b\,x^{4}-a\right)^{2}a}+\frac{cx}{8\left(b\,x^{4}-a\right)^{2}a}-\frac{3d\,x^{2}}{16\left(b\,x^{4}-a\right)a^{2}}-\frac{7cx}{32\left(b\,x^{4}-a\right)a^{2}}-\frac{3d\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{32\sqrt{ab}\,a^{2}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4-a)^2-7/32*c/a^2*x/(b*x^4-a)+21/128*c/a^3*(a/b)^(1/4)*ln((x
+(a/b)^(1/4))/(x-(a/b)^(1/4)))+21/64*c/a^3*(a/b)^(1/4)*arctan(1/(a/b)^(1/4)
*x)+1/8*d*x^2/a/(b*x^4-a)^2-3/16*d/a^2*x^2/(b*x^4-a)-3/32*d/a^2/(a*b)^(1/2)
*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))
```

maxima [A] time = 3.01, size = 186, normalized size = 1.37

$$-\frac{6\,bdx^{6}+7\,bcx^{5}-10\,adx^{2}-11\,acx}{32\left(a^{2}b^{2}x^{8}-2\,a^{3}bx^{4}+a^{4}\right)}+\frac{3\left(\frac{14\,c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{4\,d\log\left(\sqrt{b}x^{2}+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{4\,d\log\left(\sqrt{b}x^{2}-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{7\,c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right)}{128\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/(a^2*b^2*x^8 - 2*a^3*
b*x^4 + a^4) + 3/128*(14*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(b))) + 4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b))
- 4*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 7*c*log((sqrt(b)*x - s
qrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))))/a^2
```

mupad [B] time = 4.98, size = 315, normalized size = 2.32

$$\frac{\frac{5\,d\,x^2}{16\,a}+\frac{11\,c\,x}{32\,a}-\frac{7\,b\,c\,x^5}{32\,a^2}-\frac{3\,b\,d\,x^6}{16\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8}+\left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\,\left(63\,c\,d^2+36\,d^3\,x+\mathrm{root}\left(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k\right)^2\,a^5\,b\,c\,7168+\mathrm{root}\left(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k\right)\,a^2\,b\,c^2\,x\,1176-\mathrm{root}\left(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k\right)^2\,a^5\,b\,d\,x\,4096\right)\,3}{a^6\,2048}\right)\,\mathrm{root}\left(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^2) - (3*b*d*x^6)/(1
6*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(-(3*b^2*(63*c*d^2 + 36*d^3
*x + 7168*root(268435456*a^11*b^2*z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*a^3
*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4, z, k)^2*a^5*b*c + 1176*root(268435
456*a^11*b^2*z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b
*c^4 + 20736*a*d^4, z, k)*a^2*b*c^2*x - 4096*root(268435456*a^11*b^2*z^4 -
4718592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4,
 z, k)^2*a^5*b*d*x))/(2048*a^6))*root(268435456*a^11*b^2*z^4 - 4718592*a^6*
b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4, z, k), k, 1
, 4)
```

sympy [A] time = 1.97, size = 194, normalized size = 1.43

$$-\operatorname{RootSum}\left(268435456t^4a^{11}b^2-4718592t^2a^6bd^2-2709504ta^3bc^2d+20736ad^4-194481bc^4,\left(t\mapsto t\log\left(x+\frac{-67108864t^3a^9bd^2+9633792t^2a^6bc^2d+589824ta^4d^4-2765952ta^3bc^4+423360ac^2d^3}{193536acd^4+453789bc^5}\right)\right)\right)-\frac{-11acx-10adx^2+7bcx^5+6bdx^6}{32a^4-64a^3bx^4+32a^2b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**2 - 4718592*_t**2*a**6*b*d**2 - 2709504*_
t*a**3*b*c**2*d + 20736*a*d**4 - 194481*b*c**4, Lambda(_t, _t*log(x + (-671
08864*_t**3*a**9*b*d**2 + 9633792*_t**2*a**6*b*c**2*d + 589824*_t*a**4*d**4
 - 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 + 453789*b
*c**5)))) - (-11*a*c*x - 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 -
64*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

3.73 $\int \frac{c+dx}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=266

$$-\frac{21c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21c\tan^{-1}}{64\sqrt{}}$$

Rubi [A] time = 0.23, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}-\frac{21c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] (x*(c + d*x))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a + b*x^4))
+ (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - (21*c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) + (21*c*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) - (21*c*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/
4)*b^(1/4)) + (21*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(128*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^4\right)^3}\,dx = \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} - \frac{\int \frac{-7c-6dx}{\left(a+bx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{\int \frac{21c+12dx}{a+bx^4}\,dx}{32a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{\int \left(\frac{21c}{a+bx^4} + \frac{12dx}{a+bx^4}\right)dx}{32a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{(21c)\int \frac{1}{a+bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a+bx^4}\,dx}{8a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{(21c)\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{64a^{5/2}} + \frac{(21c)\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{64a^{5/2}} + \frac{(3d)\,\mathrm{Subst}\Big(\int}{1\ldots}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{(21c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{128a^{5/2}\sqrt{b}} + \frac{(21c)\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+\ldots}}{128a^{5\ldots}}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{21c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{b}} + \ldots$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}} + \frac{21c\tan^{-1}\Big(1+\ldots}{64\sqrt{2}\,a^{11\ldots}}$$

Mathematica [A] time = 0.26, size = 249, normalized size = 0.94

$$\frac{\frac{32a^{7/4}x(c+dx)}{\left(a+bx^4\right)^2} + \frac{8a^{3/4}x(7c+6dx)}{a+bx^4} - \frac{6\left(8\sqrt[4]{a}\,d+7\sqrt{2}\sqrt[4]{b}\,c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6\left(7\sqrt{2}\sqrt[4]{b}\,c-8\sqrt[4]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{\sqrt{b}} - \frac{21\sqrt{2}\,c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt[4]{b}} + \frac{21\sqrt{2}\,c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt[4]{b}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(c + d*x))/(a + b*x^4)^2 + (8*a^(3/4)*x*(7*c + 6*d*x))/(a +
b*x^4) - (6*(7*Sqrt[2]*b^(1/4)*c + 8*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/Sqrt[b] + (6*(7*Sqrt[2]*b^(1/4)*c - 8*a^(1/4)*d)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (21*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (21*Sqrt[2]*c*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(256*a^(11/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{\left(a+bx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.18, size = 256, normalized size = 0.96

$$\frac{21\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256a^{3}b}-\frac{21\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256a^{3}b}+\frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7\left(ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^{3}b^{2}}+\frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7\left(ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^{3}b^{2}}+\frac{6bdx^{6}+7bcx^{5}+10adx^{2}+11acx}{32\left(bx^{4}+a\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(a^3*b) - 21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1
/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b
^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1/4)*b*c)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^2) + 1/32*(6*b*
d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/((b*x^4 + a)^2*a^2)
```

maple [A] time = 0.05, size = 222, normalized size = 0.83

$$\frac{dx^{2}}{8\left(bx^{4}+a\right)^{2}a}+\frac{cx}{8\left(bx^{4}+a\right)^{2}a}+\frac{3dx^{2}}{16\left(bx^{4}+a\right)a^{2}}+\frac{7cx}{32\left(bx^{4}+a\right)a^{2}}+\frac{3d\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{16\sqrt{ab}\,a^{2}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*c/a^3*(a/b)^(1/4)*2^(1/
2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a
/b)^(1/2)))+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1
)+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/8*d*x^
2/a/(b*x^4+a)^2+3/16*d/a^2*x^2/(b*x^4+a)+3/16*d/a^2/(a*b)^(1/2)*arctan((1/a
*b)^(1/2)*x^2)
```

maxima [A] time = 3.06, size = 269, normalized size = 1.01

$$\frac{6bdx^{6}+7bcx^{5}+10adx^{2}+11acx}{32\left(a^{2}b^{2}x^{8}+2a^{3}bx^{4}+a^{4}\right)}+\frac{3\left(\frac{7\sqrt{2}c\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{7\sqrt{2}c\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\left(7\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-8\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}+\frac{2\left(7\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c+8\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}\right)}{256a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(6*b*d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/(a^2*b^2*x^8 + 2*a^3*b
*x^4 + a^4) + 3/256*(7*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*
x + sqrt(a))/(a^(3/4)*b^(1/4)) - 7*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*b^(1/4)*c
 - 8*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/
sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 2*(7*sqrt(
```

```
2)*a^(1/4)*b^(1/4)*c + 8*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b
^(1/4)))/a^2
```

mupad [B] time = 4.99, size = 315, normalized size = 1.18

$$\frac{\frac{5\,d\,x^2}{16\,a}+\frac{11\,c\,x}{32\,a}+\frac{7\,b\,c\,x^5}{32\,a^2}+\frac{3\,b\,d\,x^6}{16\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8}+\left(\sum_{k=1}^{4}\ln\left(\frac{b^2\,\left(63\,c\,d^2+36\,d^3\,x-\mathrm{root}\left(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k\right)^2\,a^5\,b\,c\,7168-\mathrm{root}\left(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k\right)\,a^2\,b\,c^2\,x\,1176+\mathrm{root}\left(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k\right)^2\,a^5\,b\,d\,x\,4096\right)\,3}{a^6\,2048}\right)\,\mathrm{root}\left(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^2) + (3*b*d*x^6)/(1
6*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log((3*b^2*(63*c*d^2 + 36*d^3*
x - 7168*root(268435456*a^11*b^2*z^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a^3*
b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4, z, k)^2*a^5*b*c - 1176*root(2684354
56*a^11*b^2*z^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*
c^4 + 20736*a*d^4, z, k)*a^2*b*c^2*x + 4096*root(268435456*a^11*b^2*z^4 + 4
718592*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4,
z, k)^2*a^5*b*d*x))/(2048*a^6))*root(268435456*a^11*b^2*z^4 + 4718592*a^6*b
*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4, z, k), k, 1,
 4)
```

sympy [A] time = 1.99, size = 192, normalized size = 0.72

$$\operatorname{RootSum}{\left(268435456 t^{4} a^{11} b^{2} + 4718592 t^{2} a^{6} b d^{2} - 2709504 t a^{3} b c^{2} d + 20736 a d^{4} + 194481 b c^{4}, \left(t \mapsto t \log{\left(x + \frac{- 67108864 t^{3} a^{9} b d^{2} - 9633792 t^{2} a^{6} b c^{2} d - 589824 t a^{4} d^{4} - 2765952 t a^{3} b c^{4} + 423360 a c^{2} d^{3}}{193536 a c d^{4} - 453789 b c^{5}} \right)} \right)\right)} + \frac{11 a c x + 10 a d x^{2} + 7 b c x^{5} + 6 b d x^{6}}{32 a^{4} + 64 a^{3} b x^{4} + 32 a^{2} b^{2} x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**2 + 4718592*_t**2*a**6*b*d**2 - 2709504*_t
*a**3*b*c**2*d + 20736*a*d**4 + 194481*b*c**4, Lambda(_t, _t*log(x + (-6710
8864*_t**3*a**9*b*d**2 - 9633792*_t**2*a**6*b*c**2*d - 589824*_t*a**4*d**4
- 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 - 453789*b*
c**5)))) + (11*a*c*x + 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 + 64
*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

3.74 $\int \frac{c+dx}{\left(a-bx^4\right)^4}\,dx$

Optimal. Leaf size=162

$$\frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(c+dx)}{12a\left(a-bx^4\right)^3}$$

Rubi [A] time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1855, 1876, 212, 208, 205, 275}

$$\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(c+dx)}{12a\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4)^4,x]
```

```
[Out] (x*(c + d*x))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a - b*x^4
)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a - b*x^4)) + (77*c*ArcTan[(b^(1/4)*x)
/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (77*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(25
6*a^(15/4)*b^(1/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt
[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\left(a-bx^4\right)^4}\,dx &= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}-\frac{\int \frac{-11c-10dx}{\left(a-bx^4\right)^3}\,dx}{12a}\\
&= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{\int \frac{77c+60dx}{\left(a-bx^4\right)^2}\,dx}{96a^2}\\
&= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}-\frac{\int \frac{-231c-120dx}{a-bx^4}\,dx}{384a^3}\\
&= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}-\frac{\int \left(-\frac{231c}{a-bx^4}-\frac{120dx}{a-bx^4}\right)dx}{384a^3}\\
&= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{(77c)\int \frac{1}{a-bx^4}\,dx}{128a^3}+\frac{(5d)\int \frac{x}{a-bx^4}\,dx}{16a^3}\\
&= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{(77c)\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx}{256a^{7/2}}+\frac{(77c)\int \frac{1}{\sqrt{a}+\sqrt{b}x}}{256a^{7/2}}\\
&= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 217, normalized size = 1.34

$$\frac{\frac{128a^3x(c+dx)}{\left(a-bx^4\right)^3}+\frac{16a^2x(11c+10dx)}{\left(a-bx^4\right)^2}+\frac{4ax(77c+60dx)}{a-bx^4}-\frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}c+40\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}c-40\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{462\sqrt[4]{a}c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{120\sqrt{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{b}}}{1536a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4)^4,x]
```

```
[Out] ((128*a^3*x*(c + d*x))/(a - b*x^4)^3 + (16*a^2*x*(11*c + 10*d*x))/(a - b*x^
4)^2 + (4*a*x*(77*c + 60*d*x))/(a - b*x^4) + (462*a^(1/4)*c*ArcTan[(b^(1/4)
*x)/a^(1/4)])/b^(1/4) - (3*(77*a^(1/4)*b^(1/4)*c + 40*Sqrt[a]*d)*Log[a^(1/4
) - b^(1/4)*x])/Sqrt[b] + (3*(77*a^(1/4)*b^(1/4)*c - 40*Sqrt[a]*d)*Log[a^(1
/4) + b^(1/4)*x])/Sqrt[b] + (120*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt
[b])/(1536*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{\left(a-bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a - b*x^4)^4,x]
[Out] IntegrateAlgebraic[(c + d*x)/(a - b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
[Out] Timed out
```

giac [B] time = 0.28, size = 296, normalized size = 1.83

$$\frac{77\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024a^{4}b}-\frac{77\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024a^{4}b}-\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}bd-77\left(-ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^{4}b^{2}}-\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}bd-77\left(-ab^{3}\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^{4}b^{2}}-\frac{60b^{2}dx^{10}+77b^{2}cx^{9}-160abdx^{6}-198abcx^{5}+132a^{2}dx^{2}+153a^{2}cx}{384\left(bx^{4}-a\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
[Out] 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/(a^4*b) - 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/(a^4*b) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d - 77
*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)
^(1/4))/(a^4*b^2) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d - 77*(-a*b^3)^
(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a
^4*b^2) - 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*c*x
^5 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

maple [A] time = 0.06, size = 177, normalized size = 1.09

$$-\frac{5d\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{64\sqrt{ab}\,a^{3}}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256a^{4}}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^{4}}+\frac{-\frac{5b^{2}d\,x^{10}}{32a^{3}}-\frac{77b^{2}c\,x^{9}}{384a^{3}}+\frac{5bd\,x^{6}}{12a^{2}}+\frac{33bc\,x^{5}}{64a^{2}}-\frac{11d\,x^{2}}{32a}-\frac{51cx}{128a}}{\left(b\,x^{4}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^4,x)
[Out] (-5/32*d/a^3*b^2*x^10-77/384*c/a^3*b^2*x^9+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x
^5-11/32*d/a*x^2-51/128*c/a*x)/(b*x^4-a)^3+77/512/a^4*c*(a/b)^(1/4)*ln((x+(
a/b)^(1/4))/(x-(a/b)^(1/4)))+77/256/a^4*c*(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*
x)-5/64/a^3*d/(a*b)^(1/2)*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))
```

maxima [A] time = 2.97, size = 223, normalized size = 1.38

$$-\frac{60b^{2}dx^{10}+77b^{2}cx^{9}-160abdx^{6}-198abcx^{5}+132a^{2}dx^{2}+153a^{2}cx}{384\left(a^{3}b^{3}x^{12}-3a^{4}b^{2}x^{8}+3a^{5}bx^{4}-a^{6}\right)}+\frac{\frac{154c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{40d\log\left(\sqrt{b}x^{2}+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40d\log\left(\sqrt{b}x^{2}-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{77c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{512a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
[Out] -1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 132*
a^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 - 3*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^6)
 + 1/512*(154*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + 40*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log
(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 77*c*log((sqrt(b)*x - sqrt(sqrt
```

```
(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(b))))/a^3
```

mupad [B] time = 4.97, size = 351, normalized size = 2.17

$$\left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\left(1925\,c\,d^2+1000\,d^3\,x+\mathrm{root}\left(68719476736\,a^{15}\,b^2\,z^4-838860800\,a^8\,b\,d^2\,z^2+485703680\,a^4\,b\,c^2\,d\,z-35153041\,b\,c^4+2560000\,a\,d^4,z,k\right)^2\,a^7\,b\,c\,315392+\mathrm{root}\left(68719476736\,a^{15}\,b^2\,z^4-838860800\,a^8\,b\,d^2\,z^2+485703680\,a^4\,b\,c^2\,d\,z-35153041\,b\,c^4+2560000\,a\,d^4,z,k\right)\,a^3\,b\,c^2\,x\,47432-\mathrm{root}\left(68719476736\,a^{15}\,b^2\,z^4-838860800\,a^8\,b\,d^2\,z^2+485703680\,a^4\,b\,c^2\,d\,z-35153041\,b\,c^4+2560000\,a\,d^4,z,k\right)^2\,a^7\,b\,d\,x\,163840\right)}{a^9\,32768}\right)\mathrm{root}\left(68719476736\,a^{15}\,b^2\,z^4-838860800\,a^8\,b\,d^2\,z^2+485703680\,a^4\,b\,c^2\,d\,z-35153041\,b\,c^4+2560000\,a\,d^4,z,k\right)\right)+\frac{\frac{11\,d\,x^2}{32\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}-\frac{33\,b\,c\,x^5}{64\,a^2}-\frac{5\,b\,d\,x^6}{12\,a^2}}{a^3-3\,a^2\,b\,x^4+3\,a\,b^2\,x^8-b^3\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(-(b^2*(1925*c*d^2 + 1000*d^3*x + 315392*root(68719476736*a^15*b^
2*z^4 - 838860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4
+ 2560000*a*d^4, z, k)^2*a^7*b*c + 47432*root(68719476736*a^15*b^2*z^4 - 83
8860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*
a*d^4, z, k)*a^3*b*c^2*x - 163840*root(68719476736*a^15*b^2*z^4 - 838860800
*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*a*d^4,
z, k)^2*a^7*b*d*x))/(32768*a^9))*root(68719476736*a^15*b^2*z^4 - 838860800*
a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*a*d^4, z
, k), k, 1, 4) + ((11*d*x^2)/(32*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(38
4*a^3) + (5*b^2*d*x^10)/(32*a^3) - (33*b*c*x^5)/(64*a^2) - (5*b*d*x^6)/(12*
a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

sympy [A] time = 2.06, size = 231, normalized size = 1.43

$$\operatorname{RootSum}\left(68719476736t^4a^{15}b^2-838860800t^2a^8bd^2+485703680ta^4bc^2d+2560000ad^4-35153041bc^4,\left(t\mapsto t\log\left(x+\frac{429496729600t^3a^{12}bd^2+62170071040t^2a^8bc^2d-2621440000ta^5d^4+17998356992ta^4bc^4+1897280000ac^2d^3}{788480000acd^4+2706784157bc^5}\right)\right)\right)+\frac{-153a^2cx-132a^2dx^2+198abcx^5+160abdx^6-77b^2cx^9-60b^2dx^{10}}{-384a^6+1152a^5bx^4-1152a^4b^2x^8+384a^3b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**2 - 838860800*_t**2*a**8*b*d**2 + 485703
680*_t*a**4*b*c**2*d + 2560000*a*d**4 - 35153041*b*c**4, Lambda(_t, _t*log(
x + (429496729600*_t**3*a**12*b*d**2 + 62170071040*_t**2*a**8*b*c**2*d - 26
21440000*_t*a**5*d**4 + 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*d**3
)/(788480000*a*c*d**4 + 2706784157*b*c**5)))) + (-153*a**2*c*x - 132*a**2*d
*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 - 77*b**2*c*x**9 - 60*b**2*d*x**10)
/(-384*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

3.75 $\int \frac{c+dx}{\left(a+bx^4\right)^4} \, dx$

Optimal. Leaf size=291

$$-\frac{77c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{77c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{77c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{77c\,\mathrm{t}}{2}$$

Rubi [A] time = 0.27, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2}-\frac{77c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{77c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{77c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{77c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4)^4,x]
```

```
[Out] (x*(c + d*x))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a + b*x^4
)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (77*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b
^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^4\right)^4}\,dx = \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} - \frac{\int \frac{-11c-10dx}{\left(a+bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{\int \frac{77c+60dx}{\left(a+bx^4\right)^2}\,dx}{96a^2}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} - \frac{\int \frac{-231c-120dx}{a+bx^4}\,dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} - \frac{\int \left(-\frac{231c}{a+bx^4} - \frac{120dx}{a+bx^4}\right)dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{(77c)\int \frac{1}{a+bx^4}\,dx}{128a^3} + \frac{(5d)\int \frac{x}{a+bx^4}\,dx}{16a^3}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{(77c)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{256a^{7/2}} + \frac{(77c)\int \frac{\sqrt{a}+ \cdots}{a+ \cdots}}{256a^{7 \cdots}}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{(77c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\cdots}{\sqrt[4]{\cdots}}}}{512a^{7/2}\sqrt{\cdots}}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\log\left(\sqrt{a} - \cdots\right.}{512\sqrt{\cdots}}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\tan^{-1}\left(1 - \cdots\right.}{256\sqrt{2}\,a^{15/4}\cdots}$$

Mathematica [A] time = 0.34, size = 274, normalized size = 0.94

$$\frac{\frac{256a^{11/4}x(c+dx)}{\left(a+bx^4\right)^3} + \frac{32a^{7/4}x(11c+10dx)}{\left(a+bx^4\right)^2} + \frac{8a^{3/4}x(77c+60dx)}{a+bx^4} - \frac{6\left(80\sqrt[4]{a}d+77\sqrt{2}\sqrt[4]{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6\left(77\sqrt{2}\sqrt[4]{b}c-80\sqrt[4]{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{b}} - \frac{231\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{b}} + \frac{231\sqrt{2}c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{b}}}{3072a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4)^4,x]
```

```
[Out] ((256*a^(11/4)*x*(c + d*x))/(a + b*x^4)^3 + (32*a^(7/4)*x*(11*c + 10*d*x))/
(a + b*x^4)^2 + (8*a^(3/4)*x*(77*c + 60*d*x))/(a + b*x^4) - (6*(77*Sqrt[2]*
b^(1/4)*c + 80*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b]
+ (6*(77*Sqrt[2]*b^(1/4)*c - 80*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/Sqrt[b] - (231*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2])/b^(1/4) + (231*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/b^(1/4))/(3072*a^(15/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{\left(a+bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.18, size = 280, normalized size = 0.96

$$\frac{77\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b}-\frac{77\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,bd+77\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,bd+77\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2}+\frac{60\,b^2dx^{10}+77\,b^2cx^9+160\,abdx^6+198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\left(bx^4+a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^4*b) - 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
 + sqrt(a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3
)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^4*b^2) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3)^(1/4)*b*c)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^2) + 1/38
4*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*a^2*d
*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

maple [A] time = 0.07, size = 225, normalized size = 0.77

$$\frac{5d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{32\sqrt{ab}\,a^3}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{512a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{512a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024a^4}+\frac{\frac{5b^2d\,x^{10}}{32a^3}+\frac{77b^2c\,x^9}{384a^3}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{\left(b\,x^4+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/32/a^3*b^2*d*x^10+77/384/a^3*b^2*c*x^9+5/12/a^2*b*d*x^6+33/64/a^2*b*c*x^
5+11/32/a*d*x^2+51/128/a*c*x)/(b*x^4+a)^3+77/1024/a^4*c*(a/b)^(1/4)*2^(1/2)
*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2)))+77/512/a^4*c*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+
77/512/a^4*c*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+5/32/a^3*d
/(a*b)^(1/2)*arctan((1/a*b)^(1/2)*x^2)
```

maxima [A] time = 3.20, size = 304, normalized size = 1.04

$$\frac{60\,b^2dx^{10}+77\,b^2cx^9+160\,abdx^6+198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\left(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6\right)}+\frac{\frac{77\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{77\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-80\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c+80\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{1}{4}}}}{1024\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*a
^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6)
+ 1/1024*(77*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
```

```
))/(a^(3/4)*b^(1/4)) - 77*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(1/4)*c - 80*sq
rt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 2*(77*sqrt(2)*a^(
1/4)*b^(1/4)*c + 80*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^
(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4
)))/a^3
```

mupad [B] time = 0.31, size = 350, normalized size = 1.20

$$\left(\sum_{k=1}^{4}\ln\left(\frac{b^2\left(1925\,c\,d^2+1000\,d^3\,x-\mathrm{root}\left(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k\right)^2\,a^7\,b\,c\,315392-\mathrm{root}\left(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k\right)\,a^3\,b\,c^2\,x\,47432+\mathrm{root}\left(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k\right)^2\,a^7\,b\,d\,x\,163840\right)}{a^9\,32768}\right)\mathrm{root}\left(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k\right)\right)+\frac{\frac{11\,d\,x^2}{32\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{33\,b\,c\,x^5}{64\,a^2}+\frac{5\,b\,d\,x^6}{12\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log((b^2*(1925*c*d^2 + 1000*d^3*x - 315392*root(68719476736*a^15*b^2
*z^4 + 838860800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 +
 2560000*a*d^4, z, k)^2*a^7*b*c - 47432*root(68719476736*a^15*b^2*z^4 + 838
860800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a
*d^4, z, k)*a^3*b*c^2*x + 163840*root(68719476736*a^15*b^2*z^4 + 838860800*
a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a*d^4, z
, k)^2*a^7*b*d*x))/(32768*a^9))*root(68719476736*a^15*b^2*z^4 + 838860800*a
^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a*d^4, z,
 k), k, 1, 4) + ((11*d*x^2)/(32*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(384
*a^3) + (5*b^2*d*x^10)/(32*a^3) + (33*b*c*x^5)/(64*a^2) + (5*b*d*x^6)/(12*a
^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

sympy [A] time = 1.81, size = 231, normalized size = 0.79

$$\operatorname{RootSum}{\left(68719476736 t^{4} a^{15} b^{2} + 838860800 t^{2} a^{8} b d^{2} - 485703680 t a^{4} b c^{2} d + 2560000 a d^{4} + 35153041 b c^{4}, \left(t \mapsto t \log{\left(x + \frac{- 429496729600 t^{3} a^{12} b d^{2} - 62170071040 t^{2} a^{8} b c^{2} d - 2621440000 t a^{5} d^{4} - 17998356992 t a^{4} b c^{4} + 1897280000 a c^{2} d^{3}}{788480000 a c d^{4} - 2706784157 b c^{5}} \right)} \right)\right)} + \frac{153 a^{2} c x + 132 a^{2} d x^{2} + 198 a b c x^{5} + 160 a b d x^{6} + 77 b^{2} c x^{9} + 60 b^{2} d x^{10}}{384 a^{6} + 1152 a^{5} b x^{4} + 1152 a^{4} b^{2} x^{8} + 384 a^{3} b^{3} x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**2 + 838860800*_t**2*a**8*b*d**2 - 485703
680*_t*a**4*b*c**2*d + 2560000*a*d**4 + 35153041*b*c**4, Lambda(_t, _t*log(
x + (-429496729600*_t**3*a**12*b*d**2 - 62170071040*_t**2*a**8*b*c**2*d - 2
621440000*_t*a**5*d**4 - 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*d**
3)/(788480000*a*c*d**4 - 2706784157*b*c**5)))) + (153*a**2*c*x + 132*a**2*d
*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*x**10)
/(384*a**6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

3.76 $\int \frac{c+dx}{1-x^4}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}c\tan^{-1}(x)+\frac{1}{2}c\tanh^{-1}(x)+\frac{1}{2}d\tanh^{-1}\left(x^2\right)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1876, 212, 206, 203, 275}

$$\frac{1}{2}c\tan^{-1}(x)+\frac{1}{2}c\tanh^{-1}(x)+\frac{1}{2}d\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(1 - x^4),x]
```

```
[Out] (c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{1-x^4}\,dx &= \int \left(\frac{c}{1-x^4}+\frac{dx}{1-x^4}\right)dx \\
&= c\int \frac{1}{1-x^4}\,dx + d\int \frac{x}{1-x^4}\,dx \\
&= \frac{1}{2}c\int \frac{1}{1-x^2}\,dx + \frac{1}{2}c\int \frac{1}{1+x^2}\,dx + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}c\tan^{-1}(x) + \frac{1}{2}c\tanh^{-1}(x) + \frac{1}{2}d\tanh^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.02, size = 42, normalized size = 1.75

$$\frac{1}{4}\left(-(c+d)\log(1-x)+c\log(x+1)+2c\tan^{-1}(x)+d\log\left(x^2+1\right)-d\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(1 - x^4),x]
```

```
[Out] (2*c*ArcTan[x] - (c + d)*Log[1 - x] + c*Log[1 + x] - d*Log[1 + x] + d*Log[1
 + x^2])/4
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{1-x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(1 - x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(1 - x^4), x]
```

fricas [**A**] time = 0.41, size = 35, normalized size = 1.46

$$\frac{1}{2}c\arctan(x)+\frac{1}{4}d\log\left(x^2+1\right)+\frac{1}{4}(c-d)\log(x+1)-\frac{1}{4}(c+d)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d)
*log(x - 1)
```

giac [**B**] time = 0.15, size = 37, normalized size = 1.54

$$\frac{1}{2}c\arctan(x)+\frac{1}{4}d\log\left(x^2+1\right)+\frac{1}{4}(c-d)\log(|x+1|)-\frac{1}{4}(c+d)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(abs(x + 1)) - 1/4*(c
 + d)*log(abs(x - 1))
```

maple [**B**] time = 0.04, size = 44, normalized size = 1.83

$$\frac{c\arctan(x)}{2}-\frac{c\ln(x-1)}{4}+\frac{c\ln(x+1)}{4}-\frac{d\ln(x-1)}{4}-\frac{d\ln(x+1)}{4}+\frac{d\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-x^4+1),x)
```

```
[Out] -1/4*c*ln(x-1)-1/4*ln(x-1)*d+1/4*ln(x+1)*c-1/4*ln(x+1)*d+1/4*d*ln(x^2+1)+1/
2*c*arctan(x)
```

maxima [A] time = 3.04, size = 35, normalized size = 1.46

$$\frac{1}{2}\,c\arctan(x)+\frac{1}{4}\,d\log\left(x^2+1\right)+\frac{1}{4}\,(c-d)\log\left(x+1\right)-\frac{1}{4}\,(c+d)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d)
*log(x - 1)
```

mupad [B] time = 4.92, size = 100, normalized size = 4.17

$$-\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{3/4}\,\sqrt{2}\,x+1\right)\left(\sqrt{2}\,c+2\,(-1)^{1/4}\,d\right)}{4}-\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{3/4}\,\sqrt{2}\,x-1\right)\left(2\,\sqrt{2}\,c-4\,(-1)^{1/4}\,d\right)}{8}+\frac{(-1)^{1/4}\,\sqrt{2}\,c\,\ln\left(\frac{x^2+(-1)^{1/4}\,\sqrt{2}\,x+1\mathrm{i}}{x^2-(-1)^{1/4}\,\sqrt{2}\,x+1\mathrm{i}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(c + d*x)/(x^4 - 1),x)
```

```
[Out] ((-1)^(1/4)*2^(1/2)*c*log((x^2 + (-1)^(1/4)*2^(1/2)*x + 1i)/(x^2 - (-1)^(1/
4)*2^(1/2)*x + 1i)))/8 - ((-1)^(1/4)*atan((-1)^(3/4)*2^(1/2)*x - 1)*(2*2^(1
/2)*c - 4*(-1)^(1/4)*d))/8 - ((-1)^(1/4)*atan((-1)^(3/4)*2^(1/2)*x + 1)*(2^
(1/2)*c + 2*(-1)^(1/4)*d))/4
```

sympy [C] time = 0.92, size = 313, normalized size = 13.04

$$\frac{(c-d)\log\left(x+\frac{c^4(c-d)+5c^2d^3+c^2d(c-d)^2-2d^4(c-d)+2d^2(c-d)^3}{c^5+4cd^4}\right)}{4}-\frac{(c+d)\log\left(x+\frac{-c^4(c+d)+5c^2d^3+c^2d(c+d)^2+2d^4(c+d)-2d^2(c+d)^3}{c^5+4cd^4}\right)}{4}-\left(-\frac{ic}{4}-\frac{d}{4}\right)\log\left(x+\frac{-4c^4\left(-\frac{ic}{4}-\frac{d}{4}\right)+5c^2d^3+16c^2d\left(-\frac{ic}{4}-\frac{d}{4}\right)^2+8d^4\left(-\frac{ic}{4}-\frac{d}{4}\right)-128d^2\left(-\frac{ic}{4}-\frac{d}{4}\right)^3}{c^5+4cd^4}\right)-\left(\frac{ic}{4}-\frac{d}{4}\right)\log\left(x+\frac{-4c^4\left(\frac{ic}{4}-\frac{d}{4}\right)+5c^2d^3+16c^2d\left(\frac{ic}{4}-\frac{d}{4}\right)^2+8d^4\left(\frac{ic}{4}-\frac{d}{4}\right)-128d^2\left(\frac{ic}{4}-\frac{d}{4}\right)^3}{c^5+4cd^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x**4+1),x)
```

```
[Out] (c - d)*log(x + (c**4*(c - d) + 5*c**2*d**3 + c**2*d*(c - d)**2 - 2*d**4*(c
 - d) + 2*d**2*(c - d)**3)/(c**5 + 4*c*d**4))/4 - (c + d)*log(x + (-c**4*(c
 + d) + 5*c**2*d**3 + c**2*d*(c + d)**2 + 2*d**4*(c + d) - 2*d**2*(c + d)**
3)/(c**5 + 4*c*d**4))/4 - (-I*c/4 - d/4)*log(x + (-4*c**4*(-I*c/4 - d/4) +
5*c**2*d**3 + 16*c**2*d*(-I*c/4 - d/4)**2 + 8*d**4*(-I*c/4 - d/4) - 128*d**
2*(-I*c/4 - d/4)**3)/(c**5 + 4*c*d**4)) - (I*c/4 - d/4)*log(x + (-4*c**4*(I
*c/4 - d/4) + 5*c**2*d**3 + 16*c**2*d*(I*c/4 - d/4)**2 + 8*d**4*(I*c/4 - d/
4) - 128*d**2*(I*c/4 - d/4)**3)/(c**5 + 4*c*d**4))
```

3.77 $\int \frac{c+dx}{1+x^4} dx$

Optimal. Leaf size=98

$$-\frac{c\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{c\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{c\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{c\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}+\frac{1}{2}d\tan^{-1}\left(x^2\right)$$

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 203}

$$-\frac{c\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{c\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{c\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{c\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}+\frac{1}{2}d\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(1 + x^4),x]
```

```
[Out] (d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*ArcTan[1 + S
qrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(4*Sqrt[2]) + (c*Log[
1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{1+x^4}\,dx &= \int \left(\frac{c}{1+x^4}+\frac{dx}{1+x^4}\right)dx \\
&= c\int \frac{1}{1+x^4}\,dx + d\int \frac{x}{1+x^4}\,dx \\
&= \frac{1}{2}c\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{2}c\int \frac{1+x^2}{1+x^4}\,dx + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}d\tan^{-1}\left(x^2\right) + \frac{1}{4}c\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{4}c\int \frac{1}{1+\sqrt{2}x+x^2}\,dx - \frac{c\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} - \cdots \\
&= \frac{1}{2}d\tan^{-1}\left(x^2\right) - \frac{c\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{c\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{c\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{2\sqrt{2}} \cdots \\
&= \frac{1}{2}d\tan^{-1}\left(x^2\right) - \frac{c\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{c\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{c\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{c\log\left(1+\sqrt{2}\cdots\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.13, size = 99, normalized size = 1.01

$$\frac{1}{4}\left(-\left(\left(\sqrt[4]{-1}c+id\right)\log\left(\sqrt[4]{-1}-x\right)\right)+\left(-(-1)^{3/4}c+id\right)\log\left((-1)^{3/4}-x\right)+\left(\sqrt[4]{-1}c-id\right)\log\left(x+\sqrt[4]{-1}\right)+\left((-1)^{3/4}c+id\right)\log\left(x+(-1)^{3/4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(1 + x^4),x]
```

```
[Out] (-(((-1)^(1/4)*c + I*d)*Log[(-1)^(1/4) - x]) + (-((-1)^(3/4)*c) + I*d)*Log[
(-1)^(3/4) - x] + ((-1)^(1/4)*c - I*d)*Log[(-1)^(1/4) + x] + ((-1)^(3/4)*c
+ I*d)*Log[(-1)^(3/4) + x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{1+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(1 + x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(1 + x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.17, size = 86, normalized size = 0.88

$$\frac{1}{8}\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right)+\frac{1}{4}\left(\sqrt{2}c-2d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\left(\sqrt{2}c+2d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
+ 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqr
t(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

maple [A] time = 0.05, size = 68, normalized size = 0.69

$$\frac{\sqrt{2}\,c\arctan\left(\sqrt{2}\,x-1\right)}{4}+\frac{\sqrt{2}\,c\arctan\left(\sqrt{2}\,x+1\right)}{4}+\frac{\sqrt{2}\,c\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{8}+\frac{d\arctan\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(x^4+1),x)
```

```
[Out] 1/4*c*arctan(2^(1/2)*x-1)*2^(1/2)+1/8*c*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2
^(1/2)*x+1))+1/4*c*arctan(2^(1/2)*x+1)*2^(1/2)+1/2*d*arctan(x^2)
```

maxima [A] time = 3.00, size = 86, normalized size = 0.88

$$\frac{1}{8}\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right)+\frac{1}{4}\left(\sqrt{2}c-2d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\left(\sqrt{2}c+2d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
+ 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqr
t(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

mupad [B] time = 0.09, size = 71, normalized size = 0.72

$$\mathrm{atan}\left(\sqrt{2}\,x-1\right)\left(\frac{d}{2}+\frac{\sqrt{2}\,c}{4}\right)-\mathrm{atan}\left(\sqrt{2}\,x+1\right)\left(\frac{d}{2}-\frac{\sqrt{2}\,c}{4}\right)+\frac{\sqrt{2}\,c\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(x^4 + 1),x)
```

```
[Out] atan(2^(1/2)*x - 1)*(d/2 + (2^(1/2)*c)/4) - atan(2^(1/2)*x + 1)*(d/2 - (2^(
1/2)*c)/4) + (2^(1/2)*c*log((2^(1/2)*x + x^2 + 1)/(x^2 - 2^(1/2)*x + 1)))/8
```

sympy [A] time = 0.71, size = 83, normalized size = 0.85

$$\operatorname{RootSum}\left(256t^4 + 32t^2d^2 - 16tc^2d + c^4 + d^4, \left(t \mapsto t \log\left(x + \frac{128t^3d^2 + 16t^2c^2d + 4tc^4 + 8td^4 - 5c^2d^3}{c^5 - 4cd^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x**4+1),x)
```

```
[Out] RootSum(256*_t**4 + 32*_t**2*d**2 - 16*_t*c**2*d + c**4 + d**4, Lambda(_t,
_t*log(x + (128*_t**3*d**2 + 16*_t**2*c**2*d + 4*_t*c**4 + 8*_t*d**4 - 5*c*
*2*d**3)/(c**5 - 4*c*d**4))))
```

3.78 $\int \frac{c+dx+ex^2}{a-bx^4}\,dx$

Optimal. Leaf size=116

$$\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1876, 275, 208, 1167, 205}

$$\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
 + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{a-bx^4}\,dx &= \int \left(\frac{dx}{a-bx^4}+\frac{c+ex^2}{a-bx^4}\right)dx \\
&= d\int \frac{x}{a-bx^4}\,dx+\int \frac{c+ex^2}{a-bx^4}\,dx \\
&= \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(-\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{}{\sqrt{}} \\
&= \frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 187, normalized size = 1.61

$$
\frac{-\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)\left(\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{a}\,e+\sqrt{b}\,c\right)+2\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt{b}\,c\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+\sqrt[4]{a}\,\sqrt[4]{b}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)-\sqrt[4]{a}\,\sqrt[4]{b}\,d\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)+\sqrt{a}\,e\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)}{4a^{3/4}b^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + a^(1/
4)*b^(1/4)*d + Sqrt[a]*e)*Log[a^(1/4) - b^(1/4)*x] + Sqrt[b]*c*Log[a^(1/4)
+ b^(1/4)*x] - a^(1/4)*b^(1/4)*d*Log[a^(1/4) + b^(1/4)*x] + Sqrt[a]*e*Log[a
^(1/4) + b^(1/4)*x] + a^(1/4)*b^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3
/4)*b^(3/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{c+dx+ex^2}{a-bx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a - b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.18, size = 263, normalized size = 2.27

$$
-\frac{\sqrt{2}\left(b^2c-\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c+\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*
sqrt(2)*(b^2*c + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)*arctan(1/2*sq
```

```
rt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(
2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

maple [B] time = 0.04, size = 161, normalized size = 1.39

$$-\frac{d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a}-\frac{e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*(a/b)^(1/4)/a*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*(a/b)^(1/4)/a*c
*arctan(1/(a/b)^(1/4)*x)-1/4/(a*b)^(1/2)*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(
1/2)*x^2-a))-1/2*e/b/(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)+1/4*e/b/(a/b)^(1/4
)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 2.91, size = 153, normalized size = 1.32

$$\frac{d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}}-\frac{d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*d*log(sqrt(b)*x^2
- sqrt(a))/(sqrt(a)*sqrt(b)) + 1/2*(sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt
(b)*c + sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqr
t(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

mupad [B] time = 5.14, size = 725, normalized size = 6.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a - b*x^4),x)
```

```
[Out] symsum(log(b^2*c^2*e - b^2*c*d^2 - b^2*d^3*x - a*b*e^3 - 16*root(256*a^3*b^
3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b
^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z
, k)^2*a*b^3*c - 4*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d
^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^
2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*b^3*c^2*x + 16*root(256*a^3*b^3*z^4
- 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2
*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)^2
*a*b^3*d*x - 4*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z
^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 +
a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*a*b^2*e^2*x + 8*root(256*a^3*b^3*z^4 - 6
4*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*
z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*a*b^
2*d*e + 2*b^2*c*d*e*x)*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b
^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^
2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k), k, 1, 4)
```

sympy [B] time = 11.04, size = 471, normalized size = 4.06

$$-\operatorname{RootSum}\left(256t^4a^3b^3+t^2\left(-64a^2b^2ce-32a^2b^2d^2\right)+t\left(-16a^2bde^2-16ab^2c^2d\right)-a^2e^4+2abc^2e^2-4abcd^2e+abd^4-b^2c^4,\left(t\mapsto t\log\left(x+\frac{-64t^3a^4b^2e^3-64t^3a^3b^3c^2e+128t^3a^3b^3cd^2+48t^2a^3b^2cde^2-32t^2a^3b^2d^3e-16t^2a^2b^3c^3d+12ta^3bce^4+12ta^3bd^2e^3+16ta^2b^2c^3e^2-36ta^2b^2c^2d^2e-8ta^2b^2cd^4+4tab^3c^5+3a^3de^5-5a^2bcd^3e^2+2a^2bd^5e+5ab^2c^4de-5ab^2c^3d^3}{a^3e^6+a^2bc^2e^4-8a^2bcd^2e^3+4a^2bd^4e^2-ab^2c^4e^2+8ab^2c^3d^2e-4ab^2c^2d^4-b^3c^6}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 + _t**2*(-64*a**2*b**2*c*e - 32*a**2*b**2*d**2
) + _t*(-16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) - a**2*e**4 + 2*a*b*c**2*e**2
 - 4*a*b*c*d**2*e + a*b*d**4 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*
a**4*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 4
8*_t**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e - 16*_t**2*a**2*b**3
*c**3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 + 16*_t*a**2*b**2*c*
*3*e**2 - 36*_t*a**2*b**2*c**2*d**2*e - 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b**3
*c**5 + 3*a**3*d*e**5 - 5*a**2*b*c*d**3*e**2 + 2*a**2*b*d**5*e + 5*a*b**2*c
**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 + a**2*b*c**2*e**4 - 8*a**2*b*c*d*
*2*e**3 + 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*e - 4*
a*b**2*c**2*d**4 - b**3*c**6))))
```

3.79 $\int \frac{c+dx+ex^2}{a+bx^4} \, dx$

Optimal. Leaf size=277

$$-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}\,e+\sqrt{}\right.}{}$$

Rubi [A] time = 0.20, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}\,e+\sqrt{b}\,c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sq
rt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \int \left(\frac{dx}{a+bx^4}+\frac{c+ex^2}{a+bx^4}\right)dx$$

$$= d\int \frac{x}{a+bx^4}\,dx+\int \frac{c+ex^2}{a+bx^4}\,dx$$

$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\,\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b}-\frac{\left(\sqrt{b}\,c-\sqrt{\right.}}{}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{a}\right.}{4\sqrt{2}\,a}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-$$

Mathematica [A] time = 0.12, size = 229, normalized size = 0.83

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(2\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{2}\,\sqrt{a}\,e+\sqrt{2}\,\sqrt{b}\,c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(-2\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{2}\,\sqrt{a}\,e+\sqrt{2}\,\sqrt{b}\,c\right)-\sqrt{2}\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\left(\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)-\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\right)}{8a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1
- (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d
 + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(Sq
rt[b]*c - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/4)*b^(
3/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2}{a + bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.17, size = 275, normalized size = 0.99

$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d - \left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d - \left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1
/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8
*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/
4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

maple [A] time = 0.05, size = 280, normalized size = 1.01

$$\frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{2\sqrt{ab}} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a} + \frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b} + \frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b} + \frac{\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/8*(a/b)^(1/4)*2^(1/2)/a*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2
-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4*(a/b)^(1/4)*2^(1/2)/a*c*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)+1/4*(a/b)^(1/4)*2^(1/2)/a*c*arctan(2^(1/2)/(a/b)^(1/4
)*x-1)+1/2/(a*b)^(1/2)*d*arctan((1/a*b)^(1/2)*x^2)+1/8*e/b/(a/b)^(1/4)*2^(1
/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2)))+1/4*e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/
4*e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.04, size = 257, normalized size = 0.93

$$\frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-2\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e+2\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*e)*log
(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 1/4
*(sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 2*sqrt(a)*sqrt(b)
*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)
*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 1/4*(sqrt(2)*a^(1/4)*b
^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e + 2*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt
(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)
*sqrt(sqrt(a)*sqrt(b))*b^(3/4))
```

mupad [B] time = 5.09, size = 712, normalized size = 2.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^4),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - a*b*e^3 - 16*root(256*a^3*b^
3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b
^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z
, k)^2*a*b^3*c - 4*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d
^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^
2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*b^3*c^2*x + 16*root(256*a^3*b^3*z^4
+ 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2
*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)^2
*a*b^3*d*x + 4*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z
^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 +
a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*a*b^2*e^2*x - 8*root(256*a^3*b^3*z^4 + 6
4*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*
z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*a*b^
2*d*e - 2*b^2*c*d*e*x)*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b
^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^
2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k), k, 1, 4)
```

sympy [A] time = 10.54, size = 466, normalized size = 1.68

$$\operatorname{RootSum}\left(256t^4a^3b^3+t^2\left(64a^2b^2ce+32a^2b^2d^2\right)+t\left(16a^2bde^2-16ab^2c^2d\right)+a^2e^4+2abc^2e^2-4abcd^2e+abd^4+b^2c^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^4b^2e^3-64t^3a^3b^3c^2e+128t^3a^3b^3cd^2+48t^2a^3b^2cde^2-32t^2a^3b^2d^3e+16t^2a^2b^3c^3d+12ta^3bce^4+12ta^3bd^2e^3-16ta^2b^2c^3e^2+36ta^2b^2c^2d^2e+8ta^2b^2cd^4+4tab^3c^5+3a^3de^5+5a^2bcd^3e^2-2a^2bd^5e+5ab^2c^4de-5ab^2c^3d^3}{a^3e^6-a^2bc^2e^4+8a^2bcd^2e^3-4a^2bd^4e^2-ab^2c^4e^2+8ab^2c^3d^2e-4ab^2c^2d^4+b^3c^6}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + _t**2*(64*a**2*b**2*c*e + 32*a**2*b**2*d**2)
+ _t*(16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) + a**2*e**4 + 2*a*b*c**2*e**2 -
4*a*b*c*d**2*e + a*b*d**4 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**4
*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 48*_t
**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e + 16*_t**2*a**2*b**3*c**
3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 - 16*_t*a**2*b**2*c**3*e
**2 + 36*_t*a**2*b**2*c**2*d**2*e + 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b**3*c**
5 + 3*a**3*d*e**5 + 5*a**2*b*c*d**3*e**2 - 2*a**2*b*d**5*e + 5*a*b**2*c**4*
d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 - a**2*b*c**2*e**4 + 8*a**2*b*c*d**2*e
**3 - 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*e - 4*a*b*
*2*c**2*d**4 + b**3*c**6))))
```

3.80 $\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^2} \, dx$

Optimal. Leaf size=146

$$\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}$$

Rubi [A] time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1855, 1876, 275, 208, 1167, 205}

$$\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*ArcTan
[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*Arc
Tanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
```

```
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^2}\,dx = \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)} - \frac{\int \frac{-3c-2dx-ex^2}{a-bx^4}\,dx}{4a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)} - \frac{\int \left(-\frac{2dx}{a-bx^4}+\frac{-3c-ex^2}{a-bx^4}\right)dx}{4a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)} - \frac{\int \frac{-3c-ex^2}{a-bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a-bx^4}\,dx}{2a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{4a} - \frac{\left(\frac{3\sqrt{b}\,c}{\sqrt{a}}-e\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx}{8a} + \frac{\left(3\sqrt{b}\,c+\sqrt{a}\right.}{}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)} + \frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d\tanh^{-1}}{4a^{3/}}$$

Mathematica [A] time = 0.28, size = 211, normalized size = 1.45

$$\frac{-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{b}c-2\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}-\frac{2\sqrt[4]{a}\left(\sqrt{a}e-3\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{4ax(c+x(d+ex))}{a-bx^4}+\frac{2\sqrt{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{b}}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*x*(c + x*(d + e*x)))/(a - b*x^4) - (2*a^(1/4)*(-3*Sqrt[b]*c + Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((3*a^(1/4)*Sqrt[b]*c + 2*Sqrt[a
]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x]])/b^(3/4) + ((3*a^(1/4)*Sq
rt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x]])/b^(3/4
) + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a - b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.18, size = 311, normalized size = 2.13

$$-\frac{\sqrt{2}\left(3\,b^{2}c-2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c+2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}+\frac{\sqrt{2}\left(3\,b^{2}c-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{x^{3}e+dx^{2}+cx}{4\left(bx^{4}-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*
a) - 1/16*sqrt(2)*(3*b^2*c + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3
/4)*a) - 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)
^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b
)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) -
1/4*(x^3*e + d*x^2 + c*x)/((b*x^4 - a)*a)
```

maple [B] time = 0.06, size = 228, normalized size = 1.56

$$-\frac{e\,x^{3}}{4\left(b\,x^{4}-a\right)a}-\frac{d\,x^{2}}{4\left(b\,x^{4}-a\right)a}-\frac{cx}{4\left(b\,x^{4}-a\right)a}-\frac{d\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{8\sqrt{ab}\,a}-\frac{e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] -1/4/(b*x^4-a)/a*c*x+3/16*(a/b)^(1/4)/a^2*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))+3/8*(a/b)^(1/4)/a^2*c*arctan(1/(a/b)^(1/4)*x)-1/4/(b*x^4-a)/a*d*x^2-1/
8/(a*b)^(1/2)/a*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-1/4*e*x^3/a/
(b*x^4-a)-1/8*e/a/b/(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)+1/16*e/a/b/(a/b)^(1
/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 2.94, size = 191, normalized size = 1.31

$$-\frac{ex^{3}+dx^{2}+cx}{4\left(abx^{4}-a^{2}\right)}+\frac{\frac{2\,d\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{2\,d\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(3\,\sqrt{b}\,c-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(3\,\sqrt{b}\,c+\sqrt{a}\,e\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(e*x^3 + d*x^2 + c*x)/(a*b*x^4 - a^2) + 1/16*(2*d*log(sqrt(b)*x^2 + sq
rt(a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b))
 + 2*(3*sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*sqrt(b)*c + sqrt(a)*e)*log((sqrt(b
)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*
sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a
```

mupad [B] time = 4.98, size = 477, normalized size = 3.27

$$\frac{\frac{d\,x^{2}}{4\,a}+\frac{e\,x^{3}}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^{4}}+\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^{7}\,b^{3}\,z^{4}-3072\,a^{4}\,b^{2}\,c\,e\,z^{2}-2048\,a^{4}\,b^{2}\,d^{2}\,z^{2}+1152\,a^{2}\,b^{2}\,c^{2}\,d\,z+128\,a^{3}\,b\,d\,e^{2}\,z-48\,a\,b\,c\,d^{2}\,e+18\,a\,b\,c^{2}\,e^{2}+16\,a\,b\,d^{4}-81\,b^{2}\,c^{4}-a^{2}\,e^{4},z,k\right)\left(\mathrm{root}\left(65536\,a^{7}\,b^{3}\,z^{4}-3072\,a^{4}\,b^{2}\,c\,e\,z^{2}-2048\,a^{4}\,b^{2}\,d^{2}\,z^{2}+1152\,a^{2}\,b^{2}\,c^{2}\,d\,z+128\,a^{3}\,b\,d\,e^{2}\,z-48\,a\,b\,c\,d^{2}\,e+18\,a\,b\,c^{2}\,e^{2}+16\,a\,b\,d^{4}-81\,b^{2}\,c^{4}-a^{2}\,e^{4},z,k\right)\left(12\,b^{3}\,c-8\,b^{3}\,d\,x\right)+\frac{x\left(4\,a^{2}\,b^{2}\,e^{2}+36\,a\,b^{3}\,c^{2}\right)}{16\,a^{3}}-\frac{b^{2}\,d\,e}{a}\right)-\frac{-9\,b^{2}\,c^{2}\,e+12\,b^{2}\,c\,d^{2}+a\,b\,e^{3}}{64\,a^{3}}-\frac{x\left(2\,b^{2}\,d^{3}-3\,b^{2}\,c\,d\,e\right)}{16\,a^{3}}\right)\mathrm{root}\left(65536\,a^{7}\,b^{3}\,z^{4}-3072\,a^{4}\,b^{2}\,c\,e\,z^{2}-2048\,a^{4}\,b^{2}\,d^{2}\,z^{2}+1152\,a^{2}\,b^{2}\,c^{2}\,d\,z+128\,a^{3}\,b\,d\,e^{2}\,z-48\,a\,b\,c\,d^{2}\,e+18\,a\,b\,c^{2}\,e^{2}+16\,a\,b\,d^{4}-81\,b^{2}\,c^{4}-a^{2}\,e^{4},z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a - b*x^4)^2,x)
```

```
[Out] ((d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a - b*x^4) + symsum(log(- ro
ot(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a
```

```
^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a
*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4 - 3072*a^4*b^2
*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z
- 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k
)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 + 4*a^2*b^2*e^2))/(16*a^3) - (b
^2*d*e)/a) - (12*b^2*c*d^2 - 9*b^2*c^2*e + a*b*e^3)/(64*a^3) - (x*(2*b^2*d^
3 - 3*b^2*c*d*e))/(16*a^3))*root(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 -
 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c
*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k), k, 1, 4
)
```

<u>**sympy**</u> **[B]** time = 13.74, size = 508, normalized size = 3.48

$$\operatorname{RootSum}\left(65536t^4a^7b^3 + t^2\left(-3072a^4b^2ce - 2048a^4b^2d^2\right) + t\left(128a^3bde^2 + 1152a^2b^2c^2d\right) - a^2e^4 + 18abc^2e^2 - 48abcd^2e + 16abd^4 - 81b^2c^4, \left(t \mapsto t\log\left(x + \frac{4096t^3a^7b^2e^3 + 36864t^3a^6b^3c^2e - 98304t^3a^6b^3cd^2 + 4608t^2a^5b^2cde^2 - 4096t^2a^5b^2d^3e - 13824t^2a^4b^3c^3d - 144ta^4bce^4 - 192ta^4bd^2e^3 - 1728ta^3b^2c^3e^2 + 5184ta^3b^2c^2d^2e + 1536ta^3b^2cd^4 - 3888ta^2b^3c^5 + 6a^3de^5 - 120a^2bcd^3e^2 + 64a^2bd^5e + 810ab^2c^4de - 1080ab^2c^3d^3}{a^3e^6 + 9a^2bc^2e^4 - 96a^2bcd^2e^3 + 64a^2bd^4e^2 - 81ab^2c^4e^2 + 864ab^2c^3d^2e - 576ab^2c^2d^4 - 729b^3c^6}\right)\right)\right) + \frac{-cx - dx^2 - ex^3}{-4a^2 + 4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*a*
b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _t*l
og(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 98304*_t
**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2
*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4
*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e
+ 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 - 120*a
**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c*
*3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3 + 64*a**2*
b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**
2*d**4 - 729*b**3*c**6)))) + (-c*x - d*x**2 - e*x**3)/(-4*a**2 + 4*a*b*x**4
)
```

3.81 $\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}\,e\right.}{}$$

Rubi [A] time = 0.25, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])
/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sq
rt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^2}\,dx = \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} - \frac{\int \frac{-3c-2dx-ex^2}{a+bx^4}\,dx}{4a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} - \frac{\int \left(-\frac{2dx}{a+bx^4}+\frac{-3c-ex^2}{a+bx^4}\right)dx}{4a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} - \frac{\int \frac{-3c-ex^2}{a+bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{4a} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{8ab} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\cdots}{\cdots}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16ab} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{\cdots}{\frac{\sqrt{a}}{\sqrt{b}}+\cdots}}{16ab}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \cdots$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}\cdots\right.}{8\cdots}$$

Mathematica [A] time = 0.55, size = 305, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(a^{3/4}e-3\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}+\frac{\sqrt{2}\left(3\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}-\frac{2\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e+3\sqrt{2}\sqrt{b}c\right)}{b^{3/4}}+\frac{2\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e+3\sqrt{2}\sqrt{b}c\right)}{b^{3/4}}+\frac{8ax(c+x(d+ex))}{a+bx^4}}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

```
[Out] ((8*a*x*(c + x*(d + e*x)))/(a + b*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c +
4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/b^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4)*b^(1/4)*d + Sq
rt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2
]*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(32*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.18, size = 306, normalized size = 0.99

$$\frac{x^3e + dx^2 + cx}{4\left(bx^4 + a\right)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d + 3\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d + 3\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32a^2b^3} - \frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e + d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*
b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sq
rt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^
3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(
x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

maple [A] time = 0.05, size = 344, normalized size = 1.12

$$\frac{ex^3}{4\left(bx^4 + a\right)a} + \frac{dx^2}{4\left(bx^4 + a\right)a} + \frac{cx}{4\left(bx^4 + a\right)a} + \frac{d\arctan\left(\sqrt{\frac{b}{a}}x^2\right)}{4\sqrt{ab}\,a} + \frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{32\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16a^2} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{16a^2} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{b}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] 1/4/(b*x^4+a)/a*c*x+3/32*(a/b)^(1/4)*2^(1/2)/a^2*c*ln((x^2+(a/b)^(1/4)*2^(1
/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/16*(a/b)^(1/4
)*2^(1/2)/a^2*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/16*(a/b)^(1/4)*2^(1/2)/a^
2*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4/(b*x^4+a)/a*d*x^2+1/4/(a*b)^(1/2)/a
*d*arctan((1/a*b)^(1/2)*x^2)+1/4*e*x^3/a/(b*x^4+a)+1/32*e/a/b/(a/b)^(1/4)*2
^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*
x+(a/b)^(1/2)))+1/16*e/a/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
+1)+1/16*e/a/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.10, size = 294, normalized size = 0.95

$$\frac{ex^3 + dx^2 + cx}{4\left(abx^4 + a^2\right)} + \frac{\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-4\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}} + \frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e+4\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e*x^3 + d*x^2 + c*x)/(a*b*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(b)*c - sq
rt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^
(3/4)) - sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c +
 sqrt(2)*a^(3/4)*b^(1/4)*e - 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqr
t(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt
(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^
(1/4)*e + 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^
(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4
)))/a
```

mupad [**B**] time = 0.33, size = 472, normalized size = 1.53

$$\frac{\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}+\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\left(\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\left(12\,b^3\,c-8\,b^3\,d\,x\right)+\frac{x\left(36\,a\,b^3\,c^2-4\,a^2\,b^2\,e^2\right)}{16\,a^3}+\frac{b^2\,d\,e}{a}\right)-\frac{9\,b^2\,c^2\,e-12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}+\frac{x\left(2\,b^2\,d^3-3\,b^2\,c\,d\,e\right)}{16\,a^3}\right)\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^2,x)
```

```
[Out] ((d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a + b*x^4) + symsum(log((x*(
2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3) - (9*b^2*c^2*e - 12*b^2*c*d^2 + a*b*e^3)
/(64*a^3) - root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^
2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*
c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4
+ 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*
a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 +
 a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 - 4*a^2*b^2*e^2))
/(16*a^3) + (b^2*d*e)/a))*root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2
048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d
^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k), k, 1, 4)
```

sympy [**A**] time = 11.55, size = 505, normalized size = 1.64

$$\mathrm{RootSum}\left(65536t^4a^7b^3+t^2\left(3072a^4b^2ce+2048a^4b^2d^2\right)+t\left(128a^3bde^2-1152a^2b^2c^2d\right)+a^2e^4+18abc^2e^2-48abcd^2e+16abd^4+81b^2c^4,\left(t\mapsto t\log\left(x+\frac{4096t^3a^7b^2e^3-36864t^3a^6b^3c^2e+98304t^3a^6b^3cd^2+4608t^2a^5b^2cde^2-4096t^2a^5b^2d^3e+13824t^2a^4b^3c^3d+144ta^4bce^4+192ta^4bd^2e^3-1728ta^3b^2c^3e^2+5184ta^3b^2c^2d^2e+1536ta^3b^2cd^4+3888ta^2b^3c^5+6a^3de^5+120a^2bcd^3e^2-64a^2bd^5e+810ab^2c^4de-1080ab^2c^3d^3}{a^3e^6-9a^2bc^2e^4+96a^2bcd^2e^3-64a^2bd^4e^2-81ab^2c^4e^2+864ab^2c^3d^2e-576ab^2c^2d^4+729b^3c^6}\right)\right)\right)+\frac{cx+dx^2+ex^3}{4a^2+4abx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2*
d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a*b
*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t*lo
g(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304*_t*
*3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*
d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*
b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e +
 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 + 120*a*
*2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**
3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 - 64*a**2*b
*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2
*d**4 + 729*b**3*c**6)))) + (c*x + d*x**2 + e*x**3)/(4*a**2 + 4*a*b*x**4)
```

3.82 $\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^3}\,dx$

Optimal. Leaf size=179

$$\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{x\left(c+\right.}{8a\left(\right.}$$

Rubi [A] time = 0.17, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1855, 1876, 275, 208, 1167, 205}

$$\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32
*a^2*(a - b*x^4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4
)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*
x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(
16*a^(5/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^3}\,dx &= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}-\frac{\int \frac{-7c-6dx-5ex^2}{\left(a-bx^4\right)^2}\,dx}{8a}\\
&= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{\int \frac{21c+12dx+5ex^2}{a-bx^4}\,dx}{32a^2}\\
&= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{\int \left(\frac{12dx}{a-bx^4}+\frac{21c+5ex^2}{a-bx^4}\right)dx}{32a^2}\\
&= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{\int \frac{21c+5ex^2}{a-bx^4}\,dx}{32a^2}+\frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}\\
&= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{(3d)\,\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{16a^2}-\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\ldots}{64\ldots}\\
&= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{3/4}}+\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\ldots}{64a^{11/4}\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.29, size = 244, normalized size = 1.36

$$
\frac{-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{b}c+12\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{b}c-12\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{16a^2x(c+x(d+ex))}{\left(a-bx^4\right)^2}+\frac{2\sqrt[4]{a}\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{4ax(7c+x(6d+5ex))}{a-bx^4}+\frac{12\sqrt{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{b}}}{128a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

```
[Out] ((16*a^2*x*(c + x*(d + e*x)))/(a - b*x^4)^2 + (4*a*x*(7*c + x*(6*d + 5*e*x)
))/(a - b*x^4) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)
/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1/4)*d + 5*a^(3
/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[b]*c - 12*Sqrt
[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (12*Sqrt[a
]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a - b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.23, size = 340, normalized size = 1.90

$$-\frac{\sqrt{2}\left(21\,b^{2}c-12\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c+12\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}+\frac{\sqrt{2}\left(21\,b^{2}c-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{5\,bx^{7}e+6\,bdx^{6}+7\,bcx^{5}-9\,ax^{3}e-10\,adx^{2}-11\,acx}{32\left(bx^{4}-a\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt(-a*b)*b*e
)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(
3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 5*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2
+ sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/256*sqrt(2)
*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*x^3
*e - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2)
```

maple [B] time = 0.05, size = 286, normalized size = 1.60

$$\frac{e\,x^{3}}{8\left(b\,x^{4}-a\right)^{2}a}+\frac{d\,x^{2}}{8\left(b\,x^{4}-a\right)^{2}a}-\frac{5e\,x^{3}}{32\left(b\,x^{4}-a\right)a^{2}}+\frac{cx}{8\left(b\,x^{4}-a\right)^{2}a}-\frac{3d\,x^{2}}{16\left(b\,x^{4}-a\right)a^{2}}-\frac{7cx}{32\left(b\,x^{4}-a\right)a^{2}}-\frac{3d\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{32\sqrt{ab}\,a^{2}}-\frac{5e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{5e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] 1/8/(b*x^4-a)^2/a*c*x-7/32/(b*x^4-a)/a^2*c*x+21/128*(a/b)^(1/4)/a^3*c*ln((x
+(a/b)^(1/4))/(x-(a/b)^(1/4)))+21/64*(a/b)^(1/4)/a^3*c*arctan(1/(a/b)^(1/4)
*x)+1/8/(b*x^4-a)^2/a*d*x^2-3/16/(b*x^4-a)/a^2*d*x^2-3/32/(a*b)^(1/2)/a^2*d
*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))+1/8*e*x^3/a/(b*x^4-a)^2-5/32*
e/a^2*x^3/(b*x^4-a)-5/64*e/a^2/b/(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)+5/128*
e/a^2/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 3.12, size = 230, normalized size = 1.28

$$-\frac{5\,bex^{7}+6\,bdx^{6}+7\,bcx^{5}-9\,aex^{3}-10\,adx^{2}-11\,acx}{32\left(a^{2}b^{2}x^{8}-2\,a^{3}bx^{4}+a^{4}\right)}+\frac{\frac{12\,d\log\left(\sqrt{b}x^{2}+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{12\,d\log\left(\sqrt{b}x^{2}-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,\sqrt{b}c+5\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*e*x^3 - 10*a*d*x^2 - 11*a*c*
x)/(a^2*b^2*x^8 - 2*a^3*b*x^4 + a^4) + 1/128*(12*d*log(sqrt(b)*x^2 + sqrt(a
))/(sqrt(a)*sqrt(b)) - 12*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) +
2*(21*sqrt(b)*c - 5*sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*sqrt(b)*c + 5*sqrt(a)*e)*log((sqr
t(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^2
```

mupad [B] time = 5.11, size = 826, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a - b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^
2) - (3*b*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^
4) + symsum(log(-(b*(125*a*e^3 + 3024*b*c*d^2 - 2205*b*c^2*e + 1728*b*d^3*x
 + 344064*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a
^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b
*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4
, z, k)^2*a^5*b^2*c + 3200*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*
e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*
e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4
 - 194481*b^2*c^4, z, k)*a^3*b*e^2*x - 2520*b*c*d*e*x + 56448*root(26843545
6*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 270950
4*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^
2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x -
 196608*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6
*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c
*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4,
z, k)^2*a^5*b^2*d*x - 15360*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c
*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d
*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^
4 - 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*
z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c
^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 207
36*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k), k, 1, 4)
```

sympy [B] time = 45.34, size = 563, normalized size = 3.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**3 + _t**2*(-6881280*a**6*b**2*c*e - 47185
92*a**6*b**2*d**2) + _t*(-153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d)
- 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4
 - 194481*b**2*c**4, Lambda(_t, _t*log(x + (-262144000*_t**3*a**10*b**2*e**
3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2
+ 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e - 1
820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a*
*5*b*d**2*e**3 + 118540800*_t*a**4*b**2*c**3*e**2 - 365783040*_t*a**4*b**2*
c**2*d**2*e - 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 +
 112500*a**3*d*e**5 - 4536000*a**2*b*c*d**3*e**2 + 2488320*a**2*b*d**5*e +
58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 + 27
5625*a**2*b*c**2*e**4 - 3024000*a**2*b*c*d**2*e**3 + 2073600*a**2*b*d**4*e*
*2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b*
*2*c**2*d**4 - 85766121*b**3*c**6)))) - (-11*a*c*x - 10*a*d*x**2 - 9*a*e*x*
*3 + 7*b*c*x**5 + 6*b*d*x**6 + 5*b*e*x**7)/(32*a**4 - 64*a**3*b*x**4 + 32*a
**2*b**2*x**8)
```

3.83 $\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3} \, dx$

Optimal. Leaf size=341

$$-\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}}-\frac{(5\sqrt{a}}{}$$

Rubi [A] time = 0.31, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}}-\frac{\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32
*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]
) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c
- 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128
*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx = \frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2} - \frac{\int \frac{-7c-6dx-5ex^2}{\left(a+bx^4\right)^2}\,dx}{8a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2} + \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} + \frac{\int \frac{21c+12dx+5ex^2}{a+bx^4}\,dx}{32a^2}$$

$$= \frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2} + \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} + \frac{\int \left(\frac{12dx}{a+bx^4} + \frac{21c+5ex^2}{a+bx^4}\right)dx}{32a^2}$$

$$= \frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2} + \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} + \frac{\int \frac{21c+5ex^2}{a+bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a+bx^4}\,dx}{8a^2}$$

$$= \frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2} + \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} + \frac{(3d)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{16a^2} + \frac{\left(\frac{21\sqrt{b}\,c}{\sqrt{a}} - 5e\right)\int \dots}{64a^2b \dots}$$

$$= \frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2} + \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}\,c}{\sqrt{a}} - 5e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2} \dots}{128\sqrt{2}\,a^{9/4}b^{3/4} \dots}$$

$$= \frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2} + \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}\,c}{\sqrt{a}} - 5e\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{\dots}\right.}{128\sqrt{2}\,a^{9/4}b^{3/4} \dots}$$

$$= \frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2} + \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}\,c+5\sqrt{a}\,e\right)\tan^{-1}\left(1 - \dots\right.}{64\sqrt{2}\,a^{11/4}b^{3/4}}$$

Mathematica [A] time = 0.42, size = 337, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(5a^{3/4}e-21\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{b}c-5a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{32a^2x(c+x(d+ex))}{\left(a+bx^4\right)^2} - \frac{2\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(24\sqrt[4]{a}\sqrt[4]{b}d+5\sqrt{2}\sqrt{a}e+21\sqrt{2}\sqrt{b}c\right)}{b^{3/4}} + \frac{2\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-24\sqrt[4]{a}\sqrt[4]{b}d+5\sqrt{2}\sqrt{a}e+21\sqrt{2}\sqrt{b}c\right)}{b^{3/4}} + \frac{8ax(7c+x(6d+5ex))}{a+bx^4}}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(c + x*(d + e*x)))/(a + b*x^4)^2 + (8*a*x*(7*c + x*(6*d + 5*e*x)
))/(a + b*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4)*b^(1/4)*d +
5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (2*
a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-21*a^(1/4)*S
qrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.19, size = 336, normalized size = 0.99

$$\frac{5\,bx^{7}e+6\,bdx^{6}+7\,bcx^{5}+9\,ax^{3}e+10\,adx^{2}+11\,acx}{32\,\left(bx^{4}+a\right)^{2}a^{2}}+\frac{\sqrt{2}\left(12\,\sqrt{2}\,\sqrt{ab}\,b^{2}d+21\,\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+5\,\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^{3}b^{3}}+\frac{\sqrt{2}\left(12\,\sqrt{2}\,\sqrt{ab}\,b^{2}d+21\,\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+5\,\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^{3}b^{3}}+\frac{\sqrt{2}\left(21\,\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c-5\,\left(ab^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^{3}b^{3}}-\frac{\sqrt{2}\left(21\,\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c-5\,\left(ab^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^{3}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
[Out] 1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*x^3*e + 10*a*d*x^2 + 11*a*c*x
)/((b*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b
^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d
 + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)
*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3)
```

maple [A] time = 0.05, size = 396, normalized size = 1.16

$$\frac{e\,x^{3}}{8\left(b\,x^{4}+a\right)^{2}a}+\frac{d\,x^{2}}{8\left(b\,x^{4}+a\right)^{2}a}+\frac{5e\,x^{3}}{32\left(b\,x^{4}+a\right)a^{2}}+\frac{cx}{8\left(b\,x^{4}+a\right)^{2}a}+\frac{3d\,x^{2}}{16\left(b\,x^{4}+a\right)a^{2}}+\frac{7cx}{32\left(b\,x^{4}+a\right)a^{2}}+\frac{3d\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{16\sqrt{ab}\,a^{2}}+\frac{5\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{5\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{5\sqrt{2}\,e\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^3,x)
[Out] 1/8/(b*x^4+a)^2/a*c*x+7/32/(b*x^4+a)/a^2*c*x+21/256*(a/b)^(1/4)*2^(1/2)/a^3
*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a
/b)^(1/2)))+21/128*(a/b)^(1/4)*2^(1/2)/a^3*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1
)+21/128*(a/b)^(1/4)*2^(1/2)/a^3*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/8/(b*x
^4+a)^2/a*d*x^2+3/16/(b*x^4+a)/a^2*d*x^2+3/16/(a*b)^(1/2)/a^2*d*arctan((1/a
*b)^(1/2)*x^2)+1/8*e*x^3/a/(b*x^4+a)^2+5/32*e/a^2*x^3/(b*x^4+a)+5/256*e/a^2
/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b
)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+5/128*e/a^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x+1)+5/128*e/a^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x-1)
```

maxima [A] time = 3.09, size = 336, normalized size = 0.99

$$\frac{5\,bex^{7}+6\,bdx^{6}+7\,bcx^{5}+9\,aex^{3}+10\,adx^{2}+11\,acx}{32\left(a^{2}b^{2}x^{8}+2\,a^{3}bx^{4}+a^{4}\right)}+\frac{\frac{\sqrt{2}\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-24\,\sqrt{a}\,\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e+24\,\sqrt{a}\,\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}}{256\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
[Out] 1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*e*x^3 + 10*a*d*x^2 + 11*a*c*x
)/(a^2*b^2*x^8 + 2*a^3*b*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqrt(b)*c - 5*sqrt
```

```
(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3
/4)) - sqrt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(3/4)*c
 + 5*sqrt(2)*a^(3/4)*b^(1/4)*e - 24*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(3/4)*c + 5*sqrt(2)*a^
(3/4)*b^(1/4)*e + 24*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - s
qrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b
))*b^(3/4)))/a^2
```

mupad [**B**] time = 5.05, size = 826, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^
2) + (3*b*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^
4) + symsum(log(-(b*(125*a*e^3 - 3024*b*c*d^2 + 2205*b*c^2*e - 1728*b*d^3*x
 + 344064*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a
^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b
*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4
, z, k)^2*a^5*b^2*c - 3200*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*
e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*
e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4
 + 194481*b^2*c^4, z, k)*a^3*b*e^2*x + 2520*b*c*d*e*x + 56448*root(26843545
6*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 270950
4*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^
2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x -
 196608*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6
*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c
*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4,
z, k)^2*a^5*b^2*d*x + 15360*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c
*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d
*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^
4 + 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*
z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c
^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 207
36*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k), k, 1, 4)
```

sympy [**A**] time = 40.86, size = 558, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**3 + _t**2*(6881280*a**6*b**2*c*e + 4718592
*a**6*b**2*d**2) + _t*(153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d) + 6
25*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4 +
194481*b**2*c**4, Lambda(_t, _t*log(x + (262144000*_t**3*a**10*b**2*e**3 -
4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 30
9657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e + 18207
86688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a**5*b
*d**2*e**3 - 118540800*_t*a**4*b**2*c**3*e**2 + 365783040*_t*a**4*b**2*c**2
*d**2*e + 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 + 112
500*a**3*d*e**5 + 4536000*a**2*b*c*d**3*e**2 - 2488320*a**2*b*d**5*e + 5834
4300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 - 275625
```

```
*a**2*b*c**2*e**4 + 3024000*a**2*b*c*d**2*e**3 - 2073600*a**2*b*d**4*e**2 -
 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b**2*c
**2*d**4 + 85766121*b**3*c**6)))) + (11*a*c*x + 10*a*d*x**2 + 9*a*e*x**3 +
7*b*c*x**5 + 6*b*d*x**6 + 5*b*e*x**7)/(32*a**4 + 64*a**3*b*x**4 + 32*a**2*b
**2*x**8)
```

3.84 $\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx$

Optimal. Leaf size=211

$$\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+$$

Rubi [A] time = 0.21, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1855, 1876, 275, 208, 1167, 205}

$$\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/
(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4
)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15
/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*
Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx = \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{\left(a-bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{\int \frac{77c+60dx+45ex^2}{\left(a-bx^4\right)^2}\,dx}{96a^2}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} - \frac{\int \frac{-231c-120dx-45ex}{a-bx^4}}{384a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} - \frac{\int \left(-\frac{120dx}{a-bx^4} + \frac{-231c}{a-}\right.}{384a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} - \frac{\int \frac{-231c-45ex^2}{a-bx^4}\,dx}{384a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} + \frac{(5d)\,\mathrm{Subst}\left(\int \frac{1}{a-b}\right.}{32a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} + \frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right.}{256a^{15/}}$$

Mathematica [A] time = 0.28, size = 276, normalized size = 1.31

$$\frac{-\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c+40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}} + \frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c-40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}} + \frac{128a^3x(c+x(d+ex))}{\left(a-bx^4\right)^3} + \frac{16a^2x(11c+x(10d+9ex))}{\left(a-bx^4\right)^2} + \frac{6\sqrt[4]{a}\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{4ax(77c+15x(4d+3ex))}{a-bx^4} + \frac{120\sqrt{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{b}}}{1536a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] ((128*a^3*x*(c + x*(d + e*x)))/(a - b*x^4)^3 + (4*a*x*(77*c + 15*x*(4*d + 3
*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a - b*x^4)^2 +
(6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/
4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^
(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*Sqrt[a]*b^(1/4)
*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*Sqrt[a]*d*Log[S
qrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a - b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.22, size = 377, normalized size = 1.79

$$-\frac{\sqrt{2}\left(77\,b^{2}c-40\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^{3}\right)^{\frac{3}{4}}a^{3}}-\frac{\sqrt{2}\left(77\,b^{2}c+40\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^{3}\right)^{\frac{3}{4}}a^{3}}-\frac{\sqrt{2}\left(77\,b^{2}c-15\,\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^{3}\right)^{\frac{3}{4}}a^{3}}+\frac{\sqrt{2}\left(77\,b^{2}c-15\,\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^{3}\right)^{\frac{3}{4}}a^{3}}-\frac{45\,b^{2}x^{11}e+60\,b^{2}dx^{10}+77\,b^{2}cx^{9}-126\,abx^{7}e-160\,abdx^{6}-198\,abcx^{5}+113\,a^{2}x^{3}e+132\,a^{2}dx^{2}+153\,a^{2}cx}{384\left(bx^{4}-a\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^
(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 15*s
qrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4)
)/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(
x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/1024*sq
rt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqr
t(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^2*x^11*e + 60*b^2*d*x^10 + 77*b
^2*c*x^9 - 126*a*b*x^7*e - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 113*a^2*x^3*e +
132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

maple [A] time = 0.06, size = 274, normalized size = 1.30

$$-\frac{5d\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{64\sqrt{ab}\,a^{3}}-\frac{15e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}b}+\frac{15e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}b}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256a^{4}}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^{4}}+\frac{-\frac{15b^{2}e\,x^{11}}{128a^{3}}-\frac{5b^{2}d\,x^{10}}{32a^{3}}-\frac{77b^{2}c\,x^{9}}{384a^{3}}+\frac{21be\,x^{7}}{64a^{2}}+\frac{5bd\,x^{6}}{12a^{2}}+\frac{33bc\,x^{5}}{64a^{2}}-\frac{113e\,x^{3}}{384a}-\frac{11d\,x^{2}}{32a}-\frac{51cx}{128a}}{\left(b\,x^{4}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128*e/a^3*b^2*x^11-5/32/a^3*b^2*d*x^10-77/384/a^3*b^2*c*x^9+21/64/a^2*
b*e*x^7+5/12/a^2*b*d*x^6+33/64/a^2*b*c*x^5-113/384/a*e*x^3-11/32/a*d*x^2-51
/128/a*c*x)/(b*x^4-a)^3+77/512*(a/b)^(1/4)/a^4*c*ln((x+(a/b)^(1/4))/(x-(a/b
)^(1/4)))+77/256*(a/b)^(1/4)/a^4*c*arctan(1/(a/b)^(1/4)*x)-5/64/(a*b)^(1/2)
/a^3*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-15/256/a^3*e/b/(a/b)^(1
/4)*arctan(1/(a/b)^(1/4)*x)+15/512/a^3*e/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(
x-(a/b)^(1/4)))
```

maxima [A] time = 3.02, size = 279, normalized size = 1.32

$$-\frac{45\,b^{2}ex^{11}+60\,b^{2}dx^{10}+77\,b^{2}cx^{9}-126\,abex^{7}-160\,abdx^{6}-198\,abcx^{5}+113\,a^{2}ex^{3}+132\,a^{2}dx^{2}+153\,a^{2}cx}{384\left(a^{3}b^{3}x^{12}-3\,a^{4}b^{2}x^{8}+3\,a^{5}bx^{4}-a^{6}\right)}+\frac{\frac{40\,d\log\left(\sqrt{b}x^{2}+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^{2}-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,\sqrt{b}c+15\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(45*b^2*e*x^11 + 60*b^2*d*x^10 + 77*b^2*c*x^9 - 126*a*b*e*x^7 - 160*
a*b*d*x^6 - 198*a*b*c*x^5 + 113*a^2*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/(a
^3*b^3*x^12 - 3*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^6) + 1/512*(40*d*log(sqrt(b)*
```

```
x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)
*sqrt(b)) + 2*(77*sqrt(b)*c - 15*sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*sqrt(b)*c + 15*sqrt(
a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqr
t(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^3
```

mupad [B] time = 5.22, size = 874, normalized size = 4.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a - b*x^4)^4,x)
```

```
[Out] ((11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9
)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) - (33*b*c
*x^5)/(64*a^2) - (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x
^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log(-(b*(3375*a*e^3 + 123200*b*c*
d^2 - 88935*b*c^2*e + 64000*b*d^3*x + 20185088*root(68719476736*a^15*b^3*z^
4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2
*c + 115200*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 83
8860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*
b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*b*e^2*x - 92400*b*c*d*e*x + 3035648*root
(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*
d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*
c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*
a^2*e^4, z, k)*a^3*b^2*c^2*x - 10485760*root(68719476736*a^15*b^3*z^4 - 121
1105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2
*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 +
 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x -
614400*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 8388608
00*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7
392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c
^4 - 50625*a^2*e^4, z, k)*a^4*b*d*e))/(2097152*a^9))*root(68719476736*a^15*
b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 48570368
0*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*
a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k), k,
 1, 4)
```

sympy [B] time = 59.74, size = 612, normalized size = 2.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(-1211105280*a**8*b**2*c*e - 8
38860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 + 485703680*a**4*b**2
*c**2*d) - 50625*a**2*e**4 + 2668050*a*b*c**2*e**2 - 7392000*a*b*c*d**2*e +
 2560000*a*b*d**4 - 35153041*b**2*c**4, Lambda(_t, _t*log(x + (452984832000
*_t**3*a**13*b**2*e**3 + 11936653639680*_t**3*a**12*b**3*c**2*e - 330712481
79200*_t**3*a**12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2 - 503
316480000*_t**2*a**9*b**2*d**3*e - 4787095470080*_t**2*a**8*b**3*c**3*d - 5
987520000*_t*a**6*b*c*e**4 - 8294400000*_t*a**6*b*d**2*e**3 - 210370406400*
_t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*c**2*d**2*e + 2018508800
00*_t*a**5*b**2*c*d**4 - 1385873488384*_t*a**4*b**3*c**5 + 91125000*a**3*d*
e**5 - 5544000000*a**2*b*c*d**3*e**2 + 3072000000*a**2*b*d**5*e + 105459123
```

```
000*a*b**2*c**4*d*e - 146090560000*a*b**2*c**3*d**3)/(11390625*a**3*e**6 +
300155625*a**2*b*c**2*e**4 - 3326400000*a**2*b*c*d**2*e**3 + 2304000000*a**
2*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b**2*c**3*d**2*
e - 60712960000*a*b**2*c**2*d**4 - 208422380089*b**3*c**6)))) + (-153*a**2*
c*x - 132*a**2*d*x**2 - 113*a**2*e*x**3 + 198*a*b*c*x**5 + 160*a*b*d*x**6 +
 126*a*b*e*x**7 - 77*b**2*c*x**9 - 60*b**2*d*x**10 - 45*b**2*e*x**11)/(-384
*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

3.85 $\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^4}\,dx$

Optimal. Leaf size=372

$$-\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}-$$

Rubi [A] time = 0.38, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}-\frac{\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/
(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4
)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b
]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a
^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a
]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a
^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^4}\,dx = \frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{\left(a+bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{\int \frac{77c+60dx+45ex^2}{\left(a+bx^4\right)^2}\,dx}{96a^2}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{\int \frac{-231c-120dx-45ex}{a+bx^4}}{384a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{\int \left(-\frac{120dx}{a+bx^4} + \frac{-231c}{a+}\right.}{384a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{\int \frac{-231c-45ex^2}{a+bx^4}\,dx}{384a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} + \frac{(5d)\,\text{Subst}\left(\int \frac{1}{a+b}\right.}{32a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.58, size = 369, normalized size = 0.99

$$\frac{\frac{3\sqrt{2}\left(15a^{3/4}e-77\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{3\sqrt{2}\left(77\sqrt[4]{a}\sqrt{b}c-15a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{256a^3x(c+x(d+ex))}{\left(a+bx^4\right)^3} + \frac{32a^2x(11c+x(10d+9ex))}{\left(a+bx^4\right)^2} - \frac{6\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e+77\sqrt{2}\sqrt{b}c\right)}{b^{3/4}} + \frac{6\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e+77\sqrt{2}\sqrt{b}c\right)}{b^{3/4}} + \frac{8ax(77c+15x(4d+3ex))}{a+bx^4}}{3072a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

```
[Out] ((256*a^3*x*(c + x*(d + e*x)))/(a + b*x^4)^3 + (8*a*x*(77*c + 15*x*(4*d + 3
*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a + b*x^4)^2 -
(6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a
]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^(1/4)*(77*Sqrt
[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*Sqrt[b]*c + 15
*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4)
 + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 373, normalized size = 1.00

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d+77\left(ab^3\right)^{\frac{1}{4}}b^2c+15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d+77\left(ab^3\right)^{\frac{1}{4}}b^2c+15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}}b^2c-15\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}-\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}}b^2c-15\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}+\frac{45\,b^2x^{11}e+60\,b^2dx^{10}+77\,b^2cx^9+126\,abx^7e+160\,abdx^6+198\,abcx^5+113\,a^2x^3e+132\,a^2dx^2+153\,a^2cx}{384\left(bx^4+a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2
*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqr
t(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^2*x^11*e + 60*b^2*d*x^10 + 77*
b^2*c*x^9 + 126*a*b*x^7*e + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 113*a^2*x^3*e +
 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

maple [A] time = 0.06, size = 394, normalized size = 1.06

$$\frac{5d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{32\sqrt{ab}\,a^3}+\frac{15\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}+\frac{15\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}+\frac{15\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{512a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{512a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024a^4}+\frac{\frac{15b^2e\,x^{11}}{128a^3}+\frac{5b^2d\,x^{10}}{32a^3}+\frac{77b^2c\,x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{\left(b\,x^4+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (15/128/a^3*b^2*e*x^11+5/32/a^3*b^2*d*x^10+77/384/a^3*b^2*c*x^9+21/64/a^2*b
*e*x^7+5/12/a^2*b*d*x^6+33/64/a^2*b*c*x^5+113/384/a*e*x^3+11/32/a*d*x^2+51/
128/a*c*x)/(b*x^4+a)^3+77/1024*(a/b)^(1/4)*2^(1/2)/a^4*c*ln((x^2+(a/b)^(1/4
)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+77/512*(a
/b)^(1/4)*2^(1/2)/a^4*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+77/512*(a/b)^(1/4)*
2^(1/2)/a^4*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+5/32/(a*b)^(1/2)/a^3*d*arctan
((1/a*b)^(1/2)*x^2)+15/1024/a^3*e/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)
*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+15/512/a^3
*e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+15/512/a^3*e/b/(a/
b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.11, size = 383, normalized size = 1.03

$$\frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9+126\,abex^7+160\,abdx^6+198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153\,a^2cx}{384\left(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6\right)}+\frac{\frac{\sqrt{2}\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e-80\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(77\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e+80\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{1024\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(45*b^2*e*x^11 + 60*b^2*d*x^10 + 77*b^2*c*x^9 + 126*a*b*e*x^7 + 160*a
*b*d*x^6 + 198*a*b*c*x^5 + 113*a^2*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/(a^
3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6) + 1/1024*(sqrt(2)*(77*sqrt(
b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2
- sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^
(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e - 80*sqrt(a)*sqrt(b)*d)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*
c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e + 80*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)
*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(b))*b^(3/4)))/a^3
```

mupad [B] time = 5.14, size = 873, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^4)^4,x)
```

```
[Out] ((11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9
)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (33*b*c
*x^5)/(64*a^2) + (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x
^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log(-(b*(3375*a*e^3 - 123200*b*c*
d^2 + 88935*b*c^2*e - 64000*b*d^3*x + 20185088*root(68719476736*a^15*b^3*z^
4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2
*c - 115200*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 83
8860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*
b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*b*e^2*x + 92400*b*c*d*e*x + 3035648*root
(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*
d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*
c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*
a^2*e^4, z, k)*a^3*b^2*c^2*x - 10485760*root(68719476736*a^15*b^3*z^4 + 121
1105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2
*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 +
 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x +
614400*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 8388608
00*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7
392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c
^4 + 50625*a^2*e^4, z, k)*a^4*b*d*e))/(2097152*a^9))*root(68719476736*a^15*
b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 48570368
0*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*
a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k), k,
 1, 4)
```

sympy [A] time = 63.47, size = 610, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(1211105280*a**8*b**2*c*e + 83
8860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 - 485703680*a**4*b**2*
c**2*d) + 50625*a**2*e**4 + 2668050*a*b*c**2*e**2 - 7392000*a*b*c*d**2*e +
2560000*a*b*d**4 + 35153041*b**2*c**4, Lambda(_t, _t*log(x + (452984832000*
_t**3*a**13*b**2*e**3 - 11936653639680*_t**3*a**12*b**3*c**2*e + 3307124817
9200*_t**3*a**12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2 - 5033
16480000*_t**2*a**9*b**2*d**3*e + 4787095470080*_t**2*a**8*b**3*c**3*d + 59
87520000*_t*a**6*b*c*e**4 + 8294400000*_t*a**6*b*d**2*e**3 - 210370406400*_
t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*c**2*d**2*e + 20185088000
0*_t*a**5*b**2*c*d**4 + 1385873488384*_t*a**4*b**3*c**5 + 91125000*a**3*d*e
**5 + 5544000000*a**2*b*c*d**3*e**2 - 3072000000*a**2*b*d**5*e + 1054591230
00*a*b**2*c**4*d*e - 146090560000*a*b**2*c**3*d**3)/(11390625*a**3*e**6 - 3
00155625*a**2*b*c**2*e**4 + 3326400000*a**2*b*c*d**2*e**3 - 2304000000*a**2
*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b**2*c**3*d**2*e
 - 60712960000*a*b**2*c**2*d**4 + 208422380089*b**3*c**6)))) + (153*a**2*c*
x + 132*a**2*d*x**2 + 113*a**2*e*x**3 + 198*a*b*c*x**5 + 160*a*b*d*x**6 + 1
26*a*b*e*x**7 + 77*b**2*c*x**9 + 60*b**2*d*x**10 + 45*b**2*e*x**11)/(384*a*
*6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

3.86 $\int a\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=28

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 194}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[a*(e + f*x^4)^2,x]
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int a\left(e+fx^4\right)^2\,dx &= a\int\left(e+fx^4\right)^2\,dx\\ &= a\int\left(e^2+2efx^4+f^2x^8\right)\,dx\\ &= ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 0.96

$$a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*(e + f*x^4)^2,x]
[Out] a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int a\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a*(e + f*x^4)^2,x]
[Out] IntegrateAlgebraic[a*(e + f*x^4)^2, x]
```

fricas [A]   time = 0.36, size = 24, normalized size = 0.86

$$\frac{1}{9}x^9f^2a + \frac{2}{5}x^5fea + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="fricas")
[Out] 1/9*x^9*f^2*a + 2/5*x^5*f*e*a + x*e^2*a
```

giac [A]   time = 0.14, size = 25, normalized size = 0.89

$$\frac{1}{45}\left(5f^2x^9 + 18fx^5e + 45xe^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="giac")
[Out] 1/45*(5*f^2*x^9 + 18*f*x^5*e + 45*x*e^2)*a
```

maple [A]   time = 0.04, size = 24, normalized size = 0.86

$$\left(\frac{1}{9}f^2x^9 + \frac{2}{5}efx^5 + e^2x\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*(f*x^4+e)^2,x)
[Out] a*(1/9*f^2*x^9+2/5*e*f*x^5+e^2*x)
```

maxima [A]   time = 1.37, size = 25, normalized size = 0.89

$$\frac{1}{45}\left(5f^2x^9 + 18efx^5 + 45e^2x\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="maxima")
[Out] 1/45*(5*f^2*x^9 + 18*e*f*x^5 + 45*e^2*x)*a
```

mupad [B]   time = 4.67, size = 25, normalized size = 0.89

$$\frac{ax\left(45e^2 + 18efx^4 + 5f^2x^8\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*(e + f*x^4)^2,x)
[Out] (a*x*(45*e^2 + 5*f^2*x^8 + 18*e*f*x^4))/45
```

sympy [A]   time = 0.12, size = 27, normalized size = 0.96

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x**4+e)**2,x)
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9
```

3.87 $\int bx\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {12, 270}

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[b*x*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int bx\left(e+fx^4\right)^2\,dx &= b\int x\left(e+fx^4\right)^2\,dx\\ &= b\int\left(e^2x+2efx^5+f^2x^9\right)dx\\ &= \frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 0.97

$$b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x*(e + f*x^4)^2,x]
```

```
[Out] b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int bx\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[b*x*(e + f*x^4)^2,x]
```

[Out] IntegrateAlgebraic[b*x*(e + f*x^4)^2, x]

fricas [A] time = 0.35, size = 27, normalized size = 0.82

$$\frac{1}{10}x^{10}f^2b + \frac{1}{3}x^6feb + \frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="fricas")
[Out] 1/10*x^10*f^2*b + 1/3*x^6*f*e*b + 1/2*x^2*e^2*b
```

giac [A] time = 0.17, size = 27, normalized size = 0.82

$$\frac{1}{30}\left(3f^2x^{10} + 10fx^6e + 15x^2e^2\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="giac")
[Out] 1/30*(3*f^2*x^10 + 10*f*x^6*e + 15*x^2*e^2)*b
```

maple [A] time = 0.04, size = 27, normalized size = 0.82

$$\left(\frac{1}{10}f^2x^{10} + \frac{1}{3}efx^6 + \frac{1}{2}e^2x^2\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x*(f*x^4+e)^2,x)
[Out] b*(1/10*f^2*x^10+1/3*e*f*x^6+1/2*e^2*x^2)
```

maxima [A] time = 1.40, size = 27, normalized size = 0.82

$$\frac{1}{30}\left(3f^2x^{10} + 10efx^6 + 15e^2x^2\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="maxima")
[Out] 1/30*(3*f^2*x^10 + 10*e*f*x^6 + 15*e^2*x^2)*b
```

mupad [B] time = 0.03, size = 27, normalized size = 0.82

$$\frac{b\,x^2\,\left(15\,e^2 + 10\,e\,f\,x^4 + 3\,f^2\,x^8\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x*(e + f*x^4)^2,x)
[Out] (b*x^2*(15*e^2 + 3*f^2*x^8 + 10*e*f*x^4))/30
```

sympy [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x**4+e)**2,x)
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10
```

3.88 $\int (a+bx)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=60

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1850}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(e+fx^4\right)^2\,dx &= \int \left(ae^2+be^2x+2aefx^4+2befx^5+af^2x^8+bf^2x^9\right)dx\\ &= ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.00

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(e + f*x^4)^2, x]
```

fricas [A] time = 0.37, size = 50, normalized size = 0.83

$$\frac{1}{10}x^{10}f^2b+\frac{1}{9}x^9f^2a+\frac{1}{3}x^6feb+\frac{2}{5}x^5fea+\frac{1}{2}x^2e^2b+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*f^2*b + 1/9*x^9*f^2*a + 1/3*x^6*f*e*b + 2/5*x^5*f*e*a + 1/2*x^2*e
^2*b + x*e^2*a
```

giac [A] time = 0.15, size = 50, normalized size = 0.83

$$\frac{1}{10}\,bf^2x^{10} + \frac{1}{9}\,af^2x^9 + \frac{1}{3}\,bfx^6e + \frac{2}{5}\,afx^5e + \frac{1}{2}\,bx^2e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*f*x^6*e + 2/5*a*f*x^5*e + 1/2*b*x^2
*e^2 + a*x*e^2
```

maple [A] time = 0.05, size = 51, normalized size = 0.85

$$\frac{1}{10}b\,f^2x^{10} + \frac{1}{9}a\,f^2x^9 + \frac{1}{3}bef\,x^6 + \frac{2}{5}aef\,x^5 + \frac{1}{2}b\,e^2x^2 + a\,e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(f*x^4+e)^2,x)
```

```
[Out] a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*
x^10
```

maxima [A] time = 1.36, size = 50, normalized size = 0.83

$$\frac{1}{10}\,bf^2x^{10} + \frac{1}{9}\,af^2x^9 + \frac{1}{3}\,befx^6 + \frac{2}{5}\,aefx^5 + \frac{1}{2}\,be^2x^2 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^2
*x^2 + a*e^2*x
```

mupad [B] time = 0.02, size = 50, normalized size = 0.83

$$\frac{b\,e^2\,x^2}{2} + a\,e^2\,x + \frac{b\,e\,f\,x^6}{3} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{b\,f^2\,x^{10}}{10} + \frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(a + b*x),x)
```

```
[Out] (b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + a*e^2*x + (2*a*e*f*x^5)/5
 + (b*e*f*x^6)/3
```

sympy [A] time = 0.11, size = 58, normalized size = 0.97

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10
```

3.89 $\int cx^2 \left(e+fx^4\right)^2 \, dx$

Optimal. Leaf size=33

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 270}

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[c*x^2*(e + f*x^4)^2,x]
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int cx^2\left(e+fx^4\right)^2\,dx &= c\int x^2\left(e+fx^4\right)^2\,dx\\ &= c\int\left(e^2x^2+2efx^6+f^2x^{10}\right)dx\\ &= \frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[c*x^2*(e + f*x^4)^2,x]
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int cx^2\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[c*x^2*(e + f*x^4)^2,x]
[Out] IntegrateAlgebraic[c*x^2*(e + f*x^4)^2, x]
```

fricas [A] time = 0.37, size = 27, normalized size = 0.82

$$\frac{1}{11}x^{11}f^2c + \frac{2}{7}x^7fec + \frac{1}{3}x^3e^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="fricas")
[Out] 1/11*x^11*f^2*c + 2/7*x^7*f*e*c + 1/3*x^3*e^2*c
```

giac [A] time = 0.20, size = 27, normalized size = 0.82

$$\frac{1}{231}\left(21\,f^2x^{11} + 66\,fx^7e + 77\,x^3e^2\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="giac")
[Out] 1/231*(21*f^2*x^11 + 66*f*x^7*e + 77*x^3*e^2)*c
```

maple [A] time = 0.04, size = 27, normalized size = 0.82

$$\left(\frac{1}{11}f^2x^{11} + \frac{2}{7}ef\,x^7 + \frac{1}{3}e^2x^3\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2*(f*x^4+e)^2,x)
[Out] c*(1/11*f^2*x^11+2/7*e*f*x^7+1/3*e^2*x^3)
```

maxima [A] time = 1.36, size = 27, normalized size = 0.82

$$\frac{1}{231}\left(21\,f^2x^{11} + 66\,efx^7 + 77\,e^2x^3\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="maxima")
[Out] 1/231*(21*f^2*x^11 + 66*e*f*x^7 + 77*e^2*x^3)*c
```

mupad [B] time = 0.04, size = 27, normalized size = 0.82

$$\frac{c\,x^3\,\left(77\,e^2 + 66\,e\,f\,x^4 + 21\,f^2\,x^8\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2*(e + f*x^4)^2,x)
[Out] (c*x^3*(77*e^2 + 21*f^2*x^8 + 66*e*f*x^4))/231
```

sympy [A] time = 0.13, size = 31, normalized size = 0.94

$$\frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2*(f*x**4+e)**2,x)
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

3.90 $\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=60

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(e + f*x^4)^2,x]

[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx&=\int \left(ae^2+ce^2x^2+2aefx^4+2cefx^6+af^2x^8+cf^2x^{10}\right)dx\\&=ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.00

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(e + f*x^4)^2,x]

[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)*(e + f*x^4)^2,x]

[Out] IntegrateAlgebraic[(a + c*x^2)*(e + f*x^4)^2, x]
```

fricas [A] time = 0.37, size = 50, normalized size = 0.83

$$\frac{1}{11}x^{11}f^2c+\frac{1}{9}x^9f^2a+\frac{2}{7}x^7fec+\frac{2}{5}x^5fea+\frac{1}{3}x^3e^2c+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 1/9*x^9*f^2*a + 2/7*x^7*f*e*c + 2/5*x^5*f*e*a + 1/3*x^3*e
^2*c + x*e^2*a
```

giac [A] time = 0.15, size = 50, normalized size = 0.83

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{9}\,af^2x^9 + \frac{2}{7}\,cfx^7e + \frac{2}{5}\,afx^5e + \frac{1}{3}\,cx^3e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 2/5*a*f*x^5*e + 1/3*c*x^3
*e^2 + a*x*e^2
```

maple [A] time = 0.04, size = 51, normalized size = 0.85

$$\frac{1}{11}c\,f^2x^{11} + \frac{1}{9}a\,f^2x^9 + \frac{2}{7}cef\,x^7 + \frac{2}{5}aef\,x^5 + \frac{1}{3}c\,e^2x^3 + a\,e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(f*x^4+e)^2,x)
```

```
[Out] a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2*
x^11
```

maxima [A] time = 1.32, size = 50, normalized size = 0.83

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{9}\,af^2x^9 + \frac{2}{7}\,cefx^7 + \frac{2}{5}\,aefx^5 + \frac{1}{3}\,ce^2x^3 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^2
*x^3 + a*e^2*x
```

mupad [B] time = 0.03, size = 50, normalized size = 0.83

$$\frac{c\,e^2\,x^3}{3} + a\,e^2\,x + \frac{2\,c\,e\,f\,x^7}{7} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{c\,f^2\,x^{11}}{11} + \frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(e + f*x^4)^2,x)
```

```
[Out] (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (c*f^2*x^11)/11 + a*e^2*x + (2*a*e*f*x^5)/5
 + (2*c*e*f*x^7)/7
```

sympy [A] time = 0.08, size = 60, normalized size = 1.00

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
+ c*f**2*x**11/11
```

3.91 $\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Rubi [A] time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1593, 1620}

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx &= \int x(b+cx)\left(e+fx^4\right)^2\,dx\\
&= \int \left(be^2x+ce^2x^2+2befx^5+2cefx^6+bf^2x^9+cf^2x^{10}\right)dx\\
&= \frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(b*x + c*x^2)*(e + f*x^4)^2, x]
```

fricas [A] time = 0.37, size = 53, normalized size = 0.82

$$\frac{1}{11}x^{11}f^2c + \frac{1}{10}x^{10}f^2b + \frac{2}{7}x^7fec + \frac{1}{3}x^6feb + \frac{1}{3}x^3e^2c + \frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 1/10*x^10*f^2*b + 2/7*x^7*f*e*c + 1/3*x^6*f*e*b + 1/3*x^3
*e^2*c + 1/2*x^2*e^2*b
```

giac [A] time = 0.15, size = 53, normalized size = 0.82

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{10}\,bf^2x^{10} + \frac{2}{7}\,cfx^7e + \frac{1}{3}\,bfx^6e + \frac{1}{3}\,cx^3e^2 + \frac{1}{2}\,bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*f*x^7*e + 1/3*b*f*x^6*e + 1/3*c*x
^3*e^2 + 1/2*b*x^2*e^2
```

maple [A] time = 0.04, size = 54, normalized size = 0.83

$$\frac{1}{11}c\,f^2x^{11} + \frac{1}{10}b\,f^2x^{10} + \frac{2}{7}cef\,x^7 + \frac{1}{3}bef\,x^6 + \frac{1}{3}c\,e^2x^3 + \frac{1}{2}b\,e^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(f*x^4+e)^2,x)
```

```
[Out] 1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/1
1*c*f^2*x^11
```

maxima [A] time = 1.35, size = 53, normalized size = 0.82

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{10}\,bf^2x^{10} + \frac{2}{7}\,cefx^7 + \frac{1}{3}\,befx^6 + \frac{1}{3}\,ce^2x^3 + \frac{1}{2}\,be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*e
^2*x^3 + 1/2*b*e^2*x^2
```

mupad [B] time = 0.03, size = 53, normalized size = 0.82

$$\frac{c\,e^2\,x^3}{3} + \frac{b\,e^2\,x^2}{2} + \frac{2\,c\,e\,f\,x^7}{7} + \frac{b\,e\,f\,x^6}{3} + \frac{c\,f^2\,x^{11}}{11} + \frac{b\,f^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)*(e + f*x^4)^2,x)
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (b*e*f*
x^6)/3 + (2*c*e*f*x^7)/7
```

sympy [A] time = 0.14, size = 61, normalized size = 0.94

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x*
*7/7 + c*f**2*x**11/11
```

3.92 $\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=92

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1657}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2\,dx &= \int \left(ae^2+be^2x+ce^2x^2+2aefx^4+2befx^5+2cefx^6+af^2x^8+bf^2x^9+cf^2x^{10}\right.\\ &= ae^2x+\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 92, normalized size = 1.00

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)*(e + f*x^4)^2, x]
```

fricas [A] time = 0.37, size = 76, normalized size = 0.83

$$\frac{1}{11}x^{11}f^2c+\frac{1}{10}x^{10}f^2b+\frac{1}{9}x^9f^2a+\frac{2}{7}x^7fec+\frac{1}{3}x^6feb+\frac{2}{5}x^5fea+\frac{1}{3}x^3e^2c+\frac{1}{2}x^2e^2b+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 1/10*x^10*f^2*b + 1/9*x^9*f^2*a + 2/7*x^7*f*e*c + 1/3*x^6
*f*e*b + 2/5*x^5*f*e*a + 1/3*x^3*e^2*c + 1/2*x^2*e^2*b + x*e^2*a
```

giac [A] time = 0.15, size = 76, normalized size = 0.83

$$\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cfx^7e+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{3}\,cx^3e^2+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 1/3*b*f
*x^6*e + 2/5*a*f*x^5*e + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2 + a*x*e^2
```

maple [A] time = 0.04, size = 77, normalized size = 0.84

$$\frac{1}{11}c\,f^2x^{11}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{2}{7}cef\,x^7+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{3}c\,e^2x^3+\frac{1}{2}b\,e^2x^2+a\,e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(f*x^4+e)^2,x)
```

```
[Out] a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2/7*c*e*f*x
^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

maxima [A] time = 1.43, size = 76, normalized size = 0.83

$$\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*e
*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x
```

mupad [B] time = 0.04, size = 76, normalized size = 0.83

$$\frac{c\,e^2\,x^3}{3}+\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{2\,c\,e\,f\,x^7}{7}+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{c\,f^2\,x^{11}}{11}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(a + b*x + c*x^2),x)
```

```
[Out] (b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (c*f^2*x^
11)/11 + a*e^2*x + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7
```

sympy [A] time = 0.16, size = 90, normalized size = 0.98

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

3.93 $\int dx^3 \left(e+fx^4\right)^2 dx$

Optimal. Leaf size=17

$$\frac{d\left(e+fx^4\right)^3}{12f}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 261}

$$\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[d*x^3*(e + f*x^4)^2,x]
```

```
[Out] (d*(e + f*x^4)^3)/(12*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int dx^3 \left(e+fx^4\right)^2 dx &= d\int x^3 \left(e+fx^4\right)^2 dx\\ &= \frac{d\left(e+fx^4\right)^3}{12f}\end{aligned}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.94

$$\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[d*x^3*(e + f*x^4)^2,x]
```

```
[Out] (d*e^2*x^4)/4 + (d*e*f*x^8)/4 + (d*f^2*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int dx^3 \left(e+fx^4\right)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[d*x^3*(e + f*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[d*x^3*(e + f*x^4)^2, x]
```

fricas [A] time = 0.38, size = 27, normalized size = 1.59

$$\frac{1}{12}x^{12}f^2d + \frac{1}{4}x^8fed + \frac{1}{4}x^4e^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="fricas")
[Out] 1/12*x^12*f^2*d + 1/4*x^8*f*e*d + 1/4*x^4*e^2*d
```

giac [A] time = 0.14, size = 16, normalized size = 0.94

$$\frac{\left(fx^4 + e\right)^3 d}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="giac")
[Out] 1/12*(f*x^4 + e)^3*d/f
```

maple [A] time = 0.04, size = 27, normalized size = 1.59

$$\left(\frac{1}{12}f^2x^{12} + \frac{1}{4}ef\,x^8 + \frac{1}{4}e^2x^4\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3*(f*x^4+e)^2,x)
[Out] d*(1/12*f^2*x^12+1/4*e*f*x^8+1/4*e^2*x^4)
```

maxima [A] time = 1.39, size = 15, normalized size = 0.88

$$\frac{\left(fx^4 + e\right)^3 d}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="maxima")
[Out] 1/12*(f*x^4 + e)^3*d/f
```

mupad [B] time = 0.03, size = 26, normalized size = 1.53

$$\frac{d\,x^4\,\left(3\,e^2 + 3\,e\,f\,x^4 + f^2\,x^8\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3*(e + f*x^4)^2,x)
[Out] (d*x^4*(3*e^2 + f^2*x^8 + 3*e*f*x^4))/12
```

sympy [B] time = 0.24, size = 29, normalized size = 1.71

$$\frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3*(f*x**4+e)**2,x)
[Out] d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

3.94 $\int \left(a+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=45

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d\left(e+fx^4\right)^3}{12f}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1582, 12, 194}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}\int \left(a+dx^3\right)\left(e+fx^4\right)^2\,dx &= \frac{d\left(e+fx^4\right)^3}{12f}+\int a\left(e+fx^4\right)^2\,dx\\ &= \frac{d\left(e+fx^4\right)^3}{12f}+a\int \left(e+fx^4\right)^2\,dx\\ &= \frac{d\left(e+fx^4\right)^3}{12f}+a\int \left(e^2+2efx^4+f^2x^8\right)dx\\ &= ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d\left(e+fx^4\right)^3}{12f}\end{aligned}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.33

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 +
 (d*f^2*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+dx^3\right)\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + d*x^3)*(e + f*x^4)^2, x]
```

fricas [A] time = 0.35, size = 50, normalized size = 1.11

$$\frac{1}{12}x^{12}f^2d+\frac{1}{9}x^9f^2a+\frac{1}{4}x^8fed+\frac{2}{5}x^5fea+\frac{1}{4}x^4e^2d+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/9*x^9*f^2*a + 1/4*x^8*f*e*d + 2/5*x^5*f*e*a + 1/4*x^4*e
^2*d + x*e^2*a
```

giac [A] time = 0.20, size = 50, normalized size = 1.11

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,dfx^8e+\frac{2}{5}\,afx^5e+\frac{1}{4}\,dx^4e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/5*a*f*x^5*e + 1/4*d*x^4
*e^2 + a*x*e^2
```

maple [A] time = 0.04, size = 51, normalized size = 1.13

$$\frac{1}{12}d\,f^2x^{12}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+a\,e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+a)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/5*a*e*f*x^5+1/4*d*e^2*x^4+a*e
^2*x
```

maxima [A] time = 1.31, size = 50, normalized size = 1.11

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^2
*x^4 + a*e^2*x
```

mupad [**B**] time = 0.02, size = 50, normalized size = 1.11

$$\frac{d\,e^2\,x^4}{4}+a\,e^2\,x+\frac{d\,e\,f\,x^8}{4}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{d\,f^2\,x^{12}}{12}+\frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + d*x^3)*(e + f*x^4)^2,x)
```

```
[Out] (a*f^2*x^9)/9 + (d*e^2*x^4)/4 + (d*f^2*x^12)/12 + a*e^2*x + (2*a*e*f*x^5)/5
 + (d*e*f*x^8)/4
```

sympy [**A**] time = 0.08, size = 58, normalized size = 1.29

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + d*e**2*x**4/4 + d*e*f*x**8/4 +
d*f**2*x**12/12
```

3.95 $\int \left(bx + dx^3\right) \left(e + fx^4\right)^2 \, dx$

Optimal. Leaf size=50

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d\left(e + fx^4\right)^3}{12f}$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1582, 12, 270}

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d\left(e + fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}
\int \left(bx + dx^3\right) \left(e + fx^4\right)^2 \, dx &= \frac{d\left(e + fx^4\right)^3}{12f} + \int bx\left(e + fx^4\right)^2 \, dx \\
&= \frac{d\left(e + fx^4\right)^3}{12f} + b\int x\left(e + fx^4\right)^2 \, dx \\
&= \frac{d\left(e + fx^4\right)^3}{12f} + b\int \left(e^2x + 2efx^5 + f^2x^9\right) \, dx \\
&= \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d\left(e + fx^4\right)^3}{12f}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.30

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4 + (b*f^2*x^10
)/10 + (d*f^2*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx+dx^3\right)\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(b*x + d*x^3)*(e + f*x^4)^2, x]
```

fricas [A] time = 0.36, size = 53, normalized size = 1.06

$$\frac{1}{12}x^{12}f^2d+\frac{1}{10}x^{10}f^2b+\frac{1}{4}x^8fed+\frac{1}{3}x^6feb+\frac{1}{4}x^4e^2d+\frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/10*x^10*f^2*b + 1/4*x^8*f*e*d + 1/3*x^6*f*e*b + 1/4*x^4
*e^2*d + 1/2*x^2*e^2*b
```

giac [A] time = 0.16, size = 53, normalized size = 1.06

$$\frac{1}{12}df^2x^{12}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}dfx^8e+\frac{1}{3}bfx^6e+\frac{1}{4}dx^4e^2+\frac{1}{2}bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 1/3*b*f*x^6*e + 1/4*d*x
^4*e^2 + 1/2*b*x^2*e^2
```

maple [A] time = 0.04, size = 54, normalized size = 1.08

$$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/4*d*e*f*x^8+1/3*b*e*f*x^6+1/4*d*e^2*x^4+1
/2*b*e^2*x^2
```

maxima [A] time = 1.32, size = 53, normalized size = 1.06

$$\frac{1}{12}df^2x^{12}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{2}be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*e
^2*x^4 + 1/2*b*e^2*x^2
```

mupad [B] time = 0.03, size = 53, normalized size = 1.06

$$\frac{d\,e^2\,x^4}{4}+\frac{b\,e^2\,x^2}{2}+\frac{d\,e\,f\,x^8}{4}+\frac{b\,e\,f\,x^6}{3}+\frac{d\,f^2\,x^{12}}{12}+\frac{b\,f^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)*(e + f*x^4)^2,x)
```

```
[Out] (b*e^2*x^2)/2 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (d*f^2*x^12)/12 + (b*e*f*
x^6)/3 + (d*e*f*x^8)/4
```

sympy [A] time = 0.12, size = 60, normalized size = 1.20

$$\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8
/4 + d*f**2*x**12/12
```

3.96 $\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=77

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1582, 1850}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx &= \frac{d\left(e+fx^4\right)^3}{12f}+\int (a+bx)\left(e+fx^4\right)^2\,dx\\ &= \frac{d\left(e+fx^4\right)^3}{12f}+\int \left(ae^2+be^2x+2aefx^4+2befx^5+af^2x^8+bf^2x^9\right)\,dx\\ &= ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{d\left(e+fx^4\right)^3}{12f}\end{aligned}$$

Mathematica [A] time = 0.00, size = 92, normalized size = 1.19

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*f^2*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + d*x^3)*(e + f*x^4)^2, x]
```

fricas [A] time = 0.37, size = 76, normalized size = 0.99

$$\frac{1}{12}x^{12}f^2d+\frac{1}{10}x^{10}f^2b+\frac{1}{9}x^9f^2a+\frac{1}{4}x^8fed+\frac{1}{3}x^6feb+\frac{2}{5}x^5fea+\frac{1}{4}x^4e^2d+\frac{1}{2}x^2e^2b+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/10*x^10*f^2*b + 1/9*x^9*f^2*a + 1/4*x^8*f*e*d + 1/3*x^6
*f*e*b + 2/5*x^5*f*e*a + 1/4*x^4*e^2*d + 1/2*x^2*e^2*b + x*e^2*a
```

giac [A] time = 0.15, size = 76, normalized size = 0.99

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,dfx^8e+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{4}\,dx^4e^2+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 1/3*b*f
*x^6*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/2*b*x^2*e^2 + a*x*e^2
```

maple [A] time = 0.04, size = 77, normalized size = 1.00

$$\frac{1}{12}df^2x^{12}+\frac{1}{10}bf^2x^{10}+\frac{1}{9}af^2x^9+\frac{1}{4}defx^8+\frac{1}{3}befx^6+\frac{2}{5}aefx^5+\frac{1}{4}de^2x^4+\frac{1}{2}be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x+a)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/9*a*f^2*x^9+1/4*d*e*f*x^8+1/3*b*e*f*x^6+2
/5*a*e*f*x^5+1/4*d*e^2*x^4+1/2*b*e^2*x^2+a*e^2*x
```

maxima [A] time = 1.36, size = 76, normalized size = 0.99

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*e
*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

mupad [B] time = 0.04, size = 76, normalized size = 0.99

$$\frac{d\,e^2\,x^4}{4}+\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{d\,e\,f\,x^8}{4}+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{d\,f^2\,x^{12}}{12}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(a + b*x + d*x^3),x)
```

```
[Out] (b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (d*f^2*x^
12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4
```

sympy [A] time = 0.08, size = 88, normalized size = 1.14

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

3.97 $\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1582, 12, 270}

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}
\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx &= \frac{d\left(e+fx^4\right)^3}{12f}+\int cx^2\left(e+fx^4\right)^2\,dx\\
&= \frac{d\left(e+fx^4\right)^3}{12f}+c\int x^2\left(e+fx^4\right)^2\,dx\\
&= \frac{d\left(e+fx^4\right)^3}{12f}+c\int \left(e^2x^2+2efx^6+f^2x^{10}\right)dx\\
&= \frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.30

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (c*f^2*x^
11)/11 + (d*f^2*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c*x^2 + d*x^3)*(e + f*x^4)^2, x]
```

fricas [A] time = 0.36, size = 53, normalized size = 1.06

$$\frac{1}{12}x^{12}f^2d+\frac{1}{11}x^{11}f^2c+\frac{1}{4}x^8fed+\frac{2}{7}x^7fec+\frac{1}{4}x^4e^2d+\frac{1}{3}x^3e^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/11*x^11*f^2*c + 1/4*x^8*f*e*d + 2/7*x^7*f*e*c + 1/4*x^4
*e^2*d + 1/3*x^3*e^2*c
```

giac [A] time = 0.15, size = 53, normalized size = 1.06

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,dfx^8e+\frac{2}{7}\,cfx^7e+\frac{1}{4}\,dx^4e^2+\frac{1}{3}\,cx^3e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*f*x^8*e + 2/7*c*f*x^7*e + 1/4*d*x
^4*e^2 + 1/3*c*x^3*e^2
```

maple [A] time = 0.04, size = 54, normalized size = 1.08

$$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{1}{4}d\,e^2x^4+\frac{1}{3}c\,e^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/4*d*e*f*x^8+2/7*c*e*f*x^7+1/4*d*e^2*x^4+1
/3*c*e^2*x^3
```

maxima [A] time = 1.36, size = 53, normalized size = 1.06

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*e
^2*x^4 + 1/3*c*e^2*x^3
```

mupad [B] time = 0.03, size = 53, normalized size = 1.06

$$\frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(c*x^2 + d*x^3),x)
```

```
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12 + (2*c*e*
f*x^7)/7 + (d*e*f*x^8)/4
```

sympy [A] time = 0.08, size = 61, normalized size = 1.22

$$\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2)*(f*x**4+e)**2,x)
```

```
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x*
*8/4 + d*f**2*x**12/12
```

3.98 $\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=77

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1582, 1154}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx &= \frac{d\left(e+fx^4\right)^3}{12f}+\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx\\
&= \frac{d\left(e+fx^4\right)^3}{12f}+\int \left(ae^2+ce^2x^2+2aefx^4+2cefx^6+af^2x^8+cf^2x^{10}\right)dx\\
&= ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 92, normalized size = 1.19

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7
 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^2 + d*x^3)*(e + f*x^4)^2, x]
```

fricas [A] time = 0.37, size = 76, normalized size = 0.99

$$\frac{1}{12}x^{12}f^2d+\frac{1}{11}x^{11}f^2c+\frac{1}{9}x^9f^2a+\frac{1}{4}x^8fed+\frac{2}{7}x^7fec+\frac{2}{5}x^5fea+\frac{1}{4}x^4e^2d+\frac{1}{3}x^3e^2c+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/11*x^11*f^2*c + 1/9*x^9*f^2*a + 1/4*x^8*f*e*d + 2/7*x^7
*f*e*c + 2/5*x^5*f*e*a + 1/4*x^4*e^2*d + 1/3*x^3*e^2*c + x*e^2*a
```

giac [A] time = 0.20, size = 76, normalized size = 0.99

$$\frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{9}af^2x^9+\frac{1}{4}dfx^8e+\frac{2}{7}cfx^7e+\frac{2}{5}afx^5e+\frac{1}{4}dx^4e^2+\frac{1}{3}cx^3e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/7*c*f
*x^7*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2 + a*x*e^2
```

maple [A] time = 0.04, size = 77, normalized size = 1.00

$$\frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{9}af^2x^9+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{2}{5}aefx^5+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+a)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/7*c*e*f*x^7+2
/5*a*e*f*x^5+1/4*d*e^2*x^4+1/3*c*e^2*x^3+a*e^2*x
```

maxima [A] time = 1.34, size = 76, normalized size = 0.99

$$\frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{9}af^2x^9+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{2}{5}aefx^5+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*e
*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

mupad [B] time = 0.04, size = 76, normalized size = 0.99

$$\frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+a\,e^2\,x+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}+\frac{a\,f^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(a + c*x^2 + d*x^3),x)
```

```
[Out] (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (c*f^2*x^11)/11 + (d*f^2*x^
12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4
```

sympy [A] time = 0.15, size = 90, normalized size = 1.17

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
+ c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

3.99 $\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1582, 1593, 1620}

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx &= \frac{d\left(e+fx^4\right)^3}{12f}+\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx\\
&= \frac{d\left(e+fx^4\right)^3}{12f}+\int x(b+cx)\left(e+fx^4\right)^2\,dx\\
&= \frac{d\left(e+fx^4\right)^3}{12f}+\int \left(be^2x+ce^2x^2+2befx^5+2cefx^6+bf^2x^9+cf^2x^{10}\right)\,dx\\
&= \frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 97, normalized size = 1.18

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11} + \frac{1}{4}de^2x^4 + \frac{1}{4}defx^8 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (2*c*e*f*x^
7)/7 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx + cx^2 + dx^3\right)\left(e + fx^4\right)^2\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2, x]
```

fricas [A] time = 0.35, size = 79, normalized size = 0.96

$$\frac{1}{12}x^{12}f^2d + \frac{1}{11}x^{11}f^2c + \frac{1}{10}x^{10}f^2b + \frac{1}{4}x^8fed + \frac{2}{7}x^7fec + \frac{1}{3}x^6feb + \frac{1}{4}x^4e^2d + \frac{1}{3}x^3e^2c + \frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/11*x^11*f^2*c + 1/10*x^10*f^2*b + 1/4*x^8*f*e*d + 2/7*x
^7*f*e*c + 1/3*x^6*f*e*b + 1/4*x^4*e^2*d + 1/3*x^3*e^2*c + 1/2*x^2*e^2*b
```

giac [A] time = 0.16, size = 79, normalized size = 0.96

$$\frac{1}{12}df^2x^{12} + \frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{1}{4}dfx^8e + \frac{2}{7}cfx^7e + \frac{1}{3}bfx^6e + \frac{1}{4}dx^4e^2 + \frac{1}{3}cx^3e^2 + \frac{1}{2}bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 2/7*c
*f*x^7*e + 1/3*b*f*x^6*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2
```

maple [A] time = 0.04, size = 80, normalized size = 0.98

$$\frac{1}{12}df^2x^{12} + \frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{1}{4}defx^8 + \frac{2}{7}cefx^7 + \frac{1}{3}befx^6 + \frac{1}{4}de^2x^4 + \frac{1}{3}ce^2x^3 + \frac{1}{2}be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/10*b*f^2*x^10+1/4*d*e*f*x^8+2/7*c*e*f*x^7
+1/3*b*e*f*x^6+1/4*d*e^2*x^4+1/3*c*e^2*x^3+1/2*b*e^2*x^2
```

maxima [A] time = 1.42, size = 79, normalized size = 0.96

$$\frac{1}{12}df^2x^{12} + \frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{1}{4}defx^8 + \frac{2}{7}cefx^7 + \frac{1}{3}befx^6 + \frac{1}{4}de^2x^4 + \frac{1}{3}ce^2x^3 + \frac{1}{2}be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*c
*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

mupad [B] time = 0.04, size = 79, normalized size = 0.96

$$\frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+\frac{b\,e^2\,x^2}{2}+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{b\,e\,f\,x^6}{3}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}+\frac{b\,f^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x^4)^2*(b*x + c*x^2 + d*x^3),x)
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (c*f^2*x^
11)/11 + (d*f^2*x^12)/12 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4
```

sympy [A] time = 0.11, size = 92, normalized size = 1.12

$$\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x*
*7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

3.100 $\int (c + dx + ex^2 + fx^3) (a + bx^4)^2 \, dx$

Optimal. Leaf size=109

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f(a+bx^4)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Rubi [A] time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1582, 1657}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f(a+bx^4)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 +
 (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(
a + b*x^4)^3)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx &= \frac{f(a+bx^4)^3}{12b}+\int (c+dx+ex^2)(a+bx^4)^2\,dx\\ &= \frac{f(a+bx^4)^3}{12b}+\int (a^2c+a^2dx+a^2ex^2+2abcx^4+2abdx^5+2abex^6+b^2\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9+\frac{1}{10}b\end{aligned}$$

Mathematica [A] time = 0.01, size = 124, normalized size = 1.14

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x
^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx + ex^2 + fx^3\right)\left(a + bx^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2, x]
```

fricas [A] time = 0.35, size = 102, normalized size = 0.94

$$\frac{1}{12}x^{12}fb^2 + \frac{1}{11}x^{11}eb^2 + \frac{1}{10}x^{10}db^2 + \frac{1}{9}x^9cb^2 + \frac{1}{4}x^8fba + \frac{2}{7}x^7eba + \frac{1}{3}x^6dba + \frac{2}{5}x^5cba + \frac{1}{4}x^4fa^2 + \frac{1}{3}x^3ea^2 + \frac{1}{2}x^2da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
[Out] 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b^2 + 1/4*x
^8*f*b*a + 2/7*x^7*e*b*a + 1/3*x^6*d*b*a + 2/5*x^5*c*b*a + 1/4*x^4*f*a^2 +
1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2
```

giac [A] time = 0.17, size = 105, normalized size = 0.96

$$\frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2x^{11}e + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abx^7e + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2x^3e + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

maple [A] time = 0.04, size = 103, normalized size = 0.94

$$\frac{1}{12}b^2f\,x^{12} + \frac{1}{11}b^2e\,x^{11} + \frac{1}{10}b^2d\,x^{10} + \frac{1}{9}b^2c\,x^9 + \frac{1}{4}abf\,x^8 + \frac{2}{7}abe\,x^7 + \frac{1}{3}abd\,x^6 + \frac{2}{5}abc\,x^5 + \frac{1}{4}a^2f\,x^4 + \frac{1}{3}a^2e\,x^3 + \frac{1}{2}a^2d\,x^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*f*a*b*x^8
+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*f*a^2*x^4+1/3*a^2*e*x^3+1/2*
a^2*d*x^2+a^2*c*x
```

maxima [A] time = 1.39, size = 102, normalized size = 0.94

$$\frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2ex^{11} + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abex^7 + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

mupad [B] time = 4.68, size = 102, normalized size = 0.94

$$\frac{f\,a^2\,x^4}{4}+\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{f\,a\,b\,x^8}{4}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{f\,b^2\,x^{12}}{12}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^9)/9 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x^
4)/4 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12 + a^2*c*x + (2*a*b*c*x^5)/5 + (a*b
*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4
```

sympy [A] time = 0.09, size = 121, normalized size = 1.11

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{a^2fx^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{abfx^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{b^2fx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5 +
 a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**1
0/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

3.101 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx$

Optimal. Leaf size=151

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b^3cx$$

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1582, 1657}

$$\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)
/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e
*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b
*x^4)^4)/(16*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx &= \frac{f\left(a+bx^4\right)^4}{16b}+\int \left(c+dx+ex^2\right)\left(a+bx^4\right)^3\,dx\\ &= \frac{f\left(a+bx^4\right)^4}{16b}+\int \left(a^3c+a^3dx+a^3ex^2+3a^2bcx^4+3a^2bdx^5+3a^2bex\right.\\ &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx\end{aligned}$$

Mathematica [A] time = 0.01, size = 180, normalized size = 1.19

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/5
 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9)/
3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*c*x
^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx + ex^2 + fx^3\right)\left(a + bx^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3, x]
```

fricas [A] time = 0.34, size = 150, normalized size = 0.99

$$\frac{1}{16}x^{16}fb^3 + \frac{1}{15}x^{15}eb^3 + \frac{1}{14}x^{14}db^3 + \frac{1}{13}x^{13}cb^3 + \frac{1}{4}x^{12}fb^2a + \frac{3}{11}x^{11}eb^2a + \frac{3}{10}x^{10}db^2a + \frac{1}{3}x^9cb^2a + \frac{3}{8}x^8fba^2 + \frac{3}{7}x^7eba^2 + \frac{1}{2}x^6dba^2 + \frac{3}{5}x^5cba^2 + \frac{1}{4}x^4fa^3 + \frac{1}{3}x^3ea^3 + \frac{1}{2}x^2da^3 + xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*f*b^3 + 1/15*x^15*e*b^3 + 1/14*x^14*d*b^3 + 1/13*x^13*c*b^3 + 1/4
*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/10*x^10*d*b^2*a + 1/3*x^9*c*b^2*a + 3
/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*
x^4*f*a^3 + 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3
```

giac [A] time = 0.17, size = 154, normalized size = 1.02

$$\frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3x^{15}e + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2x^{11}e + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bx^7e + \frac{1}{2}a^2bdx^6 + \frac{3}{5}a^2bcx^5 + \frac{1}{4}a^3fx^4 + \frac{1}{3}a^3x^3e + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

maple [A] time = 0.05, size = 151, normalized size = 1.00

$$\frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3ex^{15} + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2ex^{11} + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^2bdx^6 + \frac{3}{5}a^2bcx^5 + \frac{1}{4}a^3fx^4 + \frac{1}{3}a^3ex^3 + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/16*b^3*f*x^16+1/15*b^3*e*x^15+1/14*b^3*d*x^14+1/13*b^3*c*x^13+1/4*a*b^2*f
*x^12+3/11*a*b^2*e*x^11+3/10*a*b^2*d*x^10+1/3*a*b^2*c*x^9+3/8*f*a^2*b*x^8+3
/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*f*a^3*x^4+1/3*a^3*e*x^3+
1/2*a^3*d*x^2+a^3*c*x
```

maxima [A] time = 1.34, size = 150, normalized size = 0.99

$$\frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3ex^{15} + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2ex^{11} + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^2bdx^6 + \frac{3}{5}a^2bcx^5 + \frac{1}{4}a^3fx^4 + \frac{1}{3}a^3ex^3 + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
```

```
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

mupad [B] time = 4.86, size = 150, normalized size = 0.99

$$\frac{f\,a^3\,x^4}{4}+\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,f\,a^2\,b\,x^8}{8}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{f\,a\,b^2\,x^{12}}{4}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{f\,b^3\,x^{16}}{16}+\frac{e\,b^3\,x^{15}}{15}+\frac{d\,b^3\,x^{14}}{14}+\frac{c\,b^3\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^13)/13 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f*
x^4)/4 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16 + a^3*c*x + (3*a^2*b*c*x^5)/5 +
(a*b^2*c*x^9)/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7)/7
 + (3*a*b^2*e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

sympy [A] time = 0.13, size = 180, normalized size = 1.19

$$a^{3}cx+\frac{a^{3}dx^{2}}{2}+\frac{a^{3}ex^{3}}{3}+\frac{a^{3}fx^{4}}{4}+\frac{3a^{2}bcx^{5}}{5}+\frac{a^{2}bdx^{6}}{2}+\frac{3a^{2}bex^{7}}{7}+\frac{3a^{2}bfx^{8}}{8}+\frac{ab^{2}cx^{9}}{3}+\frac{3ab^{2}dx^{10}}{10}+\frac{3ab^{2}ex^{11}}{11}+\frac{ab^{2}fx^{12}}{4}+\frac{b^{3}cx^{13}}{13}+\frac{b^{3}dx^{14}}{14}+\frac{b^{3}ex^{15}}{15}+\frac{b^{3}fx^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5/
5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**9
/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3*c*
x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

3.102 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2} dx$

Optimal. Leaf size=155

$$\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)}$$

Rubi [A] time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1854, 1876, 275, 208, 1167, 205}

$$\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

```
[Out] (a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt
[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^2}\,dx = \frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-3c-2dx-ex^2}{a-bx^4}\,dx}{4a}$$

$$= \frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)} - \frac{\int \left(-\frac{2dx}{a-bx^4}+\frac{-3c-ex^2}{a-bx^4}\right)dx}{4a}$$

$$= \frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-3c-ex^2}{a-bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a-bx^4}\,dx}{2a}$$

$$= \frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)} + \frac{d\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)}{4a} - \frac{\left(\frac{3\sqrt{b}\,c}{\sqrt{a}}-e\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx}{8a}$$

$$= \frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)} + \frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\right.}{8a^{7/4}b^{3/4}}$$

Mathematica [A] time = 0.23, size = 220, normalized size = 1.42

$$\frac{-\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)\left(a^{3/4}e+3\sqrt[4]{a}\,\sqrt{b}\,c+2\sqrt{a}\,\sqrt[4]{b}\,d\right)+\sqrt[4]{b}\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)\left(a^{3/4}e+3\sqrt[4]{a}\,\sqrt{b}\,c-2\sqrt{a}\,\sqrt[4]{b}\,d\right)+\frac{4a(af+bx(c+x(d+ex)))}{a-bx^4}-2\sqrt[4]{a}\,\sqrt[4]{b}\left(\sqrt{a}\,e-3\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\sqrt{a}\,\sqrt{b}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{16a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*(a*f + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*a^(1/4)*b^(1/4)*(-3*Sq
rt[b]*c + Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - b^(1/4)*(3*a^(1/4)*Sqrt[
b]*c + 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*
(3*a^(1/4)*Sqrt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/
4)*x] + 2*Sqrt[a]*Sqrt[b]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^2*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.23, size = 320, normalized size = 2.06

$$-\frac{\sqrt{2}\left(3\,b^{2}c-2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c+2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}+\frac{\sqrt{2}\left(3\,b^{2}c-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{bx^{3}e+bdx^{2}+bcx+af}{4\left(bx^{4}-a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*
a) - 1/16*sqrt(2)*(3*b^2*c + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3
/4)*a) - 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)
^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b
)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) -
1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*f)/((b*x^4 - a)*a*b)
```

maple [B] time = 0.05, size = 248, normalized size = 1.60

$$-\frac{f\,x^{4}}{4\left(b\,x^{4}-a\right)a}-\frac{e\,x^{3}}{4\left(b\,x^{4}-a\right)a}-\frac{d\,x^{2}}{4\left(b\,x^{4}-a\right)a}-\frac{cx}{4\left(b\,x^{4}-a\right)a}-\frac{d\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{8\sqrt{ab}\,a}-\frac{e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] -1/4/(b*x^4-a)/a*c*x+3/16*(a/b)^(1/4)/a^2*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))+3/8*(a/b)^(1/4)/a^2*c*arctan(1/(a/b)^(1/4)*x)-1/4/(b*x^4-a)/a*d*x^2-1/
8/(a*b)^(1/2)/a*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-1/4/(b*x^4-a
)/a*e*x^3-1/8/(a/b)^(1/4)/a/b*e*arctan(1/(a/b)^(1/4)*x)+1/16/(a/b)^(1/4)/a/
b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4*f*x^4/a/(b*x^4-a)
```

maxima [A] time = 3.00, size = 200, normalized size = 1.29

$$-\frac{bex^{3}+bdx^{2}+bcx+af}{4\left(ab^{2}x^{4}-a^{2}b\right)}+\frac{\frac{2\,d\log\left(\sqrt{b}x^{2}+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{2\,d\log\left(\sqrt{b}x^{2}-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b*e*x^3 + b*d*x^2 + b*c*x + a*f)/(a*b^2*x^4 - a^2*b) + 1/16*(2*d*log(
sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(
sqrt(a)*sqrt(b)) + 2*(3*sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*sqrt(b)*c + sqrt(a
)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt
(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a
```

mupad [B] time = 0.41, size = 483, normalized size = 3.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(- root(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d
^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b
```

```
*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4
 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128
*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4
- a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 + 4*a^2*b^2*e^2)
)/(16*a^3) - (b^2*d*e)/a) - (12*b^2*c*d^2 - 9*b^2*c^2*e + a*b*e^3)/(64*a^3)
 - (x*(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3))*root(65536*a^7*b^3*z^4 - 3072*a^
4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e
^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4,
 z, k), k, 1, 4) + (f/(4*b) + (d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/
(a - b*x^4)
```

sympy [B] time = 24.17, size = 520, normalized size = 3.35

$$\operatorname{RootSum}\left(65536t^4a^7b^3 + t^2\left(-3072a^4b^2ce - 2048a^4b^2d^2\right) + t\left(128a^3bde^2 + 1152a^2b^2c^2d\right) - a^2e^4 + 18abc^2e^2 - 48abcd^2e + 16abd^4 - 81b^2c^4, \left(t \mapsto t\log\left(x + \frac{4096t^3a^7b^2e^3 + 36864t^3a^6b^3c^2e - 98304t^3a^6b^3cd^2 + 4608t^2a^5b^2cde^2 - 4096t^2a^5b^2d^3e - 13824t^2a^4b^3c^3d - 144ta^4bce^4 - 192ta^4bd^2e^3 - 1728ta^3b^2c^3e^2 + 5184ta^3b^2c^2d^2e + 1536ta^3b^2cd^4 - 3888ta^2b^3c^5 + 6a^3de^5 - 120a^2bcd^3e^2 + 64a^2bd^5e + 810ab^2c^4de - 1080ab^2c^3d^3}{a^3e^6 + 9a^2bc^2e^4 - 96a^2bcd^2e^3 + 64a^2bd^4e^2 - 81ab^2c^4e^2 + 864ab^2c^3d^2e - 576ab^2c^2d^4 - 729b^3c^6}\right)\right)\right) + \frac{-af - bcx - bdx^2 - bex^3}{-4a^2b + 4ab^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*a*
b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _t*l
og(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 98304*_t
**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2
*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4
*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e
+ 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 - 120*a
**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c*
*3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3 + 64*a**2*
b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**
2*d**4 - 729*b**3*c**6)))) + (-a*f - b*c*x - b*d*x**2 - b*e*x**3)/(-4*a**2*
b + 4*a*b**2*x**4)
```

3.103 $\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx$

Optimal. Leaf size=188

$$\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+}{8}$$

Rubi [A] time = 0.15, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*
x^2))/(8*a*b*(a - b*x^4)^2) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)
*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh
[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx &= \frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}-\frac{\int \frac{-7c-6dx-5ex^2}{\left(a-bx^4\right)^2}\,dx}{8a}\\
&= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{\int \frac{21c+12dx+5ex^2}{a-bx^4}\,dx}{32a^2}\\
&= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{\int \left(\frac{12dx}{a-bx^4}+\frac{21c+5ex^2}{a-bx^4}\right)dx}{32a^2}\\
&= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{\int \frac{21c+5ex^2}{a-bx^4}\,dx}{32a^2}+\frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}\\
&= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{(3d)\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{16a^2}-\ldots\\
&= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{\left(\frac{21\sqrt{b}\,c}{\sqrt{a}}-5e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{3/4}}+\ldots
\end{aligned}
$$

Mathematica [A] time = 0.26, size = 253, normalized size = 1.35

$$
\frac{-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{b}\,c+12\sqrt{a}\sqrt[4]{b}\,d\right)}{b^{3/4}}+\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{b}\,c-12\sqrt{a}\sqrt[4]{b}\,d\right)}{b^{3/4}}+\frac{16a^2(af+bx(c+x(d+ex)))}{b\left(a-bx^4\right)^2}+\frac{2\sqrt[4]{a}\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{4ax(7c+x(6d+5ex))}{a-bx^4}+\frac{12\sqrt{a}\,d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt{b}}}{128a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a*x*(7*c + x*(6*d + 5*e*x)))/(a - b*x^4) + (16*a^2*(a*f + b*x*(c + x*(d
 + e*x))))/(b*(a - b*x^4)^2) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcT
an[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1
/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[
b]*c - 12*Sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4
) + (12*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [B] time = 0.20, size = 358, normalized size = 1.90

$$-\frac{\sqrt{2}\left(21\,b^{2}c-12\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c+12\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{\sqrt{2}\left(21\,b^{2}c-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}+\frac{\sqrt{2}\left(21\,b^{2}c-5\,\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^{3}\right)^{\frac{3}{4}}a^{2}}-\frac{5\,b^{2}x^{7}e+6\,b^{2}dx^{6}+7\,b^{2}cx^{5}-9\,abx^{3}e-10\,abdx^{2}-11\,abcx-4\,a^{2}f}{32\left(bx^{4}-a\right)^{2}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
[Out] -1/128*sqrt(2)*(21*b^2*c - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt(-a*b)*b*e
)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(
3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 5*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2
+ sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/256*sqrt(2)
*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 - 9
*a*b*x^3*e - 10*a*b*d*x^2 - 11*a*b*c*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

maple [B] time = 0.05, size = 326, normalized size = 1.73

$$\frac{f\,x^{4}}{8\left(b\,x^{4}-a\right)^{2}a}+\frac{e\,x^{3}}{8\left(b\,x^{4}-a\right)^{2}a}-\frac{f\,x^{4}}{8\left(b\,x^{4}-a\right)a^{2}}+\frac{d\,x^{2}}{8\left(b\,x^{4}-a\right)^{2}a}-\frac{5e\,x^{3}}{32\left(b\,x^{4}-a\right)a^{2}}+\frac{cx}{8\left(b\,x^{4}-a\right)^{2}a}-\frac{3d\,x^{2}}{16\left(b\,x^{4}-a\right)a^{2}}-\frac{7cx}{32\left(b\,x^{4}-a\right)a^{2}}-\frac{3d\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{32\sqrt{ab}\,a^{2}}-\frac{5e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{5e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
[Out] 1/8/(b*x^4-a)^2/a*c*x-7/32/(b*x^4-a)/a^2*c*x+21/128*(a/b)^(1/4)/a^3*c*ln((x
+(a/b)^(1/4))/(x-(a/b)^(1/4)))+21/64*(a/b)^(1/4)/a^3*c*arctan(1/(a/b)^(1/4)
*x)+1/8/(b*x^4-a)^2/a*d*x^2-3/16/(b*x^4-a)/a^2*d*x^2-3/32/(a*b)^(1/2)/a^2*d
*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))+1/8/(b*x^4-a)^2/a*e*x^3-5/32/
(b*x^4-a)/a^2*e*x^3-5/64/(a/b)^(1/4)/a^2/b*e*arctan(1/(a/b)^(1/4)*x)+5/128/
(a/b)^(1/4)/a^2/b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/8*f*x^4/a/(b*x^4-
a)^2-1/8*f/a^2*x^4/(b*x^4-a)
```

maxima [A] time = 2.96, size = 249, normalized size = 1.32

$$-\frac{5\,b^{2}ex^{7}+6\,b^{2}dx^{6}+7\,b^{2}cx^{5}-9\,abex^{3}-10\,abdx^{2}-11\,abcx-4\,a^{2}f}{32\left(a^{2}b^{3}x^{8}-2\,a^{3}b^{2}x^{4}+a^{4}b\right)}+\frac{\frac{12\,d\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{12\,d\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(21\,\sqrt{b}\,c-5\,\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(21\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{128\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
[Out] -1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 - 9*a*b*e*x^3 - 10*a*b*d*x^2
 - 11*a*b*c*x - 4*a^2*f)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(12*
```

```
d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 12*d*log(sqrt(b)*x^2 - sqr
t(a))/(sqrt(a)*sqrt(b)) + 2*(21*sqrt(b)*c - 5*sqrt(a)*e)*arctan(sqrt(b)*x/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*sqrt(b)
*c + 5*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt
(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^2
```

mupad [B] time = 5.18, size = 832, normalized size = 4.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x)
```

```
[Out] symsum(log(-(b*(125*a*e^3 + 3024*b*c*d^2 - 2205*b*c^2*e + 1728*b*d^3*x + 34
4064*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^
2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^
2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z,
k)^2*a^5*b^2*c + 3200*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2
 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z
 - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 19
4481*b^2*c^4, z, k)*a^3*b*e^2*x - 2520*b*c*d*e*x + 56448*root(268435456*a^1
1*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3
*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2
 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x - 1966
08*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*
d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*
e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)
^2*a^5*b^2*d*x - 15360*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^
2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*
z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 1
94481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*z^4 -
 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*
z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*
b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k), k, 1, 4) + (f/(8*b) + (5*d*x^2
)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^2) - (3*b
*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4)
```

sympy [B] time = 116.92, size = 583, normalized size = 3.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**3 + _t**2*(-6881280*a**6*b**2*c*e - 47185
92*a**6*b**2*d**2) + _t*(-153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d)
- 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4
 - 194481*b**2*c**4, Lambda(_t, _t*log(x + (-262144000*_t**3*a**10*b**2*e**
3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2
+ 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e - 1
820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a*
*5*b*d**2*e**3 + 118540800*_t*a**4*b**2*c**3*e**2 - 365783040*_t*a**4*b**2*
c**2*d**2*e - 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 +
 112500*a**3*d*e**5 - 4536000*a**2*b*c*d**3*e**2 + 2488320*a**2*b*d**5*e +
58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 + 27
5625*a**2*b*c**2*e**4 - 3024000*a**2*b*c*d**2*e**3 + 2073600*a**2*b*d**4*e*
*2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b*
*2*c**2*d**4 - 85766121*b**3*c**6)))) - (-4*a**2*f - 11*a*b*c*x - 10*a*b*d*
x**2 - 9*a*b*e*x**3 + 7*b**2*c*x**5 + 6*b**2*d*x**6 + 5*b**2*e*x**7)/(32*a*
*4*b - 64*a**3*b**2*x**4 + 32*a**2*b**3*x**8)
```

3.104 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=220

$$\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+$$

Rubi [A] time = 0.19, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x +
45*e*x^2))/(384*a^3*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*x^2))/(12*a*b*(a
 - b*x^4)^3) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(256*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)
/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(3
2*a^(7/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^4}\,dx &= \frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}-\frac{\int \frac{-11c-10dx-9ex^2}{\left(a-bx^4\right)^3}\,dx}{12a}\\
&= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}+\frac{\int \frac{77c+60dx+45ex^2}{\left(a-bx^4\right)^2}\,dx}{96a^2}\\
&= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}-\int \cdots\\
&= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}-\int \Big(\cdots\\
&= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}-\int \cdots\\
&= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}+\frac{(5d)\cdots}{\cdots}\\
&= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}+\frac{(77\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.50, size = 286, normalized size = 1.30

$$
\frac{-\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c+40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c-40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}-\frac{128a^3(af+bx(c+x(d+ex)))}{b\left(bx^4-a\right)^3}+\frac{16a^2x(11c+x(10d+9ex))}{\left(a-bx^4\right)^2}+\frac{6\sqrt[4]{a}\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{4ax(77c+15x(4d+3ex))}{a-bx^4}+\frac{120\sqrt{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{b}}}{1536a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*
d + 9*e*x)))/(a - b*x^4)^2 - (128*a^3*(a*f + b*x*(c + x*(d + e*x))))/(b*(-a
 + b*x^4)^3) + (6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/b^(3/4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^
(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*
```

```
Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x]])/b^(3/4) + (120*
Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.19, size = 395, normalized size = 1.80

$$-\frac{\sqrt{2}\left(77\,b^2c-40\,\sqrt{2}\,\left(-ab^3\right)^{\frac{1}{4}}bd+15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{\sqrt{2}\left(77\,b^2c+40\,\sqrt{2}\,\left(-ab^3\right)^{\frac{1}{4}}bd-15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{\sqrt{2}\left(77\,b^2c-15\,\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,\left(-ab^3\right)^{\frac{3}{4}}a^3}+\frac{\sqrt{2}\left(77\,b^2c-15\,\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{45\,b^3x^{11}e+60\,b^3dx^{10}+77\,b^3cx^9-126\,ab^2x^7e-160\,ab^2dx^6-198\,ab^2cx^5+113\,a^2bx^3e+132\,a^2bdx^2+153\,a^2bcx+32\,a^3f}{384\,\left(bx^4-a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^
(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 15*s
qrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4)
)/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(
x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/1024*sq
rt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqr
t(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*b
^3*c*x^9 - 126*a*b^2*x^7*e - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 113*a^2*b*
x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

maple [A] time = 0.06, size = 280, normalized size = 1.27

$$-\frac{5d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{64\sqrt{ab}\,a^3}-\frac{15e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}+\frac{15e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4}+\frac{-\frac{15b^2e\,x^{11}}{128a^3}-\frac{5b^2d\,x^{10}}{32a^3}-\frac{77b^2c\,x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}-\frac{113e\,x^3}{384a}-\frac{11d\,x^2}{32a}-\frac{51cx}{128a}-\frac{f}{12b}}{\left(b\,x^4-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128/a^3*b^2*e*x^11-5/32/a^3*b^2*d*x^10-77/384/a^3*b^2*c*x^9+21/64/a^2*
b*e*x^7+5/12/a^2*b*d*x^6+33/64/a^2*b*c*x^5-113/384/a*e*x^3-11/32/a*d*x^2-51
/128/a*c*x-1/12*f/b)/(b*x^4-a)^3+77/512*(a/b)^(1/4)/a^4*c*ln((x+(a/b)^(1/4)
)/(x-(a/b)^(1/4)))+77/256*(a/b)^(1/4)/a^4*c*arctan(1/(a/b)^(1/4)*x)-5/64/(a
*b)^(1/2)/a^3*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-15/256/(a/b)^(
1/4)/a^3/b*e*arctan(1/(a/b)^(1/4)*x)+15/512/(a/b)^(1/4)/a^3/b*e*ln((x+(a/b)
^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 3.06, size = 297, normalized size = 1.35

$$-\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9-126\,ab^2ex^7-160\,ab^2dx^6-198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+153\,a^2bcx+32\,a^3f}{384\left(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b\right)}+\frac{\frac{40\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 126*a*b^2*e*x^7 - 16
0*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*a
^2*b*c*x + 32*a^3*f)/(a^3*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b)
 + 1/512*(40*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log(sqrt
(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*sqrt(b)*c - 15*sqrt(a)*e)*arct
an(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
 - (77*sqrt(b)*c + 15*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(s
qrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))
/a^3
```

mupad [B] time = 5.25, size = 880, normalized size = 4.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(-(b*(3375*a*e^3 + 123200*b*c*d^2 - 88935*b*c^2*e + 64000*b*d^3*x
 + 20185088*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 83
8860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*
b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2*c + 115200*root(68719476736*a^15*b
^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680
*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a
*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*
b*e^2*x - 92400*b*c*d*e*x + 3035648*root(68719476736*a^15*b^3*z^4 - 1211105
280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z
 + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 256
0000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^3*b^2*c^2*x - 1048
5760*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800
*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 739
2000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4
 - 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x - 614400*root(68719476736*a^15*b^3*z^
4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*b*d*e
))/(2097152*a^9))*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^
2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*
d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 351
53041*b^2*c^4 - 50625*a^2*e^4, z, k), k, 1, 4) + (f/(12*b) + (11*d*x^2)/(32
*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(384*a^3) + (
5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) - (33*b*c*x^5)/(64*a^2)
- (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x
^4 + 3*a*b^2*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

3.105 $\int \frac{a}{2+3x^4} \, dx$

Optimal. Leaf size=101

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}$$

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {12, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[a/(2 + 3*x^4),x]
```

```
[Out] -(a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/
4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{a}{2+3x^4}\,dx = a\int \frac{1}{2+3x^4}\,dx$$
$$= \frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} + \frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}}$$
$$= \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}}$$
$$= -\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} -$$
$$= -\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+\right.}{8\sqrt[4]{6}}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.77

$$\frac{a\left(-\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\tan^{-1}\left(\sqrt[4]{6}x+1\right)\right)}{8\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a/(2 + 3*x^4),x]
[Out] (a*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] - Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(1/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a/(2 + 3*x^4),x]
[Out] IntegrateAlgebraic[a/(2 + 3*x^4), x]
```

fricas [B] time = 0.45, size = 284, normalized size = 2.81

$$-\frac{1}{48}\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}\arctan\left(-\frac{4a^3+2\cdot 24^{\frac{1}{4}}\sqrt{2}\left(a^4\right)^{\frac{3}{4}}x-24^{\frac{1}{4}}\sqrt{2}\sqrt{\frac{1}{3}}\left(a^4\right)^{\frac{3}{4}}\sqrt{\frac{12a^2x^2+24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}ax+4\sqrt{6}\sqrt{a^4}}{a^2}}}{4a^3}\right)-\frac{1}{48}\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}\arctan\left(\frac{4a^3-2\cdot 24^{\frac{1}{4}}\sqrt{2}\left(a^4\right)^{\frac{3}{4}}x+24^{\frac{1}{4}}\sqrt{2}\sqrt{\frac{1}{3}}\left(a^4\right)^{\frac{3}{4}}\sqrt{\frac{12a^2x^2-24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}ax+4\sqrt{6}\sqrt{a^4}}{a^2}}}{4a^3}\right)+\frac{1}{192}\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}\log\left(12a^2x^2+24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}ax+4\sqrt{6}\sqrt{a^4}\right)-\frac{1}{192}\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}\log\left(12a^2x^2-24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}ax+4\sqrt{6}\sqrt{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/48*24^(3/4)*sqrt(2)*(a^4)^(1/4)*arctan(-1/4*(4*a^3 + 2*24^(1/4)*sqrt(2)*
(a^4)^(3/4)*x - 24^(1/4)*sqrt(2)*sqrt(1/3)*(a^4)^(3/4)*sqrt((12*a^2*x^2 + 2
4^(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4))/a^2))/a^3) - 1/48*24
^(3/4)*sqrt(2)*(a^4)^(1/4)*arctan(1/4*(4*a^3 - 2*24^(1/4)*sqrt(2)*(a^4)^(3/
4)*x + 24^(1/4)*sqrt(2)*sqrt(1/3)*(a^4)^(3/4)*sqrt((12*a^2*x^2 - 24^(3/4)*s
qrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4))/a^2))/a^3) + 1/192*24^(3/4)*s
qrt(2)*(a^4)^(1/4)*log(12*a^2*x^2 + 24^(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sq
rt(6)*sqrt(a^4)) - 1/192*24^(3/4)*sqrt(2)*(a^4)^(1/4)*log(12*a^2*x^2 - 24^(
3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4))
```

giac [A] time = 0.20, size = 97, normalized size = 0.96

$$\frac{1}{48}\left(2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+6^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-6^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/48*(2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) +
6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 6^(3/4)*log(x^2 - sq
rt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*a
```

maple [A] time = 0.04, size = 114, normalized size = 1.13

$$\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/24
*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48*a*3
^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(
x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))
```

maxima [A] time = 2.94, size = 123, normalized size = 1.22

$$\frac{1}{48}\left(2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+3^{\frac{3}{4}}2^{\frac{3}{4}}\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-3^{\frac{3}{4}}2^{\frac{3}{4}}\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/48*(2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
/4)*2^(3/4))) + 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(
2)) - 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*a
```

mupad [B] time = 0.12, size = 36, normalized size = 0.36

$$-\frac{(-1)^{1/4}\,6144^{3/4}\,a\,\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\,6144^{1/4}\,x}{8}\right)\,1\mathrm{i}+\mathrm{atanh}\left(\frac{(-1)^{1/4}\,6144^{1/4}\,x}{8}\right)\,1\mathrm{i}\right)}{3072}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a/(3*x^4 + 2),x)
```

[Out] -((-1)^(1/4)*6144^(3/4)*a*(atan(((-1)^(1/4)*6144^(1/4)*x)/8)*1i + atanh(((-1)^(1/4)*6144^(1/4)*x)/8)*1i))/3072

sympy [A] time = 0.44, size = 88, normalized size = 0.87

$$a\left(-\frac{6^{\frac{3}{4}}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}\,x-1\right)}{24}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}\,x+1\right)}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a/(3*x**4+2),x)

[Out] a*(-6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*log(x**2 + 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*atan(6**(1/4)*x - 1)/24 + 6**(3/4)*atan(6**(1/4)*x + 1)/24)

3.106 $\int \frac{bx}{2+3x^4} dx$

Optimal. Leaf size=22

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}} x^2\right)}{2\sqrt{6}}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {12, 275, 203}

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}} x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{bx}{2+3x^4} dx &= b \int \frac{x}{2+3x^4} dx \\ &= \frac{1}{2} b \operatorname{Subst}\left(\int \frac{1}{2+3x^2} dx, x, x^2\right) \\ &= \frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}} x^2\right)}{2\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}} x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x)/(2 + 3*x^4),x]
[Out] IntegrateAlgebraic[(b*x)/(2 + 3*x^4), x]
```

fricas [A] time = 0.41, size = 15, normalized size = 0.68

$$\frac{1}{12}\,\sqrt{6}\,b\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

giac [A] time = 0.17, size = 15, normalized size = 0.68

$$\frac{1}{12}\,\sqrt{6}\,b\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="giac")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

maple [A] time = 0.04, size = 16, normalized size = 0.73

$$\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x/(3*x^4+2),x)
[Out] 1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)
```

maxima [A] time = 2.88, size = 15, normalized size = 0.68

$$\frac{1}{12}\,\sqrt{6}\,b\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="maxima")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

mupad [B] time = 4.77, size = 15, normalized size = 0.68

$$\frac{\sqrt{6}\,b\,\mathrm{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)/(3*x^4 + 2),x)
```

```
[Out] (6^(1/2)*b*atan((6^(1/2)*x^2)/2))/12
```

sympy [**A**] time = 0.13, size = 19, normalized size = 0.86

$$\frac{\sqrt{6}\, b\, \mathrm{atan}\left(\frac{\sqrt{6}\, x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x**4+2),x)
```

```
[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12
```

3.107 $\int \frac{a+bx}{2+3x^4} \, dx$

Optimal. Leaf size=123

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\right.}{2\sqrt{6}}$$

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 203}

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4
)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{a+bx}{2+3x^4}\,dx = \int \left(\frac{a}{2+3x^4}+\frac{bx}{2+3x^4}\right)dx$$

$$= a\int \frac{1}{2+3x^4}\,dx + b\int \frac{x}{2+3x^4}\,dx$$

$$= \frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} + \frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}} \cdots}{-\sqrt{\frac{2}{3}}+\cdots}}{8\sqrt[4]{6}\cdots}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, \cdots\right.}{4\sqrt[4]{6}}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log \cdots}{}$$

Mathematica [A] time = 0.06, size = 107, normalized size = 0.87

$$\frac{-2\left(\sqrt[4]{6}a+2b\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt[4]{6}a-2b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)+\sqrt[4]{6}a\left(\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)\right)}{8\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(2 + 3*x^4),x]
```

```
[Out] (-2*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*(6^(1/4)*a - 2*b)*ArcTan[1
+ 6^(1/4)*x] + 6^(1/4)*a*(-Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + Log[2 + 2*6
^(1/4)*x + Sqrt[6]*x^2]))/(8*Sqrt[6])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(2 + 3*x^4),x]
[Out] IntegrateAlgebraic[(a + b*x)/(2 + 3*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.19, size = 115, normalized size = 0.93

$$\frac{1}{48}\cdot 6^{\frac{3}{4}} a \log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}} x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\cdot 6^{\frac{3}{4}} a \log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}} x+\sqrt{\frac{2}{3}}\right)+\frac{1}{24}\left(6^{\frac{3}{4}} a-2\sqrt{6} b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\left(6^{\frac{3}{4}} a+2\sqrt{6} b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="giac")
[Out] 1/48*6^(3/4)*a*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*6^(3/4)*
a*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(3/4)*a - 2*sqrt(6
)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^
(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^
(1/4)))
```

maple [A] time = 0.05, size = 129, normalized size = 1.05

$$\frac{\sqrt{3}\, 6^{\frac{1}{4}}\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\sqrt{3}\, 6^{\frac{3}{4}} x}{6}-1\right)}{24}+\frac{\sqrt{3}\, 6^{\frac{1}{4}}\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\sqrt{3}\, 6^{\frac{3}{4}} x}{6}+1\right)}{24}+\frac{\sqrt{3}\, 6^{\frac{1}{4}}\sqrt{2}\, a \ln\left(\frac{x^2+\frac{\sqrt{3}\, 6^{\frac{1}{4}}\sqrt{2}\, x}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\, 6^{\frac{1}{4}}\sqrt{2}\, x}{3}+\frac{\sqrt{6}}{3}}\right)}{48}+\frac{\sqrt{6}\, b \arctan\left(\frac{\sqrt{6}\, x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(3*x^4+2),x)
[Out] 1/24*3^(1/2)*6^(1/4)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/24
*3^(1/2)*6^(1/4)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48*3^(
1/2)*6^(1/4)*2^(1/2)*a*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(
x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/12*6^(1/2)*b*arctan(1/2*6
^(1/2)*x^2)
```

maxima [A] time = 2.91, size = 147, normalized size = 1.20

$$\frac{1}{48}\cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} a \log\left(\sqrt{3} x^2+3^{\frac{1}{4}} 2^{\frac{3}{4}} x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} a \log\left(\sqrt{3} x^2-3^{\frac{1}{4}} 2^{\frac{3}{4}} x+\sqrt{2}\right)+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}} 2^{\frac{3}{4}} a-2\sqrt{2} b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}}\left(2\sqrt{3} x+3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}} 2^{\frac{3}{4}} a+2\sqrt{2} b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}}\left(2\sqrt{3} x-3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="maxima")
[Out] 1/48*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) - 1/4
8*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*s
qrt(3)*(3^(1/4)*2^(3/4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt
```

```
(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*
arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

mupad [B] time = 0.20, size = 119, normalized size = 0.97

$$\frac{2^{3/4}\,3^{3/4}\,a\,\ln\left(x^2+\frac{6^{3/4}\,x}{3}+\frac{\sqrt{6}}{3}\right)}{48}-\frac{2^{3/4}\,3^{3/4}\,a\,\ln\left(x^2-\frac{6^{3/4}\,x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{2^{3/4}\,3^{3/4}\,a\,\mathrm{atan}\left(6^{1/4}\,x-1\right)}{24}+\frac{2^{3/4}\,3^{3/4}\,a\,\mathrm{atan}\left(6^{1/4}\,x+1\right)}{24}+\frac{\sqrt{2}\,\sqrt{3}\,b\,\mathrm{atan}\left(6^{1/4}\,x-1\right)}{12}-\frac{\sqrt{2}\,\sqrt{3}\,b\,\mathrm{atan}\left(6^{1/4}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(3*x^4 + 2),x)
```

```
[Out] (2^(3/4)*3^(3/4)*a*log((6^(3/4)*x)/3 + 6^(1/2)/3 + x^2))/48 - (2^(3/4)*3^(3
/4)*a*log(6^(1/2)/3 - (6^(3/4)*x)/3 + x^2))/48 + (2^(3/4)*3^(3/4)*a*atan(6^
(1/4)*x - 1))/24 + (2^(3/4)*3^(3/4)*a*atan(6^(1/4)*x + 1))/24 + (2^(1/2)*3^
(1/2)*b*atan(6^(1/4)*x - 1))/12 - (2^(1/2)*3^(1/2)*b*atan(6^(1/4)*x + 1))/1
2
```

sympy [A] time = 0.72, size = 88, normalized size = 0.72

$$\mathrm{RootSum}\left(18432t^4+384t^2b^2-96ta^2b+3a^4+2b^4,\left(t\mapsto t\log\left(x+\frac{3072t^3b^2+192t^2a^2b+24ta^4+32tb^4-10a^2b^3}{3a^5-8ab^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(18432*_t**4 + 384*_t**2*b**2 - 96*_t*a**2*b + 3*a**4 + 2*b**4, Lamb
da(_t, _t*log(x + (3072*_t**3*b**2 + 192*_t**2*a**2*b + 24*_t*a**4 + 32*_t*
b**4 - 10*a**2*b**3)/(3*a**5 - 8*a*b**4))))
```

3.108 $\int \frac{cx^2}{2+3x^4}\,dx$

Optimal. Leaf size=101

$$\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{2\,6^{3/4}}$$

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {12, 297, 1162, 617, 204, 1165, 628}

$$\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{2\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)/(2 + 3*x^4),x]
```

```
[Out] -(c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/
4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{cx^2}{2+3x^4}\,dx = c\int \frac{x^2}{2+3x^4}\,dx$$
$$= -\frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}$$
$$= \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\,6^{3/4}} + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-}}{4\,6^{3/4}}$$
$$= \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\,6^{3/4}} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}x\right)}{2\,6^{3/4}} - \frac{c\,\mathrm{Su}}{}$$
$$= -\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\,6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\,6^{3/4}}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.77

$$\frac{c\left(\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\tan^{-1}\left(\sqrt[4]{6}x+1\right)\right)}{4\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (c*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] + Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(4*6^(3/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2)/(2 + 3*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c*x^2)/(2 + 3*x^4), x]
```

fricas [B] time = 0.41, size = 278, normalized size = 2.75

$$-\frac{1}{108}\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^4\right)^{\frac{1}{4}}\arctan\left(-\frac{54^{\frac{3}{4}}\sqrt{2}\left(c^4\right)^{\frac{1}{4}}x-54^{\frac{3}{4}}\sqrt{2}\sqrt{\frac{1}{3}}\left(c^4\right)^{\frac{1}{4}}\sqrt{\frac{3c^3x^2+54^{\frac{1}{4}}\sqrt{2}\left(c^4\right)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c}{c^3}}+18c}{18c}\right)-\frac{1}{108}\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^4\right)^{\frac{1}{4}}\arctan\left(-\frac{54^{\frac{3}{4}}\sqrt{2}\left(c^4\right)^{\frac{1}{4}}x-54^{\frac{3}{4}}\sqrt{2}\sqrt{\frac{1}{3}}\left(c^4\right)^{\frac{1}{4}}\sqrt{\frac{3c^3x^2-54^{\frac{1}{4}}\sqrt{2}\left(c^4\right)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c}{c^3}}-18c}{18c}\right)-\frac{1}{432}\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^4\right)^{\frac{1}{4}}\log\left(9c^3x^2+3\cdot 54^{\frac{1}{4}}\sqrt{2}\left(c^4\right)^{\frac{3}{4}}x+3\sqrt{6}\sqrt{c^4}c\right)+\frac{1}{432}\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^4\right)^{\frac{1}{4}}\log\left(9c^3x^2-3\cdot 54^{\frac{1}{4}}\sqrt{2}\left(c^4\right)^{\frac{3}{4}}x+3\sqrt{6}\sqrt{c^4}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/108*54^(3/4)*sqrt(2)*(c^4)^(1/4)*arctan(-1/18*(54^(3/4)*sqrt(2)*(c^4)^(1
/4)*x - 54^(3/4)*sqrt(2)*sqrt(1/3)*(c^4)^(1/4)*sqrt((3*c^3*x^2 + 54^(1/4)*s
qrt(2)*(c^4)^(3/4)*x + sqrt(6)*sqrt(c^4)*c)/c^3) + 18*c)/c) - 1/108*54^(3/4
)*sqrt(2)*(c^4)^(1/4)*arctan(-1/18*(54^(3/4)*sqrt(2)*(c^4)^(1/4)*x - 54^(3/
4)*sqrt(2)*sqrt(1/3)*(c^4)^(1/4)*sqrt((3*c^3*x^2 - 54^(1/4)*sqrt(2)*(c^4)^(
3/4)*x + sqrt(6)*sqrt(c^4)*c)/c^3) - 18*c)/c) - 1/432*54^(3/4)*sqrt(2)*(c^4
)^(1/4)*log(9*c^3*x^2 + 3*54^(1/4)*sqrt(2)*(c^4)^(3/4)*x + 3*sqrt(6)*sqrt(c
^4)*c) + 1/432*54^(3/4)*sqrt(2)*(c^4)^(1/4)*log(9*c^3*x^2 - 3*54^(1/4)*sqrt
(2)*(c^4)^(3/4)*x + 3*sqrt(6)*sqrt(c^4)*c)
```

giac [A] time = 0.20, size = 97, normalized size = 0.96

$$\frac{1}{24}\left(2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-6^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)+6^{\frac{1}{4}}\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) -
6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 6^(1/4)*log(x^2 - sq
rt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*c
```

maple [A] time = 0.04, size = 114, normalized size = 1.13

$$\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{72}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{72}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2/(3*x^4+2),x)
```

```
[Out] 1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72
*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*c*
3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/
(x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))
```

maxima [A] time = 3.04, size = 123, normalized size = 1.22

$$\frac{1}{24}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*(2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
/4)*2^(3/4))) - 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(
2)) + 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*c
```

mupad [B] time = 4.97, size = 32, normalized size = 0.32

$$\frac{(-1)^{1/4}\,24^{1/4}\,c\,\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\,24^{1/4}\,x}{2}\right)-\mathrm{atanh}\left(\frac{(-1)^{1/4}\,24^{1/4}\,x}{2}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)/(3*x^4 + 2),x)
```

```
[Out] ((-1)^(1/4)*24^(1/4)*c*(atan(((-1)^(1/4)*24^(1/4)*x)/2) - atanh(((-1)^(1/4)
*24^(1/4)*x)/2)))/12
```

sympy [A] time = 0.43, size = 88, normalized size = 0.87

$$c\left(\frac{\sqrt[4]{6}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}-\frac{\sqrt[4]{6}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}+\frac{\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2/(3*x**4+2),x)
```

```
[Out] c*(6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/24 - 6**(1/4)*log(x**2 + 6
**(3/4)*x/3 + sqrt(6)/3)/24 + 6**(1/4)*atan(6**(1/4)*x - 1)/12 + 6**(1/4)*a
tan(6**(1/4)*x + 1)/12)
```

3.109 $\int \frac{a+cx^2}{2+3x^4} dx$

Optimal. Leaf size=141

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{\right.}}{}$$

Rubi [A] time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rubi steps

$$\int \frac{a+cx^2}{2+3x^4}\,dx = \frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx + \frac{1}{12}\left(\sqrt{6}a+2c\right)\int \frac{\sqrt{6}+3x^2}{2+3x^4}\,dx$$

$$= -\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\,6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\,6^{3/4}} + \frac{1}{24}\left(\sqrt{6}a+2c\right)\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}}{\sqrt[4]{3}}}$$

$$= -\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\,6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\,6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\text{Subs}}{}$$

$$= -\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\,6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\,6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+\right.}{8\,6^{3/4}}$$

Mathematica [A] time = 0.06, size = 113, normalized size = 0.80

$$\frac{-\left(\sqrt{6}a-2c\right)\left(\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)\right)-2\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{8\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a + 2*c)*ArcTan[1
+ 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - Log[
2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+cx^2}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^2)/(2 + 3*x^4), x]
```

fricas [B] time = 0.48, size = 2278, normalized size = 16.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/144*(2*sqrt(6)*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 -
12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a
^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*arctan(-1/12*(sqrt(2)*s
qrt(1/3)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2
*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 -
12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2
*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*sqrt
((3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 + sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^
```

```
4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*s
qrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)
/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2
+ 2*c^2))/(9*a^4 + 12*a^2*c^2 + 4*c^4)) - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24
*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2
*c^2 + 4*c^4)*a*x - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2
*c^3)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24
*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 2*sqrt(6)*sqrt(54*a^4 + 72*a^2*c
^2 + 24*c^4)*(9*a^4 + 12*a^2*c^2 + 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))
/(81*a^8 + 108*a^6*c^2 - 48*a^2*c^6 - 16*c^8)) + 2*sqrt(6)*sqrt(2)*(54*a^4
+ 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 +
 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12
*a^2*c^2 + 4*c^4))*arctan(-1/12*(sqrt(2)*sqrt(1/3)*(54*a^4 + 72*a^2*c^2 + 2
4*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^
2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*
c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^
4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*sqrt((3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*
x^2 - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^
2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 +
 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) +
sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2))/(9*a^4 + 12*a^2*c^2 + 4
*c^4)) - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4
+ 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a*x - 2*sqrt(6)*sqr
t(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3)*x)*sqrt((9*a^4 + 12*a^2*c^2
 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 +
4*c^4)) - 2*sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(9*a^4 + 12*a^2*c^2
+ 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))/(81*a^8 + 108*a^6*c^2 - 48*a^2*c
^6 - 16*c^8)) - 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(9*a^4 + 12*
a^2*c^2 + 4*c^4 - 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*sqrt((9*a^4 + 1
2*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a
^2*c^2 + 4*c^4))*log(3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 + sqrt(2)*(54*a^4 +
 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*c*x - 3*(3*
a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2
*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + sqrt(54*a^4 + 72*a^2*c^
2 + 24*c^4)*(3*a^2 + 2*c^2)) + 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/
4)*(9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*
sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c
)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*log(3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 - sq
rt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^2 + 24*c
^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(
54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + sqrt(54*
a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2)))/(9*a^4 + 12*a^2*c^2 + 4*c^4)
```

giac [A] time = 0.20, size = 131, normalized size = 0.93

$$\frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x^2
+ sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x
^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

maple [B] time = 0.04, size = 226, normalized size = 1.60

$$\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{48}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{72}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{72}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(3*x^4+2),x)
```

```
[Out] 1/24*3^(1/2)*6^(1/4)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/24
*3^(1/2)*6^(1/4)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48*3^(
1/2)*6^(1/4)*2^(1/2)*a*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(
x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/72*3^(1/2)*6^(3/4)*2^(1/2
)*c*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*
arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*3^(1/2)*6^(3/4)*2^(1/2)*c*ln(
(x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*2^
(1/2)*x+1/3*6^(1/2)))
```

maxima [A] time = 3.04, size = 167, normalized size = 1.18

$$\frac{1}{24} \cdot 3^{\frac{1}{4}} 2^{\frac{3}{4}} \left(\sqrt{3} a + \sqrt{2} c\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2 \sqrt{3} x + 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{24} \cdot 3^{\frac{1}{4}} 2^{\frac{3}{4}} \left(\sqrt{3} a + \sqrt{2} c\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2 \sqrt{3} x - 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{48} \cdot 3^{\frac{1}{4}} 2^{\frac{3}{4}} \left(\sqrt{3} a - \sqrt{2} c\right) \log\left(\sqrt{3} x^{2} + 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) - \frac{1}{48} \cdot 3^{\frac{1}{4}} 2^{\frac{3}{4}} \left(\sqrt{3} a - \sqrt{2} c\right) \log\left(\sqrt{3} x^{2} - 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*
sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c
)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/48*3^(1/4
)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqr
t(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 - 3^(1
/4)*2^(3/4)*x + sqrt(2))
```

mupad [B] time = 5.11, size = 315, normalized size = 2.23

$$-2\,\mathrm{atanh}\left(\frac{216\,a^2\,x\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3+18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2-12\,c^3}-\frac{144\,c^2\,x\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3+18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2-12\,c^3}\right)\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}+2\,\mathrm{atanh}\left(\frac{216\,a^2\,x\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3-18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2+12\,c^3}-\frac{144\,c^2\,x\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3-18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2+12\,c^3}\right)\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/(3*x^4 + 2),x)
```

```
[Out] 2*atanh((216*a^2*x*((6^(1/2)*a^2*1i)/192 - (a*c)/48 - (6^(1/2)*c^2*1i)/288)
^(1/2))/(6^(1/2)*a^3*9i - 18*a^2*c + 12*c^3 - 6^(1/2)*a*c^2*6i) - (144*c^2*
x*((6^(1/2)*a^2*1i)/192 - (a*c)/48 - (6^(1/2)*c^2*1i)/288)^(1/2))/(6^(1/2)*
a^3*9i - 18*a^2*c + 12*c^3 - 6^(1/2)*a*c^2*6i))*((6^(1/2)*a^2*1i)/192 - (a*
c)/48 - (6^(1/2)*c^2*1i)/288)^(1/2) - 2*atanh((216*a^2*x*((6^(1/2)*c^2*1i)/
288 - (6^(1/2)*a^2*1i)/192 - (a*c)/48)^(1/2))/(6^(1/2)*a^3*9i + 18*a^2*c -
12*c^3 - 6^(1/2)*a*c^2*6i) - (144*c^2*x*((6^(1/2)*c^2*1i)/288 - (6^(1/2)*a^
2*1i)/192 - (a*c)/48)^(1/2))/(6^(1/2)*a^3*9i + 18*a^2*c - 12*c^3 - 6^(1/2)*
a*c^2*6i))*((6^(1/2)*c^2*1i)/288 - (6^(1/2)*a^2*1i)/192 - (a*c)/48)^(1/2)
```

sympy [A] time = 0.57, size = 68, normalized size = 0.48

$$\operatorname{RootSum}\left(55296t^{4} + 2304t^{2}ac + 9a^{4} + 12a^{2}c^{2} + 4c^{4}, \left(t \mapsto t\log\left(x + \frac{-4608t^{3}c + 72ta^{3} - 144tac^{2}}{9a^{4} - 4c^{4}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(3*x**4+2),x)
```

```
[Out] RootSum(55296*_t**4 + 2304*_t**2*a*c + 9*a**4 + 12*a**2*c**2 + 4*c**4, Lamb
da(_t, _t*log(x + (-4608*_t**3*c + 72*_t*a**3 - 144*_t*a*c**2)/(9*a**4 - 4*
c**4))))
```

3.110 $\int \frac{bx+cx^2}{2+3x^4}\,dx$

Optimal. Leaf size=123

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\,6^{3/4}}$$

Rubi [A] time = 0.12, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1593, 1831, 275, 203, 297, 1162, 617, 204, 1165, 628}

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4
)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{bx+cx^2}{2+3x^4}\,dx &= \int \frac{x(b+cx)}{2+3x^4}\,dx\\
&= \int \left(\frac{bx}{2+3x^4}+\frac{cx^2}{2+3x^4}\right)dx\\
&= b\int \frac{x}{2+3x^4}\,dx + c\int \frac{x^2}{2+3x^4}\,dx\\
&= \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right) - \frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}\\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}}{4\,6^{3/4}}\\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\,6^{3/4}} + \frac{c\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx\right.}{2\,6^{3/4}}\\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\,6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}} - \frac{c\log}{}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 99, normalized size = 0.80

$$\frac{-2\left(\sqrt[4]{6}b+c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(c-\sqrt[4]{6}b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)+c\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-c\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)}{4\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*(-(6^(1/4)*b) + c)*ArcTan[1 +
 6^(1/4)*x] + c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - c*Log[2 + 2*6^(1/4)*x
+ Sqrt[6]*x^2])/(4*6^(3/4))
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + cx^2}{2 + 3x^4}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] IntegrateAlgebraic[(b*x + c*x^2)/(2 + 3*x^4), x]
```

fricas [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A]   time = 0.19, size = 114, normalized size = 0.93

$$-\frac{1}{24} \cdot 6^{\frac{1}{4}} c \log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}} x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24} \cdot 6^{\frac{1}{4}} c \log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}} x + \sqrt{\frac{2}{3}}\right) - \frac{1}{12}\left(\sqrt{6} b - 6^{\frac{1}{4}} c\right) \arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\left(\sqrt{6} b + 6^{\frac{1}{4}} c\right) \arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/24*6^(1/4)*c*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*6^(1/4)
*c*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/12*(sqrt(6)*b - 6^(1/4)
*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/12*(sqr
t(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/
4)))
```

maple [A]   time = 0.05, size = 129, normalized size = 1.05

$$\frac{\sqrt{6}\, b \arctan\left(\frac{\sqrt{6}\, x^2}{2}\right)}{12} + \frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, \sqrt{3}\, 6^{\frac{3}{4}} x}{6} - 1\right)}{72} + \frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, \sqrt{3}\, 6^{\frac{3}{4}} x}{6} + 1\right)}{72} + \frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{2}\, c \ln\left(\frac{x^2 - \frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, x}{3} + \frac{\sqrt{6}}{3}}{x^2 + \frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, x}{3} + \frac{\sqrt{6}}{3}}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(3*x^4+2),x)
```

```
[Out] 1/12*6^(1/2)*b*arctan(1/2*6^(1/2)*x^2)+1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*arcta
n(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*arctan(1/
6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*3^(1/2)*6^(3/4)*2^(1/2)*c*ln((x^2-1/3*
3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1
/3*6^(1/2)))
```

maxima [A]   time = 3.09, size = 147, normalized size = 1.20

$$\frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}} 2^{\frac{3}{4}} c - 2\sqrt{3}\, b\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}}\left(2\sqrt{3}\, x + 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}} 2^{\frac{3}{4}} c + 2\sqrt{3}\, b\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}}\left(2\sqrt{3}\, x - 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) - \frac{1}{24} \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} c \log\left(\sqrt{3}\, x^2 + 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{24} \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} c \log\left(\sqrt{3}\, x^2 - 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c - 2*sqrt(3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(
2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c + 2*sqrt(
3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) - 1/24*3^
(1/4)*2^(1/4)*c*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*3^(1/
4)*2^(1/4)*c*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

mupad [B] time = 0.22, size = 162, normalized size = 1.32

$$\sum_{k=1}^{4}\ln\left(9\,b^3\,x-6\,c^3-\mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\,b\,c\,144+\mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)^2\,b\,x\,864+\mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\,c^2\,x\,72\right)\,\mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(9*b^3*x - 6*c^3 - 144*root(z^4 + (b^2*z^2)/48 + (b*c^2*z)/288 +
c^4/13824 + b^4/9216, z, k)*b*c + 864*root(z^4 + (b^2*z^2)/48 + (b*c^2*z)/2
88 + c^4/13824 + b^4/9216, z, k)^2*b*x + 72*root(z^4 + (b^2*z^2)/48 + (b*c^
2*z)/288 + c^4/13824 + b^4/9216, z, k)*c^2*x)*root(z^4 + (b^2*z^2)/48 + (b*
c^2*z)/288 + c^4/13824 + b^4/9216, z, k), k, 1, 4)
```

sympy [A] time = 0.77, size = 85, normalized size = 0.69

$$\operatorname{RootSum}\left(27648t^4+576t^2b^2+96tbc^2+3b^4+2c^4,\left(t\mapsto t\log\left(x+\frac{-1152t^3c^2+288t^2b^3-36tb^2c^2+3b^5-3bc^4}{6b^4c-c^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] RootSum(27648*_t**4 + 576*_t**2*b**2 + 96*_t*b*c**2 + 3*b**4 + 2*c**4, Lamb
da(_t, _t*log(x + (-1152*_t**3*c**2 + 288*_t**2*b**3 - 36*_t*b**2*c**2 + 3*
b**5 - 3*b*c**4)/(6*b**4*c - c**5))))
```

3.111 $\int \frac{a+bx+cx^2}{2+3x^4} \, dx$

Optimal. Leaf size=163

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{\right.}}{}$$

Rubi [A] time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1876, 275, 203, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\,6^{3/4}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/
4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4))
- ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt
[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \int \left(\frac{bx}{2+3x^4}+\frac{a+cx^2}{2+3x^4}\right) dx$$

$$= b\int \frac{x}{2+3x^4}\,dx + \int \frac{a+cx^2}{2+3x^4}\,dx$$

$$= \frac{1}{2} b\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right) + \frac{1}{12}\left(\sqrt{6}\,a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx + \frac{1}{12}\left(\sqrt{6}\,a+2c\right)\int \frac{\sqrt{6}}{2+}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}\,a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\;6^{3/4}} - \frac{\left(\sqrt{6}\,a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\;6^{3/4}} + \frac{1}{24}(\sqrt{}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\;6^{3/4}} + \frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x\right.}{8\;6^{3/4}}$$

$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\;6^{3/4}} + \frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\;6^{3/4}} - \frac{\left(\sqrt{6}\,a-\right.}{}$$

Mathematica [A] time = 0.08, size = 129, normalized size = 0.79

$$\frac{-2\tan^{-1}\left(1-\sqrt[4]{6}x\right)\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)+2\tan^{-1}\left(\sqrt[4]{6}x+1\right)\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)-\left(\sqrt{6}a-2c\right)\left(\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)\right)}{8\;6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a -
2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(
1/4)*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{2 + 3x^4}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(2 + 3*x^4),x]
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)/(2 + 3*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.22, size = 143, normalized size = 0.88

$$\frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)
*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)*c
)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/
4)*a - 2*6^(1/4)*c)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^
(3/4)*a - 2*6^(1/4)*c)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

maple [B] time = 0.05, size = 241, normalized size = 1.48

$$\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6} - 1\right)}{24} + \frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6} + 1\right)}{24} + \frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^2 + \frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3} + \frac{\sqrt{6}}{3}}{x^2 - \frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3} + \frac{\sqrt{6}}{3}}\right)}{48} + \frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12} + \frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6} - 1\right)}{72} + \frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6} + 1\right)}{72} + \frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\ln\left(\frac{x^2 - \frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3} + \frac{\sqrt{6}}{3}}{x^2 + \frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3} + \frac{\sqrt{6}}{3}}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(3*x^4+2),x)
[Out] 1/24*3^(1/2)*6^(1/4)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/24
*3^(1/2)*6^(1/4)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48*3^(
1/2)*6^(1/4)*2^(1/2)*a*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(
x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/12*6^(1/2)*b*arctan(1/2*6
^(1/2)*x^2)+1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/
4)*x+1)+1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x
-1)+1/144*3^(1/2)*6^(3/4)*2^(1/2)*c*ln((x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1
/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))
```

maxima [A] time = 3.06, size = 187, normalized size = 1.15

$$\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a - \sqrt{2}c\right)\log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) - \frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a - \sqrt{2}c\right)\log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) + \frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a - 2\sqrt{3}\sqrt{2}b + 2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x + 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a + 2\sqrt{3}\sqrt{2}b + 2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x - 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
[Out] 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3
/4)*x + sqrt(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)
```

```
*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*(3^(3/4)*2^(3/4)*a - 2*sqrt(3)*s
qrt(2)*b + 2*3^(1/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3
^(1/4)*2^(3/4))) + 1/24*(3^(3/4)*2^(3/4)*a + 2*sqrt(3)*sqrt(2)*b + 2*3^(1/4
)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

mupad [**B**] time = 5.52, size = 270, normalized size = 1.66

$$\sum_{k=1}^{4}\ln\left(9\,a\,b^2-9\,a^2\,c-\mathrm{root}\left(z^4+\frac{z^2\,\left(2304\,a\,c+1152\,b^2\right)}{55296}-\frac{z\,\left(288\,a^2\,b-192\,b\,c^2\right)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)\left(\mathrm{root}\left(z^4+\frac{z^2\,\left(2304\,a\,c+1152\,b^2\right)}{55296}-\frac{z\,\left(288\,a^2\,b-192\,b\,c^2\right)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)(864\,a-864\,b\,x)+144\,b\,c+x\,\left(108\,a^2-72\,c^2\right)\right)-6\,c^3+x\,\left(9\,b^3-18\,a\,b\,c\right)\right)\mathrm{root}\left(z^4+\frac{z^2\,\left(2304\,a\,c+1152\,b^2\right)}{55296}-\frac{z\,\left(288\,a^2\,b-192\,b\,c^2\right)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(9*a*b^2 - 9*a^2*c - root(z^4 + (z^2*(2304*a*c + 1152*b^2))/55296
 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304 + (a^2*c^2)/4608 + c^
4/13824 + b^4/9216 + a^4/6144, z, k)*(root(z^4 + (z^2*(2304*a*c + 1152*b^2)
)/55296 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304 + (a^2*c^2)/46
08 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*(864*a - 864*b*x) + 144*b*c + x
*(108*a^2 - 72*c^2)) - 6*c^3 + x*(9*b^3 - 18*a*b*c))*root(z^4 + (z^2*(2304*
a*c + 1152*b^2))/55296 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304
 + (a^2*c^2)/4608 + c^4/13824 + b^4/9216 + a^4/6144, z, k), k, 1, 4)
```

sympy [**B**] time = 5.07, size = 292, normalized size = 1.79

$$\mathrm{RootSum}\left(55296t^4+t^2\left(2304ac+1152b^2\right)+t\left(-288a^2b+192bc^2\right)+9a^4+12a^2c^2-24ab^2c+6b^4+4c^4,\left(t\mapsto t\log\left(x+\frac{-13824t^3a^2c+27648t^3ab^2+9216t^3c^3+1728t^2a^3b+3456t^2abc^2-2304t^2b^3c+216ta^5-576ta^3c^2+1296ta^2b^2c+288tab^4+288tac^4+288tb^2c^3+90a^4bc-90a^3b^3+60ab^3c^2-24b^5c+24bc^5}{27a^6-18a^4c^2+144a^3b^2c-72a^2b^4-12a^2c^4+96ab^2c^3-48b^4c^2+8c^6}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(55296*_t**4 + _t**2*(2304*a*c + 1152*b**2) + _t*(-288*a**2*b + 192*
b*c**2) + 9*a**4 + 12*a**2*c**2 - 24*a*b**2*c + 6*b**4 + 4*c**4, Lambda(_t,
 _t*log(x + (-13824*_t**3*a**2*c + 27648*_t**3*a*b**2 + 9216*_t**3*c**3 + 1
728*_t**2*a**3*b + 3456*_t**2*a*b*c**2 - 2304*_t**2*b**3*c + 216*_t*a**5 -
576*_t*a**3*c**2 + 1296*_t*a**2*b**2*c + 288*_t*a*b**4 + 288*_t*a*c**4 + 28
8*_t*b**2*c**3 + 90*a**4*b*c - 90*a**3*b**3 + 60*a*b**3*c**2 - 24*b**5*c +
24*b*c**5)/(27*a**6 - 18*a**4*c**2 + 144*a**3*b**2*c - 72*a**2*b**4 - 12*a*
*2*c**4 + 96*a*b**2*c**3 - 48*b**4*c**2 + 8*c**6)))
```

3.112 $\int \frac{dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 260}

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (d*Log[2 + 3*x^4])/12
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{dx^3}{2+3x^4}\,dx &= d\int \frac{x^3}{2+3x^4}\,dx\\ &= \frac{1}{12}d\log\left(2+3x^4\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (d*Log[2 + 3*x^4])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x^3)/(2 + 3*x^4),x]
```

```
[Out] IntegrateAlgebraic[(d*x^3)/(2 + 3*x^4), x]
```

fricas [A] time = 0.39, size = 11, normalized size = 0.85

$$\frac{1}{12} d \log \left(3 x^4 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="fricas")
[Out] 1/12*d*log(3*x^4 + 2)
```

giac [A] time = 0.16, size = 11, normalized size = 0.85

$$\frac{1}{12} d \log \left(3 x^4 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="giac")
[Out] 1/12*d*log(3*x^4 + 2)
```

maple [A] time = 0.05, size = 12, normalized size = 0.92

$$\frac{d \ln \left(3x^4 + 2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3/(3*x^4+2),x)
[Out] 1/12*d*ln(3*x^4+2)
```

maxima [A] time = 1.32, size = 11, normalized size = 0.85

$$\frac{1}{12} d \log \left(3 x^4 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="maxima")
[Out] 1/12*d*log(3*x^4 + 2)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.69

$$\frac{d \ln \left(x^4 + \frac{2}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3)/(3*x^4 + 2),x)
[Out] (d*log(x^4 + 2/3))/12
```

sympy [A] time = 0.09, size = 10, normalized size = 0.77

$$\frac{d \log \left(3x^4 + 2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3/(3*x**4+2),x)
[Out] d*log(3*x**4 + 2)/12
```

3.113 $\int \frac{a+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=114

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{1}{12}d\log\left(3x^4\right.$$

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 260}

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -(a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/
4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \int \left(\frac{a}{2+3x^4}+\frac{dx^3}{2+3x^4}\right)dx$$

$$= a\int \frac{1}{2+3x^4}\,dx + d\int \frac{x^3}{2+3x^4}\,dx$$

$$= \frac{1}{12}d\log\left(2+3x^4\right) + \frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} + \frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}}$$

$$= \frac{1}{12}d\log\left(2+3x^4\right) + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}} - \frac{a\int \frac{2^{3/}}{-\sqrt{\frac{2}{3}}+}}{8}$$

$$= -\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{1}{12}d\log\left(2+3x^4\right) + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\right.}{4\sqrt[4]{}}$$

$$= -\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x\right.}{8\sqrt[4]{6}}$$

Mathematica [A] time = 0.03, size = 108, normalized size = 0.95

$$\frac{1}{48}\left(-6^{3/4}a\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+6^{3/4}a\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\,6^{3/4}a\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\,6^{3/4}a\tan^{-1}\left(\sqrt[4]{6}x+1\right)+4d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(3/4)*a*ArcTan[1 - 6^(1/4)*x] + 2*6^(3/4)*a*ArcTan[1 + 6^(1/4)*x] - 6
^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x
 + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+dx^3}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + d*x^3)/(2 + 3*x^4), x]
```

fricas [B] time = 0.43, size = 359, normalized size = 3.15

$$-\frac{4\cdot 6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a^{4}\arctan\left(-\frac{6^{\frac{3}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a^{4}x-6^{\frac{3}{4}}\sqrt{3}\sqrt{2}\sqrt{\frac{1}{3}}\left(a^{4}\right)^{\frac{3}{4}}a^{4}\sqrt{\frac{3a^{2}x^{2}+6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}ax+\sqrt{6}\sqrt{a^{4}}}{a^{2}}}+6a^{7}}{6a^{7}}\right)+4\cdot 6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a^{4}\arctan\left(-\frac{6^{\frac{3}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a^{4}x-6^{\frac{3}{4}}\sqrt{3}\sqrt{2}\sqrt{\frac{1}{3}}\left(a^{4}\right)^{\frac{3}{4}}a^{4}\sqrt{\frac{3a^{2}x^{2}-6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}ax+\sqrt{6}\sqrt{a^{4}}}{a^{2}}}-6a^{7}}{6a^{7}}\right)-\left(6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a^{4}+4a^{4}d\right)\log\left(3a^{2}x^{2}+6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}ax+\sqrt{6}\sqrt{a^{4}}\right)+\left(6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a^{4}-4a^{4}d\right)\log\left(3a^{2}x^{2}-6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}ax+\sqrt{6}\sqrt{a^{4}}\right)}{48a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/48*(4*6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a^4*arctan(-1/6*(6^(3/4)*sqrt(
3)*sqrt(2)*(a^4)^(3/4)*a^4*x - 6^(3/4)*sqrt(3)*sqrt(2)*sqrt(1/3)*(a^4)^(3/4
)*a^4*sqrt((3*a^2*x^2 + 6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a*x + sqrt(6)*s
qrt(a^4))/a^2) + 6*a^7)/a^7) + 4*6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a^4*ar
ctan(-1/6*(6^(3/4)*sqrt(3)*sqrt(2)*(a^4)^(3/4)*a^4*x - 6^(3/4)*sqrt(3)*sqrt
(2)*sqrt(1/3)*(a^4)^(3/4)*a^4*sqrt((3*a^2*x^2 - 6^(1/4)*sqrt(3)*sqrt(2)*(a^
4)^(1/4)*a*x + sqrt(6)*sqrt(a^4))/a^2) - 6*a^7)/a^7) - (6^(1/4)*sqrt(3)*sqr
t(2)*(a^4)^(1/4)*a^4 + 4*a^4*d)*log(3*a^2*x^2 + 6^(1/4)*sqrt(3)*sqrt(2)*(a^
4)^(1/4)*a*x + sqrt(6)*sqrt(a^4)) + (6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a^
4 - 4*a^4*d)*log(3*a^2*x^2 - 6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a*x + sqrt
(6)*sqrt(a^4)))/a^4
```

giac [A] time = 0.20, size = 109, normalized size = 0.96

$$\frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\left(6^{\frac{3}{4}}a+4d\right)\log\left(x^{2}+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\left(6^{\frac{3}{4}}a-4d\right)\log\left(x^{2}-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
+ 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))
) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1
/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

maple [A] time = 0.05, size = 125, normalized size = 1.10

$$\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{48}+\frac{d\ln\left(3x^{4}+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+a)/(3*x^4+2),x)
```

```
[Out] 1/24*3^(1/2)*6^(1/4)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/24
*3^(1/2)*6^(1/4)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48*3^(
1/2)*6^(1/4)*2^(1/2)*a*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(
x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/12*d*ln(3*x^4+2)
```

maxima [A] time = 3.06, size = 149, normalized size = 1.31

$$\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3a\right)\log\left(\sqrt{3}x^{2}+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^{2}-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^
(3/4))) + 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x -
3^(1/4)*2^(3/4))) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(s
```

```
qrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4
)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

mupad [B] time = 0.28, size = 117, normalized size = 1.03

$$\ln\left(x-\frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}-\frac{6^{1/4}\,\sqrt{\frac{3}{4}\mathrm{i}}\,a}{12}\right)+\ln\left(x+\frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}+\frac{6^{1/4}\,\sqrt{\frac{3}{4}\mathrm{i}}\,a}{12}\right)+\ln\left(x-\frac{(-1)^{3/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}+\frac{6^{1/4}\,\sqrt{-\frac{3}{4}\mathrm{i}}\,a}{12}\right)+\ln\left(x+\frac{(-1)^{3/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}-\frac{6^{1/4}\,\sqrt{-\frac{3}{4}\mathrm{i}}\,a}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] log(x - ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(3i/4)^(1/2)*a)/12
) + log(x + ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(3i/4)^(1/2)*a
)/12) + log(x - ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(-3i/4)^(1
/2)*a)/12) + log(x + ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(-3i/
4)^(1/2)*a)/12)
```

sympy [A] time = 0.42, size = 51, normalized size = 0.45

$$\operatorname{RootSum}\left(165888t^4-55296t^3d+6912t^2d^2-384td^3+27a^4+8d^4,\left(t\mapsto t\log\left(x+\frac{24t-2d}{3a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + 6912*_t**2*d**2 - 384*_t*d**3 + 27*a
**4 + 8*d**4, Lambda(_t, _t*log(x + (24*_t - 2*d)/(3*a))))
```

3.114 $\int \frac{bx+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=36

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1593, 1248, 635, 203, 260}

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) + (d*Log[2 + 3*x^4])/12
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = \int \frac{x\left(b+dx^2\right)}{2+3x^4}\,dx$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right) + \frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{x}{2+3x^2}\,dx, x, x^2\right)$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

Mathematica [C] time = 0.04, size = 65, normalized size = 1.81

$$\frac{1}{24}\left(2d+i\sqrt{6}\,b\right)\log\left(\sqrt{6}\,-3ix^2\right)+\frac{1}{24}\left(2d-i\sqrt{6}\,b\right)\log\left(\sqrt{6}\,+3ix^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)/(2 + 3*x^4),x]
[Out] ((I*Sqrt[6]*b + 2*d)*Log[Sqrt[6] - (3*I)*x^2])/24 + (((-I)*Sqrt[6]*b + 2*d)
*Log[Sqrt[6] + (3*I)*x^2])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+dx^3}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + d*x^3)/(2 + 3*x^4),x]
[Out] IntegrateAlgebraic[(b*x + d*x^3)/(2 + 3*x^4), x]
```

fricas [A] time = 0.39, size = 27, normalized size = 0.75

$$\frac{1}{12}\,\sqrt{6}\,b\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right)+\frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2) + 1/12*d*log(3*x^4 + 2)
```

giac [B] time = 0.18, size = 93, normalized size = 2.58

$$-\frac{1}{12}\,\sqrt{6}\,b\arctan\left(\frac{3}{4}\,\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\,\sqrt{6}\,b\arctan\left(\frac{3}{4}\,\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\,d\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)+\frac{1}{12}\,d\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="giac")
[Out] -1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)
)) + 1/12*d*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/12*d*log(x^2 -
 sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

maple [A] time = 0.05, size = 28, normalized size = 0.78

$$\frac{\sqrt{6}\, b \arctan\left(\frac{\sqrt{6}\, x^2}{2}\right)}{12}+\frac{d \ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x)/(3*x^4+2),x)
```

```
[Out] 1/12*d*ln(3*x^4+2)+1/12*6^(1/2)*b*arctan(1/2*6^(1/2)*x^2)
```

maxima [B] time = 3.06, size = 113, normalized size = 3.14

$$-\frac{1}{12}\sqrt{3}\sqrt{2}\, b \arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}}\left(2\sqrt{3}\, x+3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right)+\frac{1}{12}\sqrt{3}\sqrt{2}\, b \arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}}\left(2\sqrt{3}\, x-3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right)+\frac{1}{12}\, d \log\left(\sqrt{3}\, x^2+3^{\frac{1}{4}} 2^{\frac{3}{4}} x+\sqrt{2}\right)+\frac{1}{12}\, d \log\left(\sqrt{3}\, x^2-3^{\frac{1}{4}} 2^{\frac{3}{4}} x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x -
 3^(1/4)*2^(3/4))) + 1/12*d*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2))
+ 1/12*d*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

mupad [B] time = 0.06, size = 25, normalized size = 0.69

$$\frac{d\, \ln\left(x^4+\frac{2}{3}\right)}{12}+\frac{\sqrt{6}\, b\, \mathrm{atan}\left(\frac{\sqrt{6}\, x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] (d*log(x^4 + 2/3))/12 + (6^(1/2)*b*atan((6^(1/2)*x^2)/2))/12
```

sympy [C] time = 0.41, size = 53, normalized size = 1.47

$$\left(-\frac{\sqrt{6}\, i b}{24}+\frac{d}{12}\right)\log\left(x^2-\frac{\sqrt{6}\, i}{3}\right)+\left(\frac{\sqrt{6}\, i b}{24}+\frac{d}{12}\right)\log\left(x^2+\frac{\sqrt{6}\, i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)/(3*x**4+2),x)
```

```
[Out] (-sqrt(6)*I*b/24 + d/12)*log(x**2 - sqrt(6)*I/3) + (sqrt(6)*I*b/24 + d/12)*
log(x**2 + sqrt(6)*I/3)
```

3.115 $\int \frac{a+bx+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=136

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

Rubi [A] time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 1248, 635, 203, 260}

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4
)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*
Log[2 + 3*x^4])/12
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+dx^3}{2+3x^4}\,dx &= \int \left(\frac{a}{2+3x^4}+\frac{x\left(b+dx^2\right)}{2+3x^4}\right)dx \\
&= a\int \frac{1}{2+3x^4}\,dx+\int \frac{x\left(b+dx^2\right)}{2+3x^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx,x,x^2\right)+\frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}}+\frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} \\
&= \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}}+\frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}}-\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}}-\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}}+ \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{1}{12}d\log\left(2+\right. \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 128, normalized size = 0.94

$$\frac{1}{48}\left(-2\sqrt{6}\left(\sqrt[4]{6}a+2b\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\sqrt{6}\left(\sqrt[4]{6}a-2b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)-6^{3/4}a\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+6^{3/4}a\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)+4d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*Sqrt[6]*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*Sqrt[6]*(6^(1/4)*a
- 2*b)*ArcTan[1 + 6^(1/4)*x] - 6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2]
 + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + d*x^3)/(2 + 3*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.20, size = 125, normalized size = 0.92

$$\frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\left(6^{\frac{3}{4}}a+4d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\left(6^{\frac{3}{4}}a-4d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2
)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/
3)^(1/4)*x + sqrt(2/3))
```

maple [A] time = 0.04, size = 140, normalized size = 1.03

$$\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{48}+\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}+\frac{d\ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x+a)/(3*x^4+2),x)
```

```
[Out] 1/24*3^(1/2)*6^(1/4)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/24
*3^(1/2)*6^(1/4)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48*3^(
1/2)*6^(1/4)*2^(1/2)*a*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(
x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/12*6^(1/2)*b*arctan(1/2*6
^(1/2)*x^2)+1/12*d*ln(3*x^4+2)
```

maxima [A] time = 3.05, size = 171, normalized size = 1.26

$$\frac{1}{144} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \left(2 \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} d + 3 \, a\right) \log\left(\sqrt{3} x^{2} + 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{144} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \left(2 \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} d - 3 \, a\right) \log\left(\sqrt{3} x^{2} - 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{24} \sqrt{3} \left(3^{\frac{1}{4}} 2^{\frac{3}{4}} a - 2 \sqrt{2} b\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2 \sqrt{3} x + 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{24} \sqrt{3} \left(3^{\frac{1}{4}} 2^{\frac{3}{4}} a + 2 \sqrt{2} b\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2 \sqrt{3} x - 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(sqrt(3)*x^2 + 3^(1/4)
*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d - 3*a)*l
og(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*sqrt(3)*(3^(1/4)*2^(3/
4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/
4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^
(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

mupad [B] time = 5.50, size = 307, normalized size = 2.26

$$\sum_{k=1}^{4} \ln\left(x\,\left(9\,a^2\,d+9\,b^3+6\,b\,d^2\right)+9\,a\,b^2-6\,a\,d^2-\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\,\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)\,\left(\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\,\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)\,\left(864\,a-864\,b\,x\right)-144\,a\,d+x\,\left(144\,b\,d+108\,a^2\right)\right)\right)\,\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\,\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(x*(9*a^2*d + 6*b*d^2 + 9*b^3) + 9*a*b^2 - 6*a*d^2 - root(z^4 - (
d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^2))/165888 - (z*(864*a^2*b + 576*b^2*d +
 384*d^3))/165888 + (a^2*b*d)/2304 + (b^2*d^2)/6912 + d^4/20736 + b^4/9216
+ a^4/6144, z, k)*(root(z^4 - (d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^2))/16588
8 - (z*(864*a^2*b + 576*b^2*d + 384*d^3))/165888 + (a^2*b*d)/2304 + (b^2*d^
2)/6912 + d^4/20736 + b^4/9216 + a^4/6144, z, k)*(864*a - 864*b*x) - 144*a*
d + x*(144*b*d + 108*a^2)))*root(z^4 - (d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^
2))/165888 - (z*(864*a^2*b + 576*b^2*d + 384*d^3))/165888 + (a^2*b*d)/2304
+ (b^2*d^2)/6912 + d^4/20736 + b^4/9216 + a^4/6144, z, k), k, 1, 4)
```

sympy [A] time = 1.68, size = 199, normalized size = 1.46

$$\operatorname{RootSum} {\left(165888 t^{4} - 55296 t^{3} d + t^{2} \left(3456 b^{2} + 6912 d^{2}\right) + t \left(- 864 a^{2} b - 576 b^{2} d - 384 d^{3}\right) + 27 a^{4} + 72 a^{2} b d + 18 b^{4} + 24 b^{2} d^{2} + 8 d^{4}, \left(t \mapsto t \log{\left(x + \frac{27648 t^{3} b^{2} + 1728 t^{2} a^{2} b - 6912 t^{2} b^{2} d + 216 t a^{4} - 288 t a^{2} b d + 288 t b^{4} + 576 t b^{2} d^{2} - 18 a^{4} d - 90 a^{2} b^{3} + 12 a^{2} b d^{2} - 24 b^{4} d - 16 b^{2} d^{3}}{27 a^{5} - 72 a b^{4}} \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(3456*b**2 + 6912*d**2) + _t*(
-864*a**2*b - 576*b**2*d - 384*d**3) + 27*a**4 + 72*a**2*b*d + 18*b**4 + 24
*b**2*d**2 + 8*d**4, Lambda(_t, _t*log(x + (27648*_t**3*b**2 + 1728*_t**2*a
**2*b - 6912*_t**2*b**2*d + 216*_t*a**4 - 288*_t*a**2*b*d + 288*_t*b**4 + 5
76*_t*b**2*d**2 - 18*a**4*d - 90*a**2*b**3 + 12*a**2*b*d**2 - 24*b**4*d - 1
6*b**2*d**3)/(27*a**5 - 72*a*b**4))))
```

3.116 $\int \frac{cx^2+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=114

$$\frac{c \log \left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c \log \left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c \tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\ 6^{3/4}}+\frac{c \tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{2\ 6^{3/4}}+\frac{1}{12} d \log \left(3x^4+2\right)$$

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1593, 1831, 297, 1162, 617, 204, 1165, 628, 260}

$$\frac{c \log \left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c \log \left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c \tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\ 6^{3/4}}+\frac{c \tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{2\ 6^{3/4}}+\frac{1}{12} d \log \left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -(c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/
4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{cx^2+dx^3}{2+3x^4}\,dx &= \int \frac{x^2(c+dx)}{2+3x^4}\,dx \\
&= \int \left(\frac{cx^2}{2+3x^4}+\frac{dx^3}{2+3x^4}\right)dx \\
&= c\int \frac{x^2}{2+3x^4}\,dx + d\int \frac{x^3}{2+3x^4}\,dx \\
&= \frac{1}{12}d\log\left(2+3x^4\right) - \frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} \\
&= \frac{1}{12}d\log\left(2+3x^4\right) + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+\cdots}{-\sqrt{\frac{2}{3}}-\frac{2^{3}\cdots}{\cdots}}}{4\,6^{3\cdots}} \\
&= \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\,6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right) + \frac{c\,\text{Subst}\left(\int \frac{1}{-1\cdots}\cdots\right)}{2\cdots} \\
&= -\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\,6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x\cdots\right)}{4\,6^{3/4}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 108, normalized size = 0.95

$$\frac{1}{24}\left(\sqrt[4]{6}c\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\sqrt[4]{6}c\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\sqrt[4]{6}c\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}c\tan^{-1}\left(\sqrt[4]{6}x+1\right)+2d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*c*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*c*ArcTan[1 + 6^(1/4)*x] + 6
^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - 6^(1/4)*c*Log[2 + 2*6^(1/4)*x
 + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2 + dx^3}{2 + 3x^4}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c*x^2 + d*x^3)/(2 + 3*x^4), x]
```

fricas [B] time = 0.45, size = 272, normalized size = 2.39

$$-\frac{4\cdot 6^{\frac{1}{4}}\left(c^4\right)^{\frac{1}{4}}c^4\arctan\left(-\frac{c^5+6^{\frac{1}{4}}\left(c^4\right)^{\frac{5}{4}}x-6^{\frac{1}{4}}\sqrt{\frac{1}{3}}\left(c^4\right)^{\frac{5}{4}}\sqrt{\frac{3c^3x^2+6^{\frac{3}{4}}\left(c^4\right)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c}{c^3}}}{c^5}\right)+4\cdot 6^{\frac{1}{4}}\left(c^4\right)^{\frac{1}{4}}c^4\arctan\left(\frac{c^5-6^{\frac{1}{4}}\left(c^4\right)^{\frac{5}{4}}x+6^{\frac{1}{4}}\sqrt{\frac{1}{3}}\left(c^4\right)^{\frac{5}{4}}\sqrt{\frac{3c^3x^2-6^{\frac{3}{4}}\left(c^4\right)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c}{c^3}}}{c^5}\right)-\left(2c^4d-6^{\frac{1}{4}}\left(c^4\right)^{\frac{1}{4}}c^4\right)\log\left(3c^3x^2+6^{\frac{3}{4}}\left(c^4\right)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c\right)-\left(2c^4d+6^{\frac{1}{4}}\left(c^4\right)^{\frac{1}{4}}c^4\right)\log\left(3c^3x^2-6^{\frac{3}{4}}\left(c^4\right)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c\right)}{24c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/24*(4*6^(1/4)*(c^4)^(1/4)*c^4*arctan(-(c^5 + 6^(1/4)*(c^4)^(5/4)*x - 6^(
1/4)*sqrt(1/3)*(c^4)^(5/4)*sqrt((3*c^3*x^2 + 6^(3/4)*(c^4)^(3/4)*x + sqrt(6
)*sqrt(c^4)*c)/c^3))/c^5) + 4*6^(1/4)*(c^4)^(1/4)*c^4*arctan((c^5 - 6^(1/4)
*(c^4)^(5/4)*x + 6^(1/4)*sqrt(1/3)*(c^4)^(5/4)*sqrt((3*c^3*x^2 - 6^(3/4)*(c
^4)^(3/4)*x + sqrt(6)*sqrt(c^4)*c)/c^3))/c^5) - (2*c^4*d - 6^(1/4)*(c^4)^(1
/4)*c^4)*log(3*c^3*x^2 + 6^(3/4)*(c^4)^(3/4)*x + sqrt(6)*sqrt(c^4)*c) - (2*
c^4*d + 6^(1/4)*(c^4)^(1/4)*c^4)*log(3*c^3*x^2 - 6^(3/4)*(c^4)^(3/4)*x + sq
rt(6)*sqrt(c^4)*c))/c^4
```

giac [A] time = 0.28, size = 109, normalized size = 0.96

$$\frac{1}{12}\cdot 6^{\frac{1}{4}}c\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 6^{\frac{1}{4}}c\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-\frac{1}{24}\left(6^{\frac{1}{4}}c-2d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)+\frac{1}{24}\left(6^{\frac{1}{4}}c+2d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
+ 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))
) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1
/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

maple [A] time = 0.04, size = 125, normalized size = 1.10

$$\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{72}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{72}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{144}+\frac{d\ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2)/(3*x^4+2),x)
```

```
[Out] 1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72
*3^(1/2)*6^(3/4)*2^(1/2)*c*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*3^
(1/2)*6^(3/4)*2^(1/2)*c*ln((x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/
(x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/12*d*ln(3*x^4+2)
```

maxima [A] time = 3.03, size = 152, normalized size = 1.33

$$\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d-\sqrt{3}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d+\sqrt{3}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1
/4)*2^(3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3
)*c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/12*3^(1/4)*2^(1/4)*
c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/12*3^(1/4
)*2^(1/4)*c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

mupad [B] time = 0.37, size = 117, normalized size = 1.03

$$\ln\left(x-\frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}+\frac{6^{1/4}\,\sqrt{-\frac{1}{2}\mathrm{i}}\,c}{12}\right)+\ln\left(x+\frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}-\frac{6^{1/4}\,\sqrt{-\frac{1}{2}\mathrm{i}}\,c}{12}\right)+\ln\left(x-\frac{(-1)^{3/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}-\frac{6^{1/4}\,\sqrt{\frac{1}{2}\mathrm{i}}\,c}{12}\right)+\ln\left(x+\frac{(-1)^{3/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}+\frac{6^{1/4}\,\sqrt{\frac{1}{2}\mathrm{i}}\,c}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] log(x - ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(-1i/2)^(1/2)*c)/1
2) + log(x + ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(-1i/2)^(1/2)
*c)/12) + log(x - ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(1i/2)^(
1/2)*c)/12) + log(x + ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(1i/
2)^(1/2)*c)/12)
```

sympy [A] time = 0.42, size = 70, normalized size = 0.61

$$\operatorname{RootSum}\left(41472t^4-13824t^3d+1728t^2d^2-96td^3+3c^4+2d^4,\left(t\mapsto t\log\left(x+\frac{3456t^3-864t^2d+72td^2-2d^3}{3c^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2)/(3*x**4+2),x)
```

```
[Out] RootSum(41472*_t**4 - 13824*_t**3*d + 1728*_t**2*d**2 - 96*_t*d**3 + 3*c**4
 + 2*d**4, Lambda(_t, _t*log(x + (3456*_t**3 - 864*_t**2*d + 72*_t*d**2 - 2
*d**3)/(3*c**3))))
```

3.117 $\int \frac{a+cx^2+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=154

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}a\right.}{}$$

Rubi [A] time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1876, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\,6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx &= \int \left(\frac{dx^3}{2+3x^4}+\frac{a+cx^2}{2+3x^4}\right)dx \\
&= d\int \frac{x^3}{2+3x^4}\,dx+\int \frac{a+cx^2}{2+3x^4}\,dx \\
&= \frac{1}{12}d\log\left(2+3x^4\right)+\frac{1}{12}\left(\sqrt{6}\,a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx+\frac{1}{12}\left(\sqrt{6}\,a+2c\right)\int \frac{\sqrt{6}+3x^2}{2+3x^4}\,dx \\
&= \frac{1}{12}d\log\left(2+3x^4\right)-\frac{\left(\sqrt{6}\,a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\,6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\,6^{3/4}}+ \\
&= -\frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\,6^{3/4}}+\frac{1}{12}d\log \\
&= -\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\,6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}\right.}{8\,6^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 148, normalized size = 0.96

$$\frac{1}{48}\left(-\sqrt[4]{6}\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}\,x^2-2\sqrt[4]{6}\,x+2\right)+\sqrt[4]{6}\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}\,x^2+2\sqrt[4]{6}\,x+2\right)-2\sqrt[4]{6}\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\sqrt[4]{6}\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)+4d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(Sqrt[6]*a
+ 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 + 2*6^(1/4)*x + Sqrt[6]*
x^2] + 4*d*Log[2 + 3*x^4])/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^2 + d*x^3)/(2 + 3*x^4), x]
```

fricas [B] time = 0.49, size = 2326, normalized size = 15.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/144*(2*sqrt(6)*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 -
12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a
^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*arctan(-1/12*(sqrt(2)*s
qrt(1/3)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2
*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 -
12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2
*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*sqrt
((3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 + sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^
4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*s
qrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)
/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2
+ 2*c^2))/(9*a^4 + 12*a^2*c^2 + 4*c^4)) - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24
*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2
*c^2 + 4*c^4)*a*x - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2
*c^3)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24
*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 2*sqrt(6)*sqrt(54*a^4 + 72*a^2*c
^2 + 24*c^4)*(9*a^4 + 12*a^2*c^2 + 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))
/(81*a^8 + 108*a^6*c^2 - 48*a^2*c^6 - 16*c^8)) + 2*sqrt(6)*sqrt(2)*(54*a^4
+ 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 +
 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12
*a^2*c^2 + 4*c^4))*arctan(-1/12*(sqrt(2)*sqrt(1/3)*(54*a^4 + 72*a^2*c^2 + 2
4*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^
2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*
c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^
4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*sqrt((3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*
x^2 - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^
2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 +
 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) +
sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2))/(9*a^4 + 12*a^2*c^2 + 4
*c^4)) - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4
+ 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a*x - 2*sqrt(6)*sqr
t(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3)*x)*sqrt((9*a^4 + 12*a^2*c^2
 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 +
4*c^4)) - 2*sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(9*a^4 + 12*a^2*c^2
+ 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))/(81*a^8 + 108*a^6*c^2 - 48*a^2*c
^6 - 16*c^8)) - 3*(sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(9*a^4 + 12
*a^2*c^2 + 4*c^4 - 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*sqrt((9*a^4 +
12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*
a^2*c^2 + 4*c^4)) - 4*(9*a^4 + 12*a^2*c^2 + 4*c^4)*d)*log(3*(9*a^4 + 12*a^2
*c^2 + 4*c^4)*x^2 + sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a
^4 + 72*a^2*c^2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2
*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^
2 + 4*c^4)) + sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2)) + 3*(sqrt
(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sq
rt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*
sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 4*(
9*a^4 + 12*a^2*c^2 + 4*c^4)*d)*log(3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 - sqr
t(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^2 + 24*c^
4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(5
4*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + sqrt(54*a
^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2)))/(9*a^4 + 12*a^2*c^2 + 4*c^4)
```

giac [A] time = 0.21, size = 137, normalized size = 0.89

$$\frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c+4d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c-4d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c + 4*d)*lo
g(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c
- 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

maple [B] time = 0.05, size = 237, normalized size = 1.54

$$\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{48}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{72}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{72}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,c\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{144}+\frac{d\ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+a)/(3*x^4+2),x)
```

```
[Out] 1/48*3^(1/2)*6^(1/4)*2^(1/2)*a*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^
(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/24*3^(1/2)*6^(1/4
)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/24*3^(1/2)*6^(1/4)*2^
(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/72*3^(1/2)*6^(3/4)*2^(1/2
)*c*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*
arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*3^(1/2)*6^(3/4)*2^(1/2)*c*ln(
(x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*2^
(1/2)*x+1/3*6^(1/2)))+1/12*d*ln(3*x^4+2)
```

maxima [A] time = 2.99, size = 195, normalized size = 1.27

$$-\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c-2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log(
sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(3)
*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x
 + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arct
an(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*(3*3
^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(
3)*x - 3^(1/4)*2^(3/4)))
```

mupad [B] time = 5.81, size = 286, normalized size = 1.86

$$\ln\left(-2c+\sqrt{6}\,a\,1\mathrm{i}+x\sqrt{3\mathrm{i}\sqrt{6}\,a^2-12ac-2\mathrm{i}\sqrt{6}\,c^2}\right)\left(\frac{d}{12}+\frac{\sqrt{\frac{3\mathrm{i}\sqrt{6}\,a^2}{4}-3ac-\frac{1\mathrm{i}\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2c-\sqrt{6}\,a\,1\mathrm{i}+x\sqrt{3\mathrm{i}\sqrt{6}\,a^2-12ac-2\mathrm{i}\sqrt{6}\,c^2}\right)\left(\frac{d}{12}-\frac{\sqrt{\frac{3\mathrm{i}\sqrt{6}\,a^2}{4}-3ac-\frac{1\mathrm{i}\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2c+\sqrt{6}\,a\,1\mathrm{i}+x\sqrt{-3\mathrm{i}\sqrt{6}\,a^2-12ac+2\mathrm{i}\sqrt{6}\,c^2}\right)\left(\frac{d}{12}-\frac{\sqrt{-\frac{3\mathrm{i}\sqrt{6}\,a^2}{4}-3ac+\frac{1\mathrm{i}\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2c+\sqrt{6}\,a\,1\mathrm{i}-x\sqrt{-3\mathrm{i}\sqrt{6}\,a^2-12ac+2\mathrm{i}\sqrt{6}\,c^2}\right)\left(\frac{d}{12}+\frac{\sqrt{-\frac{3\mathrm{i}\sqrt{6}\,a^2}{4}-3ac+\frac{1\mathrm{i}\sqrt{6}\,c^2}{2}}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] log(6^(1/2)*a*1i - 2*c + x*(6^(1/2)*a^2*3i - 12*a*c - 6^(1/2)*c^2*2i)^(1/2)
)*(d/12 + ((6^(1/2)*a^2*3i)/4 - 3*a*c - (6^(1/2)*c^2*1i)/2)^(1/2)/12) + log
(2*c - 6^(1/2)*a*1i + x*(6^(1/2)*a^2*3i - 12*a*c - 6^(1/2)*c^2*2i)^(1/2))*(
d/12 - ((6^(1/2)*a^2*3i)/4 - 3*a*c - (6^(1/2)*c^2*1i)/2)^(1/2)/12) + log(2*
c + 6^(1/2)*a*1i + x*(6^(1/2)*c^2*2i - 6^(1/2)*a^2*3i - 12*a*c)^(1/2))*(d/1
2 - ((6^(1/2)*c^2*1i)/2 - (6^(1/2)*a^2*3i)/4 - 3*a*c)^(1/2)/12) + log(2*c +
```

```
6^(1/2)*a*1i - x*(6^(1/2)*c^2*2i - 6^(1/2)*a^2*3i - 12*a*c)^(1/2))*(d/12 +
((6^(1/2)*c^2*1i)/2 - (6^(1/2)*a^2*3i)/4 - 3*a*c)^(1/2)/12)
```

sympy [A] time = 1.38, size = 148, normalized size = 0.96

$$\mathrm{RootSum}\left(165888t^4-55296t^3d+t^2\left(6912ac+6912d^2\right)+t\left(-1152acd-384d^3\right)+27a^4+36a^2c^2+48acd^2+12c^4+8d^4,\left(t\mapsto t\log\left(x+\frac{-13824t^3c+3456t^2cd+216ta^3-432tac^2-288tcd^2-18a^3d+36ac^2d+8cd^3}{27a^4-12c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 6912*d**2) + _t*(-
1152*a*c*d - 384*d**3) + 27*a**4 + 36*a**2*c**2 + 48*a*c*d**2 + 12*c**4 + 8
*d**4, Lambda(_t, _t*log(x + (-13824*_t**3*c + 3456*_t**2*c*d + 216*_t*a**3
 - 432*_t*a*c**2 - 288*_t*c*d**2 - 18*a**3*d + 36*a*c**2*d + 8*c*d**3)/(27*
a**4 - 12*c**4))))
```

3.118 $\int \frac{bx+cx^2+dx^3}{2+3x^4} dx$

Optimal. Leaf size=136

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right.}{2\,6^{3/4}}$$

Rubi [A] time = 0.14, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1594, 1831, 297, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\,6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4
)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*
Log[2 + 3*x^4])/12
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx &= \int \frac{x\left(b+cx+dx^2\right)}{2+3x^4}\,dx \\
&= \int \left(\frac{cx^2}{2+3x^4}+\frac{x\left(b+dx^2\right)}{2+3x^4}\right)dx \\
&= c\int \frac{x^2}{2+3x^4}\,dx+\int \frac{x\left(b+dx^2\right)}{2+3x^4}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx,x,x^2\right)-\frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}+\frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right)+\frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx+\frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}}-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\,6^{3/4}}+\frac{1}{12}d\log\left(2 \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right.}{4\,6^{3/4}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 125, normalized size = 0.92

$$\frac{1}{24}\left(-2\sqrt[4]{6}\left(\sqrt[4]{6}b+c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}\left(c-\sqrt[4]{6}b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)+\sqrt[4]{6}c\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\sqrt[4]{6}c\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)+2d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(-(6^(1/4)*b)
 + c)*ArcTan[1 + 6^(1/4)*x] + 6^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2]
- 6^(1/4)*c*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] IntegrateAlgebraic[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.20, size = 124, normalized size = 0.91

$$-\frac{1}{12}\left(\sqrt{6}b-6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\left(\sqrt{6}b+6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-\frac{1}{24}\left(6^{\frac{1}{4}}c-2d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)+\frac{1}{24}\left(6^{\frac{1}{4}}c+2d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(6)*b - 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)
*(2/3)^(1/4))) + 1/12*(sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4
)*(2*x - sqrt(2)*(2/3)^(1/4))) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(
2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^
(1/4)*x + sqrt(2/3))
```

maple [A] time = 0.05, size = 140, normalized size = 1.03

$$\frac{\sqrt{6}\, b \arctan\left(\frac{\sqrt{6}\, x^2}{2}\right)}{12}+\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, \sqrt{3}\, 6^{\frac{3}{4}} x}{6}-1\right)}{72}+\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, \sqrt{3}\, 6^{\frac{3}{4}} x}{6}+1\right)}{72}+\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{2}\, c \ln\left(\frac{x^2-\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, x}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, x}{3}+\frac{\sqrt{6}}{3}}\right)}{144}+\frac{d \ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x)/(3*x^4+2),x)
```

```
[Out] 1/12*6^(1/2)*b*arctan(1/2*6^(1/2)*x^2)+1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*arcta
n(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*arctan(1/
6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*3^(1/2)*6^(3/4)*2^(1/2)*c*ln((x^2-1/3*
3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1
/3*6^(1/2)))+1/12*d*ln(3*x^4+2)
```

maxima [A] time = 3.02, size = 174, normalized size = 1.28

$$\frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c-6b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c+6b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d-\sqrt{3}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d+\sqrt{3}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c - 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(
2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c +
 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/72*3^
(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(
3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3)*c)*lo
g(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

mupad [B] time = 5.39, size = 300, normalized size = 2.21

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(z^4-\frac{d z^3}{3}+\frac{z^2\left(1728b^2+3456d^2\right)}{82944}-\frac{z\left(-288bc^2+288b^2d+192d^3\right)}{82944}-\frac{bc^2d}{3456}+\frac{b^2d^2}{6912}+\frac{d^4}{20736}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\left(144bc+x\left(144bd-72c^2\right)-\mathrm{root}\left(z^4-\frac{d z^3}{3}+\frac{z^2\left(1728b^2+3456d^2\right)}{82944}-\frac{z\left(-288bc^2+288b^2d+192d^3\right)}{82944}-\frac{bc^2d}{3456}+\frac{b^2d^2}{6912}+\frac{d^4}{20736}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)bx864\right)+x\left(9b^3+6bd^2-6c^2d\right)-6c^3+12bcd\right)\mathrm{root}\left(z^4-\frac{d z^3}{3}+\frac{z^2\left(1728b^2+3456d^2\right)}{82944}-\frac{z\left(-288bc^2+288b^2d+192d^3\right)}{82944}-\frac{bc^2d}{3456}+\frac{b^2d^2}{6912}+\frac{d^4}{20736}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(x*(6*b*d^2 - 6*c^2*d + 9*b^3) - root(z^4 - (d*z^3)/3 + (z^2*(172
8*b^2 + 3456*d^2))/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 -
(b*c^2*d)/3456 + (b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k)*(
144*b*c + x*(144*b*d - 72*c^2) - 864*root(z^4 - (d*z^3)/3 + (z^2*(1728*b^2
+ 3456*d^2))/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 - (b*c^2
*d)/3456 + (b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k)*b*x) -
6*c^3 + 12*b*c*d)*root(z^4 - (d*z^3)/3 + (z^2*(1728*b^2 + 3456*d^2))/82944
- (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 - (b*c^2*d)/3456 + (b^2*d^2
)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k), k, 1, 4)
```

sympy [A] time = 1.99, size = 189, normalized size = 1.39

$$\mathrm{RootSum}\left(82944t^4-27648t^3d+t^2\left(1728b^2+3456d^2\right)+t\left(-288b^2d+288bc^2-192d^3\right)+9b^4+12b^2d^2-24bc^2d+6c^4+4d^4,\left(t\mapsto t\log\left(x+\frac{-3456t^3c^2+864t^2b^3+864t^2c^2d-144tb^3d-108tb^2c^2-72tc^2d^2+9b^5+6b^3d^2+9b^2c^2d-9bc^4+2c^2d^3}{18b^4c-3c^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] RootSum(82944*_t**4 - 27648*_t**3*d + _t**2*(1728*b**2 + 3456*d**2) + _t*(-
288*b**2*d + 288*b*c**2 - 192*d**3) + 9*b**4 + 12*b**2*d**2 - 24*b*c**2*d +
 6*c**4 + 4*d**4, Lambda(_t, _t*log(x + (-3456*_t**3*c**2 + 864*_t**2*b**3
+ 864*_t**2*c**2*d - 144*_t*b**3*d - 108*_t*b**2*c**2 - 72*_t*c**2*d**2 + 9
*b**5 + 6*b**3*d**2 + 9*b**2*c**2*d - 9*b*c**4 + 2*c**2*d**3)/(18*b**4*c -
3*c**5))))
```

3.119 $\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}-\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}\,a\right.}{}$$

Rubi [A] time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$-\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}\,a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,6^{3/4}}-\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{4\,6^{3/4}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/
4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4))
- ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt
[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x
^4])/12
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx &= \int \left(\frac{a+cx^2}{2+3x^4}+\frac{x\left(b+dx^2\right)}{2+3x^4}\right)dx \\
&= \int \frac{a+cx^2}{2+3x^4}\,dx+\int \frac{x\left(b+dx^2\right)}{2+3x^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx,x,x^2\right)+\frac{1}{12}\left(\sqrt{6}\,a-2c\right)\int \frac{\sqrt{6}\,-3x^2}{2+3x^4}\,dx+\frac{1}{12}\left(\sqrt{6}\,a+2c\right)\int \\
&= \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right)-\frac{\left(\sqrt{6}\,a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\,6^{3/4}}-\frac{\left(\sqrt{6}\,a-2c\right)\int \frac{}{-\sqrt{}}}{8\,6^{3/4}} \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}\,-6^{3/4}x+3x^2\right)}{8\,6^{3/4}}+\frac{\left(\sqrt{6}\,a-2c\right)\log\left(\sqrt{6}\,+6^{3/4}\right.}{8\,6^{3/4}} \\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\,6^{3/4}}+\frac{\left(\sqrt{6}\,a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\,6^{3/4}}-\frac{\left(\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 164, normalized size = 0.93

$$\frac{1}{48}\left(-2\sqrt[4]{6}\tan^{-1}\left(1-\sqrt[4]{6}x\right)\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)+2\sqrt[4]{6}\tan^{-1}\left(\sqrt[4]{6}x+1\right)\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)-\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)+4d\log\left(3x^4+2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/
4)*(Sqrt[6]*a - 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]
*a - 2*c)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*Lo
g[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 149, normalized size = 0.85

$$\frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b+2\cdot6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b+2\cdot6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot6^{\frac{1}{4}}c+4d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot6^{\frac{1}{4}}c-4d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)
*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)*c
)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/
4)*a - 2*6^(1/4)*c + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/
48*(6^(3/4)*a - 2*6^(1/4)*c - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2
/3))
```

maple [A] time = 0.05, size = 252, normalized size = 1.43

$$\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\,\sqrt{3}\, 6^{\frac{3}{4}} x}{6}-1\right)}{24}+\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\,\sqrt{3}\, 6^{\frac{3}{4}} x}{6}+1\right)}{24}+\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, a \ln\left(\frac{x^2+\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, x}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, x}{3}+\frac{\sqrt{6}}{3}}\right)}{48}+\frac{\sqrt{6}\, b \arctan\left(\frac{\sqrt{6}\, x^2}{2}\right)}{12}+\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\,\sqrt{3}\, 6^{\frac{3}{4}} x}{6}-1\right)}{72}+\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\,\sqrt{3}\, 6^{\frac{3}{4}} x}{6}+1\right)}{72}+\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{2}\, c \ln\left(\frac{x^2-\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, x}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{2}\, x}{3}+\frac{\sqrt{6}}{3}}\right)}{144}+\frac{d \ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x)
```

```
[Out] 1/48*3^(1/2)*6^(1/4)*2^(1/2)*a*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^
(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/24*3^(1/2)*6^(1/4
)*2^(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/24*3^(1/2)*6^(1/4)*2^
(1/2)*a*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/12*6^(1/2)*b*arctan(1/2*6
^(1/2)*x^2)+1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/
4)*x+1)+1/72*3^(1/2)*6^(3/4)*2^(1/2)*c*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x
-1)+1/144*3^(1/2)*6^(3/4)*2^(1/2)*c*ln((x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1
/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/12*d*ln(3*x^
4+2)
```

maxima [A] time = 2.99, size = 207, normalized size = 1.18

$$-\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\,\sqrt{2}\,c-2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}\,x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\,\sqrt{2}\,c+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}\,x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c-6\,\sqrt{2}\,b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\,\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c+6\,\sqrt{2}\,b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\,\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log(
sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(3)
*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x
 + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c - 6*s
qrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/7
2*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c + 6*sqrt(2)*b)*arctan(
1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

mupad [B] time = 5.64, size = 1168, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

```
[Out] symsum(log(9*a*b^2 - 864*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24
 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)
/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*
b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4
/9216 + a^4/6144, z, k)^2*a - 9*a^2*c - 6*a*d^2 + 9*b^3*x - 6*c^3 + 144*roo
t(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/
144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*
d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912
 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*a*d
- 144*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (
a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432
- (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*
```

```
d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z
, k)*b*c + 12*b*c*d - 108*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/2
4 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z
)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a
*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^
4/9216 + a^4/6144, z, k)*a^2*x + 864*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 +
(d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288
 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d
)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4
/13824 + b^4/9216 + a^4/6144, z, k)^2*b*x + 72*root(z^4 - (d*z^3)/3 + (a*c*
z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*
c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456
+ (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/2
0736 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*c^2*x + 9*a^2*d*x + 6*b*d^2*x
 - 6*c^2*d*x - 144*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^
2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 -
 (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)
/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216
+ a^4/6144, z, k)*b*d*x - 18*a*b*c*x)*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 +
 (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/28
8 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*
d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^
4/13824 + b^4/9216 + a^4/6144, z, k), k, 1, 4)
```

sympy [B] time = 13.07, size = 580, normalized size = 3.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 3456*b**2 + 6912*d
**2) + _t*(-864*a**2*b - 1152*a*c*d - 576*b**2*d + 576*b*c**2 - 384*d**3) +
 27*a**4 + 72*a**2*b*d + 36*a**2*c**2 - 72*a*b**2*c + 48*a*c*d**2 + 18*b**4
 + 24*b**2*d**2 - 48*b*c**2*d + 12*c**4 + 8*d**4, Lambda(_t, _t*log(x + (-4
1472*_t**3*a**2*c + 82944*_t**3*a*b**2 + 27648*_t**3*c**3 + 5184*_t**2*a**3
*b + 10368*_t**2*a**2*c*d - 20736*_t**2*a*b**2*d + 10368*_t**2*a*b*c**2 - 6
912*_t**2*b**3*c - 6912*_t**2*c**3*d + 648*_t*a**5 - 864*_t*a**3*b*d - 1728
*_t*a**3*c**2 + 3888*_t*a**2*b**2*c - 864*_t*a**2*c*d**2 + 864*_t*a*b**4 +
1728*_t*a*b**2*d**2 - 1728*_t*a*b*c**2*d + 864*_t*a*c**4 + 1152*_t*b**3*c*d
 + 864*_t*b**2*c**3 + 576*_t*c**3*d**2 - 54*a**5*d + 270*a**4*b*c - 270*a**
3*b**3 + 36*a**3*b*d**2 + 144*a**3*c**2*d - 324*a**2*b**2*c*d + 24*a**2*c*d
**3 - 72*a*b**4*d + 180*a*b**3*c**2 - 48*a*b**2*d**3 + 72*a*b*c**2*d**2 - 7
2*a*c**4*d - 72*b**5*c - 48*b**3*c*d**2 - 72*b**2*c**3*d + 72*b*c**5 - 16*c
**3*d**3)/(81*a**6 - 54*a**4*c**2 + 432*a**3*b**2*c - 216*a**2*b**4 - 36*a*
*2*c**4 + 288*a*b**2*c**3 - 144*b**4*c**2 + 24*c**6))))
```

3.120 $\int \frac{1+x+x^2+x^3}{1-x^4} dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(1 - x^4),x]
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2+x^3}{1-x^4} dx &= \int \frac{1}{1-x} dx\\ &= -\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]
[Out] -Log[1 - x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2+x^3}{1-x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2 + x^3)/(1 - x^4),x]
[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3)/(1 - x^4), x]
```

fricas [A] time = 0.40, size = 6, normalized size = 0.75

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

giac [A] time = 0.15, size = 7, normalized size = 0.88

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
```

```
[Out] -log(abs(x - 1))
```

maple [A] time = 0.04, size = 7, normalized size = 0.88

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(-x^4+1),x)
```

```
[Out] -ln(x-1)
```

maxima [A] time = 1.35, size = 6, normalized size = 0.75

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -log(x - 1)
```

mupad [B] time = 0.02, size = 6, normalized size = 0.75

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2 + x^3 + 1)/(x^4 - 1),x)
```

```
[Out] -log(x - 1)
```

sympy [A] time = 0.07, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(-x**4+1),x)
```

```
[Out] -log(x - 1)
```

3.121 $\int \frac{1+x+x^2+x^3}{1+x^4} dx$

Optimal. Leaf size=53

$$\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}$$

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1876, 1162, 617, 204, 1248, 635, 203, 260}

$$\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/Sqrt[
2] + Log[1 + x^4]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2+x^3}{1+x^4}\,dx &= \int \left(\frac{1+x^2}{1+x^4}+\frac{x\left(1+x^2\right)}{1+x^4}\right)dx\\
&= \int \frac{1+x^2}{1+x^4}\,dx+\int \frac{x\left(1+x^2\right)}{1+x^4}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x}{1+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,x^2\right)+\frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\right.}{\sqrt{2}}\\
&= \frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{1}{4}\log\left(1+x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.94

$$\frac{1}{4}\left(\log\left(x^4+1\right)-2\left(1+\sqrt{2}\right)\tan^{-1}\left(1-\sqrt{2}\,x\right)+2\left(\sqrt{2}-1\right)\tan^{-1}\left(\sqrt{2}\,x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

```
[Out] (-2*(1 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(-1 + Sqrt[2])*ArcTan[1 + Sqrt[
2]*x] + Log[1 + x^4])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

```
[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3)/(1 + x^4), x]
```

fricas [B] time = 0.43, size = 145, normalized size = 2.74

$$-\sqrt{-2\sqrt{2}+3}\arctan\left(\sqrt{x^2+\sqrt{2}x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+3}-\left(\sqrt{2}(x+1)+2x+1\right)\sqrt{-2\sqrt{2}+3}\right)+\sqrt{2\sqrt{2}+3}\arctan\left(-\left(\sqrt{2}(x+1)-\sqrt{x^2-\sqrt{2}x+1}\left(\sqrt{2}-2\right)-2x-1\right)\sqrt{2\sqrt{2}+3}\right)+\frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*sqrt(2) + 3)*arctan(sqrt(x^2 + sqrt(2)*x + 1)*(sqrt(2) + 2)*sqrt(-
2*sqrt(2) + 3) - (sqrt(2)*(x + 1) + 2*x + 1)*sqrt(-2*sqrt(2) + 3)) + sqrt(2
*sqrt(2) + 3)*arctan(-(sqrt(2)*(x + 1) - sqrt(x^2 - sqrt(2)*x + 1)*(sqrt(2)
 - 2) - 2*x - 1)*sqrt(2*sqrt(2) + 3)) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*
log(x^2 - sqrt(2)*x + 1)
```

giac [A] time = 0.15, size = 70, normalized size = 1.32

$$\frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*(sqrt(2) + 1)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*log
(x^2 - sqrt(2)*x + 1)
```

maple [B] time = 0.05, size = 102, normalized size = 1.92

$$\frac{\arctan\left(x^2\right)}{2}+\frac{\sqrt{2}\arctan\left(\sqrt{2}x-1\right)}{2}+\frac{\sqrt{2}\arctan\left(\sqrt{2}x+1\right)}{2}+\frac{\sqrt{2}\ln\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right)}{8}+\frac{\sqrt{2}\ln\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)}{8}+\frac{\ln\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(x^4+1),x)
```

```
[Out] 1/2*2^(1/2)*arctan(2^(1/2)*x-1)+1/8*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/
2)*x+1))+1/2*2^(1/2)*arctan(2^(1/2)*x+1)+1/2*arctan(x^2)+1/8*2^(1/2)*ln((x^
2-2^(1/2)*x+1)/(x^2+2^(1/2)*x+1))+1/4*ln(x^4+1)
```

maxima [A] time = 3.00, size = 76, normalized size = 1.43

$$-\frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2
)*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)
*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)
```

mupad [B] time = 0.40, size = 156, normalized size = 2.94

$$\ln\left((16x-16)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}+\frac{1}{4}\right)-8x\right)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}+\frac{1}{4}\right)-\ln\left(8x+(16x-16)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}-\frac{1}{4}\right)\right)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}-\frac{1}{4}\right)-\ln\left(8x+(16x-16)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}-\frac{1}{4}\right)\right)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}-\frac{1}{4}\right)+\ln\left(8x-(16x-16)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}+\frac{1}{4}\right)\right)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 1)/(x^4 + 1),x)
```

```
[Out] log((16*x - 16)*((- 2*2^(1/2) - 3)^(1/2)/4 + 1/4) - 8*x)*((- 2*2^(1/2) - 3)
^(1/2)/4 + 1/4) - log(8*x + (16*x - 16)*((- 2*2^(1/2) - 3)^(1/2)/4 - 1/4))*
((- 2*2^(1/2) - 3)^(1/2)/4 - 1/4) - log(8*x + (16*x - 16)*((2*2^(1/2) - 3)^
(1/2)/4 - 1/4))*((2*2^(1/2) - 3)^(1/2)/4 - 1/4) + log(8*x - (16*x - 16)*((2
*2^(1/2) - 3)^(1/2)/4 + 1/4))*((2*2^(1/2) - 3)^(1/2)/4 + 1/4)
```

sympy [A] time = 0.43, size = 73, normalized size = 1.38

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4}+2\left(\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x-1\right)+2\left(-\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(x**4+1),x)
```

```
[Out] log(x**2 - sqrt(2)*x + 1)/4 + log(x**2 + sqrt(2)*x + 1)/4 + 2*(1/4 + sqrt(2
)/4)*atan(sqrt(2)*x - 1) + 2*(-1/4 + sqrt(2)/4)*atan(sqrt(2)*x + 1)
```

3.122 $\int \frac{1+x+x^2+x^3}{a-bx^4} dx$

Optimal. Leaf size=124

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}-\frac{\log\left(a-bx^4\right)}{4b}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1876, 1167, 205, 208, 1248, 635, 260}

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}-\frac{\log\left(a-bx^4\right)}{4b}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] -((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((
Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - Log[a - b*x^4]/(4*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx &= \int \left(\frac{1+x^2}{a-bx^4}+\frac{x\left(1+x^2\right)}{a-bx^4}\right)dx \\
&= \int \frac{1+x^2}{a-bx^4}\,dx+\int \frac{x\left(1+x^2\right)}{a-bx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{}{\sqrt{a}} \\
&= -\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right) \\
&= -\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\log\left(a-bx^4\right.}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 203, normalized size = 1.64

$$
-\frac{\left(a^{3/4}+\sqrt{a}\sqrt[4]{b}+\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{4ab^{3/4}}-\frac{\left(-a^{3/4}+\sqrt{a}\sqrt[4]{b}-\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4ab^{3/4}}+\frac{\left(\sqrt[4]{a}\sqrt{b}-a^{3/4}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2ab^{3/4}}-\frac{\log\left(a-bx^4\right)}{4b}+\frac{\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{a}\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] ((-a^(3/4) + a^(1/4)*Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4)) -
((a^(3/4) + Sqrt[a]*b^(1/4) + a^(1/4)*Sqrt[b])*Log[a^(1/4) - b^(1/4)*x])/(4
*a*b^(3/4)) - ((-a^(3/4) + Sqrt[a]*b^(1/4) - a^(1/4)*Sqrt[b])*Log[a^(1/4) +
 b^(1/4)*x])/(4*a*b^(3/4)) + Log[Sqrt[a] + Sqrt[b]*x^2]/(4*Sqrt[a]*Sqrt[b])
 - Log[a - b*x^4]/(4*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3)/(a - b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.17, size = 290, normalized size = 2.34

$$-\frac{\log\left(|bx^4-a|\right)}{4\,b}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\sqrt{2}\sqrt{-ab^3}\,b+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\sqrt{2}\sqrt{-ab^3}\,b+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(b*x^4 - a))/b + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt
(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)
)/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(-a
*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(
-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*lo
g(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3
)^(1/4)*b^2 - (-a*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a*b^3)
```

maple [B] time = 0.05, size = 171, normalized size = 1.38

$$-\frac{\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a}-\frac{\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{\ln\left(b\,x^4-a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(-b*x^4+a),x)
```

```
[Out] 1/4*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*(a/b)^(1/4)/a*arc
tan(1/(a/b)^(1/4)*x)-1/4/(a*b)^(1/2)*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x
^2-a))-1/2/b/(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)+1/4/b/(a/b)^(1/4)*ln((x+(a
/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b*ln(b*x^4-a)
```

maxima [A] time = 3.02, size = 160, normalized size = 1.29

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{4\sqrt{a}\,b}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{4\sqrt{a}\,b}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(a) - sqrt(b))*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(a) - sqrt(b))*log(sqrt(b)*x^2 + s
qrt(a))/(sqrt(a)*b) - 1/4*(sqrt(a) + sqrt(b))*log(sqrt(b)*x^2 - sqrt(a))/(s
qrt(a)*b) - 1/4*(sqrt(a) + sqrt(b))*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))
/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b
))
```

mupad [B] time = 5.03, size = 312, normalized size = 2.52

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3+a^3,z,k\right)\left(\mathrm{root}\left(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3+a^3,z,k\right)\left(16\,a\,b^3-16\,a\,b^3\,x\right)-x\left(4\,a\,b^2-4\,b^3\right)\right)\right)\mathrm{root}\left(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3+a^3,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 1)/(a - b*x^4),x)
```

```
[Out] symsum(log(-root(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 96*a^
2*b^3*z^2 + 16*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2 - b^
3 + a^3, z, k)*(root(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 9
```

```
6*a^2*b^3*z^2 + 16*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2
- b^3 + a^3, z, k)*(16*a*b^3 - 16*a*b^3*x) - x*(4*a*b^2 - 4*b^3)))*root(256
*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 96*a^2*b^3*z^2 + 16*a^3*b
*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2 - b^3 + a^3, z, k), k, 1
, 4)
```

sympy [**A**] time = 2.28, size = 187, normalized size = 1.51

$$-\operatorname{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2\left(96a^3b^2 - 96a^2b^3\right) + t\left(-16a^3b + 32a^2b^2 - 16ab^3\right) + a^3 - 3a^2b + 3ab^2 - b^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^3b^3 + 48t^2a^3b^2 + 16t^2a^2b^3 - 12ta^3b + 16ta^2b^2 - 4tab^3 + a^3 - 2a^2b + ab^2}{a^2b - 2ab^2 + b^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 -
96*a**2*b**3) + _t*(-16*a**3*b + 32*a**2*b**2 - 16*a*b**3) + a**3 - 3*a**2*
b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**3 + 48*_t**2*
a**3*b**2 + 16*_t**2*a**2*b**3 - 12*_t*a**3*b + 16*_t*a**2*b**2 - 4*_t*a*b*
*3 + a**3 - 2*a**2*b + a*b**2)/(a**2*b - 2*a*b**2 + b**3))))
```

3.123 $\int \frac{1+x+x^2+x^3}{a+bx^4} dx$

Optimal. Leaf size=277

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan}{2\sqrt{2}}$$

Rubi [A] time = 0.20, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\log\left(a+bx^4\right)}{4b}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt
[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) +
Log[a + b*x^4]/(4*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx &= \int \left(\frac{1+x^2}{a+bx^4}+\frac{x\left(1+x^2\right)}{a+bx^4}\right)dx \\
&= \int \frac{1+x^2}{a+bx^4}\,dx+\int \frac{x\left(1+x^2\right)}{a+bx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{a+bx^2}\,dx,x,x^2\right)-\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b} \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}}}{4b} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}\right.}{4\sqrt{2}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+
\end{aligned}$$

Mathematica [A] time = 0.24, size = 283, normalized size = 1.02

$$\frac{\sqrt{2}\sqrt[4]{b}\left(a^{3/4}-\sqrt[4]{a}\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{b}-a^{3/4}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+2a\log\left(a+bx^4\right)-2\sqrt[4]{a}\sqrt[4]{b}\left(2\sqrt[4]{a}\sqrt[4]{b}+\sqrt{2}\sqrt{a}+\sqrt{2}\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{b}\left(-2\sqrt[4]{a}\sqrt[4]{b}+\sqrt{2}\sqrt{a}+\sqrt{2}\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

```
[Out] (-2*a^(1/4)*(Sqrt[2]*Sqrt[a] + 2*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*(Sqrt[2]*Sqrt[a] - 2*a
^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)] + Sqrt[2]*(a^(3/4) - a^(1/4)*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(-a^(3/4) + a^(1/4)*Sqrt[b])*b^(
1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a*Log[a + b
*x^4])/(8*a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3)/(a + b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 270, normalized size = 0.97

$$\frac{\log\left(|bx^4+a|\right)}{4b}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2-\sqrt{2}\sqrt{ab^3}\,b+\left(ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2+\sqrt{2}\sqrt{ab^3}\,b+\left(ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2-\left(ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2-\left(ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt(a
*b^3)*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(a*b^3)*b
+ (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a
*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

maple [A] time = 0.05, size = 286, normalized size = 1.03

$$\frac{\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{2\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\ln\left(bx^4+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(b*x^4+a),x)
```

```
[Out] 1/8*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(
a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x+1)+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
+1/2/(a*b)^(1/2)*arctan((1/a*b)^(1/2)*x^2)+1/8/b/(a/b)^(1/4)*2^(1/2)*ln((x^
2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)
))+1/4/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/b/(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*ln(b*x^4+a)/b
```

maxima [A] time = 2.98, size = 296, normalized size = 1.07

$$\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}-\sqrt{a}\sqrt{b}+b\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}+\sqrt{a}\sqrt{b}-b\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\left(\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{a}\right)b+\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}+2a\right)\sqrt{b}-2a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{\left(\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{a}\right)b+\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}-2a\right)\sqrt{b}+2a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*b^(1/4) - sqrt(a)*sqrt(b) + b)*log(sqrt(b)*x^2
 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/8*sqrt(2)*(sq
rt(2)*a^(3/4)*b^(1/4) + sqrt(a)*sqrt(b) - b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/4*((sqrt(2)*a^(1/4)*b^(1/4)
- 2*sqrt(a))*b + (sqrt(2)*a^(3/4)*b^(1/4) + 2*a)*sqrt(b) - 2*a*sqrt(b))*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 1/4*((sqrt(2)*a^(1/4)*b^(1/4)
 + 2*sqrt(a))*b + (sqrt(2)*a^(3/4)*b^(1/4) - 2*a)*sqrt(b) + 2*a*sqrt(b))*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4))
```

mupad [B] time = 5.04, size = 305, normalized size = 1.10

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(256a^3b^4z^4-256a^3b^3z^3+96a^3b^2z^2+96a^2b^3z^2-16a^3bz-16ab^3z-32a^2b^2z+3a^2b+3ab^2+b^3+a^3,z,k\right)\left(\mathrm{root}\left(256a^3b^4z^4-256a^3b^3z^3+96a^3b^2z^2+96a^2b^3z^2-16a^3bz-16ab^3z-32a^2b^2z+3a^2b+3ab^2+b^3+a^3,z,k\right)\left(16ab^3-16ab^3x\right)+x\left(4b^3+4ab^2\right)\right)\right)\mathrm{root}\left(256a^3b^4z^4-256a^3b^3z^3+96a^3b^2z^2+96a^2b^3z^2-16a^3bz-16ab^3z-32a^2b^2z+3a^2b+3ab^2+b^3+a^3,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 1)/(a + b*x^4),x)
```

```
[Out] symsum(log(-root(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 96*a^
2*b^3*z^2 - 16*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2 + b^
3 + a^3, z, k)*(root(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 9
6*a^2*b^3*z^2 - 16*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2
+ b^3 + a^3, z, k)*(16*a*b^3 - 16*a*b^3*x) + x*(4*a*b^2 + 4*b^3)))*root(256
*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 96*a^2*b^3*z^2 - 16*a^3*b
*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2 + b^3 + a^3, z, k), k, 1
, 4)
```

sympy [A] time = 2.27, size = 187, normalized size = 0.68

$$\operatorname{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2\left(96a^3b^2 + 96a^2b^3\right) + t\left(-16a^3b - 32a^2b^2 - 16ab^3\right) + a^3 + 3a^2b + 3ab^2 + b^3, \left(t \mapsto t\log\left(x + \frac{64t^3a^3b^3 - 48t^2a^3b^2 + 16t^2a^2b^3 + 12ta^3b + 16ta^2b^2 + 4tab^3 - a^3 - 2a^2b - ab^2}{a^2b + 2ab^2 + b^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 + 9
6*a**2*b**3) + _t*(-16*a**3*b - 32*a**2*b**2 - 16*a*b**3) + a**3 + 3*a**2*b
 + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**3 - 48*_t**2*a*
*3*b**2 + 16*_t**2*a**2*b**3 + 12*_t*a**3*b + 16*_t*a**2*b**2 + 4*_t*a*b**3
 - a**3 - 2*a**2*b - a*b**2)/(a**2*b + 2*a*b**2 + b**3))))
```

3.124 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4} dx$

Optimal. Leaf size=148

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}$$

Rubi [A] time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1885, 1248, 635, 208, 260, 1887, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[
a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx &= \int \left(\frac{x\left(d+fx^2\right)}{a-bx^4}+\frac{c+ex^2+gx^4}{a-bx^4}\right)dx\\ &= \int \frac{x\left(d+fx^2\right)}{a-bx^4}\,dx+\int \frac{c+ex^2+gx^4}{a-bx^4}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+fx}{a-bx^2}\,dx,x,x^2\right)+\int \left(-\frac{g}{b}+\frac{bc+ag+bex^2}{b\left(a-bx^4\right)}\right)dx\\ &= -\frac{gx}{b}+\frac{\int \frac{bc+ag+bex^2}{a-bx^4}\,dx}{b}+\frac{1}{2}d\operatorname{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}f\operatorname{Subst}\Bigg(\int \frac{}{a}\\ &= -\frac{gx}{b}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}+\frac{1}{2}\left(e-\frac{bc+ag}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-}\\ &= -\frac{gx}{b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right)\tanh^{-1}\Bigg(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}}{2a^{3/4}b^{5/4}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 249, normalized size = 1.68

$$\frac{-a^{3/4}\sqrt[4]{b}f\log\left(a-bx^4\right)-4a^{3/4}\sqrt[4]{b}gx-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e+ag+bc\right)+\sqrt[4]{a}b^{3/4}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)-\sqrt[4]{a}b^{3/4}d\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)+2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)+bc\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)+\sqrt{a}\sqrt{b}e\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)+ag\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] (-4*a^(3/4)*b^(1/4)*g*x + 2*(b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)
*x)/a^(1/4)] - (b*c + a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e + a*g)*Log[a^(1
/4) - b^(1/4)*x] + b*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*b^(3/4)*d*Log[a^(
1/4) + b^(1/4)*x] + Sqrt[a]*Sqrt[b]*e*Log[a^(1/4) + b^(1/4)*x] + a*g*Log[a^
(1/4) + b^(1/4)*x] + a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3/4)
*b^(1/4)*f*Log[a - b*x^4])/(4*a^(3/4)*b^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.19, size = 303, normalized size = 2.05

$$-\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+abg+\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^{2}c+abg-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{gx}{b}-\frac{f\log\left(\left|bx^{4}-a\right|\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*
arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4
) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(
3/4) - 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a
/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(
-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) -
g*x/b - 1/4*f*log(abs(b*x^4 - a))/b
```

maple [B] time = 0.05, size = 244, normalized size = 1.65

$$-\frac{d\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{4\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a}-\frac{e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{f\ln\left(bx^{4}-a\right)}{4b}-\frac{gx}{b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -g*x/b+1/2/b*(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)*g+1/2*c*(a/b)^(1/4)/a*arct
an(1/(a/b)^(1/4)*x)+1/4/b*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g
+1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4*d/(a*b)^(1/2)*
ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-1/2*e/b/(a/b)^(1/4)*arctan(1/(
a/b)^(1/4)*x)+1/4*e/b/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b
*f*ln(b*x^4-a)
```

maxima [A] time = 3.10, size = 202, normalized size = 1.36

$$-\frac{gx}{b}+\frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}\,be+a\sqrt{b}\,g\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}\,bf\right)\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}\,bf\right)\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}c+\sqrt{a}\,be+a\sqrt{b}\,g\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -g*x/b + 1/4*(2*(b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*arctan(sqrt(b)*x/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d -
 sqrt(a)*b*f)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt(a)
*b*f)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (b^(3/2)*c + sqrt(a)*b*e + a
*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(
a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

mupad [B] time = 5.51, size = 5082, normalized size = 34.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x)
```

```
[Out] symsum(log(b^2*c^2*e - b^2*c*d^2 + a^2*e*g^2 - a^2*f^2*g - b^2*d^3*x - a*b*
e^3 - a*b*c*f^2 - a*b*d^2*g - 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 -
 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*
d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16
*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d
*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d
*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*
d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g
 - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^
2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)^2*a*b^3*
c - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^
2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*
z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b
^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a
^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g
 - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f -
 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^
2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 -
 a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*b^3*c^2*x - b^2*c^2*f*x - a^2*f*g^2
*x - 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*
a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*
g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2
*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8
*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*
g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f
 - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*
a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4
 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)^2*a^2*b^2*g + 16*root(256*a^3*b^5
*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3
*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z
 + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^
3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2
*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 -
4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a
^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*
b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4
 - b^4*c^4, z, k)^2*a*b^3*d*x - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3
- 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4
*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 1
6*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*
d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*
d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b
*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*
g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e
^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a*b^2*e
^2*x - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64
*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f
*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^
2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z +
8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2
*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*
f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2
*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^
4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a^2*b*g^2*x + 2*a*b*c*e*g + 2*a*
```

```
b*d*e*f - 8*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 -
 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*
e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16
*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z
 + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*
e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2
*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2
- 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3
*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a*b^2*c*f + 8*root(256*a^3*b^
5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^
3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*
z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b
^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^
2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 -
 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*
a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3
*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^
4 - b^4*c^4, z, k)*a*b^2*d*e - 8*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 -
 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*
d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16
*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d
*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d
*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*
d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g
 - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^
2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a^2*b*f*
g + a*b*d*f^2*x - a*b*e^2*f*x + 2*b^2*c*d*e*x - 8*root(256*a^3*b^5*z^4 + 25
6*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*
z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2
*b^3*c*d*g*z + 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z
 + 16*a*b^4*c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*
e*g + 4*a^2*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e
*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^
3 - 4*a*b^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2
 + 2*a*b^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^
4, z, k)*a*b^2*c*g*x + 8*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*
b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2
- 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2
*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*
a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f +
 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2
+ 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2
*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*
b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*a*b^2*d*f*x - 2*
a*b*c*f*g*x + 2*a*b*d*e*g*x)*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*
a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*
z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3
*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z +
 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2
*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*
g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6
*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 +
a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k), k, 1, 4) -
(g*x)/b
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)

[Out] Timed out
```

3.125 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2} \, dx$

Optimal. Leaf size=172

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(ag+bc+bdx+\ldots\right.}{4ab\left(a-\ldots\right.}$$

Rubi [A] time = 0.16, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1858, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + ((3*b*c -
 Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4))
+ ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/
4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^2}\,dx &= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{\int \frac{3bc-ag+2bdx+bex^2}{a-bx^4}\,dx}{4ab}\\
&= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{\int \left(\frac{2bdx}{a-bx^4}+\frac{3bc-ag+bex^2}{a-bx^4}\right)dx}{4ab}\\
&= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{\int \frac{3bc-ag+bex^2}{a-bx^4}\,dx}{4ab}+\frac{d\int \frac{x}{a-bx^4}\,dx}{2a}\\
&= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{d\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{4a}-\frac{\left(3bc-\sqrt{a}\right.}{}\\
&= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}}+\frac{(3}{}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 221, normalized size = 1.28

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{a-bx^4}-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e-ag+3bc\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-2\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e-ag+3bc\right)+2\sqrt[4]{a}b^{3/4}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)-2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag-3bc\right)}{16a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*
(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (3*b*c + 2
*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) - b^(1/4)*x] + (3
*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) + b^(1/4)
*x] + 2*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.18, size = 344, normalized size = 2.00

$$-\frac{\sqrt{2}\left(3\,b^{2}c-abg-2\,\sqrt{2}\,\left(-ab^{3}\right)^{\frac{1}{4}}bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-abg+2\,\sqrt{2}\,\left(-ab^{3}\right)^{\frac{1}{4}}bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-abg-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,\left(-ab^{3}\right)^{\frac{3}{4}}a}+\frac{\sqrt{2}\left(3\,b^{2}c-abg-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{bx^{3}e+bdx^{2}+bcx+agx+af}{4\left(bx^{4}-a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*
b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d -
 sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/((-a*b^3)^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*lo
g(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt
(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + s
qrt(-a/b))/((-a*b^3)^(3/4)*a) - 1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*g*x + a*
f)/((b*x^4 - a)*a*b)
```

maple [B] time = 0.05, size = 289, normalized size = 1.68

$$-\frac{d\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{8\sqrt{ab}\,a}-\frac{e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}}+\frac{-\frac{e\,x^{3}}{4a}-\frac{d\,x^{2}}{4a}-\frac{f}{4b}-\frac{\left(ag+bc\right)x}{4ab}}{b\,x^{4}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4/a*e*x^3-1/4/a*d*x^2-1/4*(a*g+b*c)/a/b*x-1/4/b*f)/(b*x^4-a)-1/8/b/a*(a
/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)*g+3/8*(a/b)^(1/4)/a^2*c*arctan(1/(a/b)^(1
/4)*x)-1/16/b/a*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+3/16*(a/b
)^(1/4)/a^2*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/8/(a*b)^(1/2)/a*d*ln(((
a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-1/8/(a/b)^(1/4)/a/b*e*arctan(1/(a/b
)^(1/4)*x)+1/16/(a/b)^(1/4)/a/b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 3.11, size = 224, normalized size = 1.30

$$-\frac{bex^{3}+bdx^{2}+af+\left(bc+ag\right)x}{4\left(ab^{2}x^{4}-a^{2}b\right)}+\frac{\frac{2\,\sqrt{b}\,d\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}}-\frac{2\,\sqrt{b}\,d\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}\,be-a\sqrt{b}\,g\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(3\,b^{\frac{3}{2}}c+\sqrt{a}\,be-a\sqrt{b}\,g\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b*e*x^3 + b*d*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 - a^2*b) + 1/16*(
2*sqrt(b)*d*log(sqrt(b)*x^2 + sqrt(a))/sqrt(a) - 2*sqrt(b)*d*log(sqrt(b)*x^
2 - sqrt(a))/sqrt(a) + 2*(3*b^(3/2)*c - sqrt(a)*b*e - a*sqrt(b)*g)*arctan(s
qrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (
3*b^(3/2)*c + sqrt(a)*b*e - a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt
(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)))/(a*b)
```

mupad [B] time = 5.56, size = 1393, normalized size = 8.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(- (12*b^2*c*d^2 - 9*b^2*c^2*e - a^2*e*g^2 + a*b*e^3 - 4*a*b*d^2*
g + 6*a*b*c*e*g)/(64*a^3) - (root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2
- 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a
^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^
2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*
c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*
d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*b*(9*b^2*c^2*x + a^2*g^2*x
- 16*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 -
 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3
*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2
*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g
^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b
^2*e^4 - a^4*g^4, z, k)*a^3*b*g + a*b*e^2*x + 48*root(65536*a^7*b^5*z^4 + 1
024*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3
*b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2
*d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b
^3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3
*c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*a^2*b^2
*c - 4*a*b*d*e - 32*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 - 3072*a^
4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a^4*b^2*d*
g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^2*e*g - 1
2*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 - 5
4*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 - 81*
b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*a^2*b^2*d*x - 6*a*b*c*g*x))/(4*a^2)
- (b*d*x*(2*b*d^2 - 3*b*c*e + a*e*g))/(16*a^3))*root(65536*a^7*b^5*z^4 + 10
24*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*
b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*
d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^
3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*
c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k), k, 1, 4
) + (f/(4*b) + (d*x^2)/(4*a) + (e*x^3)/(4*a) + (x*(b*c + a*g))/(4*a*b))/(a
- b*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.126 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^3}\,dx$

Optimal. Leaf size=221

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(-ag+7\right.}{}$$

Rubi [A] time = 0.26, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{x\left(-ag+7bc+6bdx+5bex^2\right)+4af}{32a^2b\left(a-bx^4\right)}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f
+ x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c
- 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^
(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4
)])/(64*a^(11/4)*b^(5/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2
)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^3}\,dx = \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{\int \frac{7bc-ag+6bdx+5bex^2+4bfx^3}{\left(a-bx^4\right)^2}\,dx}{8ab}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x\left(7bc-ag+6bdx+5bex^2\right)}{32a^2b\left(a-bx^4\right)}-\int$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x\left(7bc-ag+6bdx+5bex^2\right)}{32a^2b\left(a-bx^4\right)}-\int$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x\left(7bc-ag+6bdx+5bex^2\right)}{32a^2b\left(a-bx^4\right)}-\int$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x\left(7bc-ag+6bdx+5bex^2\right)}{32a^2b\left(a-bx^4\right)}+\frac{(3}{}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x\left(7bc-ag+6bdx+5bex^2\right)}{32a^2b\left(a-bx^4\right)}+\frac{(2}{}$$

Mathematica [A] time = 0.77, size = 263, normalized size = 1.19

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\left(a^2(4f+3gx)+abx\left(11c+x\left(10d+9ex+gx^3\right)\right)-b^2x^5(7c+x(6d+5ex))\right)}{\left(a-bx^4\right)^2}-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(12\sqrt[4]{a}b^{3/4}d+5\sqrt{a}\sqrt{b}e-3ag+21bc\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-12\sqrt[4]{a}b^{3/4}d+5\sqrt{a}\sqrt{b}e-3ag+21bc\right)+12\sqrt[4]{a}b^{3/4}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)+2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{128a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(a^2*(4*f + 3*g*x) - b^2*x^5*(7*c + x*(6*d + 5*e*x)) +
a*b*x*(11*c + x*(10*d + 9*e*x + g*x^3))))/(a - b*x^4)^2 + 2*(21*b*c - 5*Sqr
t[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (21*b*c + 12*a^(1/4)*
b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) - b^(1/4)*x] + (21*b*c
 - 12*a^(1/4)*b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) + b^(1/4
)*x] + 12*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^(5/
4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{\left(a - bx^4\right)^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.20, size = 393, normalized size = 1.78

$$-\frac{\sqrt{2}\left(21\, b^2 c - 3\, abg - 12\, \sqrt{2}\, \left(-ab^3\right)^{\frac{1}{4}} bd + 5\, \sqrt{-ab}\, be\right) \arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\, \left(-ab^3\right)^{\frac{3}{4}} a^2} - \frac{\sqrt{2}\left(21\, b^2 c - 3\, abg + 12\, \sqrt{2}\, \left(-ab^3\right)^{\frac{1}{4}} bd - 5\, \sqrt{-ab}\, be\right) \arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\, \left(-ab^3\right)^{\frac{3}{4}} a^2} - \frac{\sqrt{2}\left(21\, b^2 c - 3\, abg - 5\, \sqrt{-ab}\, be\right) \log\left(x^2 + \sqrt{2}\, x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256\, \left(-ab^3\right)^{\frac{3}{4}} a^2} + \frac{\sqrt{2}\left(21\, b^2 c - 3\, abg - 5\, \sqrt{-ab}\, be\right) \log\left(x^2 - \sqrt{2}\, x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256\, \left(-ab^3\right)^{\frac{3}{4}} a^2} - \frac{5\, b^2 x^7 e + 6\, b^2 dx^6 + 7\, b^2 cx^5 - abgx^5 - 9\, abx^3 e - 10\, abdx^2 - 11\, abcx - 3\, a^2 gx - 4\, a^2 f}{32\, \left(bx^4 - a\right)^2 a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt
(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(
(-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c - 3*a*b*g + 12*sqrt(2)*(-a*b^
3)^(1/4)*b*d - 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(
1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*
g - 5*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b
^3)^(3/4)*a^2) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b
^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*x^3*e - 10*a*b*d*x
^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

maple [A] time = 0.07, size = 328, normalized size = 1.48

$$-\frac{3 d \ln\left(\frac{\sqrt{ab}\, x^2 - a}{-\sqrt{ab}\, x^2 - a}\right)}{32 \sqrt{ab}\, a^2} - \frac{5 e \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^2 b} + \frac{5 e \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^2 b} - \frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} g \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^2 b} - \frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} g \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128 a^2 b} + \frac{21 \left(\frac{a}{b}\right)^{\frac{1}{4}} c \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^3} + \frac{21 \left(\frac{a}{b}\right)^{\frac{1}{4}} c \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128 a^3} - \frac{\frac{5 b e x^7}{32 a^2} + \frac{3 b d x^6}{16 a^2} - \frac{9 e x^3}{32 a} - \frac{\left(a g - 7 b c\right) x^5}{32 a^2} - \frac{5 d x^2}{16 a} - \frac{f}{8 b} - \frac{\left(3 a g + 11 b c\right) x}{32 a b}}{\left(b x^4 - a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(5/32/a^2*b*e*x^7+3/16/a^2*b*d*x^6-1/32*(a*g-7*b*c)/a^2*x^5-9/32/a*e*x^3-5
/16/a*d*x^2-1/32*(3*a*g+11*b*c)/a/b*x-1/8/b*f)/(b*x^4-a)^2-3/64/a^2/b*(a/b)
^(1/4)*arctan(1/(a/b)^(1/4)*x)*g+21/64*(a/b)^(1/4)/a^3*c*arctan(1/(a/b)^(1/
4)*x)-3/128/a^2/b*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+21/128*
(a/b)^(1/4)/a^3*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-3/32/(a*b)^(1/2)/a^2*
d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-5/64/(a/b)^(1/4)/a^2/b*e*arc
tan(1/(a/b)^(1/4)*x)+5/128/(a/b)^(1/4)/a^2/b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^
(1/4)))
```

maxima [A] time = 3.00, size = 284, normalized size = 1.29

$$-\frac{5\, b^2 e x^7 + 6\, b^2 d x^6 - 9\, abe x^3 + \left(7\, b^2 c - abg\right) x^5 - 10\, abd x^2 - 4\, a^2 f - \left(11\, abc + 3\, a^2 g\right) x}{32\, \left(a^2 b^3 x^8 - 2\, a^3 b^2 x^4 + a^4 b\right)} + \frac{\frac{12\, \sqrt{b}\, d \log\left(\sqrt{b}\, x^2 + \sqrt{a}\right)}{\sqrt{a}} - \frac{12\, \sqrt{b}\, d \log\left(\sqrt{b}\, x^2 - \sqrt{a}\right)}{\sqrt{a}} + \frac{2 \left(21\, b^{\frac{3}{2}} c - 5\, \sqrt{a}\, be - 3\, a \sqrt{b}\, g\right) \arctan\left(\frac{\sqrt{b}\, x}{\sqrt{\sqrt{a}\, \sqrt{b}}}\right)}{\sqrt{a}\, \sqrt{\sqrt{a}\, \sqrt{b}}\, \sqrt{b}} - \frac{\left(21\, b^{\frac{3}{2}} c + 5\, \sqrt{a}\, be - 3\, a \sqrt{b}\, g\right) \log\left(\frac{\sqrt{b}\, x - \sqrt{\sqrt{a}\, \sqrt{b}}}{\sqrt{b}\, x + \sqrt{\sqrt{a}\, \sqrt{b}}}\right)}{\sqrt{a}\, \sqrt{\sqrt{a}\, \sqrt{b}}\, \sqrt{b}}}{128\, a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 - 9*a*b*e*x^3 + (7*b^2*c - a*b*g)*x^5 - 10
*a*b*d*x^2 - 4*a^2*f - (11*a*b*c + 3*a^2*g)*x)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4
 + a^4*b) + 1/128*(12*sqrt(b)*d*log(sqrt(b)*x^2 + sqrt(a))/sqrt(a) - 12*sqr
t(b)*d*log(sqrt(b)*x^2 - sqrt(a))/sqrt(a) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*e
 - 3*a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))*sqrt(b)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt(b)*g)*log
((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

mupad [B] time = 5.44, size = 1002, normalized size = 4.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x)
```

```
[Out] (f/(8*b) + (5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) - (x^5*(7*b*c - a*g))/(32*a^
2) + (x*(11*b*c + 3*a*g))/(32*a*b) - (3*b*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32
*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^5*z
^4 + 983040*a^7*b^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4*d^2
*z^2 - 774144*a^4*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^
2*z + 2709504*a^3*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c*e^2*g
 - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 23814*a^2*
b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2*e^4 + 2
0736*a*b^3*d^4 - 81*a^4*g^4 - 194481*b^4*c^4, z, k)*(root(268435456*a^11*b^
5*z^4 + 983040*a^7*b^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4*
d^2*z^2 - 774144*a^4*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d
*e^2*z + 2709504*a^3*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c*e^
2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 23814*a
^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2*e^4
+ 20736*a*b^3*d^4 - 81*a^4*g^4 - 194481*b^4*c^4, z, k)*((344064*a^5*b^3*c -
 49152*a^6*b^2*g)/(32768*a^6) - (6*b^3*d*x)/a) + (x*(144*a^4*b*g^2 + 7056*a
^2*b^3*c^2 + 400*a^3*b^2*e^2 - 2016*a^3*b^2*c*g))/(4096*a^6) - (15*b^2*d*e)
/(32*a^3)) - (3024*b^2*c*d^2 - 2205*b^2*c^2*e - 45*a^2*e*g^2 + 125*a*b*e^3
- 432*a*b*d^2*g + 630*a*b*c*e*g)/(32768*a^6) - (x*(216*b^2*d^3 - 315*b^2*c*
d*e + 45*a*b*d*e*g))/(4096*a^6))*root(268435456*a^11*b^5*z^4 + 983040*a^7*b
^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4*d^2*z^2 - 774144*a^4
*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^2*z + 2709504*a^3
*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c*e^2*g - 60480*a*b^3*c*
d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 23814*a^2*b^2*c^2*g^2 + 450
*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2*e^4 + 20736*a*b^3*d^4 -
81*a^4*g^4 - 194481*b^4*c^4, z, k), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.127 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4} \, dx$

Optimal. Leaf size=266

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(7(11b}{}$$

Rubi [A] time = 0.32, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{x\left(-ag+11bc+10bdx+9bex^2\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*
(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f +
 x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b
*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/
4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(256*a^(15/4)*b^(5/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32
*a^(7/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^4}\,dx = \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}+\frac{\int \frac{11bc-ag+10bdx+9bex^2+8bfx^3}{\left(a-bx^4\right)^3}\,dx}{12ab}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}+\frac{8af+x\left(11bc-ag+10bdx+9bex^2\right)}{96a^2b\left(a-bx^4\right)^2}-$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+ 8a$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+ 8a$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+ 8a$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+ 8a$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+ 8a$$

Mathematica [A] time = 0.39, size = 313, normalized size = 1.18

$$\frac{\frac{128a^{11/4}\sqrt[4]{b}\left(a(f+gx)+bx(c+x(d+ex))\right)}{\left(a-bx^4\right)^3}+\frac{16a^{7/4}\sqrt[4]{b}x(-ag+11bc+bx(10d+9ex))}{\left(a-bx^4\right)^2}+\frac{4a^{3/4}\sqrt[4]{b}x(-7ag+77bc+15bx(4d+3ex))}{a-bx^4}-3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(40\sqrt[4]{a}b^{3/4}d+15\sqrt{a}\sqrt{b}e-7ag+77bc\right)+3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-40\sqrt[4]{a}b^{3/4}d+15\sqrt{a}\sqrt{b}e-7ag+77bc\right)+120\sqrt[4]{a}b^{3/4}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)+6\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{1536a^{15/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*x*(77*b*c - 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a - b*x^4)
+ (16*a^(7/4)*b^(1/4)*x*(11*b*c - a*g + b*x*(10*d + 9*e*x)))/(a - b*x^4)^2
+ (128*a^(11/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4)^
3 + 6*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] -
 3*(77*b*c + 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*Log[a^(1/
4) - b^(1/4)*x] + 3*(77*b*c - 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e -
 7*a*g)*Log[a^(1/4) + b^(1/4)*x] + 120*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt
[b]*x^2])/(1536*a^(15/4)*b^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{\left(a - bx^4\right)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 442, normalized size = 1.66

$$-\frac{\sqrt{2}\left(77\,b^2c - 7\,abg - 40\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd + 15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^3\right)^{\frac{3}{4}}a^3} - \frac{\sqrt{2}\left(77\,b^2c - 7\,abg + 40\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd - 15\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^3\right)^{\frac{3}{4}}a^3} - \frac{\sqrt{2}\left(77\,b^2c - 7\,abg - 15\,\sqrt{-ab}\,be\right)\log\left(x^2 + \sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac{3}{4}}a^3} + \frac{\sqrt{2}\left(77\,b^2c - 7\,abg - 15\,\sqrt{-ab}\,be\right)\log\left(x^2 - \sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac{3}{4}}a^3} - \frac{45\,b^3x^{11}e + 60\,b^3dx^{10} + 77\,b^3cx^9 - 7\,ab^2gx^9 - 126\,ab^2x^7e - 160\,ab^2dx^6 - 198\,ab^2cx^5 + 18\,a^2bgx^5 + 113\,a^2bx^3e + 132\,a^2bdx^2 + 153\,a^2bcx + 21\,a^3gx + 32\,a^3f}{384\left(bx^4 - a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c - 7*a*b*g + 40*sqrt(2)*(-a*b
^3)^(1/4)*b*d - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a
*b*g - 15*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((
-a*b^3)^(3/4)*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e
)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/3
84*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126*a*b^
2*x^7*e - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^
3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)
^3*a^3*b)
```

maple [A] time = 0.06, size = 368, normalized size = 1.38

$$-\frac{5d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{64\sqrt{ab}\,a^3} - \frac{15e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b} + \frac{15e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b} - \frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256a^3b} - \frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^3b} + \frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256a^4} + \frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4} + \frac{-\frac{15b^2e\,x^{11}}{128a^3} - \frac{5b^2d\,x^{10}}{32a^3} + \frac{21be\,x^7}{64a^2} + \frac{7\left(ag-11bc\right)b\,x^9}{384a^3} + \frac{5bd\,x^6}{12a^2} - \frac{113e\,x^3}{384a} - \frac{3\left(ag-11bc\right)x^5}{64a^2} - \frac{11d\,x^2}{32a} - \frac{f}{12b} - \frac{\left(7ag+51bc\right)x}{128ab}}{\left(b\,x^4-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128/a^3*b^2*e*x^11-5/32/a^3*b^2*d*x^10+7/384*(a*g-11*b*c)/a^3*b*x^9+21
/64/a^2*b*e*x^7+5/12/a^2*b*d*x^6-3/64/a^2*(a*g-11*b*c)*x^5-113/384/a*e*x^3-
11/32/a*d*x^2-1/128*(7*a*g+51*b*c)/a/b*x-1/12/b*f)/(b*x^4-a)^3-7/256/a^3/b*
(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)*g+77/256*(a/b)^(1/4)/a^4*c*arctan(1/(a/
b)^(1/4)*x)-7/512/a^3/b*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g+7
7/512*(a/b)^(1/4)/a^4*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-5/64/(a*b)^(1/2
)/a^3*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-15/256/(a/b)^(1/4)/a^3
/b*e*arctan(1/(a/b)^(1/4)*x)+15/512/(a/b)^(1/4)/a^3/b*e*ln((x+(a/b)^(1/4))/
(x-(a/b)^(1/4)))
```

maxima [A] time = 3.18, size = 345, normalized size = 1.30

$$-\frac{45\,b^3ex^{11}+60\,b^3dx^{10}-126\,ab^2ex^7-160\,ab^2dx^6+7\left(11\,b^3c-ab^2g\right)x^9+113\,a^2bex^3+132\,a^2bdx^2-18\left(11\,ab^2c-a^2bg\right)x^5+32\,a^3f+3\left(51\,a^2bc+7\,a^3g\right)x}{384\left(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b\right)}+\frac{\frac{40\,\sqrt{b}d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}}-\frac{40\,\sqrt{b}d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 - 126*a*b^2*e*x^7 - 160*a*b^2*d*x^6 +
 7*(11*b^3*c - a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 - 18*(11*a*
b^2*c - a^2*b*g)*x^5 + 32*a^3*f + 3*(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x^12
 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b) + 1/512*(40*sqrt(b)*d*log(sqrt(b)
*x^2 + sqrt(a))/sqrt(a) - 40*sqrt(b)*d*log(sqrt(b)*x^2 - sqrt(a))/sqrt(a) +
 2*(77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c + 1
5*sqrt(a)*b*e - 7*a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqr
t(b)*x + sqrt(sqrt(a)*sqrt(b)))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(
a^3*b)
```

mupad [B] time = 5.66, size = 1056, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 + 1
10100480*a^9*b^3*e*g*z^2 - 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d
*g*z + 485703680*a^4*b^4*c^2*d*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b
^3*d*e^2*z + 672000*a^2*b^2*d^2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*a*b^
3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^2*c^2
*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^4 + 25
60000*a*b^3*d^4 - 2401*a^4*g^4 - 35153041*b^4*c^4, z, k)*(root(68719476736*
a^15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 - 838
860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z + 485703680*a^4*b^4*c^2*d
*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z + 672000*a^2*b^2*d^
2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3
*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2
668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 - 2401*a^4*g^4
 - 35153041*b^4*c^4, z, k)*((20185088*a^7*b^3*c - 1835008*a^8*b^2*g)/(20971
52*a^9) - (5*b^3*d*x)/a^2) + (x*(1568*a^5*b*g^2 + 189728*a^3*b^3*c^2 + 7200
*a^4*b^2*e^2 - 34496*a^4*b^2*c*g))/(131072*a^9) - (75*b^2*d*e)/(256*a^5)) -
 (123200*b^2*c*d^2 - 88935*b^2*c^2*e - 735*a^2*e*g^2 + 3375*a*b*e^3 - 11200
*a*b*d^2*g + 16170*a*b*c*e*g)/(2097152*a^9) - (x*(4000*b^2*d^3 - 5775*b^2*c
*d*e + 525*a*b*d*e*g))/(131072*a^9))*root(68719476736*a^15*b^5*z^4 - 121110
5280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 - 838860800*a^8*b^4*d^2*z^
2 - 88309760*a^5*b^3*c*d*g*z + 485703680*a^4*b^4*c^2*d*z + 4014080*a^6*b^2*
d*g^2*z + 18432000*a^5*b^3*d*e^2*z + 672000*a^2*b^2*d^2*e*g - 485100*a^2*b^
2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g
^3 - 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2
```

```
- 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 - 2401*a^4*g^4 - 35153041*b^4*c^4,
z, k), k, 1, 4) + (f/(12*b) + (11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) - (3*
x^5*(11*b*c - a*g))/(64*a^2) + (7*b*x^9*(11*b*c - a*g))/(384*a^3) + (x*(51*
b*c + 7*a*g))/(128*a*b) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^
3) - (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x^12 - 3*a^2*
b*x^4 + 3*a*b^2*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

3.128 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$

Optimal. Leaf size=319

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+\right.}{4\sqrt{2}a^{3/4}b^{5/4}}$$

Rubi [A] time = 0.35, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1885, 1248, 635, 205, 260, 1887, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((b*c + S
qrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e
- a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a + b*
x^4])/(4*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx &= \int \left(\frac{x\left(d+fx^2\right)}{a+bx^4}+\frac{c+ex^2+gx^4}{a+bx^4}\right)dx \\
&= \int \frac{x\left(d+fx^2\right)}{a+bx^4}\,dx+\int \frac{c+ex^2+gx^4}{a+bx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+fx}{a+bx^2}\,dx,x,x^2\right)+\int \left(\frac{g}{b}+\frac{bc-ag+bex^2}{b\left(a+bx^4\right)}\right)dx \\
&= \frac{gx}{b}+\frac{\int \frac{bc-ag+bex^2}{a+bx^4}\,dx}{b}+\frac{1}{2}d\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{1}{2}f\operatorname{Subst}\left(\int \frac{}{a+}\right. \\
&= \frac{gx}{b}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}}{2\sqrt{a}b^{3/2}} \\
&= \frac{gx}{b}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}}}{4\sqrt{2}a^{3/4}b^{5/4}} \\
&= \frac{gx}{b}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right.}{4\sqrt{2}a^{3/4}b^{5/4}} \\
&= \frac{gx}{b}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.41, size = 311, normalized size = 0.97

$$\frac{2a^{3/4}\sqrt[4]{b}f\log\left(a+bx^4\right)+8a^{3/4}\sqrt[4]{b}gx-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e-\sqrt{2}ag+\sqrt{2}bc\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-2\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e-\sqrt{2}ag+\sqrt{2}bc\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}\sqrt{b}e+ag-bc\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{8a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*g*x - 2*(Sqrt[2]*b*c + 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqr
t[a]*Sqrt[b]*e - Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[2]*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e - Sqrt[2]*a*g
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(b*c) + Sqrt[a]*Sqrt[
b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2
]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*f*Log[a + b*x^4])/(8*a^(3/4)*b^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.27, size = 340, normalized size = 1.07

$$\frac{gx}{b}+\frac{f\log\left(|bx^4+a|\right)}{4\,b}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d+\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{1}{4}}abg+\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d+\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{1}{4}}abg+\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{1}{4}}abg-\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{1}{4}}abg-\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] g*x/b + 1/4*f*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d
+ (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt
(2)*sqrt(a*b)*b^2*d + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(
3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3)
 + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e
)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^
3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

maple [A] time = 0.06, size = 429, normalized size = 1.34

$$\frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{2\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{f\ln\left(bx^4+a\right)}{4b}+\frac{gx}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/b*g*x-1/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*g+1/4*(a/
b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c-1/8/b*(a/b)^(1/4)*2^(1
/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2)))*g+1/8*(a/b)^(1/4)*2^(1/2)/a*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a
/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))-1/4/b*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x+1)*c+1/2*d/(a*b)^(1/2)*arctan((1/a*b)^(1/2)*x^2)+1/8/b*e/(
a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1
/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4/(a/b)^(1/4)*2^(1/2)/b*e*arctan(2^(1/2)/(a/b
)^(1/4)*x+1)+1/4/b*e/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/
4*f*ln(b*x^4+a)/b
```

maxima [A] time = 3.03, size = 328, normalized size = 1.03

$$\frac{gx}{b}+\frac{\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^2c-\sqrt{a}\,b^{\frac{3}{2}}e-abg\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^2c+\sqrt{a}\,b^{\frac{3}{2}}e+abg\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g-2\sqrt{a}\,b^2d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g+2\sqrt{a}\,b^2d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] g*x/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f + b^2*c - sqrt(a)*b^(3/2)*e
 - a*b*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b
^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - b^2*c + sqrt(a)*b^(3/2)*e +
a*b*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5
/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a
^(5/4)*b^(5/4)*g - 2*sqrt(a)*b^2*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b
^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2
```

```
)*a^(5/4)*b^(5/4)*g + 2*sqrt(a)*b^2*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sq
rt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b)
)*b^(5/4)))/b
```

mupad [B] time = 5.59, size = 5042, normalized size = 15.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e - a^2*e*g^2 + a^2*f^2*g + b^2*d^3*x - a*b*
e^3 - a*b*c*f^2 - a*b*d^2*g - 16*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 -
 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*
d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16
*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d
*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d
*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*
d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g
 + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^
2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)^2*a*b^3*
c - 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^
2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*
z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b
^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a
^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g
+ 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f -
 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^
2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4
 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*b^3*c^2*x + b^2*c^2*f*x + a^2*f*g^2
*x + 16*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*
a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*
g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2
*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8
*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*
g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f
 - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*
a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f
^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)^2*a^2*b^2*g + 16*root(256*a^3*b^5
*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3
*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z
 + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^
3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2
*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 -
4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a
^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*
b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4
 + b^4*c^4, z, k)^2*a*b^3*d*x + 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3
- 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4
*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 1
6*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*
d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*
d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b
*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*
g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e
^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*e
^2*x - 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64
*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f
*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^
2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z -
```

8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a^2*b*g^2*x + 2*a*b*c*e*g + 2*a*b*d*e*f + 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*c*f - 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*d*e - 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a^2*b*f*g + a*b*d*f^2*x - a*b*e^2*f*x - 2*b^2*c*d*e*x + 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*c*g*x - 8*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*a*b^2*d*f*x - 2*a*b*c*f*g*x + 2*a*b*d*e*g*x)*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k), k, 1, 4) + (g*x)/b

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

3.129 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^2} \, dx$

Optimal. Leaf size=341

$$-\frac{\log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x+\sqrt{a}+\sqrt{b} x^2\right) \left(-\sqrt{a} \sqrt{b} e+a g+3 b c\right)}{16 \sqrt{2} a^{7/4} b^{5/4}}+\frac{\log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x+\sqrt{a}+\sqrt{b} x^2\right) \left(-\sqrt{a} \sqrt{b} e+a g+\right.}{16 \sqrt{2} a^{7/4} b^{5/4}}$$

Rubi [A] time = 0.31, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1858, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x+\sqrt{a}+\sqrt{b} x^2\right) \left(-\sqrt{a} \sqrt{b} e+a g+3 b c\right)}{16 \sqrt{2} a^{7/4} b^{5/4}}+\frac{\log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x+\sqrt{a}+\sqrt{b} x^2\right) \left(-\sqrt{a} \sqrt{b} e+a g+3 b c\right)}{16 \sqrt{2} a^{7/4} b^{5/4}}-\frac{\tan ^{-1}\left(1-\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right) \left(\sqrt{a} \sqrt{b} e+a g+3 b c\right)}{8 \sqrt{2} a^{7/4} b^{5/4}}+\frac{\tan ^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}+1\right) \left(\sqrt{a} \sqrt{b} e+a g+3 b c\right)}{8 \sqrt{2} a^{7/4} b^{5/4}}+\frac{d \tan ^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right)}{4 a^{3/2} \sqrt{b}}+\frac{x \left(-a g+b c+b d x+b e x^2+b f x^3\right)}{4 a b \left(a+b x^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (d*ArcTan
[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*b*c + Sqrt[a]*Sqrt[b]*e
+ a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
 + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/
4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^2}\,dx = \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-3bc-ag-2bdx-bex^2}{a+bx^4}\,dx}{4ab}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \left(-\frac{2bdx}{a+bx^4} + \frac{-3bc-ag-bex^2}{a+bx^4}\right)dx}{4ab}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-3bc-ag-bex^2}{a+bx^4}\,dx}{4ab} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a} + \frac{\left(3bc-\sqrt{a}\sqrt{b}\right.}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\int -}{16\sqrt{2}a^{7/4}b}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log}{16\sqrt{}}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\tan}{8\sqrt{2}a^{7/4}b^{5}}$$

Mathematica [A] time = 0.23, size = 319, normalized size = 0.94

$$\frac{-\frac{8a^{3/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{a+bx^4} - 2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e+\sqrt{2}ag+3\sqrt{2}bc\right) + 2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-4\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e+\sqrt{2}ag+3\sqrt{2}bc\right) + \sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}\sqrt{b}e-ag-3bc\right) + \sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{32a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2
*(3*Sqrt[2]*b*c + 4*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]
*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b*c - 4*a^(1/4
)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]*a*g)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c - Sqrt[a]*Sqr
t[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a
^(7/4)*b^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.20, size = 365, normalized size = 1.07

$$\frac{bx^{3}e+bdx^{2}+bcx-agx-af}{4\left(bx^{4}+a\right)ab}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+\left(ab^{3}\right)^{\frac{1}{4}}abg+\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{3}}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+\left(ab^{3}\right)^{\frac{1}{4}}abg+\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{3}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+\left(ab^{3}\right)^{\frac{1}{4}}abg-\left(ab^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{3}}-\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+\left(ab^{3}\right)^{\frac{1}{4}}abg-\left(ab^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt
(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*
g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)
*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*
b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^
(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(a^2*b^3)
```

maple [A] time = 0.05, size = 482, normalized size = 1.41

$$\frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{4\sqrt{ab}\,a}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\sqrt{2}\,e\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32a^{2}}+\frac{\frac{ex^{3}}{4a}+\frac{dx^{2}}{4a}-\frac{f}{4b}-\frac{(ag-bc)x}{4ab}}{bx^{4}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (1/4/a*e*x^3+1/4/a*d*x^2-1/4*(a*g-b*c)/a/b*x-1/4/b*f)/(b*x^4+a)+1/16/b/a*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*g+3/16*(a/b)^(1/4)*2^(1/2
)/a^2*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/32/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^
2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)
))*g+3/32*(a/b)^(1/4)*2^(1/2)/a^2*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/
2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/16/b/a*(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+3/16*(a/b)^(1/4)*2^(1/2)/a^2*c*arctan(2^(1
/2)/(a/b)^(1/4)*x+1)+1/4/(a*b)^(1/2)/a*d*arctan((1/a*b)^(1/2)*x^2)+1/32/(a/
b)^(1/4)*2^(1/2)/a/b*e*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b
)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/16/(a/b)^(1/4)*2^(1/2)/a/b*e*arctan(2^(1/
2)/(a/b)^(1/4)*x+1)+1/16/(a/b)^(1/4)*2^(1/2)/a/b*e*arctan(2^(1/2)/(a/b)^(1/
4)*x-1)
```

maxima [A] time = 3.00, size = 350, normalized size = 1.03

$$\frac{bex^{3}+bdx^{2}-af+\left(bc-ag\right)x}{4\left(ab^{2}x^{4}+a^{2}b\right)}+\frac{\frac{\sqrt{2}\left(3b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g-4\sqrt{a}b^{\frac{3}{2}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+4\sqrt{a}b^{\frac{3}{2}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*e*x^3 + b*d*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 + a^2*b) + 1/32*(s
qrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*b^(3/2)*c - sqr
t(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/4)*b^
(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g - 4*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(
```

```
2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*
sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a
^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g + 4*sqrt(a)*b^(3/2)*d)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a*b)
```

mupad [B] time = 5.59, size = 1383, normalized size = 4.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log(- (9*b^2*c^2*e - 12*b^2*c*d^2 + a^2*e*g^2 + a*b*e^3 - 4*a*b*d^2*
g + 6*a*b*c*e*g)/(64*a^3) - (root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2
+ 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a
^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^
2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*
c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*
d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*b*(9*b^2*c^2*x + a^2*g^2*x
+ 16*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 +
 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3
*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2
*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g
^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b
^2*e^4 + a^4*g^4, z, k)*a^3*b*g - a*b*e^2*x + 48*root(65536*a^7*b^5*z^4 + 1
024*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3
*b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2
*d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b
^3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3
*c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*a^2*b^2
*c + 4*a*b*d*e - 32*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 + 3072*a^
4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a^4*b^2*d*
g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^2*e*g + 1
2*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 + 5
4*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 + 81*
b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*a^2*b^2*d*x + 6*a*b*c*g*x))/(4*a^2)
- (b*d*x*(3*b*c*e - 2*b*d^2 + a*e*g))/(16*a^3))*root(65536*a^7*b^5*z^4 + 10
24*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*
b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*
d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^
3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*
c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k), k, 1, 4
) + ((d*x^2)/(4*a) - f/(4*b) + (e*x^3)/(4*a) + (x*(b*c - a*g))/(4*a*b))/(a
+ b*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.130 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=394

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e\right.}{128\sqrt{2}a^{11/4}b^{5/4}}$$

Rubi [A] time = 0.44, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}-\frac{4af-x\left(ag+7bc+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f
- x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a + b*x^4)) + (3*d*ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*b*c + 5*Sqrt[a]*Sqrt
[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4
)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqr
t[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(12
8*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^
(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^3}\,dx = \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{\int \frac{-7bc-ag-6bdx-5bex^2-4bfx^3}{\left(a+bx^4\right)^2}\,dx}{8ab}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{\int}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{\int}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{\int}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{(3}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{3a}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{3a}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{3a}{}$$

Mathematica [A] time = 0.39, size = 366, normalized size = 0.93

$$\frac{-\frac{32a^{7/4}\sqrt[4]{b}\left(a(f+gx)-bx(c+x(d+ex))\right)}{\left(a+bx^4\right)^2}+\frac{8a^{3/4}\sqrt[4]{b}x\left(ag+7bc+bx(6d+5ex)\right)}{a+bx^4}-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(24\sqrt[4]{a}b^{3/4}d+5\sqrt{2}\sqrt{a}\sqrt{b}e+3\sqrt{2}ag+21\sqrt{2}bc\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-24\sqrt[4]{a}b^{3/4}d+5\sqrt{2}\sqrt{a}\sqrt{b}e+3\sqrt{2}ag+21\sqrt{2}bc\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(5\sqrt{a}\sqrt{b}e-3ag-21bc\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{256a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*b^(1/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x)))/(a + b*x^4) - (32*
a^(7/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4)^2 - 2*(2
1*Sqrt[2]*b*c + 24*a^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt
[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b*c - 24*a
^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt[2]*a*g)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a
*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(21*b*
c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(256*a^(11/4)*b^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.20, size = 416, normalized size = 1.06

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d + 21\left(ab^3\right)^{\frac{1}{4}}b^2c + 3\left(ab^3\right)^{\frac{1}{4}}abg + 5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3} + \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^2d + 21\left(ab^3\right)^{\frac{1}{4}}b^2c + 3\left(ab^3\right)^{\frac{1}{4}}abg + 5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3} + \frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c + 3\left(ab^3\right)^{\frac{1}{4}}abg - 5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256a^3b^3} - \frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c + 3\left(ab^3\right)^{\frac{1}{4}}abg - 5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256a^3b^3} + \frac{5b^2x^7e + 6b^2dx^6 + 7b^2cx^5 + abgx^5 + 9abx^3e + 10abdx^2 + 11abcx - 3a^2gx - 4a^2f}{32\left(bx^4+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b
^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d
 + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*s
qrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21
*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 -
 sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d
*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2 + 11*a*b*c*x -
3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

maple [A] time = 0.06, size = 519, normalized size = 1.32

$$\frac{3d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{16\sqrt{ab}\,a^2} + \frac{5\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b} + \frac{5\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b} + \frac{5\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128a^2b} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128a^2b} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256a^2b} + \frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128a^3} + \frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128a^3} + \frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256a^3} + \frac{\frac{5bex^7}{32a^2}+\frac{3bdx^6}{16a^2}+\frac{9ex^3}{32a}+\frac{(ag+7bc)x^5}{32a^2}+\frac{5dx^2}{16a}-\frac{f}{8b}-\frac{(3ag-11bc)x}{32ab}}{\left(bx^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
[Out] (5/32/a^2*b*e*x^7+3/16/a^2*b*d*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a*e*x^3+5/
16/a*d*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8/b*f)/(b*x^4+a)^2+3/128/a^2/b*(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+21/128*c/a^3*(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/128/a^2/b*(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x-1)*g+21/128*c/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x-1)+3/256/a^2/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(
1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*g+21/256*c/a^3
*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^
(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/16/(a*b)^(1/2)/a^2*d*arctan((1/a*b)^(1/2)*x
^2)+5/256/a^2/b*e/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(
1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+5/128/(a/b)^(1/4)*2^(1/2)/a^
2/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+5/128/(a/b)^(1/4)*2^(1/2)/a^2/b*e*arc
tan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.03, size = 412, normalized size = 1.05

$$\frac{5b^2ex^7 + 6b^2dx^6 + 9abex^3 + \left(7b^2c + abg\right)x^5 + 10abdx^2 - 4a^2f + \left(11abc - 3a^2g\right)x}{32\left(a^2b^3x^8 + 2a^3b^2x^4 + a^4b\right)} + \frac{\frac{\sqrt{2}\left(21b^{\frac{3}{2}}c - 5\sqrt{a}\,be + 3a\sqrt{b}\,g\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21b^{\frac{3}{2}}c - 5\sqrt{a}\,be + 3a\sqrt{b}\,g\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(21\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c + 5\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e + 3\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g - 24\sqrt{a}\,b^{\frac{3}{2}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}} + \frac{2\left(21\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c + 5\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e + 3\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g + 24\sqrt{a}\,b^{\frac{3}{2}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{256a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 9*a*b*e*x^3 + (7*b^2*c + a*b*g)*x^5 + 10*
a*b*d*x^2 - 4*a^2*f + (11*a*b*c - 3*a^2*g)*x)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4
+ a^4*b) + 1/256*(sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g)*lo
g(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sq
rt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt
(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*
b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g - 24*
sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*s
qrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*
b^(3/4)*g + 24*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)
*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(
3/4)))/(a^2*b)
```

mupad [B] time = 0.71, size = 1001, normalized size = 2.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x)
```

```
[Out] ((5*d*x^2)/(16*a) - f/(8*b) + (9*e*x^3)/(32*a) + (x^5*(7*b*c + a*g))/(32*a^
2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (3*b*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32
*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^5*z
^4 + 983040*a^7*b^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4*d^2
*z^2 - 774144*a^4*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^
2*z - 2709504*a^3*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c*e^2*g
 - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 23814*a^2*
b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2*e^4 + 2
0736*a*b^3*d^4 + 81*a^4*g^4 + 194481*b^4*c^4, z, k)*(root(268435456*a^11*b^
5*z^4 + 983040*a^7*b^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4*
d^2*z^2 - 774144*a^4*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d
*e^2*z - 2709504*a^3*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c*e^
2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 23814*a
^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2*e^4
+ 20736*a*b^3*d^4 + 81*a^4*g^4 + 194481*b^4*c^4, z, k)*((344064*a^5*b^3*c +
 49152*a^6*b^2*g)/(32768*a^6) - (6*b^3*d*x)/a) + (x*(144*a^4*b*g^2 + 7056*a
^2*b^3*c^2 - 400*a^3*b^2*e^2 + 2016*a^3*b^2*c*g))/(4096*a^6) + (15*b^2*d*e)
/(32*a^3)) - (2205*b^2*c^2*e - 3024*b^2*c*d^2 + 45*a^2*e*g^2 + 125*a*b*e^3
- 432*a*b*d^2*g + 630*a*b*c*e*g)/(32768*a^6) - (x*(315*b^2*c*d*e - 216*b^2*
d^3 + 45*a*b*d*e*g))/(4096*a^6))*root(268435456*a^11*b^5*z^4 + 983040*a^7*b
^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4*d^2*z^2 - 774144*a^4
*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3*d*e^2*z - 2709504*a^3
*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c*e^2*g - 60480*a*b^3*c*
d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 23814*a^2*b^2*c^2*g^2 + 450
*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2*e^4 + 20736*a*b^3*d^4 +
81*a^4*g^4 + 194481*b^4*c^4, z, k), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.131 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=437

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-15\sqrt{a}\sqrt{b}\,e+7ag+77bc\right)}{512\sqrt{2}\,a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-15\sqrt{a}\sqrt{b}\,e\right.}{512\sqrt{2}\,a^{15/4}b^{5/4}}$$

Rubi [A] time = 0.53, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-15\sqrt{a}\sqrt{b}\,e+7ag+77bc\right)}{512\sqrt{2}\,a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-15\sqrt{a}\sqrt{b}\,e+7ag+77bc\right)}{512\sqrt{2}\,a^{15/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}\,e+7ag+77bc\right)}{256\sqrt{2}\,a^{15/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}\,e+7ag+77bc\right)}{256\sqrt{2}\,a^{15/4}b^{5/4}}-\frac{8af-x\left(ag+11bc+10bdx+9bex^2\right)}{96a^2b\left(a+bx^4\right)^2}+\frac{x\left(7(ag+11bc)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*
(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f -
 x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + (5*d*A
rcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*b*c + 15*Sqrt[a]*
Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^
(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqr
t[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a
*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^
(15/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^4}\,dx = \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-11bc-ag-10bdx-9bex^2-8bfx^3}{\left(a+bx^4\right)^3}\,dx}{12ab}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{8af-x\left(11bc+ag+10bdx+9bex^2\right)}{96a^2b\left(a+bx^4\right)^2} + \cdots$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af\cdots}{\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af\cdots}{\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af\cdots}{\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af\cdots}{\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af\cdots}{\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af\cdots}{\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af\cdots}{\cdots}$$

Mathematica [A] time = 0.52, size = 411, normalized size = 0.94

$$\frac{-\frac{256a^{11/4}\sqrt[4]{b}\left(a(f+gx)-bx(c+x(d+ex))\right)}{\left(a+bx^4\right)^3}+\frac{32a^{7/4}\sqrt[4]{b}\,x(ag+11bc+bx(10d+9ex))}{\left(a+bx^4\right)^2}+\frac{8a^{3/4}\sqrt[4]{b}\,x(7ag+77bc+15bx(4d+3ex))}{a+bx^4}-6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(80\sqrt[4]{a}b^{3/4}d+15\sqrt{2}\sqrt{a}\sqrt{b}e+7\sqrt{2}ag+77\sqrt{2}bc\right)+6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-80\sqrt[4]{a}b^{3/4}d+15\sqrt{2}\sqrt{a}\sqrt{b}e+7\sqrt{2}ag+77\sqrt{2}bc\right)-3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)+3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{3072a^{15/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*b^(1/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a + b*x^4)
+ (32*a^(7/4)*b^(1/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x)))/(a + b*x^4)^2
- (256*a^(11/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4)^
3 - 6*(77*Sqrt[2]*b*c + 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e
 + 7*Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b
*c - 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 7*Sqrt[2]*a*g)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqr
t[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*
Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{\left(a + bx^4\right)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.19, size = 466, normalized size = 1.07

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d + 77\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\left(ab^3\right)^{\frac{1}{4}}abg + 15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d + 77\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\left(ab^3\right)^{\frac{1}{4}}abg + 15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3} + \frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\left(ab^3\right)^{\frac{1}{4}}abg - 15\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3} - \frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}}b^2c + 7\left(ab^3\right)^{\frac{1}{4}}abg - 15\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3} + \frac{45\,b^3x^{11}e + 60\,b^3dx^{10} + 77\,b^3cx^9 + 7\,ab^2gx^9 + 126\,ab^2x^7e + 160\,ab^2dx^6 + 198\,ab^2cx^5 + 18\,a^2bgx^5 + 113\,a^2bx^3e + 132\,a^2bdx^2 + 153\,a^2bcx - 21\,a^3gx - 32\,a^3f}{384\left(bx^4+a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b
^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*
d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/102
4*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4
)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2
)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log
(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e
+ 60*b^3*d*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*x^7*e + 160*a*b^
2*d*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*
x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

maple [A] time = 0.06, size = 560, normalized size = 1.28

$$\frac{5d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{32\sqrt{ab}\,a^3} + \frac{15\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b} + \frac{15\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b} + \frac{15\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b} + \frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{512a^3b} + \frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{512a^3b} + \frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024a^3b} + \frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{512a^4} + \frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{512a^4} + \frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024a^4} + \frac{\frac{15b^2ex^{11}}{128a^3} + \frac{5b^2dx^{10}}{32a^3} + \frac{21bex^7}{64a^2} + \frac{7\left(ag+11bc\right)bx^9}{384a^3} + \frac{5bdx^6}{12a^2} + \frac{113ex^3}{384a} + \frac{3\left(ag+11bc\right)x^5}{64a^2} + \frac{11dx^2}{32a} - \frac{f}{12b} - \frac{\left(7ag-51bc\right)x}{128ab}}{\left(bx^4+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
[Out] (15/128/a^3*b^2*e*x^11+5/32/a^3*b^2*d*x^10+7/384*(a*g+11*b*c)/a^3*b*x^9+21/
64/a^2*b*e*x^7+5/12/a^2*b*d*x^6+3/64/a^2*(a*g+11*b*c)*x^5+113/384/a*e*x^3+1
1/32/a*d*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12/b*f)/(b*x^4+a)^3+7/512/a^3/b*(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+77/512*(a/b)^(1/4)*2^(
1/2)/a^4*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+7/512/a^3/b*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x-1)*g+77/512*(a/b)^(1/4)*2^(1/2)/a^4*c*arctan(2
^(1/2)/(a/b)^(1/4)*x-1)+7/1024/a^3/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4
)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*g+77/1024
*(a/b)^(1/4)*2^(1/2)/a^4*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-
(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+5/32/(a*b)^(1/2)/a^3*d*arctan((1/a*b)^(
1/2)*x^2)+15/1024/a^3*e/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x
```

```
+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+15/512/a^3*e/b/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+15/512/(a/b)^(1/4)*2^(1/2)/a
^3/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.12, size = 472, normalized size = 1.08

$$\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+126\,ab^2ex^7+160\,ab^2dx^6+7\left(11\,b^3c+ab^2g\right)x^9+113\,a^2bex^3+132\,a^2bdx^2+18\left(11\,ab^2c+a^2bg\right)x^5-32\,a^3f+3\left(51\,a^2bc-7\,a^3g\right)x}{384\left(a^3b^4x^{12}+3\,a^4b^3x^8+3\,a^5b^2x^4+a^6b\right)}+\frac{\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+7\,\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g-80\,\sqrt{a}b^{\frac{3}{2}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+7\,\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+80\,\sqrt{a}b^{\frac{3}{2}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{1024\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 126*a*b^2*e*x^7 + 160*a*b^2*d*x^6 +
7*(11*b^3*c + a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 18*(11*a*b
^2*c + a^2*b*g)*x^5 - 32*a^3*f + 3*(51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^12
+ 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*c -
15*sqrt(a)*b*e + 7*a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*
a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4
)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4)*e
 + 7*sqrt(2)*a^(5/4)*b^(3/4)*g - 80*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a
^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 80*sqrt(a)*b^(3/2)*d)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

mupad [B] time = 5.56, size = 1053, normalized size = 2.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 1
10100480*a^9*b^3*e*g*z^2 + 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d
*g*z - 485703680*a^4*b^4*c^2*d*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b
^3*d*e^2*z - 672000*a^2*b^2*d^2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*a*b^
3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^2*c^2
*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^4 + 25
60000*a*b^3*d^4 + 2401*a^4*g^4 + 35153041*b^4*c^4, z, k)*(root(68719476736*
a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 + 838
860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z - 485703680*a^4*b^4*c^2*d
*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z - 672000*a^2*b^2*d^
2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3
*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2
668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 + 2401*a^4*g^4
 + 35153041*b^4*c^4, z, k)*((20185088*a^7*b^3*c + 1835008*a^8*b^2*g)/(20971
52*a^9) - (5*b^3*d*x)/a^2) + (x*(1568*a^5*b*g^2 + 189728*a^3*b^3*c^2 - 7200
*a^4*b^2*e^2 + 34496*a^4*b^2*c*g))/(131072*a^9) + (75*b^2*d*e)/(256*a^5)) -
 (88935*b^2*c^2*e - 123200*b^2*c*d^2 + 735*a^2*e*g^2 + 3375*a*b*e^3 - 11200
*a*b*d^2*g + 16170*a*b*c*e*g)/(2097152*a^9) - (x*(5775*b^2*c*d*e - 4000*b^2
*d^3 + 525*a*b*d*e*g))/(131072*a^9))*root(68719476736*a^15*b^5*z^4 + 121110
5280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 + 838860800*a^8*b^4*d^2*z^
2 - 88309760*a^5*b^3*c*d*g*z - 485703680*a^4*b^4*c^2*d*z - 4014080*a^6*b^2*
d*g^2*z + 18432000*a^5*b^3*d*e^2*z - 672000*a^2*b^2*d^2*e*g + 485100*a^2*b^
2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g
^3 + 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2
+ 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 + 2401*a^4*g^4 + 35153041*b^4*c^4,
z, k), k, 1, 4) + ((11*d*x^2)/(32*a) - f/(12*b) + (113*e*x^3)/(384*a) + (3*
```

```
x^5*(11*b*c + a*g))/(64*a^2) + (7*b*x^9*(11*b*c + a*g))/(384*a^3) + (x*(51*
b*c - 7*a*g))/(128*a*b) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^
3) + (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x^12 + 3*a^2*
b*x^4 + 3*a*b^2*x^8)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

3.132 $\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3} \, dx$

Optimal. Leaf size=11

$$-\frac{1}{4}(1-x)^4$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 32}

$$-\frac{1}{4}(1-x)^4$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

```
[Out] -(1 - x)^4/4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3} \, dx &= \int (1-x)^3 \, dx \\ &= -\frac{1}{4}(1-x)^4 \end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.82

$$-\frac{1}{4}(x-1)^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

```
[Out] -1/4*(-1 + x)^4
```

IntegrateAlgebraic [A] time = 0.03, size = 9, normalized size = 0.82

$$-\frac{1}{4}(x-1)^4$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

```
[Out] -1/4*(-1 + x)^4
```

fricas [B] time = 0.40, size = 15, normalized size = 1.36

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="fricas")
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

giac [B] time = 0.24, size = 15, normalized size = 1.36

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="giac")
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

maple [A] time = 0.05, size = 8, normalized size = 0.73

$$-\frac{(x-1)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^3/(x^3+x^2+x+1)^3,x)
[Out] -1/4*(x-1)^4
```

maxima [B] time = 1.29, size = 15, normalized size = 1.36

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="maxima")
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

mupad [B] time = 0.03, size = 15, normalized size = 1.36

$$-\frac{x^4}{4} + x^3 - \frac{3x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)^3/(x + x^2 + x^3 + 1)^3,x)
[Out] x - (3*x^2)/2 + x^3 - x^4/4
```

sympy [B] time = 0.09, size = 15, normalized size = 1.36

$$-\frac{x^4}{4} + x^3 - \frac{3x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**3/(x**3+x**2+x+1)**3,x)
[Out] -x**4/4 + x**3 - 3*x**2/2 + x
```

3.133 $\int \frac{\left(1-x^4\right)^2}{\left(1+x+x^2+x^3\right)^2} \, dx$

Optimal. Leaf size=11

$$-\frac{1}{3}(1-x)^3$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 32}

$$-\frac{1}{3}(1-x)^3$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]
```

```
[Out] -(1 - x)^3/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-x^4\right)^2}{\left(1+x+x^2+x^3\right)^2} \, dx &= \int (1-x)^2 \, dx \\
&= -\frac{1}{3}(1-x)^3
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.27

$$\frac{x^3}{3}-x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]
```

```
[Out] x - x^2 + x^3/3
```

IntegrateAlgebraic [A] time = 0.02, size = 9, normalized size = 0.82

$$\frac{1}{3}(x-1)^3$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]
```

```
[Out] (-1 + x)^3/3
```

fricas [A] time = 0.40, size = 12, normalized size = 1.09

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="fricas")
[Out] 1/3*x^3 - x^2 + x
```

giac [A] time = 0.16, size = 12, normalized size = 1.09

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="giac")
[Out] 1/3*x^3 - x^2 + x
```

maple [A] time = 0.04, size = 8, normalized size = 0.73

$$\frac{(x-1)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^2/(x^3+x^2+x+1)^2,x)
[Out] 1/3*(x-1)^3
```

maxima [A] time = 1.29, size = 12, normalized size = 1.09

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="maxima")
[Out] 1/3*x^3 - x^2 + x
```

mupad [B] time = 0.02, size = 11, normalized size = 1.00

$$\frac{x\left(x^2 - 3x + 3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^2/(x + x^2 + x^3 + 1)^2,x)
[Out] (x*(x^2 - 3*x + 3))/3
```

sympy [A] time = 0.08, size = 8, normalized size = 0.73

$$\frac{x^3}{3} - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**2/(x**3+x**2+x+1)**2,x)
[Out] x**3/3 - x**2 + x
```

3.134 $\int \frac{1-x^4}{1+x+x^2+x^3} dx$

Optimal. Leaf size=9

$$x-\frac{x^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1586}

$$x-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + x + x^2 + x^3),x]
```

```
[Out] x - x^2/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^4}{1+x+x^2+x^3} \, dx &= \int (1-x) \, dx \\ &= x-\frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$x-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + x + x^2 + x^3),x]
```

```
[Out] x - x^2/2
```

IntegrateAlgebraic [A] time = 0.02, size = 11, normalized size = 1.22

$$-\frac{1}{2}(1-x)^2$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 + x + x^2 + x^3),x]
```

```
[Out] -1/2*(1 - x)^2
```

fricas [A] time = 0.41, size = 7, normalized size = 0.78

$$-\frac{1}{2}\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="fricas")
[Out] -1/2*x^2 + x
```

giac [A] time = 0.15, size = 7, normalized size = 0.78

$$-\frac{1}{2}\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="giac")
[Out] -1/2*x^2 + x
```

maple [A] time = 0.04, size = 8, normalized size = 0.89

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^3+x^2+x+1),x)
[Out] x-1/2*x^2
```

maxima [A] time = 1.29, size = 7, normalized size = 0.78

$$-\frac{1}{2}\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="maxima")
[Out] -1/2*x^2 + x
```

mupad [B] time = 0.02, size = 6, normalized size = 0.67

$$-\frac{x\,(x-2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x + x^2 + x^3 + 1),x)
[Out] -(x*(x - 2))/2
```

sympy [A] time = 0.07, size = 5, normalized size = 0.56

$$-\frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**3+x**2+x+1),x)
[Out] -x**2/2 + x
```

3.135 $\int \frac{1+x+x^2+x^3}{1-x^4} \, dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(1 - x^4),x]
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2+x^3}{1-x^4} \, dx &= \int \frac{1}{1-x} \, dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]
[Out] -Log[1 - x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2 + x^3)/(1 - x^4),x]
[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3)/(1 - x^4), x]
```

fricas [A] time = 0.40, size = 6, normalized size = 0.75

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
[Out] -log(x - 1)
```

giac [A] time = 0.17, size = 7, normalized size = 0.88

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
[Out] -log(abs(x - 1))
```

maple [A] time = 0.04, size = 7, normalized size = 0.88

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(-x^4+1),x)
[Out] -ln(x-1)
```

maxima [A] time = 1.37, size = 6, normalized size = 0.75

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
[Out] -log(x - 1)
```

mupad [B] time = 0.00, size = 6, normalized size = 0.75

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2 + x^3 + 1)/(x^4 - 1),x)
[Out] -log(x - 1)
```

sympy [A] time = 0.07, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(-x**4+1),x)
[Out] -log(x - 1)
```

3.136 $\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} \, dx$

Optimal. Leaf size=7

$$\frac{1}{1-x}$$

Rubi [A] time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 32}

$$\frac{1}{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
[Out] (1 - x)^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} \, dx &= \int \frac{1}{(1-x)^2} \, dx \\ &= \frac{1}{1-x} \end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
[Out] -(-1 + x)^(-1)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3)^2/(1 - x^4)^2, x]
```

fricas [A] time = 0.40, size = 7, normalized size = 1.00

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="fricas")
[Out] -1/(x - 1)
```

giac [A] time = 0.21, size = 7, normalized size = 1.00

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="giac")
[Out] -1/(x - 1)
```

maple [A] time = 0.04, size = 8, normalized size = 1.14

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)^2/(-x^4+1)^2,x)
[Out] -1/(x-1)
```

maxima [A] time = 1.30, size = 7, normalized size = 1.00

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="maxima")
[Out] -1/(x - 1)
```

mupad [B] time = 0.03, size = 7, normalized size = 1.00

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 1)^2/(x^4 - 1)^2,x)
[Out] -1/(x - 1)
```

sympy [A] time = 0.11, size = 5, normalized size = 0.71

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)**2/(-x**4+1)**2,x)
[Out] -1/(x - 1)
```

3.137 $\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} dx$

Optimal. Leaf size=11

$$\frac{1}{2(1-x)^2}$$

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 32}

$$\frac{1}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

```
[Out] 1/(2*(1 - x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} dx &= \int \frac{1}{(1-x)^3} dx \\ &= \frac{1}{2(1-x)^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.82

$$\frac{1}{2(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

```
[Out] 1/(2*(-1 + x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3)^3/(1 - x^4)^3, x]
```

fricas [A] time = 0.40, size = 12, normalized size = 1.09

$$\frac{1}{2\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="fricas")
[Out] 1/2/(x^2 - 2*x + 1)
```

giac [A] time = 0.16, size = 7, normalized size = 0.64

$$\frac{1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="giac")
[Out] 1/2/(x - 1)^2
```

maple [A] time = 0.04, size = 8, normalized size = 0.73

$$\frac{1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)^3/(-x^4+1)^3,x)
[Out] 1/2/(x-1)^2
```

maxima [A] time = 1.30, size = 12, normalized size = 1.09

$$\frac{1}{2\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="maxima")
[Out] 1/2/(x^2 - 2*x + 1)
```

mupad [B] time = 4.84, size = 7, normalized size = 0.64

$$\frac{1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2 + x^3 + 1)^3/(x^4 - 1)^3,x)
[Out] 1/(2*(x - 1)^2)
```

sympy [A] time = 0.21, size = 10, normalized size = 0.91

$$\frac{1}{2x^2-4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)**3/(-x**4+1)**3,x)
[Out] 1/(2*x**2 - 4*x + 2)
```

3.138 $\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} \, dx$

Optimal. Leaf size=11

$$\frac{1}{3(1-x)^3}$$

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 32}

$$\frac{1}{3(1-x)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
[Out] 1/(3*(1 - x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} \, dx &= \int \frac{1}{(1-x)^4} \, dx \\ &= \frac{1}{3(1-x)^3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.82

$$-\frac{1}{3(x-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
[Out] -1/3*1/(-1 + x)^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3)^4/(1 - x^4)^4, x]
```

fricas [B] time = 0.39, size = 17, normalized size = 1.55

$$-\frac{1}{3\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="fricas")
[Out] -1/3/(x^3 - 3*x^2 + 3*x - 1)
```

giac [A] time = 0.16, size = 7, normalized size = 0.64

$$-\frac{1}{3\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="giac")
[Out] -1/3/(x - 1)^3
```

maple [A] time = 0.04, size = 8, normalized size = 0.73

$$-\frac{1}{3\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)^4/(-x^4+1)^4,x)
[Out] -1/3/(x-1)^3
```

maxima [B] time = 1.32, size = 17, normalized size = 1.55

$$-\frac{1}{3\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="maxima")
[Out] -1/3/(x^3 - 3*x^2 + 3*x - 1)
```

mupad [B] time = 4.81, size = 7, normalized size = 0.64

$$-\frac{1}{3\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 1)^4/(x^4 - 1)^4,x)
[Out] -1/(3*(x - 1)^3)
```

sympy [B] time = 0.15, size = 17, normalized size = 1.55

$$-\frac{1}{3x^3-9x^2+9x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)**4/(-x**4+1)**4,x)
[Out] -1/(3*x**3 - 9*x**2 + 9*x - 3)
```

3.139 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4} dx$

Optimal. Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

Rubi [A] time = 0.26, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1885, 1887, 1167, 205, 208, 1819, 1810, 635, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}-\frac{hx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/
4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcT
anh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - (f*Log[a - b*x^4])/(4*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx = \int \left(\frac{c+ex^2+gx^4}{a-bx^4}+\frac{x\left(d+fx^2+hx^4\right)}{a-bx^4}\right)dx$$

$$= \int \frac{c+ex^2+gx^4}{a-bx^4}\,dx + \int \frac{x\left(d+fx^2+hx^4\right)}{a-bx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a-bx^2}\,dx, x, x^2\right) + \int \left(-\frac{g}{b}+\frac{bc+ag+bex^2}{b\left(a-bx^4\right)}\right)dx$$

$$= -\frac{gx}{b}+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{h}{b}+\frac{bd+ah+bfx}{b\left(a-bx^2\right)}\right)dx, x, x^2\right)+\frac{\int \frac{bc+ag+bex^2}{a-bx^4}\,d}{b}$$

$$= -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\text{Subst}\left(\int \frac{bd+ah+bfx}{a-bx^2}\,dx, x, x^2\right)}{2b}+\frac{1}{2}\left(e-\frac{bc+ag}{\sqrt{a}\sqrt{b}}\right)\int \frac{}{-\sqrt{a}}$$

$$= -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right.}{2a^{3/4}b}$$

$$= -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right.}{2a^{3/4}b}$$

Mathematica [A] time = 0.43, size = 256, normalized size = 1.55

$$\frac{-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{5/4}h+\sqrt{a}b^{3/4}e+\sqrt[4]{a}bd+a\sqrt[4]{b}g+b^{5/4}c\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(a^{5/4}(-h)+\sqrt{a}b^{3/4}e-\sqrt[4]{a}bd+a\sqrt[4]{b}g+b^{5/4}c\right)-a^{3/4}\sqrt{b}f\log\left(a-bx^4\right)-4a^{3/4}\sqrt{b}gx-2a^{3/4}\sqrt{b}hx^2+2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)+\sqrt[4]{a}(ah+bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{4a^{3/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

```
[Out] (-4*a^(3/4)*Sqrt[b]*g*x - 2*a^(3/4)*Sqrt[b]*h*x^2 + 2*b^(1/4)*(b*c - Sqrt[a
]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(5/4)*c + a^(1/4)*b*d +
 Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (b
^(5/4)*c - a^(1/4)*b*d + Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g - a^(5/4)*h)*Log[a
^(1/4) + b^(1/4)*x] + a^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3
/4)*Sqrt[b]*f*Log[a - b*x^4])/(4*a^(3/4)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4), x
]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.20, size = 342, normalized size = 2.07

$$-\frac{\sqrt{2}\left(b^2c+abg-\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c+abg+\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c+abg-\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^2c+abg-\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}-\frac{f\log\left(\left|bx^4-a\right|\right)}{4b}-\frac{bhx^2+2bgx}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(2)*(-a*b^3)
^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)
)/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b
^3)^(1/4)*b*d + sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b*e)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(2
)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(
-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^
2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log(abs(b*x
^4 - a))/b - 1/2*(b*h*x^2 + 2*b*g*x)/b^2
```

maple [B] time = 0.05, size = 296, normalized size = 1.79

$$-\frac{ah\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}\,b}-\frac{hx^2}{2b}-\frac{d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a}-\frac{e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{f\ln\left(bx^4-a\right)}{4b}-\frac{gx}{b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/2*h*x^2/b-1/b*g*x+1/2*(a/b)^(1/4)/b*g*arctan(1/(a/b)^(1/4)*x)+1/2*(a/b)^
(1/4)/a*c*arctan(1/(a/b)^(1/4)*x)+1/4*(a/b)^(1/4)/b*g*ln((x+(a/b)^(1/4))/(x
-(a/b)^(1/4)))+1/4*(a/b)^(1/4)/a*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/
b/(a*b)^(1/2)*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))*a*h-1/4/(a*b)^(1
/2)*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-1/2/(a/b)^(1/4)/b*e*arct
an(1/(a/b)^(1/4)*x)+1/4/(a/b)^(1/4)/b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
-1/4/b*f*ln(b*x^4-a)
```

maxima [A] time = 3.04, size = 222, normalized size = 1.35

$$-\frac{hx^2+2\,gx}{2\,b}+\frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}\,be+a\sqrt{b}\,g\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}\,bf+a\sqrt{b}\,h\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}\,bf+a\sqrt{b}\,h\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}c+\sqrt{a}\,be+a\sqrt{b}\,g\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*(h*x^2 + 2*g*x)/b + 1/4*(2*(b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*arc
tan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)
) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b)*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqr
t(a)*b) - (b^(3/2)*d + sqrt(a)*b*f + a*sqrt(b)*h)*log(sqrt(b)*x^2 - sqrt(a)
)/(sqrt(a)*b) - (b^(3/2)*c + sqrt(a)*b*e + a*sqrt(b)*g)*log((sqrt(b)*x - sq
rt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))*sqrt(b)))/b
```

mupad [B] time = 5.54, size = 2478, normalized size = 15.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x)
```

```
[Out] symsum(log(- root(256*a^3*b^6*z^4 + 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*z^2
- 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32*a^4*b^3*h^2*z^2 + 96*a^3*b^4
*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*f*g*z - 32*a^3*b^3*d*f*h*z + 3
2*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^2*g^
2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*e^2*h*z + 16*a^3*b^3*d*g^2*z + 16*a
^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z + 16*a*b^5*c^2*d*z
 + 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h + 8*a^2*b^3*c*d
*f*g - 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*a^3*b^
2*d*f^2*h + 4*a^3*b^2*d*f*g^2 + 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2 - 4*a
^2*b^3*d^2*e*g + 4*a^2*b^3*d*e^2*f + 4*a^2*b^3*c*e^2*g - 4*a^2*b^3*c*e*f^2
+ 4*a^4*b*f*g^2*h - 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2*e + 4
*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 6*a^
2*b^3*c^2*g^2 - 2*a^2*b^3*d^2*f^2 - 2*a^4*b*f^2*h^2 + 4*a^2*b^3*d^3*h - 4*a
^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a*b^4*d^4 + a^5*h^4 - a^2*b^
3*e^4 - a^4*b*g^4 - b^5*c^4, z, k)*((8*a*b^3*c*f - 8*a*b^3*d*e - 8*a^2*b^2*
e*h + 8*a^2*b^2*f*g)/b + root(256*a^3*b^6*z^4 + 256*a^3*b^5*f*z^3 - 64*a^3*
b^4*e*g*z^2 - 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32*a^4*b^3*h^2*z^2
+ 96*a^3*b^4*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*f*g*z - 32*a^3*b^3
*d*f*h*z + 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 1
6*a^4*b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*e^2*h*z + 16*a^3*b^3*d*
g^2*z + 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z + 16*a
*b^5*c^2*d*z + 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h + 8
*a^2*b^3*c*d*f*g - 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*
g - 4*a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 + 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c
*e*h^2 - 4*a^2*b^3*d^2*e*g + 4*a^2*b^3*d*e^2*f + 4*a^2*b^3*c*e^2*g - 4*a^2*
b^3*c*e*f^2 + 4*a^4*b*f*g^2*h - 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4
*c*d^2*e + 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^
2*g^2 - 6*a^2*b^3*c^2*g^2 - 2*a^2*b^3*d^2*f^2 - 2*a^4*b*f^2*h^2 + 4*a^2*b^3
*d^3*h - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a*b^4*d^4 + a^5*
h^4 - a^2*b^3*e^4 - a^4*b*g^4 - b^5*c^4, z, k)*((16*a^2*b^3*g + 16*a*b^4*c)
/b - (x*(16*a^2*b^3*h + 16*a*b^4*d))/b) + (x*(4*b^4*c^2 + 4*a*b^3*e^2 + 4*a
^2*b^2*g^2 + 8*a*b^3*c*g - 8*a*b^3*d*f - 8*a^2*b^2*f*h))/b) - (a*b^2*e^3 +
b^3*c*d^2 - b^3*c^2*e + a^3*g*h^2 + a*b^2*c*f^2 + a*b^2*d^2*g + a^2*b*c*h^2
 - a^2*b*e*g^2 + a^2*b*f^2*g + 2*a*b^2*c*d*h - 2*a*b^2*c*e*g - 2*a*b^2*d*e*
f + 2*a^2*b*d*g*h - 2*a^2*b*e*f*h)/b - (x*(b^3*d^3 + a^3*h^3 + b^3*c^2*f -
2*b^3*c*d*e - a*b^2*d*f^2 + a*b^2*e^2*f + 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 + a
^2*b*f*g^2 - a^2*b*f^2*h - 2*a*b^2*c*e*h + 2*a*b^2*c*f*g - 2*a*b^2*d*e*g -
2*a^2*b*e*g*h))/b)*root(256*a^3*b^6*z^4 + 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*
g*z^2 - 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32*a^4*b^3*h^2*z^2 + 96*a
^3*b^4*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*f*g*z - 32*a^3*b^3*d*f*h
*z + 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*
b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*e^2*h*z + 16*a^3*b^3*d*g^2*z
```

```
+ 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z + 16*a*b^5*c
^2*d*z + 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h + 8*a^2*b
^3*c*d*f*g - 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*
a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 + 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2
 - 4*a^2*b^3*d^2*e*g + 4*a^2*b^3*d*e^2*f + 4*a^2*b^3*c*e^2*g - 4*a^2*b^3*c*
e*f^2 + 4*a^4*b*f*g^2*h - 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2
*e + 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2
- 6*a^2*b^3*c^2*g^2 - 2*a^2*b^3*d^2*f^2 - 2*a^4*b*f^2*h^2 + 4*a^2*b^3*d^3*h
 - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a*b^4*d^4 + a^5*h^4 -
a^2*b^3*e^4 - a^4*b*g^4 - b^5*c^4, z, k), k, 1, 4) - (h*x^2)/(2*b) - (g*x)/
b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

3.140 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4} dx$

Optimal. Leaf size=188

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a\right.}{4}$$

Rubi [A] time = 0.33, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.220, Rules used = {1885, 1819, 1810, 635, 208, 260, 1887, 1167, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - ((b*e - (Sqrt[b]*(b*c + a*g))/
Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*e + (
Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4
)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2
)) - (f*Log[a - b*x^4])/(4*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+140x^6}{a-bx^4}\,dx &= \int \left(\frac{x\left(d+fx^2+hx^4\right)}{a-bx^4}+\frac{c+ex^2+gx^4+140x^6}{a-bx^4}\right)dx \\
&= \int \frac{x\left(d+fx^2+hx^4\right)}{a-bx^4}\,dx+\int \frac{c+ex^2+gx^4+140x^6}{a-bx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a-bx^2}\,dx,x,x^2\right)+\int \left(-\frac{g}{b}-\frac{140x^2}{b}+\frac{bc+a \\
&= -\frac{gx}{b}-\frac{140x^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{h}{b}+\frac{bd+ah+bfx}{b\left(a-bx^2\right)}\right)dx,x,x^2\right)+ \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{140x^3}{3b}+\frac{\text{Subst}\left(\int \frac{bd+ah+bfx}{a-bx^2}\,dx,x,x^2\right)}{2b}+\frac{\left(140a+\right. \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{140x^3}{3b}-\frac{\left(140a+be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,b^{7/4}}+\frac{\left(\right.}{} \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{140x^3}{3b}-\frac{\left(140a+be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,b^{7/4}}+\frac{\left(\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.55, size = 301, normalized size = 1.60

$$\frac{-\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{5/4}\sqrt[4]{b}h+a^{3/2}i+\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-a^{5/4}\sqrt[4]{b}h+a^{3/2}i-\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(a^{3/2}(-i)-\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}-3b^{3/4}f\log\left(a-bx^4\right)+\frac{3\sqrt[4]{b}(ah+bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{a}}-12b^{3/4}gx-6b^{3/4}hx^2-4b^{3/4}ix^3}{12b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 + (6*(b^(3/2)*c - Sqrt
[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(3/4) - (
3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g + a^(5/4)*b^(1
/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(3/4) + (3*(b^(3/2)*c - a^(1
/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/4)*h + a^(3/2)*i)*
Log[a^(1/4) + b^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sq
rt[b]*x^2])/Sqrt[a] - 3*b^(3/4)*f*Log[a - b*x^4])/(12*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b
*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b
*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [B] time = 0.21, size = 541, normalized size = 2.88

$$\frac{1}{8}\,i\left(\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{b^4}\right)+\frac{1}{8}\,i\left(\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{b^4}+\frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{b^4}\right)-\frac{\sqrt{2}\left(b^2c+abg-\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c+abg+\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c+abg-\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^2c+abg-\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}-\frac{f\log\left(\left|bx^4-a\right|\right)}{4b}-\frac{2b^2ix^3+3b^2hx^2+6b^2gx}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="gi
ac")
```

```
[Out] 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1
/4))/(-a/b)^(1/4))/b^4 - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/b^4) + 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 + sqrt(2)*(-a*b^3)^(3/4)*
log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4) - 1/4*sqrt(2)*(b^2*c +
a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(2)*(-a*b^3)^(1/4)*a*h + sqrt(-a*b
)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^
3)^(3/4) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(2
)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(2)*(b^2*c + a*b*g - sqr
t(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
+ 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(
1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log(abs(b*x^4 - a))/b - 1/6*(2*b^
2*i*x^3 + 3*b^2*h*x^2 + 6*b^2*g*x)/b^3
```

maple [B] time = 0.05, size = 367, normalized size = 1.95

$$-\frac{i\,x^3}{3b}-\frac{ah\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}\,b}-\frac{h\,x^2}{2b}-\frac{d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}}-\frac{ai\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}+\frac{ai\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a}-\frac{e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{f\ln\left(bx^4-a\right)}{4b}-\frac{gx}{b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/3*i*x^3/b-1/2/b*h*x^2-1/b*g*x+1/4*(a/b)^(1/4)/b*g*ln((x+(a/b)^(1/4))/(x-
(a/b)^(1/4)))+1/4*(a/b)^(1/4)/a*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*(
a/b)^(1/4)/b*g*arctan(1/(a/b)^(1/4)*x)+1/2*(a/b)^(1/4)/a*c*arctan(1/(a/b)^(
1/4)*x)-1/4/(a*b)^(1/2)*a/b*h*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-
1/4/(a*b)^(1/2)*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))+1/4/b^2/(a/b
```

```
)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*a*i+1/4/(a/b)^(1/4)/b*e*ln((x+(
a/b)^(1/4))/(x-(a/b)^(1/4)))-1/2/b^2/(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)*a*
i-1/2/(a/b)^(1/4)/b*e*arctan(1/(a/b)^(1/4)*x)-1/4/b*f*ln(b*x^4-a)
```

maxima [A] time = 3.03, size = 240, normalized size = 1.28

$$-\frac{2\,ix^3+3\,hx^2+6\,gx}{6\,b}+\frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}\,be+a\sqrt{b}\,g-a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}\,bf+a\sqrt{b}\,h\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}\,bf+a\sqrt{b}\,h\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}c+\sqrt{a}\,be+a\sqrt{b}\,g+a^{\frac{3}{2}}i\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="ma
xima")
```

```
[Out] -1/6*(2*i*x^3 + 3*h*x^2 + 6*g*x)/b + 1/4*(2*(b^(3/2)*c - sqrt(a)*b*e + a*sq
rt(b)*g - a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b)*h)*log(sqr
t(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt(a)*b*f + a*sqrt(b)*h)*l
og(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (b^(3/2)*c + sqrt(a)*b*e + a*sqrt(b
)*g + a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(
sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

mupad [B] time = 5.07, size = 3810, normalized size = 20.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x)
```

```
[Out] symsum(log(- (a^4*i^3 + a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^2*b^2*c*h^2 -
 a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i + a*b^3*c*f^2 + a*b^3*d^2*
g - a*b^3*c^2*i + 3*a^3*b*e*i^2 + a^3*b*g*h^2 - a^3*b*g^2*i - 2*a^2*b^2*c*g
*i - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h + 2*a*b^3*c*d*h -
2*a*b^3*c*e*g - 2*a*b^3*d*e*f - 2*a^3*b*f*h*i)/b^2 - root(256*a^3*b^7*z^4 +
 256*a^3*b^6*f*z^3 - 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d
*h*z^2 - 64*a^3*b^5*c*i*z^2 - 64*a^2*b^6*c*e*z^2 - 32*a^4*b^4*h^2*z^2 + 96*
a^3*b^5*f^2*z^2 - 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*
i*z - 32*a^3*b^4*e*f*g*z - 32*a^3*b^4*d*f*h*z + 32*a^3*b^4*d*e*i*z + 32*a^3
*b^4*c*g*h*z - 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z
 + 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^
3*d*i^2*z + 16*a^3*b^4*e^2*h*z + 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z -
16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z + 16*a*b^6*c^2*d*z + 16*a^3*b^4*f^3
*z + 8*a^4*b^2*e*f*h*i - 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*
d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*c*d*h*i + 8*a^2
*b^4*c*d*f*g - 8*a^2*b^4*c*d*e*h - 4*a^4*b^2*f^2*g*i + 4*a^4*b^2*f*g^2*h +
4*a^4*b^2*e*g^2*i - 4*a^4*b^2*e*g*h^2 - 4*a^4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g
*i + 4*a^4*b^2*d*f*i^2 + 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*
e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*b^3*d*f*g^2 + 4*a^2
*b^4*c^2*f*h + 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 - 4*a^2*b^4*d^2*e*g -
4*a^2*b^4*c*d^2*i + 4*a^2*b^4*d*e^2*f + 4*a^2*b^4*c*e^2*g - 4*a^2*b^4*c*e*f
^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d*f - 4*a*b^5*c*d^2*e
- 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g - 6*a^4*b^2*e^2*i^2 - 2*a^4*b^2*f^2*h^2 + 6
*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i^2 - 6*a^2*b^4*c^2*g^
2 - 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^3*i + 4*a^4*b^2*d*h^3
 + 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2 + a^3*b^3*f^4 + a^5*
b*h^4 + a*b^5*d^4 - a^4*b^2*g^4 - a^2*b^4*e^4 - a^6*i^4 - b^6*c^4, z, l)*(r
oot(256*a^3*b^7*z^4 + 256*a^3*b^6*f*z^3 - 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e
*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 - 64*a^2*b^6*c*e*z^2 - 32*
a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 - 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*
i*z + 32*a^4*b^3*e*h*i*z - 32*a^3*b^4*e*f*g*z - 32*a^3*b^4*d*f*h*z + 32*a^3
*b^4*d*e*i*z + 32*a^3*b^4*c*g*h*z - 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z
```

```
 + 32*a^2*b^5*c*d*g*z + 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^
3*f*h^2*z + 16*a^4*b^3*d*i^2*z + 16*a^3*b^4*e^2*h*z + 16*a^3*b^4*d*g^2*z +
16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z + 16*a*b^6*c^2
*d*z + 16*a^3*b^4*f^3*z + 8*a^4*b^2*e*f*h*i - 8*a^4*b^2*d*g*h*i - 8*a^3*b^3
*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^
3*b^3*c*d*h*i + 8*a^2*b^4*c*d*f*g - 8*a^2*b^4*c*d*e*h - 4*a^4*b^2*f^2*g*i +
 4*a^4*b^2*f*g^2*h + 4*a^4*b^2*e*g^2*i - 4*a^4*b^2*e*g*h^2 - 4*a^4*b^2*c*h^
2*i - 4*a^3*b^3*d^2*g*i + 4*a^4*b^2*d*f*i^2 + 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3
*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^
3*b^3*d*f*g^2 + 4*a^2*b^4*c^2*f*h + 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 -
 4*a^2*b^4*d^2*e*g - 4*a^2*b^4*c*d^2*i + 4*a^2*b^4*d*e^2*f + 4*a^2*b^4*c*e^
2*g - 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d
*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g - 6*a^4*b^2*e^2*i^2 -
2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i
^2 - 6*a^2*b^4*c^2*g^2 - 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^
3*i + 4*a^4*b^2*d*h^3 + 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2
 + a^3*b^3*f^4 + a^5*b*h^4 + a*b^5*d^4 - a^4*b^2*g^4 - a^2*b^4*e^4 - a^6*i^
4 - b^6*c^4, z, l)*((16*a^2*b^4*g + 16*a*b^5*c)/b^2 - (x*(16*a^2*b^3*h + 16
*a*b^4*d))/b) - (8*a*b^4*d*e - 8*a*b^4*c*f + 8*a^2*b^3*d*i + 8*a^2*b^3*e*h
- 8*a^2*b^3*f*g + 8*a^3*b^2*h*i)/b^2 + (x*(4*b^4*c^2 + 4*a*b^3*e^2 + 4*a^3*
b*i^2 + 4*a^2*b^2*g^2 + 8*a*b^3*c*g - 8*a*b^3*d*f + 8*a^2*b^2*e*i - 8*a^2*b
^2*f*h))/b) - (x*(b^3*d^3 + a^3*h^3 + b^3*c^2*f + a^3*f*i^2 - 2*b^3*c*d*e -
 2*a^3*g*h*i - a*b^2*d*f^2 + a*b^2*e^2*f + 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 +
a^2*b*f*g^2 - a^2*b*f^2*h - 2*a*b^2*c*d*i - 2*a*b^2*c*e*h + 2*a*b^2*c*f*g -
 2*a*b^2*d*e*g - 2*a^2*b*c*h*i - 2*a^2*b*d*g*i + 2*a^2*b*e*f*i - 2*a^2*b*e*
g*h))/b)*root(256*a^3*b^7*z^4 + 256*a^3*b^6*f*z^3 - 64*a^4*b^4*g*i*z^2 - 64
*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 - 64*a^2*b^6*c*e
*z^2 - 32*a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 - 32*a^2*b^6*d^2*z^2 - 32*a^
4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*z - 32*a^3*b^4*e*f*g*z - 32*a^3*b^4*d*f*h*
z + 32*a^3*b^4*d*e*i*z + 32*a^3*b^4*c*g*h*z - 32*a^3*b^4*c*f*i*z - 32*a^2*b
^5*c*e*f*z + 32*a^2*b^5*c*d*g*z + 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z -
 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*d*i^2*z + 16*a^3*b^4*e^2*h*z + 16*a^3*b^4*
d*g^2*z + 16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z + 16
*a*b^6*c^2*d*z + 16*a^3*b^4*f^3*z + 8*a^4*b^2*e*f*h*i - 8*a^4*b^2*d*g*h*i -
 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*
g*i - 8*a^3*b^3*c*d*h*i + 8*a^2*b^4*c*d*f*g - 8*a^2*b^4*c*d*e*h - 4*a^4*b^2
*f^2*g*i + 4*a^4*b^2*f*g^2*h + 4*a^4*b^2*e*g^2*i - 4*a^4*b^2*e*g*h^2 - 4*a^
4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g*i + 4*a^4*b^2*d*f*i^2 + 4*a^4*b^2*c*g*i^2 +
 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^
2*i + 4*a^3*b^3*d*f*g^2 + 4*a^2*b^4*c^2*f*h + 4*a^2*b^4*c^2*e*i - 4*a^3*b^3
*c*e*h^2 - 4*a^2*b^4*d^2*e*g - 4*a^2*b^4*c*d^2*i + 4*a^2*b^4*d*e^2*f + 4*a^
2*b^4*c*e^2*g - 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a
*b^5*c^2*d*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g - 6*a^4*b^2*
e^2*i^2 - 2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3
*b^3*c^2*i^2 - 6*a^2*b^4*c^2*g^2 - 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*
a^3*b^3*e^3*i + 4*a^4*b^2*d*h^3 + 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b
^5*c^2*e^2 + a^3*b^3*f^4 + a^5*b*h^4 + a*b^5*d^4 - a^4*b^2*g^4 - a^2*b^4*e^
4 - a^6*i^4 - b^6*c^4, z, l), l, 1, 4) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (g
*x)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

3.141 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4} dx$

Optimal. Leaf size=205

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{(aj+bf)\log\ldots}{4\ldots}$$

Rubi [A] time = 0.31, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.196, Rules used = {1885, 1887, 1167, 205, 208, 1819, 1810, 635, 260}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{(aj+bf)\log\left(a-bx^4\right)}{4b^2}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{jx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),x
]
```

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (j*x^4)/(4*b) - ((b*e - (Sqrt[
b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7
/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(
1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(
2*Sqrt[a]*b^(3/2)) - ((b*f + a*j)*Log[a - b*x^4])/(4*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+141x^6+jx^7}{a-bx^4}\,dx = \int \left(\frac{c+ex^2+gx^4+141x^6}{a-bx^4}+\frac{x\left(d+fx^2+hx^4+jx^6\right)}{a-bx^4}\right)$$

$$= \int \frac{c+ex^2+gx^4+141x^6}{a-bx^4}\,dx+\int \frac{x\left(d+fx^2+hx^4+jx\right.}{a-bx^4}$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2+jx^3}{a-bx^2}\,dx,x,x^2\right)+\int\left(-\frac{g}{b}-\frac{1}{}\right.$$

$$= -\frac{gx}{b}-\frac{47x^3}{b}+\frac{1}{2}\text{Subst}\left(\int\left(-\frac{h}{b}-\frac{jx}{b}+\frac{bd+ah+(bf}{b\left(a-bx^2\right.}\right.\right.$$

$$= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{47x^3}{b}-\frac{jx^4}{4b}+\frac{\text{Subst}\left(\int \frac{bd+ah+(bf+aj)x}{a-bx^2}\,dx,\right.}{2b}$$

$$= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{47x^3}{b}-\frac{jx^4}{4b}-\frac{\left(141a+be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}\right)\tan}{2\sqrt[4]{a}\,b^{7/4}}$$

$$= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{47x^3}{b}-\frac{jx^4}{4b}-\frac{\left(141a+be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}\right)\tan}{2\sqrt[4]{a}\,b^{7/4}}$$

Mathematica [A] time = 0.51, size = 318, normalized size = 1.55

$$\frac{-\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{5/4}\sqrt[4]{b}h+a^{3/2}i+\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-a^{5/4}\sqrt[4]{b}h+a^{3/2}i-\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(a^{3/2}(-i)-\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{3\sqrt[4]{b}(ah+bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{a}}-\frac{3(aj+bf)\log\left(a-bx^4\right)}{\sqrt[4]{b}}-12b^{3/4}gx-6b^{3/4}hx^2-4b^{3/4}ix^3-3b^{3/4}jx^4}{12b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4),x]
```

```
[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 - 3*b^(3/4)*j*x^4 + (6
*(b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(
1/4)])/a^(3/4) - (3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b
]*g + a^(5/4)*b^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(3/4) + (3
*(b^(3/2)*c - a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/
```

```
4)*h + a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d + a*h
)*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[a] - (3*(b*f + a*j)*Log[a - b*x^4])/b^(1
/4))/(12*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{a - bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a - b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a - b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.21, size = 556, normalized size = 2.71

$$\frac{1}{8} i \left(\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{b^4} - \frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}} \log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{b^4}\right) + \frac{1}{8} i \left(\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{b^4} + \frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}} \log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{b^4}\right) - \frac{\sqrt{2}\left(b^2c + abg - \sqrt{2}\left(-ab^3\right)^{\frac{1}{4}} bd - \sqrt{2}\left(-ab^3\right)^{\frac{1}{4}} ah + \sqrt{-ab}\,be\right) \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c + abg + \sqrt{2}\left(-ab^3\right)^{\frac{1}{4}} bd + \sqrt{2}\left(-ab^3\right)^{\frac{1}{4}} ah - \sqrt{-ab}\,be\right) \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c + abg - \sqrt{-ab}\,be\right) \log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(b^2c + abg - \sqrt{-ab}\,be\right) \log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} - \frac{(bf + aj)\log\left(\left|bx^4 - a\right|\right)}{4b^2} - \frac{3b^3jx^4 + 4b^3ix^3 + 6b^3hx^2 + 12b^3gx}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="giac")
```

```
[Out] 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1
/4))/(-a/b)^(1/4))/b^4 - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/b^4) + 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 + sqrt(2)*(-a*b^3)^(3/4)*
log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4) - 1/4*sqrt(2)*(b^2*c +
a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(2)*(-a*b^3)^(1/4)*a*h + sqrt(-a*b
)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^
3)^(3/4) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(2
)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(2)*(b^2*c + a*b*g - sqr
t(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
+ 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(
1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*(b*f + a*j)*log(abs(b*x^4 - a))/b^2
 - 1/12*(3*b^3*j*x^4 + 4*b^3*i*x^3 + 6*b^3*h*x^2 + 12*b^3*g*x)/b^4
```

maple [B] time = 0.05, size = 393, normalized size = 1.92

$$-\frac{jx^4}{4b} - \frac{ix^3}{3b} - \frac{ah\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}\,b} - \frac{hx^2}{2b} - \frac{d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}} - \frac{ai\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}} b^2} + \frac{ai\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}} b^2} - \frac{aj\ln\left(bx^4-a\right)}{4b^2} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a} - \frac{e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}} b} + \frac{e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}} b} - \frac{f\ln\left(bx^4-a\right)}{4b} - \frac{gx}{b} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/4*j*x^4/b-1/3/b*i*x^3-1/2/b*h*x^2-1/b*g*x+1/2*(a/b)^(1/4)/b*g*arctan(1/(
a/b)^(1/4)*x)+1/2*(a/b)^(1/4)/a*c*arctan(1/(a/b)^(1/4)*x)+1/4*(a/b)^(1/4)/b
```

```
*g*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/4*(a/b)^(1/4)/a*c*ln((x+(a/b)^(1/4
))/(x-(a/b)^(1/4)))-1/4/(a*b)^(1/2)*a/b*h*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1
/2)*x^2-a))-1/4/(a*b)^(1/2)*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-
1/2/(a/b)^(1/4)*a/b^2*i*arctan(1/(a/b)^(1/4)*x)-1/2/(a/b)^(1/4)/b*e*arctan(
1/(a/b)^(1/4)*x)+1/4/(a/b)^(1/4)*a/b^2*i*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4))
)+1/4/(a/b)^(1/4)/b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b^2*ln(b*x^4-
a)*a*j-1/4/b*f*ln(b*x^4-a)
```

maxima [A] time = 3.07, size = 257, normalized size = 1.25

$$-\frac{3\,jx^4+4\,ix^3+6\,hx^2+12\,gx}{12\,b}+\frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}\,be+a\sqrt{b}\,g-a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}\,bf+a\sqrt{b}\,h-a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}\,bf+a\sqrt{b}\,h+a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,b}-\frac{\left(b^{\frac{3}{2}}c+\sqrt{a}\,be+a\sqrt{b}\,g+a^{\frac{3}{2}}i\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="maxima")
```

```
[Out] -1/12*(3*j*x^4 + 4*i*x^3 + 6*h*x^2 + 12*g*x)/b + 1/4*(2*(b^(3/2)*c - sqrt(a
)*b*e + a*sqrt(b)*g - a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b
)*h - a^(3/2)*j)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt
(a)*b*f + a*sqrt(b)*h + a^(3/2)*j)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) -
 (b^(3/2)*c + sqrt(a)*b*e + a*sqrt(b)*g + a^(3/2)*i)*log((sqrt(b)*x - sqrt(
sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a
)*sqrt(b))*sqrt(b)))/b
```

mupad [B] time = 5.16, size = 5673, normalized size = 27.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),x
)
```

```
[Out] symsum(log(- (a^4*i^3 + a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^4*g*j^2 + a^2
*b^2*c*h^2 - a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - 2*a^4*h*i*j
+ a*b^3*c*f^2 + a*b^3*d^2*g - a*b^3*c^2*i + a^3*b*c*j^2 + 3*a^3*b*e*i^2 + a
^3*b*g*h^2 - a^3*b*g^2*i + 2*a^2*b^2*c*f*j - 2*a^2*b^2*c*g*i - 2*a^2*b^2*d*
e*j - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h + 2*a*b^3*c*d*h -
 2*a*b^3*c*e*g - 2*a*b^3*d*e*f - 2*a^3*b*d*i*j - 2*a^3*b*e*h*j + 2*a^3*b*f*
g*j - 2*a^3*b*f*h*i)/b^2 - root(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 + 256*a
^3*b^7*f*z^3 + 192*a^4*b^5*f*j*z^2 - 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^
2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*z^2 - 64*a^2*b^7*c*e*z^2 + 96*a^5*b
^4*j^2*z^2 - 32*a^4*b^5*h^2*z^2 + 96*a^3*b^6*f^2*z^2 - 32*a^2*b^7*d^2*z^2 -
 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*
e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^4*c*i*j*z - 32*a^3*b^5*e*f*g*z - 32
*a^3*b^5*d*f*h*z + 32*a^3*b^5*d*e*i*z + 32*a^3*b^5*c*g*h*z - 32*a^3*b^5*c*f
*i*z - 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z - 16*a^
5*b^3*h^2*j*z + 16*a^5*b^3*h*i^2*z + 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*
z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^2*z - 16*a^3*b^5*d^2*j*z + 16*a^4*b
^4*d*i^2*z + 16*a^3*b^5*e^2*h*z + 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z -
 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z + 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^
3*z + 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i*j + 8*a^5*b^2*e*h*i*j + 8*a^4*b^3*
e*f*h*i - 8*a^4*b^3*e*f*g*j - 8*a^4*b^3*d*g*h*i - 8*a^4*b^3*d*f*h*j + 8*a^4
*b^3*d*e*i*j + 8*a^4*b^3*c*g*h*j - 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h +
8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f
*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d*g*j + 8*a^2*b^5*c*d*f*g - 8*a^2*b^5*
c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5
*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j + 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 -
4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 - 4*a^4*b^3*f^2*g*i + 4*a^4*b^3*f*g^2
```

*h + 4*a^4*b^3*e*g^2*i + 4*a^4*b^3*d*g^2*j + 4*a^3*b^4*c^2*h*j - 4*a^4*b^3*
e*g*h^2 - 4*a^4*b^3*c*h^2*i - 4*a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j + 4*a^4
*b^3*d*f*i^2 + 4*a^4*b^3*c*g*i^2 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j -
4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2
*i + 4*a^3*b^4*d*f*g^2 + 4*a^2*b^5*c^2*f*h + 4*a^2*b^5*c^2*e*i + 4*a^2*b^5*
c^2*d*j - 4*a^3*b^4*c*e*h^2 - 4*a^2*b^5*d^2*e*g - 4*a^2*b^5*c*d^2*i + 4*a^2
*b^5*d*e^2*f + 4*a^2*b^5*c*e^2*g - 4*a^2*b^5*c*e*f^2 + 4*a^6*b*h*i^2*j - 4*
a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a*b^6*c*d^2*e + 4*a^6*b*f*j^3 - 4*a*b^6
*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2*g^2*i^2 - 6*a^4*b^3*e^2*i^2 - 2*a^4*
b^3*f^2*h^2 - 2*a^4*b^3*d^2*j^2 + 6*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2
*a^3*b^4*c^2*i^2 - 6*a^2*b^5*c^2*g^2 - 2*a^2*b^5*d^2*f^2 - 2*a^6*b*h^2*j^2
+ 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 - 4*a^3*b^4*e^3*i + 4*a^4*b^3*d*h^3 + 4
*a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a*b^6*c^2*e^2 + a^5*b^2*h^4 + a^3*b^4*
f^4 + a*b^6*d^4 + a^7*j^4 - a^4*b^3*g^4 - a^2*b^5*e^4 - a^6*b*i^4 - b^7*c^4
, z, m)*((8*a*b^4*c*f - 8*a*b^4*d*e + 8*a^2*b^3*c*j - 8*a^2*b^3*d*i - 8*a^2
*b^3*e*h + 8*a^2*b^3*f*g + 8*a^3*b^2*g*j - 8*a^3*b^2*h*i)/b^2 + root(256*a^
3*b^8*z^4 + 256*a^4*b^6*j*z^3 + 256*a^3*b^7*f*z^3 + 192*a^4*b^5*f*j*z^2 - 6
4*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*
i*z^2 - 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4*j^2*z^2 - 32*a^4*b^5*h^2*z^2 + 96*a
^3*b^6*f^2*z^2 - 32*a^2*b^7*d^2*z^2 - 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i
*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*
b^4*c*i*j*z - 32*a^3*b^5*e*f*g*z - 32*a^3*b^5*d*f*h*z + 32*a^3*b^5*d*e*i*z
+ 32*a^3*b^5*c*g*h*z - 32*a^3*b^5*c*f*i*z - 32*a^3*b^5*c*e*j*z - 32*a^2*b^6
*c*e*f*z + 32*a^2*b^6*c*d*g*z - 16*a^5*b^3*h^2*j*z + 16*a^5*b^3*h*i^2*z + 4
8*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*
h^2*z - 16*a^3*b^5*d^2*j*z + 16*a^4*b^4*d*i^2*z + 16*a^3*b^5*e^2*h*z + 16*a
^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2
*z + 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*z + 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g
*i*j + 8*a^5*b^2*e*h*i*j + 8*a^4*b^3*e*f*h*i - 8*a^4*b^3*e*f*g*j - 8*a^4*b^
3*d*g*h*i - 8*a^4*b^3*d*f*h*j + 8*a^4*b^3*d*e*i*j + 8*a^4*b^3*c*g*h*j - 8*a
^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h
+ 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d
*g*j + 8*a^2*b^5*c*d*f*g - 8*a^2*b^5*c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^
2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j + 4*a
^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2
- 4*a^4*b^3*f^2*g*i + 4*a^4*b^3*f*g^2*h + 4*a^4*b^3*e*g^2*i + 4*a^4*b^3*d*g
^2*j + 4*a^3*b^4*c^2*h*j - 4*a^4*b^3*e*g*h^2 - 4*a^4*b^3*c*h^2*i - 4*a^3*b^
4*d^2*g*i - 4*a^3*b^4*d^2*f*j + 4*a^4*b^3*d*f*i^2 + 4*a^4*b^3*c*g*i^2 + 4*a
^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j - 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g
- 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 + 4*a^2*b^5*c^2
*f*h + 4*a^2*b^5*c^2*e*i + 4*a^2*b^5*c^2*d*j - 4*a^3*b^4*c*e*h^2 - 4*a^2*b^
5*d^2*e*g - 4*a^2*b^5*c*d^2*i + 4*a^2*b^5*d*e^2*f + 4*a^2*b^5*c*e^2*g - 4*a
^2*b^5*c*e*f^2 + 4*a^6*b*h*i^2*j - 4*a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a*
b^6*c*d^2*e + 4*a^6*b*f*j^3 - 4*a*b^6*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2
*g^2*i^2 - 6*a^4*b^3*e^2*i^2 - 2*a^4*b^3*f^2*h^2 - 2*a^4*b^3*d^2*j^2 + 6*a^
3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a^3*b^4*c^2*i^2 - 6*a^2*b^5*c^2*g^2 -
2*a^2*b^5*d^2*f^2 - 2*a^6*b*h^2*j^2 + 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 -
4*a^3*b^4*e^3*i + 4*a^4*b^3*d*h^3 + 4*a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a
*b^6*c^2*e^2 + a^5*b^2*h^4 + a^3*b^4*f^4 + a*b^6*d^4 + a^7*j^4 - a^4*b^3*g^
4 - a^2*b^5*e^4 - a^6*b*i^4 - b^7*c^4, z, m)*((16*a^2*b^4*g + 16*a*b^5*c)/b
^2 - (x*(16*a^2*b^4*h + 16*a*b^5*d))/b^2) + (x*(4*b^5*c^2 + 4*a*b^4*e^2 + 4
*a^2*b^3*g^2 + 4*a^3*b^2*i^2 + 8*a*b^4*c*g - 8*a*b^4*d*f - 8*a^2*b^3*d*j +
8*a^2*b^3*e*i - 8*a^2*b^3*f*h - 8*a^3*b^2*h*j))/b^2) - (x*(b^4*d^3 + a^3*b*
h^3 + b^4*c^2*f - a^4*h*j^2 + a^4*i^2*j + 3*a^2*b^2*d*h^2 + a^2*b^2*f*g^2 -
a^2*b^2*f^2*h + a^2*b^2*e^2*j - 2*b^4*c*d*e - a*b^3*d*f^2 + a*b^3*e^2*f +
3*a*b^3*d^2*h + a*b^3*c^2*j - a^3*b*d*j^2 + a^3*b*f*i^2 + a^3*b*g^2*j + 2*a
^2*b^2*c*g*j - 2*a^2*b^2*c*h*i - 2*a^2*b^2*d*f*j - 2*a^2*b^2*d*g*i + 2*a^2*
b^2*e*f*i - 2*a^2*b^2*e*g*h - 2*a*b^3*c*d*i - 2*a*b^3*c*e*h + 2*a*b^3*c*f*g
- 2*a*b^3*d*e*g + 2*a^3*b*e*i*j - 2*a^3*b*f*h*j - 2*a^3*b*g*h*i))/b^2)*roo

```
t(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 + 256*a^3*b^7*f*z^3 + 192*a^4*b^5*f*j
*z^2 - 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^
3*b^6*c*i*z^2 - 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4*j^2*z^2 - 32*a^4*b^5*h^2*z^
2 + 96*a^3*b^6*f^2*z^2 - 32*a^2*b^7*d^2*z^2 - 32*a^5*b^3*g*i*j*z - 32*a^4*b
^4*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z -
 32*a^4*b^4*c*i*j*z - 32*a^3*b^5*e*f*g*z - 32*a^3*b^5*d*f*h*z + 32*a^3*b^5*
d*e*i*z + 32*a^3*b^5*c*g*h*z - 32*a^3*b^5*c*f*i*z - 32*a^3*b^5*c*e*j*z - 32
*a^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z - 16*a^5*b^3*h^2*j*z + 16*a^5*b^3*h*i
^2*z + 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^
4*b^4*f*h^2*z - 16*a^3*b^5*d^2*j*z + 16*a^4*b^4*d*i^2*z + 16*a^3*b^5*e^2*h*
z + 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b
^6*d*e^2*z + 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*z + 16*a^3*b^5*f^3*z - 8*a^5
*b^2*f*g*i*j + 8*a^5*b^2*e*h*i*j + 8*a^4*b^3*e*f*h*i - 8*a^4*b^3*e*f*g*j -
8*a^4*b^3*d*g*h*i - 8*a^4*b^3*d*f*h*j + 8*a^4*b^3*d*e*i*j + 8*a^4*b^3*c*g*h
*j - 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*a^3*b^4*d*e*f*i + 8*a^3*b^4*
c*f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j - 8*a^3*b^4*c*d*h*i + 8*a^3
*b^4*c*d*g*j + 8*a^2*b^5*c*d*f*g - 8*a^2*b^5*c*d*e*h + 4*a^5*b^2*g^2*h*j -
4*a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2
*j + 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*
c*i*j^2 - 4*a^4*b^3*f^2*g*i + 4*a^4*b^3*f*g^2*h + 4*a^4*b^3*e*g^2*i + 4*a^4
*b^3*d*g^2*j + 4*a^3*b^4*c^2*h*j - 4*a^4*b^3*e*g*h^2 - 4*a^4*b^3*c*h^2*i -
4*a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j + 4*a^4*b^3*d*f*i^2 + 4*a^4*b^3*c*g*i
^2 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j - 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*
e*f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 + 4*a^2
*b^5*c^2*f*h + 4*a^2*b^5*c^2*e*i + 4*a^2*b^5*c^2*d*j - 4*a^3*b^4*c*e*h^2 -
4*a^2*b^5*d^2*e*g - 4*a^2*b^5*c*d^2*i + 4*a^2*b^5*d*e^2*f + 4*a^2*b^5*c*e^2
*g - 4*a^2*b^5*c*e*f^2 + 4*a^6*b*h*i^2*j - 4*a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*
f - 4*a*b^6*c*d^2*e + 4*a^6*b*f*j^3 - 4*a*b^6*c^3*g + 6*a^5*b^2*f^2*j^2 + 2
*a^5*b^2*g^2*i^2 - 6*a^4*b^3*e^2*i^2 - 2*a^4*b^3*f^2*h^2 - 2*a^4*b^3*d^2*j^
2 + 6*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a^3*b^4*c^2*i^2 - 6*a^2*b^5*c
^2*g^2 - 2*a^2*b^5*d^2*f^2 - 2*a^6*b*h^2*j^2 + 4*a^4*b^3*f^3*j - 4*a^5*b^2*
e*i^3 - 4*a^3*b^4*e^3*i + 4*a^4*b^3*d*h^3 + 4*a^2*b^5*d^3*h - 4*a^3*b^4*c*g
^3 + 2*a*b^6*c^2*e^2 + a^5*b^2*h^4 + a^3*b^4*f^4 + a*b^6*d^4 + a^7*j^4 - a^
4*b^3*g^4 - a^2*b^5*e^4 - a^6*b*i^4 - b^7*c^4, z, m), m, 1, 4) - (h*x^2)/(2
*b) - (i*x^3)/(3*b) - (j*x^4)/(4*b) - (g*x)/b
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
[Out] Timed out
```

3.142 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4} dx$

Optimal. Leaf size=337

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

Rubi [A] time = 0.40, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {1885, 1887, 1168, 1162, 617, 204, 1165, 628, 1819, 1810, 635, 205, 260}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sq
rt[a]*b^(3/2)) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a
*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) -
((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4
)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \int \left(\frac{c+ex^2+gx^4}{a+bx^4}+\frac{x\left(d+fx^2+hx^4\right)}{a+bx^4}\right)dx$$

$$= \int \frac{c+ex^2+gx^4}{a+bx^4}\,dx + \int \frac{x\left(d+fx^2+hx^4\right)}{a+bx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a+bx^2}\,dx,x,x^2\right)+\int \left(\frac{g}{b}+\frac{bc-ag+bex^2}{b\left(a+bx^4\right)}\right)dx$$

$$= \frac{gx}{b}+\frac{1}{2}\text{Subst}\left(\int \left(\frac{h}{b}+\frac{bd-ah+bfx}{b\left(a+bx^2\right)}\right)dx,x,x^2\right)+\frac{\int \frac{bc-ag+bex^2}{a+bx^4}dx}{b}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{\text{Subst}\left(\int \frac{bd-ah+bfx}{a+bx^2}dx,x,x^2\right)}{2b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\sqrt{a}\sqrt{b}}{a+b}}{2\sqrt{a}\,b^{3/2}}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{1}{2}f\,\text{Subst}\left(\int \frac{x}{a+bx^2}dx,x,x^2\right)-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int}{4\sqrt{2}\,a^{3/4}b^{5/4}}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\right.}{4\sqrt{2}\,a^{3/4}b^{5/4}}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}\,a^{3/4}b^{5/4}}$$

Mathematica [A] time = 0.49, size = 342, normalized size = 1.01

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-2a^{5/4}h+\sqrt{2}\sqrt{a}b^{3/4}e+2\sqrt[4]{a}bd-\sqrt{2}a\sqrt[4]{b}g+\sqrt{2}b^{5/4}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(2a^{5/4}h+\sqrt{2}\sqrt{a}b^{3/4}e-2\sqrt[4]{a}bd-\sqrt{2}a\sqrt[4]{b}g+\sqrt{2}b^{5/4}c\right)+\sqrt[4]{b}\left(2a^{3/4}\sqrt[4]{b}\left(f\log\left(a+bx^4\right)+2x(2g+hx)\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{a}\sqrt{b}e+ag-bc\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)\right)}{8a^{3/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*b^(5/4)*c + 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2
]*a*b^(1/4)*g - 2*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(S
qrt[2]*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2]*a*b^
(1/4)*g + 2*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + b^(1/4)*(S
qrt[2]*(-(b*c) + Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*(2*x*(2*g +
 h*x) + f*Log[a + b*x^4])))/(8*a^(3/4)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4), x
]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 375, normalized size = 1.11

$$\frac{f\log\left(|bx^4+a|\right)}{4b}+\frac{bhx^2+2bgx}{2b^2}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d+\sqrt{2}\sqrt{ab}\,abh+(ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{1}{4}}abg+(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d+\sqrt{2}\sqrt{ab}\,abh+(ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{1}{4}}abg+(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{1}{4}}abg-(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{1}{4}}abg-(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*f*log(abs(b*x^4 + a))/b + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 + 1/4*sqrt(2)*(sq
rt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*
b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)
^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(
2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3
/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3)
+ 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3
)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*
(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

maple [A] time = 0.05, size = 462, normalized size = 1.37

$$-\frac{ah\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{2\sqrt{ab}\,b}+\frac{hx^2}{2b}+\frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{2\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{f\ln\left(bx^4+a\right)}{4b}+\frac{gx}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/2/b*h*x^2+1/b*g*x-1/4*(a/b)^(1/4)*2^(1/2)/b*g*arctan(2^(1/2)/(a/b)^(1/4)*
x-1)+1/4*(a/b)^(1/4)*2^(1/2)/a*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/8*(a/b)^
(1/4)*2^(1/2)/b*g*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/
4)*2^(1/2)*x+(a/b)^(1/2)))+1/8*(a/b)^(1/4)*2^(1/2)/a*c*ln((x^2+(a/b)^(1/4)*
2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))-1/4*(a/b)^(
1/4)*2^(1/2)/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*(a/b)^(1/4)*2^(1/2)/a*
c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/2/b/(a*b)^(1/2)*arctan((1/a*b)^(1/2)*x^
2)*a*h+1/2/(a*b)^(1/2)*d*arctan((1/a*b)^(1/2)*x^2)+1/8/(a/b)^(1/4)*2^(1/2)/
b*e*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2)))+1/4/(a/b)^(1/4)*2^(1/2)/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/
4/(a/b)^(1/4)*2^(1/2)/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4/b*f*ln(b*x^4+
a)
```

maxima [A] time = 3.05, size = 351, normalized size = 1.04

$$\frac{hx^2+2gx}{2b}+\frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^2c-\sqrt{a}b^{\frac{3}{2}}e-abg\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^2c+\sqrt{a}b^{\frac{3}{2}}e+abg\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{7}{4}}e-\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}g-2\sqrt{a}b^2d+2a^{\frac{3}{2}}bh\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{7}{4}}e-\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}g+2\sqrt{a}b^2d-2a^{\frac{3}{2}}bh\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(h*x^2 + 2*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f + b^2*c - s
qrt(a)*b^(3/2)*e - a*b*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqr
t(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - b^2*c + sqrt
(a)*b^(3/2)*e + a*b*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
))/(a^(3/4)*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/
4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h)*arctan(
```

```
1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sq
rt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g + 2*sqrt(a)*b^2*d - 2*a
^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt
(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/b
```

mupad [B] time = 5.54, size = 2469, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x)
```

```
[Out] symsum(log(root(256*a^3*b^6*z^4 - 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*z^2 -
64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a^4*b^3*h^2*z^2 + 96*a^3*b^4*f
^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*g*z + 32*a^3*b^3*d*f*h*z - 32*
a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^2*g^2*
h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^2*h*z - 16*a^3*b^3*d*g^2*z + 16*a^2
*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z - 16*a*b^5*c^2*d*z -
 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h - 8*a^2*b^3*c*d*f
*g + 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*a^3*b^2*
d*f^2*h + 4*a^3*b^2*d*f*g^2 - 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2 + 4*a^2
*b^3*d^2*e*g - 4*a^2*b^3*d*e^2*f - 4*a^2*b^3*c*e^2*g + 4*a^2*b^3*c*e*f^2 -
4*a^4*b*f*g^2*h + 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2*e - 4*a
^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 6*a^2*
b^3*c^2*g^2 + 2*a^2*b^3*d^2*f^2 + 2*a^4*b*f^2*h^2 - 4*a^2*b^3*d^3*h - 4*a^3
*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a^2*b^3*e^4 + a^4*b*g^4 + a*b^
4*d^4 + a^5*h^4 + b^5*c^4, z, k)*((8*a*b^3*c*f - 8*a*b^3*d*e + 8*a^2*b^2*e*
h - 8*a^2*b^2*f*g)/b + root(256*a^3*b^6*z^4 - 256*a^3*b^5*f*z^3 - 64*a^3*b^
4*e*g*z^2 - 64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a^4*b^3*h^2*z^2 +
96*a^3*b^4*f^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*g*z + 32*a^3*b^3*d
*f*h*z - 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*
a^4*b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^2*h*z - 16*a^3*b^3*d*g^
2*z + 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z - 16*a*b
^5*c^2*d*z - 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h - 8*a
^2*b^3*c*d*f*g + 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g
- 4*a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 - 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e
*h^2 + 4*a^2*b^3*d^2*e*g - 4*a^2*b^3*d*e^2*f - 4*a^2*b^3*c*e^2*g + 4*a^2*b^
3*c*e*f^2 - 4*a^4*b*f*g^2*h + 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c
*d^2*e - 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*
g^2 + 6*a^2*b^3*c^2*g^2 + 2*a^2*b^3*d^2*f^2 + 2*a^4*b*f^2*h^2 - 4*a^2*b^3*d
^3*h - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a^2*b^3*e^4 + a^4*
b*g^4 + a*b^4*d^4 + a^5*h^4 + b^5*c^4, z, k)*((16*a^2*b^3*g - 16*a*b^4*c)/b
 - (x*(16*a^2*b^3*h - 16*a*b^4*d))/b) - (x*(4*b^4*c^2 - 4*a*b^3*e^2 + 4*a^2
*b^2*g^2 - 8*a*b^3*c*g + 8*a*b^3*d*f - 8*a^2*b^2*f*h))/b) - (a*b^2*e^3 - b^
3*c*d^2 + b^3*c^2*e + a^3*g*h^2 + a*b^2*c*f^2 + a*b^2*d^2*g - a^2*b*c*h^2 +
 a^2*b*e*g^2 - a^2*b*f^2*g + 2*a*b^2*c*d*h - 2*a*b^2*c*e*g - 2*a*b^2*d*e*f
- 2*a^2*b*d*g*h + 2*a^2*b*e*f*h)/b + (x*(b^3*d^3 - a^3*h^3 + b^3*c^2*f - 2*
b^3*c*d*e + a*b^2*d*f^2 - a*b^2*e^2*f - 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 + a^2
*b*f*g^2 - a^2*b*f^2*h + 2*a*b^2*c*e*h - 2*a*b^2*c*f*g + 2*a*b^2*d*e*g - 2*
a^2*b*e*g*h))/b)*root(256*a^3*b^6*z^4 - 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*
z^2 - 64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a^4*b^3*h^2*z^2 + 96*a^3
*b^4*f^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*g*z + 32*a^3*b^3*d*f*h*z
 - 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^
2*g^2*h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^2*h*z - 16*a^3*b^3*d*g^2*z +
16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z - 16*a*b^5*c^2
*d*z - 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h - 8*a^2*b^3
*c*d*f*g + 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*a^
3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 - 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^2 +
 4*a^2*b^3*d^2*e*g - 4*a^2*b^3*d*e^2*f - 4*a^2*b^3*c*e^2*g + 4*a^2*b^3*c*e*
```

```
f^2 - 4*a^4*b*f*g^2*h + 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*d^2*e
 - 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 +
6*a^2*b^3*c^2*g^2 + 2*a^2*b^3*d^2*f^2 + 2*a^4*b*f^2*h^2 - 4*a^2*b^3*d^3*h -
 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a^2*b^3*e^4 + a^4*b*g^4
+ a*b^4*d^4 + a^5*h^4 + b^5*c^4, z, k), k, 1, 4) + (h*x^2)/(2*b) + (g*x)/b
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

3.143 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx$

Optimal. Leaf size=384

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag\right.}{4\sqrt{2}a^{3/4}b^{7/4}}$$

Rubi [A] time = 0.57, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.325, Rules used = {1885, 1819, 1810, 635, 205, 260, 1887, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)
/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i
))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + (
(Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*
e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2
]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4))
+ (f*Log[a + b*x^4])/(4*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+143x^6}{a+bx^4}\,dx = \int \left(\frac{x\left(d+fx^2+hx^4\right)}{a+bx^4}+\frac{c+ex^2+gx^4+143x^6}{a+bx^4}\right)dx$$

$$= \int \frac{x\left(d+fx^2+hx^4\right)}{a+bx^4}\,dx + \int \frac{c+ex^2+gx^4+143x^6}{a+bx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a+bx^2}\,dx, x, x^2\right) + \int \left(\frac{g}{b}+\frac{143x^2}{b}+\frac{bc-ag}{b \cdots}\right.$$

$$= \frac{gx}{b}+\frac{143x^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \left(\frac{h}{b}+\frac{bd-ah+bfx}{b\left(a+bx^2\right)}\right)dx, x, x^2\right)+\int \cdots$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{143x^3}{3b}+\frac{\text{Subst}\left(\int \frac{bd-ah+bfx}{a+bx^2}\,dx, x, x^2\right)}{2b}-\frac{\left(143a-b\cdots\right.}{\cdots}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{143x^3}{3b}+\frac{1}{2}f\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, x^2\right)-\frac{\left(143a-\cdots\right.}{\cdots}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{143x^3}{3b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{\left(143a-be+\frac{\sqrt{b}\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{143x^3}{3b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}+\frac{\left(143a-be-\frac{\sqrt{b}\cdots}{\cdots}\right.}{2\cdots}$$

Mathematica [A] time = 0.37, size = 427, normalized size = 1.11

$$\frac{\frac{6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2a^{5/4}\sqrt[4]{b}h+\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g-\sqrt{2}b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(2a^{5/4}\sqrt[4]{b}h-\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be-\sqrt{2}a\sqrt{b}g+\sqrt{2}b^{3/2}c\right)}{a^{3/4}}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(a^{3/2}i-\sqrt{a}be-a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(a^{3/2}i-\sqrt{a}be-a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+6b^{3/4}f\log\left(a+bx^4\right)+24b^{3/4}gx+12b^{3/4}hx^2+8b^{3/4}ix^3}{24b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + (6*(-(Sqrt[2]*b^(3/2
)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 2*
a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a
]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[2]*a^(3/2)*i)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]*(b^(3/2)*c - Sqrt
[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g
 + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/
4) + 6*b^(3/4)*f*Log[a + b*x^4])/(24*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b
*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b
*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 562, normalized size = 1.46

$$-\frac{1}{8}\, i \left(\frac{2\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{b^{4}}-\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{b^{4}}\right)-\frac{1}{8}\, i\left(\frac{2\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{b^{4}}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{b^{4}}\right)+\frac{f\log\left(\left|bx^{4}+a\right|\right)}{4b}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^{2}d+\sqrt{2}\sqrt{ab}\,abh+\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c-\left(ab^{3}\right)^{\frac{1}{4}}abg+\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^{3}}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^{2}d+\sqrt{2}\sqrt{ab}\,abh+\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c-\left(ab^{3}\right)^{\frac{1}{4}}abg+\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^{3}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c-\left(ab^{3}\right)^{\frac{1}{4}}abg-\left(ab^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^{3}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c-\left(ab^{3}\right)^{\frac{1}{4}}abg-\left(ab^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^{3}}+\frac{2b^{2}ix^{3}+3b^{2}hx^{2}+6b^{2}gx}{6b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="gia
c")
```

```
[Out] -1/8*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/b^4 - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/b^4) - 1/8*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^4 + sqrt(2)*(a*b^3)^(3/4)*log(x^2 -
 sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4) + 1/4*f*log(abs(b*x^4 + a))/b + 1/
4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4
)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x +
sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*
b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g
 + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*
b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sq
rt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/6*(2*b^2*i*x^3 + 3*b^2*h*
x^2 + 6*b^2*g*x)/b^3
```

maple [B] time = 0.06, size = 603, normalized size = 1.57

$$\frac{ix^{3}}{3b}-\frac{ah\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{2\sqrt{ab}\,b}+\frac{hx^{2}}{2b}+\frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{2\sqrt{ab}}-\frac{\sqrt{2}\,ai\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{\sqrt{2}\,ai\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{\sqrt{2}\,ai\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,e\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{f\ln\left(bx^{4}+a\right)}{4b}+\frac{gx}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/3/b*i*x^3+1/2/b*h*x^2+1/b*g*x-1/4*(a/b)^(1/4)*2^(1/2)/b*g*arctan(2^(1/2)/
(a/b)^(1/4)*x-1)+1/4*(a/b)^(1/4)*2^(1/2)/a*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1
)-1/8*(a/b)^(1/4)*2^(1/2)/b*g*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x
^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/8*(a/b)^(1/4)*2^(1/2)/a*c*ln((x^2+
(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))
-1/4*(a/b)^(1/4)*2^(1/2)/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*(a/b)^(1/4
)*2^(1/2)/a*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/2/(a*b)^(1/2)*a/b*h*arctan(
(1/a*b)^(1/2)*x^2)+1/2/(a*b)^(1/2)*d*arctan((1/a*b)^(1/2)*x^2)-1/8/b^2/(a/b
)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)
*2^(1/2)*x+(a/b)^(1/2)))*a*i+1/8/(a/b)^(1/4)*2^(1/2)/b*e*ln((x^2-(a/b)^(1/4
)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))-1/4/b^2/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*a*i+1/4/(a/b)^(1/4)*2^(1
/2)/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/4/b^2/(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x+1)*a*i+1/4/(a/b)^(1/4)*2^(1/2)/b*e*arctan(2^(1/2)/(a/
b)^(1/4)*x+1)+1/4/b*f*ln(b*x^4+a)
```

maxima [A] time = 3.08, size = 399, normalized size = 1.04

$$\frac{2ix^{3}+3hx^{2}+6gx}{6b}+\frac{\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^{2}c-\sqrt{a}\,b^{\frac{3}{2}}e-abg+a^{\frac{3}{2}}\sqrt{b}\,i\right)\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^{2}c+\sqrt{a}\,b^{\frac{3}{2}}e+abg-a^{\frac{3}{2}}\sqrt{b}\,i\right)\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g-\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{3}{4}}i-2\sqrt{a}\,b^{2}d+2a^{\frac{3}{2}}bh\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g-\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{3}{4}}i+2\sqrt{a}\,b^{2}d-2a^{\frac{3}{2}}bh\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="max
ima")
```

```
[Out] 1/6*(2*i*x^3 + 3*h*x^2 + 6*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f
 + b^2*c - sqrt(a)*b^(3/2)*e - a*b*g + a^(3/2)*sqrt(b)*i)*log(sqrt(b)*x^2 +
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*
a^(3/4)*b^(5/4)*f - b^2*c + sqrt(a)*b^(3/2)*e + a*b*g - a^(3/2)*sqrt(b)*i)*
log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) +
2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*
b^(5/4)*g - sqrt(2)*a^(7/4)*b^(3/4)*i - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c
 + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g - sqrt(2)*a^(7/4)*
b^(3/4)*i + 2*sqrt(a)*b^2*d - 2*a^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*
x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*s
qrt(b))*b^(5/4)))/b
```

mupad [B] time = 5.05, size = 3798, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x)
```

```
[Out] symsum(log((a^4*i^3 - a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^2*b^2*c*h^2 - a
^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - a*b^3*c*f^2 - a*b^3*d^2*g
+ a*b^3*c^2*i - 3*a^3*b*e*i^2 - a^3*b*g*h^2 + a^3*b*g^2*i - 2*a^2*b^2*c*g*i
 - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h - 2*a*b^3*c*d*h + 2*
a*b^3*c*e*g + 2*a*b^3*d*e*f + 2*a^3*b*f*h*i)/b^2 + root(256*a^3*b^7*z^4 - 2
56*a^3*b^6*f*z^3 + 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h
*z^2 - 64*a^3*b^5*c*i*z^2 + 64*a^2*b^6*c*e*z^2 + 32*a^4*b^4*h^2*z^2 + 96*a^
3*b^5*f^2*z^2 + 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*
z + 32*a^3*b^4*e*f*g*z + 32*a^3*b^4*d*f*h*z - 32*a^3*b^4*d*e*i*z - 32*a^3*b
^4*c*g*h*z + 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z -
 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*
d*i^2*z - 16*a^3*b^4*e^2*h*z - 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z - 16
*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z - 16*a*b^6*c^2*d*z - 16*a^3*b^4*f^3*z
 - 8*a^4*b^2*e*f*h*i + 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*
e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*c*d*h*i - 8*a^2*b
^4*c*d*f*g + 8*a^2*b^4*c*d*e*h + 4*a^4*b^2*f^2*g*i - 4*a^4*b^2*f*g^2*h - 4*
a^4*b^2*e*g^2*i + 4*a^4*b^2*e*g*h^2 + 4*a^4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g*i
 - 4*a^4*b^2*d*f*i^2 - 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*e*
f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*b^3*d*f*g^2 - 4*a^2*b
^4*c^2*f*h - 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 + 4*a^2*b^4*d^2*e*g + 4*
a^2*b^4*c*d^2*i - 4*a^2*b^4*d*e^2*f - 4*a^2*b^4*c*e^2*g + 4*a^2*b^4*c*e*f^2
 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b^5*c^2*d*f - 4*a*b^5*c*d^2*e -
4*a^5*b*e*i^3 - 4*a*b^5*c^3*g + 6*a^4*b^2*e^2*i^2 + 2*a^4*b^2*f^2*h^2 + 6*a
^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b^3*c^2*i^2 + 6*a^2*b^4*c^2*g^2
+ 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^3*b^3*e^3*i - 4*a^4*b^2*d*h^3 -
 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5*c^2*e^2 + a^4*b^2*g^4 + a^3*b^
3*f^4 + a^2*b^4*e^4 + a^5*b*h^4 + a*b^5*d^4 + a^6*i^4 + b^6*c^4, z, l)*((8*
a*b^4*c*f - 8*a*b^4*d*e + 8*a^2*b^3*d*i + 8*a^2*b^3*e*h - 8*a^2*b^3*f*g - 8
*a^3*b^2*h*i)/b^2 + root(256*a^3*b^7*z^4 - 256*a^3*b^6*f*z^3 + 64*a^4*b^4*g
*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 + 64*
a^2*b^6*c*e*z^2 + 32*a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 + 32*a^2*b^6*d^2*
z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^3*e*h*i*z + 32*a^3*b^4*e*f*g*z + 32*a^3
*b^4*d*f*h*z - 32*a^3*b^4*d*e*i*z - 32*a^3*b^4*c*g*h*z + 32*a^3*b^4*c*f*i*z
 - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5*c*d*g*z - 16*a^5*b^2*h*i^2*z + 16*a^4*b^
3*g^2*h*z - 16*a^4*b^3*f*h^2*z + 16*a^4*b^3*d*i^2*z - 16*a^3*b^4*e^2*h*z -
```

```
16*a^3*b^4*d*g^2*z + 16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d
*e^2*z - 16*a*b^6*c^2*d*z - 16*a^3*b^4*f^3*z - 8*a^4*b^2*e*f*h*i + 8*a^4*b^
2*d*g*h*i - 8*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a
^3*b^3*c*e*g*i - 8*a^3*b^3*c*d*h*i - 8*a^2*b^4*c*d*f*g + 8*a^2*b^4*c*d*e*h
+ 4*a^4*b^2*f^2*g*i - 4*a^4*b^2*f*g^2*h - 4*a^4*b^2*e*g^2*i + 4*a^4*b^2*e*g
*h^2 + 4*a^4*b^2*c*h^2*i - 4*a^3*b^3*d^2*g*i - 4*a^4*b^2*d*f*i^2 - 4*a^4*b^
2*c*g*i^2 + 4*a^3*b^3*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a
^3*b^3*c*f^2*i + 4*a^3*b^3*d*f*g^2 - 4*a^2*b^4*c^2*f*h - 4*a^2*b^4*c^2*e*i
- 4*a^3*b^3*c*e*h^2 + 4*a^2*b^4*d^2*e*g + 4*a^2*b^4*c*d^2*i - 4*a^2*b^4*d*e
^2*f - 4*a^2*b^4*c*e^2*g + 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*
h*i^2 + 4*a*b^5*c^2*d*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g +
 6*a^4*b^2*e^2*i^2 + 2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*
g^2 + 2*a^3*b^3*c^2*i^2 + 6*a^2*b^4*c^2*g^2 + 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g
^2*i^2 - 4*a^3*b^3*e^3*i - 4*a^4*b^2*d*h^3 - 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*
g^3 + 2*a*b^5*c^2*e^2 + a^4*b^2*g^4 + a^3*b^3*f^4 + a^2*b^4*e^4 + a^5*b*h^4
 + a*b^5*d^4 + a^6*i^4 + b^6*c^4, z, l)*((16*a^2*b^4*g - 16*a*b^5*c)/b^2 -
(x*(16*a^2*b^3*h - 16*a*b^4*d))/b) - (x*(4*b^4*c^2 - 4*a*b^3*e^2 - 4*a^3*b*
i^2 + 4*a^2*b^2*g^2 - 8*a*b^3*c*g + 8*a*b^3*d*f + 8*a^2*b^2*e*i - 8*a^2*b^2
*f*h))/b) + (x*(b^3*d^3 - a^3*h^3 + b^3*c^2*f - a^3*f*i^2 - 2*b^3*c*d*e + 2
*a^3*g*h*i + a*b^2*d*f^2 - a*b^2*e^2*f - 3*a*b^2*d^2*h + 3*a^2*b*d*h^2 + a^
2*b*f*g^2 - a^2*b*f^2*h + 2*a*b^2*c*d*i + 2*a*b^2*c*e*h - 2*a*b^2*c*f*g + 2
*a*b^2*d*e*g - 2*a^2*b*c*h*i - 2*a^2*b*d*g*i + 2*a^2*b*e*f*i - 2*a^2*b*e*g*
h))/b)*root(256*a^3*b^7*z^4 - 256*a^3*b^6*f*z^3 + 64*a^4*b^4*g*i*z^2 - 64*a
^3*b^5*e*g*z^2 - 64*a^3*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 + 64*a^2*b^6*c*e*z
^2 + 32*a^4*b^4*h^2*z^2 + 96*a^3*b^5*f^2*z^2 + 32*a^2*b^6*d^2*z^2 - 32*a^4*
b^3*f*g*i*z + 32*a^4*b^3*e*h*i*z + 32*a^3*b^4*e*f*g*z + 32*a^3*b^4*d*f*h*z
- 32*a^3*b^4*d*e*i*z - 32*a^3*b^4*c*g*h*z + 32*a^3*b^4*c*f*i*z - 32*a^2*b^5
*c*e*f*z + 32*a^2*b^5*c*d*g*z - 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 1
6*a^4*b^3*f*h^2*z + 16*a^4*b^3*d*i^2*z - 16*a^3*b^4*e^2*h*z - 16*a^3*b^4*d*
g^2*z + 16*a^2*b^5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z - 16*a
*b^6*c^2*d*z - 16*a^3*b^4*f^3*z - 8*a^4*b^2*e*f*h*i + 8*a^4*b^2*d*g*h*i - 8
*a^3*b^3*d*e*g*h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*
i - 8*a^3*b^3*c*d*h*i - 8*a^2*b^4*c*d*f*g + 8*a^2*b^4*c*d*e*h + 4*a^4*b^2*f
^2*g*i - 4*a^4*b^2*f*g^2*h - 4*a^4*b^2*e*g^2*i + 4*a^4*b^2*e*g*h^2 + 4*a^4*
b^2*c*h^2*i - 4*a^3*b^3*d^2*g*i - 4*a^4*b^2*d*f*i^2 - 4*a^4*b^2*c*g*i^2 + 4
*a^3*b^3*e^2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*
i + 4*a^3*b^3*d*f*g^2 - 4*a^2*b^4*c^2*f*h - 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c
*e*h^2 + 4*a^2*b^4*d^2*e*g + 4*a^2*b^4*c*d^2*i - 4*a^2*b^4*d*e^2*f - 4*a^2*
b^4*c*e^2*g + 4*a^2*b^4*c*e*f^2 - 4*a^5*b*g*h^2*i + 4*a^5*b*f*h*i^2 + 4*a*b
^5*c^2*d*f - 4*a*b^5*c*d^2*e - 4*a^5*b*e*i^3 - 4*a*b^5*c^3*g + 6*a^4*b^2*e^
2*i^2 + 2*a^4*b^2*f^2*h^2 + 6*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 2*a^3*b
^3*c^2*i^2 + 6*a^2*b^4*c^2*g^2 + 2*a^2*b^4*d^2*f^2 + 2*a^5*b*g^2*i^2 - 4*a^
3*b^3*e^3*i - 4*a^4*b^2*d*h^3 - 4*a^2*b^4*d^3*h - 4*a^3*b^3*c*g^3 + 2*a*b^5
*c^2*e^2 + a^4*b^2*g^4 + a^3*b^3*f^4 + a^2*b^4*e^4 + a^5*b*h^4 + a*b^5*d^4
+ a^6*i^4 + b^6*c^4, z, l), l, 1, 4) + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (g*x
)/b
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

3.144 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx$

Optimal. Leaf size=402

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{b}\,(bc-ag)-\sqrt{a}\,(be-ai)\right)}{4\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{b}\,(bc-ag\right.}{4\sqrt{2}\,a^{3/4}b^{7/4}}$$

Rubi [A] time = 0.57, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {1885, 1887, 1168, 1162, 617, 204, 1165, 628, 1819, 1810, 635, 205, 260}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{b}\,(bc-ag)-\sqrt{a}\,(be-ai)\right)}{4\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{b}\,(bc-ag)-\sqrt{a}\,(be-ai)\right)}{4\sqrt{2}\,a^{3/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{b}\,(bc-ag)+\sqrt{a}\,(be-ai)\right)}{2\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}\,(bc-ag)+\sqrt{a}\,(be-ai)\right)}{2\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}+\frac{(bf-aj)\log\left(a+bx^4\right)}{4b^2}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),x
]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + ((b*d - a*h)*ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sq
rt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*
g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e -
 a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(7/4)) + ((b*f - a*j)*Log[a + b*x^4])/(4*b^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+144x^6+jx^7}{a+bx^4}\,dx = \int \left(\frac{c+ex^2+gx^4+144x^6}{a+bx^4}+\frac{x\left(d+fx^2+hx^4+jx^6\right)}{a+bx^4}\right) dx$$

$$= \int \frac{c+ex^2+gx^4+144x^6}{a+bx^4}\,dx + \int \frac{x\left(d+fx^2+hx^4+jx^6\right)}{a+bx^4}$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{d+fx+hx^2+jx^3}{a+bx^2}\,dx, x, x^2\right) + \int \left(\frac{g}{b}+\frac{144x}{b}\right.$$

$$= \frac{gx}{b}+\frac{48x^3}{b}+\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{h}{b}+\frac{jx}{b}+\frac{bd-ah+(bf-aj)x}{b\left(a+bx^2\right)}\right.\right.$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{48x^3}{b}+\frac{jx^4}{4b}+\frac{\text{Subst}\left(\int \frac{bd-ah+(bf-aj)x}{a+bx^2}\,dx, x, x^2\right.}{2b}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{48x^3}{b}+\frac{jx^4}{4b}-\frac{\left(144a-be-\frac{\sqrt{b}\,(bc-ag)}{\sqrt{a}}\right)\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}-}}{4b^2}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{48x^3}{b}+\frac{jx^4}{4b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{(144a}{}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{48x^3}{b}+\frac{jx^4}{4b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}+\frac{(144}{}$$

Mathematica [A] time = 0.42, size = 445, normalized size = 1.11

$$\frac{\frac{6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2a^{5/4}\sqrt[4]{b}h+\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g-\sqrt{2}b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(2a^{5/4}\sqrt[4]{b}h-\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be-\sqrt{2}a\sqrt{b}g+\sqrt{2}b^{3/2}c\right)}{a^{3/4}}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(a^{3/2}i-\sqrt{a}be-a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(a^{3/2}i-\sqrt{a}be-a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{6(bf-aj)\log\left(a+bx^4\right)}{\sqrt[4]{b}}+24b^{3/4}gx+12b^{3/4}hx^2+8b^{3/4}ix^3+6b^{3/4}jx^4}{24b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4),x]
```

```
[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + 6*b^(3/4)*j*x^4 + (6
*(-(Sqrt[2]*b^(3/2)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2
]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*
d + Sqrt[2]*Sqrt[a]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[
2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]
*(b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]
*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/a^(3/4) + (6*(b*f - a*j)*Log[a + b*x^4])/b^(1/4))/(24*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a + b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.20, size = 578, normalized size = 1.44

$$-\frac{1}{8} i \left(\frac{2 \sqrt{2} \left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(2 x+\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{b^{4}} - \frac{\sqrt{2} \left(ab^{3}\right)^{\frac{3}{4}} \log\left(x^{2}+\sqrt{2} x \left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{b^{4}}\right) - \frac{1}{8} i \left(\frac{2 \sqrt{2} \left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(2 x-\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{b^{4}} + \frac{\sqrt{2} \left(ab^{3}\right)^{\frac{3}{4}} \log\left(x^{2}-\sqrt{2} x \left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{b^{4}}\right) + \frac{\left(bf-aj\right) \log\left(\left|bx^{4}+a\right|\right)}{4 b^{2}} + \frac{\sqrt{2} \left(\sqrt{2} \sqrt{ab} b^{2} d+\sqrt{2} \sqrt{ab} abh+\left(ab^{3}\right)^{\frac{1}{4}} b^{2} c-\left(ab^{3}\right)^{\frac{1}{4}} abg+\left(ab^{3}\right)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2} \left(2 x+\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4 ab^{3}} + \frac{\sqrt{2} \left(\sqrt{2} \sqrt{ab} b^{2} d+\sqrt{2} \sqrt{ab} abh+\left(ab^{3}\right)^{\frac{1}{4}} b^{2} c-\left(ab^{3}\right)^{\frac{1}{4}} abg+\left(ab^{3}\right)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2} \left(2 x-\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4 ab^{3}} + \frac{\sqrt{2} \left(\left(ab^{3}\right)^{\frac{1}{4}} b^{2} c-\left(ab^{3}\right)^{\frac{1}{4}} abg-\left(ab^{3}\right)^{\frac{3}{4}} e\right) \log\left(x^{2}+\sqrt{2} x \left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8 ab^{3}} - \frac{\sqrt{2} \left(\left(ab^{3}\right)^{\frac{1}{4}} b^{2} c-\left(ab^{3}\right)^{\frac{1}{4}} abg-\left(ab^{3}\right)^{\frac{3}{4}} e\right) \log\left(x^{2}-\sqrt{2} x \left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8 ab^{3}} + \frac{3 b^{3} jx^{4}+4 b^{3} ix^{3}+6 b^{3} hx^{2}+12 b^{3} gx}{12 b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="giac")
```

```
[Out] -1/8*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/b^4 - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/b^4) - 1/8*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^4 + sqrt(2)*(a*b^3)^(3/4)*log(x^2 -
 sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4) + 1/4*(b*f - a*j)*log(abs(b*x^4 +
a))/b^2 + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h +
(a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2
)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*b^3)
^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)
*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b
^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4
)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/12*(3*b^3*j*x
^4 + 4*b^3*i*x^3 + 6*b^3*h*x^2 + 12*b^3*g*x)/b^4
```

maple [B] time = 0.05, size = 627, normalized size = 1.56

$$\frac{j x^{4}}{4 b}+\frac{i x^{3}}{3 b}-\frac{a h \arctan\left(\sqrt{\frac{b}{a}}\, x^{2}\right)}{2 \sqrt{a b}\, b}+\frac{h x^{2}}{2 b}+\frac{d \arctan\left(\sqrt{\frac{b}{a}}\, x^{2}\right)}{2 \sqrt{a b}}-\frac{\sqrt{2}\, a i \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}-\frac{\sqrt{2}\, a i \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}-\frac{\sqrt{2}\, a i \ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{8 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}-\frac{a j \ln\left(b x^{4}+a\right)}{4 b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4 a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4 a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, c \ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{8 a}+\frac{\sqrt{2}\, e \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\sqrt{2}\, e \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\sqrt{2}\, e \ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{8 \left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{f \ln\left(b x^{4}+a\right)}{4 b}+\frac{g x}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, g \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4 b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, g \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4 b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, g \ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/4/b*j*x^4+1/3/b*i*x^3+1/2/b*h*x^2+1/b*g*x-1/4*(a/b)^(1/4)*2^(1/2)/b*g*arc
tan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*(a/b)^(1/4)*2^(1/2)/a*c*arctan(2^(1/2)/(a/
b)^(1/4)*x-1)-1/8*(a/b)^(1/4)*2^(1/2)/b*g*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/
b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/8*(a/b)^(1/4)*2^(1/2)/
a*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2)))-1/4*(a/b)^(1/4)*2^(1/2)/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/
4*(a/b)^(1/4)*2^(1/2)/a*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/2/(a*b)^(1/2)*a
/b*h*arctan((1/a*b)^(1/2)*x^2)+1/2/(a*b)^(1/2)*d*arctan((1/a*b)^(1/2)*x^2)-
1/4/(a/b)^(1/4)*2^(1/2)*a/b^2*i*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4/(a/b)^(
1/4)*2^(1/2)/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/4/(a/b)^(1/4)*2^(1/2)*a/
b^2*i*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/(a/b)^(1/4)*2^(1/2)/b*e*arctan(2^
(1/2)/(a/b)^(1/4)*x+1)-1/8/(a/b)^(1/4)*2^(1/2)*a/b^2*i*ln((x^2-(a/b)^(1/4)*
2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/8/(a/b)^(
1/4)*2^(1/2)/b*e*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4
)*2^(1/2)*x+(a/b)^(1/2)))-1/4/b^2*ln(b*x^4+a)*a*j+1/4/b*f*ln(b*x^4+a)
```

maxima [A] time = 3.16, size = 429, normalized size = 1.07

$$\frac{3\,jx^4+4\,ix^3+6\,hx^2+12\,gx}{12\,b}+\frac{\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}f-\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}j+b^2c-\sqrt{a}\,b^{\frac{3}{2}}e-abg+a^{\frac{3}{2}}\sqrt{b}\,i\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}f-\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{1}{4}}j-b^2c+\sqrt{a}\,b^{\frac{3}{2}}e+abg-a^{\frac{3}{2}}\sqrt{b}\,i\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g-\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{3}{4}}i-2\,\sqrt{a}\,b^2d+2\,a^{\frac{3}{2}}bh\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}e-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}g-\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{3}{4}}i+2\,\sqrt{a}\,b^2d-2\,a^{\frac{3}{2}}bh\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{5}{4}}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="maxima")
```

```
[Out] 1/12*(3*j*x^4 + 4*i*x^3 + 6*h*x^2 + 12*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/
4)*b^(5/4)*f - sqrt(2)*a^(7/4)*b^(1/4)*j + b^2*c - sqrt(a)*b^(3/2)*e - a*b*
g + a^(3/2)*sqrt(b)*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - sqrt(2)*a^(7/4)
*b^(1/4)*j - b^2*c + sqrt(a)*b^(3/2)*e + a*b*g - a^(3/2)*sqrt(b)*i)*log(sqr
t(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 2*(sqrt
(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)
*g - sqrt(2)*a^(7/4)*b^(3/4)*i - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt
(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g - sqrt(2)*a^(7/4)*b^(3/4)
*i + 2*sqrt(a)*b^2*d - 2*a^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqr
t(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))
*b^(5/4)))/b
```

mupad [B] time = 5.20, size = 5664, normalized size = 14.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),x
)
```

```
[Out] symsum(log((a^4*i^3 - a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^4*g*j^2 + a^2*b
^2*c*h^2 - a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - 2*a^4*h*i*j -
a*b^3*c*f^2 - a*b^3*d^2*g + a*b^3*c^2*i - a^3*b*c*j^2 - 3*a^3*b*e*i^2 - a^3
*b*g*h^2 + a^3*b*g^2*i + 2*a^2*b^2*c*f*j - 2*a^2*b^2*c*g*i - 2*a^2*b^2*d*e*
j - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h - 2*a*b^3*c*d*h + 2
*a*b^3*c*e*g + 2*a*b^3*d*e*f + 2*a^3*b*d*i*j + 2*a^3*b*e*h*j - 2*a^3*b*f*g*
j + 2*a^3*b*f*h*i)/b^2 + root(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 - 256*a^3
*b^7*f*z^3 - 192*a^4*b^5*f*j*z^2 + 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2
- 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*z^2 + 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4
*j^2*z^2 + 32*a^4*b^5*h^2*z^2 + 96*a^3*b^6*f^2*z^2 + 32*a^2*b^7*d^2*z^2 + 3
2*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*
g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^4*c*i*j*z + 32*a^3*b^5*e*f*g*z + 32*a
^3*b^5*d*f*h*z - 32*a^3*b^5*d*e*i*z - 32*a^3*b^5*c*g*h*z + 32*a^3*b^5*c*f*i
*z + 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z + 16*a^5*
b^3*h^2*j*z - 16*a^5*b^3*h*i^2*z - 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z
+ 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^2*z + 16*a^3*b^5*d^2*j*z + 16*a^4*b^4
*d*i^2*z - 16*a^3*b^5*e^2*h*z - 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 1
6*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z - 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*
z - 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i*j + 8*a^5*b^2*e*h*i*j - 8*a^4*b^3*e*
f*h*i + 8*a^4*b^3*e*f*g*j + 8*a^4*b^3*d*g*h*i + 8*a^4*b^3*d*f*h*j - 8*a^4*b
^3*d*e*i*j - 8*a^4*b^3*c*g*h*j + 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*
a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j
 - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d*g*j - 8*a^2*b^5*c*d*f*g + 8*a^2*b^5*c*
d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b
^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j - 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*
a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 + 4*a^4*b^3*f^2*g*i - 4*a^4*b^3*f*g^2*h
 - 4*a^4*b^3*e*g^2*i - 4*a^4*b^3*d*g^2*j + 4*a^3*b^4*c^2*h*j + 4*a^4*b^3*e*
g*h^2 + 4*a^4*b^3*c*h^2*i - 4*a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j - 4*a^4*b
```

^3*d*f*i^2 - 4*a^4*b^3*c*g*i^2 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j + 4*
a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i
 + 4*a^3*b^4*d*f*g^2 - 4*a^2*b^5*c^2*f*h - 4*a^2*b^5*c^2*e*i - 4*a^2*b^5*c^
2*d*j - 4*a^3*b^4*c*e*h^2 + 4*a^2*b^5*d^2*e*g + 4*a^2*b^5*c*d^2*i - 4*a^2*b
^5*d*e^2*f - 4*a^2*b^5*c*e^2*g + 4*a^2*b^5*c*e*f^2 - 4*a^6*b*h*i^2*j + 4*a^
6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a*b^6*c*d^2*e - 4*a^6*b*f*j^3 - 4*a*b^6*c
^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2*g^2*i^2 + 6*a^4*b^3*e^2*i^2 + 2*a^4*b^
3*f^2*h^2 + 2*a^4*b^3*d^2*j^2 + 6*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a
^3*b^4*c^2*i^2 + 6*a^2*b^5*c^2*g^2 + 2*a^2*b^5*d^2*f^2 + 2*a^6*b*h^2*j^2 -
4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 - 4*a^3*b^4*e^3*i - 4*a^4*b^3*d*h^3 - 4*a
^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a*b^6*c^2*e^2 + a^5*b^2*h^4 + a^4*b^3*g^
4 + a^3*b^4*f^4 + a^2*b^5*e^4 + a^6*b*i^4 + a*b^6*d^4 + a^7*j^4 + b^7*c^4,
z, m)*((8*a*b^4*c*f - 8*a*b^4*d*e - 8*a^2*b^3*c*j + 8*a^2*b^3*d*i + 8*a^2*b
^3*e*h - 8*a^2*b^3*f*g + 8*a^3*b^2*g*j - 8*a^3*b^2*h*i)/b^2 + root(256*a^3*
b^8*z^4 + 256*a^4*b^6*j*z^3 - 256*a^3*b^7*f*z^3 - 192*a^4*b^5*f*j*z^2 + 64*
a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*
z^2 + 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4*j^2*z^2 + 32*a^4*b^5*h^2*z^2 + 96*a^3
*b^6*f^2*z^2 + 32*a^2*b^7*d^2*z^2 + 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z
 + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^
4*c*i*j*z + 32*a^3*b^5*e*f*g*z + 32*a^3*b^5*d*f*h*z - 32*a^3*b^5*d*e*i*z -
32*a^3*b^5*c*g*h*z + 32*a^3*b^5*c*f*i*z + 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c
*e*f*z + 32*a^2*b^6*c*d*g*z + 16*a^5*b^3*h^2*j*z - 16*a^5*b^3*h*i^2*z - 48*
a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^
2*z + 16*a^3*b^5*d^2*j*z + 16*a^4*b^4*d*i^2*z - 16*a^3*b^5*e^2*h*z - 16*a^3
*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z
 - 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*z - 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i
*j + 8*a^5*b^2*e*h*i*j - 8*a^4*b^3*e*f*h*i + 8*a^4*b^3*e*f*g*j + 8*a^4*b^3*
d*g*h*i + 8*a^4*b^3*d*f*h*j - 8*a^4*b^3*d*e*i*j - 8*a^4*b^3*c*g*h*j + 8*a^4
*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*f*g*h +
8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b^4*c*d*g
*j - 8*a^2*b^5*c*d*f*g + 8*a^2*b^5*c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*a^5*b^2*
g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j - 4*a^4
*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*i*j^2 +
4*a^4*b^3*f^2*g*i - 4*a^4*b^3*f*g^2*h - 4*a^4*b^3*e*g^2*i - 4*a^4*b^3*d*g^2
*j + 4*a^3*b^4*c^2*h*j + 4*a^4*b^3*e*g*h^2 + 4*a^4*b^3*c*h^2*i - 4*a^3*b^4*
d^2*g*i - 4*a^3*b^4*d^2*f*j - 4*a^4*b^3*d*f*i^2 - 4*a^4*b^3*c*g*i^2 + 4*a^3
*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j + 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*f^2*g -
4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 - 4*a^2*b^5*c^2*f
*h - 4*a^2*b^5*c^2*e*i - 4*a^2*b^5*c^2*d*j - 4*a^3*b^4*c*e*h^2 + 4*a^2*b^5*
d^2*e*g + 4*a^2*b^5*c*d^2*i - 4*a^2*b^5*d*e^2*f - 4*a^2*b^5*c*e^2*g + 4*a^2
*b^5*c*e*f^2 - 4*a^6*b*h*i^2*j + 4*a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f - 4*a*b^
6*c*d^2*e - 4*a^6*b*f*j^3 - 4*a*b^6*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a^5*b^2*g
^2*i^2 + 6*a^4*b^3*e^2*i^2 + 2*a^4*b^3*f^2*h^2 + 2*a^4*b^3*d^2*j^2 + 6*a^3*
b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a^3*b^4*c^2*i^2 + 6*a^2*b^5*c^2*g^2 + 2
*a^2*b^5*d^2*f^2 + 2*a^6*b*h^2*j^2 - 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*i^3 - 4*
a^3*b^4*e^3*i - 4*a^4*b^3*d*h^3 - 4*a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3 + 2*a*b
^6*c^2*e^2 + a^5*b^2*h^4 + a^4*b^3*g^4 + a^3*b^4*f^4 + a^2*b^5*e^4 + a^6*b*
i^4 + a*b^6*d^4 + a^7*j^4 + b^7*c^4, z, m)*((16*a^2*b^4*g - 16*a*b^5*c)/b^2
 - (x*(16*a^2*b^4*h - 16*a*b^5*d))/b^2) - (x*(4*b^5*c^2 - 4*a*b^4*e^2 + 4*a
^2*b^3*g^2 - 4*a^3*b^2*i^2 - 8*a*b^4*c*g + 8*a*b^4*d*f - 8*a^2*b^3*d*j + 8*
a^2*b^3*e*i - 8*a^2*b^3*f*h + 8*a^3*b^2*h*j))/b^2) + (x*(b^4*d^3 - a^3*b*h^
3 + b^4*c^2*f - a^4*h*j^2 + a^4*i^2*j + 3*a^2*b^2*d*h^2 + a^2*b^2*f*g^2 - a
^2*b^2*f^2*h + a^2*b^2*e^2*j - 2*b^4*c*d*e + a*b^3*d*f^2 - a*b^3*e^2*f - 3*
a*b^3*d^2*h - a*b^3*c^2*j + a^3*b*d*j^2 - a^3*b*f*i^2 - a^3*b*g^2*j + 2*a^2
*b^2*c*g*j - 2*a^2*b^2*c*h*i - 2*a^2*b^2*d*f*j - 2*a^2*b^2*d*g*i + 2*a^2*b^
2*e*f*i - 2*a^2*b^2*e*g*h + 2*a*b^3*c*d*i + 2*a*b^3*c*e*h - 2*a*b^3*c*f*g +
 2*a*b^3*d*e*g - 2*a^3*b*e*i*j + 2*a^3*b*f*h*j + 2*a^3*b*g*h*i))/b^2)*root(
256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 - 256*a^3*b^7*f*z^3 - 192*a^4*b^5*f*j*z
^2 + 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*

```
b^6*c*i*z^2 + 64*a^2*b^7*c*e*z^2 + 96*a^5*b^4*j^2*z^2 + 32*a^4*b^5*h^2*z^2
+ 96*a^3*b^6*f^2*z^2 + 32*a^2*b^7*d^2*z^2 + 32*a^5*b^3*g*i*j*z - 32*a^4*b^4
*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 3
2*a^4*b^4*c*i*j*z + 32*a^3*b^5*e*f*g*z + 32*a^3*b^5*d*f*h*z - 32*a^3*b^5*d*
e*i*z - 32*a^3*b^5*c*g*h*z + 32*a^3*b^5*c*f*i*z + 32*a^3*b^5*c*e*j*z - 32*a
^2*b^6*c*e*f*z + 32*a^2*b^6*c*d*g*z + 16*a^5*b^3*h^2*j*z - 16*a^5*b^3*h*i^2
*z - 48*a^5*b^3*f*j^2*z + 48*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*
b^4*f*h^2*z + 16*a^3*b^5*d^2*j*z + 16*a^4*b^4*d*i^2*z - 16*a^3*b^5*e^2*h*z
- 16*a^3*b^5*d*g^2*z + 16*a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6
*d*e^2*z - 16*a*b^7*c^2*d*z + 16*a^6*b^2*j^3*z - 16*a^3*b^5*f^3*z - 8*a^5*b
^2*f*g*i*j + 8*a^5*b^2*e*h*i*j - 8*a^4*b^3*e*f*h*i + 8*a^4*b^3*e*f*g*j + 8*
a^4*b^3*d*g*h*i + 8*a^4*b^3*d*f*h*j - 8*a^4*b^3*d*e*i*j - 8*a^4*b^3*c*g*h*j
 + 8*a^4*b^3*c*f*i*j - 8*a^3*b^4*d*e*g*h + 8*a^3*b^4*d*e*f*i + 8*a^3*b^4*c*
f*g*h + 8*a^3*b^4*c*e*g*i - 8*a^3*b^4*c*e*f*j - 8*a^3*b^4*c*d*h*i + 8*a^3*b
^4*c*d*g*j - 8*a^2*b^5*c*d*f*g + 8*a^2*b^5*c*d*e*h + 4*a^5*b^2*g^2*h*j - 4*
a^5*b^2*g*h^2*i - 4*a^5*b^2*f*h^2*j + 4*a^5*b^2*f*h*i^2 + 4*a^5*b^2*d*i^2*j
 - 4*a^4*b^3*e^2*h*j - 4*a^5*b^2*e*g*j^2 - 4*a^5*b^2*d*h*j^2 - 4*a^5*b^2*c*
i*j^2 + 4*a^4*b^3*f^2*g*i - 4*a^4*b^3*f*g^2*h - 4*a^4*b^3*e*g^2*i - 4*a^4*b
^3*d*g^2*j + 4*a^3*b^4*c^2*h*j + 4*a^4*b^3*e*g*h^2 + 4*a^4*b^3*c*h^2*i - 4*
a^3*b^4*d^2*g*i - 4*a^3*b^4*d^2*f*j - 4*a^4*b^3*d*f*i^2 - 4*a^4*b^3*c*g*i^2
 + 4*a^3*b^4*e^2*f*h + 4*a^3*b^4*d*e^2*j + 4*a^4*b^3*c*e*j^2 - 4*a^3*b^4*e*
f^2*g - 4*a^3*b^4*d*f^2*h - 4*a^3*b^4*c*f^2*i + 4*a^3*b^4*d*f*g^2 - 4*a^2*b
^5*c^2*f*h - 4*a^2*b^5*c^2*e*i - 4*a^2*b^5*c^2*d*j - 4*a^3*b^4*c*e*h^2 + 4*
a^2*b^5*d^2*e*g + 4*a^2*b^5*c*d^2*i - 4*a^2*b^5*d*e^2*f - 4*a^2*b^5*c*e^2*g
 + 4*a^2*b^5*c*e*f^2 - 4*a^6*b*h*i^2*j + 4*a^6*b*g*i*j^2 + 4*a*b^6*c^2*d*f
- 4*a*b^6*c*d^2*e - 4*a^6*b*f*j^3 - 4*a*b^6*c^3*g + 6*a^5*b^2*f^2*j^2 + 2*a
^5*b^2*g^2*i^2 + 6*a^4*b^3*e^2*i^2 + 2*a^4*b^3*f^2*h^2 + 2*a^4*b^3*d^2*j^2
+ 6*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 2*a^3*b^4*c^2*i^2 + 6*a^2*b^5*c^2
*g^2 + 2*a^2*b^5*d^2*f^2 + 2*a^6*b*h^2*j^2 - 4*a^4*b^3*f^3*j - 4*a^5*b^2*e*
i^3 - 4*a^3*b^4*e^3*i - 4*a^4*b^3*d*h^3 - 4*a^2*b^5*d^3*h - 4*a^3*b^4*c*g^3
 + 2*a*b^6*c^2*e^2 + a^5*b^2*h^4 + a^4*b^3*g^4 + a^3*b^4*f^4 + a^2*b^5*e^4
+ a^6*b*i^4 + a*b^6*d^4 + a^7*j^4 + b^7*c^4, z, m), m, 1, 4) + (h*x^2)/(2*b
) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + (g*x)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

3.145 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^2} dx$

Optimal. Leaf size=184

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah\right.}{}$$

Rubi [A] time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1858, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + (
(3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b
^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/
(8*a^(7/4)*b^(5/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/
2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^2}\,dx = \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3bc-ag)-2b(bd-ah)x-b^2ex^2}{a-bx^4}\,dx}{4ab^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \left(-\frac{2b(bd-ah)x}{a-bx^4} + \frac{-b(3bc-ag)-b^2ex}{a-bx^4}\right.}{4ab^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3bc-ag)-b^2ex^2}{a-bx^4}\,dx}{4ab^2} + \frac{(bd-a}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\left(3bc-\sqrt{a}\sqrt{b}\,e-ag\right)\int \frac{1}{-\sqrt{a}\sqrt{b}}}{8a^{3/2}\sqrt{b}}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)} + \frac{\left(3bc-\sqrt{a}\sqrt{b}\,e-ag\right)\tan^{-1}\left(\right.}{8a^{7/4}b^{5/4}}$$

Mathematica [A] time = 0.28, size = 257, normalized size = 1.40

$$\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2a^{5/4}h-\sqrt{a}b^{3/4}e-2\sqrt[4]{a}bd+a\sqrt[4]{b}g-3b^{5/4}c\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(2a^{5/4}h+\sqrt{a}b^{3/4}e-2\sqrt[4]{a}bd-a\sqrt[4]{b}g+3b^{5/4}c\right)+\frac{4a^{3/4}\sqrt{b}(a(f+x(g+hx))+bx(c+x(d+ex)))}{a-bx^4}-2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag-3bc\right)-2\sqrt[4]{a}(ah-bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{16a^{7/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a - b*x
^4) - 2*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/
4)] + (-3*b^(5/4)*c - 2*a^(1/4)*b*d - Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + 2*a
^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (3*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[a]
*b^(3/4)*e - a*b^(1/4)*g + 2*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4
)*(-(b*d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,
x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,
 x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [B] time = 0.19, size = 380, normalized size = 2.07

$$-\frac{\sqrt{2}\left(3\,b^{2}c-abg-2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd+2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-abg+2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}bd-2\,\sqrt{2}\left(-ab^{3}\right)^{\frac{1}{4}}ah-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{\sqrt{2}\left(3\,b^{2}c-abg-\sqrt{-ab}\,be\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}+\frac{\sqrt{2}\left(3\,b^{2}c-abg-\sqrt{-ab}\,be\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^{3}\right)^{\frac{3}{4}}a}-\frac{bx^{3}e+bdx^{2}+ahx^{2}+bcx+agx+af}{4\left(bx^{4}-a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + 2*sqrt(2)*(
-a*b^3)^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g +
 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - 2*sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)
^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c
- a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((
-a*b^3)^(3/4)*a) - 1/4*(b*x^3*e + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/
((b*x^4 - a)*a*b)
```

maple [B] time = 0.05, size = 340, normalized size = 1.85

$$-\frac{d\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{8\sqrt{ab}\,a}+\frac{h\ln\left(\frac{\sqrt{ab}\,x^{2}-a}{-\sqrt{ab}\,x^{2}-a}\right)}{8\sqrt{ab}\,b}-\frac{e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}}+\frac{-\frac{e\,x^{3}}{4a}-\frac{(ah+bd)x^{2}}{4ab}-\frac{f}{4b}-\frac{\left(ag+bc\right)x}{4ab}}{b\,x^{4}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4/a*e*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x-1/4/b*f)/(b*x^4-a)-
1/8*(a/b)^(1/4)/a/b*g*arctan(1/(a/b)^(1/4)*x)+3/8*(a/b)^(1/4)/a^2*c*arctan(
1/(a/b)^(1/4)*x)-1/16/b/a*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*g
+3/16/a^2*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*c+1/8/b/(a*b)^(1/
2)*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))*h-1/8/(a*b)^(1/2)/a*d*ln(((
a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-1/8/b/a*e/(a/b)^(1/4)*arctan(1/(a/b
)^(1/4)*x)+1/16/(a/b)^(1/4)/a/b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 3.08, size = 243, normalized size = 1.32

$$-\frac{bex^{3}+(bd+ah)x^{2}+af+\left(bc+ag\right)x}{4\left(ab^{2}x^{4}-a^{2}b\right)}+\frac{\frac{2\,(bd-ah)\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{2\,(bd-ah)\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}\,be-a\sqrt{b}\,g\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(3\,b^{\frac{3}{2}}c+\sqrt{a}\,be-a\sqrt{b}\,g\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(b*e*x^3 + (b*d + a*h)*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 - a^2*b)
+ 1/16*(2*(b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 2*(b*d
 - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(3*b^(3/2)*c - sqr
t(a)*b*e - a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*b^(3/2)*c + sqrt(a)*b*e - a*sqrt(b)*g)*lo
g((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a*b)
```

mupad [B] time = 5.61, size = 1626, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log(- root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 1024*a^5*b^4*e
*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^2 - 2048*a^4*b^5*d^2*z^2
 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2*h*z - 128*a^
4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^3*d*g^2*z + 128*a^3*b^4*d*
e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h + 96*a^2*b^3*c*d*e*h - 48
*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*b^3*c*e^2*g + 16*a^4*b*e*g*h
^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*a^3*b^2*d^2*h
^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 - 64*a^2*b^3*d^3*h + 12*a^3*b^2
*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 - 81*b^5*c^4 - a^2*b^
3*e^4 - a^4*b*g^4, z, k)*(root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 1
024*a^5*b^4*e*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^2 - 2048*a^
4*b^5*d^2*z^2 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2
*h*z - 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^3*d*g^2*z + 1
28*a^3*b^4*d*e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h + 96*a^2*b^3
*c*d*e*h - 48*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*b^3*c*e^2*g + 1
6*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*
a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 - 64*a^2*b^3*d^3*h
 + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 - 81*b^5
*c^4 - a^2*b^3*e^4 - a^4*b*g^4, z, k)*((768*a^3*b^4*c - 256*a^4*b^3*g)/(64*
a^3*b) - (x*(128*a^3*b^4*d - 128*a^4*b^3*h))/(16*a^3*b)) - (64*a^2*b^3*d*e
- 64*a^3*b^2*e*h)/(64*a^3*b) + (x*(36*a*b^4*c^2 + 4*a^2*b^3*e^2 + 4*a^3*b^2
*g^2 - 24*a^2*b^3*c*g))/(16*a^3*b)) - (a*b^2*e^3 + 12*b^3*c*d^2 - 9*b^3*c^2
*e - 4*a^3*g*h^2 - 4*a*b^2*d^2*g + 12*a^2*b*c*h^2 - a^2*b*e*g^2 - 24*a*b^2*
c*d*h + 6*a*b^2*c*e*g + 8*a^2*b*d*g*h)/(64*a^3*b) - (x*(2*b^3*d^3 - 2*a^3*h
^3 - 3*b^3*c*d*e - 6*a*b^2*d^2*h + 6*a^2*b*d*h^2 + 3*a*b^2*c*e*h + a*b^2*d*
e*g - a^2*b*e*g*h))/(16*a^3*b))*root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z
^2 + 1024*a^5*b^4*e*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^2 - 2
048*a^4*b^5*d^2*z^2 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b
^2*g^2*h*z - 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^3*d*g^2
*z + 128*a^3*b^4*d*e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h + 96*a
^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*b^3*c*e^2
*g + 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g
 + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 - 64*a^2*b^3
*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 -
81*b^5*c^4 - a^2*b^3*e^4 - a^4*b*g^4, z, k), k, 1, 4) + (f/(4*b) + (e*x^3)/
(4*a) + (x*(b*c + a*g))/(4*a*b) + (x^2*(b*d + a*h))/(4*a*b))/(a - b*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.146 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a-bx^4\right)^2}\,dx$

Optimal. Leaf size=203

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x\right.}{}$$

Rubi [A] time = 0.27, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1858, 1876, 275, 208, 1167, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(4*a*b*(a - b*x
^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3
*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+146x^6}{\left(a-bx^4\right)^2}\,dx = \frac{x\left(bc+ag+(bd+ah)x+(146a+be)x^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \int \frac{-b(3bc-ag)-}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(146a+be)x^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \int \left(-\frac{2b(bd-ah}{a-bx^4}\right.$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(146a+be)x^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3bc-ag)+}{a-}}{4}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(146a+be)x^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\left(438a-be\right.}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(146a+be)x^2+bfx^3\right)}{4ab\left(a-bx^4\right)} + \frac{\left(438a-be\right.}{}$$

Mathematica [A] time = 0.28, size = 302, normalized size = 1.49

$$\frac{\frac{4a^{3/4}b^{3/4}(a(f+x(g+x(h+ix)))+bx(c+x(d+ex)))}{a-bx^4}+\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{a}be+a\sqrt{b}g-3b^{3/2}c\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(2a^{5/4}\sqrt[4]{b}h-3a^{3/2}i-2\sqrt[4]{a}b^{5/4}d+\sqrt{a}be-a\sqrt{b}g+3b^{3/2}c\right)+2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3a^{3/2}i-\sqrt{a}be-a\sqrt{b}g+3b^{3/2}c\right)-2\sqrt[4]{a}\sqrt[4]{b}(ah-bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{16a^{7/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x
]
```

```
[Out] ((4*a^(3/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + x*(h + i*x)))))/
(a - b*x^4) + 2*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Arc
Tan[(b^(1/4)*x)/a^(1/4)] + (-3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d - Sqrt[a]*b*
e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*
x] + (3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(
5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4)*b^(1/4)*
(-(b*d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a-bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b
*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b
*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.20, size = 583, normalized size = 2.87

$$-\frac{3}{32} i \left(\frac{2 \sqrt{2} \left(-a b^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(2 x + \sqrt{2} \left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a b^{4}} - \frac{\sqrt{2} \left(-a b^{3}\right)^{\frac{3}{4}} \log\left(x^{2} + \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{a b^{4}}\right) - \frac{3}{32} i \left(\frac{2 \sqrt{2} \left(-a b^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(2 x - \sqrt{2} \left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a b^{4}} + \frac{\sqrt{2} \left(-a b^{3}\right)^{\frac{3}{4}} \log\left(x^{2} - \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{a b^{4}}\right) - \frac{\sqrt{2} \left(3 b^{2} c - a b g - 2 \sqrt{2} \left(-a b^{3}\right)^{\frac{1}{4}} b d + 2 \sqrt{2} \left(-a b^{3}\right)^{\frac{1}{4}} a h + \sqrt{-a b} b e\right) \arctan\left(\frac{\sqrt{2} \left(2 x + \sqrt{2} \left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 \left(-a b^{3}\right)^{\frac{3}{4}} a} - \frac{\sqrt{2} \left(3 b^{2} c - a b g + 2 \sqrt{2} \left(-a b^{3}\right)^{\frac{1}{4}} b d - 2 \sqrt{2} \left(-a b^{3}\right)^{\frac{1}{4}} a h - \sqrt{-a b} b e\right) \arctan\left(\frac{\sqrt{2} \left(2 x - \sqrt{2} \left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2 \left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 \left(-a b^{3}\right)^{\frac{3}{4}} a} - \frac{\sqrt{2} \left(3 b^{2} c - a b g - \sqrt{-a b} b e\right) \log\left(x^{2} + \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32 \left(-a b^{3}\right)^{\frac{3}{4}} a} + \frac{\sqrt{2} \left(3 b^{2} c - a b g - \sqrt{-a b} b e\right) \log\left(x^{2} - \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32 \left(-a b^{3}\right)^{\frac{3}{4}} a} - \frac{a i x^{3} + b x^{3} e + b d x^{2} + a h x^{2} + b c x + a g x + a f}{4 \left(b x^{4} - a\right) a b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
giac")
```

```
[Out] -3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^
(1/4))/(-a/b)^(1/4))/(a*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) - 3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^4) + sqrt(2)*
(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) - 1/
16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + 2*sqrt(2)*(-a*
b^3)^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(
1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g + 2*
sqrt(2)*(-a*b^3)^(1/4)*b*d - 2*sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3
/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*
x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - a
*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*
b^3)^(3/4)*a) - 1/4*(a*i*x^3 + b*x^3*e + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x
+ a*f)/((b*x^4 - a)*a*b)
```

maple [B] time = 0.05, size = 409, normalized size = 2.01

$$-\frac{d \ln\left(\frac{\sqrt{a b}\, x^{2}-a}{-\sqrt{a b}\, x^{2}-a}\right)}{8 \sqrt{a b}\, a}+\frac{h \ln\left(\frac{\sqrt{a b}\, x^{2}-a}{-\sqrt{a b}\, x^{2}-a}\right)}{8 \sqrt{a b}\, b}-\frac{e \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 \left(\frac{a}{b}\right)^{\frac{1}{4}} a b}+\frac{e \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 \left(\frac{a}{b}\right)^{\frac{1}{4}} a b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} g \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 a b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} g \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 a b}+\frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} c \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 a^{2}}+\frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} c \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 a^{2}}+\frac{3 i \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}-\frac{3 i \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}+\frac{-\frac{\left(a i+b e\right) x^{3}}{4 a b}-\frac{\left(a h+b d\right) x^{2}}{4 a b}-\frac{f}{4 b}-\frac{\left(a g+b c\right) x}{4 a b}}{b\, x^{4}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i+b*e)/a/b*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x-1/4/b*f)/
(b*x^4-a)-1/8*(a/b)^(1/4)/a/b*g*arctan(1/(a/b)^(1/4)*x)+3/8*(a/b)^(1/4)/a^2
*c*arctan(1/(a/b)^(1/4)*x)-1/16*(a/b)^(1/4)/a/b*g*ln((x+(a/b)^(1/4))/(x-(a/
b)^(1/4)))+3/16*(a/b)^(1/4)/a^2*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/8/(
a*b)^(1/2)/b*h*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-1/8/(a*b)^(1/2)
/a*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))+3/8/b^2/(a/b)^(1/4)*arcta
n(1/(a/b)^(1/4)*x)*i-1/8/(a/b)^(1/4)/a/b*e*arctan(1/(a/b)^(1/4)*x)-3/16/b^2
/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))*i+1/16/(a/b)^(1/4)/a/b*e*l
n((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 3.06, size = 260, normalized size = 1.28

$$-\frac{(be + ai)x^{3} + (bd + ah)x^{2} + af + \left(bc + ag\right)x}{4 \left(ab^{2}x^{4} - a^{2}b\right)} + \frac{\frac{2\,(bd-ah)\log\left(\sqrt{b}\,x^{2}+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{2\,(bd-ah)\log\left(\sqrt{b}\,x^{2}-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}} + \frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}\,be-a\sqrt{b}\,g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}} - \frac{\left(3\,b^{\frac{3}{2}}c+\sqrt{a}\,be-a\sqrt{b}\,g-3\,a^{\frac{3}{2}}i\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
maxima")
```

```
[Out] -1/4*((b*e + a*i)*x^3 + (b*d + a*h)*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 -
 a^2*b) + 1/16*(2*(b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b))
- 2*(b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(3*b^(3/2)
*c - sqrt(a)*b*e - a*sqrt(b)*g + 3*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*b^(3/2)*c + sqrt(a)
*b*e - a*sqrt(b)*g - 3*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(
sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
)/(a*b)
```

mupad [B] time = 5.67, size = 2611, normalized size = 12.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x)
```

```
[Out] symsum(log((27*a^4*i^3 - a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e - 12*a^2*b^
2*c*h^2 + a^2*b^2*e*g^2 + 9*a^2*b^2*e^2*i + 4*a*b^3*d^2*g - 27*a*b^3*c^2*i
- 27*a^3*b*e*i^2 + 4*a^3*b*g*h^2 - 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 8*a^2
*b^2*d*g*h + 24*a*b^3*c*d*h - 6*a*b^3*c*e*g)/(64*a^3*b^2) - root(65536*a^7*
b^7*z^4 - 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*i*z^2 + 4096*a^5*b^5*d*h*z^
2 + 1024*a^5*b^5*e*g*z^2 - 3072*a^4*b^6*c*e*z^2 - 2048*a^6*b^4*h^2*z^2 - 20
48*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z - 768*a^4*b^4*d*e*i*z + 768*a^4*b^
4*c*g*h*z - 768*a^3*b^5*c*d*g*z - 1152*a^6*b^2*h*i^2*z - 128*a^5*b^3*g^2*h*
z + 1152*a^5*b^3*d*i^2*z - 128*a^4*b^4*e^2*h*z - 1152*a^3*b^5*c^2*h*z + 128
*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z + 1152*a^2*b^6*c^2*d*z + 96*a^4*b^2*
d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*g*i - 32*a^3*b^3*d*e*g*h + 9
6*a^2*b^4*c*d*e*h - 12*a^4*b^2*e*g^2*i + 144*a^4*b^2*c*h^2*i - 48*a^3*b^3*d
^2*g*i + 16*a^4*b^2*e*g*h^2 - 108*a^4*b^2*c*g*i^2 - 108*a^2*b^4*c^2*e*i + 1
44*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 + 16*a^2*b^4*d^2*e*g - 12*a^2*b^4*c
*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e + 108*a^5*b*e*i^3 + 108*a*b^5*
c^3*g - 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^2 + 96*a^3*b^3*d^2*h^2 + 2*a
^3*b^3*e^2*g^2 - 54*a^2*b^4*c^2*g^2 + 18*a^5*b*g^2*i^2 + 12*a^3*b^3*e^3*i -
 64*a^4*b^2*d*h^3 - 64*a^2*b^4*d^3*h + 12*a^3*b^3*c*g^3 + 18*a*b^5*c^2*e^2
+ 16*a^5*b*h^4 + 16*a*b^5*d^4 - 81*a^6*i^4 - 81*b^6*c^4 - a^4*b^2*g^4 - a^2
*b^4*e^4, z, l)*(root(65536*a^7*b^7*z^4 - 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b
^5*c*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 - 3072*a^4*b^6*c*e
*z^2 - 2048*a^6*b^4*h^2*z^2 - 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z -
768*a^4*b^4*d*e*i*z + 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z - 1152*a^6*
b^2*h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z - 128*a^4*b^4*e^2*
h*z - 1152*a^3*b^5*c^2*h*z + 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z + 11
52*a^2*b^6*c^2*d*z + 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*
c*e*g*i - 32*a^3*b^3*d*e*g*h + 96*a^2*b^4*c*d*e*h - 12*a^4*b^2*e*g^2*i + 14
4*a^4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i + 16*a^4*b^2*e*g*h^2 - 108*a^4*b^2*c
*g*i^2 - 108*a^2*b^4*c^2*e*i + 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 + 1
6*a^2*b^4*d^2*e*g - 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*
e + 108*a^5*b*e*i^3 + 108*a*b^5*c^3*g - 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^
2*i^2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 - 54*a^2*b^4*c^2*g^2 + 18*a^
5*b*g^2*i^2 + 12*a^3*b^3*e^3*i - 64*a^4*b^2*d*h^3 - 64*a^2*b^4*d^3*h + 12*a
^3*b^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 - 81*a^6*i^4
- 81*b^6*c^4 - a^4*b^2*g^4 - a^2*b^4*e^4, z, l)*((768*a^3*b^5*c - 256*a^4*b
^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^4*d - 128*a^4*b^3*h))/(16*a^3*b)) - (64*
a^2*b^4*d*e - 192*a^3*b^3*d*i - 64*a^3*b^3*e*h + 192*a^4*b^2*h*i)/(64*a^3*b
^2) + (x*(36*a*b^4*c^2 + 36*a^4*b*i^2 + 4*a^2*b^3*e^2 + 4*a^3*b^2*g^2 - 24*
a^2*b^3*c*g - 24*a^3*b^2*e*i))/(16*a^3*b)) - (x*(2*b^3*d^3 - 2*a^3*h^3 - 3*
b^3*c*d*e + 3*a^3*g*h*i - 6*a*b^2*d^2*h + 6*a^2*b*d*h^2 + 9*a*b^2*c*d*i + 3
*a*b^2*c*e*h + a*b^2*d*e*g - 9*a^2*b*c*h*i - 3*a^2*b*d*g*i - a^2*b*e*g*h))/
(16*a^3*b))*root(65536*a^7*b^7*z^4 - 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*
```

```
i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 - 3072*a^4*b^6*c*e*z^2
- 2048*a^6*b^4*h^2*z^2 - 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z - 768*a
^4*b^4*d*e*i*z + 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z - 1152*a^6*b^2*h
*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z - 128*a^4*b^4*e^2*h*z -
 1152*a^3*b^5*c^2*h*z + 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z + 1152*a^
2*b^6*c^2*d*z + 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*g
*i - 32*a^3*b^3*d*e*g*h + 96*a^2*b^4*c*d*e*h - 12*a^4*b^2*e*g^2*i + 144*a^4
*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i + 16*a^4*b^2*e*g*h^2 - 108*a^4*b^2*c*g*i^
2 - 108*a^2*b^4*c^2*e*i + 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 + 16*a^2
*b^4*d^2*e*g - 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e + 1
08*a^5*b*e*i^3 + 108*a*b^5*c^3*g - 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^2
 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 - 54*a^2*b^4*c^2*g^2 + 18*a^5*b*g
^2*i^2 + 12*a^3*b^3*e^3*i - 64*a^4*b^2*d*h^3 - 64*a^2*b^4*d^3*h + 12*a^3*b^
3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 - 81*a^6*i^4 - 81*
b^6*c^4 - a^4*b^2*g^4 - a^2*b^4*e^4, z, l), l, 1, 4) + (f/(4*b) + (x*(b*c +
 a*g))/(4*a*b) + (x^2*(b*d + a*h))/(4*a*b) + (x^3*(b*e + a*i))/(4*a*b))/(a
- b*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
[Out] Timed out
```

3.147 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a-bx^4\right)^2}\,dx$

Optimal. Leaf size=225

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{j\log\left(a\right.}{4}$$

Rubi [A] time = 0.31, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1858, 1876, 1167, 205, 208, 1248, 635, 260}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}\,(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{j\log\left(a-bx^4\right)}{4b^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2
,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(4*a*b*
(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(
1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqr
t[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a
*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) + (j*Log[a - b*x^4]
)/(4*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+147x^6+jx^7}{\left(a-bx^4\right)^2}\,dx = \frac{x\left(bc+ag+(bd+ah)x+(147a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(147a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(147a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(147a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(147a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(147a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)}$$

Mathematica [A] time = 0.25, size = 338, normalized size = 1.50

$$\frac{\frac{\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{a}be+a\sqrt{b}g-3b^{3/2}c\right)}{a^{7/4}}+\frac{\sqrt[4]{b}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(2a^{5/4}\sqrt[4]{b}h-3a^{3/2}i-2\sqrt[4]{a}b^{5/4}d+\sqrt{a}be-a\sqrt{b}g+3b^{3/2}c\right)}{a^{7/4}}+\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3a^{3/2}i-\sqrt{a}be-a\sqrt{b}g+3b^{3/2}c\right)}{a^{7/4}}+\frac{2\sqrt{b}(bd-ah)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{a^{3/2}}+\frac{4\left(a^2j+ab(f+x(g+x(h+ix)))+b^2x(c+x(d+ex))\right)}{a\left(a-bx^4\right)}+4j\log\left(a-bx^4\right)}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^2,x]
```

```
[Out] ((4*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a*(
a - b*x^4)) + (2*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/
2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(7/4) + (b^(1/4)*(-3*b^(3/2)*c - 2*a^(
1/4)*b^(5/4)*d - Sqrt[a]*b*e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2
```

```
)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(7/4) + (b^(1/4)*(3*b^(3/2)*c - 2*a^(1/4)*
b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*
Log[a^(1/4) + b^(1/4)*x])/a^(7/4) + (2*Sqrt[b]*(b*d - a*h)*Log[Sqrt[a] + Sq
rt[b]*x^2])/a^(3/2) + 4*j*Log[a - b*x^4])/(16*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{\left(a - bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a - b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a - b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.23, size = 610, normalized size = 2.71

$$-\frac{3}{32} i \left(\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^4} - \frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}} \log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{ab^4}\right) - \frac{3}{32} i \left(\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^4} + \frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}} \log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{ab^4}\right) - \frac{\sqrt{2}\left(3b^2c - abg - 2\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}} bd + 2\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}} ah + \sqrt{-ab}\, be\right) \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^3\right)^{\frac{3}{4}} a} - \frac{\sqrt{2}\left(3b^2c - abg + 2\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}} bd - 2\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}} ah - \sqrt{-ab}\, be\right) \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(-ab^3\right)^{\frac{3}{4}} a} - \frac{\sqrt{2}\left(3b^2c - abg - \sqrt{-ab}\, be\right) \log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^3\right)^{\frac{3}{4}} a} + \frac{\sqrt{2}\left(3b^2c - abg - \sqrt{-ab}\, be\right) \log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\left(-ab^3\right)^{\frac{3}{4}} a} + \frac{j \log\left(\left|bx^4-a\right|\right)}{4b^2} - \frac{(ai+be)x^3 + (bd+ah)x^2 + (bc+ag)x + \frac{abf+a^2j}{b}}{4\left(bx^4-a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="giac")
```

```
[Out] -3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^
(1/4))/(-a/b)^(1/4))/(a*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) - 3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^4) + sqrt(2)*
(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) - 1/
16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + 2*sqrt(2)*(-a*
b^3)^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(
1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g + 2*
sqrt(2)*(-a*b^3)^(1/4)*b*d - 2*sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3
/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*
x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - a
*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*
b^3)^(3/4)*a) + 1/4*j*log(abs(b*x^4 - a))/b^2 - 1/4*((a*i + b*e)*x^3 + (b*d
 + a*h)*x^2 + (b*c + a*g)*x + (a*b*f + a^2*j)/b)/((b*x^4 - a)*a*b)
```

maple [B] time = 0.06, size = 431, normalized size = 1.92

$$-\frac{d \ln\left(\frac{\sqrt{ab}\, x^2-a}{-\sqrt{ab}\, x^2-a}\right)}{8\sqrt{ab}\, a} + \frac{h \ln\left(\frac{\sqrt{ab}\, x^2-a}{-\sqrt{ab}\, x^2-a}\right)}{8\sqrt{ab}\, b} - \frac{e \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}} ab} + \frac{e \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}} ab} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} g \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} g \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16ab} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}} c \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}} c \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2} + \frac{3i \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}} b^2} - \frac{3i \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}} b^2} + \frac{j \ln\left(b x^4 - a\right)}{4b^2} + \frac{-\frac{(ai+be)x^3}{4ab} - \frac{(ah+bd)x^2}{4ab} - \frac{(ag+bc)x}{4ab} - \frac{aj+bf}{4b^2}}{b x^4 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

[Out] (-1/4*(a*i+b*e)/a/b*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x-1/4*(a*j+b*f)/b^2)/(b*x^4-a)-1/8*(a/b)^(1/4)/a/b*g*arctan(1/(a/b)^(1/4)*x)+3/8*(a/b)^(1/4)/a^2*c*arctan(1/(a/b)^(1/4)*x)-1/16*(a/b)^(1/4)/a/b*g*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+3/16*(a/b)^(1/4)/a^2*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/8/(a*b)^(1/2)/b*h*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-1/8/(a*b)^(1/2)/a*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))+3/8/(a/b)^(1/4)/b^2*i*arctan(1/(a/b)^(1/4)*x)-1/8/(a/b)^(1/4)/a/b*e*arctan(1/(a/b)^(1/4)*x)-3/16/(a/b)^(1/4)/b^2*i*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/16/(a/b)^(1/4)/a/b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/4/b^2*j*ln(b*x^4-a)

maxima [A] time = 3.15, size = 299, normalized size = 1.33

$$-\frac{(b^2e+abi)x^3+abf+a^2j+(b^2d+abh)x^2+(b^2c+abg)x}{4(ab^3x^4-a^2b^2)}+\frac{\frac{2\left(3b^{\frac{3}{2}}c-\sqrt{a}be-a\sqrt{b}g+3a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\left(b^{\frac{3}{2}}d-a\sqrt{b}h+2a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b}-\frac{2\left(b^{\frac{3}{2}}d-a\sqrt{b}h-2a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b}-\frac{\left(3b^{\frac{3}{2}}c+\sqrt{a}be-a\sqrt{b}g-3a^{\frac{3}{2}}i\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")

[Out] -1/4*((b^2*e + a*b*i)*x^3 + a*b*f + a^2*j + (b^2*d + a*b*h)*x^2 + (b^2*c + a*b*g)*x)/(a*b^3*x^4 - a^2*b^2) + 1/16*(2*(3*b^(3/2)*c - sqrt(a)*b*e - a*sqrt(b)*g + 3*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*(b^(3/2)*d - a*sqrt(b)*h + 2*a^(3/2)*j)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - 2*(b^(3/2)*d - a*sqrt(b)*h - 2*a^(3/2)*j)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (3*b^(3/2)*c + sqrt(a)*b*e - a*sqrt(b)*g - 3*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a*b)

mupad [B] time = 5.91, size = 3943, normalized size = 17.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2,x)

[Out] ((b*f + a*j)/(4*b^2) + (x*(b*c + a*g))/(4*a*b) + (x^2*(b*d + a*h))/(4*a*b) + (x^3*(b*e + a*i))/(4*a*b))/(a - b*x^4) + symsum(log((27*a^4*i^3 - a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e + 16*a^4*g*j^2 - 12*a^2*b^2*c*h^2 + a^2*b^2*e*g^2 + 9*a^2*b^2*e^2*i - 48*a^4*h*i*j + 4*a*b^3*d^2*g - 27*a*b^3*c^2*i - 48*a^3*b*c*j^2 - 27*a^3*b*e*i^2 + 4*a^3*b*g*h^2 - 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 16*a^2*b^2*d*e*j - 8*a^2*b^2*d*g*h + 24*a*b^3*c*d*h - 6*a*b^3*c*e*g + 48*a^3*b*d*i*j + 16*a^3*b*e*h*j)/(64*a^3*b^2) - root(65536*a^7*b^8*z^4 - 65536*a^7*b^6*j*z^3 - 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 - 3072*a^4*b^7*c*e*z^2 + 24576*a^7*b^4*j^2*z^2 - 2048*a^6*b^5*h^2*z^2 - 2048*a^4*b^7*d^2*z^2 + 1536*a^6*b^3*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*z - 512*a^5*b^4*e*g*j*z + 1536*a^4*b^5*c*e*j*z - 768*a^4*b^5*d*e*i*z + 768*a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z + 1024*a^6*b^3*h^2*j*z - 1152*a^6*b^3*h*i^2*z - 128*a^5*b^4*g^2*h*z + 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*z - 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z + 128*a^4*b^5*d*g^2*z + 128*a^3*b^6*d*e^2*z + 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e*h*i*j + 192*a^4*b^3*d*e*i*j - 192*a^4*b^3*c*g*h*j + 96*a^4*b^3*d*g*h*i - 288*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*d*e*g*h + 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 288*a^5*b^2*d*i^2*j + 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j - 32*a^4*b^3*d*g^2*j - 12*a^4*b^3*e*g^2*i + 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i + 16*a^4*b^3*e*g*h^2 - 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j - 192*a^4*b^3*c*e*j^2 - 288*a^2*b^5*c^2*d*j - 108*a^2*b^5*c^2*e*i + 144*a^2*b^5*c*d^2*i - 48*a^3*b^4*c*

```
e*h^2 + 16*a^2*b^5*d^2*e*g - 12*a^2*b^5*c*e^2*g + 288*a^6*b*h*i^2*j - 192*a
^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 - 12
8*a^4*b^3*d^2*j^2 - 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^4*d
^2*h^2 + 2*a^3*b^4*e^2*g^2 - 54*a^2*b^5*c^2*g^2 - 128*a^6*b*h^2*j^2 + 108*a
^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i - 64*a^4*b^3*d*h^3 - 64*a^2*b^5*d^3*h + 12*
a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 - 81*a^6*b*i^4 + 16*a*b^6
*d^4 + 256*a^7*j^4 - 81*b^7*c^4 - a^4*b^3*g^4 - a^2*b^5*e^4, z, m)*(root(65
536*a^7*b^8*z^4 - 65536*a^7*b^6*j*z^3 - 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6
*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 - 3072*a^4*b^7*c*e*z
^2 + 24576*a^7*b^4*j^2*z^2 - 2048*a^6*b^5*h^2*z^2 - 2048*a^4*b^7*d^2*z^2 +
1536*a^6*b^3*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^
5*b^4*e*h*i*z - 512*a^5*b^4*e*g*j*z + 1536*a^4*b^5*c*e*j*z - 768*a^4*b^5*d*
e*i*z + 768*a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z + 1024*a^6*b^3*h^2*j*z -
1152*a^6*b^3*h*i^2*z - 128*a^5*b^4*g^2*h*z + 1024*a^4*b^5*d^2*j*z + 1152*a^
5*b^4*d*i^2*z - 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z + 128*a^4*b^5*d*
g^2*z + 128*a^3*b^6*d*e^2*z + 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 1
92*a^5*b^2*e*h*i*j + 192*a^4*b^3*d*e*i*j - 192*a^4*b^3*c*g*h*j + 96*a^4*b^3
*d*g*h*i - 288*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i -
 32*a^3*b^4*d*e*g*h + 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*
g*h^2*i - 288*a^5*b^2*d*i^2*j + 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 +
256*a^5*b^2*d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j - 32*a^4*b^3
*d*g^2*j - 12*a^4*b^3*e*g^2*i + 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i +
16*a^4*b^3*e*g*h^2 - 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j - 192*a^4*b^3
*c*e*j^2 - 288*a^2*b^5*c^2*d*j - 108*a^2*b^5*c^2*e*i + 144*a^2*b^5*c*d^2*i
- 48*a^3*b^4*c*e*h^2 + 16*a^2*b^5*d^2*e*g - 12*a^2*b^5*c*e^2*g + 288*a^6*b*
h*i^2*j - 192*a^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b
^2*g^2*i^2 - 128*a^4*b^3*d^2*j^2 - 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2
 + 96*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 - 54*a^2*b^5*c^2*g^2 - 128*a^6*b*
h^2*j^2 + 108*a^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i - 64*a^4*b^3*d*h^3 - 64*a^2*
b^5*d^3*h + 12*a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 - 81*a^6*b
*i^4 + 16*a*b^6*d^4 + 256*a^7*j^4 - 81*b^7*c^4 - a^4*b^3*g^4 - a^2*b^5*e^4,
 z, m)*((768*a^3*b^5*c - 256*a^4*b^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^5*d -
128*a^4*b^4*h))/(16*a^3*b^2)) - (64*a^2*b^4*d*e + 384*a^3*b^3*c*j - 192*a^3
*b^3*d*i - 64*a^3*b^3*e*h - 128*a^4*b^2*g*j + 192*a^4*b^2*h*i)/(64*a^3*b^2)
 + (x*(36*a*b^5*c^2 + 4*a^2*b^4*e^2 + 4*a^3*b^3*g^2 + 36*a^4*b^2*i^2 - 24*a
^2*b^4*c*g + 64*a^3*b^3*d*j - 24*a^3*b^3*e*i - 64*a^4*b^2*h*j))/(16*a^3*b^2
)) + (x*(2*a^3*b*h^3 - 2*b^4*d^3 - 8*a^4*h*j^2 + 9*a^4*i^2*j - 6*a^2*b^2*d*
h^2 + a^2*b^2*e^2*j + 3*b^4*c*d*e + 6*a*b^3*d^2*h + 9*a*b^3*c^2*j + 8*a^3*b
*d*j^2 + a^3*b*g^2*j - 6*a^2*b^2*c*g*j + 9*a^2*b^2*c*h*i + 3*a^2*b^2*d*g*i
+ a^2*b^2*e*g*h - 9*a*b^3*c*d*i - 3*a*b^3*c*e*h - a*b^3*d*e*g - 6*a^3*b*e*i
*j - 3*a^3*b*g*h*i))/(16*a^3*b^2))*root(65536*a^7*b^8*z^4 - 65536*a^7*b^6*j
*z^3 - 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 +
 1024*a^5*b^6*e*g*z^2 - 3072*a^4*b^7*c*e*z^2 + 24576*a^7*b^4*j^2*z^2 - 2048
*a^6*b^5*h^2*z^2 - 2048*a^4*b^7*d^2*z^2 + 1536*a^6*b^3*g*i*j*z - 4608*a^5*b
^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*z - 512*a^5*b^4*e*g*j
*z + 1536*a^4*b^5*c*e*j*z - 768*a^4*b^5*d*e*i*z + 768*a^4*b^5*c*g*h*z - 768
*a^3*b^6*c*d*g*z + 1024*a^6*b^3*h^2*j*z - 1152*a^6*b^3*h*i^2*z - 128*a^5*b^
4*g^2*h*z + 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*z - 128*a^4*b^5*e^2*h
*z - 1152*a^3*b^6*c^2*h*z + 128*a^4*b^5*d*g^2*z + 128*a^3*b^6*d*e^2*z + 115
2*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e*h*i*j + 192*a^4*b^3*
d*e*i*j - 192*a^4*b^3*c*g*h*j + 96*a^4*b^3*d*g*h*i - 288*a^3*b^4*c*d*h*i +
192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*d*e*g*h + 96*a^2*b^5*
c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 288*a^5*b^2*d*i^2*j + 3
2*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*d*h*j^2 + 64*a^5*b^2*
e*g*j^2 + 288*a^3*b^4*c^2*h*j - 32*a^4*b^3*d*g^2*j - 12*a^4*b^3*e*g^2*i + 1
44*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i + 16*a^4*b^3*e*g*h^2 - 108*a^4*b^3*
c*g*i^2 - 32*a^3*b^4*d*e^2*j - 192*a^4*b^3*c*e*j^2 - 288*a^2*b^5*c^2*d*j -
108*a^2*b^5*c^2*e*i + 144*a^2*b^5*c*d^2*i - 48*a^3*b^4*c*e*h^2 + 16*a^2*b^5
*d^2*e*g - 12*a^2*b^5*c*e^2*g + 288*a^6*b*h*i^2*j - 192*a^6*b*g*i*j^2 - 48*
```

```
a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 - 128*a^4*b^3*d^2*j^2
- 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^4*d^2*h^2 + 2*a^3*b^4
*e^2*g^2 - 54*a^2*b^5*c^2*g^2 - 128*a^6*b*h^2*j^2 + 108*a^5*b^2*e*i^3 + 12*
a^3*b^4*e^3*i - 64*a^4*b^3*d*h^3 - 64*a^2*b^5*d^3*h + 12*a^3*b^4*c*g^3 + 18
*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 - 81*a^6*b*i^4 + 16*a*b^6*d^4 + 256*a^7*j^4
 - 81*b^7*c^4 - a^4*b^3*g^4 - a^2*b^5*e^4, z, m), m, 1, 4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,
x)

[Out] Timed out
```

3.148 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=353

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+\right.}{16\sqrt{2}a^{7/4}b^{5/4}}$$

Rubi [A] time = 0.34, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1858, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{(ah+bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (
(b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((3*b*c +
Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[
b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqr
t[2]*a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^2}\,dx = \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+ag)-2b(bd+ah)x-b^2ex^2}{a+bx^4}\,dx}{4ab^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \left(-\frac{2b(bd+ah)x}{a+bx^4} + \frac{-b(3bc+ag)-b^2ex}{a+bx^4}\right.}{4ab^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+ag)-b^2ex^2}{a+bx^4}\,dx}{4ab^2} + \frac{(bd+a}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{\left(3bc-\sqrt{a}\sqrt{b}\,e+ag\right)\int \frac{\sqrt{a}\sqrt{b}-}{a+bx}}{8a^{3/2}b^{3/2}}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{(3bc}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{(3bc}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{(3bc}{}$$

Mathematica [A] time = 0.30, size = 359, normalized size = 1.02

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(4a^{5/4}h+\sqrt{2}\sqrt{a}\,b^{3/4}e+4\sqrt[4]{a}\,bd+\sqrt{2}\,a\sqrt[4]{b}\,g+3\sqrt{2}\,b^{5/4}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(-4a^{5/4}h+\sqrt{2}\sqrt{a}\,b^{3/4}e-4\sqrt[4]{a}\,bd+\sqrt{2}\,a\sqrt[4]{b}\,g+3\sqrt{2}\,b^{5/4}c\right)-\frac{8a^{3/4}\sqrt{b}\,(a(f+x(g+hx))-bx(c+x(d+ex)))}{a+bx^4}+\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(\sqrt{a}\sqrt{b}\,e-ag-3bc\right)+\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(-\sqrt{a}\sqrt{b}\,e+ag+3bc\right)}{32a^{7/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(a +
 b*x^4) - 2*(3*Sqrt[2]*b^(5/4)*c + 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*
e + Sqrt[2]*a*b^(1/4)*g + 4*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)] + 2*(3*Sqrt[2]*b^(5/4)*c - 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e +
Sqrt[2]*a*b^(1/4)*g - 4*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
+ Sqrt[2]*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*b*c - Sqrt[a]*Sqrt[b]
*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/
4)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,
x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,
 x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 398, normalized size = 1.13

$$\frac{bx^3e+bdx^2-ahx^2+bcx-agx-af}{4\left(bx^4+a\right)ab}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d+2\sqrt{2}\sqrt{ab}\,abh+3\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{1}{4}}abg+\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d+2\sqrt{2}\sqrt{ab}\,abh+3\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{1}{4}}abg+\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3}+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{1}{4}}abg-\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b^3}-\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{1}{4}}abg-\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 - a*h*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) +
 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a
*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqr
t(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c +
(a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a
/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (
a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b
*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3
)
```

maple [A] time = 0.06, size = 515, normalized size = 1.46

$$\frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{4\sqrt{ab}\,a}+\frac{h\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{4\sqrt{ab}\,b}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16a^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16a^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32a^2}+\frac{\frac{e\,x^3}{4a}-\frac{(ah-bd)x^2}{4ab}-\frac{f}{4b}-\frac{(ag-bc)x}{4ab}}{b\,x^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (1/4/a*e*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4/b*f)/(b*x^4+a)+1
/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*g+3/16*(a/b)^(1
/4)*2^(1/2)/a^2*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/16/b/a*(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*g+3/16*(a/b)^(1/4)*2^(1/2)/a^2*c*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)+1/32*(a/b)^(1/4)*2^(1/2)/a/b*g*ln((x^2+(a/b)^(1/4
)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/32*(a/b
)^(1/4)*2^(1/2)/a^2*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)
^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4/b/(a*b)^(1/2)*arctan((1/a*b)^(1/2)*x^2)*
h+1/4/(a*b)^(1/2)/a*d*arctan((1/a*b)^(1/2)*x^2)+1/32/(a/b)^(1/4)*2^(1/2)/a/
b*e*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2)))+1/16/(a/b)^(1/4)*2^(1/2)/a/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x+1)
+1/16/(a/b)^(1/4)*2^(1/2)/a/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.19, size = 374, normalized size = 1.06

$$\frac{bex^3+(bd-ah)x^2-af+\left(bc-ag\right)x}{4\left(ab^2x^4+a^2b\right)}+\frac{\frac{\sqrt{2}\left(3b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g-4\sqrt{a}b^{\frac{3}{2}}d-4a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+4\sqrt{a}b^{\frac{3}{2}}d+4a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*(b*e*x^3 + (b*d - a*h)*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 + a^2*b) +
 1/32*(sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 +
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*b^(3/2)
*c - sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^
(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g - 4*sqrt(a)*b^(3/2)*d - 4*a^(3/
2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sq
rt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)
*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g
+ 4*sqrt(a)*b^(3/2)*d + 4*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*
x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*s
qrt(b))*b^(3/4)))/(a*b)
```

mupad [B] time = 5.58, size = 1623, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((12*b^3*c*d^2 - a*b^2*e^3 - 9*b^3*c^2*e + 4*a^3*g*h^2 + 4*a*b^2*
d^2*g + 12*a^2*b*c*h^2 - a^2*b*e*g^2 + 24*a*b^2*c*d*h - 6*a*b^2*c*e*g + 8*a
^2*b*d*g*h)/(64*a^3*b) - root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 10
24*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*a^6*b^3*h^2*z^2 + 2048*a^4
*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2*
h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z - 128*a^4*b^3*d*g^2*z + 12
8*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h - 96*a^2*b^3*
c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2*e*g + 12*a^2*b^3*c*e^2*g - 16
*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*a
^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2*b^3*c^2*g^2 + 64*a^2*b^3*d^3*h
+ 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 + 81*b^5*
c^4 + a^2*b^3*e^4 + a^4*b*g^4, z, k)*(root(65536*a^7*b^6*z^4 + 4096*a^5*b^4
*d*h*z^2 + 1024*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*a^6*b^3*h^2*z
^2 + 2048*a^4*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128
*a^5*b^2*g^2*h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z - 128*a^4*b^3
*d*g^2*z + 128*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h
- 96*a^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2*e*g + 12*a^2*b^3
*c*e^2*g - 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b*d*h^3 + 108*a*b^4
*c^3*g + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2*b^3*c^2*g^2 + 64*a
^2*b^3*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*
h^4 + 81*b^5*c^4 + a^2*b^3*e^4 + a^4*b*g^4, z, k)*((768*a^3*b^4*c + 256*a^4
*b^3*g)/(64*a^3*b) - (x*(128*a^3*b^4*d + 128*a^4*b^3*h))/(16*a^3*b)) + (64*
a^2*b^3*d*e + 64*a^3*b^2*e*h)/(64*a^3*b) + (x*(36*a*b^4*c^2 - 4*a^2*b^3*e^2
 + 4*a^3*b^2*g^2 + 24*a^2*b^3*c*g))/(16*a^3*b)) + (x*(2*b^3*d^3 + 2*a^3*h^3
 - 3*b^3*c*d*e + 6*a*b^2*d^2*h + 6*a^2*b*d*h^2 - 3*a*b^2*c*e*h - a*b^2*d*e*
g - a^2*b*e*g*h))/(16*a^3*b))*root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2
 + 1024*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*a^6*b^3*h^2*z^2 + 204
8*a^4*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2
*g^2*h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z - 128*a^4*b^3*d*g^2*z
 + 128*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h - 96*a^2
*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2*e*g + 12*a^2*b^3*c*e^2*g
 - 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g +
 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2*b^3*c^2*g^2 + 64*a^2*b^3*d
^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 + 81
*b^5*c^4 + a^2*b^3*e^4 + a^4*b*g^4, z, k), k, 1, 4) + ((e*x^3)/(4*a) - f/(4
*b) + (x*(b*c - a*g))/(4*a*b) + (x^2*(b*d - a*h))/(4*a*b))/(a + b*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.149 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a+bx^4\right)^2} \, dx$

Optimal. Leaf size=395

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+\right.}{16\sqrt{2}a^{7/4}b^{7/4}}$$

Rubi [A] time = 0.49, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1858, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{(ah+bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(4*a*b*(a + b*x
^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((
Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a
]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (
(Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+149x^6}{\left(a+bx^4\right)^2}\,dx = \frac{x\left(bc-ag+(bd-ah)x-(149a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \int \frac{-b(3bc+ag)-\cdots}{\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(149a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \int\left(-\frac{2b(bd+ah)}{a+bx^4}\cdots\right.$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(149a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+ag)-\cdots}{a+\cdots}}{4a\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(149a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\left(447a+be-\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(149a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan\cdots}{4a^{3/2}\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(149a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan\cdots}{4a^{3/2}\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(149a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan\cdots}{4a^{3/2}\cdots}$$

Mathematica [A] time = 0.47, size = 415, normalized size = 1.05

$$\frac{-\frac{8a^{3/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{a+bx^4}-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i+4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+3\sqrt{2}b^{3/2}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i-4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+3\sqrt{2}b^{3/2}c\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3a^{3/2}i+\sqrt{a}be-a\sqrt{b}g-3b^{3/2}c\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-3a^{3/2}i-\sqrt{a}be+a\sqrt{b}g+3b^{3/2}c\right)}{32a^{7/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x
]
```

```
[Out] ((-8*a^(3/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x))
)))/(a + b*x^4) - 2*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sq
rt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b^(3/2)*c - 4*a^(1
/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g - 4*a^(5/4)*b^(1/
4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[
2]*(-3*b^(3/2)*c + Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b^(3/2)*c - Sqrt[a]*b*
e + a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(32*a^(7/4)*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b
*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b
*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [A] time = 0.60, size = 589, normalized size = 1.49

$$\frac{3}{32}\,i\left(\frac{2\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^{4}}-\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{ab^{4}}\right)+\frac{3}{32}\,i\left(\frac{2\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^{4}}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{ab^{4}}\right)-\frac{aix^{3}-bx^{3}e-bdx^{2}+ahx^{2}-bcx+agx+af}{4\left(bx^{4}+a\right)ab}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^{2}d+2\sqrt{2}\sqrt{ab}abh+3\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+\left(ab^{3}\right)^{\frac{1}{4}}abg+\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{3}}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^{2}d+2\sqrt{2}\sqrt{ab}abh+3\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+\left(ab^{3}\right)^{\frac{1}{4}}abg+\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{3}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+\left(ab^{3}\right)^{\frac{1}{4}}abg-\left(ab^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{3}}-\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+\left(ab^{3}\right)^{\frac{1}{4}}abg-\left(ab^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="g
iac")
```

```
[Out] 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(a*b^4)) + 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + sqrt(2)*(a*b^3)^(3
/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)) - 1/4*(a*i*x^3 -
b*x^3*e - b*d*x^2 + a*h*x^2 - b*c*x + a*g*x + a*f)/((b*x^4 + a)*a*b) + 1/16
*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)
^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*
sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^
3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3
)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b
))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g -
(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

maple [B] time = 0.05, size = 654, normalized size = 1.66

$$\frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{4\sqrt{ab}\,a}+\frac{h\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{4\sqrt{ab}\,b}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\sqrt{2}\,e\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32a^{2}}+\frac{3\sqrt{2}\,i\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{3\sqrt{2}\,i\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{3\sqrt{2}\,i\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{-\frac{(ai-be)x^{3}}{4ab}-\frac{(ah-bd)x^{2}}{4ab}-\frac{f}{4b}-\frac{(ag-bc)x}{4ab}}{b\,x^{4}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4/b*f)/
(b*x^4+a)+1/16*(a/b)^(1/4)*2^(1/2)/a/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+3/
16*(a/b)^(1/4)*2^(1/2)/a^2*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/32*(a/b)^(1/
4)*2^(1/2)/a/b*g*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4
)*2^(1/2)*x+(a/b)^(1/2)))+3/32*(a/b)^(1/4)*2^(1/2)/a^2*c*ln((x^2+(a/b)^(1/4
)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/16*(a/b
)^(1/4)*2^(1/2)/a/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/16*(a/b)^(1/4)*2^(1
/2)/a^2*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/(a*b)^(1/2)/b*h*arctan((1/a*b
)^(1/2)*x^2)+1/4/(a*b)^(1/2)/a*d*arctan((1/a*b)^(1/2)*x^2)+3/32/b^2/(a/b)^(
1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^
(1/2)*x+(a/b)^(1/2)))*i+1/32/(a/b)^(1/4)*2^(1/2)/a/b*e*ln((x^2-(a/b)^(1/4)*
2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/16/b^2/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*i+1/16/(a/b)^(1/4)*2^(1/2
)/a/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+3/16/b^2/(a/b)^(1/4)*2^(1/2)*arctan
```

```
(2^(1/2)/(a/b)^(1/4)*x+1)*i+1/16/(a/b)^(1/4)*2^(1/2)/a/b*e*arctan(2^(1/2)/(
a/b)^(1/4)*x+1)
```

maxima [A] time = 3.19, size = 416, normalized size = 1.05

$$\frac{(be-ai)x^3+(bd-ah)x^2-af+\left(bc-ag\right)x}{4\left(ab^2x^4+a^2b\right)}+\frac{\frac{\sqrt{2}\left(3b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g-3a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g-3a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+3\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}i-4\sqrt{a}b^{\frac{3}{2}}d-4a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+3\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}i+4\sqrt{a}b^{\frac{3}{2}}d+4a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="m
axima")
```

```
[Out] 1/4*((b*e - a*i)*x^3 + (b*d - a*h)*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 +
a^2*b) + 1/32*(sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g - 3*a^(3/2)
*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)
) - sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqr
t(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sq
rt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/
4)*g + 3*sqrt(2)*a^(7/4)*b^(1/4)*i - 4*sqrt(a)*b^(3/2)*d - 4*a^(3/2)*sqrt(b
)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a
)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*
b^(7/4)*c + sqrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqrt(
2)*a^(7/4)*b^(1/4)*i + 4*sqrt(a)*b^(3/2)*d + 4*a^(3/2)*sqrt(b)*h)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a*b)
```

mupad [B] time = 5.70, size = 2605, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log(- root(65536*a^7*b^7*z^4 + 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c
*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 + 3072*a^4*b^6*c*e*z^2
 + 2048*a^6*b^4*h^2*z^2 + 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z + 768*
a^4*b^4*d*e*i*z - 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z + 1152*a^6*b^2*
h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z + 128*a^4*b^4*e^2*h*z
- 1152*a^3*b^5*c^2*h*z - 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z - 1152*a
^2*b^6*c^2*d*z - 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*
g*i - 32*a^3*b^3*d*e*g*h - 96*a^2*b^4*c*d*e*h + 12*a^4*b^2*e*g^2*i - 144*a^
4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i - 16*a^4*b^2*e*g*h^2 + 108*a^4*b^2*c*g*i
^2 + 108*a^2*b^4*c^2*e*i - 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 - 16*a^
2*b^4*d^2*e*g + 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e +
108*a^5*b*e*i^3 + 108*a*b^5*c^3*g + 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^
2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 54*a^2*b^4*c^2*g^2 + 18*a^5*b*
g^2*i^2 + 12*a^3*b^3*e^3*i + 64*a^4*b^2*d*h^3 + 64*a^2*b^4*d^3*h + 12*a^3*b
^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 + 81*a^6*i^4 + 81
*b^6*c^4 + a^4*b^2*g^4 + a^2*b^4*e^4, z, l)*(root(65536*a^7*b^7*z^4 + 3072*
a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^
5*e*g*z^2 + 3072*a^4*b^6*c*e*z^2 + 2048*a^6*b^4*h^2*z^2 + 2048*a^4*b^6*d^2*
z^2 + 768*a^5*b^3*e*h*i*z + 768*a^4*b^4*d*e*i*z - 768*a^4*b^4*c*g*h*z - 768
*a^3*b^5*c*d*g*z + 1152*a^6*b^2*h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^
3*d*i^2*z + 128*a^4*b^4*e^2*h*z - 1152*a^3*b^5*c^2*h*z - 128*a^4*b^4*d*g^2*
z + 128*a^3*b^5*d*e^2*z - 1152*a^2*b^6*c^2*d*z - 96*a^4*b^2*d*g*h*i - 288*a
^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*g*i - 32*a^3*b^3*d*e*g*h - 96*a^2*b^4*c*d*e
*h + 12*a^4*b^2*e*g^2*i - 144*a^4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i - 16*a^4
*b^2*e*g*h^2 + 108*a^4*b^2*c*g*i^2 + 108*a^2*b^4*c^2*e*i - 144*a^2*b^4*c*d^
2*i - 48*a^3*b^3*c*e*h^2 - 16*a^2*b^4*d^2*e*g + 12*a^2*b^4*c*e^2*g - 48*a^5
*b*g*h^2*i - 48*a*b^5*c*d^2*e + 108*a^5*b*e*i^3 + 108*a*b^5*c^3*g + 54*a^4*
b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2
```

```
+ 54*a^2*b^4*c^2*g^2 + 18*a^5*b*g^2*i^2 + 12*a^3*b^3*e^3*i + 64*a^4*b^2*d*h
^3 + 64*a^2*b^4*d^3*h + 12*a^3*b^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4
+ 16*a*b^5*d^4 + 81*a^6*i^4 + 81*b^6*c^4 + a^4*b^2*g^4 + a^2*b^4*e^4, z, l)
*((768*a^3*b^5*c + 256*a^4*b^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^4*d + 128*a^
4*b^3*h))/(16*a^3*b)) + (64*a^2*b^4*d*e + 192*a^3*b^3*d*i + 64*a^3*b^3*e*h
+ 192*a^4*b^2*h*i)/(64*a^3*b^2) + (x*(36*a*b^4*c^2 - 36*a^4*b*i^2 - 4*a^2*b
^3*e^2 + 4*a^3*b^2*g^2 + 24*a^2*b^3*c*g - 24*a^3*b^2*e*i))/(16*a^3*b)) - (2
7*a^4*i^3 + a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e - 12*a^2*b^2*c*h^2 + a^2
*b^2*e*g^2 + 9*a^2*b^2*e^2*i - 4*a*b^3*d^2*g + 27*a*b^3*c^2*i + 27*a^3*b*e*
i^2 - 4*a^3*b*g*h^2 + 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 8*a^2*b^2*d*g*h -
24*a*b^3*c*d*h + 6*a*b^3*c*e*g)/(64*a^3*b^2) - (x*(3*b^3*c*d*e - 2*a^3*h^3
- 2*b^3*d^3 + 3*a^3*g*h*i - 6*a*b^2*d^2*h - 6*a^2*b*d*h^2 + 9*a*b^2*c*d*i +
 3*a*b^2*c*e*h + a*b^2*d*e*g + 9*a^2*b*c*h*i + 3*a^2*b*d*g*i + a^2*b*e*g*h)
)/(16*a^3*b))*root(65536*a^7*b^7*z^4 + 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*
c*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 + 3072*a^4*b^6*c*e*z^
2 + 2048*a^6*b^4*h^2*z^2 + 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z + 768
*a^4*b^4*d*e*i*z - 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z + 1152*a^6*b^2
*h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z + 128*a^4*b^4*e^2*h*z
 - 1152*a^3*b^5*c^2*h*z - 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z - 1152*
a^2*b^6*c^2*d*z - 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e
*g*i - 32*a^3*b^3*d*e*g*h - 96*a^2*b^4*c*d*e*h + 12*a^4*b^2*e*g^2*i - 144*a
^4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i - 16*a^4*b^2*e*g*h^2 + 108*a^4*b^2*c*g*
i^2 + 108*a^2*b^4*c^2*e*i - 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 - 16*a
^2*b^4*d^2*e*g + 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e +
 108*a^5*b*e*i^3 + 108*a*b^5*c^3*g + 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i
^2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 54*a^2*b^4*c^2*g^2 + 18*a^5*b
*g^2*i^2 + 12*a^3*b^3*e^3*i + 64*a^4*b^2*d*h^3 + 64*a^2*b^4*d^3*h + 12*a^3*
b^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 + 81*a^6*i^4 + 8
1*b^6*c^4 + a^4*b^2*g^4 + a^2*b^4*e^4, z, l), l, 1, 4) + ((x*(b*c - a*g))/(
4*a*b) - f/(4*b) + (x^2*(b*d - a*h))/(4*a*b) + (x^3*(b*e - a*i))/(4*a*b))/(
a + b*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
[Out] Timed out
```

3.150 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=417

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+\right.}{16\sqrt{2}a^{7/4}b^{7/4}}$$

Rubi [A] time = 0.54, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1858, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{(ah+bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{j\log\left(a+bx^4\right)}{4b^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2
,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(4*a*b*
(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/
2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g)
+ Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7
/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (j*Log[
a + b*x^4])/(4*b^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+150x^6+jx^7}{\left(a+bx^4\right)^2}\,dx = \frac{x\left(bc-ag+(bd-ah)x-(150a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} - \cdots$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(150a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} - \cdots$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(150a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} - \cdots$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(150a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} - \cdots$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(150a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} + \cdots$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(150a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} + \cdots$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(150a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} + \cdots$$

Mathematica [A] time = 0.44, size = 460, normalized size = 1.10

$$\frac{-\frac{2\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i+4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+3\sqrt{2}b^{3/2}c\right)}{a^{7/4}}+\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i-4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+3\sqrt{2}b^{3/2}c\right)}{a^{7/4}}+\frac{\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3a^{3/2}i+\sqrt{a}be-a\sqrt{b}g-3b^{3/2}c\right)}{a^{7/4}}+\frac{\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-3a^{3/2}i-\sqrt{a}be+a\sqrt{b}g+3b^{3/2}c\right)}{a^{7/4}}+\frac{8\left(a^2j-ab(f+x(g+x(h+ix)))+b^2x(c+x(d+ex))\right)}{a\left(a+bx^4\right)}+8j\log\left(a+bx^4\right)}{32b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^2,x]
```

```
[Out] ((8*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a*(
a + b*x^4)) - (2*b^(1/4)*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[
2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3
/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*b^(1/4)*(3*Sqr
t[2]*b^(3/2)*c - 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt
[b]*g - 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*b^(1/4)*(-3*b^(3/2)*c + Sqrt[a]*b*e - a
*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/a^(7/4) + (Sqrt[2]*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g
- 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7
/4) + 8*j*Log[a + b*x^4])/(32*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a + b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 617, normalized size = 1.48

$$\frac{3}{32} i \left(\frac{2 \sqrt{2} \left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2 x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^{4}} - \frac{\sqrt{2} \left(ab^{3}\right)^{\frac{3}{4}} \log\left(x^{2} + \sqrt{2} x \left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{ab^{4}}\right) + \frac{3}{32} i \left(\frac{2 \sqrt{2} \left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2 x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^{4}} + \frac{\sqrt{2} \left(ab^{3}\right)^{\frac{3}{4}} \log\left(x^{2} - \sqrt{2} x \left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{ab^{4}}\right) + \frac{j \log\left(\left|bx^{4} + a\right|\right)}{4 b^{2}} - \frac{(ai - be)x^{3} - (bd - ah)x^{2} - (bc - ag)x + \frac{abf - a^{2} j}{b}}{4\left(bx^{4} + a\right)ab} + \frac{\sqrt{2}\left(2 \sqrt{2} \sqrt{ab} b^{2} d + 2 \sqrt{2} \sqrt{ab} abh + 3 \left(ab^{3}\right)^{\frac{1}{4}} b^{2} c + \left(ab^{3}\right)^{\frac{1}{4}} abg + \left(ab^{3}\right)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2}\left(2 x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 a^{2} b^{3}} + \frac{\sqrt{2}\left(2 \sqrt{2} \sqrt{ab} b^{2} d + 2 \sqrt{2} \sqrt{ab} abh + 3 \left(ab^{3}\right)^{\frac{1}{4}} b^{2} c + \left(ab^{3}\right)^{\frac{1}{4}} abg + \left(ab^{3}\right)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2}\left(2 x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16 a^{2} b^{3}} + \frac{\sqrt{2}\left(3 \left(ab^{3}\right)^{\frac{1}{4}} b^{2} c + \left(ab^{3}\right)^{\frac{1}{4}} abg - \left(ab^{3}\right)^{\frac{3}{4}} e\right) \log\left(x^{2} + \sqrt{2} x \left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32 a^{2} b^{3}} - \frac{\sqrt{2}\left(3 \left(ab^{3}\right)^{\frac{1}{4}} b^{2} c + \left(ab^{3}\right)^{\frac{1}{4}} abg - \left(ab^{3}\right)^{\frac{3}{4}} e\right) \log\left(x^{2} - \sqrt{2} x \left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32 a^{2} b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="giac")
```

```
[Out] 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(a*b^4)) + 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + sqrt(2)*(a*b^3)^(3
/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)) + 1/4*j*log(abs(b
*x^4 + a))/b^2 - 1/4*((a*i - b*e)*x^3 - (b*d - a*h)*x^2 - (b*c - a*g)*x + (
a*b*f - a^2*j)/b)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2
*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*
g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(
a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/
32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a
*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2
)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

maple [B] time = 0.06, size = 675, normalized size = 1.62

$$\frac{d \arctan\left(\sqrt{\frac{b}{a}}\, x^{2}\right)}{4\sqrt{ab}\; a} + \frac{h \arctan\left(\sqrt{\frac{b}{a}}\, x^{2}\right)}{4\sqrt{ab}\; b} + \frac{\sqrt{2}\, e \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16 \left(\frac{a}{b}\right)^{\frac{1}{4}} ab} + \frac{\sqrt{2}\, e \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{16 \left(\frac{a}{b}\right)^{\frac{1}{4}} ab} + \frac{\sqrt{2}\, e \ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{32 \left(\frac{a}{b}\right)^{\frac{1}{4}} ab} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, g \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16ab} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, g \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{16ab} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, g \ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{32ab} + \frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16a^{2}} + \frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{16a^{2}} + \frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, c \ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{32a^{2}} + \frac{3\sqrt{2}\, i \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}} + \frac{3\sqrt{2}\, i \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{16 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}} + \frac{3\sqrt{2}\, i \ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{32 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}} + \frac{j \ln\left(b x^{4} + a\right)}{4b^{2}} + \frac{-\frac{(ai-be)x^{3}}{4ab} - \frac{(ah-bd)x^{2}}{4ab} - \frac{(ag-bc)x}{4ab} + \frac{aj-bf}{4b^{2}}}{b x^{4} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x+1/4*(a*j-
b*f)/b^2)/(b*x^4+a)+1/16*(a/b)^(1/4)*2^(1/2)/a/b*g*arctan(2^(1/2)/(a/b)^(1/
4)*x+1)+3/16*(a/b)^(1/4)*2^(1/2)/a^2*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/16
*(a/b)^(1/4)*2^(1/2)/a/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+3/16*(a/b)^(1/4)
*2^(1/2)/a^2*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/32*(a/b)^(1/4)*2^(1/2)/a/b
*g*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a
/b)^(1/2)))+3/32*(a/b)^(1/4)*2^(1/2)/a^2*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a
/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4/(a*b)^(1/2)/b*h*arc
tan((1/a*b)^(1/2)*x^2)+1/4/(a*b)^(1/2)/a*d*arctan((1/a*b)^(1/2)*x^2)+3/16/(
a/b)^(1/4)*2^(1/2)/b^2*i*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/16/(a/b)^(1/4)*2
^(1/2)/a/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/32/(a/b)^(1/4)*2^(1/2)/b^2*i
*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2)))+1/32/(a/b)^(1/4)*2^(1/2)/a/b*e*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/16/(a/b)^(1/4)*2^(1/2)/
```

```
b^2*i*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/16/(a/b)^(1/4)*2^(1/2)/a/b*e*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*j*ln(b*x^4+a)/b^2
```

maxima [A] time = 3.21, size = 458, normalized size = 1.10

$$\frac{\left(b^2e-abi\right)x^3-abf+a^2j+\left(b^2d-abh\right)x^2+\left(b^2c-abg\right)x}{4\left(ab^3x^4+a^2b^2\right)}+\frac{\frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j+3b^2c-\sqrt{a}b^{\frac{3}{2}}e+abg-3a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j-3b^2c+\sqrt{a}b^{\frac{3}{2}}e-abg+3a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{7}{4}}e+\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}g+3\sqrt{2}a^{\frac{7}{4}}b^{\frac{3}{4}}i-4\sqrt{a}b^2d-4a^{\frac{3}{2}}bh\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{9}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{7}{4}}e+\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}g+3\sqrt{2}a^{\frac{7}{4}}b^{\frac{3}{4}}i+4\sqrt{a}b^2d+4a^{\frac{3}{2}}bh\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{32\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="maxima")
```

```
[Out] 1/4*((b^2*e - a*b*i)*x^3 - a*b*f + a^2*j + (b^2*d - a*b*h)*x^2 + (b^2*c - a
*b*g)*x)/(a*b^3*x^4 + a^2*b^2) + 1/32*(sqrt(2)*(4*sqrt(2)*a^(7/4)*b^(1/4)*j
 + 3*b^2*c - sqrt(a)*b^(3/2)*e + a*b*g - 3*a^(3/2)*sqrt(b)*i)*log(sqrt(b)*x
^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(4*sq
rt(2)*a^(7/4)*b^(1/4)*j - 3*b^2*c + sqrt(a)*b^(3/2)*e - a*b*g + 3*a^(3/2)*s
qrt(b)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b
^(5/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e + sqrt
(2)*a^(5/4)*b^(5/4)*g + 3*sqrt(2)*a^(7/4)*b^(3/4)*i - 4*sqrt(a)*b^2*d - 4*a
^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt
(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(3*sqrt(2)*a
^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e + sqrt(2)*a^(5/4)*b^(5/4)*g +
3*sqrt(2)*a^(7/4)*b^(3/4)*i + 4*sqrt(a)*b^2*d + 4*a^(3/2)*b*h)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3
/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/(a*b)
```

mupad [B] time = 5.84, size = 3939, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2
,x)
```

```
[Out] ((x*(b*c - a*g))/(4*a*b) - (b*f - a*j)/(4*b^2) + (x^2*(b*d - a*h))/(4*a*b)
 + (x^3*(b*e - a*i))/(4*a*b))/(a + b*x^4) + symsum(log(- root(65536*a^7*b^8*
z^4 - 65536*a^7*b^6*j*z^3 + 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4
096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 + 3072*a^4*b^7*c*e*z^2 + 24576*a
^7*b^4*j^2*z^2 + 2048*a^6*b^5*h^2*z^2 + 2048*a^4*b^7*d^2*z^2 - 1536*a^6*b^3
*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*
z - 512*a^5*b^4*e*g*j*z - 1536*a^4*b^5*c*e*j*z + 768*a^4*b^5*d*e*i*z - 768*
a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z - 1024*a^6*b^3*h^2*j*z + 1152*a^6*b^3
*h*i^2*z - 128*a^5*b^4*g^2*h*z - 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*
z + 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z - 128*a^4*b^5*d*g^2*z + 128*
a^3*b^6*d*e^2*z - 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e
*h*i*j - 192*a^4*b^3*d*e*i*j + 192*a^4*b^3*c*g*h*j - 96*a^4*b^3*d*g*h*i - 2
88*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*
d*e*g*h - 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 28
8*a^5*b^2*d*i^2*j - 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*
d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j + 32*a^4*b^3*d*g^2*j + 1
2*a^4*b^3*e*g^2*i - 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i - 16*a^4*b^3*e
*g*h^2 + 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j + 192*a^4*b^3*c*e*j^2 + 2
88*a^2*b^5*c^2*d*j + 108*a^2*b^5*c^2*e*i - 144*a^2*b^5*c*d^2*i - 48*a^3*b^4
*c*e*h^2 - 16*a^2*b^5*d^2*e*g + 12*a^2*b^5*c*e^2*g - 288*a^6*b*h*i^2*j + 19
2*a^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 +
 128*a^4*b^3*d^2*j^2 + 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^
4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 54*a^2*b^5*c^2*g^2 + 128*a^6*b*h^2*j^2 + 10
8*a^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i + 64*a^4*b^3*d*h^3 + 64*a^2*b^5*d^3*h +
12*a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 + 81*a^6*b*i^4 + 16*a*
b^6*d^4 + 256*a^7*j^4 + 81*b^7*c^4 + a^4*b^3*g^4 + a^2*b^5*e^4, z, m)*(root
```

(65536*a^7*b^8*z^4 - 65536*a^7*b^6*j*z^3 + 3072*a^6*b^5*g*i*z^2 + 9216*a^5*
b^6*c*i*z^2 + 4096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 + 3072*a^4*b^7*c*
e*z^2 + 24576*a^7*b^4*j^2*z^2 + 2048*a^6*b^5*h^2*z^2 + 2048*a^4*b^7*d^2*z^2
 - 1536*a^6*b^3*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768
*a^5*b^4*e*h*i*z - 512*a^5*b^4*e*g*j*z - 1536*a^4*b^5*c*e*j*z + 768*a^4*b^5
*d*e*i*z - 768*a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z - 1024*a^6*b^3*h^2*j*z
 + 1152*a^6*b^3*h*i^2*z - 128*a^5*b^4*g^2*h*z - 1024*a^4*b^5*d^2*j*z + 1152
*a^5*b^4*d*i^2*z + 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z - 128*a^4*b^5
*d*g^2*z + 128*a^3*b^6*d*e^2*z - 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z
 - 192*a^5*b^2*e*h*i*j - 192*a^4*b^3*d*e*i*j + 192*a^4*b^3*c*g*h*j - 96*a^4*
b^3*d*g*h*i - 288*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*
i - 32*a^3*b^4*d*e*g*h - 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b
^2*g*h^2*i - 288*a^5*b^2*d*i^2*j - 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2
 + 256*a^5*b^2*d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j + 32*a^4*
b^3*d*g^2*j + 12*a^4*b^3*e*g^2*i - 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i
 - 16*a^4*b^3*e*g*h^2 + 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j + 192*a^4*
b^3*c*e*j^2 + 288*a^2*b^5*c^2*d*j + 108*a^2*b^5*c^2*e*i - 144*a^2*b^5*c*d^2
*i - 48*a^3*b^4*c*e*h^2 - 16*a^2*b^5*d^2*e*g + 12*a^2*b^5*c*e^2*g - 288*a^6
*b*h*i^2*j + 192*a^6*b*g*i*j^2 - 48*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^
5*b^2*g^2*i^2 + 128*a^4*b^3*d^2*j^2 + 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*
i^2 + 96*a^3*b^4*d^2*h^2 + 2*a^3*b^4*e^2*g^2 + 54*a^2*b^5*c^2*g^2 + 128*a^6
*b*h^2*j^2 + 108*a^5*b^2*e*i^3 + 12*a^3*b^4*e^3*i + 64*a^4*b^3*d*h^3 + 64*a
^2*b^5*d^3*h + 12*a^3*b^4*c*g^3 + 18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 + 81*a^
6*b*i^4 + 16*a*b^6*d^4 + 256*a^7*j^4 + 81*b^7*c^4 + a^4*b^3*g^4 + a^2*b^5*e
^4, z, m)*((768*a^3*b^5*c + 256*a^4*b^4*g)/(64*a^3*b^2) - (x*(128*a^3*b^5*d
 + 128*a^4*b^4*h))/(16*a^3*b^2)) + (64*a^2*b^4*d*e - 384*a^3*b^3*c*j + 192*
a^3*b^3*d*i + 64*a^3*b^3*e*h - 128*a^4*b^2*g*j + 192*a^4*b^2*h*i)/(64*a^3*b
^2) + (x*(36*a*b^5*c^2 - 4*a^2*b^4*e^2 + 4*a^3*b^3*g^2 - 36*a^4*b^2*i^2 + 2
4*a^2*b^4*c*g + 64*a^3*b^3*d*j - 24*a^3*b^3*e*i + 64*a^4*b^2*h*j))/(16*a^3*
b^2)) - (27*a^4*i^3 + a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e + 16*a^4*g*j^2
 - 12*a^2*b^2*c*h^2 + a^2*b^2*e*g^2 + 9*a^2*b^2*e^2*i - 48*a^4*h*i*j - 4*a*
b^3*d^2*g + 27*a*b^3*c^2*i + 48*a^3*b*c*j^2 + 27*a^3*b*e*i^2 - 4*a^3*b*g*h^
2 + 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 16*a^2*b^2*d*e*j - 8*a^2*b^2*d*g*h -
 24*a*b^3*c*d*h + 6*a*b^3*c*e*g - 48*a^3*b*d*i*j - 16*a^3*b*e*h*j)/(64*a^3*
b^2) - (x*(9*a^4*i^2*j - 2*a^3*b*h^3 - 8*a^4*h*j^2 - 2*b^4*d^3 - 6*a^2*b^2*
d*h^2 + a^2*b^2*e^2*j + 3*b^4*c*d*e - 6*a*b^3*d^2*h - 9*a*b^3*c^2*j - 8*a^3
*b*d*j^2 - a^3*b*g^2*j - 6*a^2*b^2*c*g*j + 9*a^2*b^2*c*h*i + 3*a^2*b^2*d*g*
i + a^2*b^2*e*g*h + 9*a*b^3*c*d*i + 3*a*b^3*c*e*h + a*b^3*d*e*g + 6*a^3*b*e
*i*j + 3*a^3*b*g*h*i))/(16*a^3*b^2))*root(65536*a^7*b^8*z^4 - 65536*a^7*b^6
*j*z^3 + 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2 + 4096*a^5*b^6*d*h*z^2
 + 1024*a^5*b^6*e*g*z^2 + 3072*a^4*b^7*c*e*z^2 + 24576*a^7*b^4*j^2*z^2 + 20
48*a^6*b^5*h^2*z^2 + 2048*a^4*b^7*d^2*z^2 - 1536*a^6*b^3*g*i*j*z - 4608*a^5
*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*e*h*i*z - 512*a^5*b^4*e*g
*j*z - 1536*a^4*b^5*c*e*j*z + 768*a^4*b^5*d*e*i*z - 768*a^4*b^5*c*g*h*z - 7
68*a^3*b^6*c*d*g*z - 1024*a^6*b^3*h^2*j*z + 1152*a^6*b^3*h*i^2*z - 128*a^5*
b^4*g^2*h*z - 1024*a^4*b^5*d^2*j*z + 1152*a^5*b^4*d*i^2*z + 128*a^4*b^5*e^2
*h*z - 1152*a^3*b^6*c^2*h*z - 128*a^4*b^5*d*g^2*z + 128*a^3*b^6*d*e^2*z - 1
152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 192*a^5*b^2*e*h*i*j - 192*a^4*b^
3*d*e*i*j + 192*a^4*b^3*c*g*h*j - 96*a^4*b^3*d*g*h*i - 288*a^3*b^4*c*d*h*i
+ 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i - 32*a^3*b^4*d*e*g*h - 96*a^2*b^
5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^2*g*h^2*i - 288*a^5*b^2*d*i^2*j -
 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2 + 256*a^5*b^2*d*h*j^2 + 64*a^5*b^
2*e*g*j^2 + 288*a^3*b^4*c^2*h*j + 32*a^4*b^3*d*g^2*j + 12*a^4*b^3*e*g^2*i -
 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g*i - 16*a^4*b^3*e*g*h^2 + 108*a^4*b^
3*c*g*i^2 - 32*a^3*b^4*d*e^2*j + 192*a^4*b^3*c*e*j^2 + 288*a^2*b^5*c^2*d*j
+ 108*a^2*b^5*c^2*e*i - 144*a^2*b^5*c*d^2*i - 48*a^3*b^4*c*e*h^2 - 16*a^2*b
^5*d^2*e*g + 12*a^2*b^5*c*e^2*g - 288*a^6*b*h*i^2*j + 192*a^6*b*g*i*j^2 - 4
8*a*b^6*c*d^2*e + 108*a*b^6*c^3*g + 18*a^5*b^2*g^2*i^2 + 128*a^4*b^3*d^2*j^
2 + 54*a^4*b^3*e^2*i^2 + 162*a^3*b^4*c^2*i^2 + 96*a^3*b^4*d^2*h^2 + 2*a^3*b

```
^4*e^2*g^2 + 54*a^2*b^5*c^2*g^2 + 128*a^6*b*h^2*j^2 + 108*a^5*b^2*e*i^3 + 1
2*a^3*b^4*e^3*i + 64*a^4*b^3*d*h^3 + 64*a^2*b^5*d^3*h + 12*a^3*b^4*c*g^3 +
18*a*b^6*c^2*e^2 + 16*a^5*b^2*h^4 + 81*a^6*b*i^4 + 16*a*b^6*d^4 + 256*a^7*j
^4 + 81*b^7*c^4 + a^4*b^3*g^4 + a^2*b^5*e^4, z, m), m, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x
)
```

```
[Out] Timed out
```

3.151 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=241

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+$$

Rubi [A] time = 0.34, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\,\sqrt{b}\,e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x\left(2x(3bd-ah)-ag+7bc+5bex^2\right)+4af}{32a^2b\left(a-bx^4\right)}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) +
 (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a - b
*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)
])/(64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^3}\,dx = \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} - \frac{\int \frac{-b(7bc-ag)-2b(3bd-ah)x-5b^2ex^2-4\cdots}{\left(a-bx^4\right)^2}}{8ab^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+2(3bd-\cdots\right.}{32a^2b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+2(3bd-\cdots\right.}{32a^2b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+2(3bd-\cdots\right.}{32a^2b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+2(3bd-\cdots\right.}{32a^2b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+2(3bd-\cdots\right.}{32a^2b\left(a-bx^4\right)}$$

Mathematica [A] time = 0.42, size = 309, normalized size = 1.28

$$\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(4a^{5/4}h-5\sqrt{a}b^{3/4}e-12\sqrt[4]{a}bd+3a\sqrt[4]{b}g-21b^{5/4}c\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(4a^{5/4}h+5\sqrt{a}b^{3/4}e-12\sqrt[4]{a}bd-3a\sqrt[4]{b}g+21b^{5/4}c\right)+\frac{16a^{7/4}\sqrt{b}(a(f+x(g+hx))+bx(c+x(d+ex)))}{\left(a-bx^4\right)^2}+\frac{4a^{3/4}\sqrt{b}x(-a(g+2hx)+7bc+bx(6d+5ex))}{a-bx^4}+2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)-4\sqrt[4]{a}(ah-3bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{128a^{11/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) - a*(g + 2*h*x)))/(a - b*x
^4) + (16*a^(7/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a
 - b*x^4)^2 + 2*b^(1/4)*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1
/4)*x)/a^(1/4)] + (-21*b^(5/4)*c - 12*a^(1/4)*b*d - 5*Sqrt[a]*b^(3/4)*e + 3
*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(5/4)*c - 12*a
^(1/4)*b*d + 5*Sqrt[a]*b^(3/4)*e - 3*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4)
 + b^(1/4)*x] - 4*a^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a
^(11/4)*b^(3/2))
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,
x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,
 x]
```

fricas [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [B]   time = 0.21, size = 440, normalized size = 1.83

$$-\frac{\sqrt{2}\left(21\,b^2c-3\,abg-12\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+4\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah+5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^3\right)^{\frac{3}{4}}a^2}-\frac{\sqrt{2}\left(21\,b^2c-3\,abg+12\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-4\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah-5\,\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^3\right)^{\frac{3}{4}}a^2}-\frac{\sqrt{2}\left(21\,b^2c-3\,abg-5\,\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^3\right)^{\frac{3}{4}}a^2}+\frac{\sqrt{2}\left(21\,b^2c-3\,abg-5\,\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^3\right)^{\frac{3}{4}}a^2}-\frac{5\,b^2x^7e+6\,b^2dx^6-2\,abhx^6+7\,b^2cx^5-abgx^5-9\,abx^3e-10\,abdx^2-2\,a^2hx^2-11\,abcx-3\,a^2gx-4\,a^2f}{32\left(bx^4-a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 4*sqrt
(2)*(-a*b^3)^(1/4)*a*h + 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*
c - 3*a*b*g + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 4*sqrt(2)*(-a*b^3)^(1/4)*a*h
- 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^
(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a
*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2
) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)
*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b^2*x^7*e + 6*
b^2*d*x^6 - 2*a*b*h*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*x^3*e - 10*a*b*d*
x^2 - 2*a^2*h*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

maple [A]   time = 0.06, size = 389, normalized size = 1.61

$$\frac{h\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{32\sqrt{ab}\,ab}-\frac{3d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{32\sqrt{ab}\,a^2}-\frac{5e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b}+\frac{5e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^2b}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^2b}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^3}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3}-\frac{\frac{5be\,x^7}{32a^2}-\frac{(ah-3bd)x^6}{16a^2}-\frac{9e\,x^3}{32a}-\frac{(ag-7bc)x^5}{32a^2}-\frac{(ah+5bd)x^2}{16ab}-\frac{f}{8b}-\frac{(3ag+11bc)x}{32ab}}{\left(b\,x^4-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(5/32/a^2*b*e*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-7*b*c)/a^2*x^5-9/32/a
*e*x^3-1/16*(a*h+5*b*d)/a/b*x^2-1/32*(3*a*g+11*b*c)/a/b*x-1/8/b*f)/(b*x^4-a
)^2-3/64*(a/b)^(1/4)/a^2/b*g*arctan(1/(a/b)^(1/4)*x)+21/64*(a/b)^(1/4)/a^3*
c*arctan(1/(a/b)^(1/4)*x)-3/128*(a/b)^(1/4)/a^2/b*g*ln((x+(a/b)^(1/4))/(x-(
a/b)^(1/4)))+21/128*(a/b)^(1/4)/a^3*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1
/32/a/b/(a*b)^(1/2)*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))*h-3/32/(a*
b)^(1/2)/a^2*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-5/64/(a/b)^(1/4
)/a^2/b*e*arctan(1/(a/b)^(1/4)*x)+5/128/(a/b)^(1/4)/a^2/b*e*ln((x+(a/b)^(1/
4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 2.96, size = 316, normalized size = 1.31

$$-\frac{5\,b^2ex^7+2\,\left(3\,b^2d-abh\right)x^6-9\,abex^3+\left(7\,b^2c-abg\right)x^5-4\,a^2f-2\,\left(5\,abd+a^2h\right)x^2-\left(11\,abc+3\,a^2g\right)x}{32\,\left(a^2b^3x^8-2\,a^3b^2x^4+a^4b\right)}+\frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be-3\,a\sqrt{b}g\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima
")
```

```
[Out] -1/32*(5*b^2*e*x^7 + 2*(3*b^2*d - a*b*h)*x^6 - 9*a*b*e*x^3 + (7*b^2*c - a*b
*g)*x^5 - 4*a^2*f - 2*(5*a*b*d + a^2*h)*x^2 - (11*a*b*c + 3*a^2*g)*x)/(a^2*
b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(4*(3*b*d - a*h)*log(sqrt(b)*x^2 +
 sqrt(a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(s
qrt(a)*sqrt(b)) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*e - 3*a*sqrt(b)*g)*arctan(s
qrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (
21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)
*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))*sqrt(b)))/(a^2*b)
```

mupad [B] time = 5.73, size = 1687, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x)
```

```
[Out] (f/(8*b) + (9*e*x^3)/(32*a) - (x^5*(7*b*c - a*g))/(32*a^2) - (x^6*(3*b*d -
a*h))/(16*a^2) + (x*(11*b*c + 3*a*g))/(32*a*b) + (x^2*(5*b*d + a*h))/(16*a*
b) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(- root(
268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 -
 6881280*a^6*b^5*c*e*z^2 - 524288*a^8*b^3*h^2*z^2 - 4718592*a^6*b^5*d^2*z^2
 + 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z
- 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z + 55296*a^5*b^3*d*g^2*z +
153600*a^4*b^4*d*e^2*z + 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h + 4
0320*a^2*b^3*c*d*e*h + 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 - 6300*a
^2*b^3*c*e^2*g + 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e - 3072*a^4*b*d*h^3
 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 - 23814
*a^2*b^3*c^2*g^2 - 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c
^2*e^2 - 625*a^2*b^3*e^4 - 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 - 1
94481*b^5*c^4, z, k)*(root(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2
 + 983040*a^7*b^4*e*g*z^2 - 6881280*a^6*b^5*c*e*z^2 - 524288*a^8*b^3*h^2*z^
2 - 4718592*a^6*b^5*d^2*z^2 + 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g
*z - 18432*a^6*b^2*g^2*h*z - 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z
 + 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z + 2709504*a^3*b^5*c^2*d*z
 - 5760*a^3*b^2*d*e*g*h + 40320*a^2*b^3*c*d*e*h + 8640*a^2*b^3*d^2*e*g - 67
20*a^3*b^2*c*e*h^2 - 6300*a^2*b^3*c*e^2*g + 960*a^4*b*e*g*h^2 - 60480*a*b^4
*c*d^2*e - 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 +
450*a^3*b^2*e^2*g^2 - 23814*a^2*b^3*c^2*g^2 - 27648*a^2*b^3*d^3*h + 2268*a^
3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 - 625*a^2*b^3*e^4 - 81*a^4*b*g^4 + 20736*
a*b^4*d^4 + 256*a^5*h^4 - 194481*b^5*c^4, z, k)*((344064*a^5*b^4*c - 49152*
a^6*b^3*g)/(32768*a^6*b) - (x*(24576*a^5*b^4*d - 8192*a^6*b^3*h))/(4096*a^6
*b)) - (15360*a^3*b^3*d*e - 5120*a^4*b^2*e*h)/(32768*a^6*b) + (x*(7056*a^2*
b^4*c^2 + 400*a^3*b^3*e^2 + 144*a^4*b^2*g^2 - 2016*a^3*b^3*c*g))/(4096*a^6*
b)) - (125*a*b^2*e^3 + 3024*b^3*c*d^2 - 2205*b^3*c^2*e - 48*a^3*g*h^2 - 432
*a*b^2*d^2*g + 336*a^2*b*c*h^2 - 45*a^2*b*e*g^2 - 2016*a*b^2*c*d*h + 630*a*
b^2*c*e*g + 288*a^2*b*d*g*h)/(32768*a^6*b) - (x*(216*b^3*d^3 - 8*a^3*h^3 -
315*b^3*c*d*e - 216*a*b^2*d^2*h + 72*a^2*b*d*h^2 + 105*a*b^2*c*e*h + 45*a*b
^2*d*e*g - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^6*z^4 + 314
5728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 - 6881280*a^6*b^5*c*e*z^2 - 5
24288*a^8*b^3*h^2*z^2 - 4718592*a^6*b^5*d^2*z^2 + 258048*a^5*b^3*c*g*h*z -
```

```
774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z - 51200*a^5*b^3*e^2*h*z - 90
3168*a^4*b^4*c^2*h*z + 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z + 270
9504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h + 40320*a^2*b^3*c*d*e*h + 8640*
a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 - 6300*a^2*b^3*c*e^2*g + 960*a^4*b*e
*g*h^2 - 60480*a*b^4*c*d^2*e - 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 1382
4*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 - 23814*a^2*b^3*c^2*g^2 - 27648*a^2
*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 - 625*a^2*b^3*e^4 - 8
1*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 - 194481*b^5*c^4, z, k), k, 1,
4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.152 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a-bx^4\right)^3}\,dx$

Optimal. Leaf size=268

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x}{\cdots}+\cdots$$

Rubi [A] time = 0.43, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x\left(2x(3bd-ah)+x^2(5be-3ai)-ag+7bc\right)+4af}{32a^2b\left(a-bx^4\right)}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(8*a*b*(a - b*x
^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + (5*b*e - 3*a*i)*x^2)
)/(32*a^2*b*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*
a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sqrt[
b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4
)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(
3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+152x^6}{\left(a-bx^4\right)^3}\,dx = \frac{x\left(bc+ag+(bd+ah)x+(152a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} - \frac{\int \frac{-b(7bc-}{}}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(152a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(152a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(152a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(152a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(152a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x}{}$$

Mathematica [A] time = 0.40, size = 359, normalized size = 1.34

$$\frac{\frac{16a^{7/4}b^{3/4}(a(f+x(g+x(h+ix)))+bx(c+x(d+ex)))}{\left(a-bx^4\right)^2} - \frac{4a^{3/4}b^{3/4}x(a(g+x(2h+3ix))-b(7c+x(6d+5ex)))}{a-bx^4} + \log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(4a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-12\sqrt[4]{a}b^{5/4}d-5\sqrt{a}be+3a\sqrt{b}g-21b^{3/2}c\right) + \log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(4a^{5/4}\sqrt[4]{b}h-3a^{3/2}i-12\sqrt[4]{a}b^{5/4}d+5\sqrt{a}be-3a\sqrt{b}g+21b^{3/2}c\right) + 2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3a^{3/2}i-5\sqrt{a}be-3a\sqrt{b}g+21b^{3/2}c\right) - 4\sqrt[4]{a}\sqrt[4]{b}(ah-3bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{128a^{11/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x
]
```

```
[Out] ((-4*a^(3/4)*b^(3/4)*x*(-(b*(7*c + x*(6*d + 5*e*x))) + a*(g + x*(2*h + 3*i*
x))))/(a - b*x^4) + (16*a^(7/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(
g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*(21*b^(3/2)*c - 5*Sqrt[a]*b*e - 3*a*S
qrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-21*b^(3/2)*c - 12*a
^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*
a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d
```

+ 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 4*a^(1/4)*b^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^(7/4))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a-bx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b
*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b
*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.28, size = 652, normalized size = 2.43

$$-\frac{3}{256}\,i\left(\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4}-\frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{a^2b^4}\right)-\frac{3}{256}\,i\left(\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4}+\frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{a^2b^4}\right)-\frac{\sqrt{2}\left(21b^2c-3abg-12\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+4\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah+5\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^3\right)^{\frac{3}{4}}a^2}-\frac{\sqrt{2}\left(21b^2c-3abg+12\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-4\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah-5\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(-ab^3\right)^{\frac{3}{4}}a^2}-\frac{\sqrt{2}\left(21b^2c-3abg-5\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^3\right)^{\frac{3}{4}}a^2}+\frac{\sqrt{2}\left(21b^2c-3abg-5\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\left(-ab^3\right)^{\frac{3}{4}}a^2}+\frac{3abix^7-5b^2x^7e-6b^2dx^6+2abhx^6-7b^2cx^5+abgx^5+a^2ix^3+9abx^3e+10abdx^2+2a^2hx^2+11abcx+3a^2gx+4a^2f}{32\left(bx^4-a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
giac")
```

```
[Out] -3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*
x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4)) - 3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) +
sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*
b^4)) - 1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d +
 4*sqrt(2)*(-a*b^3)^(1/4)*a*h + 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x +
 sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(
21*b^2*c - 3*a*b*g + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 4*sqrt(2)*(-a*b^3)^(1/
4)*a*h - 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/
(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*
sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3
/4)*a^2) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(x^2 -
sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/32*(3*a*b*i*x
^7 - 5*b^2*x^7*e - 6*b^2*d*x^6 + 2*a*b*h*x^6 - 7*b^2*c*x^5 + a*b*g*x^5 + a^
2*i*x^3 + 9*a*b*x^3*e + 10*a*b*d*x^2 + 2*a^2*h*x^2 + 11*a*b*c*x + 3*a^2*g*x
 + 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

maple [B] time = 0.06, size = 472, normalized size = 1.76

$$\frac{h\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{32\sqrt{ab}\,ab}-\frac{3d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{32\sqrt{ab}\,a^2}+\frac{3i\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2}-\frac{3i\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2}-\frac{5e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b}+\frac{5e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^2b}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^2b}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^3}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3}-\frac{-\frac{(3ai-5be)x^7}{32a^2}-\frac{(ah-3bd)x^6}{16a^2}-\frac{(ag-7bc)x^5}{32a^2}-\frac{(ai+9be)x^3}{32ab}-\frac{(ah+5bd)x^2}{16ab}-\frac{f}{8b}-\frac{(3ag+11bc)x}{32ab}}{\left(b\,x^4-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(-1/32*(3*a*i-5*b*e)/a^2*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-7*b*c)/a^2
*x^5-1/32*(a*i+9*b*e)/a/b*x^3-1/16*(a*h+5*b*d)/a/b*x^2-1/32*(3*a*g+11*b*c)/
a/b*x-1/8/b*f)/(b*x^4-a)^2-3/64*(a/b)^(1/4)/a^2/b*g*arctan(1/(a/b)^(1/4)*x)
+21/64*(a/b)^(1/4)/a^3*c*arctan(1/(a/b)^(1/4)*x)-3/128*(a/b)^(1/4)/a^2/b*g*
ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+21/128*(a/b)^(1/4)/a^3*c*ln((x+(a/b)^(1
/4))/(x-(a/b)^(1/4)))+1/32/(a*b)^(1/2)/a/b*h*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)
^(1/2)*x^2-a))-3/32/(a*b)^(1/2)/a^2*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*
x^2-a))+3/64/a/b^2/(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)*i-5/64/(a/b)^(1/4)/a
^2/b*e*arctan(1/(a/b)^(1/4)*x)-3/128/a/b^2/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(
x-(a/b)^(1/4)))*i+5/128/(a/b)^(1/4)/a^2/b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))
```

maxima [A] time = 3.08, size = 343, normalized size = 1.28

$$-\frac{\left(5\,b^{2}e-3\,abi\right)x^{7}+2\left(3\,b^{2}d-abh\right)x^{6}+\left(7\,b^{2}c-abg\right)x^{5}-\left(9\,abe+a^{2}i\right)x^{3}-4\,a^{2}f-2\left(5\,abd+a^{2}h\right)x^{2}-\left(11\,abc+3\,a^{2}g\right)x}{32\left(a^{2}b^{3}x^{8}-2\,a^{3}b^{2}x^{4}+a^{4}b\right)}+\frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^{2}+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^{2}-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be-3\,a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
maxima")
```

```
[Out] -1/32*((5*b^2*e - 3*a*b*i)*x^7 + 2*(3*b^2*d - a*b*h)*x^6 + (7*b^2*c - a*b*g
)*x^5 - (9*a*b*e + a^2*i)*x^3 - 4*a^2*f - 2*(5*a*b*d + a^2*h)*x^2 - (11*a*b
*c + 3*a^2*g)*x)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(4*(3*b*d -
a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqr
t(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*e - 3
*a*sqrt(b)*g + 3*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt
(b)*g - 3*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + s
qrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

mupad [B] time = 5.80, size = 2680, normalized size = 10.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x)
```

```
[Out] symsum(log((27*a^4*i^3 - 125*a*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e -
336*a^2*b^2*c*h^2 + 45*a^2*b^2*e*g^2 + 225*a^2*b^2*e^2*i + 432*a*b^3*d^2*g
- 1323*a*b^3*c^2*i - 135*a^3*b*e*i^2 + 48*a^3*b*g*h^2 - 27*a^3*b*g^2*i + 37
8*a^2*b^2*c*g*i - 288*a^2*b^2*d*g*h + 2016*a*b^3*c*d*h - 630*a*b^3*c*e*g)/(
32768*a^6*b^2) - root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2 + 412
8768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 - 6
881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*z^2 +
 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i*z -
774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55
296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z + 5529
6*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z + 3456
*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*
b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^
2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*i^2 -
 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g - 672
0*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*
c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2 + 138
24*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 23814*a^2
*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*d^3*h
 - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b
^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 1
```

```
94481*b^6*c^4, z, l)*(root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2
+ 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^
2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*
z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i
*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z
 + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z +
 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z +
 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760
*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b
^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*
i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g
- 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a
*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2
+ 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 2381
4*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*
d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*
a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^
4 - 194481*b^6*c^4, z, l)*((344064*a^5*b^5*c - 49152*a^6*b^4*g)/(32768*a^6*
b^2) - (x*(24576*a^5*b^4*d - 8192*a^6*b^3*h))/(4096*a^6*b)) - (15360*a^3*b^
4*d*e - 9216*a^4*b^3*d*i - 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*
b^2) + (x*(144*a^5*b*i^2 + 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 + 144*a^4*b^2
*g^2 - 2016*a^3*b^3*c*g - 480*a^4*b^2*e*i))/(4096*a^6*b)) - (x*(216*b^3*d^3
 - 8*a^3*h^3 - 315*b^3*c*d*e + 9*a^3*g*h*i - 216*a*b^2*d^2*h + 72*a^2*b*d*h
^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g - 63*a^2*b*c*h*i -
27*a^2*b*d*g*i - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z^4
 - 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z
^2 + 983040*a^7*b^5*e*g*z^2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*
z^2 - 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*
h*z - 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2
*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z
- 903168*a^4*b^5*c^2*h*z + 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z +
 2709504*a^3*b^6*c^2*d*z + 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7
560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^
4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e
*g*h^2 - 2268*a^4*b^2*c*g*i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2
*i + 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 5
76*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3
*g - 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 +
450*a^3*b^3*e^2*g^2 - 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*
b^3*e^3*i - 27648*a^2*b^4*d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 +
 22050*a*b^5*c^2*e^2 - 81*a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 2
0736*a*b^5*d^4 - 81*a^6*i^4 - 194481*b^6*c^4, z, l), l, 1, 4) + (f/(8*b) -
(x^5*(7*b*c - a*g))/(32*a^2) - (x^6*(3*b*d - a*h))/(16*a^2) - (x^7*(5*b*e -
 3*a*i))/(32*a^2) + (x*(11*b*c + 3*a*g))/(32*a*b) + (x^2*(5*b*d + a*h))/(16
*a*b) + (x^3*(9*b*e + a*i))/(32*a*b))/(a^2 + b^2*x^8 - 2*a*b*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.153 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3} \, dx$

Optimal. Leaf size=285

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

Rubi [A] time = 0.39, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4a(bf-aj)}{32a^2b^2\left(a-bx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}\,(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{8ab\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3
,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(8*a*b*
(a - b*x^4)^2) + (4*a*(b*f - a*j) + x*(b*(7*b*c - a*g) + 2*b*(3*b*d - a*h)*
x + b*(5*b*e - 3*a*i)*x^2))/(32*a^2*b^2*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]
*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b
^(7/4)) + ((5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(
1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^
2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
```

```
0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+153x^6+jx^7}{\left(a-bx^4\right)^3}\,dx = \frac{x\left(bc+ag+(bd+ah)x+(153a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} -$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(153a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} +$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(153a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} +$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(153a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} +$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(153a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} +$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(153a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} +$$

Mathematica [A] time = 0.34, size = 380, normalized size = 1.33

$$\frac{\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(4a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-12\sqrt[4]{a}b^{5/4}d-5\sqrt{a}be+3a\sqrt{b}g-21b^{3/2}c\right)+\sqrt[4]{b}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(4a^{5/4}\sqrt[4]{b}h-3a^{3/2}i-12\sqrt[4]{a}b^{5/4}d+5\sqrt{a}be-3a\sqrt{b}g+21b^{3/2}c\right)+2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3a^{3/2}i-5\sqrt{a}be-3a\sqrt{b}g+21b^{3/2}c\right)+\frac{16a^{7/4}\left(a^2j+ab(f+x(g+x(h+ix)))+b^2x(c+x(d+ex))\right)}{\left(a-bx^4\right)^2}-\frac{4a^{3/4}\left(8a^2j+abx(g+x(2h+3ix))-b^2x(7c+x(6d+5ex))\right)}{a-bx^4}-4\sqrt[4]{a}\sqrt{b}(ah-3bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{128a^{11/4}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^3,x]
```

```
[Out] ((-4*a^(3/4)*(8*a^2*j - b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h +
 3*i*x))))/(a - b*x^4) + (16*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b
*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*b^(1/4)*(21*b^(3/2)*c - 5*Sq
rt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + b^(1
```

```
/4)*(-21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g +
 4*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*(21*
b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4
)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 4*a^(1/4)*Sqrt[b]*(-3
*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{\left(a - bx^4\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a - b*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a - b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.21, size = 684, normalized size = 2.40

$$-\frac{3}{256} i \left(\frac{2 \sqrt{2} \left(-a b^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2 x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^2 b^4} - \frac{\sqrt{2} \left(-a b^3\right)^{\frac{3}{4}} \log\left(x^2 + \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{a^2 b^4}\right) - \frac{3}{256} i \left(\frac{2 \sqrt{2} \left(-a b^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2 x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^2 b^4} + \frac{\sqrt{2} \left(-a b^3\right)^{\frac{3}{4}} \log\left(x^2 - \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{a^2 b^4}\right) - \frac{\sqrt{2}\left(21 b^2 c - 3 a b g - 12 \sqrt{2} \left(-a b^3\right)^{\frac{1}{4}} b d + 4 \sqrt{2} \left(-a b^3\right)^{\frac{1}{4}} a h + 5 \sqrt{-a b}\, b e\right) \arctan\left(\frac{\sqrt{2}\left(2 x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128 \left(-a b^3\right)^{\frac{3}{4}} a^2} - \frac{\sqrt{2}\left(21 b^2 c - 3 a b g + 12 \sqrt{2} \left(-a b^3\right)^{\frac{1}{4}} b d - 4 \sqrt{2} \left(-a b^3\right)^{\frac{1}{4}} a h - 5 \sqrt{-a b}\, b e\right) \arctan\left(\frac{\sqrt{2}\left(2 x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128 \left(-a b^3\right)^{\frac{3}{4}} a^2} - \frac{\sqrt{2}\left(21 b^2 c - 3 a b g - 5 \sqrt{-a b}\, b e\right) \log\left(x^2 + \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256 \left(-a b^3\right)^{\frac{3}{4}} a^2} + \frac{\sqrt{2}\left(21 b^2 c - 3 a b g - 5 \sqrt{-a b}\, b e\right) \log\left(x^2 - \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256 \left(-a b^3\right)^{\frac{3}{4}} a^2} + \frac{3 a b^2 i x^7 - 5 b^3 x^7 e - 6 b^3 d x^6 + 2 a b^2 h x^6 - 7 b^3 c x^5 + a b^2 g x^5 + 8 a^2 b j x^4 + a^2 b i x^3 + 9 a b^2 x^3 e + 10 a b^2 d x^2 + 2 a^2 b h x^2 + 11 a b^2 c x + 3 a^2 b g x + 4 a^2 b f - 4 a^3 j}{32 \left(b x^4 - a\right)^2 a^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="giac")
```

```
[Out] -3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*
x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4)) - 3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) +
sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*
b^4)) - 1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d +
 4*sqrt(2)*(-a*b^3)^(1/4)*a*h + 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x +
 sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(
21*b^2*c - 3*a*b*g + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 4*sqrt(2)*(-a*b^3)^(1/
4)*a*h - 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/
(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*
sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3
/4)*a^2) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(x^2 -
sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/32*(3*a*b^2*i
*x^7 - 5*b^3*x^7*e - 6*b^3*d*x^6 + 2*a*b^2*h*x^6 - 7*b^3*c*x^5 + a*b^2*g*x^
5 + 8*a^2*b*j*x^4 + a^2*b*i*x^3 + 9*a*b^2*x^3*e + 10*a*b^2*d*x^2 + 2*a^2*b*
h*x^2 + 11*a*b^2*c*x + 3*a^2*b*g*x + 4*a^2*b*f - 4*a^3*j)/((b*x^4 - a)^2*a^
2*b^2)
```

maple [B] time = 0.06, size = 488, normalized size = 1.71

$$\frac{h \ln\left(\frac{\sqrt{a b}\, x^2 - a}{-\sqrt{a b}\, x^2 - a}\right)}{32 \sqrt{a b}\, a b} - \frac{3 d \ln\left(\frac{\sqrt{a b}\, x^2 - a}{-\sqrt{a b}\, x^2 - a}\right)}{32 \sqrt{a b}\, a^2} + \frac{3 i \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a b^2} - \frac{3 i \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128 \left(\frac{a}{b}\right)^{\frac{1}{4}} a b^2} - \frac{5 e \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^2 b} + \frac{5 e \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^2 b} - \frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} g \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^2 b} - \frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} g \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128 a^2 b} + \frac{21 \left(\frac{a}{b}\right)^{\frac{1}{4}} c \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^3} + \frac{21 \left(\frac{a}{b}\right)^{\frac{1}{4}} c \ln\left(\frac{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}}{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128 a^3} - \frac{-\frac{(3ai - 5be) x^7}{32 a^2} - \frac{(ah - 3bd) x^6}{16 a^2} - \frac{j x^4}{4 b} - \frac{(ag - 7bc) x^5}{32 a^2} - \frac{(ai + 9be) x^3}{32 a b} - \frac{(ah + 5bd) x^2}{16 a b} - \frac{(3ag + 11bc) x}{32 a b} + \frac{aj - bf}{8 b^2}}{\left(b x^4 - a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(-1/32*(3*a*i-5*b*e)/a^2*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-7*b*c)/a^2
*x^5-1/4/b*j*x^4-1/32*(a*i+9*b*e)/a/b*x^3-1/16*(a*h+5*b*d)/a/b*x^2-1/32*(3*
a*g+11*b*c)/a/b*x+1/8*(a*j-b*f)/b^2)/(b*x^4-a)^2-3/64*(a/b)^(1/4)/a^2/b*g*a
rctan(1/(a/b)^(1/4)*x)+21/64*(a/b)^(1/4)/a^3*c*arctan(1/(a/b)^(1/4)*x)-3/12
8*(a/b)^(1/4)/a^2/b*g*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+21/128*(a/b)^(1/4
)/a^3*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/32/(a*b)^(1/2)/a/b*h*ln(((a*b
)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-3/32/(a*b)^(1/2)/a^2*d*ln(((a*b)^(1/2)
*x^2-a)/(-(a*b)^(1/2)*x^2-a))+3/64/(a/b)^(1/4)/a/b^2*i*arctan(1/(a/b)^(1/4)
*x)-5/64/(a/b)^(1/4)/a^2/b*e*arctan(1/(a/b)^(1/4)*x)-3/128/(a/b)^(1/4)/a/b^
2*i*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+5/128/(a/b)^(1/4)/a^2/b*e*ln((x+(a/
b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 3.13, size = 377, normalized size = 1.32

$$\frac{8\,a^2bjx^4-\left(5\,b^3e-3\,ab^2i\right)x^7-2\left(3\,b^3d-ab^2h\right)x^6-\left(7\,b^3c-ab^2g\right)x^5+4\,a^2bf-4\,a^3j+\left(9\,ab^2e+a^2bi\right)x^3+2\left(5\,ab^2d+a^2bh\right)x^2+\left(11\,ab^2c+3\,a^2bg\right)x}{32\left(a^2b^4x^8-2\,a^3b^3x^4+a^4b^2\right)}+\frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be-3\,a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*a^2*b*j*x^4 - (5*b^3*e - 3*a*b^2*i)*x^7 - 2*(3*b^3*d - a*b^2*h)*x^6
 - (7*b^3*c - a*b^2*g)*x^5 + 4*a^2*b*f - 4*a^3*j + (9*a*b^2*e + a^2*b*i)*x^
3 + 2*(5*a*b^2*d + a^2*b*h)*x^2 + (11*a*b^2*c + 3*a^2*b*g)*x)/(a^2*b^4*x^8
- 2*a^3*b^3*x^4 + a^4*b^2) + 1/128*(4*(3*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(
a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)
*sqrt(b)) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*e - 3*a*sqrt(b)*g + 3*a^(3/2)*i)*
arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt
(b)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log((sq
rt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

mupad [B] time = 5.91, size = 2696, normalized size = 9.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3
,x)
```

```
[Out] symsum(log((27*a^4*i^3 - 125*a*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e -
336*a^2*b^2*c*h^2 + 45*a^2*b^2*e*g^2 + 225*a^2*b^2*e^2*i + 432*a*b^3*d^2*g
- 1323*a*b^3*c^2*i - 135*a^3*b*e*i^2 + 48*a^3*b*g*h^2 - 27*a^3*b*g^2*i + 37
8*a^2*b^2*c*g*i - 288*a^2*b^2*d*g*h + 2016*a*b^3*c*d*h - 630*a*b^3*c*e*g)/(
32768*a^6*b^2) - root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2 + 412
8768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 - 6
881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*z^2 +
 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i*z -
774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55
296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z + 5529
6*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z + 3456
*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*
b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^
2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*i^2 -
 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g - 672
0*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*
c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2 + 138
```

```
24*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 23814*a^2
*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*d^3*h
 - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*a^4*b
^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^4 - 1
94481*b^6*c^4, z, m)*(root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2
+ 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^
2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^2*
z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*e*i
*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z
 + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z +
 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2*d*z +
 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760
*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 5184*a^3*b
^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4*b^2*c*g*
i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^4*d^2*e*g
- 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a
*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*b^2*e^2*i^2
+ 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 - 2381
4*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i - 27648*a^2*b^4*
d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 - 81*
a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 - 81*a^6*i^
4 - 194481*b^6*c^4, z, m)*((344064*a^5*b^5*c - 49152*a^6*b^4*g)/(32768*a^6*
b^2) - (x*(24576*a^5*b^4*d - 8192*a^6*b^3*h))/(4096*a^6*b)) - (15360*a^3*b^
4*d*e - 9216*a^4*b^3*d*i - 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*
b^2) + (x*(144*a^5*b*i^2 + 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 + 144*a^4*b^2
*g^2 - 2016*a^3*b^3*c*g - 480*a^4*b^2*e*i))/(4096*a^6*b)) - (x*(216*b^3*d^3
 - 8*a^3*h^3 - 315*b^3*c*d*e + 9*a^3*g*h*i - 216*a*b^2*d^2*h + 72*a^2*b*d*h
^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g - 63*a^2*b*c*h*i -
27*a^2*b*d*g*i - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z^4
 - 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z
^2 + 983040*a^7*b^5*e*g*z^2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*
z^2 - 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*
h*z - 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2
*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z
- 903168*a^4*b^5*c^2*h*z + 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z +
 2709504*a^3*b^6*c^2*d*z + 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7
560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^
4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e
*g*h^2 - 2268*a^4*b^2*c*g*i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2
*i + 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 5
76*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3
*g - 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 +
450*a^3*b^3*e^2*g^2 - 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*
b^3*e^3*i - 27648*a^2*b^4*d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 +
 22050*a*b^5*c^2*e^2 - 81*a^4*b^2*g^4 - 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 2
0736*a*b^5*d^4 - 81*a^6*i^4 - 194481*b^6*c^4, z, m), m, 1, 4) + ((b*f - a*j
)/(8*b^2) + (j*x^4)/(4*b) - (x^5*(7*b*c - a*g))/(32*a^2) - (x^6*(3*b*d - a*
h))/(16*a^2) - (x^7*(5*b*e - 3*a*i))/(32*a^2) + (x*(11*b*c + 3*a*g))/(32*a*
b) + (x^2*(5*b*d + a*h))/(16*a*b) + (x^3*(9*b*e + a*i))/(32*a*b))/(a^2 + b^
2*x^8 - 2*a*b*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,
x)
```

```
[Out] Timed out
```

3.154 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^3}\,dx$

Optimal. Leaf size=413

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3\right.}{128\sqrt{2}a^{11/4}b^{5/4}}$$

Rubi [A] time = 0.49, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{64\sqrt{2}a^{11/4}b^{5/4}}+\frac{(ah+3bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}-\frac{4af-x\left(2x(ah+3bd)+ag+7bc+5bex^2\right)}{32a^2b\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) -
 (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a + b
*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
 - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3
*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)
) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt
[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx = \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{\int \frac{-b(7bc+ag)-2b(3bd+ah)x-5b^2ex^2-4\ldots}{(a+bx^4)^2}}{8ab^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+a\ldots\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+a\ldots\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+a\ldots\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+a\ldots\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+a\ldots\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+a\ldots\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+a\ldots\right.}{32a^2b\left(a+bx^4\right)}$$

Mathematica [A] time = 0.43, size = 411, normalized size = 1.00

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8a^{5/4}h+5\sqrt{2}\sqrt{a}b^{3/4}e+24\sqrt[4]{a}bd+3\sqrt{2}a\sqrt[4]{b}g+21\sqrt{2}b^{5/4}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-8a^{5/4}h+5\sqrt{2}\sqrt{a}b^{3/4}e-24\sqrt[4]{a}bd+3\sqrt{2}a\sqrt[4]{b}g+21\sqrt{2}b^{5/4}c\right)-\frac{32a^{7/4}\sqrt{b}(a(f+x(g+hx))-bx(c+x(d+ex)))}{(a+bx^4)^2}+\frac{8a^{3/4}\sqrt{b}x(a(g+2hx)+7bc+bx(6d+5ex))}{a+bx^4}+\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(5\sqrt{a}\sqrt{b}e-3ag-21bc\right)+\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{256a^{11/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) + a*(g + 2*h*x)))/(a + b*x
^4) - (32*a^(7/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))
/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(5/4)*c + 24*a^(1/4)*b*d + 5*Sqrt[2]*Sqrt[
a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*g + 8*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b^(5/4)*c - 24*a^(1/4)*b*d + 5*Sqrt[2]*Sqr
t[a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*g - 8*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*
(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,
x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,
 x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [A] time = 0.20, size = 459, normalized size = 1.11

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^{2}d+4\sqrt{2}\sqrt{ab}\,abh+21\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+3\left(ab^{3}\right)^{\frac{1}{4}}abg+5\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^{3}b^{3}}+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}\,b^{2}d+4\sqrt{2}\sqrt{ab}\,abh+21\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+3\left(ab^{3}\right)^{\frac{1}{4}}abg+5\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^{3}b^{3}}+\frac{\sqrt{2}\left(21\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+3\left(ab^{3}\right)^{\frac{1}{4}}abg-5\left(ab^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^{3}b^{3}}-\frac{\sqrt{2}\left(21\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+3\left(ab^{3}\right)^{\frac{1}{4}}abg-5\left(ab^{3}\right)^{\frac{3}{4}}e\right)\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^{3}b^{3}}+\frac{5b^{2}ex^{7}+6b^{2}dx^{6}+2abhx^{6}+7b^{2}cx^{5}+abgx^{5}+9abex^{3}+10abdx^{2}-2a^{2}hx^{2}+11abcx-3a^{2}gx-4a^{2}f}{32\left(bx^{4}+a\right)^{2}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*
(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)
*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)
*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^
(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*
c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 2*a*b*h*x^6
+ 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2 - 2*a^2*h*x^2 + 11*a
*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

maple [A] time = 0.06, size = 561, normalized size = 1.36

$$\frac{h\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{16\sqrt{ab}\,ab}+\frac{3d\arctan\left(\sqrt{\frac{b}{a}}\,x^{2}\right)}{16\sqrt{ab}\,a^{2}}+\frac{5\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{5\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{5\sqrt{2}\,e\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128a^{2}b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128a^{2}b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256a^{2}b}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256a^{3}}+\frac{\frac{5bex^{7}}{32a^{2}}+\frac{(ah+3bd)x^{6}}{16a^{2}}+\frac{9ex^{3}}{32a}+\frac{(ag+7bc)x^{5}}{32a^{2}}-\frac{(ah-5bd)x^{2}}{16ab}-\frac{f}{8b}-\frac{(3ag-11bc)x}{32ab}}{\left(bx^{4}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (5/32/a^2*b*e*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a*
e*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8/b*f)/(b*x^4+a)
^2+3/128*(a/b)^(1/4)*2^(1/2)/a^2/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+21/128
*(a/b)^(1/4)*2^(1/2)/a^3*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+3/256*(a/b)^(1/4
)*2^(1/2)/a^2/b*g*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/
4)*2^(1/2)*x+(a/b)^(1/2)))+21/256*(a/b)^(1/4)*2^(1/2)/a^3*c*ln((x^2+(a/b)^(
1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/128*
(a/b)^(1/4)*2^(1/2)/a^2/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+21/128*(a/b)^(1
/4)*2^(1/2)/a^3*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/16/a/b/(a*b)^(1/2)*arct
an((1/a*b)^(1/2)*x^2)*h+3/16/(a*b)^(1/2)/a^2*d*arctan((1/a*b)^(1/2)*x^2)+5/
256/(a/b)^(1/4)*2^(1/2)/a^2/b*e*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/
(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+5/128/(a/b)^(1/4)*2^(1/2)/a^2/b*e*
arctan(2^(1/2)/(a/b)^(1/4)*x+1)+5/128/(a/b)^(1/4)*2^(1/2)/a^2/b*e*arctan(2^
(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.07, size = 446, normalized size = 1.08

$$\frac{5b^{2}ex^{7}+2\left(3b^{2}d+abh\right)x^{6}+9abex^{3}+\left(7b^{2}c+abg\right)x^{5}-4a^{2}f+2\left(5abd-a^{2}h\right)x^{2}+\left(11abc-3a^{2}g\right)x}{32\left(a^{2}b^{3}x^{8}+2a^{3}b^{2}x^{4}+a^{4}b\right)}+\frac{\frac{\sqrt{2}\left(21b^{\frac{3}{2}}c-5\sqrt{a}be+3a\sqrt{b}g\right)\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21b^{\frac{3}{2}}c-5\sqrt{a}be+3a\sqrt{b}g\right)\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+5\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+3\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g-24\sqrt{a}b^{\frac{3}{2}}d-8a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+5\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+3\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+24\sqrt{a}b^{\frac{3}{2}}d+8a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{256a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima"
)
```

```
[Out] 1/32*(5*b^2*e*x^7 + 2*(3*b^2*d + a*b*h)*x^6 + 9*a*b*e*x^3 + (7*b^2*c + a*b*
g)*x^5 - 4*a^2*f + 2*(5*a*b*d - a^2*h)*x^2 + (11*a*b*c - 3*a^2*g)*x)/(a^2*b
^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*
b*e + 3*a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g)
*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) +
 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*
a^(5/4)*b^(3/4)*g - 24*sqrt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*s
qrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 24*sqrt(a)*b^(3/2)
*d + 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(
a^2*b)
```

mupad [B] time = 5.69, size = 1686, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x)
```

```
[Out] ((9*e*x^3)/(32*a) - f/(8*b) + (x^5*(7*b*c + a*g))/(32*a^2) + (x^6*(3*b*d +
a*h))/(16*a^2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (x^2*(5*b*d - a*h))/(16*a*
b) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log((3024*b
^3*c*d^2 - 125*a*b^2*e^3 - 2205*b^3*c^2*e + 48*a^3*g*h^2 + 432*a*b^2*d^2*g
+ 336*a^2*b*c*h^2 - 45*a^2*b*e*g^2 + 2016*a*b^2*c*d*h - 630*a*b^2*c*e*g + 2
88*a^2*b*d*g*h)/(32768*a^6*b) - root(268435456*a^11*b^6*z^4 + 3145728*a^7*b
^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 + 6881280*a^6*b^5*c*e*z^2 + 524288*a^8*
b^3*h^2*z^2 + 4718592*a^6*b^5*d^2*z^2 - 258048*a^5*b^3*c*g*h*z - 774144*a^4
*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z + 51200*a^5*b^3*e^2*h*z - 903168*a^4*b
^4*c^2*h*z - 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z - 2709504*a^3*b
^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h - 40320*a^2*b^3*c*d*e*h - 8640*a^2*b^3*d^
2*e*g - 6720*a^3*b^2*c*e*h^2 + 6300*a^2*b^3*c*e^2*g - 960*a^4*b*e*g*h^2 - 6
0480*a*b^4*c*d^2*e + 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*
d^2*h^2 + 450*a^3*b^2*e^2*g^2 + 23814*a^2*b^3*c^2*g^2 + 27648*a^2*b^3*d^3*h
 + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 + 625*a^2*b^3*e^4 + 81*a^4*b*g^
4 + 20736*a*b^4*d^4 + 256*a^5*h^4 + 194481*b^5*c^4, z, k)*(root(268435456*a
^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 + 6881280*a^
6*b^5*c*e*z^2 + 524288*a^8*b^3*h^2*z^2 + 4718592*a^6*b^5*d^2*z^2 - 258048*a
^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z + 51200*a^5
*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z - 55296*a^5*b^3*d*g^2*z + 153600*a^4*
b^4*d*e^2*z - 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h - 40320*a^2*b^
3*c*d*e*h - 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 + 6300*a^2*b^3*c*e^
2*g - 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e + 3072*a^4*b*d*h^3 + 111132*a
*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 + 23814*a^2*b^3*c^
2*g^2 + 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 + 62
5*a^2*b^3*e^4 + 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 + 194481*b^5*c
^4, z, k)*((344064*a^5*b^4*c + 49152*a^6*b^3*g)/(32768*a^6*b) - (x*(24576*a
^5*b^4*d + 8192*a^6*b^3*h))/(4096*a^6*b)) + (15360*a^3*b^3*d*e + 5120*a^4*b
^2*e*h)/(32768*a^6*b) + (x*(7056*a^2*b^4*c^2 - 400*a^3*b^3*e^2 + 144*a^4*b^
2*g^2 + 2016*a^3*b^3*c*g))/(4096*a^6*b)) + (x*(216*b^3*d^3 + 8*a^3*h^3 - 31
5*b^3*c*d*e + 216*a*b^2*d^2*h + 72*a^2*b*d*h^2 - 105*a*b^2*c*e*h - 45*a*b^2
*d*e*g - 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^6*z^4 + 31457
28*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 + 6881280*a^6*b^5*c*e*z^2 + 524
288*a^8*b^3*h^2*z^2 + 4718592*a^6*b^5*d^2*z^2 - 258048*a^5*b^3*c*g*h*z - 77
```

```
4144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z + 51200*a^5*b^3*e^2*h*z - 9031
68*a^4*b^4*c^2*h*z - 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z - 27095
04*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h - 40320*a^2*b^3*c*d*e*h - 8640*a^
2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 + 6300*a^2*b^3*c*e^2*g - 960*a^4*b*e*g
*h^2 - 60480*a*b^4*c*d^2*e + 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*
a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 + 23814*a^2*b^3*c^2*g^2 + 27648*a^2*b
^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 + 625*a^2*b^3*e^4 + 81*
a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 + 194481*b^5*c^4, z, k), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
[Out] Timed out
```

3.155 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=463

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(3\sqrt{b}\,(ag+7bc)-\sqrt{a}\,(3ai+5be)\right)}{128\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(3\sqrt{b}\,(a\right.}{128\sqrt{2}\,a^{11/4}b^{7/}}$$

Rubi [A] time = 0.69, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(3\sqrt{b}\,(ag+7bc)-\sqrt{a}\,(3ai+5be)\right)}{128\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\left(3\sqrt{b}\,(ag+7bc)-\sqrt{a}\,(3ai+5be)\right)}{128\sqrt{2}\,a^{11/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}\,(ag+7bc)+\sqrt{a}\,(3ai+5be)\right)}{64\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}\,(ag+7bc)+\sqrt{a}\,(3ai+5be)\right)}{64\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{(ah+3bd)\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}-\frac{4af-x\left(2x(ah+3bd)+x^2(3ai+5be)+ag+7bc\right)}{32a^2b\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(8*a*b*(a + b*x
^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + (5*b*e + 3*a*i)*x^2)
)/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(1
6*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3
*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a*g) -
 Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]
*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(
128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+155x^6}{\left(a+bx^4\right)^3}\,dx = \frac{x\left(bc-ag+(bd-ah)x-(155a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{\int \frac{-b(7bc+ag)-\cdots}{\cdots}}{\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(155a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(155a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(155a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(155a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(155a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(155a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(155a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc\cdots\right.}{\cdots}$$

<u>**Mathematica**</u> **[A]** time = 0.68, size = 473, normalized size = 1.02

$$\frac{-\frac{32a^{7/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{\left(a+bx^4\right)^2}+\frac{8a^{3/4}b^{3/4}x(ag+ax(2h+3ix)+7bc+bx(6d+5ex))}{a+bx^4}-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i+24\sqrt[4]{a}b^{5/4}d+5\sqrt{2}\sqrt{a}be+3\sqrt{2}a\sqrt{b}g+21\sqrt{2}b^{3/2}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-8a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i-24\sqrt[4]{a}b^{5/4}d+5\sqrt{2}\sqrt{a}be+3\sqrt{2}a\sqrt{b}g+21\sqrt{2}b^{3/2}c\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3a^{3/2}i+5\sqrt{a}be-3a\sqrt{b}g-21b^{3/2}c\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-3a^{3/2}i-5\sqrt{a}be+3a\sqrt{b}g+21b^{3/2}c\right)}{256a^{11/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x
]
```

```
[Out] ((8*a^(3/4)*b^(3/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x) + a*x*(2*h + 3*i*x))
)/(a + b*x^4) - (32*a^(7/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g
 + x*(h + i*x)))))/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(3/2)*c + 24*a^(1/4)*b^(
5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*
h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sq
rt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*
a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b^(3/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b
]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] +
 Sqrt[2]*(21*b^(3/2)*c - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(7/4))
```

<u>**IntegrateAlgebraic**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a+bx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b
*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b
*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [A] time = 0.23, size = 661, normalized size = 1.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="g
iac")
```

```
[Out] 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + sqrt(2)*(a
*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 1/128
*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^
3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*
sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*
c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)
*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3
*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a^3*b^3) + 1/32*(3*a*b*i*x^7 + 5*b^2*x^7*e + 6*b^2*d*x^6 + 2*a
*b*h*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 - a^2*i*x^3 + 9*a*b*x^3*e + 10*a*b*d*x^2
 - 2*a^2*h*x^2 + 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

maple [A] time = 0.06, size = 716, normalized size = 1.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x
^5-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/
b*x-1/8/b*f)/(b*x^4+a)^2+3/128*(a/b)^(1/4)*2^(1/2)/a^2/b*g*arctan(2^(1/2)/(
a/b)^(1/4)*x+1)+21/128*(a/b)^(1/4)*2^(1/2)/a^3*c*arctan(2^(1/2)/(a/b)^(1/4)
*x+1)+3/128*(a/b)^(1/4)*2^(1/2)/a^2/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+21/
128*(a/b)^(1/4)*2^(1/2)/a^3*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+3/256*(a/b)^(
1/4)*2^(1/2)/a^2/b*g*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^
(1/4)*2^(1/2)*x+(a/b)^(1/2)))+21/256*(a/b)^(1/4)*2^(1/2)/a^3*c*ln((x^2+(a/b
```

```
)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/1
6/(a*b)^(1/2)/a/b*h*arctan((1/a*b)^(1/2)*x^2)+3/16/(a*b)^(1/2)/a^2*d*arctan
((1/a*b)^(1/2)*x^2)+3/128/a/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)*i+5/128/(a/b)^(1/4)*2^(1/2)/a^2/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x+1
)+3/128/a/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*i+5/128/(
a/b)^(1/4)*2^(1/2)/a^2/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+3/256/a/b^2/(a/b
)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)
*2^(1/2)*x+(a/b)^(1/2)))*i+5/256/(a/b)^(1/4)*2^(1/2)/a^2/b*e*ln((x^2-(a/b)^
(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))
```

maxima [A] time = 3.17, size = 497, normalized size = 1.07

$$\frac{\left(5\,b^{2}e+3\,abi\right)x^{7}+2\left(3\,b^{2}d+abh\right)x^{6}+\left(7\,b^{2}c+abg\right)x^{5}+\left(9\,abe-a^{2}i\right)x^{3}-4\,a^{2}f+2\left(5\,abd-a^{2}h\right)x^{2}+\left(11\,abc-3\,a^{2}g\right)x}{32\left(a^{2}b^{3}x^{8}+2\,a^{3}b^{2}x^{4}+a^{4}b\right)}+\frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+3\,\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+3\,\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}i-24\,\sqrt{a}b^{\frac{3}{2}}d-8\,a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+3\,\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+3\,\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}i+24\,\sqrt{a}b^{\frac{3}{2}}d+8\,a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{256\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="m
axima")
```

```
[Out] 1/32*((5*b^2*e + 3*a*b*i)*x^7 + 2*(3*b^2*d + a*b*h)*x^6 + (7*b^2*c + a*b*g)
*x^5 + (9*a*b*e - a^2*i)*x^3 - 4*a^2*f + 2*(5*a*b*d - a^2*h)*x^2 + (11*a*b*
c - 3*a^2*g)*x)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt(2)*(21*
b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 +
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2
)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^
(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqr
t(2)*a^(7/4)*b^(1/4)*i - 24*sqrt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*h)*arctan
(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c
+ 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqrt(2)*a^(
7/4)*b^(1/4)*i + 24*sqrt(a)*b^(3/2)*d + 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqr
t(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4
)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^2*b)
```

mupad [B] time = 5.75, size = 2680, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x)
```

```
[Out] symsum(log(- root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 4128768
*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 + 68812
80*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 614
40*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z - 7741
44*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*
a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 55296*a^
5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 3456*a^4
*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*
d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*
i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 264
60*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^
3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^
2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a
^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4
*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3
072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g
^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 + 19448
1*b^6*c^4, z, l)*(root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 41
28768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 +
```

```
6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2
+ 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z -
 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 5
5296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 552
96*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 345
6*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3
*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d
^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2
+ 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 67
20*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5
*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13
824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^
2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*
h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*
b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 +
194481*b^6*c^4, z, l)*((344064*a^5*b^5*c + 49152*a^6*b^4*g)/(32768*a^6*b^2)
 - (x*(24576*a^5*b^4*d + 8192*a^6*b^3*h))/(4096*a^6*b)) + (15360*a^3*b^4*d*
e + 9216*a^4*b^3*d*i + 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*b^2)
 - (x*(144*a^5*b*i^2 - 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 - 144*a^4*b^2*g^2
 - 2016*a^3*b^3*c*g + 480*a^4*b^2*e*i))/(4096*a^6*b)) - (27*a^4*i^3 + 125*a
*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e - 336*a^2*b^2*c*h^2 + 45*a^2*b^2
*e*g^2 + 225*a^2*b^2*e^2*i - 432*a*b^3*d^2*g + 1323*a*b^3*c^2*i + 135*a^3*b
*e*i^2 - 48*a^3*b*g*h^2 + 27*a^3*b*g^2*i + 378*a^2*b^2*c*g*i - 288*a^2*b^2*
d*g*h - 2016*a*b^3*c*d*h + 630*a*b^3*c*e*g)/(32768*a^6*b^2) - (x*(315*b^3*c
*d*e - 8*a^3*h^3 - 216*b^3*d^3 + 9*a^3*g*h*i - 216*a*b^2*d^2*h - 72*a^2*b*d
*h^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g + 63*a^2*b*c*h*i
+ 27*a^2*b*d*g*i + 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z
^4 + 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h
*z^2 + 983040*a^7*b^5*e*g*z^2 + 6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^
2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*
g*h*z + 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i
^2*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*
z - 903168*a^4*b^5*c^2*h*z - 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z
 - 2709504*a^3*b^6*c^2*d*z - 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i +
 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*
a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2
*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d
^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g -
 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c
^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2
+ 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^
3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3
 + 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 +
 20736*a*b^5*d^4 + 81*a^6*i^4 + 194481*b^6*c^4, z, l), l, 1, 4) + ((x^5*(7*
b*c + a*g))/(32*a^2) - f/(8*b) + (x^6*(3*b*d + a*h))/(16*a^2) + (x^7*(5*b*e
 + 3*a*i))/(32*a^2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (x^2*(5*b*d - a*h))/(
16*a*b) + (x^3*(9*b*e - a*i))/(32*a*b))/(a^2 + b^2*x^8 + 2*a*b*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.156 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=480

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(a\right.}{128\sqrt{2}a^{11/4}b^{7/}}$$

Rubi [A] time = 0.67, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.244, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{4a(aj+bf)-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{32a^2b^2\left(a+bx^4\right)}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{64\sqrt{2}a^{11/4}b^{7/4}}+\frac{(ah+3bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3
,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(8*a*b*
(a + b*x^4)^2) - (4*a*(b*f + a*j) - x*(b*(7*b*c + a*g) + 2*b*(3*b*d + a*h)*
x + b*(5*b*e + 3*a*i)*x^2))/(32*a^2*b^2*(a + b*x^4)) + ((3*b*d + a*h)*ArcTa
n[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g)
+ Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqr
t[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i
))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) -
 ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqr
t[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+156x^6+jx^7}{\left(a+bx^4\right)^3}\,dx = \frac{x\left(bc-ag+(bd-ah)x-(156a-be)x^2+(bf-aj)x^3\right)}{8ab\left(a+bx^4\right)^2} - $$

$$= \frac{x\left(bc-ag+(bd-ah)x-(156a-be)x^2+(bf-aj)x^3\right)}{8ab\left(a+bx^4\right)^2} - $$

$$= \frac{x\left(bc-ag+(bd-ah)x-(156a-be)x^2+(bf-aj)x^3\right)}{8ab\left(a+bx^4\right)^2} - $$

$$= \frac{x\left(bc-ag+(bd-ah)x-(156a-be)x^2+(bf-aj)x^3\right)}{8ab\left(a+bx^4\right)^2} - $$

$$= \frac{x\left(bc-ag+(bd-ah)x-(156a-be)x^2+(bf-aj)x^3\right)}{8ab\left(a+bx^4\right)^2} - $$

$$= \frac{x\left(bc-ag+(bd-ah)x-(156a-be)x^2+(bf-aj)x^3\right)}{8ab\left(a+bx^4\right)^2} - $$

$$= \frac{x\left(bc-ag+(bd-ah)x-(156a-be)x^2+(bf-aj)x^3\right)}{8ab\left(a+bx^4\right)^2} - $$

$$= \frac{x\left(bc-ag+(bd-ah)x-(156a-be)x^2+(bf-aj)x^3\right)}{8ab\left(a+bx^4\right)^2} - $$

Mathematica [A] time = 0.51, size = 500, normalized size = 1.04

$$\frac{-2\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i+24\sqrt[4]{a}b^{5/4}d+5\sqrt{2}\sqrt{a}be+3\sqrt{2}a\sqrt{b}g+21\sqrt{2}b^{3/2}c\right)+2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-8a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i-24\sqrt[4]{a}b^{5/4}d+5\sqrt{2}\sqrt{a}be+3\sqrt{2}a\sqrt{b}g+21\sqrt{2}b^{3/2}c\right)+\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3a^{3/2}i+5\sqrt{a}be-3a\sqrt{b}g-21b^{3/2}c\right)+\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-3a^{3/2}i-5\sqrt{a}be+3a\sqrt{b}g+21b^{3/2}c\right)+\frac{32a^{7/4}\left(a^2j-ab(f+x(g+x(h+ix)))+b^2x(c+x(d+ex))\right)}{\left(a+bx^4\right)^2}+\frac{8a^{3/4}\left(-8a^2j+abx(g+x(2h+3ix))+b^2x(7c+x(6d+5ex))\right)}{a+bx^4}}{256a^{11/4}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*(-8*a^2*j + b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h +
 3*i*x))))/(a + b*x^4) + (32*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b
*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^2 - 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*
c + 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g +
8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)] + 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2
]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(
3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b^(3
/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(21*b^(3/2)*c - 5*Sqrt[a
]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(256*a^(11/4)*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a+bx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a + b*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a + b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 693, normalized size = 1.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="giac")
```

```
[Out] 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + sqrt(2)*(a
*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 1/128
*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^
3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*
sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*
c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)
*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3
*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a^3*b^3) + 1/32*(3*a*b^2*i*x^7 + 5*b^3*x^7*e + 6*b^3*d*x^6 + 2
*a*b^2*h*x^6 + 7*b^3*c*x^5 + a*b^2*g*x^5 - 8*a^2*b*j*x^4 - a^2*b*i*x^3 + 9*
a*b^2*x^3*e + 10*a*b^2*d*x^2 - 2*a^2*b*h*x^2 + 11*a*b^2*c*x - 3*a^2*b*g*x -
 4*a^2*b*f - 4*a^3*j)/((b*x^4 + a)^2*a^2*b^2)
```

maple [A] time = 0.06, size = 731, normalized size = 1.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x
^5-1/4/b*j*x^4-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*
g-11*b*c)/a/b*x-1/8*(a*j+b*f)/b^2)/(b*x^4+a)^2+3/128*(a/b)^(1/4)*2^(1/2)/a^
2/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+21/128*(a/b)^(1/4)*2^(1/2)/a^3*c*arct
an(2^(1/2)/(a/b)^(1/4)*x-1)+3/256*(a/b)^(1/4)*2^(1/2)/a^2/b*g*ln((x^2+(a/b)
^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+21/2
56*(a/b)^(1/4)*2^(1/2)/a^3*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^
```

2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/128*(a/b)^(1/4)*2^(1/2)/a^2/b*g*arc
tan(2^(1/2)/(a/b)^(1/4)*x+1)+21/128*(a/b)^(1/4)*2^(1/2)/a^3*c*arctan(2^(1/2
)/(a/b)^(1/4)*x+1)+1/16/(a*b)^(1/2)/a/b*h*arctan((1/a*b)^(1/2)*x^2)+3/16/(a
*b)^(1/2)/a^2*d*arctan((1/a*b)^(1/2)*x^2)+3/256/(a/b)^(1/4)*2^(1/2)/a/b^2*i
*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2)))+5/256/(a/b)^(1/4)*2^(1/2)/a^2/b*e*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/128/(a/b)^(1/4)*2^(1
/2)/a/b^2*i*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+5/128/(a/b)^(1/4)*2^(1/2)/a^2/b
*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+3/128/(a/b)^(1/4)*2^(1/2)/a/b^2*i*arctan
(2^(1/2)/(a/b)^(1/4)*x+1)+5/128/(a/b)^(1/4)*2^(1/2)/a^2/b*e*arctan(2^(1/2)/
(a/b)^(1/4)*x+1)

maxima [A] time = 3.09, size = 535, normalized size = 1.11

$$-\frac{8\,a^{2}bjx^{4}-\left(5\,b^{3}e+3\,ab^{2}i\right)x^{7}-2\left(3\,b^{3}d+ab^{2}h\right)x^{6}-\left(7\,b^{3}c+ab^{2}g\right)x^{5}+4\,a^{2}bf+4\,a^{3}j-\left(9\,ab^{2}e-a^{2}bi\right)x^{3}-2\left(5\,ab^{2}d-a^{2}bh\right)x^{2}-\left(11\,ab^{2}c-3\,a^{2}bg\right)x}{32\left(a^{2}b^{4}x^{8}+2\,a^{3}b^{3}x^{4}+a^{4}b^{2}\right)}+\frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+3\,\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+3\,\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}i-24\,\sqrt{a}b^{\frac{3}{2}}d-8\,a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+3\,\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+3\,\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}i+24\,\sqrt{a}b^{\frac{3}{2}}d+8\,a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{256\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="maxima")

[Out] -1/32*(8*a^2*b*j*x^4 - (5*b^3*e + 3*a*b^2*i)*x^7 - 2*(3*b^3*d + a*b^2*h)*x^
6 - (7*b^3*c + a*b^2*g)*x^5 + 4*a^2*b*f + 4*a^3*j - (9*a*b^2*e - a^2*b*i)*x
^3 - 2*(5*a*b^2*d - a^2*b*h)*x^2 - (11*a*b^2*c - 3*a^2*b*g)*x)/(a^2*b^4*x^8
 + 2*a^3*b^3*x^4 + a^4*b^2) + 1/256*(sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e
+ 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
+ sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*
sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt
(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4
)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqrt(2)*a^(7/4)*b^(1/4)*i - 2
4*sqrt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x
+ sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqr
t(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4
)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqrt(2)*a^(7/4)*b^(1/4)*i + 24*sqrt(a
)*b^(3/2)*d + 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2
)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^
(3/4)))/(a^2*b)

mupad [B] time = 5.79, size = 2695, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3
,x)

[Out] symsum(log(- root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 4128768
*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 + 68812
80*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 614
40*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z - 7741
44*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 55296*
a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 55296*a^
5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 3456*a^4
*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*
d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*
i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 264
60*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^
3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^
2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a
^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4
*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3

```
072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g
^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 + 19448
1*b^6*c^4, z, m)*(root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 41
28768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 +
6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2
+ 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z -
 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 5
5296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 552
96*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 345
6*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*a^3
*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b^3*d
^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g*i^2
+ 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*g - 67
20*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 60480*a*b^5
*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*i^2 + 13
824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 + 23814*a^
2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^2*b^4*d^3*
h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^2 + 81*a^4*
b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 81*a^6*i^4 +
194481*b^6*c^4, z, m)*((344064*a^5*b^5*c + 49152*a^6*b^4*g)/(32768*a^6*b^2)
 - (x*(24576*a^5*b^4*d + 8192*a^6*b^3*h))/(4096*a^6*b)) + (15360*a^3*b^4*d*
e + 9216*a^4*b^3*d*i + 5120*a^4*b^3*e*h + 3072*a^5*b^2*h*i)/(32768*a^6*b^2)
 - (x*(144*a^5*b*i^2 - 7056*a^2*b^4*c^2 + 400*a^3*b^3*e^2 - 144*a^4*b^2*g^2
 - 2016*a^3*b^3*c*g + 480*a^4*b^2*e*i))/(4096*a^6*b)) - (27*a^4*i^3 + 125*a
*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e - 336*a^2*b^2*c*h^2 + 45*a^2*b^2
*e*g^2 + 225*a^2*b^2*e^2*i - 432*a*b^3*d^2*g + 1323*a*b^3*c^2*i + 135*a^3*b
*e*i^2 - 48*a^3*b*g*h^2 + 27*a^3*b*g^2*i + 378*a^2*b^2*c*g*i - 288*a^2*b^2*
d*g*h - 2016*a*b^3*c*d*h + 630*a*b^3*c*e*g)/(32768*a^6*b^2) - (x*(315*b^3*c
*d*e - 8*a^3*h^3 - 216*b^3*d^3 + 9*a^3*g*h*i - 216*a*b^2*d^2*h - 72*a^2*b*d
*h^2 + 189*a*b^2*c*d*i + 105*a*b^2*c*e*h + 45*a*b^2*d*e*g + 63*a^2*b*c*h*i
+ 27*a^2*b*d*g*i + 15*a^2*b*e*g*h))/(4096*a^6*b))*root(268435456*a^11*b^7*z
^4 + 589824*a^8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h
*z^2 + 983040*a^7*b^5*e*g*z^2 + 6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^
2*z^2 + 4718592*a^6*b^6*d^2*z^2 + 61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*
g*h*z + 184320*a^5*b^4*d*e*i*z - 774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i
^2*z - 18432*a^6*b^3*g^2*h*z + 55296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*
z - 903168*a^4*b^5*c^2*h*z - 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z
 - 2709504*a^3*b^6*c^2*d*z - 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i +
 7560*a^3*b^3*c*e*g*i - 5760*a^3*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*
a^4*b^2*e*g^2*i - 5184*a^3*b^3*d^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2
*e*g*h^2 + 2268*a^4*b^2*c*g*i^2 + 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d
^2*i - 8640*a^2*b^4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g -
 576*a^5*b*g*h^2*i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c
^3*g + 1350*a^4*b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2
+ 450*a^3*b^3*e^2*g^2 + 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^
3*b^3*e^3*i + 27648*a^2*b^4*d^3*h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3
 + 22050*a*b^5*c^2*e^2 + 81*a^4*b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 +
 20736*a*b^5*d^4 + 81*a^6*i^4 + 194481*b^6*c^4, z, m), m, 1, 4) + ((x^5*(7*
b*c + a*g))/(32*a^2) - (j*x^4)/(4*b) - (b*f + a*j)/(8*b^2) + (x^6*(3*b*d +
a*h))/(16*a^2) + (x^7*(5*b*e + 3*a*i))/(32*a^2) + (x*(11*b*c - 3*a*g))/(32*
a*b) + (x^2*(5*b*d - a*h))/(16*a*b) + (x^3*(9*b*e - a*i))/(32*a*b))/(a^2 +
b^2*x^8 + 2*a*b*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x
)
```

```
[Out] Timed out
```

3.157 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4} \, dx$

Optimal. Leaf size=293

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right.}{32a^{7/2}b^{3/2}}$$

Rubi [A] time = 0.43, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(2x(5bd-ah)-ag+11bc+9bex^2\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+45bex^2\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3)
+ (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a -
b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x + 9*b*e*x^2))/(96*a^
2*b*(a - b*x^4)^2) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/
4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e -
7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((5*b*d - a*h
)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^4}\,dx = \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-b(11bc-ag)-2b(5bd-ah)x-9b^2ex^2-}{\left(a-bx^4\right)^3}}{12ab^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{8af+x\left(11bc-ag+2(5bd-\right.}{96a^2b\left(a-bx^4\right)^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+12(5bd-ah\right.}{384a^3b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+12(5bd-ah\right.}{384a^3b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+12(5bd-ah\right.}{384a^3b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+12(5bd-ah\right.}{384a^3b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+12(5bd-ah\right.}{384a^3b\left(a-bx^4\right)}$$

Mathematica [A] time = 0.49, size = 360, normalized size = 1.23

$$\frac{-3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(-8a^{5/4}h+15\sqrt{a}b^{3/4}e+40\sqrt[4]{a}bd-7a\sqrt[4]{b}g+77b^{5/4}c\right)+3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(8a^{5/4}h+15\sqrt{a}b^{3/4}e-40\sqrt[4]{a}bd-7a\sqrt[4]{b}g+77b^{5/4}c\right)+\frac{128a^{11/4}\sqrt{b}(a(f+x(g+hx))+bx(c+x(d+ex)))}{\left(a-bx^4\right)^3}+\frac{16a^{7/4}\sqrt{b}x(-a(g+2hx)+11bc+bx(10d+9ex))}{\left(a-bx^4\right)^2}+\frac{4a^{3/4}\sqrt{b}x\left(-7ag-12ahx+77bc+60bdx+45bex^2\right)}{a-bx^4}+6\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)-24\sqrt[4]{a}(ah-5bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{1536a^{15/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*x*(77*b*c - 7*a*g + 60*b*d*x - 12*a*h*x + 45*b*e*x^2))/
(a - b*x^4) + (16*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) - a*(g + 2
*h*x)))/(a - b*x^4)^2 + (128*a^(11/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f
 + x*(g + h*x))))/(a - b*x^4)^3 + 6*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e
- 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(77*b^(5/4)*c + 40*a^(1/4)*b*d + 1
5*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g - 8*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x]
 + 3*(77*b^(5/4)*c - 40*a^(1/4)*b*d + 15*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g
+ 8*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 24*a^(1/4)*(-5*b*d + a*h)*Log[Sqr
t[a] + Sqrt[b]*x^2])/(1536*a^(15/4)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,
x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,
 x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [B] time = 0.26, size = 501, normalized size = 1.71

$$-\frac{\sqrt{2}\left(77b^2c-7abg-40\sqrt{2}\left(-ab^3\right)^{\frac14}bd+8\sqrt{2}\left(-ab^3\right)^{\frac14}ah+15\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{512\left(-ab^3\right)^{\frac34}a^3}-\frac{\sqrt{2}\left(77b^2c-7abg+40\sqrt{2}\left(-ab^3\right)^{\frac14}bd-8\sqrt{2}\left(-ab^3\right)^{\frac14}ah-15\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{512\left(-ab^3\right)^{\frac34}a^3}-\frac{\sqrt{2}\left(77b^2c-7abg-15\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac34}a^3}+\frac{\sqrt{2}\left(77b^2c-7abg-15\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac34}a^3}-\frac{45b^3ex^{11}+60b^3dx^{10}-12ab^2hx^{10}+77b^3cx^9-7ab^2gx^9-126ab^2ex^7-160ab^2dx^6+32a^2bhx^6-198ab^2cx^5+18a^2bgx^5+113a^2bex^3+132a^2bdx^2+12a^3hx^2+153a^2bcx+21a^3gx+32a^3f}{384\left(bx^4-a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 8*sqrt
(2)*(-a*b^3)^(1/4)*a*h + 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2
*c - 7*a*b*g + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 8*sqrt(2)*(-a*b^3)^(1/4)*a*h
 - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b
)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqr
t(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)
*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e)*log(x^2 - s
qrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^3*x^
11*e + 60*b^3*d*x^10 - 12*a*b^2*h*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126
*a*b^2*x^7*e - 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 - 198*a*b^2*c*x^5 + 18*a^2*
b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 12*a^3*h*x^2 + 153*a^2*b*c*x
+ 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

maple [A] time = 0.06, size = 434, normalized size = 1.48

$$\frac{h\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{64\sqrt{ab}\,a^2b}-\frac{5d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{64\sqrt{ab}\,a^3}-\frac{15e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac14}}\right)}{256\left(\frac{a}{b}\right)^{\frac14}a^3b}+\frac{15e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac14}}{x-\left(\frac{a}{b}\right)^{\frac14}}\right)}{512\left(\frac{a}{b}\right)^{\frac14}a^3b}-\frac{7\left(\frac{a}{b}\right)^{\frac14}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac14}}\right)}{256a^3b}-\frac{7\left(\frac{a}{b}\right)^{\frac14}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac14}}{x-\left(\frac{a}{b}\right)^{\frac14}}\right)}{512a^3b}+\frac{77\left(\frac{a}{b}\right)^{\frac14}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac14}}\right)}{256a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac14}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac14}}{x-\left(\frac{a}{b}\right)^{\frac14}}\right)}{512a^4}+\frac{-\frac{15b^2ex^{11}}{128a^3}+\frac{(ah-5bd)bx^{10}}{32a^3}+\frac{21bex^7}{64a^2}+\frac{7\left(ag-11bc\right)bx^9}{384a^3}-\frac{(ah-5bd)x^6}{12a^2}-\frac{113ex^3}{384a}-\frac{3\left(ag-11bc\right)x^5}{64a^2}-\frac{(ah+11bd)x^2}{32ab}-\frac{f}{12b}-\frac{\left(7ag+51bc\right)x}{128ab}}{\left(bx^4-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128/a^3*b^2*e*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(a*g-11*b*c)/a^3*
b*x^9+21/64/a^2*b*e*x^7-1/12/a^2*(a*h-5*b*d)*x^6-3/64*(a*g-11*b*c)/a^2*x^5-
113/384/a*e*x^3-1/32*(a*h+11*b*d)/a/b*x^2-1/128*(7*a*g+51*b*c)/a/b*x-1/12/b
*f)/(b*x^4-a)^3-7/256*(a/b)^(1/4)/a^3/b*g*arctan(1/(a/b)^(1/4)*x)+77/256*(a
/b)^(1/4)/a^4*c*arctan(1/(a/b)^(1/4)*x)-7/512*(a/b)^(1/4)/a^3/b*g*ln((x+(a/
b)^(1/4))/(x-(a/b)^(1/4)))+77/512*(a/b)^(1/4)/a^4*c*ln((x+(a/b)^(1/4))/(x-(
a/b)^(1/4)))+1/64/a^2/b/(a*b)^(1/2)*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^
2-a))*h-5/64/(a*b)^(1/2)/a^3*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))
-15/256/(a/b)^(1/4)/a^3/b*e*arctan(1/(a/b)^(1/4)*x)+15/512/(a/b)^(1/4)/a^3/
b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 3.18, size = 389, normalized size = 1.33

$$-\frac{45\,b^3ex^{11}-126\,ab^2ex^7+12\left(5\,b^3d-ab^2h\right)x^{10}+7\left(11\,b^3c-ab^2g\right)x^9+113\,a^2bex^3-32\left(5\,ab^2d-a^2bh\right)x^6-18\left(11\,ab^2c-a^2bg\right)x^5+32\,a^3f+12\left(11\,a^2bd+a^3h\right)x^2+3\left(51\,a^2bc+7\,a^3g\right)x}{384\left(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b\right)}+\frac{\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{512\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima
")
```

```
[Out] -1/384*(45*b^3*e*x^11 - 126*a*b^2*e*x^7 + 12*(5*b^3*d - a*b^2*h)*x^10 + 7*(
11*b^3*c - a*b^2*g)*x^9 + 113*a^2*b*e*x^3 - 32*(5*a*b^2*d - a^2*b*h)*x^6 -
18*(11*a*b^2*c - a^2*b*g)*x^5 + 32*a^3*f + 12*(11*a^2*b*d + a^3*h)*x^2 + 3*
(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a
^6*b) + 1/512*(8*(5*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b))
 - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*b^(
3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c + 15*sqrt(a)*
b*e - 7*a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + s
qrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

mupad [B] time = 5.99, size = 1747, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^6*z^4 - 1211105280*a^8*b^5*c*e*z^2 + 3
35544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 - 838860800*a^8*b^5*d^
2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z + 17661952*a^6
*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 80281
6*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*h*z + 4014080*a^6*b^3*d*g^2*z + 184
32000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h + 2956800*a^2*b^3*c*d*e*h +
672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 - 485100*a^2*b^3*c*e^2*g +
26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e - 81920*a^4*b*d*h^3 + 12782924*
a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 - 1743126*a^2*
b^3*c^2*g^2 - 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^4*
c^2*e^2 - 50625*a^2*b^3*e^4 - 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a^5
*h^4 - 35153041*b^5*c^4, z, k)*(root(68719476736*a^15*b^6*z^4 - 1211105280*
a^8*b^5*c*e*z^2 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 - 8
38860800*a^8*b^5*d^2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d
*g*z + 17661952*a^6*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*
b^4*c^2*h*z - 802816*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*h*z + 4014080*a^
6*b^3*d*g^2*z + 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h + 2956800
*a^2*b^3*c*d*e*h + 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 - 485100
*a^2*b^3*c*e^2*g + 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e - 81920*a^4*
b*d*h^3 + 12782924*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2
```

```
*g^2 - 1743126*a^2*b^3*c^2*g^2 - 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g
^3 + 2668050*a*b^4*c^2*e^2 - 50625*a^2*b^3*e^4 - 2401*a^4*b*g^4 + 2560000*a
*b^4*d^4 + 4096*a^5*h^4 - 35153041*b^5*c^4, z, k)*((20185088*a^7*b^4*c - 18
35008*a^8*b^3*g)/(2097152*a^9*b) - (x*(655360*a^7*b^4*d - 131072*a^8*b^3*h)
)/(131072*a^9*b)) - (614400*a^4*b^3*d*e - 122880*a^5*b^2*e*h)/(2097152*a^9*
b) + (x*(189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 + 1568*a^5*b^2*g^2 - 34496*a
^4*b^3*c*g))/(131072*a^9*b)) - (3375*a*b^2*e^3 + 123200*b^3*c*d^2 - 88935*b
^3*c^2*e - 448*a^3*g*h^2 - 11200*a*b^2*d^2*g + 4928*a^2*b*c*h^2 - 735*a^2*b
*e*g^2 - 49280*a*b^2*c*d*h + 16170*a*b^2*c*e*g + 4480*a^2*b*d*g*h)/(2097152
*a^9*b) - (x*(4000*b^3*d^3 - 32*a^3*h^3 - 5775*b^3*c*d*e - 2400*a*b^2*d^2*h
 + 480*a^2*b*d*h^2 + 1155*a*b^2*c*e*h + 525*a*b^2*d*e*g - 105*a^2*b*e*g*h))
/(131072*a^9*b))*root(68719476736*a^15*b^6*z^4 - 1211105280*a^8*b^5*c*e*z^2
 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 - 838860800*a^8*b^
5*d^2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z + 17661952
*a^6*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 8
02816*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*h*z + 4014080*a^6*b^3*d*g^2*z +
 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h + 2956800*a^2*b^3*c*d*e*
h + 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 - 485100*a^2*b^3*c*e^2*
g + 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e - 81920*a^4*b*d*h^3 + 12782
924*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 - 1743126*
a^2*b^3*c^2*g^2 - 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*
b^4*c^2*e^2 - 50625*a^2*b^3*e^4 - 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096
*a^5*h^4 - 35153041*b^5*c^4, z, k), k, 1, 4) + (f/(12*b) + (113*e*x^3)/(384
*a) - (3*x^5*(11*b*c - a*g))/(64*a^2) - (x^6*(5*b*d - a*h))/(12*a^2) + (7*b
*x^9*(11*b*c - a*g))/(384*a^3) + (x*(51*b*c + 7*a*g))/(128*a*b) + (b*x^10*(
5*b*d - a*h))/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (x^2*(11*b*d + a*h))/(
32*a*b) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^
8)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

3.158 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=331

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}\,(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}\,(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+$$

Rubi [A] time = 0.57, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}\,(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}\,(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(2x(5bd-ah)+3x^2(3be-ai)-ag+11bc\right)+8af}{96a^2b\left(a-bx^4\right)^2}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+15x^2(3be-ai)\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(12*a*b*(a - b*
x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a*i)*x^2)
)/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x +
3*(3*b*e - a*i)*x^2))/(96*a^2*b*(a - b*x^4)^2) + (((7*Sqrt[b]*(11*b*c - a*g
))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^
(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTanh[(b^
(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+158x^6}{\left(a-bx^4\right)^4}dx = \frac{x\left(bc+ag+(bd+ah)x+(158a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-b(11bc\cdots}{\cdots}}{\cdots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(158a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{8af+x\cdots}{\cdots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(158a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(158a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(158a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(158a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(158a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.54, size = 422, normalized size = 1.27

$$\frac{3\sqrt[4]{a}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(8a^{5/4}\sqrt[4]{b}h+5a^{3/2}i-40\sqrt[4]{a}b^{5/4}d-15\sqrt{a}be+7a\sqrt{b}g-77b^{3/2}c\right)-3\sqrt[4]{a}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-8a^{5/4}\sqrt[4]{b}h+5a^{3/2}i+40\sqrt[4]{a}b^{5/4}d-15\sqrt{a}be+7a\sqrt{b}g-77b^{3/2}c\right)+6\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5a^{3/2}i-15\sqrt{a}be-7a\sqrt{b}g+77b^{3/2}c\right)+\frac{128a^3b^{3/4}(a(f+x(g+x(h+ix)))+bx(c+x(d+ex)))}{\left(a-bx^4\right)^3}-\frac{16a^2b^{3/4}x(a(g+x(2h+3ix))-b(11c+x(10d+9ex)))}{\left(a-bx^4\right)^2}-\frac{4ab^{3/4}x(7ag+3ax(4h+5ix)-77bc-15bx(4d+3ex))}{a-bx^4}-24\sqrt{a}\sqrt[4]{b}(ah-5bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{1536a^4b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x
]
```

```
[Out] ((-4*a*b^(3/4)*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i
*x)))/(a - b*x^4) - (16*a^2*b^(3/4)*x*(-(b*(11*c + x*(10*d + 9*e*x))) + a*(
g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*b^(3/4)*(b*x*(c + x*(d + e*
x)) + a*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^3 + 6*a^(1/4)*(77*b^(3/2)*c
 - 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)
] + 3*a^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*
Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] - 3
*a^(1/4)*(-77*b^(3/2)*c + 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*Sqrt[
b]*g - 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 24*Sqr
t[a]*b^(1/4)*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(1536*a^4*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{\left(a - bx^4\right)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b
*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b
*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.21, size = 727, normalized size = 2.20

$$-\frac{5}{1024} i \left(\frac{2 \sqrt{2} \left(-ab^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3 b^4} - \frac{\sqrt{2} \left(-ab^3\right)^{\frac{3}{4}} \log\left(x^2 + \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{a^3 b^4} \right) - \frac{5}{1024} i \left(\frac{2 \sqrt{2} \left(-ab^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3 b^4} + \frac{\sqrt{2} \left(-ab^3\right)^{\frac{3}{4}} \log\left(x^2 - \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{a^3 b^4} \right) - \frac{\sqrt{2} \left(77 b^2 c - 7 abg - 40 \sqrt{2} \left(-ab^3\right)^{\frac{1}{4}} bd + 8 \sqrt{2} \left(-ab^3\right)^{\frac{1}{4}} ah + 15 \sqrt{-ab} be\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512 \left(-ab^3\right)^{\frac{3}{4}} a^3} - \frac{\sqrt{2} \left(77 b^2 c - 7 abg + 40 \sqrt{2} \left(-ab^3\right)^{\frac{1}{4}} bd - 8 \sqrt{2} \left(-ab^3\right)^{\frac{1}{4}} ah - 15 \sqrt{-ab} be\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512 \left(-ab^3\right)^{\frac{3}{4}} a^3} - \frac{\sqrt{2} \left(77 b^2 c - 7 abg - 15 \sqrt{-ab} be\right) \log\left(x^2 + \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024 \left(-ab^3\right)^{\frac{3}{4}} a^3} + \frac{\sqrt{2} \left(77 b^2 c - 7 abg - 15 \sqrt{-ab} be\right) \log\left(x^2 - \sqrt{2} x \left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024 \left(-ab^3\right)^{\frac{3}{4}} a^3} + \frac{15 ab^2 ix^{11} - 45 b^3 x^{11} e - 60 b^3 dx^{10} + 12 ab^2 hx^{10} - 77 b^3 cx^9 + 7 ab^2 gx^9 - 42 a^2 bix^7 + 126 ab^2 x^7 e + 160 ab^2 dx^6 - 32 a^2 bhx^6 + 198 ab^2 cx^5 - 18 a^2 bgx^5 - 5 a^3 ix^3 - 113 a^2 bx^3 e - 132 a^2 bdx^2 - 12 a^3 hx^2 - 153 a^2 bcx - 21 a^3 gx - 32 a^3 f}{384 \left(bx^4 - a\right)^3 a^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
giac")
```

```
[Out] -5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)
*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4)) - 5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^4)
+ sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^
3*b^4)) - 1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d
 + 8*sqrt(2)*(-a*b^3)^(1/4)*a*h + 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2
)*(77*b^2*c - 7*a*b*g + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 8*sqrt(2)*(-a*b^3)^
(1/4)*a*h - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/
4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g
 - 15*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b
^3)^(3/4)*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e)*lo
```

```
g(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/384*(
15*a*b^2*i*x^11 - 45*b^3*x^11*e - 60*b^3*d*x^10 + 12*a*b^2*h*x^10 - 77*b^3*
c*x^9 + 7*a*b^2*g*x^9 - 42*a^2*b*i*x^7 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6
- 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 - 18*a^2*b*g*x^5 - 5*a^3*i*x^3 - 113*a^2
*b*x^3*e - 132*a^2*b*d*x^2 - 12*a^3*h*x^2 - 153*a^2*b*c*x - 21*a^3*g*x - 32
*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

maple [A] time = 0.06, size = 522, normalized size = 1.58

$$\frac{h\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{64\sqrt{ab}\,a^2b}-\frac{5d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{64\sqrt{ab}\,a^3}+\frac{5i\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b^2}-\frac{5i\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b^2}-\frac{15e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}+\frac{15e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256a^3b}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^3b}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4}+\frac{\frac{5(ai-3be)b\,x^{11}}{128a^3}+\frac{(ah-5bd)b\,x^{10}}{32a^3}+\frac{7(ag-11bc)b\,x^9}{384a^3}-\frac{7(ai-3be)x^7}{64a^2}-\frac{(ah-5bd)x^6}{12a^2}-\frac{3(ag-11bc)x^5}{64a^2}-\frac{(5ai+113be)x^3}{384ab}-\frac{(ah+11bd)x^2}{32ab}-\frac{f}{12b}-\frac{(7ag+51bc)x}{128ab}}{\left(b\,x^4-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i-3*b*e)/a^3*b*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(a*g-11*b*c
)/a^3*b*x^9-7/64/a^2*(a*i-3*b*e)*x^7-1/12*(a*h-5*b*d)/a^2*x^6-3/64*(a*g-11*
b*c)/a^2*x^5-1/384*(5*a*i+113*b*e)/a/b*x^3-1/32*(a*h+11*b*d)/a/b*x^2-1/128*
(7*a*g+51*b*c)/a/b*x-1/12/b*f)/(b*x^4-a)^3-7/512*(a/b)^(1/4)/a^3/b*g*ln((x+
(a/b)^(1/4))/(x-(a/b)^(1/4)))+77/512*(a/b)^(1/4)/a^4*c*ln((x+(a/b)^(1/4))/(
x-(a/b)^(1/4)))-7/256*(a/b)^(1/4)/a^3/b*g*arctan(1/(a/b)^(1/4)*x)+77/256*(a
/b)^(1/4)/a^4*c*arctan(1/(a/b)^(1/4)*x)+1/64/(a*b)^(1/2)/a^2/b*h*ln(((a*b)^
(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-5/64/(a*b)^(1/2)/a^3*d*ln(((a*b)^(1/2)*x
^2-a)/(-(a*b)^(1/2)*x^2-a))-5/512/a^2/b^2/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x
-(a/b)^(1/4)))*i+15/512/(a/b)^(1/4)/a^3/b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/
4)))+5/256/a^2/b^2/(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)*i-15/256/(a/b)^(1/4)
/a^3/b*e*arctan(1/(a/b)^(1/4)*x)
```

maxima [A] time = 3.16, size = 429, normalized size = 1.30

$$-\frac{15\left(3b^3e-ab^2i\right)x^{11}+12\left(5b^3d-ab^2h\right)x^{10}+7\left(11b^3c-ab^2g\right)x^9-42\left(3ab^2e-a^2bi\right)x^7-32\left(5ab^2d-a^2bh\right)x^6-18\left(11ab^2c-a^2bg\right)x^5+32a^3f+\left(113a^2be+5a^3i\right)x^3+12\left(11a^2bd+a^3h\right)x^2+3\left(51a^2bc+7a^3g\right)x}{384\left(a^3b^4x^{12}-3a^4b^3x^8+3a^5b^2x^4-a^6b\right)}+\frac{\frac{8(5bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{8(5bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77b^{\frac{3}{2}}c-15\sqrt{a}be-7a\sqrt{b}g+5a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77b^{\frac{3}{2}}c+15\sqrt{a}be-7a\sqrt{b}g-5a^{\frac{3}{2}}i\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
maxima")
```

```
[Out] -1/384*(15*(3*b^3*e - a*b^2*i)*x^11 + 12*(5*b^3*d - a*b^2*h)*x^10 + 7*(11*b
^3*c - a*b^2*g)*x^9 - 42*(3*a*b^2*e - a^2*b*i)*x^7 - 32*(5*a*b^2*d - a^2*b*
h)*x^6 - 18*(11*a*b^2*c - a^2*b*g)*x^5 + 32*a^3*f + (113*a^2*b*e + 5*a^3*i)
*x^3 + 12*(11*a^2*b*d + a^3*h)*x^2 + 3*(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x
^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b) + 1/512*(8*(5*b*d - a*h)*log(s
qrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 -
 sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b
)*g + 5*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c + 15*sqrt(a)*b*e - 7*a*sqrt(b)*g -
5*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt
(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

mupad [B] time = 6.14, size = 2747, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x)
```

```
[Out] (f/(12*b) - (3*x^5*(11*b*c - a*g))/(64*a^2) - (x^6*(5*b*d - a*h))/(12*a^2)
- (7*x^7*(3*b*e - a*i))/(64*a^2) + (7*b*x^9*(11*b*c - a*g))/(384*a^3) + (x*
(51*b*c + 7*a*g))/(128*a*b) + (b*x^10*(5*b*d - a*h))/(32*a^3) + (5*b*x^11*(
3*b*e - a*i))/(128*a^3) + (x^2*(11*b*d + a*h))/(32*a*b) + (x^3*(113*b*e + 5
*a*i))/(384*a*b))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log
```

((125*a^4*i^3 - 3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2*e - 4928*
a^2*b^2*c*h^2 + 735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i + 11200*a*b^3*d^2*g
- 29645*a*b^3*c^2*i - 1125*a^3*b*e*i^2 + 448*a^3*b*g*h^2 - 245*a^3*b*g^2*i
+ 5390*a^2*b^2*c*g*i - 4480*a^2*b^2*d*g*h + 49280*a*b^3*c*d*h - 16170*a*b^3
*c*e*g)/(2097152*a^9*b^2) - root(68719476736*a^15*b^7*z^4 - 1211105280*a^8*
b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 11010
0480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^
2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*
c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 485703680*a
^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816
*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014
080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 98
5600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 29
56800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98
560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 17787
00*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295
680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000
*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^
2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2
*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i -
2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 26680
50*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 +
2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b^6*c^4, z, l)*(root(68719476736
*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 33
5544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i
*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*
b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 1228800
0*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97
140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z +
2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2
*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*
i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*
i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2
- 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i
+ 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g
- 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924
*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*
b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*
g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3
+ 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*
a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b
^6*c^4, z, l)*((20185088*a^7*b^5*c - 1835008*a^8*b^4*g)/(2097152*a^9*b^2) -
(x*(655360*a^7*b^4*d - 131072*a^8*b^3*h))/(131072*a^9*b)) - (614400*a^4*b^
4*d*e - 204800*a^5*b^3*d*i - 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(20971
52*a^9*b^2) + (x*(800*a^6*b*i^2 + 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 + 1
568*a^5*b^2*g^2 - 34496*a^4*b^3*c*g - 4800*a^5*b^2*e*i))/(131072*a^9*b)) -
(x*(4000*b^3*d^3 - 32*a^3*h^3 - 5775*b^3*c*d*e + 35*a^3*g*h*i - 2400*a*b^2*
d^2*h + 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h + 525*a*b^2*d
*e*g - 385*a^2*b*c*h*i - 175*a^2*b*d*g*i - 105*a^2*b*e*g*h))/(131072*a^9*b)
)*root(68719476736*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^
9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 367
00160*a^10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*
z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4
*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^
8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400
*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 1843
2000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323
400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14
700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 2688

```
0*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 24640
00*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 4851
00*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*
b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2
*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2
*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4*d^3*h - 8
1920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^
4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^
6*i^4 - 35153041*b^6*c^4, z, l), l, 1, 4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
[Out] Timed out
```

3.159 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a-bx^4\right)^4}\,dx$

Optimal. Leaf size=349

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}\,(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}\,(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+$$

Rubi [A] time = 0.52, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+4a(2bf-aj)}{96a^2b^2\left(a-bx^4\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}\,(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}\,(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(11bc-ag)+12x(5bd-ah)+15x^2(3be-ai)\right)}{384a^3b\left(a-bx^4\right)}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4
,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(12*a*b
*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a
*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (4*a*(2*b*f - a*j) + x*(b*(11*b*c - a*g
) + 2*b*(5*b*d - a*h)*x + 3*b*(3*b*e - a*i)*x^2))/(96*a^2*b^2*(a - b*x^4)^2
) + (((7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)
*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g)
)/Sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + (
(5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
```

```
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+159x^6+jx^7}{\left(a-bx^4\right)^4}\,dx = \frac{x\left(bc+ag+(bd+ah)x+(159a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(159a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(159a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(159a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(159a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(159a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(159a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3}$$

Mathematica [A] time = 0.52, size = 439, normalized size = 1.26

$$\frac{3\sqrt[4]{a}\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(8a^{5/4}\sqrt[4]{b}h+5a^{3/2}i-40\sqrt[4]{a}b^{5/4}d-15\sqrt{a}be+7a\sqrt{b}g-77b^{3/2}c\right)+3\sqrt[4]{a}\sqrt[4]{b}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(8a^{5/4}\sqrt[4]{b}h-5a^{3/2}i-40\sqrt[4]{a}b^{5/4}d+15\sqrt{a}be-7a\sqrt{b}g+77b^{3/2}c\right)+6\sqrt[4]{a}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5a^{3/2}i-15\sqrt{a}be-7a\sqrt{b}g+77b^{3/2}c\right)-\frac{16a^2\left(12a^2j-b^2x(11c+x(10d+9ex))+abx(g+x(2h+3ix))\right)}{\left(a-bx^4\right)^2}+\frac{128a^3\left(a^2j+b^2x(c+x(d+ex))+ab(f+x(g+x(h+ix)))\right)}{\left(a-bx^4\right)^3}-\frac{4abx(7ag+3ax(4h+5ix)-77bc-15bx(4d+3ex))}{a-bx^4}-24\sqrt{a}\sqrt{b}(ah-5bd)\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{1536a^4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^4,x]
```

```
[Out] ((-4*a*b*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x)))/
(a - b*x^4) - (16*a^2*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) + a*b*x*(
g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*(a^2*j + b^2*x*(c + x*(d +
e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^3 + 6*a^(1/4)*b^(1/4)*(
77*b^(3/2)*c - 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4
)*x)/a^(1/4)] + 3*a^(1/4)*b^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 1
5*Sqrt[a]*b*e + 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1
/4) - b^(1/4)*x] + 3*a^(1/4)*b^(1/4)*(77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d +
 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h - 5*a^(3/2)*i)*Log[a^
(1/4) + b^(1/4)*x] - 24*Sqrt[a]*Sqrt[b]*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b
]*x^2])/(1536*a^4*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a-bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a - b*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a - b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.21, size = 759, normalized size = 2.17

$$-\frac{5}{1024}i\left(\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}-\frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{a^3b^4}\right)-\frac{5}{1024}i\left(\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}+\frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{a^3b^4}\right)-\frac{\sqrt{2}\left(77b^2c-7abg-40\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+8\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah+15\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{\sqrt{2}\left(77b^2c-7abg+40\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-8\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}ah-15\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(-ab^3\right)^{\frac{3}{4}}a^3}-\frac{\sqrt{2}\left(77b^2c-7abg-15\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac{3}{4}}a^3}+\frac{\sqrt{2}\left(77b^2c-7abg-15\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\left(-ab^3\right)^{\frac{3}{4}}a^3}+\frac{15ab^3ix^{11}-45b^4x^{11}e-60b^4dx^{10}+12ab^3hx^{10}-77b^4cx^9+7ab^3gx^9-42a^2b^2ix^7+126ab^3x^7e+160ab^3dx^6-32a^2b^2hx^6+198ab^3cx^5-18a^2b^2gx^5-48a^3bjx^4-5a^3bix^3-113a^2b^2x^3e-132a^2b^2dx^2-12a^3bhx^2-153a^2b^2cx-21a^3bgx-32a^3bf+16a^4j}{384\left(bx^4-a\right)^3a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="giac")
```

```
[Out] -5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)
*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4)) - 5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^4)
+ sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^
3*b^4)) - 1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d
 + 8*sqrt(2)*(-a*b^3)^(1/4)*a*h + 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2
```

```
)*(77*b^2*c - 7*a*b*g + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 8*sqrt(2)*(-a*b^3)^
(1/4)*a*h - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/
4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g
 - 15*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b
^3)^(3/4)*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e)*lo
g(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/384*(
15*a*b^3*i*x^11 - 45*b^4*x^11*e - 60*b^4*d*x^10 + 12*a*b^3*h*x^10 - 77*b^4*
c*x^9 + 7*a*b^3*g*x^9 - 42*a^2*b^2*i*x^7 + 126*a*b^3*x^7*e + 160*a*b^3*d*x^
6 - 32*a^2*b^2*h*x^6 + 198*a*b^3*c*x^5 - 18*a^2*b^2*g*x^5 - 48*a^3*b*j*x^4
- 5*a^3*b*i*x^3 - 113*a^2*b^2*x^3*e - 132*a^2*b^2*d*x^2 - 12*a^3*b*h*x^2 -
153*a^2*b^2*c*x - 21*a^3*b*g*x - 32*a^3*b*f + 16*a^4*j)/((b*x^4 - a)^3*a^3*
b^2)
```

maple [A] time = 0.06, size = 538, normalized size = 1.54

$$\frac{h\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{64\sqrt{ab}\,a^2b}-\frac{5d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{64\sqrt{ab}\,a^3}+\frac{5i\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b^2}-\frac{5i\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b^2}-\frac{15e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}+\frac{15e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}g\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256a^3b}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}g\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^3b}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{256a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4}+\frac{\frac{5(ai-3be)b\,x^{11}}{128a^3}+\frac{(ah-5bd)b\,x^{10}}{32a^3}+\frac{7\left(ag-11bc\right)b\,x^9}{384a^3}-\frac{7(ai-3be)x^7}{64a^2}-\frac{(ah-5bd)x^6}{12a^2}-\frac{j\,x^4}{8b}-\frac{3\left(ag-11bc\right)x^5}{64a^2}-\frac{(5ai+113be)x^3}{384ab}-\frac{(ah+11bd)x^2}{32ab}-\frac{\left(7ag+51bc\right)x}{128ab}+\frac{aj-2bf}{24b^2}}{\left(b\,x^4-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i-3*b*e)/a^3*b*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(a*g-11*b*c
)/a^3*b*x^9-7/64*(a*i-3*b*e)/a^2*x^7-1/12*(a*h-5*b*d)/a^2*x^6-3/64*(a*g-11*
b*c)/a^2*x^5-1/8/b*j*x^4-1/384*(5*a*i+113*b*e)/a/b*x^3-1/32*(a*h+11*b*d)/a/
b*x^2-1/128*(7*a*g+51*b*c)/a/b*x+1/24*(a*j-2*b*f)/b^2)/(b*x^4-a)^3-7/256*(a
/b)^(1/4)/a^3/b*g*arctan(1/(a/b)^(1/4)*x)+77/256*(a/b)^(1/4)/a^4*c*arctan(1
/(a/b)^(1/4)*x)-7/512*(a/b)^(1/4)/a^3/b*g*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)
))+77/512*(a/b)^(1/4)/a^4*c*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/64/(a*b)^
(1/2)/a^2/b*h*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-5/64/(a*b)^(1/2)
/a^3*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))+5/256/(a/b)^(1/4)/a^2/b
^2*i*arctan(1/(a/b)^(1/4)*x)-15/256/(a/b)^(1/4)/a^3/b*e*arctan(1/(a/b)^(1/4
)*x)-5/512/(a/b)^(1/4)/a^2/b^2*i*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+15/512
/(a/b)^(1/4)/a^3/b*e*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))
```

maxima [A] time = 3.08, size = 463, normalized size = 1.33

$$-\frac{15\left(3\,b^4e-ab^3i\right)x^{11}+12\left(5\,b^4d-ab^3h\right)x^{10}+7\left(11\,b^4c-ab^3g\right)x^9+48\,a^3bjx^4-42\left(3\,ab^3e-a^2b^2i\right)x^7-32\left(5\,ab^3d-a^2b^2h\right)x^6-18\left(11\,ab^3c-a^2b^2g\right)x^5+32\,a^3bf-16\,a^4j+\left(113\,a^2b^2e+5\,a^3bi\right)x^3+12\left(11\,a^2b^2d+a^3bh\right)x^2+3\left(51\,a^2b^2c+7\,a^3bg\right)x}{384\left(a^3b^5x^{12}-3\,a^4b^4x^8+3\,a^5b^3x^4-a^6b^2\right)}+\frac{\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}\,x^2+\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}\,x^2-\sqrt{a}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}\,be-7\,a\sqrt{b}\,g+5\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{a}\,be-7\,a\sqrt{b}\,g-5\,a^{\frac{3}{2}}i\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}}{512\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="maxima")
```

```
[Out] -1/384*(15*(3*b^4*e - a*b^3*i)*x^11 + 12*(5*b^4*d - a*b^3*h)*x^10 + 7*(11*b
^4*c - a*b^3*g)*x^9 + 48*a^3*b*j*x^4 - 42*(3*a*b^3*e - a^2*b^2*i)*x^7 - 32*
(5*a*b^3*d - a^2*b^2*h)*x^6 - 18*(11*a*b^3*c - a^2*b^2*g)*x^5 + 32*a^3*b*f
- 16*a^4*j + (113*a^2*b^2*e + 5*a^3*b*i)*x^3 + 12*(11*a^2*b^2*d + a^3*b*h)*
x^2 + 3*(51*a^2*b^2*c + 7*a^3*b*g)*x)/(a^3*b^5*x^12 - 3*a^4*b^4*x^8 + 3*a^5
*b^3*x^4 - a^6*b^2) + 1/512*(8*(5*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sq
rt(a)*sqrt(b)) - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b
)) + 2*(77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b)*g + 5*a^(3/2)*i)*arctan
(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) -
 (77*b^(3/2)*c + 15*sqrt(a)*b*e - 7*a*sqrt(b)*g - 5*a^(3/2)*i)*log((sqrt(b)
*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

mupad [B] time = 6.40, size = 2764, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4
,x)
```

```
[Out] symsum(log((125*a^4*i^3 - 3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2
*e - 4928*a^2*b^2*c*h^2 + 735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i + 11200*a*
b^3*d^2*g - 29645*a*b^3*c^2*i - 1125*a^3*b*e*i^2 + 448*a^3*b*g*h^2 - 245*a^
3*b*g^2*i + 5390*a^2*b^2*c*g*i - 4480*a^2*b^2*d*g*h + 49280*a*b^3*c*d*h - 1
6170*a*b^3*c*e*g)/(2097152*a^9*b^2) - root(68719476736*a^15*b^7*z^4 - 12111
05280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z
^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^8*
b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 8830976
0*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 4
85703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*
z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^
2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*
g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*
e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^
2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i
^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2
*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i
 - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*
a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a
^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b
^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g
^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^
5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^6*i^4 - 35153041*b^6*c^4, z, m)*(root(6
8719476736*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*
i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^
10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 24
57600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z
 - 12288000*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*
i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4
*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5
*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*
b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*
b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^
2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b
^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b
^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3
+ 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 2
96450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2
450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4*d^3*h - 81920*a^4
*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 - 2401*a^4*b^2*g^
4 - 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 - 625*a^6*i^4 -
35153041*b^6*c^4, z, m)*((20185088*a^7*b^5*c - 1835008*a^8*b^4*g)/(2097152*
a^9*b^2) - (x*(655360*a^7*b^4*d - 131072*a^8*b^3*h))/(131072*a^9*b)) - (614
400*a^4*b^4*d*e - 204800*a^5*b^3*d*i - 122880*a^5*b^3*e*h + 40960*a^6*b^2*h
*i)/(2097152*a^9*b^2) + (x*(800*a^6*b*i^2 + 189728*a^3*b^4*c^2 + 7200*a^4*b
^3*e^2 + 1568*a^5*b^2*g^2 - 34496*a^4*b^3*c*g - 4800*a^5*b^2*e*i))/(131072*
a^9*b)) - (x*(4000*b^3*d^3 - 32*a^3*h^3 - 5775*b^3*c*d*e + 35*a^3*g*h*i - 2
400*a*b^2*d^2*h + 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h + 5
25*a*b^2*d*e*g - 385*a^2*b*c*h*i - 175*a^2*b*d*g*i - 105*a^2*b*e*g*h))/(131
072*a^9*b))*root(68719476736*a^15*b^7*z^4 - 1211105280*a^8*b^6*c*e*z^2 + 40
3701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g
*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^8*b^6*d^2*z^2 - 33554432*a^1
0*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z + 176619
52*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z + 485703680*a^4*b^6*c^2*d*z -
 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z
 - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^
2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d
```

```
*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*
d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^
2*i + 26880*a^4*b^2*e*g*h^2 - 53900*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e
*i + 2464000*a^2*b^4*c*d^2*i + 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*
h^2 - 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e +
 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a
^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a
^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i - 2048000*a^2*b^4
*d^3*h - 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2
 - 2401*a^4*b^2*g^4 - 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^
4 - 625*a^6*i^4 - 35153041*b^6*c^4, z, m), m, 1, 4) + ((2*b*f - a*j)/(24*b^
2) + (j*x^4)/(8*b) - (3*x^5*(11*b*c - a*g))/(64*a^2) - (x^6*(5*b*d - a*h))/
(12*a^2) - (7*x^7*(3*b*e - a*i))/(64*a^2) + (7*b*x^9*(11*b*c - a*g))/(384*a
^3) + (x*(51*b*c + 7*a*g))/(128*a*b) + (b*x^10*(5*b*d - a*h))/(32*a^3) + (5
*b*x^11*(3*b*e - a*i))/(128*a^3) + (x^2*(11*b*d + a*h))/(32*a*b) + (x^3*(11
3*b*e + 5*a*i))/(384*a*b))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,
x)
```

```
[Out] Timed out
```

3.160 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=462

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e\right.}{512\sqrt{2}a^{15/4}b^{5/4}}$$

Rubi [A] time = 0.62, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{256\sqrt{2}a^{15/4}b^{5/4}}+\frac{(ah+5bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}-\frac{8af-x\left(2x(ah+5bd)+ag+11bc+9bex^2\right)}{96a^2b\left(a+bx^4\right)^2}+\frac{x\left(7(ag+11bc)+12x(ah+5bd)+45bex^2\right)}{384a^3b\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3)
+ (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a +
b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x + 9*b*e*x^2))/(96*a^
2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7
/2)*b^(3/2)) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a
]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*
a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) +
((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^4}dx = \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-b(11bc+ag)-2b(5bd+ah)x-9b^2ex^2-\ldots}{\left(a+bx^4\right)^3}}{12ab^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{8af-x\left(11bc+ag+2(5bd+\ldots\right.}{96a^2b\left(a+bx^4\right)^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah\ldots\right.}{384a^3b\left(a+bx^4\right)\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah\ldots\right.}{384a^3b\left(a+bx^4\right)\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah\ldots\right.}{384a^3b\left(a+bx^4\right)\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah\ldots\right.}{384a^3b\left(a+bx^4\right)\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah\ldots\right.}{384a^3b\left(a+bx^4\right)\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah\ldots\right.}{384a^3b\left(a+bx^4\right)\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah\ldots\right.}{384a^3b\left(a+bx^4\right)\ldots}$$

Mathematica [A] time = 0.58, size = 461, normalized size = 1.00

$$\frac{-6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(16a^{5/4}h+15\sqrt{2}\sqrt{a}b^{3/4}e+80\sqrt[4]{a}bd+7\sqrt{2}a\sqrt[4]{b}g+77\sqrt{2}b^{5/4}c\right)+6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-16a^{5/4}h+15\sqrt{2}\sqrt{a}b^{3/4}e-80\sqrt[4]{a}bd+7\sqrt{2}a\sqrt[4]{b}g+77\sqrt{2}b^{5/4}c\right)-\frac{256a^{11/4}\sqrt{b}(a(f+x(g+hx))-bx(c+x(d+ex)))}{\left(a+bx^4\right)^3}+\frac{32a^{7/4}\sqrt{b}x(a(g+2hx)+11bc+bx(10d+9ex))}{\left(a+bx^4\right)^2}+\frac{8a^{3/4}\sqrt{b}x\left(7ag+12ahx+77bc+60bdx+45bex^2\right)}{a+bx^4}-3\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)+3\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{3072a^{15/4}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(77*b*c + 7*a*g + 60*b*d*x + 12*a*h*x + 45*b*e*x^2))/
(a + b*x^4) + (32*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) + a*(g + 2
*h*x)))/(a + b*x^4)^2 - (256*a^(11/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a
*(f + x*(g + h*x))))/(a + b*x^4)^3 - 6*(77*Sqrt[2]*b^(5/4)*c + 80*a^(1/4)*b
*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g + 16*a^(5/4)*h)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(5/4)*c - 80*a^(1/
4)*b*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g - 16*a^(5/4)*
h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*b^(1/4)*(77*b*c - 15
*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + 3*Sqrt[2]*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,
x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,
 x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [A] time = 0.20, size = 521, normalized size = 1.13

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d+8\sqrt{2}\sqrt{ab}\,abh+77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg+15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d+8\sqrt{2}\sqrt{ab}\,abh+77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg+15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg-15\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}-\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg-15\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}+\frac{45\,b^3x^{11}e+60\,b^3dx^{10}+12\,ab^2hx^{10}+77\,b^3cx^9+7\,ab^2gx^9+126\,ab^2x^7e+160\,ab^2dx^6+32\,a^2bhx^6+198\,ab^2cx^5+18\,a^2bgx^5+113\,a^2bx^3e+132\,a^2bdx^2-12\,a^3hx^2+153\,a^2bcx-21\,a^3gx-32\,a^3f}{384\left(bx^4+a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*
(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2
)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4
)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^
3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)
*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 1
2*a*b^2*h*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*x^7*e + 160*a*b^2
*d*x^6 + 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*
e + 132*a^2*b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)
/((b*x^4 + a)^3*a^3*b)
```

maple [A] time = 0.07, size = 607, normalized size = 1.31

$$\frac{h\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{32\sqrt{ab}\,a^2b}+\frac{5d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{32\sqrt{ab}\,a^3}+\frac{15\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}+\frac{15\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{512\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}+\frac{15\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3b}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{512a^3b}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{512a^3b}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,g\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024a^3b}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{512a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{512a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024a^4}+\frac{\frac{15b^2ex^{11}}{128a^3}+\frac{(ah+5bd)bx^{10}}{32a^3}+\frac{21bex^7}{64a^2}+\frac{7(ag+11bc)bx^9}{384a^3}+\frac{(ah+5bd)x^6}{12a^2}+\frac{113ex^3}{384a}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{(ah-11bd)x^2}{32ab}-\frac{f}{12b}-\frac{(7ag-51bc)x}{128ab}}{\left(bx^4+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (15/128/a^3*b^2*e*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c)/a^3*b
*x^9+21/64/a^2*b*e*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g+11*b*c)*x^5+1
13/384/a*e*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12/b*
f)/(b*x^4+a)^3+7/512/a^3/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
-1)*g+77/512/a^4*c*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+7/10
24*(a/b)^(1/4)*2^(1/2)/a^3/b*g*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(
x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+77/1024/a^4*c*(a/b)^(1/4)*2^(1/2)*l
```

```
n((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^
(1/2)))+7/512*(a/b)^(1/4)*2^(1/2)/a^3/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+7
7/512*(a/b)^(1/4)*2^(1/2)/a^4*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/32/a^2/b/
(a*b)^(1/2)*arctan((1/a*b)^(1/2)*x^2)*h+5/32/(a*b)^(1/2)/a^3*d*arctan((1/a*
b)^(1/2)*x^2)+15/1024/a^3*e/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/
2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+15/512/a^3*e/b/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+15/512/a^3*e/b/(a/b)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.13, size = 517, normalized size = 1.12

$$\frac{45\,b^3ex^{11}+126\,ab^2ex^7+12\left(5\,b^3d+ab^2h\right)x^{10}+7\left(11\,b^3c+ab^2g\right)x^9+113\,a^2bex^3+32\left(5\,ab^2d+a^2bh\right)x^6+18\left(11\,ab^2c+a^2bg\right)x^5-32\,a^3f+12\left(11\,a^2bd-a^3h\right)x^2+3\left(51\,a^2bc-7\,a^3g\right)x}{384\left(a^3b^4x^{12}+3\,a^4b^3x^8+3\,a^5b^2x^4+a^6b\right)}+\frac{\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+7\,\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g-80\,\sqrt{a}b^{\frac{3}{2}}d-16\,a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+7\,\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+80\,\sqrt{a}b^{\frac{3}{2}}d+16\,a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{3}{4}}}}{1024\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima"
)
```

```
[Out] 1/384*(45*b^3*e*x^11 + 126*a*b^2*e*x^7 + 12*(5*b^3*d + a*b^2*h)*x^10 + 7*(1
1*b^3*c + a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 32*(5*a*b^2*d + a^2*b*h)*x^6 + 1
8*(11*a*b^2*c + a^2*b*g)*x^5 - 32*a^3*f + 12*(11*a^2*b*d - a^3*h)*x^2 + 3*(
51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^
6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g)*log(
sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt
(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b
^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g - 80*
sqrt(a)*b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x +
 sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt
(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4
)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 80*sqrt(a)*b^(3/2)*d + 16*a^(3/2)*sqrt(
b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

mupad [B] time = 6.08, size = 1743, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log((123200*b^3*c*d^2 - 3375*a*b^2*e^3 - 88935*b^3*c^2*e + 448*a^3*g
*h^2 + 11200*a*b^2*d^2*g + 4928*a^2*b*c*h^2 - 735*a^2*b*e*g^2 + 49280*a*b^2
*c*d*h - 16170*a*b^2*c*e*g + 4480*a^2*b*d*g*h)/(2097152*a^9*b) - root(68719
476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*c*e*z^2 + 335544320*a^9*b^4*d*h*z^
2 + 110100480*a^9*b^4*e*g*z^2 + 838860800*a^8*b^5*d^2*z^2 + 33554432*a^10*b
^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z - 17661952*a^6*b^3*c*g*h*z - 48570368
0*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 802816*a^7*b^2*g^2*h*z + 368
6400*a^6*b^3*e^2*h*z - 4014080*a^6*b^3*d*g^2*z + 18432000*a^5*b^4*d*e^2*z -
 268800*a^3*b^2*d*e*g*h - 2956800*a^2*b^3*c*d*e*h - 672000*a^2*b^3*d^2*e*g
- 295680*a^3*b^2*c*e*h^2 + 485100*a^2*b^3*c*e^2*g - 26880*a^4*b*e*g*h^2 - 7
392000*a*b^4*c*d^2*e + 81920*a^4*b*d*h^3 + 12782924*a*b^4*c^3*g + 614400*a^
3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 + 1743126*a^2*b^3*c^2*g^2 + 2048000*a
^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^4*c^2*e^2 + 50625*a^2*b^3
*e^4 + 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a^5*h^4 + 35153041*b^5*c^4
, z, k)*(root(68719476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*c*e*z^2 + 33554
4320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 + 838860800*a^8*b^5*d^2*z^
2 + 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z - 17661952*a^6*b^3
*c*g*h*z - 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 802816*a^
7*b^2*g^2*h*z + 3686400*a^6*b^3*e^2*h*z - 4014080*a^6*b^3*d*g^2*z + 1843200
0*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h - 2956800*a^2*b^3*c*d*e*h - 6720
```

```
00*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 + 485100*a^2*b^3*c*e^2*g - 2688
0*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e + 81920*a^4*b*d*h^3 + 12782924*a*b^
4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 + 1743126*a^2*b^3*
c^2*g^2 + 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^4*c^2*
e^2 + 50625*a^2*b^3*e^4 + 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a^5*h^4
 + 35153041*b^5*c^4, z, k)*((20185088*a^7*b^4*c + 1835008*a^8*b^3*g)/(20971
52*a^9*b) - (x*(655360*a^7*b^4*d + 131072*a^8*b^3*h))/(131072*a^9*b)) + (61
4400*a^4*b^3*d*e + 122880*a^5*b^2*e*h)/(2097152*a^9*b) + (x*(189728*a^3*b^4
*c^2 - 7200*a^4*b^3*e^2 + 1568*a^5*b^2*g^2 + 34496*a^4*b^3*c*g))/(131072*a^
9*b)) + (x*(4000*b^3*d^3 + 32*a^3*h^3 - 5775*b^3*c*d*e + 2400*a*b^2*d^2*h +
 480*a^2*b*d*h^2 - 1155*a*b^2*c*e*h - 525*a*b^2*d*e*g - 105*a^2*b*e*g*h))/(
131072*a^9*b))*root(68719476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*c*e*z^2 +
 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 + 838860800*a^8*b^5*
d^2*z^2 + 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z - 17661952*a
^6*b^3*c*g*h*z - 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 802
816*a^7*b^2*g^2*h*z + 3686400*a^6*b^3*e^2*h*z - 4014080*a^6*b^3*d*g^2*z + 1
8432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h - 2956800*a^2*b^3*c*d*e*h
- 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 + 485100*a^2*b^3*c*e^2*g
- 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e + 81920*a^4*b*d*h^3 + 1278292
4*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 + 1743126*a^
2*b^3*c^2*g^2 + 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^
4*c^2*e^2 + 50625*a^2*b^3*e^4 + 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a
^5*h^4 + 35153041*b^5*c^4, z, k), k, 1, 4) + ((113*e*x^3)/(384*a) - f/(12*b
) + (3*x^5*(11*b*c + a*g))/(64*a^2) + (x^6*(5*b*d + a*h))/(12*a^2) + (7*b*x
^9*(11*b*c + a*g))/(384*a^3) + (x*(51*b*c - 7*a*g))/(128*a*b) + (b*x^10*(5*
b*d + a*h))/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (x^2*(11*b*d - a*h))/(32
*a*b) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
[Out] Timed out
```

3.161 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=516

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(\right.}{512\sqrt{2}a^{15/4}b}$$

Rubi [A] time = 0.85, antiderivative size = 516, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{(ah+5bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}-\frac{8af-x\left(2x(ah+5bd)+3x^2(ai+3be)+ag+11bc\right)}{96a^2b\left(a+bx^4\right)^2}+\frac{x\left(7(ag+11bc)+12x(ah+5bd)+15x^2(ai+3be)\right)}{384a^3b\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(12*a*b*(a + b*
x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a*i)*x^2)
)/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x +
3*(3*b*e + a*i)*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqr
t[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b*c + a*g) + 5*S
qrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]
*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) - (
(7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt
[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+161x^6}{\left(a+bx^4\right)^4}\,dx = \frac{x\left(bc-ag+(bd-ah)x-(161a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-b(11bc+ag)-}{}}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(161a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{8af-x\left(11b\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(161a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+a\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(161a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+a\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(161a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+a\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(161a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+a\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(161a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+a\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(161a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+a\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(161a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+a\right.}{}$$

Mathematica [A]   time = 1.01, size = 530, normalized size = 1.03

$$\frac{-\frac{256a^{11/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{\left(a+bx^4\right)^3}+\frac{32a^{7/4}b^{3/4}x(ag+ax(2h+3ix)+11bc+bx(10d+9ex))}{\left(a+bx^4\right)^2}+\frac{8a^{3/4}b^{3/4}x(7ag+3ax(4h+5ix)+77bc+15bx(4d+3ex))}{a+bx^4}-6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(16a^{5/4}\sqrt[4]{b}h+5\sqrt{2}a^{3/2}i+80\sqrt[4]{a}b^{5/4}d+15\sqrt{2}\sqrt{a}be+7\sqrt{2}a\sqrt{b}g+77\sqrt{2}b^{3/2}c\right)+6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-16a^{5/4}\sqrt[4]{b}h+5\sqrt{2}a^{3/2}i-80\sqrt[4]{a}b^{5/4}d+15\sqrt{2}\sqrt{a}be+7\sqrt{2}a\sqrt{b}g+77\sqrt{2}b^{3/2}c\right)+3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(5a^{3/2}i+15\sqrt{a}be-7a\sqrt{b}g-77b^{3/2}c\right)+3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5a^{3/2}i-15\sqrt{a}be+7a\sqrt{b}g+77b^{3/2}c\right)}{3072a^{15/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x
]
```

```
[Out] ((32*a^(7/4)*b^(3/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x) + a*x*(2*h + 3*i*
x)))/(a + b*x^4)^2 + (8*a^(3/4)*b^(3/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3
*e*x) + 3*a*x*(4*h + 5*i*x)))/(a + b*x^4) - (256*a^(11/4)*b^(3/4)*(-(b*x*(c
 + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^3 - 6*(77*Sqrt
[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a
*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(3/2)*c - 80*a^(1/4)*b^(5/4)*d + 15
*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/4)*b^(1/4)*h + 5*Sqr
t[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*(-77*b^
(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[
```

```
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*(77*b^(3/2)*c - 15*Sqrt[a]*
b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/(3072*a^(15/4)*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a+bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b
*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b
*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 735, normalized size = 1.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="g
iac")
```

```
[Out] 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + sqrt(2)*
(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 1/5
12*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*
b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(
40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b
^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^
(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^
2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)
^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(15*a*b^2*i*x^11 + 45*b^3*x^11*e + 60
*b^3*d*x^10 + 12*a*b^2*h*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 42*a^2*b*i*x
^7 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 +
 18*a^2*b*g*x^5 - 5*a^3*i*x^3 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 - 12*a^3*
h*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

maple [A] time = 0.07, size = 767, normalized size = 1.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c
)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12*(a*h+5*b*d)/a^2*x^6+3/64*(a*g+11*
b*c)/a^2*x^5-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*
(7*a*g-51*b*c)/a/b*x-1/12/b*f)/(b*x^4+a)^3+7/1024*(a/b)^(1/4)*2^(1/2)/a^3/b
*g*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a
/b)^(1/2)))+77/1024*(a/b)^(1/4)*2^(1/2)/a^4*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x
+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+7/512*(a/b)^(1/4)*2^
(1/2)/a^3/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+77/512*(a/b)^(1/4)*2^(1/2)/a^
4*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+7/512*(a/b)^(1/4)*2^(1/2)/a^3/b*g*arcta
n(2^(1/2)/(a/b)^(1/4)*x+1)+77/512*(a/b)^(1/4)*2^(1/2)/a^4*c*arctan(2^(1/2)/
(a/b)^(1/4)*x+1)+1/32/(a*b)^(1/2)/a^2/b*h*arctan((1/a*b)^(1/2)*x^2)+5/32/(a
*b)^(1/2)/a^3*d*arctan((1/a*b)^(1/2)*x^2)+5/1024/a^2/b^2/(a/b)^(1/4)*2^(1/2
)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/
b)^(1/2)))*i+15/1024/(a/b)^(1/4)*2^(1/2)/a^3/b*e*ln((x^2-(a/b)^(1/4)*2^(1/2
)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+5/512/a^2/b^2/(a/
b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*i+15/512/(a/b)^(1/4)*2^(1/
2)/a^3/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+5/512/a^2/b^2/(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*i+15/512/(a/b)^(1/4)*2^(1/2)/a^3/b*e*arct
an(2^(1/2)/(a/b)^(1/4)*x+1)
```

maxima [A]   time = 3.16, size = 579, normalized size = 1.12

$$\frac{15\left(3b^3e+ab^2i\right)x^{11}+12\left(5b^3d+ab^2h\right)x^{10}+7\left(11b^3c+ab^2g\right)x^9+42\left(3ab^2e+a^2bi\right)x^7+32\left(5ab^2d+a^2bh\right)x^6+18\left(11ab^2c+a^2bg\right)x^5-32a^3f+\left(113a^2be-5a^3i\right)x^3+12\left(11a^2bd-a^3h\right)x^2+3\left(51a^2bc-7a^3g\right)x}{384\left(a^3b^4x^{12}+3a^4b^3x^8+3a^5b^2x^4+a^6b\right)}+\frac{\frac{\sqrt{2}\left(77b^{\frac{3}{2}}c-15\sqrt{a}be+7a\sqrt{b}g-5a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77b^{\frac{3}{2}}c-15\sqrt{a}be+7a\sqrt{b}g-5a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+15\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+7\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}i-80\sqrt{a}b^{\frac{3}{2}}d-16a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+15\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+7\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}i+80\sqrt{a}b^{\frac{3}{2}}d+16a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{1024a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="m
axima")
```

```
[Out] 1/384*(15*(3*b^3*e + a*b^2*i)*x^11 + 12*(5*b^3*d + a*b^2*h)*x^10 + 7*(11*b^
3*c + a*b^2*g)*x^9 + 42*(3*a*b^2*e + a^2*b*i)*x^7 + 32*(5*a*b^2*d + a^2*b*h
)*x^6 + 18*(11*a*b^2*c + a^2*b*g)*x^5 - 32*a^3*f + (113*a^2*b*e - 5*a^3*i)*
x^3 + 12*(11*a^2*b*d - a^3*h)*x^2 + 3*(51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^
12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*c
 - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c - 15
*sqrt(a)*b*e + 7*a*sqrt(b)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c
 + 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a
^(7/4)*b^(1/4)*i - 80*sqrt(a)*b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*
sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(7/4)
*b^(1/4)*i + 80*sqrt(a)*b^(3/2)*d + 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2
)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*s
qrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

mupad [B]   time = 6.08, size = 2741, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x)
```

```
[Out] ((3*x^5*(11*b*c + a*g))/(64*a^2) - f/(12*b) + (x^6*(5*b*d + a*h))/(12*a^2)
+ (7*x^7*(3*b*e + a*i))/(64*a^2) + (7*b*x^9*(11*b*c + a*g))/(384*a^3) + (x*
(51*b*c - 7*a*g))/(128*a*b) + (b*x^10*(5*b*d + a*h))/(32*a^3) + (5*b*x^11*(
3*b*e + a*i))/(128*a^3) + (x^2*(11*b*d - a*h))/(32*a*b) + (x^3*(113*b*e - 5
*a*i))/(384*a*b))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log
(- root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 403701760*a
```

^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, l)*(root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, l)*((20185088*a^7*b^5*c + 1835008*a^8*b^4*g)/(2097152*a^9*b^2) - (x*(655360*a^7*b^4*d + 131072*a^8*b^3*h))/(131072*a^9*b)) + (614400*a^4*b^4*d*e + 204800*a^5*b^3*d*i + 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(2097152*a^9*b^2) - (x*(800*a^6*b*i^2 - 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 - 1568*a^5*b^2*g^2 - 34496*a^4*b^3*c*g + 4800*a^5*b^2*e*i))/(131072*a^9*b)) - (125*a^4*i^3 + 3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2*e - 4928*a^2*b^2*c*h^2 + 735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i - 11200*a*b^3*d^2*g + 29645*a*b^3*c^2*i + 1125*a^3*b*e*i^2 - 448*a^3*b*g*h^2 + 245*a^3*b*g^2*i + 5390*a^2*b^2*c*g*i - 4480*a^2*b^2*d*g*h - 49280*a*b^3*c*d*h + 16170*a*b^3*c*e*g)/(2097152*a^9*b^2) - (x*(5775*b^3*c*d*e - 32*a^3*h^3 - 4000*b^3*d^3 + 35*a^3*g*h*i - 2400*a*b^2*d^2*h - 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h + 525*a*b^2*d*e*g + 385*a^2*b*c*h*i + 175*a^2*b*d*g*i + 105*a^2*b*e*g*h))/(131072*a^9*b))*root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*e*i - 246

```
4000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 + 48
5100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^
5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d
^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126*a^2*b^4*c
^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b^4*d^3*h +
 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^2 + 2401*
a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d^4 + 625*
a^6*i^4 + 35153041*b^6*c^4, z, l), l, 1, 4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
[Out] Timed out
```

3.162 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=534

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\right.}{512\sqrt{2}a^{15}}$$

Rubi [A] time = 0.82, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{4a(aj+2bf)-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be)\right)}{96a^2b^2\left(a+bx^4\right)^2}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{256\sqrt{2}a^{15/4}b^{7/4}}+\frac{(ah+5bd)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x\left(7(ag+11bc)+12x(ah+5bd)+15x^2(ai+3be)\right)}{384a^3b\left(a+bx^4\right)}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4
,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(12*a*b
*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a
*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (4*a*(2*b*f + a*j) - x*(b*(11*b*c + a*g
) + 2*b*(5*b*d + a*h)*x + 3*b*(3*b*e + a*i)*x^2))/(96*a^2*b^2*(a + b*x^4)^2
) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((
7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*
g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256
*Sqrt[2]*a^(15/4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e
+ a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2
]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15
/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
```

```
0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+162x^6+jx^7}{\left(a+bx^4\right)^4}\,dx = \frac{x\left(bc-ag+(bd-ah)x-(162a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(162a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(162a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(162a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(162a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(162a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(162a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(162a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(162a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3}$$

Mathematica [A] time = 0.71, size = 555, normalized size = 1.04

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*b*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x
)))/(a + b*x^4) - (32*a^(7/4)*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) -
 a*b*x*(g + x*(2*h + 3*i*x))))/(a + b*x^4)^2 + (256*a^(11/4)*(a^2*j + b^2*x
*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^3 - 6*b^(1
/4)*(77*Sqrt[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e +
 7*Sqrt[2]*a*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*b^(1/4)*(77*Sqrt[2]*b^(3/2)*c - 80*a^
```

```
(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/
4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 + 3*Sqrt[2]*b^(1/4)*(-77*b^(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^
(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]
*b^(1/4)*(77*b^(3/2)*c - 15*Sqrt[a]*b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a+bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a + b*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7
)/(a + b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 767, normalized size = 1.44

$$\frac{5}{1024}\,i\left(\frac{2\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{a^3b^4}\right)+\frac{5}{1024}\,i\left(\frac{2\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{a^3b^4}\right)+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d+8\sqrt{2}\sqrt{ab}\,abh+77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg+15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2d+8\sqrt{2}\sqrt{ab}\,abh+77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg+15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg-15\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}-\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg-15\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}+\frac{15ab^3ix^{11}+45b^4x^{11}e+60b^4dx^{10}+12ab^3hx^{10}+77b^4cx^9+7ab^3gx^9+42a^2b^2ix^7+126ab^3x^7e+160ab^3dx^6+32a^2b^2hx^6+198ab^3cx^5+18a^2b^2gx^5-48a^3bjx^4-5a^3bix^3+113a^2b^2x^3e+132a^2b^2dx^2-12a^3bhx^2+153a^2b^2cx-21a^3bgx-32a^3bf-16a^4j}{384\left(bx^4+a\right)^3a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="giac")
```

```
[Out] 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + sqrt(2)*
(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 1/5
12*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*
b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(
40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b
^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^
(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^
2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)
^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(15*a*b^3*i*x^11 + 45*b^4*x^11*e + 60
*b^4*d*x^10 + 12*a*b^3*h*x^10 + 77*b^4*c*x^9 + 7*a*b^3*g*x^9 + 42*a^2*b^2*i
*x^7 + 126*a*b^3*x^7*e + 160*a*b^3*d*x^6 + 32*a^2*b^2*h*x^6 + 198*a*b^3*c*x
^5 + 18*a^2*b^2*g*x^5 - 48*a^3*b*j*x^4 - 5*a^3*b*i*x^3 + 113*a^2*b^2*x^3*e
+ 132*a^2*b^2*d*x^2 - 12*a^3*b*h*x^2 + 153*a^2*b^2*c*x - 21*a^3*b*g*x - 32*
a^3*b*f - 16*a^4*j)/((b*x^4 + a)^3*a^3*b^2)
```

maple [A] time = 0.07, size = 783, normalized size = 1.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c
)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12*(a*h+5*b*d)/a^2*x^6+3/64*(a*g+11*
b*c)/a^2*x^5-1/8/b*j*x^4-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/
b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/24*(a*j+2*b*f)/b^2)/(b*x^4+a)^3+7/512*(a
/b)^(1/4)*2^(1/2)/a^3/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+77/512*(a/b)^(1/4
)*2^(1/2)/a^4*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+7/512*(a/b)^(1/4)*2^(1/2)/a
^3/b*g*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+77/512*(a/b)^(1/4)*2^(1/2)/a^4*c*arc
tan(2^(1/2)/(a/b)^(1/4)*x-1)+7/1024*(a/b)^(1/4)*2^(1/2)/a^3/b*g*ln((x^2+(a/
b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+77
/1024*(a/b)^(1/4)*2^(1/2)/a^4*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/
(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/32/(a*b)^(1/2)/a^2/b*h*arctan((1
/a*b)^(1/2)*x^2)+5/32/(a*b)^(1/2)/a^3*d*arctan((1/a*b)^(1/2)*x^2)+5/1024/(a
/b)^(1/4)*2^(1/2)/a^2/b^2*i*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2
+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+15/1024/(a/b)^(1/4)*2^(1/2)/a^3/b*e*ln
((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(
1/2)))+5/512/(a/b)^(1/4)*2^(1/2)/a^2/b^2*i*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+
15/512/(a/b)^(1/4)*2^(1/2)/a^3/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+5/512/(a
/b)^(1/4)*2^(1/2)/a^2/b^2*i*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+15/512/(a/b)^(1
/4)*2^(1/2)/a^3/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [**A**] time = 3.23, size = 613, normalized size = 1.15

$$\frac{15\left(3b^4e+ab^3i\right)x^{11}+12\left(5b^4d+ab^3h\right)x^{10}+7\left(11b^4c+ab^3g\right)x^9-48a^3bjx^4+42\left(3ab^3e+a^2b^2i\right)x^7+32\left(5ab^3d+a^2b^2h\right)x^6+18\left(11ab^3c+a^2b^2g\right)x^5-32a^3bf-16a^4j+\left(113a^2b^2e-5a^3bi\right)x^3+12\left(11a^2b^2d-a^3bh\right)x^2+3\left(51a^2b^2c-7a^3bg\right)x}{384\left(a^3b^5x^{12}+3a^4b^4x^8+3a^5b^3x^4+a^6b^2\right)}+\frac{\frac{\sqrt{2}\left(77b^{\frac{3}{2}}c-15\sqrt{a}be+7a\sqrt{b}g-5a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77b^{\frac{3}{2}}c-15\sqrt{a}be+7a\sqrt{b}g-5a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+15\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+7\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}i-80\sqrt{a}b^{\frac{3}{2}}d-16a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+15\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}e+7\sqrt{2}a^{\frac{5}{4}}b^{\frac{3}{4}}g+5\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}i+80\sqrt{a}b^{\frac{3}{2}}d+16a^{\frac{3}{2}}\sqrt{b}h\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{1024a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="maxima")
```

```
[Out] 1/384*(15*(3*b^4*e + a*b^3*i)*x^11 + 12*(5*b^4*d + a*b^3*h)*x^10 + 7*(11*b^
4*c + a*b^3*g)*x^9 - 48*a^3*b*j*x^4 + 42*(3*a*b^3*e + a^2*b^2*i)*x^7 + 32*(
5*a*b^3*d + a^2*b^2*h)*x^6 + 18*(11*a*b^3*c + a^2*b^2*g)*x^5 - 32*a^3*b*f -
 16*a^4*j + (113*a^2*b^2*e - 5*a^3*b*i)*x^3 + 12*(11*a^2*b^2*d - a^3*b*h)*x
^2 + 3*(51*a^2*b^2*c - 7*a^3*b*g)*x)/(a^3*b^5*x^12 + 3*a^4*b^4*x^8 + 3*a^5*
b^3*x^4 + a^6*b^2) + 1/1024*(sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*a*s
qrt(b)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*a*sqrt(b
)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(
a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(
5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(7/4)*b^(1/4)*i - 80*sqr
t(a)*b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sq
rt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b)
)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4)*e
 + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(7/4)*b^(1/4)*i + 80*sqrt(a)*b
^(3/2)*d + 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3
/4)))/(a^3*b)
```

mupad [**B**] time = 6.48, size = 2757, normalized size = 5.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4
,x)
```

```
[Out] symsum(log(- root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 4
03701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*
g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^
10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 17661
952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z
+ 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*
z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*g
^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*
d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*c
*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*h
^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2*
e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e
*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e
+ 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400*
a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126*
a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b^
4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e^
2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*d
^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, m)*(root(68719476736*a^15*b^7*z^4 +
 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5
*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 83886080
0*a^8*b^6*d^2*z^2 + 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 8
8309760*a^5*b^5*c*d*g*z - 17661952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i
*z - 485703680*a^4*b^6*c^2*d*z + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*
c^2*h*z - 802816*a^7*b^3*g^2*h*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^
3*d*i^2*z - 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*
b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*
b^3*d*e*g*h - 2956800*a^2*b^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*
b^3*d^2*g*i - 98560*a^4*b^2*c*h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2
*c*g*i^2 + 1778700*a^2*b^4*c^2*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b
^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g
*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g +
33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 2
2050*a^3*b^3*e^2*g^2 + 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500
*a^3*b^3*e^3*i + 2048000*a^2*b^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b
^3*c*g^3 + 2668050*a*b^5*c^2*e^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4
096*a^5*b*h^4 + 2560000*a*b^5*d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, m)*(
(20185088*a^7*b^5*c + 1835008*a^8*b^4*g)/(2097152*a^9*b^2) - (x*(655360*a^7
*b^4*d + 131072*a^8*b^3*h))/(131072*a^9*b)) + (614400*a^4*b^4*d*e + 204800*
a^5*b^3*d*i + 122880*a^5*b^3*e*h + 40960*a^6*b^2*h*i)/(2097152*a^9*b^2) - (
x*(800*a^6*b*i^2 - 189728*a^3*b^4*c^2 + 7200*a^4*b^3*e^2 - 1568*a^5*b^2*g^2
 - 34496*a^4*b^3*c*g + 4800*a^5*b^2*e*i))/(131072*a^9*b)) - (125*a^4*i^3 +
3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2*e - 4928*a^2*b^2*c*h^2 +
735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i - 11200*a*b^3*d^2*g + 29645*a*b^3*c^
2*i + 1125*a^3*b*e*i^2 - 448*a^3*b*g*h^2 + 245*a^3*b*g^2*i + 5390*a^2*b^2*c
*g*i - 4480*a^2*b^2*d*g*h - 49280*a*b^3*c*d*h + 16170*a*b^3*c*e*g)/(2097152
*a^9*b^2) - (x*(5775*b^3*c*d*e - 32*a^3*h^3 - 4000*b^3*d^3 + 35*a^3*g*h*i -
 2400*a*b^2*d^2*h - 480*a^2*b*d*h^2 + 1925*a*b^2*c*d*i + 1155*a*b^2*c*e*h +
 525*a*b^2*d*e*g + 385*a^2*b*c*h*i + 175*a^2*b*d*g*i + 105*a^2*b*e*g*h))/(1
31072*a^9*b))*root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 +
403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e
*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a
^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 1766
1952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z
 + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h
*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d*
g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c
*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b^4*
c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^2*c*
```

```
h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^4*c^2
*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*
e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e
 + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2 + 614400
*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 + 1743126
*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i + 2048000*a^2*b
^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 2668050*a*b^5*c^2*e
^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4 + 2560000*a*b^5*
d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, m), m, 1, 4) + ((3*x^5*(11*b*c + a
*g))/(64*a^2) - (j*x^4)/(8*b) - (2*b*f + a*j)/(24*b^2) + (x^6*(5*b*d + a*h)
)/(12*a^2) + (7*x^7*(3*b*e + a*i))/(64*a^2) + (7*b*x^9*(11*b*c + a*g))/(384
*a^3) + (x*(51*b*c - 7*a*g))/(128*a*b) + (b*x^10*(5*b*d + a*h))/(32*a^3) +
(5*b*x^11*(3*b*e + a*i))/(128*a^3) + (x^2*(11*b*d - a*h))/(32*a*b) + (x^3*(
113*b*e - 5*a*i))/(384*a*b))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x
)
```

```
[Out] Timed out
```

3.163 $\int \frac{ag-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=14

$$\frac{gx}{\sqrt{a+bx^4}}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {383}

$$\frac{gx}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^4]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{ag-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx = \frac{gx}{\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{gx}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^4]
```

IntegrateAlgebraic [A] time = 0.37, size = 14, normalized size = 1.00

$$\frac{gx}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^4]
```

fricas [A] time = 0.41, size = 12, normalized size = 0.86

$$\frac{gx}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
[Out] g*x/sqrt(b*x^4 + a)
```

giac [A] time = 0.20, size = 12, normalized size = 0.86

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] g*x/sqrt(b*x^4 + a)
```

maple [A] time = 0.05, size = 13, normalized size = 0.93

$$\frac{gx}{\sqrt{b\,x^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x)
[Out] g*x/(b*x^4+a)^(1/2)
```

maxima [A] time = 1.76, size = 12, normalized size = 0.86

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] g*x/sqrt(b*x^4 + a)
```

mupad [B] time = 5.04, size = 12, normalized size = 0.86

$$\frac{g\,x}{\sqrt{b\,x^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g - b*g*x^4)/(a + b*x^4)^(3/2),x)
[Out] (g*x)/(a + b*x^4)^(1/2)
```

sympy [C] time = 9.60, size = 80, normalized size = 5.71

$$\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4}, \frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+a*g)/(b*x**4+a)**(3/2),x)
[Out] g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

3.164 $\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1856}

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

Rule 1856

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, -Simp[(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\int \frac{ag+ex-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.10, size = 27, normalized size = 0.93

$$\frac{x(2ag+ex)}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (x*(2*a*g + e*x))/(2*a*Sqrt[a + b*x^4])
```

IntegrateAlgebraic [A] time = 15.16, size = 29, normalized size = 1.00

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

fricas [A] time = 0.42, size = 34, normalized size = 1.17

$$\frac{\sqrt{bx^4+a}\left(2\,agx+ex^2\right)}{2\left(abx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*g*x + e*x^2)/(a*b*x^4 + a^2)
```

giac [A] time = 0.24, size = 23, normalized size = 0.79

$$\frac{\left(2g + \frac{xe}{a}\right)x}{2\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] 1/2*(2*g + x*e/a)*x/sqrt(b*x^4 + a)
```

maple [A] time = 0.05, size = 24, normalized size = 0.83

$$\frac{\left(2ag + ex\right)x}{2\sqrt{b\,x^4 + a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x)
[Out] 1/2*x*(2*a*g+e*x)/(b*x^4+a)^(1/2)/a
```

maxima [A] time = 1.77, size = 25, normalized size = 0.86

$$\frac{2\,agx + ex^2}{2\,\sqrt{bx^4 + a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] 1/2*(2*a*g*x + e*x^2)/(sqrt(b*x^4 + a)*a)
```

mupad [B] time = 4.91, size = 23, normalized size = 0.79

$$\frac{g\,x + \frac{e\,x^2}{2\,a}}{\sqrt{b\,x^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x)
[Out] (g*x + (e*x^2)/(2*a))/(a + b*x^4)^(1/2)
```

sympy [C] time = 12.39, size = 104, normalized size = 3.59

$$\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1 + \frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+a*g+e*x)/(b*x**4+a)**(3/2),x)
[Out] g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/
a))
```

3.165 $\int \frac{ag+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1856}

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] -(f - 2*b*g*x)/(2*b*Sqrt[a + b*x^4])
```

Rule 1856

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, -Simp[(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\int \frac{ag+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.04, size = 27, normalized size = 1.08

$$\frac{2bgx-f}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (-f + 2*b*g*x)/(2*b*Sqrt[a + b*x^4])
```

IntegrateAlgebraic [A] time = 15.85, size = 27, normalized size = 1.08

$$\frac{2bgx-f}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (-f + 2*b*g*x)/(2*b*Sqrt[a + b*x^4])
```

fricas [A] time = 0.42, size = 33, normalized size = 1.32

$$\frac{\sqrt{bx^4+a}\,\left(2\,bgx-f\right)}{2\,\left(b^2x^4+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
[Out] 1/2*sqrt(b*x^4 + a)*(2*b*g*x - f)/(b^2*x^4 + a*b)
```

giac [A] time = 0.20, size = 22, normalized size = 0.88

$$\frac{2\,gx-\frac{f}{b}}{2\,\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] 1/2*(2*g*x - f/b)/sqrt(b*x^4 + a)
```

maple [A] time = 0.05, size = 24, normalized size = 0.96

$$\frac{2bgx-f}{2\sqrt{b\,x^4+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x)
[Out] 1/2*(2*b*g*x-f)/b/(b*x^4+a)^(1/2)
```

maxima [A] time = 1.83, size = 23, normalized size = 0.92

$$\frac{2\,bgx-f}{2\,\sqrt{bx^4+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] 1/2*(2*b*g*x - f)/(sqrt(b*x^4 + a)*b)
```

mupad [B] time = 4.90, size = 20, normalized size = 0.80

$$\frac{g\,x-\frac{f}{2\,b}}{\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x)
[Out] (g*x - f/(2*b))/(a + b*x^4)^(1/2)
```

sympy [A] time = 17.80, size = 109, normalized size = 4.36

$$f\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}-\frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+f*x**3+a*g)/(b*x**4+a)**(3/2),x)
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

3.166 $\int \frac{ag+ex+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{-2abgx+af-bex^2}{2ab\sqrt{a+bx^4}}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1856}

$$-\frac{-2abgx+af-bex^2}{2ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] -(a*f - 2*a*b*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])
```

Rule 1856

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, -Simp[(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\int \frac{ag+ex+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=-\frac{af-2abgx-bex^2}{2ab\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 1.00

$$\frac{2abgx-af+bex^2}{2ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (-(a*f) + 2*a*b*g*x + b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])
```

IntegrateAlgebraic [A] time = 34.85, size = 38, normalized size = 1.00

$$\frac{2abgx-af+bex^2}{2ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (-(a*f) + 2*a*b*g*x + b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])
```

fricas [A] time = 0.41, size = 44, normalized size = 1.16

$$\frac{\sqrt{bx^4+a}\left(2\,abgx+bex^2-af\right)}{2\left(ab^2x^4+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)
```

giac [A] time = 0.22, size = 31, normalized size = 0.82

$$\frac{\left(2\,g+\frac{xe}{a}\right)x-\frac{f}{b}}{2\,\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*g + x*e/a)*x - f/b)/sqrt(b*x^4 + a)
```

maple [A] time = 0.04, size = 35, normalized size = 0.92

$$\frac{2abgx+be\,x^2-af}{2\sqrt{b\,x^4+a}\;ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*(2*a*b*g*x+b*e*x^2-a*f)/a/b/(b*x^4+a)^(1/2)
```

maxima [A] time = 1.85, size = 44, normalized size = 1.16

$$\frac{\sqrt{bx^4+a}\left(2\,abgx+bex^2-af\right)}{2\left(ab^2x^4+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)
```

mupad [B] time = 4.84, size = 29, normalized size = 0.76

$$\frac{g\,x-\frac{f}{2\,b}+\frac{e\,x^2}{2\,a}}{\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

```
[Out] (g*x - f/(2*b) + (e*x^2)/(2*a))/(a + b*x^4)^(1/2)
```

sympy [A] time = 21.51, size = 133, normalized size = 3.50

$$f\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}-\frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+f*x**3+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*
x**4/a))
```

3.167 $\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{x^4+1}}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {383}

$$-\frac{x}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(1 + x^4)^(3/2),x]
```

```
[Out] -(x/Sqrt[1 + x^4])
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{-1+x^4}{\left(1+x^4\right)^{3/2}}\,dx = -\frac{x}{\sqrt{1+x^4}}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{x}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(1 + x^4)^(3/2),x]
```

```
[Out] -(x/Sqrt[1 + x^4])
```

IntegrateAlgebraic [A] time = 0.21, size = 12, normalized size = 1.00

$$-\frac{x}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(1 + x^4)^(3/2),x]
```

```
[Out] -(x/Sqrt[1 + x^4])
```

fricas [A] time = 0.41, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] -x/sqrt(x^4 + 1)
```

giac [A] time = 0.18, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="giac")
[Out] -x/sqrt(x^4 + 1)
```

maple [A] time = 0.05, size = 11, normalized size = 0.92

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^4+1)^(3/2),x)
[Out] -1/(x^4+1)^(1/2)*x
```

maxima [A] time = 3.24, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="maxima")
[Out] -x/sqrt(x^4 + 1)
```

mupad [B] time = 4.85, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^4 + 1)^(3/2),x)
[Out] -x/(x^4 + 1)^(1/2)
```

sympy [C] time = 5.21, size = 58, normalized size = 4.83

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**4+1)**(3/2),x)
[Out] x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9/
4)) - x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma
(5/4))
```

3.168 $\int \frac{1+x}{1+x^5} \, dx$

Optimal. Leaf size=109

$$-\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(-x-(-1)^{4/5}\right)+\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left(x+(\right.$$

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1586, 2068}

$$-\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(-x-(-1)^{4/5}\right)+\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left(x+(-1)^{2/5}\right)-\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(x-(-1)^{3/5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(1 + x^5),x]
```

```
[Out] -((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^(1/5) - x])/5 + ((-1)^(4/5)*(1 - (-1
)^(4/5))*Log[-(-1)^(4/5) - x])/5 + ((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^(2
/5) + x])/5 - ((-1)^(3/5)*(1 + (-1)^(3/5))*Log[-(-1)^(3/5) + x])/5
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2068

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[1/a^
(3*p), Int[ExpandIntegrand[1/((a - b*x)^p/(a^5 - b^5*x^5)^p), x], x], x] /;
 NeQ[a, 0] && EqQ[c, b^2/a] && EqQ[d, b^3/a^2] && EqQ[e, b^4/a^3]] /; FreeQ
[p, x] && PolyQ[P4, x, 4] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{1+x^5} \, dx &= \int \frac{1}{1-x+x^2-x^3+x^4} \, dx \\
&= \int \left(\frac{-1+(-1)^{4/5}}{5\left(-1+\sqrt[5]{-1}\,x\right)}+\frac{-1-(-1)^{3/5}}{5\left(-1-(-1)^{2/5}x\right)}+\frac{-1+(-1)^{2/5}}{5\left(-1+(-1)^{3/5}x\right)}+\frac{-1-\sqrt[5]{-1}}{5\left(-1-(-1)^{4/5}x\right)}\right) dx \\
&= -\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(-(-1)^{4/5}-x\right)+\frac{1}{5}(-1)^{2/5}\left(1-(\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.47

$$\text{RootSum}\left[\#1^4-\#1^3+\#1^2-\#1+1\&,\frac{\log(x-\#1)}{4\#1^3-3\#1^2+2\#1-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(1 + x^5),x]
```

```
[Out] RootSum[1 - #1 + #1^2 - #1^3 + #1^4 & , Log[x - #1]/(-1 + 2*#1 - 3*#1^2 + 4
*#1^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x}{1+x^5} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x)/(1 + x^5),x]
```

```
[Out] IntegrateAlgebraic[(1 + x)/(1 + x^5), x]
```

fricas [B] time = 1.28, size = 835, normalized size = 7.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^5+1),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))*log(3/8*(sqrt(5) + 5*sqrt(-2/
25*sqrt(5) - 1/5))^3 + 1/8*(3*sqrt(5) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(
sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2 + 3/8*((sqrt(5) + 5*sqrt(-2/25*sqr
t(5) - 1/5))^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 11*x + 1) -
1/10*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))*log(-3/8*(sqrt(5) + 5*sqrt(-2/
25*sqrt(5) - 1/5))^3 + (sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 + 11*x - 9
/2*sqrt(5) - 45/2*sqrt(-2/25*sqrt(5) - 1/5) - 14) + 1/10*(sqrt(5) + 5*sqrt(
-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*sqrt(-
2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(sqrt(
5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2))*log(-1/8*(3*sqrt(5) + 15*sqrt(-2/25*s
qrt(5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - (sqrt(5) + 5
*sqrt(-2/25*sqrt(5) - 1/5))^2 - 3/8*((sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5)
)^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 5/4*sqrt(-3/100*(sqrt(5
) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) -
 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(sqrt(5) - 5*sqrt(-2
/25*sqrt(5) - 1/5))^2)*((3*sqrt(5) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(sqr
t(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 8*sqrt(5) + 40*sqrt(-2/25*sqrt(5) - 1
/5) + 36) + 22*x + 9/2*sqrt(5) + 45/2*sqrt(-2/25*sqrt(5) - 1/5) + 2) + 1/10
*(sqrt(5) - 5*sqrt(-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*
(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1
/5)) - 3/100*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2))*log(-1/8*(3*sqrt(5
) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5
))^2 - (sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 3/8*((sqrt(5) + 5*sqrt(-
2/25*sqrt(5) - 1/5))^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 5/4*
sqrt(-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*s
qrt(-2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(
sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2)*((3*sqrt(5) + 15*sqrt(-2/25*sqrt(
5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 8*sqrt(5) + 40*sqr
t(-2/25*sqrt(5) - 1/5) + 36) + 22*x + 9/2*sqrt(5) + 45/2*sqrt(-2/25*sqrt(5)
 - 1/5) + 2)
```

giac [A] time = 0.22, size = 101, normalized size = 0.93

$$\frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x(\sqrt{5}+1)+1\right)+\frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x(\sqrt{5}-1)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^5+1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) +
 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))
- 1/10*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/10*sqrt(5)*log(x^2 +
1/2*x*(sqrt(5) - 1) + 1)
```

maple [B] time = 0.12, size = 173, normalized size = 1.59

$$\frac{\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}}+\frac{\sqrt{5}\ \arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}}-\frac{\sqrt{5}\ \arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{\sqrt{5}\ \ln\left(2x^2-\sqrt{5}\,x-x+2\right)}{10}+\frac{\sqrt{5}\ \ln\left(2x^2+\sqrt{5}\,x-x+2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^5+1),x)
```

```
[Out] 1/10*5^(1/2)*ln(2*x^2+5^(1/2)*x-x+2)+1/(10+2*5^(1/2))^(1/2)*arctan((4*x+5^(
1/2)-1)/(10+2*5^(1/2))^(1/2))-1/5/(10+2*5^(1/2))^(1/2)*arctan((4*x+5^(1/2)-
1)/(10+2*5^(1/2))^(1/2))*5^(1/2)-1/10*5^(1/2)*ln(-5^(1/2)*x+2*x^2-x+2)+1/(1
0-2*5^(1/2))^(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))+1/5/(10-2*5
^(1/2))^(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{x^5+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^5+1),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(x^5 + 1), x)
```

mupad [B] time = 4.92, size = 64, normalized size = 0.59

$$\sum_{k=1}^{4}\ln\left(\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)\left(-4\,x+\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)\left(25\,\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)+15\,x-15\right)+1\right)\right)\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^5 + 1),x)
```

```
[Out] symsum(log(root(z^4 - z/25 + 1/125, z, k)*(root(z^4 - z/25 + 1/125, z, k)*(
25*root(z^4 - z/25 + 1/125, z, k) + 15*x - 15) - 4*x + 1))*root(z^4 - z/25
+ 1/125, z, k), k, 1, 4)
```

sympy [B] time = 1.20, size = 1287, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**5+1),x)
```

```
[Out] sqrt(5)*log(x**2 + x*(-48/11 - 21*sqrt(5)/11 + 4*sqrt(10)*sqrt(sqrt(5) + 3)
/11 + 45*sqrt(2)*sqrt(sqrt(5) + 3)/22) - 1381*sqrt(10)*sqrt(sqrt(5) + 3)/48
4 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)/10 -
sqrt(5)*log(x**2 + x*(-48/11 - 45*sqrt(2)*sqrt(3 - sqrt(5))/22 + 4*sqrt(10)
*sqrt(3 - sqrt(5))/11 + 21*sqrt(5)/11) - 2213*sqrt(5)/242 - 1381*sqrt(10)*s
qrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)/10 +
2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*sqrt(-2
*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sq
rt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
)) - 96/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*s
qrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt
(10)*sqrt(3 - sqrt(5)) + 15)) - 45*sqrt(2)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(
-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5))
 + 15)) + 8*sqrt(10)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3
- sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - s
```

```
qrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) + 42*sqrt(5)/(
-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - s
qrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt
(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5) + 3)/50 + 3/20)*atan(
44*x/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(
10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)
*sqrt(sqrt(5) + 3) + 15)) - 96/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3
) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqr
t(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)) - 42*sqrt(5)/(8*sqrt(5)
*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5
) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) +
 3) + 15)) + 8*sqrt(10)*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(
sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(1
0)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)) + 45*sqrt(2)
*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18
*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sq
rt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)))
```

3.169 $\int \frac{1-x}{1-x^5} \, dx$

Optimal. Leaf size=109

$$-\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-x-(-1)^{3/5}\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(x+\right.$$

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1586, 2068}

$$-\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-x-(-1)^{3/5}\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(x+\sqrt[5]{-1}\right)-\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(x-(-1)^{4/5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 - x^5),x]
```

```
[Out] -((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^(2/5) - x])/5 + ((-1)^(3/5)*(1 + (-1
)^(3/5))*Log[-(-1)^(3/5) - x])/5 + ((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^(1
/5) + x])/5 - ((-1)^(4/5)*(1 - (-1)^(4/5))*Log[-(-1)^(4/5) + x])/5
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2068

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[1/a^
(3*p), Int[ExpandIntegrand[1/((a - b*x)^p/(a^5 - b^5*x^5)^p), x], x], x] /;
 NeQ[a, 0] && EqQ[c, b^2/a] && EqQ[d, b^3/a^2] && EqQ[e, b^4/a^3]] /; FreeQ
[p, x] && PolyQ[P4, x, 4] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x}{1-x^5}\,dx &= \int \frac{1}{1+x+x^2+x^3+x^4}\,dx\\
&= \int \left(\frac{1-(-1)^{4/5}}{5\left(1+\sqrt[5]{-1}\,x\right)}+\frac{1+(-1)^{3/5}}{5\left(1-(-1)^{2/5}x\right)}+\frac{1-(-1)^{2/5}}{5\left(1+(-1)^{3/5}x\right)}+\frac{1+\sqrt[5]{-1}}{5\left(1-(-1)^{4/5}x\right)}\right)dx\\
&= -\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-(-1)^{3/5}-x\right)+\frac{1}{5}\sqrt[5]{-1}\left(1\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.43

$$\text{RootSum}\left[\#1^4+\#1^3+\#1^2+\#1+1\&,\frac{\log(x-\#1)}{4\#1^3+3\#1^2+2\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 - x^5),x]
```

```
[Out] RootSum[1 + #1 + #1^2 + #1^3 + #1^4 & , Log[x - #1]/(1 + 2*#1 + 3*#1^2 + 4*
#1^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x}{1-x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x)/(1 - x^5),x]
```

```
[Out] IntegrateAlgebraic[(1 - x)/(1 - x^5), x]
```

fricas [B] time = 1.27, size = 799, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(5) - sqrt(2*sqrt(5) - 5))*log(3/8*(sqrt(5) + sqrt(2*sqrt(5) - 5
))^3 + 1/8*(3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5
) - 5))^2 + 3/8*((sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*
sqrt(5) - 5)) + 11*x - 1) - 1/10*(sqrt(5) + sqrt(2*sqrt(5) - 5))*log(-3/8*(
sqrt(5) + sqrt(2*sqrt(5) - 5))^3 - (sqrt(5) + sqrt(2*sqrt(5) - 5))^2 + 11*x
 - 9/2*sqrt(5) - 9/2*sqrt(2*sqrt(5) - 5) + 14) + 1/10*(sqrt(5) + 5*sqrt(-3/
100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5)
)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2
))*log(-1/8*(3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(
5) - 5))^2 + (sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 3/8*((sqrt(5) + sqrt(2*sqr
t(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) + 5/4*sqrt(-3/100*(sqrt(
5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5)
 - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2)*((3*sqrt
(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 8*sqrt(5
) - 8*sqrt(2*sqrt(5) - 5) + 36) + 22*x + 9/2*sqrt(5) + 9/2*sqrt(2*sqrt(5) -
 5) - 2) + 1/10*(sqrt(5) - 5*sqrt(-3/100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2
- 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/
100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2))*log(-1/8*(3*sqrt(5) + 3*sqrt(2*sqrt
(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2 + (sqrt(5) + sqrt(2*sqrt(5)
 - 5))^2 - 3/8*((sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*s
qrt(5) - 5)) - 5/4*sqrt(-3/100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sq
rt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(
5) - sqrt(2*sqrt(5) - 5))^2)*((3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt
(5) - sqrt(2*sqrt(5) - 5)) - 8*sqrt(5) - 8*sqrt(2*sqrt(5) - 5) + 36) + 22*x
 + 9/2*sqrt(5) + 9/2*sqrt(2*sqrt(5) - 5) - 2)
```

giac [A] time = 0.18, size = 101, normalized size = 0.93

$$\frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-\frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5) + 10)) +
 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x + sqrt(5) + 1)/sqrt(-2*sqrt(5) + 10))
+ 1/10*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 1/10*sqrt(5)*log(x^2 -
1/2*x*(sqrt(5) - 1) + 1)
```

maple [B] time = 0.11, size = 169, normalized size = 1.55

$$\frac{\arctan\left(\frac{4x+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}}-\frac{\sqrt{5}\arctan\left(\frac{4x+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{\arctan\left(\frac{4x+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}}+\frac{\sqrt{5}\arctan\left(\frac{4x+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{\sqrt{5}\ln\left(2x^2-\sqrt{5}\,x+x+2\right)}{10}+\frac{\sqrt{5}\ln\left(2x^2+\sqrt{5}\,x+x+2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(-x^5+1),x)
```

```
[Out] -1/10*5^(1/2)*ln(-5^(1/2)*x+2*x^2+x+2)+1/(10+2*5^(1/2))^(1/2)*arctan((4*x+1
-5^(1/2))/(10+2*5^(1/2))^(1/2))-1/5/(10+2*5^(1/2))^(1/2)*arctan((4*x+1-5^(1
/2))/(10+2*5^(1/2))^(1/2))*5^(1/2)+1/10*5^(1/2)*ln(5^(1/2)*x+2*x^2+x+2)+1/(
10-2*5^(1/2))^(1/2)*arctan((4*x+1+5^(1/2))/(10-2*5^(1/2))^(1/2))+1/5/(10-2*
5^(1/2))^(1/2)*arctan((4*x+1+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{x^5-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/(x^5 - 1), x)
```

mupad [B] time = 4.98, size = 65, normalized size = 0.60

$$\sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\left(4\,x+\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\left(25\,\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)+15\,x+15\right)+1\right)\right)\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^5 - 1),x)
```

```
[Out] symsum(log(-root(z^4 + z/25 + 1/125, z, k)*(4*x + root(z^4 + z/25 + 1/125,
z, k)*(25*root(z^4 + z/25 + 1/125, z, k) + 15*x + 15) + 1))*root(z^4 + z/25
 + 1/125, z, k), k, 1, 4)
```

sympy [B] time = 1.28, size = 1287, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x**5+1),x)
```

```
[Out] sqrt(5)*log(x**2 + x*(-21*sqrt(5)/11 - 4*sqrt(10)*sqrt(3 - sqrt(5))/11 + 45
*sqrt(2)*sqrt(3 - sqrt(5))/22 + 48/11) - 2213*sqrt(5)/242 - 1381*sqrt(10)*s
qrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)/10 -
sqrt(5)*log(x**2 + x*(-45*sqrt(2)*sqrt(sqrt(5) + 3)/22 - 4*sqrt(10)*sqrt(sq
rt(5) + 3)/11 + 21*sqrt(5)/11 + 48/11) - 1381*sqrt(10)*sqrt(sqrt(5) + 3)/48
4 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)/10 +
2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*sqrt(-2
*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sq
rt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
)) - 42*sqrt(5)/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sq
rt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt
(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) - 8*sqrt(10)*sqrt(3 - sqrt(5))/(-8*sq
rt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5
))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 -
sqrt(5)) + 15)) + 45*sqrt(2)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sqrt(-2*sqrt(10)
*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sq
rt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) + 96/(
-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - s
qrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt
(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5) + 3)/50 + 3/20)*atan(
44*x/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(
10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)
*sqrt(sqrt(5) + 3) + 15)) - 45*sqrt(2)*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2
```

```
*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) +
 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15
)) - 8*sqrt(10)*sqrt(sqrt(5) + 3)/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5)
+ 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(
sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)) + 42*sqrt(5)/(8*sqrt
(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(sqr
t(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sqrt(-2*sqrt(10)*sqrt(sqrt(5
) + 3) + 15)) + 96/(8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 18
*sqrt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15) + 3*sqrt(10)*sqrt(sqrt(5) + 3)*sq
rt(-2*sqrt(10)*sqrt(sqrt(5) + 3) + 15)))
```

3.170 $\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$

Optimal. Leaf size=208

$$\frac{x^{12}\left(a^2f-abe+b^2d\right)}{12b^3}-\frac{a^3\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^7}+\frac{a^2x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6}-\frac{ax^6}{}$$

Rubi [A] time = 0.32, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^9\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{9b^4}-\frac{ax^6\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5}+\frac{a^2x^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6}-\frac{a^3\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^7}+\frac{x^{12}\left(a^2f-abe+b^2d\right)}{12b^3}+\frac{x^{15}(be-af)}{15b^2}+\frac{fx^{18}}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^6) - (a*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^6)/(6*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9
)/(9*b^4) + ((b^2*d - a*b*e + a^2*f)*x^12)/(12*b^3) + ((b*e - a*f)*x^15)/(1
5*b^2) + (f*x^18)/(18*b) - (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a +
 b*x^3])/(3*b^7)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{a+bx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^6}+\frac{a\left(-b^3c+ab^2d-a^2be+a^3\right.}{b^5}\right.\right.\\
&= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^6}-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{6b^5}+\frac{\left(b^3c-\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 187, normalized size = 0.90

$$\frac{60a^3\log\left(a+bx^3\right)\left(a^3f-a^2be+ab^2d-b^3c\right)+bx^3\left(-60a^5f+30a^4b\left(2e+fx^3\right)-10a^3b^2\left(6d+3ex^3+2fx^6\right)+5a^2b^3\left(12c+6dx^3+4ex^6+3fx^9\right)-ab^4x^3\left(30c+20dx^3+15ex^6+12fx^9\right)+b^5x^6\left(20c+15dx^3+12ex^6+10fx^9\right)\right)}{180b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(-60*a^5*f + 30*a^4*b*(2*e + f*x^3) - 10*a^3*b^2*(6*d + 3*e*x^3 + 2*
f*x^6) + 5*a^2*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^5*x^6*(20*c + 1
```

```
5*d*x^3 + 12*e*x^6 + 10*f*x^9) - a*b^4*x^3*(30*c + 20*d*x^3 + 15*e*x^6 + 12
*f*x^9)) + 60*a^3*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a + b*x^3])/(1
80*b^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]
```

fricas [A] time = 0.41, size = 210, normalized size = 1.01

$$\frac{10\,b^6fx^{18}+12\left(b^6e-ab^5f\right)x^{15}+15\left(b^6d-ab^5e+a^2b^4f\right)x^{12}+20\left(b^6c-ab^5d+a^2b^4e-a^3b^3f\right)x^9-30\left(ab^5c-a^2b^4d+a^3b^3e-a^4b^2f\right)x^6+60\left(a^2b^4c-a^3b^3d+a^4b^2e-a^5bf\right)x^3-60\left(a^3b^3c-a^4b^2d+a^5be-a^6f\right)\log\left(bx^3+a\right)}{180\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(10*b^6*f*x^18 + 12*(b^6*e - a*b^5*f)*x^15 + 15*(b^6*d - a*b^5*e + a^
2*b^4*f)*x^12 + 20*(b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*x^9 - 30*(a*b^
5*c - a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*x^6 + 60*(a^2*b^4*c - a^3*b^3*d +
a^4*b^2*e - a^5*b*f)*x^3 - 60*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log
(b*x^3 + a))/b^7
```

giac [A] time = 0.17, size = 246, normalized size = 1.18

$$\frac{10\,b^5fx^{18}-12\,ab^4fx^{15}+12\,b^5x^{15}e+15\,b^5dx^{12}+15\,a^2b^3fx^{12}-15\,ab^4x^{12}e+20\,b^5cx^9-20\,ab^4dx^9-20\,a^3b^2fx^9+20\,a^2b^3x^9e-30\,ab^4cx^6+30\,a^2b^3dx^6+30\,a^4bfx^6-30\,a^3b^2x^6e+60\,a^2b^3cx^3-60\,a^3b^2dx^3-60\,a^5fx^3+60\,a^4bx^3e}{180\,b^6}-\frac{\left(a^3b^3c-a^4b^2d-a^6f+a^5be\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/180*(10*b^5*f*x^18 - 12*a*b^4*f*x^15 + 12*b^5*x^15*e + 15*b^5*d*x^12 + 15
*a^2*b^3*f*x^12 - 15*a*b^4*x^12*e + 20*b^5*c*x^9 - 20*a*b^4*d*x^9 - 20*a^3*
b^2*f*x^9 + 20*a^2*b^3*x^9*e - 30*a*b^4*c*x^6 + 30*a^2*b^3*d*x^6 + 30*a^4*b
*f*x^6 - 30*a^3*b^2*x^6*e + 60*a^2*b^3*c*x^3 - 60*a^3*b^2*d*x^3 - 60*a^5*f*
x^3 + 60*a^4*b*x^3*e)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*l
og(abs(b*x^3 + a))/b^7
```

maple [A] time = 0.05, size = 266, normalized size = 1.28

$$\frac{fx^{18}}{18b}-\frac{afx^{15}}{15b^2}+\frac{ex^{15}}{15b}+\frac{a^2fx^{12}}{12b^3}-\frac{aex^{12}}{12b^2}+\frac{dx^{12}}{12b}-\frac{a^3fx^9}{9b^4}+\frac{a^2ex^9}{9b^3}-\frac{adx^9}{9b^2}+\frac{cx^9}{9b}+\frac{a^4fx^6}{6b^5}-\frac{a^3ex^6}{6b^4}+\frac{a^2dx^6}{6b^3}-\frac{acx^6}{6b^2}-\frac{a^5fx^3}{3b^6}+\frac{a^4ex^3}{3b^5}-\frac{a^3dx^3}{3b^4}+\frac{a^2cx^3}{3b^3}+\frac{a^6f\ln\left(bx^3+a\right)}{3b^7}-\frac{a^5e\ln\left(bx^3+a\right)}{3b^6}+\frac{a^4d\ln\left(bx^3+a\right)}{3b^5}-\frac{a^3c\ln\left(bx^3+a\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/18*f*x^18/b-1/15/b^2*x^15*a*f+1/15/b*x^15*e+1/12/b^3*x^12*a^2*f-1/12/b^2*
x^12*a*e+1/12/b*x^12*d-1/9/b^4*x^9*a^3*f+1/9/b^3*x^9*a^2*e-1/9/b^2*x^9*a*d+
1/9/b*x^9*c+1/6/b^5*x^6*a^4*f-1/6/b^4*x^6*a^3*e+1/6/b^3*x^6*a^2*d-1/6/b^2*x
^6*a*c-1/3/b^6*x^3*a^5*f+1/3/b^5*x^3*a^4*e-1/3/b^4*x^3*a^3*d+1/3/b^3*x^3*a^
2*c+1/3*a^6/b^7*ln(b*x^3+a)*f-1/3*a^5/b^6*ln(b*x^3+a)*e+1/3*a^4/b^5*ln(b*x^
3+a)*d-1/3*a^3/b^4*ln(b*x^3+a)*c
```

maxima [A] time = 1.37, size = 209, normalized size = 1.00

$$\frac{10\,b^5fx^{18}+12\left(b^5e-ab^4f\right)x^{15}+15\left(b^5d-ab^4e+a^2b^3f\right)x^{12}+20\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^9-30\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^6+60\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)x^3}{180\,b^6}-\frac{\left(a^3b^3c-a^4b^2d+a^5be-a^6f\right)\log\left(bx^3+a\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/180*(10*b^5*f*x^18 + 12*(b^5*e - a*b^4*f)*x^15 + 15*(b^5*d - a*b^4*e + a^
2*b^3*f)*x^12 + 20*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^9 - 30*(a*b^
4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^6 + 60*(a^2*b^3*c - a^3*b^2*d + a^
4*b*e - a^5*f)*x^3)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log
(b*x^3 + a)/b^7
```

mupad [B] time = 4.92, size = 237, normalized size = 1.14

$$x^{15}\left(\frac{e}{15\,b}-\frac{a\,f}{15\,b^2}\right)+x^{12}\left(\frac{d}{12\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{12\,b}\right)+x^9\left(\frac{c}{9\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{9\,b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3\right)}{3\,b^7}+\frac{f\,x^{18}}{18\,b}+\frac{a^2\,x^3\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b^2}-\frac{a\,x^6\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^15*(e/(15*b) - (a*f)/(15*b^2)) + x^12*(d/(12*b) - (a*(e/b - (a*f)/b^2))/(
12*b)) + x^9*(c/(9*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(9*b)) + (log(a
 + b*x^3)*(a^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e))/(3*b^7) + (f*x^18)/(18
*b) + (a^2*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b^2) - (a*
x^6*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(6*b)
```

sympy [A] time = 1.32, size = 216, normalized size = 1.04

$$\frac{a^3\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^7}+x^{15}\left(-\frac{af}{15b^2}+\frac{e}{15b}\right)+x^{12}\left(\frac{a^2f}{12b^3}-\frac{ae}{12b^2}+\frac{d}{12b}\right)+x^9\left(-\frac{a^3f}{9b^4}+\frac{a^2e}{9b^3}-\frac{ad}{9b^2}+\frac{c}{9b}\right)+x^6\left(\frac{a^4f}{6b^5}-\frac{a^3e}{6b^4}+\frac{a^2d}{6b^3}-\frac{ac}{6b^2}\right)+x^3\left(-\frac{a^5f}{3b^6}+\frac{a^4e}{3b^5}-\frac{a^3d}{3b^4}+\frac{a^2c}{3b^3}\right)+\frac{fx^{18}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**7) + x**
15*(-a*f/(15*b**2) + e/(15*b)) + x**12*(a**2*f/(12*b**3) - a*e/(12*b**2) +
d/(12*b)) + x**9*(-a**3*f/(9*b**4) + a**2*e/(9*b**3) - a*d/(9*b**2) + c/(9*
b)) + x**6*(a**4*f/(6*b**5) - a**3*e/(6*b**4) + a**2*d/(6*b**3) - a*c/(6*b*
*2)) + x**3*(-a**5*f/(3*b**6) + a**4*e/(3*b**5) - a**3*d/(3*b**4) + a**2*c/
(3*b**3)) + f*x**18/(18*b)
```

3.171 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=170

$$\frac{x^9\left(a^2f-abe+b^2d\right)}{9b^3}+\frac{a^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6}-\frac{ax^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5}+\frac{x^6\left(a^3(-\right.}{}$$

Rubi [A] time = 0.24, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^6\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^4}-\frac{ax^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5}+\frac{a^2\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6}+\frac{x^9\left(a^2f-abe+b^2d\right)}{9b^3}+\frac{x^{12}(be-af)}{12b^2}+\frac{fx^{15}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^6)/(6*b^4) + ((b^2*d - a*b*e + a^2*f)*x^9)/(9*b^3) + ((b
*e - a*f)*x^12)/(12*b^2) + (f*x^15)/(15*b) + (a^2*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2\left(c+dx+ex^2+fx^3\right)}{a+bx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int\left(\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{(b\right.}{}\right.\right.\\
&= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{6b^4}+\frac{\left(b^2d-abe+\right.}{9b^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 154, normalized size = 0.91

$$\frac{bx^3\left(60a^4f-30a^3b\left(2e+fx^3\right)+10a^2b^2\left(6d+3ex^3+2fx^6\right)-5ab^3\left(12c+6dx^3+4ex^6+3fx^9\right)+b^4x^3\left(30c+20dx^3+15ex^6+12fx^9\right)\right)-60a^2\log\left(a+bx^3\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{180b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(60*a^4*f - 30*a^3*b*(2*e + f*x^3) + 10*a^2*b^2*(6*d + 3*e*x^3 + 2*f
*x^6) - 5*a*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^4*x^3*(30*c + 20*d
```

```
*x^3 + 15*e*x^6 + 12*f*x^9)) - 60*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*Log[a + b*x^3])/(180*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]
```

fricas [A] time = 0.41, size = 170, normalized size = 1.00

$$\frac{12\,b^5fx^{15}+15\left(b^5e-ab^4f\right)x^{12}+20\left(b^5d-ab^4e+a^2b^3f\right)x^9+30\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^6-60\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^3+60\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)\log\left(bx^3+a\right)}{180\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^5*f*x^15 + 15*(b^5*e - a*b^4*f)*x^12 + 20*(b^5*d - a*b^4*e + a^
2*b^3*f)*x^9 + 30*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^6 - 60*(a*b^4
*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 60*(a^2*b^3*c - a^3*b^2*d + a^4
*b*e - a^5*f)*log(b*x^3 + a))/b^6
```

giac [A] time = 0.19, size = 197, normalized size = 1.16

$$\frac{12\,b^4fx^{15}-15\,ab^3fx^{12}+15\,b^4x^{12}e+20\,b^4dx^9+20\,a^2b^2fx^9-20\,ab^3x^9e+30\,b^4cx^6-30\,ab^3dx^6-30\,a^3bfx^6+30\,a^2b^2x^6e-60\,ab^3cx^3+60\,a^2b^2dx^3+60\,a^4fx^3-60\,a^3bx^3e}{180\,b^5}+\frac{\left(a^2b^3c-a^3b^2d-a^5f+a^4be\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/180*(12*b^4*f*x^15 - 15*a*b^3*f*x^12 + 15*b^4*x^12*e + 20*b^4*d*x^9 + 20*
a^2*b^2*f*x^9 - 20*a*b^3*x^9*e + 30*b^4*c*x^6 - 30*a*b^3*d*x^6 - 30*a^3*b*f
*x^6 + 30*a^2*b^2*x^6*e - 60*a*b^3*c*x^3 + 60*a^2*b^2*d*x^3 + 60*a^4*f*x^3
- 60*a^3*b*x^3*e)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*log(a
bs(b*x^3 + a))/b^6
```

maple [A] time = 0.05, size = 218, normalized size = 1.28

$$\frac{f\,x^{15}}{15b}-\frac{af\,x^{12}}{12b^2}+\frac{e\,x^{12}}{12b}+\frac{a^2f\,x^9}{9b^3}-\frac{ae\,x^9}{9b^2}+\frac{d\,x^9}{9b}-\frac{a^3f\,x^6}{6b^4}+\frac{a^2e\,x^6}{6b^3}-\frac{ad\,x^6}{6b^2}+\frac{c\,x^6}{6b}+\frac{a^4f\,x^3}{3b^5}-\frac{a^3e\,x^3}{3b^4}+\frac{a^2d\,x^3}{3b^3}-\frac{ac\,x^3}{3b^2}-\frac{a^5f\ln\left(b\,x^3+a\right)}{3b^6}+\frac{a^4e\ln\left(b\,x^3+a\right)}{3b^5}-\frac{a^3d\ln\left(b\,x^3+a\right)}{3b^4}+\frac{a^2c\ln\left(b\,x^3+a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/15*f*x^15/b-1/12/b^2*x^12*a*f+1/12/b*x^12*e+1/9/b^3*x^9*a^2*f-1/9/b^2*x^9
*a*e+1/9/b*x^9*d-1/6/b^4*x^6*a^3*f+1/6/b^3*x^6*a^2*e-1/6/b^2*x^6*a*d+1/6/b*
x^6*c+1/3/b^5*x^3*a^4*f-1/3/b^4*x^3*a^3*e+1/3/b^3*x^3*a^2*d-1/3/b^2*x^3*a*c
-1/3*a^5/b^6*ln(b*x^3+a)*f+1/3*a^4/b^5*ln(b*x^3+a)*e-1/3*a^3/b^4*ln(b*x^3+a
)*d+1/3*a^2/b^3*ln(b*x^3+a)*c
```

maxima [A] time = 1.38, size = 169, normalized size = 0.99

$$\frac{12\,b^4fx^{15}+15\left(b^4e-ab^3f\right)x^{12}+20\left(b^4d-ab^3e+a^2b^2f\right)x^9+30\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^6-60\left(ab^3c-a^2b^2d+a^3be-a^4f\right)x^3}{180\,b^5}+\frac{\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)\log\left(bx^3+a\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/180*(12*b^4*f*x^15 + 15*(b^4*e - a*b^3*f)*x^12 + 20*(b^4*d - a*b^3*e + a^
2*b^2*f)*x^9 + 30*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^6 - 60*(a*b^3*c
 - a^2*b^2*d + a^3*b*e - a^4*f)*x^3)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d + a^4
*b*e - a^5*f)*log(b*x^3 + a)/b^6
```

mupad [B] time = 4.96, size = 189, normalized size = 1.11

$$x^{12}\left(\frac{e}{12\,b}-\frac{a\,f}{12\,b^2}\right)+x^9\left(\frac{d}{9\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{9\,b}\right)+x^6\left(\frac{c}{6\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{6\,b}\right)-\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3\right)}{3\,b^6}+\frac{f\,x^{15}}{15\,b}-\frac{a\,x^3\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^12*(e/(12*b) - (a*f)/(12*b^2)) + x^9*(d/(9*b) - (a*(e/b - (a*f)/b^2))/(9*
b)) + x^6*(c/(6*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(6*b)) - (log(a +
b*x^3)*(a^5*f - a^2*b^3*c + a^3*b^2*d - a^4*b*e))/(3*b^6) + (f*x^15)/(15*b)
 - (a*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b)
```

sympy [A] time = 1.35, size = 172, normalized size = 1.01

$$-\frac{a^2\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^6}+x^{12}\left(-\frac{af}{12b^2}+\frac{e}{12b}\right)+x^9\left(\frac{a^2f}{9b^3}-\frac{ae}{9b^2}+\frac{d}{9b}\right)+x^6\left(-\frac{a^3f}{6b^4}+\frac{a^2e}{6b^3}-\frac{ad}{6b^2}+\frac{c}{6b}\right)+x^3\left(\frac{a^4f}{3b^5}-\frac{a^3e}{3b^4}+\frac{a^2d}{3b^3}-\frac{ac}{3b^2}\right)+\frac{fx^{15}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] -a**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**6) + x*
*12*(-a*f/(12*b**2) + e/(12*b)) + x**9*(a**2*f/(9*b**3) - a*e/(9*b**2) + d/
(9*b)) + x**6*(-a**3*f/(6*b**4) + a**2*e/(6*b**3) - a*d/(6*b**2) + c/(6*b))
 + x**3*(a**4*f/(3*b**5) - a**3*e/(3*b**4) + a**2*d/(3*b**3) - a*c/(3*b**2)
) + f*x**15/(15*b)
```

3.172 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=132

$$\frac{x^6\left(a^2f-abe+b^2d\right)}{6b^3}-\frac{a\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5}+\frac{x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{x^9(be-}{9b^2}$$

Rubi [A] time = 0.18, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4}-\frac{a\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5}+\frac{x^6\left(a^2f-abe+b^2d\right)}{6b^3}+\frac{x^9(be-af)}{9b^2}+\frac{fx^{12}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^6)/(6*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (f*x^12)/(12*b) - (a*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^5)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\left(c+dx+ex^2+fx^3\right)}{a+bx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^2}{b^2}\right.\right.\\
&= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^4}+\frac{\left(b^2d-abe+a^2f\right)x^6}{6b^3}+\frac{(be-af)x^9}{9b^2}+\frac{fx^{12}}{12b}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 119, normalized size = 0.90

$$\frac{12a\log\left(a+bx^3\right)\left(a^3f-a^2be+ab^2d-b^3c\right)+bx^3\left(-12a^3f+6a^2b\left(2e+fx^3\right)-2ab^2\left(6d+3ex^3+2fx^6\right)+b^3\left(12c+6dx^3+4ex^6+3fx^9\right)\right)}{36b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(-12*a^3*f + 6*a^2*b*(2*e + f*x^3) - 2*a*b^2*(6*d + 3*e*x^3 + 2*f*x^
6) + b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9)) + 12*a*(-(b^3*c) + a*b^2*d -
 a^2*b*e + a^3*f)*Log[a + b*x^3])/(36*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (c + dx^3 + ex^6 + fx^9)}{a + bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
[Out] IntegrateAlgebraic[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]
```

fricas [A] time = 0.41, size = 130, normalized size = 0.98

$$\frac{3\,b^4fx^{12} + 4\left(b^4e - ab^3f\right)x^9 + 6\left(b^4d - ab^3e + a^2b^2f\right)x^6 + 12\left(b^4c - ab^3d + a^2b^2e - a^3bf\right)x^3 - 12\left(ab^3c - a^2b^2d + a^3be - a^4f\right)\log\left(bx^3 + a\right)}{36\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
[Out] 1/36*(3*b^4*f*x^12 + 4*(b^4*e - a*b^3*f)*x^9 + 6*(b^4*d - a*b^3*e + a^2*b^2
*f)*x^6 + 12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 12*(a*b^3*c - a^
2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a))/b^5
```

giac [A] time = 0.17, size = 148, normalized size = 1.12

$$\frac{3\,b^3fx^{12} - 4\,ab^2fx^9 + 4\,b^3x^9e + 6\,b^3dx^6 + 6\,a^2bfx^6 - 6\,ab^2x^6e + 12\,b^3cx^3 - 12\,ab^2dx^3 - 12\,a^3fx^3 + 12\,a^2bx^3e}{36\,b^4} - \frac{\left(ab^3c - a^2b^2d - a^4f + a^3be\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
[Out] 1/36*(3*b^3*f*x^12 - 4*a*b^2*f*x^9 + 4*b^3*x^9*e + 6*b^3*d*x^6 + 6*a^2*b*f*
x^6 - 6*a*b^2*x^6*e + 12*b^3*c*x^3 - 12*a*b^2*d*x^3 - 12*a^3*f*x^3 + 12*a^2
*b*x^3*e)/b^4 - 1/3*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*log(abs(b*x^3 +
 a))/b^5
```

maple [A] time = 0.05, size = 170, normalized size = 1.29

$$\frac{f\,x^{12}}{12b} - \frac{af\,x^9}{9b^2} + \frac{e\,x^9}{9b} + \frac{a^2f\,x^6}{6b^3} - \frac{ae\,x^6}{6b^2} + \frac{d\,x^6}{6b} - \frac{a^3f\,x^3}{3b^4} + \frac{a^2e\,x^3}{3b^3} - \frac{ad\,x^3}{3b^2} + \frac{c\,x^3}{3b} + \frac{a^4f\ln\left(b\,x^3 + a\right)}{3b^5} - \frac{a^3e\ln\left(b\,x^3 + a\right)}{3b^4} + \frac{a^2d\ln\left(b\,x^3 + a\right)}{3b^3} - \frac{ac\ln\left(b\,x^3 + a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
[Out] 1/12*f*x^12/b-1/9/b^2*x^9*a*f+1/9/b*x^9*e+1/6/b^3*x^6*a^2*f-1/6/b^2*x^6*a*e
+1/6/b*x^6*d-1/3/b^4*x^3*a^3*f+1/3/b^3*x^3*a^2*e-1/3/b^2*x^3*a*d+1/3/b*x^3*
c+1/3*a^4/b^5*ln(b*x^3+a)*f-1/3*a^3/b^4*ln(b*x^3+a)*e+1/3*a^2/b^3*ln(b*x^3+
a)*d-1/3*a/b^2*ln(b*x^3+a)*c
```

maxima [A] time = 1.37, size = 129, normalized size = 0.98

$$\frac{3\,b^3fx^{12} + 4\left(b^3e - ab^2f\right)x^9 + 6\left(b^3d - ab^2e + a^2bf\right)x^6 + 12\left(b^3c - ab^2d + a^2be - a^3f\right)x^3}{36\,b^4} - \frac{\left(ab^3c - a^2b^2d + a^3be - a^4f\right)\log\left(bx^3 + a\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
[Out] 1/36*(3*b^3*f*x^12 + 4*(b^3*e - a*b^2*f)*x^9 + 6*(b^3*d - a*b^2*e + a^2*b*f
)*x^6 + 12*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^4 - 1/3*(a*b^3*c - a^
2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a)/b^5
```

mupad [B] time = 4.93, size = 141, normalized size = 1.07

$$x^9\left(\frac{e}{9b}-\frac{af}{9b^2}\right)+x^6\left(\frac{d}{6b}-\frac{a\left(\frac{e}{b}-\frac{af}{b^2}\right)}{6b}\right)+x^3\left(\frac{c}{3b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{af}{b^2}\right)}{b}\right)}{3b}\right)+\frac{fx^{12}}{12b}+\frac{\ln\left(bx^3+a\right)\left(fa^4-ea^3b+da^2b^2-cab^3\right)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^9*(e/(9*b) - (a*f)/(9*b^2)) + x^6*(d/(6*b) - (a*(e/b - (a*f)/b^2))/(6*b))
 + x^3*(c/(3*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(3*b)) + (f*x^12)/(12
*b) + (log(a + b*x^3)*(a^4*f + a^2*b^2*d - a*b^3*c - a^3*b*e))/(3*b^5)
```

sympy [A] time = 1.05, size = 128, normalized size = 0.97

$$\frac{a\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^5}+x^9\left(-\frac{af}{9b^2}+\frac{e}{9b}\right)+x^6\left(\frac{a^2f}{6b^3}-\frac{ae}{6b^2}+\frac{d}{6b}\right)+x^3\left(-\frac{a^3f}{3b^4}+\frac{a^2e}{3b^3}-\frac{ad}{3b^2}+\frac{c}{3b}\right)+\frac{fx^{12}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**5) + x**9*(
-a*f/(9*b**2) + e/(9*b)) + x**6*(a**2*f/(6*b**3) - a*e/(6*b**2) + d/(6*b))
+ x**3*(-a**3*f/(3*b**4) + a**2*e/(3*b**3) - a*d/(3*b**2) + c/(3*b)) + f*x*
*12/(12*b)
```

3.173 $\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$

Optimal. Leaf size=96

$$\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{x^6(be-af)}{6b^2}+\frac{fx^9}{9b}$$

Rubi [A] time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1819, 1850}

$$\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4}+\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{x^6(be-af)}{6b^2}+\frac{fx^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^6)/(6*b^2) + (f*x^9)
/(9*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^4)
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{a+bx}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x}{b^2}+\frac{fx^2}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)}\right)\right.\\ &= \frac{\left(b^2d-abe+a^2f\right)x^3}{3b^3}+\frac{(be-af)x^6}{6b^2}+\frac{fx^9}{9b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a+\right.}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.92

$$\frac{bx^3\left(6a^2f-3ab\left(2e+fx^3\right)+b^2\left(6d+3ex^3+2fx^6\right)\right)+6\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{18b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(6*a^2*f - 3*a*b*(2*e + f*x^3) + b^2*(6*d + 3*e*x^3 + 2*f*x^6)) + 6*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(c + dx^3 + ex^6 + fx^9\right)}{a + bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
[Out] IntegrateAlgebraic[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]
```

fricas [A] time = 0.41, size = 92, normalized size = 0.96

$$\frac{2\,b^3 f x^9 + 3\left(b^3 e - ab^2 f\right)x^6 + 6\left(b^3 d - ab^2 e + a^2 bf\right)x^3 + 6\left(b^3 c - ab^2 d + a^2 be - a^3 f\right)\log\left(bx^3 + a\right)}{18\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
[Out] 1/18*(2*b^3*f*x^9 + 3*(b^3*e - a*b^2*f)*x^6 + 6*(b^3*d - a*b^2*e + a^2*b*f)
*x^3 + 6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/b^4
```

giac [A] time = 0.20, size = 101, normalized size = 1.05

$$\frac{2\,b^2 f x^9 - 3\,abfx^6 + 3\,b^2 x^6 e + 6\,b^2 dx^3 + 6\,a^2 fx^3 - 6\,abx^3 e}{18\,b^3} + \frac{\left(b^3 c - ab^2 d - a^3 f + a^2 be\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
[Out] 1/18*(2*b^2*f*x^9 - 3*a*b*f*x^6 + 3*b^2*x^6*e + 6*b^2*d*x^3 + 6*a^2*f*x^3 -
 6*a*b*x^3*e)/b^3 + 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(abs(b*x^3 +
 a))/b^4
```

maple [A] time = 0.04, size = 124, normalized size = 1.29

$$\frac{f\,x^9}{9b} - \frac{af\,x^6}{6b^2} + \frac{e\,x^6}{6b} + \frac{a^2 f\,x^3}{3b^3} - \frac{ae\,x^3}{3b^2} + \frac{d\,x^3}{3b} - \frac{a^3 f \ln\left(b\,x^3 + a\right)}{3b^4} + \frac{a^2 e \ln\left(b\,x^3 + a\right)}{3b^3} - \frac{ad \ln\left(b\,x^3 + a\right)}{3b^2} + \frac{c \ln\left(b\,x^3 + a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
[Out] 1/9/b*f*x^9-1/6/b^2*x^6*a*f+1/6/b*x^6*e+1/3/b^3*x^3*a^2*f-1/3/b^2*x^3*a*e+1
/3/b*x^3*d-1/3/b^4*ln(b*x^3+a)*a^3*f+1/3/b^3*ln(b*x^3+a)*a^2*e-1/3/b^2*ln(b
*x^3+a)*a*d+1/3*c*ln(b*x^3+a)/b
```

maxima [A] time = 1.39, size = 91, normalized size = 0.95

$$\frac{2\,b^2 f x^9 + 3\left(b^2 e - abf\right)x^6 + 6\left(b^2 d - abe + a^2 f\right)x^3}{18\,b^3} + \frac{\left(b^3 c - ab^2 d + a^2 be - a^3 f\right)\log\left(bx^3 + a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
[Out] 1/18*(2*b^2*f*x^9 + 3*(b^2*e - a*b*f)*x^6 + 6*(b^2*d - a*b*e + a^2*f)*x^3)/
b^3 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/b^4
```

mupad [B] time = 4.83, size = 96, normalized size = 1.00

$$x^6\left(\frac{e}{6\,b}-\frac{a\,f}{6\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{3\,b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^4}+\frac{f\,x^9}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^6*(e/(6*b) - (a*f)/(6*b^2)) + x^3*(d/(3*b) - (a*(e/b - (a*f)/b^2))/(3*b))
 + (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^4) + (f*x^9)/(
9*b)
```

sympy [A] time = 1.13, size = 88, normalized size = 0.92

$$x^6\left(-\frac{af}{6b^2}+\frac{e}{6b}\right)+x^3\left(\frac{a^2f}{3b^3}-\frac{ae}{3b^2}+\frac{d}{3b}\right)+\frac{fx^9}{9b}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**6*(-a*f/(6*b**2) + e/(6*b)) + x**3*(a**2*f/(3*b**3) - a*e/(3*b**2) + d/(
3*b)) + f*x**9/(9*b) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x*
*3)/(3*b**4)
```

3.174 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)} dx$

Optimal. Leaf size=80

$$-\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3}+\frac{x^3(be-af)}{3b^2}+\frac{c\log(x)}{a}+\frac{fx^6}{6b}$$

Rubi [A] time = 0.12, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3ab^3}+\frac{x^3(be-af)}{3b^2}+\frac{c\log(x)}{a}+\frac{fx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

```
[Out] ((b*e - a*f)*x^3)/(3*b^2) + (f*x^6)/(6*b) + (c*Log[x])/a - ((b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a*b^3)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{be-af}{b^2}+\frac{c}{ax}+\frac{fx}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{(be-af)x^3}{3b^2}+\frac{fx^6}{6b}+\frac{c\log(x)}{a}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a+bx^3\right)}{3ab^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 0.94

$$\frac{-2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+abx^3\left(-2af+2be+bfx^3\right)+6b^3c\log(x)}{6ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

```
[Out] (a*b*x^3*(2*b*e - 2*a*f + b*f*x^3) + 6*b^3*c*Log[x] - 2*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[a + b*x^3])/(6*a*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)), x]
```

fricas [A] time = 0.46, size = 80, normalized size = 1.00

$$\frac{ab^2fx^6+6\,b^3c\log(x)+2\left(ab^2e-a^2bf\right)x^3-2\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{6\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="fricas")
[Out] 1/6*(a*b^2*f*x^6 + 6*b^3*c*log(x) + 2*(a*b^2*e - a^2*b*f)*x^3 - 2*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/(a*b^3)
```

giac [A] time = 0.21, size = 79, normalized size = 0.99

$$\frac{c\log\left(|x|\right)}{a}+\frac{bfx^6-2\,afx^3+2\,bx^3e}{6\,b^2}-\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\log\left(\left|bx^3+a\right|\right)}{3\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="giac")
[Out] c*log(abs(x))/a + 1/6*(b*f*x^6 - 2*a*f*x^3 + 2*b*x^3*e)/b^2 - 1/3*(b^3*c -
a*b^2*d - a^3*f + a^2*b*e)*log(abs(b*x^3 + a))/(a*b^3)
```

maple [A] time = 0.05, size = 97, normalized size = 1.21

$$\frac{f\,x^6}{6b}-\frac{af\,x^3}{3b^2}+\frac{e\,x^3}{3b}+\frac{a^2f\ln\left(b\,x^3+a\right)}{3b^3}-\frac{ae\ln\left(b\,x^3+a\right)}{3b^2}+\frac{c\ln(x)}{a}-\frac{c\ln\left(b\,x^3+a\right)}{3a}+\frac{d\ln\left(b\,x^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x)
[Out] 1/6*f*x^6/b-1/3/b^2*x^3*a*f+1/3*e*x^3/b+1/3*a^2/b^3*ln(b*x^3+a)*f-1/3*a*e*l
n(b*x^3+a)/b^2+1/3*d*ln(b*x^3+a)/b-1/3*c*ln(b*x^3+a)/a+c*ln(x)/a
```

maxima [A] time = 1.38, size = 77, normalized size = 0.96

$$\frac{c\log\left(x^3\right)}{3\,a}+\frac{bfx^6+2\left(be-af\right)x^3}{6\,b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*c*log(x^3)/a + 1/6*(b*f*x^6 + 2*(b*e - a*f)*x^3)/b^2 - 1/3*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/(a*b^3)
```

mupad [B] time = 4.93, size = 76, normalized size = 0.95

$$x^3\left(\frac{e}{3\,b}-\frac{a\,f}{3\,b^2}\right)+\frac{f\,x^6}{6\,b}+\frac{c\,\ln(x)}{a}-\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x)
```

```
[Out] x^3*(e/(3*b) - (a*f)/(3*b^2)) + (f*x^6)/(6*b) + (c*log(x))/a - (log(a + b*x
^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a*b^3)
```

sympy [A] time = 5.26, size = 70, normalized size = 0.88

$$x^3\left(-\frac{af}{3b^2}+\frac{e}{3b}\right)+\frac{fx^6}{6b}+\frac{c\log{(x)}}{a}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log{\left(\frac{a}{b}+x^3\right)}}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a),x)
```

```
[Out] x**3*(-a*f/(3*b**2) + e/(3*b)) + f*x**6/(6*b) + c*log(x)/a + (a**3*f - a**2
*b*e + a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a*b**3)
```

3.175 $\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=81

$$-\frac{\log(x)(bc-ad)}{a^2}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2}-\frac{c}{3ax^3}+\frac{fx^3}{3b}$$

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^2b^2}-\frac{\log(x)(bc-ad)}{a^2}-\frac{c}{3ax^3}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

```
[Out] -c/(3*a*x^3) + (f*x^3)/(3*b) - ((b*c - a*d)*Log[x])/a^2 + ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^2*b^2)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{f}{b}+\frac{c}{ax^2}+\frac{-bc+ad}{a^2x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)}\right)dx,x,x^3\right)\\
&= -\frac{c}{3ax^3}+\frac{fx^3}{3b}-\frac{(bc-ad)\log(x)}{a^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a+bx^3\right)}{3a^2b^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.95

$$\frac{1}{3}\left(\frac{3\log(x)(ad-bc)}{a^2}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^2b^2}-\frac{c}{ax^3}+\frac{fx^3}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

```
[Out] (-(c/(a*x^3)) + (f*x^3)/b + (3*(-(b*c) + a*d)*Log[x])/a^2 + ((b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(a^2*b^2))/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)), x]
```

fricas [A] time = 0.48, size = 85, normalized size = 1.05

$$\frac{a^2bfx^6+\left(b^3c-ab^2d+a^2be-a^3f\right)x^3\log\left(bx^3+a\right)-3\left(b^3c-ab^2d\right)x^3\log(x)-ab^2c}{3\,a^2b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(a^2*b*f*x^6 + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3*log(b*x^3 + a) -
 3*(b^3*c - a*b^2*d)*x^3*log(x) - a*b^2*c)/(a^2*b^2*x^3)
```

giac [A] time = 0.18, size = 95, normalized size = 1.17

$$\frac{fx^3}{3\,b}-\frac{(bc-ad)\log\left(|x|\right)}{a^2}+\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^2b^2}+\frac{bcx^3-adx^3-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*f*x^3/b - (b*c - a*d)*log(abs(x))/a^2 + 1/3*(b^3*c - a*b^2*d - a^3*f +
a^2*b*e)*log(abs(b*x^3 + a))/(a^2*b^2) + 1/3*(b*c*x^3 - a*d*x^3 - a*c)/(a^2
*x^3)
```

maple [A] time = 0.06, size = 94, normalized size = 1.16

$$\frac{f\,x^3}{3b}-\frac{af\ln\left(b\,x^3+a\right)}{3b^2}+\frac{d\ln(x)}{a}-\frac{d\ln\left(b\,x^3+a\right)}{3a}-\frac{bc\ln(x)}{a^2}+\frac{bc\ln\left(b\,x^3+a\right)}{3a^2}+\frac{e\ln\left(b\,x^3+a\right)}{3b}-\frac{c}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x)
[Out] 1/3/b*f*x^3-1/3*a/b^2*ln(b*x^3+a)*f+1/3*e*ln(b*x^3+a)/b-1/3*d*ln(b*x^3+a)/a
+1/3/a^2*b*ln(b*x^3+a)*c-1/3/a*c/x^3+d*ln(x)/a-1/a^2*ln(x)*b*c
```

maxima [A] time = 1.33, size = 77, normalized size = 0.95

$$\frac{fx^3}{3\,b}-\frac{(bc-ad)\log\left(x^3\right)}{3\,a^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,a^2b^2}-\frac{c}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*f*x^3/b - 1/3*(b*c - a*d)*log(x^3)/a^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*log(b*x^3 + a)/(a^2*b^2) - 1/3*c/(a*x^3)
```

mupad [B] time = 4.97, size = 74, normalized size = 0.91

$$\frac{f\,x^3}{3\,b}-\frac{c}{3\,a\,x^3}+\frac{\ln(x)\,\left(a\,d-b\,c\right)}{a^2}+\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x)
```

```
[Out] (f*x^3)/(3*b) - c/(3*a*x^3) + (log(x)*(a*d - b*c))/a^2 + (log(a + b*x^3)*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^2*b^2)
```

sympy [A] time = 14.56, size = 70, normalized size = 0.86

$$\frac{fx^3}{3b} - \frac{c}{3ax^3} + \frac{(ad - bc)\log(x)}{a^2} - \frac{\left(a^3f - a^2be + ab^2d - b^3c\right)\log\left(\frac{a}{b} + x^3\right)}{3a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a),x)
```

```
[Out] f*x**3/(3*b) - c/(3*a*x**3) + (a*d - b*c)*log(x)/a**2 - (a**3*f - a**2*b*e
+ a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a**2*b**2)
```

3.176 $\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=95

$$\frac{bc-ad}{3a^2x^3}+\frac{\log(x)\left(a^2e-abd+b^2c\right)}{a^3}-\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b}-\frac{c}{6ax^6}$$

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^3b}+\frac{\log(x)\left(a^2e-abd+b^2c\right)}{a^3}+\frac{bc-ad}{3a^2x^3}-\frac{c}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

```
[Out] -c/(6*a*x^6) + (b*c - a*d)/(3*a^2*x^3) + ((b^2*c - a*b*d + a^2*e)*Log[x])/a
^3 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^3*b)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)}\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^3}+\frac{-bc+ad}{a^2x^2}+\frac{b^2c-abd+a^2e}{a^3x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3(a+bx)}\right)\,dx\right.\\&=-\frac{c}{6ax^6}+\frac{bc-ad}{3a^2x^3}+\frac{\left(b^2c-abd+a^2e\right)\log(x)}{a^3}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a\right.}{3a^3b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 0.93

$$\frac{6\log(x)\left(a^2e-abd+b^2c\right)+\log\left(a+bx^3\right)\left(\frac{2a^3f}{b}-2a^2e+2abd-2b^2c\right)-\frac{a\left(ac+2adx^3-2bcx^3\right)}{x^6}}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

```
[Out] (-((a*(a*c - 2*b*c*x^3 + 2*a*d*x^3))/x^6) + 6*(b^2*c - a*b*d + a^2*e)*Log[x
] + (-2*b^2*c + 2*a*b*d - 2*a^2*e + (2*a^3*f)/b)*Log[a + b*x^3])/(6*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^7 (a + bx^3)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)), x]
```

fricas [A] time = 0.46, size = 101, normalized size = 1.06

$$-\frac{2 (b^3c - ab^2d + a^2be - a^3f)x^6 \log (bx^3 + a) - 6 (b^3c - ab^2d + a^2be)x^6 \log(x) + a^2bc - 2 (ab^2c - a^2bd)x^3}{6 a^3bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="fricas")
[Out] -1/6*(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6*log(b*x^3 + a) - 6*(b^3*c -
 a*b^2*d + a^2*b*e)*x^6*log(x) + a^2*b*c - 2*(a*b^2*c - a^2*b*d)*x^3)/(a^3*
b*x^6)
```

giac [A] time = 0.16, size = 126, normalized size = 1.33

$$\frac{(b^2c - abd + a^2e) \log (|x|)}{a^3} - \frac{(b^3c - ab^2d - a^3f + a^2be) \log (|bx^3 + a|)}{3 a^3b} - \frac{3 b^2cx^6 - 3 abdx^6 + 3 a^2x^6e - 2 abcx^3 + 2 a^2dx^3 + a^2c}{6 a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="giac")
[Out] (b^2*c - a*b*d + a^2*e)*log(abs(x))/a^3 - 1/3*(b^3*c - a*b^2*d - a^3*f + a^
2*b*e)*log(abs(b*x^3 + a))/(a^3*b) - 1/6*(3*b^2*c*x^6 - 3*a*b*d*x^6 + 3*a^2
*x^6*e - 2*a*b*c*x^3 + 2*a^2*d*x^3 + a^2*c)/(a^3*x^6)
```

maple [A] time = 0.05, size = 116, normalized size = 1.22

$$\frac{e \ln(x)}{a} - \frac{e \ln (b x^3 + a)}{3a} - \frac{bd \ln(x)}{a^2} + \frac{bd \ln (b x^3 + a)}{3a^2} + \frac{b^2c \ln(x)}{a^3} - \frac{b^2c \ln (b x^3 + a)}{3a^3} + \frac{f \ln (b x^3 + a)}{3b} - \frac{d}{3a x^3} + \frac{bc}{3a^2x^3} - \frac{c}{6a x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x)
[Out] 1/3/b*ln(b*x^3+a)*f-1/3*e*ln(b*x^3+a)/a+1/3/a^2*b*ln(b*x^3+a)*d-1/3/a^3*b^2
*ln(b*x^3+a)*c-1/6*c/a/x^6-1/3/a/x^3*d+1/3/a^2/x^3*b*c+e*ln(x)/a-1/a^2*ln(x
)*b*d+1/a^3*ln(x)*b^2*c
```

maxima [A] time = 1.36, size = 93, normalized size = 0.98

$$\frac{(b^2c - abd + a^2e) \log (x^3)}{3 a^3} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log (bx^3 + a)}{3 a^3b} + \frac{2 (bc - ad)x^3 - ac}{6 a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*(b^2*c - a*b*d + a^2*e)*log(x^3)/a^3 - 1/3*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*log(b*x^3 + a)/(a^3*b) + 1/6*(2*(b*c - a*d)*x^3 - a*c)/(a^2*x^6)
```

mupad [B] time = 4.99, size = 92, normalized size = 0.97

$$\frac{\ln(x)\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{a^3}-\frac{\frac{c}{6\,a}+\frac{x^3\,(a\,d-b\,c)}{3\,a^2}}{x^6}-\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x)
```

```
[Out] (log(x)*(b^2*c + a^2*e - a*b*d))/a^3 - (c/(6*a) + (x^3*(a*d - b*c))/(3*a^2)
)/x^6 - (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^3*b)
```

sympy [A] time = 74.00, size = 85, normalized size = 0.89

$$\frac{-ac+x^3\,(-2ad+2bc)}{6a^2x^6}+\frac{\left(a^2e-abd+b^2c\right)\log\left(x\right)}{a^3}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a}{b}+x^3\right)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a),x)
```

```
[Out] (-a*c + x**3*(-2*a*d + 2*b*c))/(6*a**2*x**6) + (a**2*e - a*b*d + b**2*c)*lo
g(x)/a**3 + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a**3
*b)
```

3.177 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=128

$$\frac{bc-ad}{6a^2x^6}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4}-\frac{\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^4}-\frac{c}{9ax^9}$$

Rubi [A] time = 0.16, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^4}-\frac{\log(x)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^4}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{bc-ad}{6a^2x^6}-\frac{c}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(6*a^2*x^6) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x^3
) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^4 + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^4}+\frac{-bc+ad}{a^2x^3}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x}-\frac{b\left(-\right.}{}\right.\right.\\ &= -\frac{c}{9ax^9}+\frac{bc-ad}{6a^2x^6}-\frac{b^2c-abd+a^2e}{3a^3x^3}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log(x)}{a^4}+\frac{\left(b^3c-ab\right.}{}\end{aligned}$$

Mathematica [A] time = 0.09, size = 128, normalized size = 1.00

$$\frac{bc-ad}{6a^2x^6}+\frac{a^2(-e)+abd-b^2c}{3a^3x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4}+\frac{\log(x)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^4}-\frac{c}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(6*a^2*x^6) + (-(b^2*c) + a*b*d - a^2*e)/(3*a^
3*x^3) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[x])/a^4 + ((b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)), x]
```

fricas [A] time = 0.45, size = 127, normalized size = 0.99

$$\frac{6\left(b^3c-ab^2d+a^2be-a^3f\right)x^9\log\left(bx^3+a\right)-18\left(b^3c-ab^2d+a^2be-a^3f\right)x^9\log(x)-6\left(ab^2c-a^2bd+a^3e\right)x^6-2\,a^3c+3\left(a^2bc-a^3d\right)x^3}{18\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(b*x^3 + a) - 18*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x^9*log(x) - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6
 - 2*a^3*c + 3*(a^2*b*c - a^3*d)*x^3)/(a^4*x^9)
```

giac [A] time = 0.18, size = 184, normalized size = 1.44

$$-\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\log\left(|x|\right)}{a^4}+\frac{\left(b^4c-ab^3d-a^3bf+a^2b^2e\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^4b}+\frac{11\,b^3cx^9-11\,ab^2dx^9-11\,a^3fx^9+11\,a^2bx^9e-6\,ab^2cx^6+6\,a^2bdx^6-6\,a^3x^6e+3\,a^2bcx^3-3\,a^3dx^3-2\,a^3c}{18\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(abs(x))/a^4 + 1/3*(b^4*c - a*b^3*d
 - a^3*b*f + a^2*b^2*e)*log(abs(b*x^3 + a))/(a^4*b) + 1/18*(11*b^3*c*x^9 -
11*a*b^2*d*x^9 - 11*a^3*f*x^9 + 11*a^2*b*x^9*e - 6*a*b^2*c*x^6 + 6*a^2*b*d*
x^6 - 6*a^3*x^6*e + 3*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^4*x^9)
```

maple [A] time = 0.05, size = 161, normalized size = 1.26

$$\frac{f\ln(x)}{a}-\frac{f\ln\left(b\,x^3+a\right)}{3a}-\frac{be\ln(x)}{a^2}+\frac{be\ln\left(b\,x^3+a\right)}{3a^2}+\frac{b^2d\ln(x)}{a^3}-\frac{b^2d\ln\left(b\,x^3+a\right)}{3a^3}-\frac{b^3c\ln(x)}{a^4}+\frac{b^3c\ln\left(b\,x^3+a\right)}{3a^4}-\frac{e}{3a\,x^3}+\frac{bd}{3a^2x^3}-\frac{b^2c}{3a^3x^3}-\frac{d}{6a\,x^6}+\frac{bc}{6a^2x^6}-\frac{c}{9a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x)
```

```
[Out] -1/3/a*ln(b*x^3+a)*f+1/3/a^2*ln(b*x^3+a)*b*e-1/3/a^3*ln(b*x^3+a)*b^2*d+1/3/
a^4*ln(b*x^3+a)*b^3*c-1/9/a*c/x^9-1/6/a/x^6*d+1/6/a^2/x^6*b*c-1/3/a/x^3*e+1
/3/a^2/x^3*b*d-1/3/a^3/x^3*b^2*c+1/a*ln(x)*f-1/a^2*ln(x)*b*e+1/a^3*ln(x)*b^
2*d-1/a^4*ln(x)*b^3*c
```

maxima [A] time = 1.36, size = 125, normalized size = 0.98

$$\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,a^4}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(x^3\right)}{3\,a^4}-\frac{6\left(b^2c-abd+a^2e\right)x^6-3\left(abc-a^2d\right)x^3+2\,a^2c}{18\,a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/a^4 - 1/3*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*log(x^3)/a^4 - 1/18*(6*(b^2*c - a*b*d + a^2*e)*x^
6 - 3*(a*b*c - a^2*d)*x^3 + 2*a^2*c)/(a^3*x^9)
```

mupad [B] time = 5.02, size = 123, normalized size = 0.96

$$\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^4}-\frac{\frac{c}{9\,a}+\frac{x^3\,\left(a\,d-b\,c\right)}{6\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,a^3}}{x^9}-\frac{\ln\left(x\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x)
```

```
[Out] (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^4) - (c/(9*a) + (
x^3*(a*d - b*c))/(6*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(3*a^3))/x^9 - (lo
g(x)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.178 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx$

Optimal. Leaf size=164

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{6a^3x^6}-\frac{b\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5}+\frac{b\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5}$$

Rubi [A] time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4x^3}-\frac{b\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5}+\frac{b\log(x)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^5}-\frac{a^2e-abd+b^2c}{6a^3x^6}+\frac{bc-ad}{9a^2x^9}-\frac{c}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

```
[Out] -c/(12*a*x^12) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(6*a^3*x
^6) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*Log[x])/a^5 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Lo
g[a + b*x^3])/(3*a^5)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)}\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^5}+\frac{-bc+ad}{a^2x^4}+\frac{b^2c-abd+a^2e}{a^3x^3}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2}-\cdots\right)\right.\\&=-\frac{c}{12ax^{12}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{6a^3x^6}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}+\frac{b\left(b^3c-ab^2d\cdots\right.}{}\end{aligned}$$

Mathematica [A] time = 0.09, size = 164, normalized size = 1.00

$$\frac{-a^4\left(3c+4dx^3+6ex^6+12fx^9\right)+2a^3bx^3\left(2c+3dx^3+6ex^6\right)-6a^2b^2x^6\left(c+2dx^3\right)+36bx^{12}\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)-12bx^{12}\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+12ab^3cx^9}{36a^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

```
[Out] (12*a*b^3*c*x^9 - 6*a^2*b^2*x^6*(c + 2*d*x^3) + 2*a^3*b*x^3*(2*c + 3*d*x^3
+ 6*e*x^6) - a^4*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + 36*b*(b^3*c - a*b^2
```

```
*d + a^2*b*e - a^3*f)*x^12*Log[x] - 12*b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*x^12*Log[a + b*x^3])/(36*a^5*x^12)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{13}(a + bx^3)}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)), x]
```

fricas [A] time = 0.49, size = 168, normalized size = 1.02

$$-\frac{12\left(b^4c - ab^3d + a^2b^2e - a^3bf\right)x^{12}\log\left(bx^3 + a\right) - 36\left(b^4c - ab^3d + a^2b^2e - a^3bf\right)x^{12}\log(x) - 12\left(ab^3c - a^2b^2d + a^3be - a^4f\right)x^9 + 6\left(a^2b^2c - a^3bd + a^4e\right)x^6 + 3a^4c - 4\left(a^3bc - a^4d\right)x^3}{36a^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(b*x^3 + a) - 36*
(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(x) - 12*(a*b^3*c - a^2*b^2
*d + a^3*b*e - a^4*f)*x^9 + 6*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 3*a^4*c -
 4*(a^3*b*c - a^4*d)*x^3)/(a^5*x^12)
```

giac [A] time = 0.17, size = 235, normalized size = 1.43

$$\frac{\left(b^4c - ab^3d - a^3bf + a^2b^2e\right)\log\left(|x|\right)}{a^5} - \frac{\left(b^5c - ab^4d - a^3b^2f + a^2b^3e\right)\log\left(\left|bx^3 + a\right|\right)}{3a^5b} - \frac{25b^4cx^{12} - 25ab^3dx^{12} - 25a^3bfx^{12} + 25a^2b^2x^{12}e - 12ab^3cx^9 + 12a^2b^2dx^9 + 12a^4fx^9 - 12a^3bx^9e + 6a^2b^2cx^6 - 6a^3bdx^6 + 6a^4x^6e - 4a^3bcx^3 + 4a^4dx^3 + 3a^4c}{36a^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="giac")
```

```
[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*log(abs(x))/a^5 - 1/3*(b^5*c - a*b^
4*d - a^3*b^2*f + a^2*b^3*e)*log(abs(b*x^3 + a))/(a^5*b) - 1/36*(25*b^4*c*x
^12 - 25*a*b^3*d*x^12 - 25*a^3*b*f*x^12 + 25*a^2*b^2*x^12*e - 12*a*b^3*c*x^
9 + 12*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 12*a^3*b*x^9*e + 6*a^2*b^2*c*x^6 - 6*
a^3*b*d*x^6 + 6*a^4*x^6*e - 4*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4*c)/(a^5*x^1
2)
```

maple [A] time = 0.06, size = 210, normalized size = 1.28

$$-\frac{bf\ln(x)}{a^2} + \frac{bf\ln\left(bx^3 + a\right)}{3a^2} + \frac{b^2e\ln(x)}{a^3} - \frac{b^2e\ln\left(bx^3 + a\right)}{3a^3} - \frac{b^3d\ln(x)}{a^4} + \frac{b^3d\ln\left(bx^3 + a\right)}{3a^4} + \frac{b^4c\ln(x)}{a^5} - \frac{b^4c\ln\left(bx^3 + a\right)}{3a^5} - \frac{f}{3ax^3} + \frac{be}{3a^2x^3} - \frac{b^2d}{3a^3x^3} + \frac{b^3c}{3a^4x^3} - \frac{e}{6ax^6} + \frac{bd}{6a^2x^6} - \frac{b^2c}{6a^3x^6} - \frac{d}{9ax^9} + \frac{bc}{9a^2x^9} - \frac{c}{12ax^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x)
```

```
[Out] 1/3*b/a^2*ln(b*x^3+a)*f-1/3*b^2/a^3*ln(b*x^3+a)*e+1/3*b^3/a^4*ln(b*x^3+a)*d
-1/3*b^4/a^5*ln(b*x^3+a)*c-1/12*c/a/x^12-1/9/a/x^9*d+1/9/a^2/x^9*b*c-1/6/a/
x^6*e+1/6/a^2/x^6*b*d-1/6/a^3/x^6*b^2*c-1/3/a/x^3*f+1/3/a^2/x^3*b*e-1/3/a^3
/x^3*b^2*d+1/3/a^4/x^3*b^3*c-1/a^2*b*ln(x)*f+1/a^3*b^2*ln(x)*e-1/a^4*b^3*ln
(x)*d+1/a^5*b^4*ln(x)*c
```

maxima [A] time = 1.36, size = 166, normalized size = 1.01

$$-\frac{\left(b^4c - ab^3d + a^2b^2e - a^3bf\right)\log\left(bx^3 + a\right)}{3a^5} + \frac{\left(b^4c - ab^3d + a^2b^2e - a^3bf\right)\log\left(x^3\right)}{3a^5} + \frac{12\left(b^3c - ab^2d + a^2be - a^3f\right)x^9 - 6\left(ab^2c - a^2bd + a^3e\right)x^6 - 3a^3c + 4\left(a^2bc - a^3d\right)x^3}{36a^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(b*x^3 + a)/a^5 + 1/3*(b^4*
c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x^3)/a^5 + 1/36*(12*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^9 - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6 - 3*a^3*c + 4*(
a^2*b*c - a^3*d)*x^3)/(a^4*x^12)
```

mupad [B] time = 5.07, size = 161, normalized size = 0.98

$$\frac{\ln(x)\left(-f\,a^3\,b+e\,a^2\,b^2-d\,a\,b^3+c\,b^4\right)}{a^5}-\frac{\ln\left(b\,x^3+a\right)\left(-f\,a^3\,b+e\,a^2\,b^2-d\,a\,b^3+c\,b^4\right)}{3\,a^5}-\frac{\frac{c}{12\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^4}+\frac{x^3\,(a\,d-b\,c)}{9\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{6\,a^3}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x)
```

```
[Out] (log(x)*(b^4*c + a^2*b^2*e - a*b^3*d - a^3*b*f))/a^5 - (log(a + b*x^3)*(b^4
*c + a^2*b^2*e - a*b^3*d - a^3*b*f))/(3*a^5) - (c/(12*a) - (x^9*(b^3*c - a^
3*f - a*b^2*d + a^2*b*e))/(3*a^4) + (x^3*(a*d - b*c))/(9*a^2) + (x^6*(b^2*c
 + a^2*e - a*b*d))/(6*a^3))/x^12
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.179 $\int \frac{c+dx^3+ex^6+fx^9}{x^{16}\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=205

$$\frac{bc-ad}{12a^2x^{12}}-\frac{a^2e-abd+b^2c}{9a^3x^9}+\frac{b^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^6}-\frac{b^2\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6}$$

Rubi [A] time = 0.21, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^4x^6}+\frac{b^2\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^6}-\frac{b^2\log(x)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^6}-\frac{a^2e-abd+b^2c}{9a^3x^9}+\frac{bc-ad}{12a^2x^{12}}-\frac{c}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] -c/(15*a*x^15) + (b*c - a*d)/(12*a^2*x^12) - (b^2*c - a*b*d + a^2*e)/(9*a^3
*x^9) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*x^6) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(3*a^5*x^3) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*Log[x])/a^6 + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(
3*a^6)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{16}\left(a+bx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^6(a+bx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^5}+\frac{b^2c-abd+a^2e}{a^3x^4}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^3}-\frac{b(-\right.\right.\\ &= -\frac{c}{15ax^{15}}+\frac{bc-ad}{12a^2x^{12}}-\frac{b^2c-abd+a^2e}{9a^3x^9}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^4x^6}-\frac{b\left(b^3c-ab^2d+\right.}{3a^5x^3}\end{aligned}$$

Mathematica [A] time = 0.24, size = 194, normalized size = 0.95

$$-\frac{-60b^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+180b^2\log(x)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+\frac{a\left(a^4\left(12c+15dx^3+20ex^6+30fx^9\right)-5a^3bx^3\left(3c+4dx^3+6ex^6+12fx^9\right)+10a^2b^2x^6\left(2c+3dx^3+6ex^6\right)-30ab^3x^9\left(c+2dx^3\right)+60b^4cx^{12}\right)}{x^{15}}}{180a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] -1/180*((a*(60*b^4*c*x^12 - 30*a*b^3*x^9*(c + 2*d*x^3) + 10*a^2*b^2*x^6*(2*
c + 3*d*x^3 + 6*e*x^6) - 5*a^3*b*x^3*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) +
 a^4*(12*c + 15*d*x^3 + 20*e*x^6 + 30*f*x^9)))/x^15 + 180*b^2*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*Log[x] - 60*b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a + b*x^3])/a^6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{16}\left(a + bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)), x]
```

fricas [A] time = 0.52, size = 210, normalized size = 1.02

$$\frac{60\left(b^5c - ab^4d + a^2b^3e - a^3b^2f\right)x^{15}\log\left(bx^3 + a\right) - 180\left(b^5c - ab^4d + a^2b^3e - a^3b^2f\right)x^{15}\log(x) - 60\left(ab^4c - a^2b^3d + a^3b^2e - a^4bf\right)x^{12} + 30\left(a^2b^3c - a^3b^2d + a^4be - a^5f\right)x^9 - 20\left(a^3b^2c - a^4bd + a^5e\right)x^6 - 12a^5c + 15\left(a^4bc - a^5d\right)x^3}{180a^6x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(60*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(b*x^3 + a) - 1
80*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(x) - 60*(a*b^4*c - a^
2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 30*(a^2*b^3*c - a^3*b^2*d + a^4*b*e -
 a^5*f)*x^9 - 20*(a^3*b^2*c - a^4*b*d + a^5*e)*x^6 - 12*a^5*c + 15*(a^4*b*c
 - a^5*d)*x^3)/(a^6*x^15)
```

giac [A] time = 0.17, size = 287, normalized size = 1.40

$$-\frac{\left(b^5c - ab^4d - a^3b^2f + a^2b^3e\right)\log\left(|x|\right)}{a^6} + \frac{\left(b^6c - ab^5d - a^3b^3f + a^2b^4e\right)\log\left(\left|bx^3 + a\right|\right)}{3a^6b} + \frac{137b^5cx^{15} - 137ab^4dx^{15} - 137a^3b^2fx^{15} + 137a^2b^3x^{15}e - 60ab^4cx^{12} + 60a^2b^3dx^{12} + 60a^4bfx^{12} - 60a^3b^2x^{12}e + 30a^2b^3cx^9 - 30a^3b^2dx^9 - 30a^5fx^9 + 30a^4bx^9e - 20a^3b^2cx^6 + 20a^4bdx^6 - 20a^5x^6e + 15a^4bcx^3 - 15a^5dx^3 - 12a^5c}{180a^6x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*log(abs(x))/a^6 + 1/3*(b^6*c - a
*b^5*d - a^3*b^3*f + a^2*b^4*e)*log(abs(b*x^3 + a))/(a^6*b) + 1/180*(137*b^
5*c*x^15 - 137*a*b^4*d*x^15 - 137*a^3*b^2*f*x^15 + 137*a^2*b^3*x^15*e - 60*
a*b^4*c*x^12 + 60*a^2*b^3*d*x^12 + 60*a^4*b*f*x^12 - 60*a^3*b^2*x^12*e + 30
*a^2*b^3*c*x^9 - 30*a^3*b^2*d*x^9 - 30*a^5*f*x^9 + 30*a^4*b*x^9*e - 20*a^3*
b^2*c*x^6 + 20*a^4*b*d*x^6 - 20*a^5*x^6*e + 15*a^4*b*c*x^3 - 15*a^5*d*x^3 -
 12*a^5*c)/(a^6*x^15)
```

maple [A] time = 0.05, size = 260, normalized size = 1.27

$$\frac{b^2f\ln(x)}{a^3} - \frac{b^2f\ln\left(bx^3 + a\right)}{3a^3} - \frac{b^3e\ln(x)}{a^4} + \frac{b^3e\ln\left(bx^3 + a\right)}{3a^4} + \frac{b^4d\ln(x)}{a^5} - \frac{b^4d\ln\left(bx^3 + a\right)}{3a^5} - \frac{b^5c\ln(x)}{a^6} + \frac{b^5c\ln\left(bx^3 + a\right)}{3a^6} + \frac{bf}{3a^2x^3} - \frac{b^2e}{3a^3x^3} + \frac{b^3d}{3a^4x^3} - \frac{b^4c}{3a^5x^3} - \frac{f}{6ax^6} + \frac{be}{6a^2x^6} - \frac{b^2d}{6a^3x^6} + \frac{b^3c}{6a^4x^6} - \frac{e}{9ax^9} + \frac{bd}{9a^2x^9} - \frac{b^2c}{9a^3x^9} - \frac{d}{12ax^{12}} + \frac{bc}{12a^2x^{12}} - \frac{c}{15ax^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x)
```

```
[Out] -1/3*b^2/a^3*ln(b*x^3+a)*f+1/3*b^3/a^4*ln(b*x^3+a)*e-1/3*b^4/a^5*ln(b*x^3+a
)*d+1/3*b^5/a^6*ln(b*x^3+a)*c-1/15*c/a/x^15-1/12/a/x^12*d+1/12/a^2/x^12*b*c
-1/9/a/x^9*e+1/9/a^2/x^9*b*d-1/9/a^3/x^9*b^2*c-1/6/a/x^6*f+1/6/a^2/x^6*b*e-
1/6/a^3/x^6*b^2*d+1/6/a^4/x^6*b^3*c+1/a^3*b^2*ln(x)*f-1/a^4*b^3*ln(x)*e+1/a
^5*b^4*ln(x)*d-1/a^6*b^5*ln(x)*c+1/3/a^2*b/x^3*f-1/3/a^3*b^2/x^3*e+1/3/a^4*
b^3/x^3*d-1/3/a^5*b^4/x^3*c
```

maxima [A] time = 1.41, size = 208, normalized size = 1.01

$$\frac{\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)\log\left(bx^3+a\right)}{3\,a^6}-\frac{\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)\log\left(x^3\right)}{3\,a^6}-\frac{60\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^{12}-30\left(ab^3c-a^2b^2d+a^3be-a^4f\right)x^9+20\left(a^2b^2c-a^3bd+a^4e\right)x^6+12\,a^4c-15\left(a^3bc-a^4d\right)x^3}{180\,a^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(b*x^3 + a)/a^6 - 1/3*(b^5
*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(x^3)/a^6 - 1/180*(60*(b^4*c - a*b
^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 30*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*
f)*x^9 + 20*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 12*a^4*c - 15*(a^3*b*c - a^
4*d)*x^3)/(a^5*x^15)
```

mupad [B] time = 0.26, size = 200, normalized size = 0.98

$$\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3\,b^2+e\,a^2\,b^3-d\,a\,b^4+c\,b^5\right)}{3\,a^6}-\frac{\frac{c}{15\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{6\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{12\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{9\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^5}}{x^{15}}-\frac{\ln(x)\,\left(-f\,a^3\,b^2+e\,a^2\,b^3-d\,a\,b^4+c\,b^5\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x)
```

```
[Out] (log(a + b*x^3)*(b^5*c + a^2*b^3*e - a^3*b^2*f - a*b^4*d))/(3*a^6) - (c/(15
*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(6*a^4) + (x^3*(a*d - b*c))
/(12*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(9*a^3) + (b*x^12*(b^3*c - a^3*f
- a*b^2*d + a^2*b*e))/(3*a^5))/x^15 - (log(x)*(b^5*c + a^2*b^3*e - a^3*b^2*
f - a*b^4*d))/a^6
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**16/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.180 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=348

$$\frac{x^{10}\left(a^2f-abe+b^2d\right)}{10b^3}+\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}-\frac{ax^4\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^5}+\frac{x^7\left(a^3(-f)+a^2b\right.}{7b}$$

Rubi [A] time = 0.33, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1836, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x^7\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{7b^4}-\frac{ax^4\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4b^5}+\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{19/3}}+\frac{a^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^6}-\frac{a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^{19/3}}+\frac{a^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}b^{19/3}}+\frac{x^{10}\left(a^2f-abe+b^2d\right)}{10b^3}+\frac{x^{13}(be-af)}{13b^2}+\frac{fx^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7)/(7*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)/(13*b^2)
 + (f*x^16)/(16*b) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) - (a^(7/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(19/3)) + (
a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*b^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx = \frac{fx^{16}}{16b}+\frac{\int \frac{x^9\left(16bc+16bdx^3+16(be-af)x^6\right)}{a+bx^3}\,dx}{16b}$$

$$= \frac{fx^{16}}{16b}+\frac{\int \left(\frac{16a^2\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^5}-\frac{16a\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{b^4}+\frac{16\left(b^3c-ab^2d+a^2be-a^3f\right)x^{\cdots}}{b^3}\cdots\right.}{16b}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6}-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^5}+\frac{\left(b^3c-ab^2d\cdots\right.}{}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6}-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^5}+\frac{\left(b^3c-ab^2d\cdots\right.}{}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6}-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^5}+\frac{\left(b^3c-ab^2d\cdots\right.}{}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6}-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^5}+\frac{\left(b^3c-ab^2d\cdots\right.}{}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6}-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^5}+\frac{\left(b^3c-ab^2d\cdots\right.}{}$$

Mathematica [A] time = 0.09, size = 351, normalized size = 1.01

$$\frac{x^{10}\left(a^2f-abe+b^2d\right)}{10b^3}-\frac{a^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^6}+\frac{ax^4\left(a^3f-a^2be+ab^2d-b^3c\right)}{4b^5}+\frac{x^7\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{7b^4}-\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^{19/3}}+\frac{a^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^{19/3}}+\frac{a^{7/3}\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt{3}b^{19/3}}+\frac{x^{13}(be-af)}{13b^2}+\frac{fx^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
```

```
x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)
/(13*b^2) + (f*x^16)/(16*b) + (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) +
 (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(19/3)) - (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(19/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 \left(c + dx^3 + ex^6 + fx^9\right)}{a + bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]
```

fricas [A] time = 0.43, size = 342, normalized size = 0.98

$$\frac{1365\,b^{5} f x^{16} + 1680\,{\left(b^{5} e - a b^{4} f\right)} x^{13} + 2184\,{\left(b^{5} d - a b^{4} e + a^{2} b^{3} f\right)} x^{10} + 3120\,{\left(b^{5} c - a b^{4} d + a^{2} b^{3} e - a^{3} b^{2} f\right)} x^{7} - 5460\,{\left(a b^{4} c - a^{2} b^{3} d + a^{3} b^{2} e - a^{4} b f\right)} x^{4} - 7280\,\sqrt{3}\,{\left(a^{2} b^{3} c - a^{3} b^{2} d + a^{4} b e - a^{5} f\right)} \left(\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2\,\sqrt{3} b x \left(\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3} a}{3\,a}\right) + 3640\,{\left(a^{2} b^{3} c - a^{3} b^{2} d + a^{4} b e - a^{5} f\right)} \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x^{2} - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 7280\,{\left(a^{2} b^{3} c - a^{3} b^{2} d + a^{4} b e - a^{5} f\right)} \left(\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 21840\,{\left(a^{2} b^{3} c - a^{3} b^{2} d + a^{4} b e - a^{5} f\right)} x}{21840\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/21840*(1365*b^5*f*x^16 + 1680*(b^5*e - a*b^4*f)*x^13 + 2184*(b^5*d - a*b^
4*e + a^2*b^3*f)*x^10 + 3120*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^7
- 5460*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4 - 7280*sqrt(3)*(a^2*
b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*
(a/b)^(2/3) - sqrt(3)*a)/a) + 3640*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f
)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 7280*(a^2*b^3*c - a^
3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 21840*(a^2*b^
3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6
```

giac [A] time = 0.18, size = 454, normalized size = 1.30

$$-\frac{\sqrt{3} {\left(\left(-a b^{2}\right)^{\frac{1}{3}} a^{2} b^{3} c - \left(-a b^{2}\right)^{\frac{1}{3}} a^{3} b^{2} d - \left(-a b^{2}\right)^{\frac{1}{3}} a^{5} f + \left(-a b^{2}\right)^{\frac{1}{3}} a^{4} b e\right)} \arctan\left(\frac{\sqrt{3} {\left(2 \, x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}}{3 \, \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 \, b^{7}} - \frac{{\left(\left(-a b^{2}\right)^{\frac{1}{3}} a^{2} b^{3} c - \left(-a b^{2}\right)^{\frac{1}{3}} a^{3} b^{2} d - \left(-a b^{2}\right)^{\frac{1}{3}} a^{5} f + \left(-a b^{2}\right)^{\frac{1}{3}} a^{4} b e\right)} \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \, b^{7}} + \frac{{\left(a^{3} b^{13} c - a^{4} b^{12} d - a^{6} b^{10} f + a^{5} b^{11} e\right)} \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left({\left| x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right|}\right)}{3 \, a b^{16}} + \frac{455 \, b^{15} f x^{16} - 560 \, a b^{14} f x^{13} + 560 \, b^{15} x^{13} e + 728 \, b^{15} d x^{10} + 728 \, a^{2} b^{13} f x^{10} - 728 \, a b^{14} x^{10} e + 1040 \, b^{15} c x^{7} - 1040 \, a b^{14} d x^{7} - 1040 \, a^{3} b^{12} f x^{7} + 1040 \, a^{2} b^{13} x^{7} e - 1820 \, a b^{14} c x^{4} + 1820 \, a^{2} b^{13} d x^{4} + 1820 \, a^{4} b^{11} f x^{4} - 1820 \, a^{3} b^{12} x^{4} e + 7280 \, a^{2} b^{13} c x - 7280 \, a^{3} b^{12} d x - 7280 \, a^{5} b^{10} f x + 7280 \, a^{4} b^{11} x e}{7280 \, b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*a^3*b^2*d - (-a*b^2
)^(1/3)*a^5*f + (-a*b^2)^(1/3)*a^4*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/b^7 - 1/6*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*a^3
*b^2*d - (-a*b^2)^(1/3)*a^5*f + (-a*b^2)^(1/3)*a^4*b*e)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/b^7 + 1/3*(a^3*b^13*c - a^4*b^12*d - a^6*b^10*f + a^5
*b^11*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^16) + 1/7280*(455*b^1
5*f*x^16 - 560*a*b^14*f*x^13 + 560*b^15*x^13*e + 728*b^15*d*x^10 + 728*a^2*
b^13*f*x^10 - 728*a*b^14*x^10*e + 1040*b^15*c*x^7 - 1040*a*b^14*d*x^7 - 104
0*a^3*b^12*f*x^7 + 1040*a^2*b^13*x^7*e - 1820*a*b^14*c*x^4 + 1820*a^2*b^13*
d*x^4 + 1820*a^4*b^11*f*x^4 - 1820*a^3*b^12*x^4*e + 7280*a^2*b^13*c*x - 728
0*a^3*b^12*d*x - 7280*a^5*b^10*f*x + 7280*a^4*b^11*x*e)/b^16
```

maple [A] time = 0.05, size = 592, normalized size = 1.70

$$\frac{f x^{16}}{16 b}-\frac{a f x^{13}}{13 b^{2}}+\frac{e x^{13}}{13 b}+\frac{a^{2} f x^{10}}{10 b^{3}}-\frac{a e x^{10}}{10 b^{2}}+\frac{d x^{10}}{10 b}-\frac{a^{3} f x^{7}}{7 b^{4}}+\frac{a^{2} e x^{7}}{7 b^{3}}-\frac{a d x^{7}}{7 b^{2}}+\frac{c x^{7}}{7 b}+\frac{a^{4} f x^{4}}{4 b^{5}}-\frac{a^{3} e x^{4}}{4 b^{4}}+\frac{a^{2} d x^{4}}{4 b^{3}}-\frac{a c x^{4}}{4 b^{2}}+\frac{\sqrt{3}\, a^{6} f \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{7}}+\frac{a^{6} f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{7}}-\frac{a^{6} f \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{7}}-\frac{\sqrt{3}\, a^{5} e \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{6}}-\frac{a^{5} e \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{6}}+\frac{a^{5} e \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{6}}-\frac{a^{5} f x}{b^{6}}+\frac{\sqrt{3}\, a^{4} d \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{5}}+\frac{a^{4} d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{5}}-\frac{a^{4} d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{5}}+\frac{a^{4} e x}{b^{5}}-\frac{\sqrt{3}\, a^{3} c \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}}-\frac{a^{3} c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}}+\frac{a^{3} c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}}-\frac{a^{3} d x}{b^{4}}+\frac{a^{2} c x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/3*a^6/b^7/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1
/3*a^5/b^6/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/
3*a^4/b^5/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3
*a^3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/10
/b^2*x^10*a*e-1/b^4*a^3*d*x-1/13/b^2*x^13*a*f+1/10/b^3*x^10*a^2*f+1/b^5*a^4
*e*x+1/4/b^3*x^4*a^2*d-1/4/b^2*x^4*a*c-1/b^6*a^5*f*x+1/4/b^5*x^4*a^4*f-1/4/
b^4*x^4*a^3*e-1/7/b^2*x^7*a*d+1/b^3*a^2*c*x-1/7/b^4*x^7*a^3*f+1/7/b^3*x^7*a
^2*e+1/3*a^6/b^7/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f+1/3*a^4/b^5/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))*d-1/3*a^3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/6*a^6/b^7/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-1/3*a^5/b^6/(a/b)^(2/3)*ln(x
+(a/b)^(1/3))*e+1/6*a^5/b^6/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e
-1/6*a^4/b^5/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/6*a^3/b^4/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/7/b*x^7*c+1/13/b*x^13*e+1/1
0/b*x^10*d+1/16*f*x^16/b
```

maxima [A]   time = 3.01, size = 351, normalized size = 1.01

$$\frac{455\,b^5 f x^{16} + 560\left(b^5 e - ab^4 f\right)x^{13} + 728\left(b^5 d - ab^4 e + a^2b^3 f\right)x^{10} + 1040\left(b^5 c - ab^4 d + a^2b^3 e - a^3b^2 f\right)x^7 - 1820\left(ab^4 c - a^2b^3 d + a^3b^2 e - a^4 b f\right)x^4 + 7280\left(a^2b^3 c - a^3b^2 d + a^4 b e - a^5 f\right)x}{7280\,b^6} - \frac{\sqrt{3}\left(a^3b^3c - a^4b^2d + a^5be - a^6f\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(a^3b^3c - a^4b^2d + a^5be - a^6f\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(a^3b^3c - a^4b^2d + a^5be - a^6f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/7280*(455*b^5*f*x^16 + 560*(b^5*e - a*b^4*f)*x^13 + 728*(b^5*d - a*b^4*e
+ a^2*b^3*f)*x^10 + 1040*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^7 - 18
20*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4 + 7280*(a^2*b^3*c - a^3*
b^2*d + a^4*b*e - a^5*f)*x)/b^6 - 1/3*sqrt(3)*(a^3*b^3*c - a^4*b^2*d + a^5*
b*e - a^6*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)
^(2/3)) + 1/6*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(b^7*(a/b)^(2/3)) - 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*
e - a^6*f)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

mupad [B]   time = 0.31, size = 358, normalized size = 1.03

$$x^{13}\left(\frac{e}{13\,b} - \frac{a\,f}{13\,b^2}\right) + x^{10}\left(\frac{d}{10\,b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{10\,b}\right) + x^7\left(\frac{c}{7\,b} - \frac{a\left(\frac{d}{b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{b}\right)}{7\,b}\right) + \frac{f\,x^{16}}{16\,b} - \frac{a^{7/3}\,\ln\left(b^{1/3}\,x + a^{1/3}\right)\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{3\,b^{19/3}} + \frac{a^2\,x\left(\frac{c}{b} - \frac{a\left(\frac{d}{b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b^2} - \frac{a\,x^4\left(\frac{c}{b} - \frac{a\left(\frac{d}{b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{4\,b} - \frac{a^{7/3}\,\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{3\,b^{19/3}} + \frac{a^{7/3}\,\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{3\,b^{19/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^13*(e/(13*b) - (a*f)/(13*b^2)) + x^10*(d/(10*b) - (a*(e/b - (a*f)/b^2))/(
10*b)) + x^7*(c/(7*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(7*b)) + (f*x^1
6)/(16*b) - (a^(7/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^
2*b*e))/(3*b^(19/3)) + (a^2*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b)
)/b^2 - (a*x^4*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(4*b) - (a^(7
/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*
(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(19/3)) + (a^(7/3)*log(3^(1/2)*a^
(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a
*b^2*d + a^2*b*e))/(3*b^(19/3))
```

sympy [A]   time = 4.35, size = 469, normalized size = 1.35

$$x^{13}\left(-\frac{af}{13b^2} + \frac{e}{13b}\right) + x^{10}\left(\frac{a^2 f}{10b^3} - \frac{ae}{10b^2} + \frac{d}{10b}\right) + x^7\left(-\frac{a^3 f}{7b^4} + \frac{a^2 e}{7b^3} - \frac{ad}{7b^2} + \frac{c}{7b}\right) + x^4\left(\frac{a^4 f}{4b^5} - \frac{a^3 e}{4b^4} + \frac{a^2 d}{4b^3} - \frac{ac}{4b^2}\right) + x\left(-\frac{a^5 f}{b^6} + \frac{a^4 e}{b^5} - \frac{a^3 d}{b^4} + \frac{a^2 c}{b^3}\right) + \operatorname{RootSum}\left(27t^3b^{19} - a^{16}f^3 + 3a^{15}bef^2 - 3a^{14}b^2df^2 - 3a^{14}b^2e^2f + 3a^{13}b^3cf^2 + 6a^{13}b^3def + a^{13}b^3e^3 - 6a^{12}b^4cef - 3a^{12}b^4d^2f - 3a^{12}b^4de^2 + 6a^{11}b^5cdf + 3a^{11}b^5ce^2 + 3a^{11}b^5d^2e - 3a^{10}b^6c^2f - 6a^{10}b^6cde - a^{10}b^6d^3 + 3a^9b^7c^2e + 3a^9b^7cd^2 - 3a^8b^8c^2d + a^7b^9c^3, \left(t \mapsto t\log\left(\frac{3tb^6}{a^5f - a^4be + a^3b^2d - a^2b^3c} + x\right)\right)\right) + \frac{fx^{16}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**13*(-a*f/(13*b**2) + e/(13*b)) + x**10*(a**2*f/(10*b**3) - a*e/(10*b**2)
 + d/(10*b)) + x**7*(-a**3*f/(7*b**4) + a**2*e/(7*b**3) - a*d/(7*b**2) + c/
```

```
(7*b)) + x**4*(a**4*f/(4*b**5) - a**3*e/(4*b**4) + a**2*d/(4*b**3) - a*c/(4
*b**2)) + x*(-a**5*f/b**6 + a**4*e/b**5 - a**3*d/b**4 + a**2*c/b**3) + Root
Sum(27*_t**3*b**19 - a**16*f**3 + 3*a**15*b*e*f**2 - 3*a**14*b**2*d*f**2 -
3*a**14*b**2*e**2*f + 3*a**13*b**3*c*f**2 + 6*a**13*b**3*d*e*f + a**13*b**3
*e**3 - 6*a**12*b**4*c*e*f - 3*a**12*b**4*d**2*f - 3*a**12*b**4*d*e**2 + 6*
a**11*b**5*c*d*f + 3*a**11*b**5*c*e**2 + 3*a**11*b**5*d**2*e - 3*a**10*b**6
*c**2*f - 6*a**10*b**6*c*d*e - a**10*b**6*d**3 + 3*a**9*b**7*c**2*e + 3*a**
9*b**7*c*d**2 - 3*a**8*b**8*c**2*d + a**7*b**9*c**3, Lambda(_t, _t*log(3*_t
*b**6/(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x))) + f*x**16/(16*
b)
```

3.181 $\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3} dx$

Optimal. Leaf size=316

$$\frac{x^8\left(a^2f-abe+b^2d\right)}{8b^3}-\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2b^5}+\frac{x^5\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{5b^4}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{}$$

Rubi [A] time = 0.31, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^5\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{5b^4}-\frac{ax^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2b^5}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{17/3}}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^{17/3}}-\frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}b^{17/3}}+\frac{x^8\left(a^2f-abe+b^2d\right)}{8b^3}+\frac{x^{11}(be-af)}{11b^2}+\frac{fx^{14}}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + ((b
*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) - (a^(5/3)*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
b^(17/3)) - (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(17/3)) + (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx = \frac{fx^{14}}{14b}+\frac{\int \frac{x^7\left(14bc+14bdx^3+14(be-af)x^6\right)}{a+bx^3}\,dx}{14b}$$

$$= \frac{fx^{14}}{14b}+\frac{\int \left(-\frac{14a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{14\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{b^3}+\frac{14\left(b^2d-abe+a^2f\right)x^7}{b^2} \cdots\right.}{14b}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^5}{5b^4}+\frac{\left(b^2d-ab \cdots\right.}{8 \cdots}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^5}{5b^4}+\frac{\left(b^2d-ab \cdots\right.}{8 \cdots}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^5}{5b^4}+\frac{\left(b^2d-ab \cdots\right.}{8 \cdots}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^5}{5b^4}+\frac{\left(b^2d-ab \cdots\right.}{8 \cdots}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^5}{5b^4}+\frac{\left(b^2d-ab \cdots\right.}{8 \cdots}$$

Mathematica [A] time = 0.11, size = 311, normalized size = 0.98

$$\frac{x^8\left(a^2f-abe+b^2d\right)}{8b^3}+\frac{ax^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{2b^5}+\frac{x^5\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{5b^4}-\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^{17/3}}+\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^{17/3}}+\frac{a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt{3}b^{17/3}}+\frac{x^{11}(be-af)}{11b^2}+\frac{fx^{14}}{14b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + (
```

```
(b*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) + (a^(5/3)*(-(b^3*c) + a*b^2*d
 - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b
^(17/3)) + (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/(3*b^(17/3)) - (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 \left(c + dx^3 + ex^6 + fx^9\right)}{a + bx^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]
```

fricas [A] time = 0.42, size = 321, normalized size = 1.02

$$\frac{660\, b^{4} f x^{14} + 840 \left(b^{4} e - a b^{3} f\right) x^{11} + 1155 \left(b^{4} d - a b^{3} e + a^{2} b^{2} f\right) x^{8} + 1848 \left(b^{4} c - a b^{3} d + a^{2} b^{2} e - a^{3} b f\right) x^{5} - 4620 \left(a b^{3} c - a^{2} b^{2} d + a^{3} b e - a^{4} f\right) x^{2} + 3080 \sqrt{3} \left(a b^{3} c - a^{2} b^{2} d + a^{3} b e - a^{4} f\right) \left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}} \arctan\left(\frac{2 \sqrt{3} b x \left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}} - \sqrt{3} a}{3 a}\right) + 1540 \left(a b^{3} c - a^{2} b^{2} d + a^{3} b e - a^{4} f\right) \left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}} \log\left(a x^{2} - b x \left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}} + a \left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right) - 3080 \left(a b^{3} c - a^{2} b^{2} d + a^{3} b e - a^{4} f\right) \left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}} \log\left(a x + b \left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}\right)}{9240\, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/9240*(660*b^4*f*x^14 + 840*(b^4*e - a*b^3*f)*x^11 + 1155*(b^4*d - a*b^3*e
 + a^2*b^2*f)*x^8 + 1848*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^5 - 4620
*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^2 + 3080*sqrt(3)*(a*b^3*c - a^2*
b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2
)^(1/3) - sqrt(3)*a)/a) + 1540*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(a^2
/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 3080*(a*
b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^
(2/3)))/b^5
```

giac [A] time = 0.18, size = 441, normalized size = 1.40

$$-\frac{\sqrt{3} \left(\left(-a b^{2}\right)^{\frac{2}{3}} a b^{3} c - \left(-a b^{2}\right)^{\frac{2}{3}} a^{2} b^{2} d - \left(-a b^{2}\right)^{\frac{2}{3}} a^{4} f + \left(-a b^{2}\right)^{\frac{2}{3}} a^{3} b e\right) \arctan\left(\frac{\sqrt{3} \left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 b^{7}} + \frac{\left(\left(-a b^{2}\right)^{\frac{2}{3}} a b^{3} c - \left(-a b^{2}\right)^{\frac{2}{3}} a^{2} b^{2} d - \left(-a b^{2}\right)^{\frac{2}{3}} a^{4} f + \left(-a b^{2}\right)^{\frac{2}{3}} a^{3} b e\right) \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b^{7}} - \frac{\left(a^{2} b^{12} c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^{3} b^{11} d \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^{5} b^{9} f \left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^{4} b^{10} \left(-\frac{a}{b}\right)^{\frac{1}{3}} e\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 a b^{14}} + \frac{220 b^{13} f x^{14} - 280 a b^{12} f x^{11} + 280 b^{13} x^{11} e + 385 b^{13} d x^{8} + 385 a^{2} b^{11} f x^{8} - 385 a b^{12} x^{8} e + 616 b^{13} c x^{5} - 616 a b^{12} d x^{5} - 616 a^{3} b^{10} f x^{5} + 616 a^{2} b^{11} x^{5} e - 1540 a b^{12} c x^{2} + 1540 a^{2} b^{11} d x^{2} + 1540 a^{4} b^{9} f x^{2} - 1540 a^{3} b^{10} x^{2} e}{3080 b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*b^2*d - (-a*b^2)^
(2/3)*a^4*f + (-a*b^2)^(2/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/b^7 + 1/6*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*b^2
*d - (-a*b^2)^(2/3)*a^4*f + (-a*b^2)^(2/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/b^7 - 1/3*(a^2*b^12*c*(-a/b)^(1/3) - a^3*b^11*d*(-a/b)^(1
/3) - a^5*b^9*f*(-a/b)^(1/3) + a^4*b^10*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a*b^14) + 1/3080*(220*b^13*f*x^14 - 280*a*b^12*f*x^11
 + 280*b^13*x^11*e + 385*b^13*d*x^8 + 385*a^2*b^11*f*x^8 - 385*a*b^12*x^8*e
 + 616*b^13*c*x^5 - 616*a*b^12*d*x^5 - 616*a^3*b^10*f*x^5 + 616*a^2*b^11*x^
5*e - 1540*a*b^12*c*x^2 + 1540*a^2*b^11*d*x^2 + 1540*a^4*b^9*f*x^2 - 1540*a
^3*b^10*x^2*e)/b^14
```

maple [B] time = 0.05, size = 554, normalized size = 1.75

$$\frac{f x^{14}}{14 b} - \frac{a f x^{11}}{11 b^{2}} + \frac{e x^{11}}{11 b} + \frac{a^{2} f x^{8}}{8 b^{3}} - \frac{a e x^{8}}{8 b^{2}} + \frac{d x^{8}}{8 b} - \frac{a^{3} f x^{5}}{5 b^{4}} + \frac{a^{2} e x^{5}}{5 b^{3}} - \frac{a d x^{5}}{5 b^{2}} + \frac{c x^{5}}{5 b} + \frac{a^{4} f x^{2}}{2 b^{5}} - \frac{a^{3} e x^{2}}{2 b^{4}} + \frac{a^{2} d x^{2}}{2 b^{3}} - \frac{a c x^{2}}{2 b^{2}} - \frac{\sqrt{3}\, a^{5} f \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{6}} + \frac{a^{5} f \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{6}} - \frac{a^{5} f \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{6}} + \frac{\sqrt{3}\, a^{4} e \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{5}} - \frac{a^{4} e \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{5}} + \frac{a^{4} e \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{5}} - \frac{\sqrt{3}\, a^{3} d \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{4}} + \frac{a^{3} d \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{4}} - \frac{a^{3} d \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{4}} + \frac{\sqrt{3}\, a^{2} c \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{3}} - \frac{a^{2} c \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{3}} + \frac{a^{2} c \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] -1/3*a^5/b^6*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-
1/3*a^3/b^4*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1
/3*a^2/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/
3*a^4/b^5*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/5
/b^4*x^5*a^3*f-1/11/b^2*x^11*a*f-1/5/b^2*x^5*a*d+1/5/b^3*x^5*a^2*e+1/8/b^3*
x^8*a^2*f-1/8/b^2*x^8*a*e-1/2/b^4*x^2*a^3*e+1/2/b^3*x^2*a^2*d+1/2/b^5*x^2*a
^4*f-1/2/b^2*x^2*a*c+1/6*a^4/b^5/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*e-1/6*a^3/b^4/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/6*a^2/b
^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/3*a^5/b^6/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))*f-1/3*a^4/b^5/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e+1/3*a^3/b^4
/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*
c-1/6*a^5/b^6/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f+1/11/b*x^11*e
+1/8/b*x^8*d+1/5/b*x^5*c+1/14*f*x^14/b
```

maxima [A] time = 2.95, size = 313, normalized size = 0.99

$$\frac{\sqrt{3}\left(a^2b^3c - a^3b^2d + a^4be - a^5f\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{220b^4fx^{14} + 280\left(b^4e - ab^3f\right)x^{11} + 385\left(b^4d - ab^3e + a^2b^2f\right)x^8 + 616\left(b^4c - ab^3d + a^2b^2e - a^3bf\right)x^5 - 1540\left(ab^3c - a^2b^2d + a^3be - a^4f\right)x^2}{3080b^5} + \frac{\left(a^2b^3c - a^3b^2d + a^4be - a^5f\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(a^2b^3c - a^3b^2d + a^4be - a^5f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/3080*(220*b^4*f*x^14 +
 280*(b^4*e - a*b^3*f)*x^11 + 385*(b^4*d - a*b^3*e + a^2*b^2*f)*x^8 + 616*(
b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^5 - 1540*(a*b^3*c - a^2*b^2*d + a^
3*b*e - a^4*f)*x^2)/b^5 + 1/6*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/b)^(1/3)) - 1/3*(a^2*b^3*c - a^
3*b^2*d + a^4*b*e - a^5*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

mupad [B] time = 5.16, size = 313, normalized size = 0.99

$$x^{11}\left(\frac{e}{11\,b} - \frac{a\,f}{11\,b^2}\right) + x^8\left(\frac{d}{8\,b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{8\,b}\right) + x^5\left(\frac{c}{5\,b} - \frac{a\left(\frac{d}{b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{b}\right)}{5\,b}\right) + \frac{f\,x^{14}}{14\,b} - \frac{a^{5/3}\ln\left(b^{1/3}\,x + a^{1/3}\right)\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{3\,b^{17/3}} - \frac{a\,x^2\left(\frac{c}{b} - \frac{a\left(\frac{d}{b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{2\,b} + \frac{a^{5/3}\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{3\,b^{17/3}} - \frac{a^{5/3}\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{3\,b^{17/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^11*(e/(11*b) - (a*f)/(11*b^2)) + x^8*(d/(8*b) - (a*(e/b - (a*f)/b^2))/(8*
b)) + x^5*(c/(5*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(5*b)) + (f*x^14)/
(14*b) - (a^(5/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b
*e))/(3*b^(17/3)) - (a*x^2*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(
2*b) + (a^(5/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(17/3)) - (a^(5/3)*lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c
 - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(17/3))
```

sympy [A] time = 4.06, size = 513, normalized size = 1.62

$$x^{11}\left(-\frac{af}{11b^2} + \frac{e}{11b}\right) + x^8\left(\frac{a^2f}{8b^3} - \frac{ae}{8b^2} + \frac{d}{8b}\right) + x^5\left(-\frac{a^3f}{5b^4} + \frac{a^2e}{5b^3} - \frac{ad}{5b^2} + \frac{c}{5b}\right) + x^2\left(\frac{a^4f}{2b^5} - \frac{a^3e}{2b^4} + \frac{a^2d}{2b^3} - \frac{ac}{2b^2}\right) + \operatorname{RootSum}\left(27t^3b^{17} - a^{14}f^3 + 3a^{13}bef^2 - 3a^{12}b^2df^2 - 3a^{12}b^2e^2f + 3a^{11}b^3cf^2 + 6a^{11}b^3def + a^{11}b^3e^3 - 6a^{10}b^4cef - 3a^{10}b^4d^2f - 3a^{10}b^4de^2 + 6a^9b^5cdf + 3a^9b^5ce^2 + 3a^9b^5d^2e - 3a^8b^6c^2f - 6a^8b^6cde - a^8b^6d^3 + 3a^7b^7c^2e + 3a^7b^7cd^2 - 3a^6b^8c^2d + a^5b^9c^3, \left(t \mapsto t\log\left(\frac{9t^2b^{11}}{a^9f^2 - 2a^8bef + 2a^7b^2df + a^7b^2e^2 - 2a^6b^3cf - 2a^6b^3de + 2a^5b^4ce + a^5b^4d^2 - 2a^4b^5cd + a^3b^6c^2} + x\right)\right)\right) + \frac{fx^{14}}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**11*(-a*f/(11*b**2) + e/(11*b)) + x**8*(a**2*f/(8*b**3) - a*e/(8*b**2) +
d/(8*b)) + x**5*(-a**3*f/(5*b**4) + a**2*e/(5*b**3) - a*d/(5*b**2) + c/(5*b
)) + x**2*(a**4*f/(2*b**5) - a**3*e/(2*b**4) + a**2*d/(2*b**3) - a*c/(2*b**
2)) + RootSum(27*_t**3*b**17 - a**14*f**3 + 3*a**13*b*e*f**2 - 3*a**12*b**2
*d*f**2 - 3*a**12*b**2*e**2*f + 3*a**11*b**3*c*f**2 + 6*a**11*b**3*d*e*f +
```

```
a**11*b**3*e**3 - 6*a**10*b**4*c*e*f - 3*a**10*b**4*d**2*f - 3*a**10*b**4*d
*e**2 + 6*a**9*b**5*c*d*f + 3*a**9*b**5*c*e**2 + 3*a**9*b**5*d**2*e - 3*a**
8*b**6*c**2*f - 6*a**8*b**6*c*d*e - a**8*b**6*d**3 + 3*a**7*b**7*c**2*e + 3
*a**7*b**7*c*d**2 - 3*a**6*b**8*c**2*d + a**5*b**9*c**3, Lambda(_t, _t*log(
9*_t**2*b**11/(a**9*f**2 - 2*a**8*b*e*f + 2*a**7*b**2*d*f + a**7*b**2*e**2
- 2*a**6*b**3*c*f - 2*a**6*b**3*d*e + 2*a**5*b**4*c*e + a**5*b**4*d**2 - 2*
a**4*b**5*c*d + a**3*b**6*c**2) + x))) + f*x**14/(14*b)
```

3.182 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=312

$$\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}-\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\frac{x^4\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^4}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\right.}{}$$

Rubi [A] time = 0.30, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1836, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x^4\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4b^4}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{16/3}}-\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^5}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^{16/3}}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}b^{16/3}}+\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}+\frac{x^{10}(be-af)}{10b^2}+\frac{fx^{13}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e -
 a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1
6/3)) + (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(16/3)) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx = \frac{fx^{13}}{13b}+\frac{\int \frac{x^6\left(13bc+13bdx^3+13(be-af)x^6\right)}{a+bx^3}\,dx}{13b}$$

$$= \frac{fx^{13}}{13b}+\frac{\int \left(-\frac{13a\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^4}+\frac{13\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{b^3}+\frac{13\left(b^2d-abe+a^2f\right)x^6}{b^2}+ \ldots\right.}{13b}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^4}+\frac{\left(b^2d-abe+\ldots\right.}{7b^3}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^4}+\frac{\left(b^2d-abe+\ldots\right.}{7b^3}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^4}+\frac{\left(b^2d-abe+\ldots\right.}{7b^3}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^4}+\frac{\left(b^2d-abe+\ldots\right.}{7b^3}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^4}+\frac{\left(b^2d-abe+\ldots\right.}{7b^3}$$

Mathematica [A] time = 0.11, size = 306, normalized size = 0.98

$$\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}+\frac{ax\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^5}+\frac{x^4\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^4}+\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^{16/3}}-\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^{16/3}}+\frac{a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt{3}b^{16/3}}+\frac{x^{10}(be-af)}{10b^2}+\frac{fx^{13}}{13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^5 + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e -
```

```
a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) + (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(16/3
)) - (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(16/3)) + (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 \left(c + dx^3 + ex^6 + fx^9\right)}{a + bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]
```

fricas [A] time = 0.42, size = 304, normalized size = 0.97

$$\frac{420 \, b^{4} f x^{13} + 546 \, {\left(b^{4} e - a b^{3} f\right)} x^{10} + 780 \, {\left(b^{4} d - a b^{3} e + a^{2} b^{2} f\right)} x^{7} + 1365 \, {\left(b^{4} c - a b^{3} d + a^{2} b^{2} e - a^{3} b f\right)} x^{4} - 1820 \, \sqrt{3} {\left(a b^{3} c - a^{2} b^{2} d + a^{3} b e - a^{4} f\right)} \left(-\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2 \, \sqrt{3} b x \left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3} a}{3 \, a}\right) + 910 \, {\left(a b^{3} c - a^{2} b^{2} d + a^{3} b e - a^{4} f\right)} \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 1820 \, {\left(a b^{3} c - a^{2} b^{2} d + a^{3} b e - a^{4} f\right)} \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) - 5460 \, {\left(a b^{3} c - a^{2} b^{2} d + a^{3} b e - a^{4} f\right)} x}{5460 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/5460*(420*b^4*f*x^13 + 546*(b^4*e - a*b^3*f)*x^10 + 780*(b^4*d - a*b^3*e
+ a^2*b^2*f)*x^7 + 1365*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^4 - 1820*
sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*arctan(1/3*(2*
sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(a*b^3*c - a^2*b^2*d + a^3*b
*e - a^4*f)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 1820*(a
*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) -
5460*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5
```

giac [A] time = 0.18, size = 401, normalized size = 1.29

$$\frac{\sqrt{3} \left(\left(-a b^{2}\right)^{\frac{1}{3}} a b^{3} c - \left(-a b^{2}\right)^{\frac{1}{3}} a^{2} b^{2} d - \left(-a b^{2}\right)^{\frac{1}{3}} a^{4} f + \left(-a b^{2}\right)^{\frac{1}{3}} a^{3} b e\right) \arctan\left(\frac{\sqrt{3} \left(2 \, x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 \, b^{6}} + \frac{\left(\left(-a b^{2}\right)^{\frac{1}{3}} a b^{3} c - \left(-a b^{2}\right)^{\frac{1}{3}} a^{2} b^{2} d - \left(-a b^{2}\right)^{\frac{1}{3}} a^{4} f + \left(-a b^{2}\right)^{\frac{1}{3}} a^{3} b e\right) \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \, b^{6}} - \frac{\left(a^{2} b^{11} c - a^{3} b^{10} d - a^{5} b^{8} f + a^{4} b^{9} e\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left({\left| x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right|}\right)}{3 \, a b^{13}} + \frac{140 \, b^{12} f x^{13} - 182 \, a b^{11} f x^{10} + 182 \, b^{12} x^{10} e + 260 \, b^{12} d x^{7} + 260 \, a^{2} b^{10} f x^{7} - 260 \, a b^{11} x^{7} e + 455 \, b^{12} c x^{4} - 455 \, a b^{11} d x^{4} - 455 \, a^{3} b^{9} f x^{4} + 455 \, a^{2} b^{10} x^{4} e - 1820 \, a b^{11} c x + 1820 \, a^{2} b^{10} d x + 1820 \, a^{4} b^{8} f x - 1820 \, a^{3} b^{9} x e}{1820 \, b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b^2*d - (-a*b^2)^(
1/3)*a^4*f + (-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/b^6 + 1/6*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b^2*
d - (-a*b^2)^(1/3)*a^4*f + (-a*b^2)^(1/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/b^6 - 1/3*(a^2*b^11*c - a^3*b^10*d - a^5*b^8*f + a^4*b^9*e
)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^13) + 1/1820*(140*b^12*f*x^1
3 - 182*a*b^11*f*x^10 + 182*b^12*x^10*e + 260*b^12*d*x^7 + 260*a^2*b^10*f*x
^7 - 260*a*b^11*x^7*e + 455*b^12*c*x^4 - 455*a*b^11*d*x^4 - 455*a^3*b^9*f*x
^4 + 455*a^2*b^10*x^4*e - 1820*a*b^11*c*x + 1820*a^2*b^10*d*x + 1820*a^4*b^
8*f*x - 1820*a^3*b^9*x*e)/b^13
```

maple [B] time = 0.04, size = 544, normalized size = 1.74

$$\frac{f x^{13}}{13 b} - \frac{a f x^{10}}{10 b^{2}} + \frac{e x^{10}}{10 b} + \frac{a^{2} f x^{7}}{7 b^{3}} - \frac{a e x^{7}}{7 b^{2}} + \frac{d x^{7}}{7 b} - \frac{a^{3} f x^{4}}{4 b^{4}} + \frac{a^{2} e x^{4}}{4 b^{3}} - \frac{a d x^{4}}{4 b^{2}} + \frac{c x^{4}}{4 b} - \frac{\sqrt{3}\, a^{5} f \arctan\left(\frac{\sqrt{3} \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{6}} - \frac{a^{5} f \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{6}} + \frac{a^{5} f \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{6}} + \frac{\sqrt{3}\, a^{4} e \arctan\left(\frac{\sqrt{3} \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{5}} + \frac{a^{4} e \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{5}} - \frac{a^{4} e \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{5}} + \frac{a^{4} f x}{b^{5}} - \frac{\sqrt{3}\, a^{3} d \arctan\left(\frac{\sqrt{3} \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}} - \frac{a^{3} d \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}} + \frac{a^{3} d \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}} - \frac{a^{3} e x}{b^{4}} + \frac{\sqrt{3}\, a^{2} c \arctan\left(\frac{\sqrt{3} \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}} + \frac{a^{2} c \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}} - \frac{a^{2} c \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}} + \frac{a^{2} d x}{b^{3}} - \frac{a c x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] -1/3*a^5/b^6/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-
1/3*a^3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1
```

```
/3*a^2/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/
3*a^4/b^5/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/7
/b^2*x^7*a*e-1/4/b^4*x^4*a^3*f-1/b^4*a^3*e*x+1/b^3*a^2*d*x-1/b^2*a*c*x+1/b^
5*a^4*f*x+1/4/b^3*x^4*a^2*e-1/4/b^2*x^4*a*d-1/10/b^2*x^10*a*f+1/7/b^3*x^7*a
^2*f-1/3*a^5/b^6/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-1/6*a^2/b^3/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c-1/3*a^3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))
*d+1/3*a^2/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c+1/6*a^5/b^6/(a/b)^(2/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f+1/3*a^4/b^5/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e
-1/6*a^4/b^5/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/6*a^3/b^4/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/10/b*x^10*e+1/7/b*x^7*d+1/4
/b*x^4*c+1/13*f*x^13/b
```

maxima [A] time = 2.97, size = 311, normalized size = 1.00

$$\frac{140\,b^4 f x^{13} + 182\left(b^4 e - a b^3 f\right)x^{10} + 260\left(b^4 d - a b^3 e + a^2 b^2 f\right)x^7 + 455\left(b^4 c - a b^3 d + a^2 b^2 e - a^3 b f\right)x^4 - 1820\left(a b^3 c - a^2 b^2 d + a^3 b e - a^4 f\right)x}{1820\,b^5} + \frac{\sqrt{3}\left(a^2 b^3 c - a^3 b^2 d + a^4 b e - a^5 f\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(a^2 b^3 c - a^3 b^2 d + a^4 b e - a^5 f\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(a^2 b^3 c - a^3 b^2 d + a^4 b e - a^5 f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/1820*(140*b^4*f*x^13 + 182*(b^4*e - a*b^3*f)*x^10 + 260*(b^4*d - a*b^3*e
+ a^2*b^2*f)*x^7 + 455*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^4 - 1820*(
a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5 + 1/3*sqrt(3)*(a^2*b^3*c - a^
3*b^2*d + a^4*b*e - a^5*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(b^6*(a/b)^(2/3)) - 1/6*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/b)^(2/3)) + 1/3*(a^2*b^3*c - a^3*
b^2*d + a^4*b*e - a^5*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

mupad [B] time = 5.19, size = 311, normalized size = 1.00

$$x^{10}\left(\frac{e}{10\,b} - \frac{a\,f}{10\,b^2}\right) + x^7\left(\frac{d}{7\,b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{7\,b}\right) + x^4\left(\frac{c}{4\,b} - \frac{a\left(\frac{d}{b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{b}\right)}{4\,b}\right) + \frac{f\,x^{13}}{13\,b} + \frac{a^{4/3}\,\ln\left(b^{1/3}\,x + a^{1/3}\right)\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{3\,b^{16/3}} - \frac{a\,x\left(\frac{c}{b} - \frac{a\left(\frac{d}{b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b} + \frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1i\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{3\,b^{16/3}} - \frac{a^{4/3}\,\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1i\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{3\,b^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^10*(e/(10*b) - (a*f)/(10*b^2)) + x^7*(d/(7*b) - (a*(e/b - (a*f)/b^2))/(7*
b)) + x^4*(c/(4*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(4*b)) + (f*x^13)/
(13*b) + (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b
*e))/(3*b^(16/3)) - (a*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b +
 (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 -
 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(16/3)) - (a^(4/3)*log(3^(1
/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3
*f - a*b^2*d + a^2*b*e))/(3*b^(16/3))
```

sympy [A] time = 3.24, size = 423, normalized size = 1.36

$$x^{10}\left(-\frac{af}{10b^2} + \frac{e}{10b}\right) + x^7\left(\frac{a^2 f}{7b^3} - \frac{ae}{7b^2} + \frac{d}{7b}\right) + x^4\left(-\frac{a^3 f}{4b^4} + \frac{a^2 e}{4b^3} - \frac{ad}{4b^2} + \frac{c}{4b}\right) + x\left(\frac{a^4 f}{b^5} - \frac{a^3 e}{b^4} + \frac{a^2 d}{b^3} - \frac{ac}{b^2}\right) + \operatorname{RootSum}\left(27t^3b^{16} + a^{13}f^3 - 3a^{12}bef^2 + 3a^{11}b^2df^2 + 3a^{11}b^2e^2f - 3a^{10}b^3cf^2 - 6a^{10}b^3def - a^{10}b^3e^3 + 6a^9b^4cef + 3a^9b^4d^2f + 3a^9b^4de^2 - 6a^8b^5cdf - 3a^8b^5ce^2 - 3a^8b^5d^2e + 3a^7b^6c^2f + 6a^7b^6cde + a^7b^6d^3 - 3a^6b^7c^2e - 3a^6b^7cd^2 + 3a^5b^8c^2d - a^4b^9c^3, \left(t \mapsto t\log\left(-\frac{3tb^5}{a^4f - a^3be + a^2b^2d - ab^3c} + x\right)\right)\right) + \frac{fx^{13}}{13b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**10*(-a*f/(10*b**2) + e/(10*b)) + x**7*(a**2*f/(7*b**3) - a*e/(7*b**2) +
d/(7*b)) + x**4*(-a**3*f/(4*b**4) + a**2*e/(4*b**3) - a*d/(4*b**2) + c/(4*b
)) + x*(a**4*f/b**5 - a**3*e/b**4 + a**2*d/b**3 - a*c/b**2) + RootSum(27*_t
**3*b**16 + a**13*f**3 - 3*a**12*b*e*f**2 + 3*a**11*b**2*d*f**2 + 3*a**11*b
**2*e**2*f - 3*a**10*b**3*c*f**2 - 6*a**10*b**3*d*e*f - a**10*b**3*e**3 + 6
*a**9*b**4*c*e*f + 3*a**9*b**4*d**2*f + 3*a**9*b**4*d*e**2 - 6*a**8*b**5*c*
d*f - 3*a**8*b**5*c*e**2 - 3*a**8*b**5*d**2*e + 3*a**7*b**6*c**2*f + 6*a**7
```

```
*b**6*c*d*e + a**7*b**6*d**3 - 3*a**6*b**7*c**2*e - 3*a**6*b**7*c*d**2 + 3*
a**5*b**8*c**2*d - a**4*b**9*c**3, Lambda(_t, _t*log(-3*_t*b**5/(a**4*f - a
**3*b*e + a**2*b**2*d - a*b**3*c) + x))) + f*x**13/(13*b)
```

3.183 $\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$

Optimal. Leaf size=279

$$\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2b^4}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d\right.}{6b^{14/3}}$$

Rubi [A] time = 0.27, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2b^4}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{14/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^{14/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}b^{14/3}}+\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}+\frac{x^8(be-af)}{8b^2}+\frac{fx^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^5)/(5*b^3) + ((b*e - a*f)*x^8)/(8*b^2) + (f*x^11)/(11*b) + (a^(2/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*b^(14/3)) + (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a^(1/3) + b^(1/3)*x])/(3*b^(14/3)) - (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx = \frac{fx^{11}}{11b}+\frac{\int \frac{x^4\left(11bc+11bdx^3+11(be-af)x^6\right)}{a+bx^3}\,dx}{11b}$$

$$= \frac{fx^{11}}{11b}+\frac{\int \left(\frac{11\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^3}+\frac{11\left(b^2d-abe+a^2f\right)x^4}{b^2}+\frac{11(be-af)x^7}{b}+\frac{11\left(-ab^3c+a^2b\right.}{b^3(a}\right.}{11b}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}$$

Mathematica [A] time = 0.12, size = 266, normalized size = 0.95

$$\frac{264b^{5/3}x^5\left(a^2f-abe+b^2d\right)+660b^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)-440a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)-440\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)+220a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)+165b^{8/3}x^8(be-af)+120b^{11/3}fx^{11}}{1320b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (660*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 + 264*b^(5/3)*(b^2*d -
 a*b*e + a^2*f)*x^5 + 165*b^(8/3)*(b*e - a*f)*x^8 + 120*b^(11/3)*f*x^11 - 4
40*Sqrt[3]*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^
```

```
3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*b^(14/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]
```

fricas [A] time = 0.42, size = 281, normalized size = 1.01

$$\frac{120\,b^3fx^{11}+165\left(b^3e-ab^2f\right)x^8+264\left(b^3d-ab^2e+a^2bf\right)x^5+660\left(b^3c-ab^2d+a^2be-a^3f\right)x^2-440\,\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)+220\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-440\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{1320\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/1320*(120*b^3*f*x^11 + 165*(b^3*e - a*b^2*f)*x^8 + 264*(b^3*d - a*b^2*e +
 a^2*b*f)*x^5 + 660*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 - 440*sqrt(3)*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b
*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/3))
- 440*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^
2/b^2)^(2/3)))/b^4
```

giac [A] time = 0.18, size = 386, normalized size = 1.38

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d-\left(-ab^2\right)^{\frac{2}{3}}a^3f+\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6}-\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d-\left(-ab^2\right)^{\frac{2}{3}}a^3f+\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6}+\frac{\left(ab^{10}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^9d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^4b^7f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3b^8\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{11}}+\frac{40\,b^{10}fx^{11}-55\,ab^9fx^8+55\,b^{10}x^8e+88\,b^{10}dx^5+88\,a^2b^8fx^5-88\,ab^9x^5e+220\,b^{10}cx^2-220\,ab^9dx^2-220\,a^3b^7fx^2+220\,a^2b^8x^2e}{440\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)
*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/b^6 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*
b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/b^6 + 1/3*(a*b^10*c*(-a/b)^(1/3) - a^2*b^9*d*(-a/b)^(1/3) - a^4*b^
7*f*(-a/b)^(1/3) + a^3*b^8*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b^11) + 1/440*(40*b^10*f*x^11 - 55*a*b^9*f*x^8 + 55*b^10*x^8*e +
 88*b^10*d*x^5 + 88*a^2*b^8*f*x^5 - 88*a*b^9*x^5*e + 220*b^10*c*x^2 - 220*a
*b^9*d*x^2 - 220*a^3*b^7*f*x^2 + 220*a^2*b^8*x^2*e)/b^11
```

maple [B] time = 0.05, size = 502, normalized size = 1.80

$$\frac{fx^{11}}{11b}-\frac{afx^8}{8b^2}+\frac{ex^8}{8b}+\frac{a^2fx^5}{5b^3}-\frac{aex^5}{5b^2}+\frac{dx^5}{5b}-\frac{a^3fx^2}{2b^4}+\frac{a^2ex^2}{2b^3}-\frac{adx^2}{2b^2}+\frac{cx^2}{2b}+\frac{\sqrt{3}\,a^4f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^5}-\frac{a^4f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^5}+\frac{a^4f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^5}-\frac{\sqrt{3}\,a^3e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4}+\frac{a^3e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4}-\frac{a^3e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4}+\frac{\sqrt{3}\,a^2d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{a^2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{a^2d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{\sqrt{3}\,ac\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{ac\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{ac\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/11/b*f*x^11-1/8/b^2*x^8*a*f+1/8/b*x^8*e+1/5/b^3*x^5*a^2*f-1/5/b^2*x^5*a*e
+1/5/b*x^5*d-1/2/b^4*x^2*a^3*f+1/2/b^3*x^2*a^2*e-1/2/b^2*x^2*a*d+1/2/b*x^2*
c-1/3*a^4/b^5/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f+1/3*a^3/b^4/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))*e-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d+1/3*a/b^2/(a/b)^
(1/3)*ln(x+(a/b)^(1/3))*c+1/6*a^4/b^5/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b
```

```
)^(2/3))*f-1/6*a^3/b^4/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/6*
a^2/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d-1/6*a/b^2/(a/b)^(1/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/3*a^4/b^5*3^(1/2)/(a/b)^(1/3)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3*a^3/b^4*3^(1/2)/(a/b)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3*a^2/b^3*3^(1/2)/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3*a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 3.02, size = 269, normalized size = 0.96

$$-\frac{\sqrt{3}\left(ab^3c - a^2b^2d + a^3be - a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{40b^3fx^{11} + 55\left(b^3e - ab^2f\right)x^8 + 88\left(b^3d - ab^2e + a^2bf\right)x^5 + 220\left(b^3c - ab^2d + a^2be - a^3f\right)x^2}{440b^4} - \frac{\left(ab^3c - a^2b^2d + a^3be - a^4f\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(ab^3c - a^2b^2d + a^3be - a^4f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/440*(40*b^3*f*x^11 + 55
*(b^3*e - a*b^2*f)*x^8 + 88*(b^3*d - a*b^2*e + a^2*b*f)*x^5 + 220*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*x^2)/b^4 - 1/6*(a*b^3*c - a^2*b^2*d + a^3*b*e -
a^4*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(1/3)) + 1/3*(a*b^
3*c - a^2*b^2*d + a^3*b*e - a^4*f)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(1/3))
```

mupad [B] time = 5.15, size = 267, normalized size = 0.96

$$x^8\left(\frac{e}{8b} - \frac{af}{8b^2}\right) + x^5\left(\frac{d}{5b} - \frac{a\left(\frac{e}{b} - \frac{af}{b^2}\right)}{5b}\right) + x^2\left(\frac{c}{2b} - \frac{a\left(\frac{d}{b} - \frac{a\left(\frac{e}{b} - \frac{af}{b^2}\right)}{b}\right)}{2b}\right) + \frac{fx^{11}}{11b} + \frac{a^{2/3}\ln\left(b^{1/3}x + a^{1/3}\right)\left(-fa^3 + ea^2b - dab^2 + cb^3\right)}{3b^{14/3}} - \frac{a^{2/3}\ln\left(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-fa^3 + ea^2b - dab^2 + cb^3\right)}{3b^{14/3}} + \frac{a^{2/3}\ln\left(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-fa^3 + ea^2b - dab^2 + cb^3\right)}{3b^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^8*(e/(8*b) - (a*f)/(8*b^2)) + x^5*(d/(5*b) - (a*(e/b - (a*f)/b^2))/(5*b))
 + x^2*(c/(2*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(2*b)) + (f*x^11)/(11
*b) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)
)/(3*b^(14/3)) - (a^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*(
(3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(14/3)) + (
a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(14/3))
```

sympy [A] time = 2.48, size = 469, normalized size = 1.68

$$x^8\left(-\frac{af}{8b^2} + \frac{e}{8b}\right) + x^5\left(\frac{a^2f}{5b^3} - \frac{ae}{5b^2} + \frac{d}{5b}\right) + x^2\left(-\frac{a^3f}{2b^4} + \frac{a^2e}{2b^3} - \frac{ad}{2b^2} + \frac{c}{2b}\right) + \operatorname{RootSum}\left(27t^3b^{14} + a^{11}f^3 - 3a^{10}bef^2 + 3a^9b^2df^2 + 3a^9b^2e^2f - 3a^8b^3cf^2 - 6a^8b^3def - a^8b^3e^3 + 6a^7b^4cef + 3a^7b^4d^2f + 3a^7b^4de^2 - 6a^6b^5cdf - 3a^6b^5ce^2 - 3a^6b^5d^2e + 3a^5b^6c^2f + 6a^5b^6cde + a^5b^6d^3 - 3a^4b^7c^2e - 3a^4b^7cd^2 + 3a^3b^8c^2d - a^2b^9c^3, \left(t \mapsto t\log\left(\frac{9t^2b^9}{a^7f^2 - 2a^6bef + 2a^5b^2df + a^5b^2e^2 - 2a^4b^3cf - 2a^4b^3de + 2a^3b^4ce + a^3b^4d^2 - 2a^2b^5cd + ab^6c^2} + x\right)\right)\right) + \frac{fx^{11}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**8*(-a*f/(8*b**2) + e/(8*b)) + x**5*(a**2*f/(5*b**3) - a*e/(5*b**2) + d/(
5*b)) + x**2*(-a**3*f/(2*b**4) + a**2*e/(2*b**3) - a*d/(2*b**2) + c/(2*b))
+ RootSum(27*_t**3*b**14 + a**11*f**3 - 3*a**10*b*e*f**2 + 3*a**9*b**2*d*f*
*2 + 3*a**9*b**2*e**2*f - 3*a**8*b**3*c*f**2 - 6*a**8*b**3*d*e*f - a**8*b**
3*e**3 + 6*a**7*b**4*c*e*f + 3*a**7*b**4*d**2*f + 3*a**7*b**4*d*e**2 - 6*a*
*6*b**5*c*d*f - 3*a**6*b**5*c*e**2 - 3*a**6*b**5*d**2*e + 3*a**5*b**6*c**2*
f + 6*a**5*b**6*c*d*e + a**5*b**6*d**3 - 3*a**4*b**7*c**2*e - 3*a**4*b**7*c
*d**2 + 3*a**3*b**8*c**2*d - a**2*b**9*c**3, Lambda(_t, _t*log(9*_t**2*b**9
/(a**7*f**2 - 2*a**6*b*e*f + 2*a**5*b**2*d*f + a**5*b**2*e**2 - 2*a**4*b**3
*c*f - 2*a**4*b**3*d*e + 2*a**3*b**4*c*e + a**3*b**4*d**2 - 2*a**2*b**5*c*d
 + a*b**6*c**2) + x))) + f*x**11/(11*b)
```

3.184 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3} \, dx$

Optimal. Leaf size=274

$$\frac{x^4\left(a^2f-abe+b^2d\right)}{4b^3}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^{13/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be\cdots\right.}{\sqrt{3}b^{13/3}}$$

Rubi [A] time = 0.27, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1836, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{13/3}}+\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^{13/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}b^{13/3}}+\frac{x^4\left(a^2f-abe+b^2d\right)}{4b^3}+\frac{x^7(be-af)}{7b^2}+\frac{fx^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^4)
/(4*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^10)/(10*b) + (a^(1/3)*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*b^(13/3)) - (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^
(1/3) + b^(1/3)*x])/(3*b^(13/3)) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx = \frac{fx^{10}}{10b}+\frac{\int \frac{x^3\left(10bc+10bdx^3+10(be-af)x^6\right)}{a+bx^3}\,dx}{10b}$$

$$= \frac{fx^{10}}{10b}+\frac{\int \left(\frac{10\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^3}+\frac{10\left(b^2d-abe+a^2f\right)x^3}{b^2}+\frac{10(be-af)x^6}{b}+\frac{10\left(-ab^3c+a^2b^2\cdots\right.}{b^3\left(a+b\cdots\right.}\right.}{10b}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\cdots$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\cdots$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\cdots$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\cdots$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}+\cdots$$

Mathematica [A] time = 0.11, size = 264, normalized size = 0.96

$$\frac{105b^{4/3}x^4\left(a^2f-abe+b^2d\right)+420\sqrt[3]{b}\,x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+140\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)-140\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)-70\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)+60b^{7/3}x^7(be-af)+42b^{10/3}fx^{10}}{420b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (420*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x + 105*b^(4/3)*(b^2*d - a
*b*e + a^2*f)*x^4 + 60*b^(7/3)*(b*e - a*f)*x^7 + 42*b^(10/3)*f*x^10 - 140*S
qrt[3]*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
```

```
*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*b^(13/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]
```

fricas [A] time = 0.44, size = 249, normalized size = 0.91

$$\frac{42\,b^3fx^{10}+60\left(b^3e-ab^2f\right)x^7+105\left(b^3d-ab^2e+a^2bf\right)x^4-140\,\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+70\left(b^3c-ab^2d+a^2be-a^3f\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-140\left(b^3c-ab^2d+a^2be-a^3f\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+420\left(b^3c-ab^2d+a^2be-a^3f\right)x}{420\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/420*(42*b^3*f*x^10 + 60*(b^3*e - a*b^2*f)*x^7 + 105*(b^3*d - a*b^2*e + a^
2*b*f)*x^4 - 140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*ar
ctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4
```

giac [A] time = 0.19, size = 346, normalized size = 1.26

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d-\left(-ab^2\right)^{\frac{1}{3}}a^3f+\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d-\left(-ab^2\right)^{\frac{1}{3}}a^3f+\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5}+\frac{\left(ab^9c-a^2b^8d-a^4b^6f+a^3b^7e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{10}}+\frac{14\,b^9fx^{10}-20\,ab^8fx^7+20\,b^9x^7e+35\,b^9dx^4+35\,a^2b^7fx^4-35\,ab^8x^4e+140\,b^9cx-140\,ab^8dx-140\,a^3b^6fx+140\,a^2b^7xe}{140\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3
)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/b^5 - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a
*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/b^5 + 1/3*(a*b^9*c - a^2*b^8*d - a^4*b^6*f + a^3*b^7*e)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*f*x^10 - 20*a*b^8*f
*x^7 + 20*b^9*x^7*e + 35*b^9*d*x^4 + 35*a^2*b^7*f*x^4 - 35*a*b^8*x^4*e + 14
0*b^9*c*x - 140*a*b^8*d*x - 140*a^3*b^6*f*x + 140*a^2*b^7*x*e)/b^10
```

maple [B] time = 0.05, size = 492, normalized size = 1.80

$$\frac{fx^{10}}{10b}-\frac{afx^7}{7b^2}+\frac{ex^7}{7b}+\frac{a^2fx^4}{4b^3}-\frac{aex^4}{4b^2}+\frac{dx^4}{4b}+\frac{\sqrt{3}\,a^4f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}+\frac{a^4f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}-\frac{a^4f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}-\frac{\sqrt{3}\,a^3e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{a^3e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{a^3e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{a^3fx}{b^4}+\frac{\sqrt{3}\,a^2d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{a^2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{a^2d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{a^2ex}{b^3}-\frac{\sqrt{3}\,ac\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{ac\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{ac\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{adx}{b^2}+\frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/10*f*x^10/b-1/7/b^2*x^7*a*f+1/7/b*x^7*e+1/4/b^3*x^4*a^2*f-1/4/b^2*x^4*a*e
+1/4/b*x^4*d-1/b^4*a^3*f*x+1/b^3*a^2*e*x-1/b^2*a*d*x+1/b*c*x+1/3*a^4/b^5/(a
/b)^(2/3)*ln(x+(a/b)^(1/3))*f-1/3*a^3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+1
/3*a^2/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)
^(1/3))*c-1/6*a^4/b^5/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f+1/6*a
^3/b^4/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-1/6*a^2/b^3/(a/b)^(2
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/6*a/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^
```

```
(1/3)*x+(a/b)^(2/3))*c+1/3*a^4/b^5/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))*f-1/3*a^3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*e+1/3*a^2/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))*d-1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))*c
```

maxima [A] time = 2.93, size = 267, normalized size = 0.97

$$\frac{14\,b^3fx^{10}+20\left(b^3e-ab^2f\right)x^7+35\left(b^3d-ab^2e+a^2bf\right)x^4+140\left(b^3c-ab^2d+a^2be-a^3f\right)x}{140\,b^4}-\frac{\sqrt{3}\left(ab^3c-a^2b^2d+a^3be-a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(ab^3c-a^2b^2d+a^3be-a^4f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(ab^3c-a^2b^2d+a^3be-a^4f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^3*f*x^10 + 20*(b^3*e - a*b^2*f)*x^7 + 35*(b^3*d - a*b^2*e + a^2
*b*f)*x^4 + 140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 - 1/3*sqrt(3)*(a
*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) + 1/6*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4
*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(2/3)) - 1/3*(a*b^3*c
 - a^2*b^2*d + a^3*b*e - a^4*f)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3))
```

mupad [B] time = 5.10, size = 264, normalized size = 0.96

$$x^7\left(\frac{e}{7\,b}-\frac{a\,f}{7\,b^2}\right)+x^4\left(\frac{d}{4\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{4\,b}\right)+x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)+\frac{f\,x^{10}}{10\,b}-\frac{a^{1/3}\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{13/3}}-\frac{a^{1/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{13/3}}+\frac{a^{1/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^7*(e/(7*b) - (a*f)/(7*b^2)) + x^4*(d/(4*b) - (a*(e/b - (a*f)/b^2))/(4*b))
 + x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b) + (f*x^10)/(10*b) - (a^(
1/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13
/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13/3)) + (a^(1/3)*lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c
 - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13/3))
```

sympy [A] time = 2.51, size = 376, normalized size = 1.37

$$x^7\left(-\frac{af}{7b^2}+\frac{e}{7b}\right)+x^4\left(\frac{a^2f}{4b^3}-\frac{ae}{4b^2}+\frac{d}{4b}\right)+x\left(-\frac{a^3f}{b^4}+\frac{a^2e}{b^3}-\frac{ad}{b^2}+\frac{c}{b}\right)+\operatorname{RootSum}\left(27t^3b^{13}-a^{10}f^3+3a^9bef^2-3a^8b^2df^2-3a^8b^2e^2f+3a^7b^3cf^2+6a^7b^3def+a^7b^3e^3-6a^6b^4cef-3a^6b^4d^2f-3a^6b^4de^2+6a^5b^5cdf+3a^5b^5ce^2+3a^5b^5d^2e-3a^4b^6c^2f-6a^4b^6cde-a^4b^6d^3+3a^3b^7c^2e+3a^3b^7cd^2-3a^2b^8c^2d+ab^9c^3,\left(t\mapsto t\log\left(\frac{3tb^4}{a^3f-a^2be+ab^2d-b^3c}+x\right)\right)\right)+\frac{fx^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**7*(-a*f/(7*b**2) + e/(7*b)) + x**4*(a**2*f/(4*b**3) - a*e/(4*b**2) + d/(
4*b)) + x*(-a**3*f/b**4 + a**2*e/b**3 - a*d/b**2 + c/b) + RootSum(27*_t**3*
b**13 - a**10*f**3 + 3*a**9*b*e*f**2 - 3*a**8*b**2*d*f**2 - 3*a**8*b**2*e**
2*f + 3*a**7*b**3*c*f**2 + 6*a**7*b**3*d*e*f + a**7*b**3*e**3 - 6*a**6*b**4
*c*e*f - 3*a**6*b**4*d**2*f - 3*a**6*b**4*d*e**2 + 6*a**5*b**5*c*d*f + 3*a*
*5*b**5*c*e**2 + 3*a**5*b**5*d**2*e - 3*a**4*b**6*c**2*f - 6*a**4*b**6*c*d*
e - a**4*b**6*d**3 + 3*a**3*b**7*c**2*e + 3*a**3*b**7*c*d**2 - 3*a**2*b**8*
c**2*d + a*b**9*c**3, Lambda(_t, _t*log(3*_t*b**4/(a**3*f - a**2*b*e + a*b*
*2*d - b**3*c) + x))) + f*x**10/(10*b)
```

3.185 $\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$

<u>**Optimal**</u>. Leaf size=245

$$\frac{x^2\left(a^2f-abe+b^2d\right)}{2b^3}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3\sqrt[3]{a}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+\right.}{\sqrt{3}\sqrt[3]{a}b^{11/3}}$$

<u>**Rubi**</u> **[A]** time = 0.22, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6\sqrt[3]{a}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3\sqrt[3]{a}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}\sqrt[3]{a}b^{11/3}}+\frac{x^2\left(a^2f-abe+b^2d\right)}{2b^3}+\frac{x^5(be-af)}{5b^2}+\frac{fx^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^2)/(2*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^8)
/(8*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(11/3)) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(11/3)) + ((b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*
a^(1/3)*b^(11/3))
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 204</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

<u>Rule 292</u>

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

<u>Rule 617</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 628</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

<u>Rule 634</u>

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx = \frac{fx^8}{8b}+\frac{\int \frac{x\left(8bc+8bdx^3+8(be-af)x^6\right)}{a+bx^3}\,dx}{8b}$$

$$= \frac{fx^8}{8b}+\frac{\int \left(\frac{8\left(b^2d-abe+a^2f\right)x}{b^2}+\frac{8(be-af)x^4}{b}+\frac{8\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^2\left(a+bx^3\right)}\right)dx}{8b}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{x}{a+b\ldots}}{b^3}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{\ldots}{\sqrt[3]{a}\ldots}}{3\sqrt[3]{a}\,b^{10/3}}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\ldots\right.}{3\sqrt[3]{a}\,b^{11/3}}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\ldots\right.}{3\sqrt[3]{a}\,b^{11/3}}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\ldots}{\sqrt{3}\,\sqrt[3]{a}\,b^{11/3}}$$

Mathematica [A] time = 0.18, size = 231, normalized size = 0.94

$$\frac{60b^{2/3}x^2\left(a^2f-abe+b^2d\right)+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt[3]{a}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt[3]{a}}+\frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt[3]{a}}+24b^{5/3}x^5(be-af)+15b^{8/3}fx^8}{120b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (60*b^(2/3)*(b^2*d - a*b*e + a^2*f)*x^2 + 24*b^(5/3)*(b*e - a*f)*x^5 + 15*b
^(8/3)*f*x^8 + (40*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(12
0*b^(11/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]
```

fricas [A] time = 0.45, size = 568, normalized size = 2.32

$$\left[\frac{15\,ab^4fx^8+24\left(ab^4e-a^2b^3f\right)x^5+60\left(ab^4d-a^2b^3e+a^3b^2f\right)x^2-60\sqrt{\frac{1}{3}}\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab-3\sqrt{\frac{1}{3}}\left(abx+2\left(ab^2\right)^{\frac{2}{3}}x^2-\left(ab^2\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}-3\left(ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+20\left(b^3c-ab^2d+a^2be-a^3f\right)\left(ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2-\left(ab^2\right)^{\frac{1}{3}}bx+\left(ab^2\right)^{\frac{2}{3}}\right)-40\left(b^3c-ab^2d+a^2be-a^3f\right)\left(ab^2\right)^{\frac{2}{3}}\log\left(bx+\left(ab^2\right)^{\frac{1}{3}}\right)}{120\,ab^5},\ \frac{15\,ab^4fx^8+24\left(ab^4e-a^2b^3f\right)x^5+60\left(ab^4d-a^2b^3e+a^3b^2f\right)x^2-120\sqrt{\frac{1}{3}}\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)\sqrt{\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2bx-\left(ab^2\right)^{\frac{1}{3}}\right)\sqrt{\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+20\left(b^3c-ab^2d+a^2be-a^3f\right)\left(ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2-\left(ab^2\right)^{\frac{1}{3}}bx+\left(ab^2\right)^{\frac{2}{3}}\right)-40\left(b^3c-ab^2d+a^2be-a^3f\right)\left(ab^2\right)^{\frac{2}{3}}\log\left(bx+\left(ab^2\right)^{\frac{1}{3}}\right)}{120\,ab^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/120*(15*a*b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b
^3*e + a^3*b^2*f)*x^2 - 60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*
(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/
3)*x)/(b*x^3 + a)) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*l
og(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5), 1/120*(15*a*
b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b^3*e + a^3*b^
2*f)*x^2 - 120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((
a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3
)/a)/b) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*log(b^2*x^2
- (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5)]
```

giac [A] time = 0.20, size = 291, normalized size = 1.19

$$\frac{\sqrt{3}\left(b^3c-ab^2d-a^3f+a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}b^3}-\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}b^3}-\frac{\left(b^8c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^7d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^6\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^8}+\frac{5\,b^7fx^8-8\,ab^6fx^5+8\,b^7x^5e+20\,b^7dx^2+20\,a^2b^5fx^2-20\,ab^6x^2e}{40\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^3) - 1/6*(b^3*c - a*b^2*d - a^
3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^3
) - 1/3*(b^8*c*(-a/b)^(1/3) - a*b^7*d*(-a/b)^(1/3) - a^3*b^5*f*(-a/b)^(1/3)
 + a^2*b^6*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^8)
+ 1/40*(5*b^7*f*x^8 - 8*a*b^6*f*x^5 + 8*b^7*x^5*e + 20*b^7*d*x^2 + 20*a^2*b
^5*f*x^2 - 20*a*b^6*x^2*e)/b^8
```

maple [B] time = 0.05, size = 450, normalized size = 1.84

$$\frac{f\,x^8}{8b}-\frac{af\,x^5}{5b^2}+\frac{e\,x^5}{5b}+\frac{a^2f\,x^2}{2b^3}-\frac{ae\,x^2}{2b^2}+\frac{d\,x^2}{2b}-\frac{\sqrt{3}\,a^3f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4}+\frac{a^3f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4}-\frac{a^3f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4}+\frac{\sqrt{3}\,a^2e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{a^2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{a^2e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{ad\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{ad\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/8*f*x^8/b-1/5/b^2*x^5*a*f+1/5/b*x^5*e+1/2/b^3*x^2*a^2*f-1/2/b^2*x^2*a*e+1
/2/b*d*x^2+1/3/b^4/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*a^3*f-1/3/b^3/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))*a^2*e+1/3/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*a*d-1/3/b/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))*c-1/6/b^4/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))*a^3*f+1/6/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*e-1
/6/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*d+1/6/b/(a/b)^(1/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c-1/3/b^4*3^(1/2)/(a/b)^(1/3)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^3*f+1/3/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*e-1/3/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))*a*d+1/3/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 3.01, size = 225, normalized size = 0.92

$$\frac{\sqrt{3}\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^{4}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5b^{2}fx^{8}+8\left(b^{2}e-abf\right)x^{5}+20\left(b^{2}d-abe+a^{2}f\right)x^{2}}{40b^{3}}+\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^{4}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^{4}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) + 1/40*(5*b^2*f*x^8 + 8*(b^2*e -
 a*b*f)*x^5 + 20*(b^2*d - a*b*e + a^2*f)*x^2)/b^3 + 1/6*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) -
 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1
/3))
```

mupad [B] time = 5.14, size = 225, normalized size = 0.92

$$x^{5}\left(\frac{e}{5b}-\frac{af}{5b^{2}}\right)+x^{2}\left(\frac{d}{2b}-\frac{a\left(\frac{e}{b}-\frac{af}{b^{2}}\right)}{2b}\right)+\frac{fx^{8}}{8b}-\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(-fa^{3}+ea^{2}b-dab^{2}+cb^{3}\right)}{3a^{1/3}b^{11/3}}+\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-fa^{3}+ea^{2}b-dab^{2}+cb^{3}\right)}{3a^{1/3}b^{11/3}}-\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-fa^{3}+ea^{2}b-dab^{2}+cb^{3}\right)}{3a^{1/3}b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

```
[Out] x^5*(e/(5*b) - (a*f)/(5*b^2)) + x^2*(d/(2*b) - (a*(e/b - (a*f)/b^2))/(2*b))
 + (f*x^8)/(8*b) - (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2
*b*e))/(3*a^(1/3)*b^(11/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/
3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(1/3)*
b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/
2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(1/3)*b^(11/3))
```

sympy [A] time = 2.38, size = 427, normalized size = 1.74

$$x^{5}\left(-\frac{af}{5b^{2}}+\frac{e}{5b}\right)+x^{2}\left(\frac{a^{2}f}{2b^{3}}-\frac{ae}{2b^{2}}+\frac{d}{2b}\right)+\operatorname{RootSum}\left(27t^{3}ab^{11}-a^{9}f^{3}+3a^{8}bef^{2}-3a^{7}b^{2}df^{2}-3a^{7}b^{2}e^{2}f+3a^{6}b^{3}cf^{2}+6a^{6}b^{3}def+a^{6}b^{3}e^{3}-6a^{5}b^{4}cef-3a^{5}b^{4}d^{2}f-3a^{5}b^{4}de^{2}+6a^{4}b^{5}cdf+3a^{4}b^{5}ce^{2}+3a^{4}b^{5}d^{2}e-3a^{3}b^{6}c^{2}f-6a^{3}b^{6}cde-a^{3}b^{6}d^{3}+3a^{2}b^{7}c^{2}e+3a^{2}b^{7}cd^{2}-3ab^{8}c^{2}d+b^{9}c^{3},\left(t\mapsto t\log\left(\frac{9t^{2}ab^{7}}{a^{6}f^{2}-2a^{5}bef+2a^{4}b^{2}df+a^{4}b^{2}e^{2}-2a^{3}b^{3}cf-2a^{3}b^{3}de+2a^{2}b^{4}ce+a^{2}b^{4}d^{2}-2ab^{5}cd+b^{6}c^{2}}+x\right)\right)\right)+\frac{fx^{8}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**5*(-a*f/(5*b**2) + e/(5*b)) + x**2*(a**2*f/(2*b**3) - a*e/(2*b**2) + d/(
2*b)) + RootSum(27*_t**3*a*b**11 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**
2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a*
*6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2
+ 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6
*c**2*f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*
```

```
b**7*c*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a*b**7
/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3
*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d +
b**6*c**2) + x))) + f*x**8/(8*b)
```

3.186 $\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$

Optimal. Leaf size=240

$$\frac{x\left(a^2f-abe+b^2d\right)}{b^3}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{2/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+\right.}{3a^{2/3}b^{10/3}}$$

Rubi [A] time = 0.15, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1887, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{2/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{2/3}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}a^{2/3}b^{10/3}}+\frac{x\left(a^2f-abe+b^2d\right)}{b^3}+\frac{x^4(be-af)}{4b^2}+\frac{fx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^4)/(4*b^2) + (f*x^7)/(7*b)
 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx = \int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x^3}{b^2}+\frac{fx^6}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\left(a+bx^3\right)}\right)dx$$

$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{a+bx^3}\,dx}{b^3}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b^3}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{3a^{2/3}b^{10/3}}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{3a^{2/3}b^{10/3}}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\sqrt[3]{}}\right.}{\sqrt{3}\,a^{2/3}b^{10/3}}$$

Mathematica [A] time = 0.17, size = 229, normalized size = 0.95

$$\frac{84\sqrt[3]{b}x\left(a^2f-abe+b^2d\right)+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{2/3}}+\frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{2/3}}+\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{2/3}}+21b^{4/3}x^4(be-af)+12b^{7/3}fx^7}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*(b^2*d - a*b*e + a^2*f)*x + 21*b^(4/3)*(b*e - a*f)*x^4 + 12*b^(
7/3)*f*x^7 + (28*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c) + a*b^2*d - a^2*
b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(84*b
^(10/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3), x]
```

fricas [A] time = 0.45, size = 600, normalized size = 2.50

$$\left[\frac{12\,a^{2}b^{3}fx^{7}+21\,(a^{2}b^{3}e-a^{3}b^{2}f)x^{4}-42\,\sqrt{\frac{1}{3}}\,(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf)\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}+3\,(-a^{2}b)^{\frac{1}{3}}ax-a^{2}-3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(-a^{2}b)^{\frac{2}{3}}x+(-a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-14\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)(-a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(-a^{2}b)^{\frac{2}{3}}x-(-a^{2}b)^{\frac{1}{3}}a\right)+28\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)(-a^{2}b)^{\frac{2}{3}}\log\left(abx+(-a^{2}b)^{\frac{2}{3}}\right)+84\,(a^{2}b^{3}d-a^{3}b^{2}e+a^{4}bf)x}{84\,a^{2}b^{4}},\ \frac{12\,a^{2}b^{3}fx^{7}+21\,(a^{2}b^{3}e-a^{3}b^{2}f)x^{4}+84\,\sqrt{\frac{1}{3}}\,(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf)\sqrt{-\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(-a^{2}b)^{\frac{2}{3}}x+(-a^{2}b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-14\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)(-a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(-a^{2}b)^{\frac{2}{3}}x-(-a^{2}b)^{\frac{1}{3}}a\right)+28\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)(-a^{2}b)^{\frac{2}{3}}\log\left(abx+(-a^{2}b)^{\frac{2}{3}}\right)+84\,(a^{2}b^{3}d-a^{3}b^{2}e+a^{4}bf)x}{84\,a^{2}b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 - 42*sqrt(1/3)*(a*
b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*
x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*
x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-
a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log
(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a^4*b*f)*x)/(a^2*b^4
), 1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 + 84*sqrt(1/3)*(
a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(s
qrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^
2) - 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-
a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a
^4*b*f)*x)/(a^2*b^4)]
```

giac [A] time = 0.19, size = 253, normalized size = 1.05

$$-\frac{\sqrt{3}\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}-\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}-\frac{\left(b^{7}c-ab^{6}d-a^{3}b^{4}f+a^{2}b^{5}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{7}}+\frac{4\,b^{6}fx^{7}-7\,ab^{5}fx^{4}+7\,b^{6}x^{4}e+28\,b^{6}dx+28\,a^{2}b^{4}fx-28\,ab^{5}xe}{28\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/6*(b^3*c - a*b^2*d - a
^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^
2) - 1/3*(b^7*c - a*b^6*d - a^3*b^4*f + a^2*b^5*e)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a*b^7) + 1/28*(4*b^6*f*x^7 - 7*a*b^5*f*x^4 + 7*b^6*x^4*e +
 28*b^6*d*x + 28*a^2*b^4*f*x - 28*a*b^5*x*e)/b^7
```

maple [B] time = 0.04, size = 442, normalized size = 1.84

$$\frac{fx^{7}}{7b}-\frac{afx^{4}}{4b^{2}}+\frac{ex^{4}}{4b}-\frac{\sqrt{3}\,a^{3}f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{4}}-\frac{a^{3}f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{4}}+\frac{a^{3}f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{4}}+\frac{\sqrt{3}\,a^{2}e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{a^{2}e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{a^{2}e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{a^{2}fx}{b^{3}}-\frac{\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{ad\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{ad\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{aex}{b^{2}}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/7/b*f*x^7-1/4/b^2*x^4*a*f+1/4/b*x^4*e+1/b^3*a^2*f*x-1/b^2*a*e*x+1/b*d*x-1
/3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^3*f+1/3/b^3/(a/b)^(2/3)*ln(x+(a/b)^(
1/3))*a^2*e-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*d+1/3/b/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))*c+1/6/b^4/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^3*
f-1/6/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*e+1/6/b^2/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*d-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))*c-1/3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))*a^3*f+1/3/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))*a^2*e-1/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))*a*d+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))*c
```

maxima [A] time = 3.01, size = 223, normalized size = 0.93

$$\frac{4\,b^{2} f x^{7} + 7\,{\left(b^{2} e - a b f\right)} x^{4} + 28\,{\left(b^{2} d - a b e + a^{2} f\right)} x}{28\,b^{3}} + \frac{\sqrt{3}\,{\left(b^{3} c - a b^{2} d + a^{2} b e - a^{3} f\right)} \arctan\left(\frac{\sqrt{3}\,{\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{4} \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{{\left(b^{3} c - a b^{2} d + a^{2} b e - a^{3} f\right)} \log\left(x^{2} - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{4} \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{{\left(b^{3} c - a b^{2} d + a^{2} b e - a^{3} f\right)} \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{4} \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^2*f*x^7 + 7*(b^2*e - a*b*f)*x^4 + 28*(b^2*d - a*b*e + a^2*f)*x)/b
^3 + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1
/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3
))
```

mupad [B] time = 5.17, size = 222, normalized size = 0.92

$$x^4\,\left(\frac{e}{4\,b}-\frac{a\,f}{4\,b^2}\right)+x\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)+\frac{f\,x^7}{7\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x)
```

```
[Out] x^4*(e/(4*b) - (a*f)/(4*b^2)) + x*(d/b - (a*(e/b - (a*f)/b^2))/b) + (f*x^7)
/(7*b) + (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*
a^(2/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1
/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(2/3)*b^(10/3))
- (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(2/3)*b^(10/3))
```

sympy [A] time = 3.41, size = 342, normalized size = 1.42

$$x^{4} \left(- \frac{a f}{4 b^{2}} + \frac{e}{4 b}\right) + x \left(\frac{a^{2} f}{b^{3}} - \frac{a e}{b^{2}} + \frac{d}{b}\right) + \operatorname{RootSum} {\left(27 t^{3} a^{2} b^{10} + a^{9} f^{3} - 3 a^{8} b e f^{2} + 3 a^{7} b^{2} d f^{2} + 3 a^{7} b^{2} e^{2} f - 3 a^{6} b^{3} c f^{2} - 6 a^{6} b^{3} d e f - a^{6} b^{3} e^{3} + 6 a^{5} b^{4} c e f + 3 a^{5} b^{4} d^{2} f + 3 a^{5} b^{4} d e^{2} - 6 a^{4} b^{5} c d f - 3 a^{4} b^{5} c e^{2} - 3 a^{4} b^{5} d^{2} e + 3 a^{3} b^{6} c^{2} f + 6 a^{3} b^{6} c d e + a^{3} b^{6} d^{3} - 3 a^{2} b^{7} c^{2} e - 3 a^{2} b^{7} c d^{2} + 3 a b^{8} c^{2} d - b^{9} c^{3}, \left(t \mapsto t \log{\left(- \frac{3 t a b^{3}}{a^{3} f - a^{2} b e + a b^{2} d - b^{3} c} + x \right)} \right)\right)} + \frac{f x^{7}}{7 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] x**4*(-a*f/(4*b**2) + e/(4*b)) + x*(a**2*f/b**3 - a*e/b**2 + d/b) + RootSum
(27*_t**3*a**2*b**10 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 + 3
*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**3*e**3
 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**
5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*
a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2
+ 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*f - a**
2*b*e + a*b**2*d - b**3*c) + x))) + f*x**7/(7*b)
```

3.187 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)} \, dx$

Optimal. Leaf size=227

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{4/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{4/3}b^{8/3}}-$$

Rubi [A] time = 0.19, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{4/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{4/3}b^{8/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}a^{4/3}b^{8/3}}+\frac{x^2(be-af)}{2b^2}-\frac{c}{ax}+\frac{fx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x^2)/(2*b^2) + (f*x^5)/(5*b) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(4/3)*b^(8/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(4/3)*b^(8/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{ab^2\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{x}{a+bx^3}\,dx}{ab^2}$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}b^{7/3}}-\frac{\left(b^3c-ab^2d\right.}{}$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{8/3}}-\frac{\left(b^3c-a\right.}{}$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{8/3}}-\frac{\left(b^3c-a\right.}{}$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{8/3}}+\frac{\left(b^3c-a\right.}{}$$

Mathematica [A] time = 0.21, size = 224, normalized size = 0.99

$$\frac{15a^{4/3}b^{2/3}x^3(be-af)+6a^{4/3}b^{5/3}fx^6+10x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+10\sqrt{3}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)-5x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)-30\sqrt[3]{a}b^{8/3}c}{30a^{4/3}b^{8/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-30*a^(1/3)*b^(8/3)*c + 15*a^(4/3)*b^(2/3)*(b*e - a*f)*x^3 + 6*a^(4/3)*b^(
5/3)*f*x^6 + 10*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*L
og[a^(1/3) + b^(1/3)*x] - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(4/3)*b^(8/3)*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)), x]
```

fricas [A] time = 0.47, size = 560, normalized size = 2.47

$$\left[\frac{6\,a^{2}b^{3}fx^{6}-30\,ab^{4}c+15\left(a^{2}b^{3}e-a^{3}b^{2}f\right)x^{3}-15\sqrt{\frac{1}{3}}\left(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf\right)x\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^{2}x^{3}-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^{2}\right)^{\frac{2}{3}}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}-3\left(-ab^{2}\right)^{\frac{2}{3}}x}{bx^{3}+a}\right)-5\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\left(-ab^{2}\right)^{\frac{2}{3}}x\log\left(b^{2}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}bx+\left(-ab^{2}\right)^{\frac{2}{3}}\right)+10\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\left(-ab^{2}\right)^{\frac{2}{3}}x\log\left(bx-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{30\,a^{2}b^{4}x},\ \frac{6\,a^{2}b^{3}fx^{6}-30\,ab^{4}c+15\left(a^{2}b^{3}e-a^{3}b^{2}f\right)x^{3}-30\sqrt{\frac{1}{3}}\left(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf\right)x\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx+\left(-ab^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}}{b}\right)-5\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\left(-ab^{2}\right)^{\frac{2}{3}}x\log\left(b^{2}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}bx+\left(-ab^{2}\right)^{\frac{2}{3}}\right)+10\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\left(-ab^{2}\right)^{\frac{2}{3}}x\log\left(bx-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{30\,a^{2}b^{4}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a^2*b^3*f*x^6 - 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 15*s
qrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt((-a*b^2)^(1/3)/
a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b
^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)
^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b
^2)^(2/3)*x*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x), 1/30*(6*a^2*b^3*f*x^6 -
 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 30*sqrt(1/3)*(a*b^4*c - a^2*
b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*
b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b*x -
(-a*b^2)^(1/3)))/(a^2*b^4*x)]
```

giac [A] time = 0.18, size = 269, normalized size = 1.19

$$-\frac{\sqrt{3}\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}}-\frac{c}{ax}+\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}}+\frac{\left(b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{2}b^{2}}+\frac{2\,b^{4}fx^{5}-5\,ab^{3}fx^{2}+5\,b^{4}x^{2}e}{10\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^2) - c/(a*x) + 1/6*(b^3*c -
 a*b^2*d - a^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b
^2)^(1/3)*a*b^2) + 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*f*(
-a/b)^(1/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))
/(a^2*b^2) + 1/10*(2*b^4*f*x^5 - 5*a*b^3*f*x^2 + 5*b^4*x^2*e)/b^5
```

maple [B] time = 0.05, size = 419, normalized size = 1.85

$$\frac{fx^{5}}{5b}-\frac{afx^{2}}{2b^{2}}+\frac{ex^{2}}{2b}+\frac{\sqrt{3}\,a^{2}f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{a^{2}f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{a^{2}f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{\sqrt{3}\,ae\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{ae\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{ae\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x)
```

```
[Out] 1/5/b*f*x^5-1/2/b^2*x^2*a*f+1/2*e*x^2/b-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^
(1/3))*f+1/3*a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-1/3/b/(a/b)^(1/3)*ln(x+(
a/b)^(1/3))*d+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/6*a^2/b^3/(a/b)^(1/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-1/6*a/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/
3)*x+(a/b)^(2/3))*e+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d-1
/6/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/3*a^2/b^3*3^(1/2)/(a
/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3*a/b^2*3^(1/2)/(a/b)
^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3/b*3^(1/2)/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/a*c/x
```

maxima [A] time = 2.96, size = 217, normalized size = 0.96

$$\frac{2\,bfx^{5}+5\left(be-af\right)x^{2}}{10\,b^{2}}-\frac{c}{ax}-\frac{\sqrt{3}\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/10*(2*b*f*x^5 + 5*(b*e - a*f)*x^2)/b^2 - c/(a*x) - 1/3*sqrt(3)*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a*b^3*(a/b)^(1/3)) - 1/6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(1/3)) + 1/3*(b^3*c - a*b^2*d + a
^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(1/3))
```

mupad [B] time = 5.37, size = 204, normalized size = 0.90

$$x^2\left(\frac{e}{2\,b}-\frac{a\,f}{2\,b^2}\right)-\frac{c}{a\,x}+\frac{f\,x^5}{5\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x)
```

```
[Out] x^2*(e/(2*b) - (a*f)/(2*b^2)) - c/(a*x) + (f*x^5)/(5*b) + (log(b^(1/3)*x +
a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(4/3)*b^(8/3)) - (log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/(3*a^(4/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i -
2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^
2*b*e))/(3*a^(4/3)*b^(8/3))
```

sympy [A] time = 4.72, size = 408, normalized size = 1.80

$$x^2\left(-\frac{af}{2b^2}+\frac{e}{2b}\right)+\operatorname{RootSum}\left(27t^3a^4b^8+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4cef+3a^5b^4d^2f+3a^5b^4de^2-6a^4b^5cdf-3a^4b^5ce^2-3a^4b^5d^2e+3a^3b^6c^2f+6a^3b^6cde+a^3b^6d^3-3a^2b^7c^2e-3a^2b^7cd^2+3ab^8c^2d-b^9c^3,\left(t\mapsto t\log\left(\frac{9t^2a^3b^5}{a^6f^2-2a^5bef+2a^4b^2df+a^4b^2e^2-2a^3b^3cf-2a^3b^3de+2a^2b^4ce+a^2b^4d^2-2ab^5cd+b^6c^2}+x\right)\right)\right)+\frac{fx^5}{5b}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a),x)
```

```
[Out] x**2*(-a*f/(2*b**2) + e/(2*b)) + RootSum(27*_t**3*a**4*b**8 + a**9*f**3 - 3
*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f*
*2 - 6*a**6*b**3*d*e*f - a**6*b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d
**2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**
4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3
*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambd
a(_t, _t*log(9*_t**2*a**3*b**5/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f
+ a**4*b**2*e**2 - 2*a**3*b**3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a*
*2*b**4*d**2 - 2*a*b**5*c*d + b**6*c**2) + x))) + f*x**5/(5*b) - c/(a*x)
```

3.188 $\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=224

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{5/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{5/3}b^{7/3}}+\cdots$$

Rubi [A] time = 0.17, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{5/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{5/3}b^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}\,a^{5/3}b^{7/3}}+\frac{x(be-af)}{b^2}-\frac{c}{2ax^2}+\frac{fx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

```
[Out] -c/(2*a*x^2) + ((b*e - a*f)*x)/b^2 + (f*x^4)/(4*b) + ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(5/3)*b^(7/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/(3*a^(5/3)*b^(7/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)}\,dx = \int \left(\frac{be-af}{b^2}+\frac{c}{ax^3}+\frac{fx^3}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^3}\,dx}{ab^2}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}b^2}-\frac{\left(b^3c-ab^2d\right.}{}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{7/3}}+\frac{\left(b^3c-\right.}{}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{7/3}}+\frac{\left(b^3c-\right.}{}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{7/3}}-\frac{\left(b^3c-\right.}{}$$

Mathematica [A] time = 0.16, size = 218, normalized size = 0.97

$$\frac{1}{12}\left(\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{5/3}b^{7/3}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{5/3}b^{7/3}}+\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{5/3}b^{7/3}}+\frac{12x(be-af)}{b^2}-\frac{6c}{ax^2}+\frac{3fx^4}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-6*c)/(a*x^2) + (12*(b*e - a*f)*x)/b^2 + (3*f*x^4)/b + (4*Sqrt[3]*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(
a^(5/3)*b^(7/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) +
b^(1/3)*x])/(a^(5/3)*b^(7/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(5/3)*b^(7/3)))/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)), x]
```

fricas [A] time = 0.45, size = 565, normalized size = 2.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a^3*b^2*f*x^6 - 6*a^2*b^3*c - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a
^3*b^2*e - a^4*b*f)*x^2*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(
1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*(a^2*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*b)^(
2/3)) + 12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2), 1/12*(3*a^3*b^2*f*x^6
- 6*a^2*b^3*c - 12*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^
2*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)
*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2
*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*b)^(2/3)) +
12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2)]
```

giac [A] time = 0.22, size = 232, normalized size = 1.04

$$\frac{\sqrt{3}\left(b^3c - ab^2d - a^3f + a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}ab} + \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}ab} + \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2b^2} - \frac{c}{2ax^2} + \frac{b^3fx^4 - 4ab^2fx + 4b^3xe}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) + 1/6*(b^3*c - a*b^2*d - a^
3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b
) + 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2) - 1/2*c/(a*x^2) + 1/4*(b^3*f*x^4 - 4*a*b^2*f*x + 4*b^3*x
*e)/b^4
```

maple [B] time = 0.06, size = 414, normalized size = 1.85

$$\frac{fx^4}{4b} + \frac{\sqrt{3}\,a^2 f \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} + \frac{a^2 f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} - \frac{a^2 f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} - \frac{\sqrt{3}\,ae\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} - \frac{ae\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} + \frac{ae\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} - \frac{afx}{b^2} - \frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a} - \frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a} + \frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a} + \frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b} + \frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b} - \frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b} + \frac{ex}{b} - \frac{c}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x)
```

```
[Out] 1/4*f*x^4/b-1/b^2*a*f*x+e*x/b+1/3*a^2/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-1
/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)*d-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/6*a^2/b^3/(a/b)^(2/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))*f+1/6*a/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))*e-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/6/a/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/3*a^2/b^3/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))*c-1/2*c/a/x^2
```

maxima [A] time = 2.98, size = 214, normalized size = 0.96

$$\frac{bfx^4 + 4\left(be - af\right)x}{4b^2} - \frac{c}{2ax^2} - \frac{\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(b^3c - ab^2d + a^2be - a^3f\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(b^3c - ab^2d + a^2be - a^3f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*f*x^4 + 4*(b*e - a*f)*x)/b^2 - 1/2*c/(a*x^2) - 1/3*sqrt(3)*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(a*b^3*(a/b)^(2/3)) + 1/6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) - 1/3*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

mupad [B] time = 0.28, size = 201, normalized size = 0.90

$$x\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)-\frac{c}{2\,a\,x^2}+\frac{f\,x^4}{4\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x)
```

```
[Out] x*(e/b - (a*f)/b^2) - c/(2*a*x^2) + (f*x^4)/(4*b) - (log(b^(1/3)*x + a^(1/3
))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(5/3)*b^(7/3)) - (log(3^(1/2)*
a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f -
 a*b^2*d + a^2*b*e))/(3*a^(5/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1
/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)
)/(3*a^(5/3)*b^(7/3))
```

sympy [A] time = 4.36, size = 326, normalized size = 1.46

$$x\left(-\frac{af}{b^{2}}+\frac{e}{b}\right)+\operatorname{RootSum}\left(27t^{3}a^{5}b^{7}-a^{9}f^{3}+3a^{8}bef^{2}-3a^{7}b^{2}df^{2}-3a^{7}b^{2}e^{2}f+3a^{6}b^{3}cf^{2}+6a^{6}b^{3}def+a^{6}b^{3}e^{3}-6a^{5}b^{4}cef-3a^{5}b^{4}d^{2}f-3a^{5}b^{4}de^{2}+6a^{4}b^{5}cdf+3a^{4}b^{5}ce^{2}+3a^{4}b^{5}d^{2}e-3a^{3}b^{6}c^{2}f-6a^{3}b^{6}cde-a^{3}b^{6}d^{3}+3a^{2}b^{7}c^{2}e+3a^{2}b^{7}cd^{2}-3ab^{8}c^{2}d+b^{9}c^{3},\left(t\mapsto t\log\left(\frac{3ta^{2}b^{2}}{a^{3}f-a^{2}be+ab^{2}d-b^{3}c}+x\right)\right)\right)+\frac{fx^{4}}{4b}-\frac{c}{2ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a),x)
```

```
[Out] x*(-a*f/b**2 + e/b) + RootSum(27*_t**3*a**5*b**7 - a**9*f**3 + 3*a**8*b*e*f
**2 - 3*a**7*b**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6
*b**3*d*e*f + a**6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a
**5*b**4*d*e**2 + 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2
*e - 3*a**3*b**6*c**2*f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*
c**2*e + 3*a**2*b**7*c*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*lo
g(3*_t*a**2*b**2/(a**3*f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**4/(4
*b) - c/(2*a*x**2)
```

3.189 $\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=227

$$\frac{bc-ad}{a^2x}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{7/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^3(-f)+a^2be-ab^2d\right.}{3a^{7/3}b^{5/3}}$$

Rubi [A] time = 0.19, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{7/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{7/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}\,a^{7/3}b^{5/3}}+\frac{bc-ad}{a^2x}-\frac{c}{4ax^4}+\frac{fx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

```
[Out] -c/(4*a*x^4) + (b*c - a*d)/(a^2*x) + (f*x^2)/(2*b) - ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(7/3)*b^(5/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/(3*a^(7/3)*b^(5/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)}\,dx &= \int \left(\frac{c}{ax^5}+\frac{-bc+ad}{a^2x^2}+\frac{fx}{b}-\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{a^2b\left(a+bx^3\right)}\right)dx \\
&= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int\frac{x}{a+bx^3}\,dx}{a^2b} \\
&= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{7/3}b^{4/3}}+\frac{\left(b^3c-ab^2d+\right.}{} \\
&= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}b^{5/3}}+\frac{\left(b^3c-ab^2\right.}{} \\
&= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}b^{5/3}}+\frac{\left(b^3c-ab^2\right.}{} \\
&= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}b^{5/3}}-\frac{\left(b^3c-ab^2\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 220, normalized size = 0.97

$$\frac{1}{12}\left(\frac{12(bc-ad)}{a^2x}+\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{7/3}b^{5/3}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{7/3}b^{5/3}}+\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{7/3}b^{5/3}}-\frac{3c}{ax^4}+\frac{6fx^2}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

```
[Out] ((-3*c)/(a*x^4) + (12*(b*c - a*d))/(a^2*x) + (6*f*x^2)/b + (4*Sqrt[3]*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/(a^(7/3)*b^(5/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/
3) + b^(1/3)*x])/(a^(7/3)*b^(5/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(5/3)))/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)), x]
```

fricas [A] time = 0.46, size = 556, normalized size = 2.45

$$\left[\frac{6a^3b^2fx^6 - 6\sqrt{\frac{1}{3}}\left(ab^4c - a^2b^3d + a^3b^2e - a^4bf\right)x^4\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3 - ab - 3\sqrt{\frac{1}{3}}\left(abx + 2(ab^2)^{\frac{2}{3}}x^2 - (ab^2)^{\frac{1}{3}}a\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}} - 3(ab^2)^{\frac{2}{3}}x}{bx^3+a}\right) + 2\left(b^3c - ab^2d + a^2be - a^3f\right)(ab^2)^{\frac{2}{3}}x^4\log\left(b^2x^2 - (ab^2)^{\frac{1}{3}}bx + (ab^2)^{\frac{2}{3}}\right) - 4\left(b^3c - ab^2d + a^2be - a^3f\right)(ab^2)^{\frac{2}{3}}x^4\log\left(bx + (ab^2)^{\frac{1}{3}}\right) - 3a^2b^3c + 12\left(ab^4c - a^2b^3d\right)x^3}{12a^3b^3x^4}, \frac{6a^3b^2fx^6 - 12\sqrt{\frac{1}{3}}\left(ab^4c - a^2b^3d + a^3b^2e - a^4bf\right)x^4\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2bx - (ab^2)^{\frac{1}{3}}\right)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{b}\right) + 2\left(b^3c - ab^2d + a^2be - a^3f\right)(ab^2)^{\frac{2}{3}}x^4\log\left(b^2x^2 - (ab^2)^{\frac{1}{3}}bx + (ab^2)^{\frac{2}{3}}\right) - 4\left(b^3c - ab^2d + a^2be - a^3f\right)(ab^2)^{\frac{2}{3}}x^4\log\left(bx + (ab^2)^{\frac{1}{3}}\right) - 3a^2b^3c + 12\left(ab^4c - a^2b^3d\right)x^3}{12a^3b^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a^3*b^2*f*x^6 - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*x^4*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x
+ 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)
^(2/3)*x)/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3
)*x^4*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*
c + 12*(a*b^4*c - a^2*b^3*d)*x^3)/(a^3*b^3*x^4), 1/12*(6*a^3*b^2*f*x^6 - 12
*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4*sqrt((a*b^2)^(1/
3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) +
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b^2*x^2 - (a*b^
2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^
2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*c + 12*(a*b^4*c - a^2*b^3
*d)*x^3)/(a^3*b^3*x^4)]
```

giac [A] time = 0.18, size = 261, normalized size = 1.15

$$\frac{fx^2}{2b} + \frac{\sqrt{3}\left(b^3c - ab^2d - a^3f + a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}a^2b} - \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}a^2b} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b} + \frac{4bcx^3 - 4adx^3 - ac}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b) - 1/6*(b^3*
c - a*b^2*d - a^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(1/3)*a^2*b) - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*
f*(-a/b)^(1/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a^3*b) + 1/4*(4*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^2*x^4)
```

maple [B] time = 0.06, size = 412, normalized size = 1.81

$$\frac{fx^2}{2b} - \frac{\sqrt{3}\,af\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{af\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{af\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a} + \frac{d\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a} - \frac{d\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a} + \frac{\sqrt{3}\,bc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} - \frac{bc\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} + \frac{bc\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2} + \frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b} - \frac{e\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b} + \frac{e\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b} - \frac{d}{ax} + \frac{bc}{a^2x} - \frac{c}{4ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x)
```

```
[Out] 1/2*f*x^2/b+1/3*a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-1/3/b/(a/b)^(1/3)*ln(
x+(a/b)^(1/3))*e+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-1/3/a^2*b/(a/b)^(1/3
)*ln(x+(a/b)^(1/3))*c-1/6*a/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))*f+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-1/6/a/(a/b)^(1/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/6/a^2*b/(a/b)^(1/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))*c-1/3*a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))*f+1/3/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))*e-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)*d+1/3/a^2*b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
-1/4*c/a/x^4-d/a/x+1/a^2/x*b*c
```

maxima [A] time = 3.04, size = 217, normalized size = 0.96

$$\frac{fx^2}{2b} + \frac{\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(b^3c - ab^2d + a^2be - a^3f\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(b^3c - ab^2d + a^2be - a^3f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{4(bc - ad)x^3 - ac}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*f*x^2/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sq
rt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/6*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2
*(a/b)^(1/3)) - 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3)
)/(a^2*b^2*(a/b)^(1/3)) + 1/4*(4*(b*c - a*d)*x^3 - a*c)/(a^2*x^4)
```

mupad [B] time = 5.16, size = 209, normalized size = 0.92

$$\frac{f\,x^2}{2\,b}-\frac{\frac{b\,c}{4\,a}+\frac{b\,x^3\,\left(a\,d-b\,c\right)}{a^2}}{b\,x^4}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x)
```

```
[Out] (f*x^2)/(2*b) - ((b*c)/(4*a) + (b*x^3*(a*d - b*c))/a^2)/(b*x^4) - (log(b^(1
/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3)) +
 (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3)) - (log(3^(1/2)*a^(1/
3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^
2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3))
```

sympy [A] time = 11.53, size = 411, normalized size = 1.81

$$\operatorname{RootSum}{\left(27 t^{3} a^{7} b^{5} - a^{9} f^{3} + 3 a^{8} b e f^{2} - 3 a^{7} b^{2} d f^{2} - 3 a^{7} b^{2} e^{2} f + 3 a^{6} b^{3} c f^{2} + 6 a^{6} b^{3} d e f + a^{6} b^{3} e^{3} - 6 a^{5} b^{4} c e f - 3 a^{5} b^{4} d^{2} f - 3 a^{5} b^{4} d e^{2} + 6 a^{4} b^{5} c d f + 3 a^{4} b^{5} c e^{2} + 3 a^{4} b^{5} d^{2} e - 3 a^{3} b^{6} c^{2} f - 6 a^{3} b^{6} c d e - a^{3} b^{6} d^{3} + 3 a^{2} b^{7} c^{2} e + 3 a^{2} b^{7} c d^{2} - 3 a b^{8} c^{2} d + b^{9} c^{3}, \left(t \mapsto t \log{\left(\frac{9 t^{2} a^{5} b^{3}}{a^{6} f^{2} - 2 a^{5} b e f + 2 a^{4} b^{2} d f + a^{4} b^{2} e^{2} - 2 a^{3} b^{3} c f - 2 a^{3} b^{3} d e + 2 a^{2} b^{4} c e + a^{2} b^{4} d^{2} - 2 a b^{5} c d + b^{6} c^{2}} + x \right)} \right)\right)} + \frac{f x^{2}}{2 b} + \frac{- a c + x^{3} \left(- 4 a d + 4 b c\right)}{4 a^{2} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**7*b**5 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f*
*2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**
3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a*
*4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*
f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c
*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**5*b**3/(a
**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*
f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**
6*c**2) + x))) + f*x**2/(2*b) + (-a*c + x**3*(-4*a*d + 4*b*c))/(4*a**2*x**4
)
```

3.190 $\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=225

$$\frac{bc-ad}{2a^2x^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^3(-f)+a^2be-ab^2d\right.}{3a^{8/3}b^{4/3}}$$

Rubi [A] time = 0.17, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{8/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}\,a^{8/3}b^{4/3}}+\frac{bc-ad}{2a^2x^2}-\frac{c}{5ax^5}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)
*x])/(3*a^(8/3)*b^(4/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)}\,dx = \int \left(\frac{f}{b}+\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{a+bx^3}\,dx}{a^2b}$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{8/3}b}+\frac{\left(b^3c-ab^2d+a\right.}{}$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}}-\frac{\left(b^3c-ab^2d\right.}{}$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}}-\frac{\left(b^3c-ab^2d\right.}{}$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}b^{4/3}}+\frac{\left(b^3c-ab^2\right.}{}$$

Mathematica [A] time = 0.12, size = 220, normalized size = 0.98

$$\frac{bc-ad}{2a^2x^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{8/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt{3}\,a^{8/3}b^{4/3}}-\frac{c}{5ax^5}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

```
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b + ((-(b^3*c) + a*b^2*d -
 a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(
8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(8/3)*b^(4/3)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)), x]
```

fricas [A] time = 0.44, size = 584, normalized size = 2.60

$$\left[\frac{30\,a^{4}bfx^{6}-15\,\sqrt{\frac{1}{3}}\left(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf\right)x^{5}\sqrt{\frac{\left(-a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}+3\left(-a^{2}b\right)^{\frac{1}{3}}ax-a^{2}-3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+\left(-a^{2}b\right)^{\frac{2}{3}}x+\left(-a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^{2}b\right)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-5\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\left(-a^{2}b\right)^{\frac{2}{3}}x^{5}\log\left(abx^{2}-\left(-a^{2}b\right)^{\frac{2}{3}}x-\left(-a^{2}b\right)^{\frac{1}{3}}a\right)+10\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\left(-a^{2}b\right)^{\frac{2}{3}}x^{5}\log\left(abx+\left(-a^{2}b\right)^{\frac{2}{3}}\right)-6\,a^{3}b^{2}c+15\left(a^{2}b^{3}c-a^{3}b^{2}d\right)x^{3}}{30\,a^{4}b^{2}x^{5}},\ \frac{30\,a^{4}bfx^{6}+30\,\sqrt{\frac{1}{3}}\left(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf\right)x^{5}\sqrt{-\frac{\left(-a^{2}b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(-a^{2}b\right)^{\frac{2}{3}}x+\left(-a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(-a^{2}b\right)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-5\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\left(-a^{2}b\right)^{\frac{2}{3}}x^{5}\log\left(abx^{2}-\left(-a^{2}b\right)^{\frac{2}{3}}x-\left(-a^{2}b\right)^{\frac{1}{3}}a\right)+10\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\left(-a^{2}b\right)^{\frac{2}{3}}x^{5}\log\left(abx+\left(-a^{2}b\right)^{\frac{2}{3}}\right)-6\,a^{3}b^{2}c+15\left(a^{2}b^{3}c-a^{3}b^{2}d\right)x^{3}}{30\,a^{4}b^{2}x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/30*(30*a^4*b*f*x^6 - 15*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*x^5*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^
2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^
2*b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b
)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^(2/3))
 - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5), 1/30*(30*a^
4*b*f*x^6 + 30*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^5*sq
rt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)
*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-
a^2*b)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^(
2/3)) - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5)]
```

giac [A] time = 0.48, size = 220, normalized size = 0.98

$$\frac{fx}{b}-\frac{\sqrt{3}\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{3}b}+\frac{5\,bcx^{3}-5\,adx^{3}-2\,ac}{10\,a^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="giac")
```

```
[Out] f*x/b - 1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/6*(b^3*c - a*b^
2*d - a^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(
2/3)*a^2) - 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^3*b) + 1/10*(5*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/(a^2*x^5)
```

maple [B] time = 0.05, size = 410, normalized size = 1.82

$$-\frac{\sqrt{3}\,af\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{af\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{af\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{\sqrt{3}\,bc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{bc\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{bc\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{fx}{b}-\frac{d}{2ax^{2}}+\frac{bc}{2a^{2}x^{2}}-\frac{c}{5ax^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x)
```

```
[Out] 1/b*f*x-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f+1/3/b/(a/b)^(2/3)*ln(x+(a
/b)^(1/3))*e-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+1/3/a^2*b/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))*c+1/6*a/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f
-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/6/a/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d-1/6/a^2*b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))*c-1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))*f+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*e-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+
1/3/a^2*b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/5
/a*c/x^5-1/2*d/a/x^2+1/2/a^2/x^2*b*c
```

maxima [A] time = 3.08, size = 214, normalized size = 0.95

$$\frac{fx}{b}+\frac{\sqrt{3}\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\left(bc-ad\right)x^{3}-2\,ac}{10\,a^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] f*x/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)
^(2/3)) + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^2
*b^2*(a/b)^(2/3)) + 1/10*(5*(b*c - a*d)*x^3 - 2*a*c)/(a^2*x^5)
```

mupad [B] time = 5.09, size = 207, normalized size = 0.92

$$\frac{f\,x}{b}-\frac{\frac{b\,c}{5\,a}+\frac{b\,x^3\,(a\,d-b\,c)}{2\,a^2}}{b\,x^5}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x)
```

```
[Out] (f*x)/b - ((b*c)/(5*a) + (b*x^3*(a*d - b*c))/(2*a^2))/(b*x^5) + (log(b^(1/3
)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(8/3)*b^(4/3)) + (
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3
*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(8/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)
*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*
d + a^2*b*e))/(3*a^(8/3)*b^(4/3))
```

sympy [A] time = 19.68, size = 328, normalized size = 1.46

$$\operatorname{RootSum}{\left(27t^3a^8b^4+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4cef+3a^5b^4d^2f+3a^5b^4de^2-6a^4b^5cdf-3a^4b^5ce^2-3a^4b^5d^2e+3a^3b^6c^2f+6a^3b^6cde+a^3b^6d^3-3a^2b^7c^2e-3a^2b^7cd^2+3ab^8c^2d-b^9c^3, \left(t \mapsto t\log{\left(-\frac{3ta^3b}{a^3f-a^2be+ab^2d-b^3c}+x \right)} \right)\right)}+\frac{fx}{b}+\frac{-2ac+x^3\left(-5ad+5bc\right)}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**8*b**4 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f*
*2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**
3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a*
*4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*
f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c
*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a**3*b/(a**3*f
 - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x/b + (-2*a*c + x**3*(-5*a*d +
5*b*c))/(10*a**2*x**5)
```

3.191 $\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=242

$$\frac{bc-ad}{4a^2x^4}-\frac{a^2e-abd+b^2c}{a^3x}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{10/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-\right.}{3a^{1\!0}}$$

Rubi [A] time = 0.19, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{10/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{10/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}a^{10/3}b^{2/3}}-\frac{a^2e-abd+b^2c}{a^3x}+\frac{bc-ad}{4a^2x^4}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(4*a^2*x^4) - (b^2*c - a*b*d + a^2*e)/(a^3*x) +
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(10/3)*b^(2/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)*b^(2/3)) - ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/3)*
b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^8}+\frac{-bc+ad}{a^2x^5}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{a^3\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{x}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{10/3}\sqrt[3]{b}}-\frac{(b^3}{}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}b^{2/3}}-$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}b^{2/3}}-$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}b^{2/3}}+$$

Mathematica [A] time = 0.15, size = 231, normalized size = 0.95

$$\frac{\frac{21a^{4/3}(bc-ad)}{x^4}-\frac{12a^{7/3}c}{x^7}-\frac{84\sqrt[3]{a}\left(a^2e-abd+b^2c\right)}{x}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{2/3}}+\frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{2/3}}+\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^{2/3}}}{84a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] ((-12*a^(7/3)*c)/x^7 + (21*a^(4/3)*(b*c - a*d))/x^4 - (84*a^(1/3)*(b^2*c -
a*b*d + a^2*e))/x + (28*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-(b^3*c) + a*b^2*d -
a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
84*a^(10/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)), x]
```

fricas [A] time = 0.45, size = 610, normalized size = 2.52

$$\left[-\frac{42\sqrt{\frac{1}{3}}\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^7\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+14\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-ab^2\right)^{\frac{2}{3}}x^7\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}bx+\left(-ab^2\right)^{\frac{2}{3}}\right)-28\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-ab^2\right)^{\frac{2}{3}}x^7\log\left(bx-\left(-ab^2\right)^{\frac{1}{3}}\right)+84\left(ab^4c-a^2b^3d+a^3b^2e\right)x^6+12a^3b^2c-21\left(a^2b^3c-a^3b^2d\right)x^3}{84a^4b^2x^7},\ -\frac{84\sqrt{\frac{1}{3}}\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^7\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+14\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-ab^2\right)^{\frac{2}{3}}x^7\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}bx+\left(-ab^2\right)^{\frac{2}{3}}\right)-28\left(b^3c-ab^2d+a^2be-a^3f\right)\left(-ab^2\right)^{\frac{2}{3}}x^7\log\left(bx-\left(-ab^2\right)^{\frac{1}{3}}\right)+84\left(ab^4c-a^2b^3d+a^3b^2e\right)x^6+12a^3b^2c-21\left(a^2b^3c-a^3b^2d\right)x^3}{84a^4b^2x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^7*sqrt((
-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/
3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*
x^3 + a)) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b
^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 84*(a*b^4*c - a^
2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^3*b^2*d)*x^3)/(
a^4*b^2*x^7), -1/84*(84*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*
f)*x^7*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sq
rt(-(-a*b^2)^(1/3)/a)/b) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^
(2/3)*x^7*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 8
4*(a*b^4*c - a^2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^
3*b^2*d)*x^3)/(a^4*b^2*x^7)]
```

giac [A] time = 0.21, size = 275, normalized size = 1.14

$$-\frac{\sqrt{3}\left(b^3c-ab^2d-a^3f+a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}a^3}+\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}a^3}+\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^4}-\frac{28b^2cx^6-28abdx^6+28a^2x^6e-7abcx^3+7a^2dx^3+4a^2c}{28a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3) + 1/6*(b^3*c - a*b^2*d - a
^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^
3) + 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3) +
a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/28*(2
8*b^2*c*x^6 - 28*a*b*d*x^6 + 28*a^2*x^6*e - 7*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a
^2*c)/(a^3*x^7)
```

maple [B] time = 0.05, size = 440, normalized size = 1.82

$$-\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{\sqrt{3}\,bd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{bd\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{bd\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{\sqrt{3}\,b^2c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{b^2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{b^2c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{e}{ax}+\frac{bd}{a^2x}-\frac{b^2c}{a^3x}-\frac{d}{4ax^4}+\frac{bc}{4a^2x^4}-\frac{c}{7ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x)
```

```
[Out] -1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*
e-1/3/a^2*b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d+1/3/a^3*b^2/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))*c+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-1/6/a/(a
/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/6/a^2*b/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))*d-1/6/a^3*b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))*c+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*f-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+
1/3/a^2*3^(1/2)*b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3
```

```
/a^3*3^(1/2)*b^2/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/7/
a*c/x^7-1/4/a/x^4*d+1/4/a^2/x^4*b*c-e/a/x+1/a^2/x*b*d-1/a^3/x*b^2*c
```

maxima [A] time = 3.07, size = 234, normalized size = 0.97

$$-\frac{\sqrt{3}\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^{3}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^{3}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^{3}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{28\left(b^{2}c-abd+a^{2}e\right)x^{6}-7\left(abc-a^{2}d\right)x^{3}+4a^{2}c}{28a^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(1/3)) - 1/6*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(1/3)) + 1
/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(1
/3)) - 1/28*(28*(b^2*c - a*b*d + a^2*e)*x^6 - 7*(a*b*c - a^2*d)*x^3 + 4*a^2
*c)/(a^3*x^7)
```

mupad [B] time = 5.20, size = 219, normalized size = 0.90

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}}-\frac{\frac{c}{7\,a}+\frac{x^3\,\left(a\,d-b\,c\right)}{4\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{a^3}}{x^7}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*
b^(2/3)) - (c/(7*a) + (x^3*(a*d - b*c))/(4*a^2) + (x^6*(b^2*c + a^2*e - a*b
*d))/a^3)/x^7 - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*b^(2/3)) + (l
og(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*
c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*b^(2/3))
```

sympy [A] time = 46.61, size = 432, normalized size = 1.79

$$\operatorname{RootSum}{\left(27t^{3}a^{10}b^{2}+a^{9}f^{3}-3a^{8}bef^{2}+3a^{7}b^{2}df^{2}+3a^{7}b^{2}e^{2}f-3a^{6}b^{3}cf^{2}-6a^{6}b^{3}def-a^{6}b^{3}e^{3}+6a^{5}b^{4}cef+3a^{5}b^{4}d^{2}f+3a^{5}b^{4}de^{2}-6a^{4}b^{5}cdf-3a^{4}b^{5}ce^{2}-3a^{4}b^{5}d^{2}e+3a^{3}b^{6}c^{2}f+6a^{3}b^{6}cde+a^{3}b^{6}d^{3}-3a^{2}b^{7}c^{2}e-3a^{2}b^{7}cd^{2}+3ab^{8}c^{2}d-b^{9}c^{3},\left(t\mapsto t\log{\left(\frac{9t^{2}a^{7}b}{a^{6}f^{2}-2a^{5}bef+2a^{4}b^{2}df+a^{4}b^{2}e^{2}-2a^{3}b^{3}cf-2a^{3}b^{3}de+2a^{2}b^{4}ce+a^{2}b^{4}d^{2}-2ab^{5}cd+b^{6}c^{2}}+x\right)}\right)\right)}+\frac{-4a^{2}c+x^{6}\left(-28a^{2}e+28abd-28b^{2}c\right)+x^{3}\left(-7a^{2}d+7abc\right)}{28a^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**10*b**2 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f
**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b*
*3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a
**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2
*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*
c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**7*b/(a**
6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f
- 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*
c**2) + x))) + (-4*a**2*c + x**6*(-28*a**2*e + 28*a*b*d - 28*b**2*c) + x**3
*(-7*a**2*d + 7*a*b*c))/(28*a**3*x**7)
```

3.192 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$

Optimal. Leaf size=244

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{2a^3x^2}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{11/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-\right.}{\sqrt{3}a^{11/3}\sqrt[3]{b}}$$

Rubi [A] time = 0.17, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{11/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}a^{11/3}\sqrt[3]{b}}-\frac{a^2e-abd+b^2c}{2a^3x^2}+\frac{bc-ad}{5a^2x^5}-\frac{c}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

```
[Out] -c/(8*a*x^8) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(2*a^3*x^2
) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3)*b^(1/3)) - ((b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(11/3)*b^(1/3)) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(11
/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^9}+\frac{-bc+ad}{a^2x^6}+\frac{b^2c-abd+a^2e}{a^3x^3}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int\frac{1}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{11/3}}-\frac{(b^3}{}$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{11/3}\sqrt[3]{b}}-$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{11/3}\sqrt[3]{b}}+$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{11/3}\sqrt[3]{b}}-$$

Mathematica [A] time = 0.25, size = 231, normalized size = 0.95

$$\frac{\frac{24a^{5/3}(bc-ad)}{x^5}-\frac{15a^{8/3}c}{x^8}-\frac{60a^{2/3}\left(a^2e-abd+b^2c\right)}{x^2}+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt[3]{b}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt[3]{b}}+\frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt[3]{b}}}{120a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

```
[Out] ((-15*a^(8/3)*c)/x^8 + (24*a^(5/3)*(b*c - a*d))/x^5 - (60*a^(2/3)*(b^2*c -
a*b*d + a^2*e))/x^2 + (40*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-(b^3*c) + a*b^2*d -
 a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20*(b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))
/(120*a^(11/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)), x]
```

fricas [A] time = 0.45, size = 595, normalized size = 2.44

$$\left[-\frac{60 \sqrt{\frac{1}{3}} \left(ab^{4} c - a^{2} b^{3} d + a^{3} b^{2} e - a^{4} b f\right) x^{8} \sqrt{-\frac{\left(a^{2} b\right)^{\frac{1}{3}}}{b}} \log\left(\frac{2 abx^{3} - 3 \left(a^{2} b\right)^{\frac{1}{3}} ax - a^{2} + 3 \sqrt{\frac{1}{3}} \left(2 abx^{2} + \left(a^{2} b\right)^{\frac{2}{3}} x - \left(a^{2} b\right)^{\frac{1}{3}} a\right) \sqrt{-\frac{\left(a^{2} b\right)^{\frac{1}{3}}}{b}}}{bx^{3} + a}\right) - 20 \left(b^{3} c - ab^{2} d + a^{2} be - a^{3} f\right) \left(a^{2} b\right)^{\frac{2}{3}} x^{8} \log\left(abx^{2} - \left(a^{2} b\right)^{\frac{2}{3}} x + \left(a^{2} b\right)^{\frac{1}{3}} a\right) + 40 \left(b^{3} c - ab^{2} d + a^{2} be - a^{3} f\right) \left(a^{2} b\right)^{\frac{2}{3}} x^{8} \log\left(abx + \left(a^{2} b\right)^{\frac{2}{3}}\right) + 60 \left(a^{2} b^{3} c - a^{3} b^{2} d + a^{4} be\right) x^{6} + 15 a^{4} bc - 24 \left(a^{3} b^{2} c - a^{4} bd\right) x^{3}}{120 a^{5} bx^{8}}, -\frac{120 \sqrt{\frac{1}{3}} \left(ab^{4} c - a^{2} b^{3} d + a^{3} b^{2} e - a^{4} b f\right) x^{8} \sqrt{\frac{\left(a^{2} b\right)^{\frac{1}{3}}}{b}} \arctan\left(\frac{\sqrt{\frac{1}{3}} \left(2 \left(a^{2} b\right)^{\frac{2}{3}} x - \left(a^{2} b\right)^{\frac{1}{3}} a\right) \sqrt{\frac{\left(a^{2} b\right)^{\frac{1}{3}}}{b}}}{a^{2}}\right) - 20 \left(b^{3} c - ab^{2} d + a^{2} be - a^{3} f\right) \left(a^{2} b\right)^{\frac{2}{3}} x^{8} \log\left(abx^{2} - \left(a^{2} b\right)^{\frac{2}{3}} x + \left(a^{2} b\right)^{\frac{1}{3}} a\right) + 40 \left(b^{3} c - ab^{2} d + a^{2} be - a^{3} f\right) \left(a^{2} b\right)^{\frac{2}{3}} x^{8} \log\left(abx + \left(a^{2} b\right)^{\frac{2}{3}}\right) + 60 \left(a^{2} b^{3} c - a^{3} b^{2} d + a^{4} be\right) x^{6} + 15 a^{4} bc - 24 \left(a^{3} b^{2} c - a^{4} bd\right) x^{3}}{120 a^{5} bx^{8}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/120*(60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*sqrt(
-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*
(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*
x^3 + a)) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*
b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) + 60*(a^2*b^3*c - a^3
*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4*b*d)*x^3)/(a^5*b*x
^8), -1/120*(120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*
sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*
b)^(2/3)*x^8*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) +
60*(a^2*b^3*c - a^3*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4
*b*d)*x^3)/(a^5*b*x^8)]
```

giac [A] time = 0.20, size = 297, normalized size = 1.22

$$\frac{\left(b^{3} c - ab^{2} d - a^{3} f + a^{2} be\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 a^{4}} - \frac{\sqrt{3} \left(\left(-ab^{2}\right)^{\frac{1}{3}} b^{3} c - \left(-ab^{2}\right)^{\frac{1}{3}} ab^{2} d - \left(-ab^{2}\right)^{\frac{1}{3}} a^{3} f + \left(-ab^{2}\right)^{\frac{1}{3}} a^{2} be\right) \arctan\left(\frac{\sqrt{3} \left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 a^{4} b} - \frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}} b^{3} c - \left(-ab^{2}\right)^{\frac{1}{3}} ab^{2} d - \left(-ab^{2}\right)^{\frac{1}{3}} a^{3} f + \left(-ab^{2}\right)^{\frac{1}{3}} a^{2} be\right) \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 a^{4} b} - \frac{20 b^{2} cx^{6} - 20 abdx^{6} + 20 a^{2} x^{6} e - 8 abcx^{3} + 8 a^{2} dx^{3} + 5 a^{2} c}{40 a^{3} x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/
3)))/a^4 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a
*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)
*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/40*(20*b^2*c*x^6 - 20*a*b*d*x^6 + 20*a^
2*x^6*e - 8*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^3*x^8)
```

maple [B] time = 0.06, size = 441, normalized size = 1.81

$$-\frac{\sqrt{3}\, e \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} a} - \frac{e \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} a} + \frac{e \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} a} + \frac{\sqrt{3}\, bd \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} a^{2}} + \frac{bd \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} a^{2}} - \frac{bd \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} a^{2}} - \frac{\sqrt{3}\, b^{2} c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} a^{3}} - \frac{b^{2} c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} a^{3}} + \frac{b^{2} c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} a^{3}} + \frac{\sqrt{3}\, f \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} + \frac{f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{f \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{e}{2a x^{2}} + \frac{bd}{2a^{2} x^{2}} - \frac{b^{2} c}{2a^{3} x^{2}} - \frac{d}{5a x^{5}} + \frac{bc}{5a^{2} x^{5}} - \frac{c}{8a x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x)
```

```
[Out] 1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e
+1/3/a^2*b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-1/3/a^3*b^2/(a/b)^(2/3)*ln(x+(a/
b)^(1/3))*c-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f+1/6/a/(a/
b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-1/6/a^2*b/(a/b)^(2/3)*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))*d+1/6/a^3*b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*c+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))*f-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1
/3/a^2*b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3/
a^3*b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/8*c
```

```
/a/x^8-1/5/a/x^5*d+1/5/a^2/x^5*b*c-1/2/a/x^2*e+1/2/a^2/x^2*b*d-1/2/a^3/x^2*
b^2*c
```

maxima [A] time = 3.01, size = 234, normalized size = 0.96

$$-\frac{\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\left(b^2c-abd+a^2e\right)x^6-8\left(abc-a^2d\right)x^3+5a^2c}{40a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/6*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 1
/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2
/3)) - 1/40*(20*(b^2*c - a*b*d + a^2*e)*x^6 - 8*(a*b*c - a^2*d)*x^3 + 5*a^2
*c)/(a^3*x^8)
```

mupad [B] time = 5.13, size = 220, normalized size = 0.90

$$-\frac{\frac{c}{8a}+\frac{x^3(ad-bc)}{5a^2}+\frac{x^6\left(ea^2-dab+cb^2\right)}{2a^3}}{x^8}-\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3a^{11/3}b^{1/3}}-\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3a^{11/3}b^{1/3}}+\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3a^{11/3}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^
3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (log(b^(1/3)*x + a
^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (c/(8*a) + (x^3*(a*d - b*
c))/(5*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(2*a^3))/x^8
```

sympy [A] time = 88.52, size = 348, normalized size = 1.43

$$\operatorname{RootSum}\left(27t^3a^{11}b-a^9f^3+3a^8bef^2-3a^7b^2df^2-3a^7b^2e^2f+3a^6b^3cf^2+6a^6b^3def+a^6b^3e^3-6a^5b^4cef-3a^5b^4d^2f-3a^5b^4de^2+6a^4b^5cdf+3a^4b^5ce^2+3a^4b^5d^2e-3a^3b^6c^2f-6a^3b^6cde-a^3b^6d^3+3a^2b^7c^2e+3a^2b^7cd^2-3ab^8c^2d+b^9c^3,\left(t\mapsto t\log\left(\frac{3ta^4}{a^3f-a^2be+ab^2d-b^3c}+x\right)\right)\right)+\frac{-5a^2c+x^6\left(-20a^2e+20abd-20b^2c\right)+x^3\left(-8a^2d+8abc\right)}{40a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**11*b - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f**2
 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**3*
e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a**4
*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f
- 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d
**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(3*_t*a**4/(a**3*f - a*
*2*b*e + a*b**2*d - b**3*c) + x))) + (-5*a**2*c + x**6*(-20*a**2*e + 20*a*b
*d - 20*b**2*c) + x**3*(-8*a**2*d + 8*a*b*c))/(40*a**3*x**8)
```

3.193 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$

Optimal. Leaf size=277

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{4a^3x^4}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{13/3}}-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)\right.}{\sqrt{3}a^{13/3}}$$

Rubi [A] time = 0.22, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{13/3}}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{a^4x}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{13/3}}-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}a^{13/3}}-\frac{a^2e-abd+b^2c}{4a^3x^4}+\frac{bc-ad}{7a^2x^7}-\frac{c}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

```
[Out] -c/(10*a*x^10) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(4*a^3*x
^4) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (b^(1/3)*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(S
qrt[3]*a^(13/3)) - (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(13/3)) + (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{11}}+\frac{-bc+ad}{a^2x^8}+\frac{b^2c-abd+a^2e}{a^3x^5}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2}-\frac{b\left(-b^3c+ab\right.}{a^4\left(\right.}\right.$$

$$= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}+\frac{\left(b\left(b^3c-ab^2d+\right.\right.}{}$$

$$= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{\left(b^{2/3}\left(b^3c-ab^2d\right.\right.}{}$$

$$= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{\sqrt[3]{b}\left(b^3c-ab^2d\right.}{}$$

$$= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{\sqrt[3]{b}\left(b^3c-ab^2d\right.}{}$$

$$= -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{\sqrt[3]{b}\left(b^3c-ab^2d\right.}{}$$

Mathematica [A] time = 0.13, size = 266, normalized size = 0.96

$$\frac{\frac{60a^{7/3}(bc-ad)}{x^7}-\frac{42a^{10/3}c}{x^{10}}-\frac{105a^{4/3}\left(a^2e-abd+b^2c\right)}{x^4}+\frac{420\sqrt[3]{a}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{x}+140\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)-140\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)+70\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{420a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

```
[Out] ((-42*a^(10/3)*c)/x^10 + (60*a^(7/3)*(b*c - a*d))/x^7 - (105*a^(4/3)*(b^2*c
 - a*b*d + a^2*e))/x^4 + (420*a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/
x - 140*Sqrt[3]*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*a^(13/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)), x]
```

fricas [A] time = 0.43, size = 262, normalized size = 0.95

$$\frac{140 \sqrt{3} \left(b^3 c - a b^2 d + a^2 b e - a^3 f\right) x^{10} \left(\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3} \sqrt{3} x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3} \sqrt{3}\right) + 70 \left(b^3 c - a b^2 d + a^2 b e - a^3 f\right) x^{10} \left(\frac{b}{a}\right)^{\frac{1}{3}} \log\left(b x^2 - a x \left(\frac{b}{a}\right)^{\frac{2}{3}} + a \left(\frac{b}{a}\right)^{\frac{1}{3}}\right) - 140 \left(b^3 c - a b^2 d + a^2 b e - a^3 f\right) x^{10} \left(\frac{b}{a}\right)^{\frac{1}{3}} \log\left(b x + a \left(\frac{b}{a}\right)^{\frac{2}{3}}\right) + 420 \left(b^3 c - a b^2 d + a^2 b e - a^3 f\right) x^9 - 105 \left(a b^2 c - a^2 b d + a^3 e\right) x^6 - 42 a^3 c + 60 \left(a^2 b c - a^3 d\right) x^3}{420 a^4 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/420*(140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*arc
tan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*x^10*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) -
140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*log(b*x + a*(b/a)^
(2/3)) + 420*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 105*(a*b^2*c - a^2*b
*d + a^3*e)*x^6 - 42*a^3*c + 60*(a^2*b*c - a^3*d)*x^3)/(a^4*x^10)
```

giac [A] time = 0.19, size = 376, normalized size = 1.36

$$-\frac{\left(b^4 c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a b^3 d \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3 b f \left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2 b^2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} e\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 a^5} - \frac{\sqrt{3} \left(\left(-a b^2\right)^{\frac{2}{3}} b^3 c - \left(-a b^2\right)^{\frac{2}{3}} a b^2 d - \left(-a b^2\right)^{\frac{2}{3}} a^3 f + \left(-a b^2\right)^{\frac{2}{3}} a^2 b e\right) \arctan\left(\frac{\sqrt{3} \left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 a^5 b} + \frac{\left(\left(-a b^2\right)^{\frac{2}{3}} b^3 c - \left(-a b^2\right)^{\frac{2}{3}} a b^2 d - \left(-a b^2\right)^{\frac{2}{3}} a^3 f + \left(-a b^2\right)^{\frac{2}{3}} a^2 b e\right) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 a^5 b} + \frac{140 b^3 c x^9 - 140 a b^2 d x^9 - 140 a^3 f x^9 + 140 a^2 b x^9 e - 35 a b^2 c x^6 + 35 a^2 b d x^6 - 35 a^3 x^6 e + 20 a^2 b c x^3 - 20 a^3 d x^3 - 14 a^3 c}{140 a^4 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(b^4*c*(-a/b)^(1/3) - a*b^3*d*(-a/b)^(1/3) - a^3*b*f*(-a/b)^(1/3) + a^
2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 1/3*sqr
t(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f
+ (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^5*b) + 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2
)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^5*b) + 1/140*(140*b^3*c*x^9 - 140*a*b^2*d*x^9 - 140*a^3*f*x^9 + 14
0*a^2*b*x^9*e - 35*a*b^2*c*x^6 + 35*a^2*b*d*x^6 - 35*a^3*x^6*e + 20*a^2*b*c
*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^4*x^10)
```

maple [B] time = 0.06, size = 491, normalized size = 1.77

$$-\frac{\sqrt{3}\, f \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a} + \frac{f \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a} - \frac{f \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} a} + \frac{\sqrt{3}\, b e \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^2} - \frac{b e \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^2} + \frac{b e \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^2} - \frac{\sqrt{3}\, b^2 d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} + \frac{b^2 d \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} - \frac{b^2 d \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} + \frac{\sqrt{3}\, b^3 c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^4} - \frac{b^3 c \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^4} + \frac{b^3 c \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^4} - \frac{f}{a x} + \frac{b e}{a^2 x} - \frac{b^2 d}{a^3 x} + \frac{b^3 c}{a^4 x} - \frac{e}{4 a x^4} + \frac{b d}{4 a^2 x^4} - \frac{b^2 c}{4 a^3 x^4} - \frac{d}{7 a x^7} + \frac{b c}{7 a^2 x^7} - \frac{c}{10 a x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x)
```

```
[Out] 1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-1/3/a^2*b/(a/b)^(1/3)*ln(x+(a/b)^(1/3
))*e+1/3/a^3*b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-1/3/a^4*b^3/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))*c-1/6/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f+1/6
/a^2*b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-1/6/a^3*b^2/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/6/a^4*b^3/(a/b)^(1/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))*c-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))*f+1/3/a^2*b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*e-1/3/a^3*b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))*d+1/3/a^4*b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*c-1/10*c/a/x^10-1/7/a/x^7*d+1/7/a^2/x^7*b*c-1/4/a/x^4*e+1/4/a^2/
x^4*b*d-1/4/a^3/x^4*b^2*c-1/a/x*f+1/a^2/x*b*e-1/a^3/x*b^2*d+1/a^4/x*b^3*c
```

maxima [A] time = 3.05, size = 260, normalized size = 0.94

$$\frac{\sqrt{3} \left(b^3 c - a b^2 d + a^2 b e - a^3 f\right) \arctan\left(\frac{\sqrt{3} \left(2 x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 a^4 \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(b^3 c - a b^2 d + a^2 b e - a^3 f\right) \log\left(x^2 - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 a^4 \left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(b^3 c - a b^2 d + a^2 b e - a^3 f\right) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 a^4 \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{140 \left(b^3 c - a b^2 d + a^2 b e - a^3 f\right) x^9 - 35 \left(a b^2 c - a^2 b d + a^3 e\right) x^6 - 14 a^3 c + 20 \left(a^2 b c - a^3 d\right) x^3}{140 a^4 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a^4*(a/b)^(1/3)) + 1/6*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(1/3)) - 1/3*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(1/3)) +
1/140*(140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 35*(a*b^2*c - a^2*b*d
+ a^3*e)*x^6 - 14*a^3*c + 20*(a^2*b*c - a^3*d)*x^3)/(a^4*x^10)
```

mupad [**B**] time = 5.33, size = 253, normalized size = 0.91

$$-\frac{\frac{c}{10\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{7\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{4\,a^3}}{x^{10}}-\frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{13/3}}+\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{13/3}}-\frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x)
```

```
[Out] (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(13/3)) - (b^(1/3)*log(b^(1/
3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(13/3)) - (c/(10*
a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4 + (x^3*(a*d - b*c))/(7*a
^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(4*a^3))/x^10 - (b^(1/3)*log(3^(1/2)*a^
(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a
*b^2*d + a^2*b*e))/(3*a^(13/3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.194 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$

Optimal. Leaf size=280

$$\frac{bc-ad}{8a^2x^8}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{14/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Rubi [A] time = 0.20, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{2a^4x^2}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{14/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{14/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}\,a^{14/3}}-\frac{a^2e-abd+b^2c}{5a^3x^5}+\frac{bc-ad}{8a^2x^8}-\frac{c}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(8*a^2*x^8) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x
^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^2) - (b^(2/3)*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(14/3)) + (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
1/3) + b^(1/3)*x])/(3*a^(14/3)) - (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{12}} + \frac{-bc+ad}{a^2x^9} + \frac{b^2c-abd+a^2e}{a^3x^6} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^3} - \frac{b\left(-b^3c+ab\right.}{a^4\left(\right.}\right.$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{\left(b\left(b^3c-ab^2d+\right.\right.}{}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{\left(b\left(b^3c-ab^2d+\right.\right.}{}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{b^{2/3}\left(b^3c-ab^2d\right.}{}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{b^{2/3}\left(b^3c-ab^2d\right.}{}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} - \frac{b^{2/3}\left(b^3c-ab^2d\right.}{}$$

Mathematica [A] time = 0.16, size = 266, normalized size = 0.95

$$\frac{\frac{165a^{8/3}(bc-ad)}{x^8} - \frac{120a^{11/3}c}{x^{11}} - \frac{264a^{5/3}\left(a^2e-abd+b^2c\right)}{x^5} + 440b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right) - 440\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right) + \frac{660a^{2/3}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{x^2} + 220b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{1320a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

```
[Out] ((-120*a^(11/3)*c)/x^11 + (165*a^(8/3)*(b*c - a*d))/x^8 - (264*a^(5/3)*(b^2
*c - a*b*d + a^2*e))/x^5 + (660*a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/x^2 - 440*Sqrt[3]*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*
e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*a^(14/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)), x]
```

fricas [A] time = 0.43, size = 295, normalized size = 1.05

$$-\frac{440\,\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)x^{11}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-220\left(b^3c-ab^2d+a^2be-a^3f\right)x^{11}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+440\left(b^3c-ab^2d+a^2be-a^3f\right)x^{11}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-660\left(b^3c-ab^2d+a^2be-a^3f\right)x^9+264\left(ab^2c-a^2bd+a^3e\right)x^6+120\,a^3c-165\left(a^2bc-a^3d\right)x^3}{1320\,a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 220*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b
^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 440*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^11*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 660*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^9 + 264*(a*b^2*c - a^2*b*d + a^3*e)*x^6 + 120*a^3
*c - 165*(a^2*b*c - a^3*d)*x^3)/(a^4*x^11)
```

giac [A] time = 0.19, size = 338, normalized size = 1.21

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d-\left(-ab^2\right)^{\frac{1}{3}}a^3f+\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^5}-\frac{\left(b^4c-ab^3d-a^3bf+a^2b^2e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^5}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d-\left(-ab^2\right)^{\frac{1}{3}}a^3f+\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^5}+\frac{220\,b^3cx^9-220\,ab^2dx^9-220\,a^3fx^9+220\,a^2bx^9e-88\,ab^2cx^6+88\,a^2bdx^6-88\,a^3x^6e+55\,a^2bcx^3-55\,a^3dx^3-40\,a^3c}{440\,a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)
*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/a^5 - 1/3*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*(-a/b)^(1/3)*
log(abs(x - (-a/b)^(1/3)))/a^5 + 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)
*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/a^5 + 1/440*(220*b^3*c*x^9 - 220*a*b^2*d*x^9 - 220*
a^3*f*x^9 + 220*a^2*b*x^9*e - 88*a*b^2*c*x^6 + 88*a^2*b*d*x^6 - 88*a^3*x^6*
e + 55*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^4*x^11)
```

maple [B] time = 0.05, size = 493, normalized size = 1.76

$$-\frac{\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{\sqrt{3}\,be\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{be\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{be\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{\sqrt{3}\,b^2d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{b^2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}+\frac{b^2d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}+\frac{\sqrt{3}\,b^3c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4}+\frac{b^3c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4}-\frac{b^3c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4}-\frac{f}{2a\,x^2}+\frac{be}{2a^2x^2}-\frac{b^2d}{2a^3x^2}+\frac{b^3c}{2a^4x^2}-\frac{e}{5a\,x^5}+\frac{bd}{5a^2x^5}-\frac{b^2c}{5a^3x^5}-\frac{d}{8a\,x^8}+\frac{bc}{8a^2x^8}-\frac{c}{11a\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x)
```

```
[Out] -1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f+1/3/a^2*b/(a/b)^(2/3)*ln(x+(a/b)^(1/
3))*e-1/3/a^3*b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+1/3/a^4*b^3/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))*c+1/6/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-1/
6/a^2*b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/6/a^3*b^2/(a/b)^(
2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d-1/6/a^4*b^3/(a/b)^(2/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))*c-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))*f+1/3/a^2*b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*e-1/3/a^3*b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*d+1/3/a^4*b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))*c-1/11/a*c/x^11-1/8/a/x^8*d+1/8/a^2/x^8*b*c-1/5/a/x^5*e+1/5/a^2
/x^5*b*d-1/5/a^3/x^5*b^2*c-1/2/a/x^2*f+1/2/a^2/x^2*b*e-1/2/a^3/x^2*b^2*d+1/
2/a^4/x^2*b^3*c
```

maxima [A] time = 2.97, size = 260, normalized size = 0.93

$$\frac{\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{220\left(b^3c-ab^2d+a^2be-a^3f\right)x^9-88\left(ab^2c-a^2bd+a^3e\right)x^6-40\,a^3c+55\left(a^2bc-a^3d\right)x^3}{440\,a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a^4*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) + 1/3*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3)) +
1/440*(220*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 88*(a*b^2*c - a^2*b*d
+ a^3*e)*x^6 - 40*a^3*c + 55*(a^2*b*c - a^3*d)*x^3)/(a^4*x^11)
```

mupad [B] time = 5.15, size = 253, normalized size = 0.90

$$\frac{b^{2/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{14/3}}-\frac{\frac{c}{11\,a}-\frac{x^9\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{2\,a^4}+\frac{x^3\,(a\,d-b\,c)}{8\,a^2}+\frac{x^6\left(e\,a^2-d\,a\,b+c\,b^2\right)}{5\,a^3}}{x^{11}}+\frac{b^{2/3}\ln\left(2\,b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{14/3}}-\frac{b^{2/3}\ln\left(a^{1/3}-2\,b^{1/3}x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a
^(14/3)) - (c/(11*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(2*a^4) +
(x^3*(a*d - b*c))/(8*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(5*a^3))/x^11 + (
b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(14/3)) - (b^(2/3)*log(3^(1/2
)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f
 - a*b^2*d + a^2*b*e))/(3*a^(14/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.195 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx$

Optimal. Leaf size=313

$$\frac{bc-ad}{10a^2x^{10}}-\frac{a^2e-abd+b^2c}{7a^3x^7}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{16/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Rubi [A] time = 0.24, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{4a^4x^4}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{16/3}}-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^5x}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{16/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}\,a^{16/3}}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{bc-ad}{10a^2x^{10}}-\frac{c}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

```
[Out] -c/(13*a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*a^3
*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(16/3)) +
(b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a
^(16/3)) - (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{14}} + \frac{-bc+ad}{a^2x^{11}} + \frac{b^2c-abd+a^2e}{a^3x^8} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^5} - \frac{b\left(-b^3c+ab\right.}{}\right.$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b\left(b^3c-ab^2d+\right.}{a^5x}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b\left(b^3c-ab^2d+\right.}{a^5x}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b\left(b^3c-ab^2d+\right.}{a^5x}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b\left(b^3c-ab^2d+\right.}{a^5x}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b\left(b^3c-ab^2d+\right.}{a^5x}$$

Mathematica [A] time = 0.13, size = 308, normalized size = 0.98

$$\frac{bc-ad}{10a^2x^{10}} - \frac{a^2e-abd+b^2c}{7a^3x^7} + \frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{6a^{16/3}} + \frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{16/3}} + \frac{b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{16/3}} + \frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5x} + \frac{a^3(-f)+a^2be-ab^2d+b^3c}{4a^4x^4} - \frac{c}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

```
[Out] -1/13*c/(a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*a
^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) + (b*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(16/3)) + (
b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^
(16/3)) + (b^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)), x]

fricas [A] time = 0.43, size = 317, normalized size = 1.01

$$-\frac{1820\sqrt{3}\left(b^{4}c - ab^{3}d + a^{2}b^{2}e - a^{3}bf\right)x^{13}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - 910\left(b^{4}c - ab^{3}d + a^{2}b^{2}e - a^{3}bf\right)x^{13}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2} - ax\left(-\frac{b}{a}\right)^{\frac{2}{3}} - a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right) + 1820\left(b^{4}c - ab^{3}d + a^{2}b^{2}e - a^{3}bf\right)x^{13}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx + a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right) + 5460\left(b^{4}c - ab^{3}d + a^{2}b^{2}e - a^{3}bf\right)x^{12} - 1365\left(ab^{3}c - a^{2}b^{2}d + a^{3}be - a^{4}f\right)x^{9} + 780\left(a^{2}b^{2}c - a^{3}bd + a^{4}e\right)x^{6} + 420a^{4}c - 546\left(a^{3}bc - a^{4}d\right)x^{3}}{5460a^{5}x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="fricas")

[Out] -1/5460*(1820*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 1820*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 5460*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 1365*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 780*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 420*a^4*c - 546*(a^3*b*c - a^4*d)*x^3)/(a^5*x^13)

giac [A] time = 0.18, size = 419, normalized size = 1.34

$$\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c - \left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d - \left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f + \left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^{6}} + \frac{\left(b^{5}c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^{4}d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^{3}b^{2}f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^{2}b^{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^{6}} - \frac{\left(\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c - \left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}d - \left(-ab^{2}\right)^{\frac{2}{3}}a^{3}f + \left(-ab^{2}\right)^{\frac{2}{3}}a^{2}be\right)\log\left(x^{2} + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^{6}} - \frac{1820b^{4}cx^{12} - 1820ab^{3}dx^{12} - 1820a^{3}bfx^{12} + 1820a^{2}b^{2}x^{12}e - 455ab^{3}cx^{9} + 455a^{2}b^{2}dx^{9} + 455a^{4}fx^{9} - 455a^{3}bx^{9}e + 260a^{2}b^{2}cx^{6} - 260a^{3}bdx^{6} + 260a^{4}x^{6}e - 182a^{3}bcx^{3} + 182a^{4}dx^{3} + 140a^{4}c}{1820a^{5}x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="giac")

[Out] 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^6 + 1/3*(b^5*c*(-a/b)^(1/3) - a*b^4*d*(-a/b)^(1/3) - a^3*b^2*f*(-a/b)^(1/3) + a^2*b^3*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 - 1/1820*(1820*b^4*c*x^12 - 1820*a*b^3*d*x^12 - 1820*a^3*b*f*x^12 + 1820*a^2*b^2*x^12*e - 455*a*b^3*c*x^9 + 455*a^2*b^2*d*x^9 + 455*a^4*f*x^9 - 455*a^3*b*x^9*e + 260*a^2*b^2*c*x^6 - 260*a^3*b*d*x^6 + 260*a^4*x^6*e - 182*a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^5*x^13)

maple [B] time = 0.05, size = 546, normalized size = 1.74

$$\frac{\sqrt{3}\,bf\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}} - \frac{bf\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}} + \frac{bf\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}} - \frac{\sqrt{3}\,b^{2}e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}} + \frac{b^{2}e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}} - \frac{b^{2}e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}} + \frac{\sqrt{3}\,b^{3}d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}} - \frac{b^{3}d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}} + \frac{b^{3}d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}} - \frac{\sqrt{3}\,b^{4}c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{5}} + \frac{b^{4}c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{5}} - \frac{b^{4}c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{5}} + \frac{bf}{a^{2}x} - \frac{b^{2}e}{a^{3}x} + \frac{b^{3}d}{a^{4}x} - \frac{b^{4}c}{a^{5}x} - \frac{f}{4ax^{4}} + \frac{be}{4a^{2}x^{4}} - \frac{b^{2}d}{4a^{3}x^{4}} + \frac{b^{3}c}{4a^{4}x^{4}} - \frac{e}{7ax^{7}} + \frac{bd}{7a^{2}x^{7}} - \frac{b^{2}c}{7a^{3}x^{7}} - \frac{d}{10ax^{10}} + \frac{bc}{10a^{2}x^{10}} - \frac{c}{13ax^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x)

[Out] 1/3/a^2*b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3/a^3*b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3/a^4*b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3/a^5*b^4*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/7/a^3/x^7*b^2*c+1/a^4*b^3/x*d-1/a^5*b^4/x*c+1/a^2*b/x*f-1/a^3*b^2/x*e+1/4/a^2/x^4*b*e-1/4/a^3/x^4*b^2*d+1/4/a^4/x^4*b^3*c+1/10/a^2/x^10*b*c+1/7/a^2/x^7*b*d-1/13*c/a/x^13-1/7/a/x^7*e-1/10/a/x^10*d+1/6/a^4*b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d-1/6/a^5*b^4/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c-1/3/a^2*b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f+1/3/a^3*b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-1/3/a^4*b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d+1/3/a^5*b^4/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/6/a^2*b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-1/6/a^3*b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-1/4/a/x^4*f

maxima [A] time = 2.99, size = 307, normalized size = 0.98

$$-\frac{\sqrt{3}\left(b^{4}c - ab^{3}d + a^{2}b^{2}e - a^{3}bf\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(b^{4}c - ab^{3}d + a^{2}b^{2}e - a^{3}bf\right)\log\left(x^{2} - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(b^{4}c - ab^{3}d + a^{2}b^{2}e - a^{3}bf\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{1820\left(b^{4}c - ab^{3}d + a^{2}b^{2}e - a^{3}bf\right)x^{12} - 455\left(ab^{3}c - a^{2}b^{2}d + a^{3}be - a^{4}f\right)x^{9} + 260\left(a^{2}b^{2}c - a^{3}bd + a^{4}e\right)x^{6} + 140a^{4}c - 182\left(a^{3}bc - a^{4}d\right)x^{3}}{1820a^{5}x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(1/3)) - 1/6*(b^4*c - a*b^3*d + a^
2*b^2*e - a^3*b*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(1/3))
 + 1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x + (a/b)^(1/3))/(a^5*(a
/b)^(1/3)) - 1/1820*(1820*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 45
5*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 260*(a^2*b^2*c - a^3*b*d +
a^4*e)*x^6 + 140*a^4*c - 182*(a^3*b*c - a^4*d)*x^3)/(a^5*x^13)
```

mupad [**B**] time = 5.23, size = 286, normalized size = 0.91

$$\frac{b^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}-\frac{\frac{c}{13\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{4\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{10\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{7\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^5}}{x^{13}}-\frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}+\frac{b^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a
^(16/3)) - (c/(13*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(4*a^4) +
(x^3*(a*d - b*c))/(10*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(7*a^3) + (b*x^1
2*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^5)/x^13 - (b^(4/3)*log(3^(1/2)*a^(
1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*
b^2*d + a^2*b*e))/(3*a^(16/3)) + (b^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3
)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/
(3*a^(16/3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.196 $\int \frac{c+dx^3+ex^6+fx^9}{x^{15}\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=315

$$\frac{bc-ad}{11a^2x^{11}}-\frac{a^2e-abd+b^2c}{8a^3x^8}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{17/3}}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Rubi [A] time = 0.23, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^5x^2}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{5a^4x^5}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{17/3}}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{17/3}}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}a^{17/3}}-\frac{a^2e-abd+b^2c}{8a^3x^8}+\frac{bc-ad}{11a^2x^{11}}-\frac{c}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

```
[Out] -c/(14*a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*a^3
*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(17/3)
) - (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{15}} + \frac{-bc+ad}{a^2x^{12}} + \frac{b^2c-abd+a^2e}{a^3x^9} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^6} - \frac{b\left(-b^3c+ab\right.}{}\right.$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+\right.}{2a^5x^2}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+\right.}{2a^5x^2}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+\right.}{2a^5x^2}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+\right.}{2a^5x^2}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+\right.}{2a^5x^2}$$

Mathematica [A] time = 0.15, size = 311, normalized size = 0.99

$$\frac{bc-ad}{11a^2x^{11}} - \frac{a^2e-abd+b^2c}{8a^3x^8} + \frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{17/3}} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^{17/3}} + \frac{b^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{17/3}} + \frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^5x^2} + \frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5} - \frac{c}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

```
[Out] -1/14*c/(a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*a
^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(17/3))
 + (b^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)), x]
```

fricas [A] time = 0.43, size = 335, normalized size = 1.06

$$-\frac{3080\,\sqrt{3}\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)x^{14}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-1540\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)x^{14}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}-abx\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)+3080\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)x^{14}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)+4620\left(b^{4}c-ab^{3}d+a^{2}b^{2}e-a^{3}bf\right)x^{12}-1848\left(ab^{3}c-a^{2}b^{2}d+a^{3}be-a^{4}f\right)x^{9}+1155\left(a^{2}b^{2}c-a^{3}bd+a^{4}e\right)x^{6}+660\,a^{4}c-840\left(a^{3}bc-a^{4}d\right)x^{3}}{9240\,a^{5}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/9240*(3080*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2
)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 1540*(b
^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*
b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 3080*(b^4*c - a*b^3*d + a^2*b^
2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 4620*(b^
4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 1848*(a*b^3*c - a^2*b^2*d + a^3
*b*e - a^4*f)*x^9 + 1155*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 660*a^4*c - 84
0*(a^3*b*c - a^4*d)*x^3)/(a^5*x^14)
```

giac [A] time = 0.19, size = 393, normalized size = 1.25

$$-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{4}c-\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}d-\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}bf+\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{2}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{6}}+\frac{\left(b^{5}c-ab^{4}d-a^{3}b^{2}f+a^{2}b^{3}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{6}}-\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{4}c-\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}d-\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}bf+\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{2}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{6}}-\frac{1540\,b^{4}cx^{12}-1540\,ab^{3}dx^{12}-1540\,a^{3}bfx^{12}+1540\,a^{2}b^{2}x^{12}e-616\,ab^{3}cx^{9}+616\,a^{2}b^{2}dx^{9}+616\,a^{4}fx^{9}-616\,a^{3}bx^{9}e+385\,a^{2}b^{2}cx^{6}-385\,a^{3}bdx^{6}+385\,a^{4}x^{6}e-280\,a^{3}bcx^{3}+280\,a^{4}dx^{3}+220\,a^{4}c}{3080\,a^{5}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^4*c - (-a*b^2)^(1/3)*a*b^3*d - (-a*b^2)^(1/3
)*a^3*b*f + (-a*b^2)^(1/3)*a^2*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/a^6 + 1/3*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 - 1/6*((-a*b^2)^(1/3)*b^4*c - (-a*b^2
)^(1/3)*a*b^3*d - (-a*b^2)^(1/3)*a^3*b*f + (-a*b^2)^(1/3)*a^2*b^2*e)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 - 1/3080*(1540*b^4*c*x^12 - 1540*a*b
^3*d*x^12 - 1540*a^3*b*f*x^12 + 1540*a^2*b^2*x^12*e - 616*a*b^3*c*x^9 + 616
*a^2*b^2*d*x^9 + 616*a^4*f*x^9 - 616*a^3*b*x^9*e + 385*a^2*b^2*c*x^6 - 385*
a^3*b*d*x^6 + 385*a^4*x^6*e - 280*a^3*b*c*x^3 + 280*a^4*d*x^3 + 220*a^4*c)/
(a^5*x^14)
```

maple [B] time = 0.06, size = 548, normalized size = 1.74

$$\frac{\sqrt{3}\,bf\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{bf\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{bf\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{\sqrt{3}\,b^{2}e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{b^{2}e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{b^{2}e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{\sqrt{3}\,b^{3}d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}+\frac{b^{3}d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}-\frac{b^{3}d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}-\frac{\sqrt{3}\,b^{4}c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{5}}-\frac{b^{4}c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{5}}+\frac{b^{4}c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{5}}+\frac{bf}{2a^{2}x^{2}}-\frac{b^{2}e}{2a^{3}x^{2}}+\frac{b^{3}d}{2a^{4}x^{2}}-\frac{b^{4}c}{2a^{5}x^{2}}-\frac{f}{5a\,x^{5}}+\frac{be}{5a^{2}x^{5}}-\frac{b^{2}d}{5a^{3}x^{5}}+\frac{b^{3}c}{5a^{4}x^{5}}-\frac{e}{8a\,x^{8}}+\frac{bd}{8a^{2}x^{8}}-\frac{b^{2}c}{8a^{3}x^{8}}-\frac{d}{11a\,x^{11}}+\frac{bc}{11a^{2}x^{11}}-\frac{c}{14a\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x)
```

```
[Out] 1/3/a^2*b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3
/a^3*b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/3/
a^4*b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-1/3/a
^5*b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/2/a^
5*b^4/x^2*c+1/11/a^2/x^11*b*c+1/8/a^2/x^8*b*d-1/8/a^3/x^8*b^2*c+1/5/a^2/x^5
*b*e+1/2/a^2*b/x^2*f-1/2/a^3*b^2/x^2*e+1/2/a^4*b^3/x^2*d-1/5/a^3/x^5*b^2*d+
1/5/a^4/x^5*b^3*c-1/11/a/x^11*d-1/14*c/a/x^14-1/3/a^5*b^4/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))*c-1/8/a/x^8*e-1/6/a^4*b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))*d+1/6/a^5*b^4/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/3
/a^2*b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-1/3/a^3*b^2/(a/b)^(2/3)*ln(x+(a/b)^(
1/3))*e-1/5/a/x^5*f-1/6/a^2*b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
*f+1/6/a^3*b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/3/a^4*b^3/
(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d
```

maxima [A] time = 3.11, size = 307, normalized size = 0.97

$$-\frac{\sqrt{3}\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{1540\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^{12}-616\left(ab^3c-a^2b^2d+a^3be-a^4f\right)x^9+385\left(a^2b^2c-a^3bd+a^4e\right)x^6+220a^4c-280\left(a^3bc-a^4d\right)x^3}{3080a^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(2/3)) + 1/6*(b^4*c - a*b^3*d + a^
2*b^2*e - a^3*b*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(2/3))
 - 1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x + (a/b)^(1/3))/(a^5*(a
/b)^(2/3)) - 1/3080*(1540*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 61
6*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 385*(a^2*b^2*c - a^3*b*d +
a^4*e)*x^6 + 220*a^4*c - 280*(a^3*b*c - a^4*d)*x^3)/(a^5*x^14)
```

mupad [B] time = 5.17, size = 287, normalized size = 0.91

$$-\frac{\frac{c}{14a}-\frac{x^9\left(-fa^3+ea^2b-dab^2+cb^3\right)}{5a^4}+\frac{x^3\left(ad-bc\right)}{11a^2}+\frac{x^6\left(ea^2-dab+cb^2\right)}{8a^3}+\frac{bx^{12}\left(-fa^3+ea^2b-dab^2+cb^3\right)}{2a^5}}{x^{14}}-\frac{b^{5/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3a^{17/3}}-\frac{b^{5/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3a^{17/3}}+\frac{b^{5/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3a^{17/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x)
```

```
[Out] (b^(5/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (b^(5/3)*log(b^(1/
3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (b^(5/3
)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (c/(14*a) - (x^9*(b^3*c -
 a^3*f - a*b^2*d + a^2*b*e))/(5*a^4) + (x^3*(a*d - b*c))/(11*a^2) + (x^6*(b
^2*c + a^2*e - a*b*d))/(8*a^3) + (b*x^12*(b^3*c - a^3*f - a*b^2*d + a^2*b*e
))/(2*a^5))/x^14
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**15/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.197 $\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx$

Optimal. Leaf size=351

$$\frac{bc-ad}{13a^2x^{13}}-\frac{a^2e-abd+b^2c}{10a^3x^{10}}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{19/3}}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Rubi [A] time = 0.26, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^5x^4}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{7a^4x^7}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{19/3}}+\frac{b^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^6x}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{19/3}}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{3}a^{19/3}}-\frac{a^2e-abd+b^2c}{10a^3x^{10}}+\frac{bc-ad}{13a^2x^{13}}-\frac{c}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

```
[Out] -c/(16*a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10*a^
3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) - (b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f))/(a^6*x) - (b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(19/3)) - (b^(7/3)*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (b^(7/
3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(6*a^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{17}} + \frac{-bc+ad}{a^2x^{14}} + \frac{b^2c-abd+a^2e}{a^3x^{11}} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^8} - \frac{b\left(-b^3c+ab\right.}{}\right.$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b\left(b^3c-ab^2d+\right.}{4a^5x}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b\left(b^3c-ab^2d+\right.}{4a^5x}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b\left(b^3c-ab^2d+\right.}{4a^5x}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b\left(b^3c-ab^2d+\right.}{4a^5x}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b\left(b^3c-ab^2d+\right.}{4a^5x}$$

Mathematica [A] time = 0.15, size = 346, normalized size = 0.99

$$\frac{bc-ad}{13a^2x^{13}} - \frac{a^2e-abd+b^2c}{10a^3x^{10}} + \frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{19/3}} + \frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^{19/3}} + \frac{b^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{\sqrt{3}a^{19/3}} + \frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x} + \frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{4a^5x^4} + \frac{a^3(-f)+a^2be-ab^2d+b^3c}{7a^4x^7} - \frac{c}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

```
[Out] -1/16*c/(a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10*
a^3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) + (b*(-(b^3*c)
+ a*b^2*d - a^2*b*e + a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f))/(a^6*x) + (b^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(19/3)) + (b^(7/3)*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (
b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*a^(19/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)), x]
```

fricas [**A**] time = 0.43, size = 355, normalized size = 1.01

$$\frac{7280\,\sqrt{3}\left(b^{5}c-ab^{4}d+a^{2}b^{3}e-a^{3}b^{2}f\right)x^{16}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+3640\left(b^{5}c-ab^{4}d+a^{2}b^{3}e-a^{3}b^{2}f\right)x^{16}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-7280\left(b^{5}c-ab^{4}d+a^{2}b^{3}e-a^{3}b^{2}f\right)x^{16}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+21840\left(b^{5}c-ab^{4}d+a^{2}b^{3}e-a^{3}b^{2}f\right)x^{15}-5460\left(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf\right)x^{12}+3120\left(a^{2}b^{3}c-a^{3}b^{2}d+a^{4}be-a^{5}f\right)x^{9}-2184\left(a^{3}b^{2}c-a^{4}bd+a^{5}e\right)x^{6}-1365\,a^{5}c+1680\left(a^{4}bc-a^{5}d\right)x^{3}}{21840\,a^{6}x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/21840*(7280*sqrt(3)*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^
(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 3640*(b^5*c - a*b^4
*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) +
a*(b/a)^(1/3)) - 7280*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^
(1/3)*log(b*x + a*(b/a)^(2/3)) + 21840*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b
^2*f)*x^15 - 5460*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 3120*(
a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^9 - 2184*(a^3*b^2*c - a^4*b*d +
a^5*e)*x^6 - 1365*a^5*c + 1680*(a^4*b*c - a^5*d)*x^3)/(a^6*x^16)
```

giac [**A**] time = 0.19, size = 474, normalized size = 1.35

$$-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{2}{3}}b^{4}c-\left(-ab^{2}\right)^{\frac{2}{3}}ab^{3}d-\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}bf+\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b^{2}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{7}}-\frac{\left(b^{6}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{5}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}b^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}b^{4}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{7}}+\frac{\left(\left(-ab^{2}\right)^{\frac{2}{3}}b^{4}c-\left(-ab^{2}\right)^{\frac{2}{3}}ab^{3}d-\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}bf+\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b^{2}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{7}}+\frac{7280\,b^{5}cx^{15}-7280\,ab^{4}dx^{15}-7280\,a^{3}b^{2}fx^{15}+7280\,a^{2}b^{3}x^{15}e-1820\,ab^{4}cx^{12}+1820\,a^{2}b^{3}dx^{12}+1820\,a^{4}bfx^{12}-1820\,a^{3}b^{2}x^{12}e+1040\,a^{2}b^{3}cx^{9}-1040\,a^{3}b^{2}dx^{9}-1040\,a^{5}fx^{9}+1040\,a^{4}bx^{9}e-728\,a^{3}b^{2}cx^{6}+728\,a^{4}bdx^{6}-728\,a^{5}x^{6}e+560\,a^{4}bcx^{3}-560\,a^{5}dx^{3}-455\,a^{5}c}{7280\,a^{6}x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d - (-a*b^2)^(2/3
)*a^3*b*f + (-a*b^2)^(2/3)*a^2*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/a^7 - 1/3*(b^6*c*(-a/b)^(1/3) - a*b^5*d*(-a/b)^(1/3) - a^3
*b^3*f*(-a/b)^(1/3) + a^2*b^4*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/a^7 + 1/6*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d - (-a*b
^2)^(2/3)*a^3*b*f + (-a*b^2)^(2/3)*a^2*b^2*e)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/a^7 + 1/7280*(7280*b^5*c*x^15 - 7280*a*b^4*d*x^15 - 7280*a^3*b^
2*f*x^15 + 7280*a^2*b^3*x^15*e - 1820*a*b^4*c*x^12 + 1820*a^2*b^3*d*x^12 +
1820*a^4*b*f*x^12 - 1820*a^3*b^2*x^12*e + 1040*a^2*b^3*c*x^9 - 1040*a^3*b^2
*d*x^9 - 1040*a^5*f*x^9 + 1040*a^4*b*x^9*e - 728*a^3*b^2*c*x^6 + 728*a^4*b*
d*x^6 - 728*a^5*x^6*e + 560*a^4*b*c*x^3 - 560*a^5*d*x^3 - 455*a^5*c)/(a^6*x
^16)
```

maple [**A**] time = 0.06, size = 600, normalized size = 1.71

$$-\frac{\sqrt{3}\,b^{2}f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{b^{2}f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}-\frac{b^{2}f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{\sqrt{3}\,b^{3}e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}}-\frac{b^{3}e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}}+\frac{b^{3}e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}}-\frac{\sqrt{3}\,b^{4}d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{5}}+\frac{b^{4}d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{5}}-\frac{b^{4}d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{5}}+\frac{\sqrt{3}\,b^{5}c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{6}}-\frac{b^{5}c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{6}}+\frac{b^{5}c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{6}}-\frac{b^{2}f}{a^{3}x}+\frac{b^{3}e}{a^{4}x}-\frac{b^{4}d}{a^{5}x}+\frac{b^{5}c}{a^{6}x}+\frac{bf}{4a^{2}x^{4}}-\frac{b^{2}e}{4a^{3}x^{4}}+\frac{b^{3}d}{4a^{4}x^{4}}-\frac{b^{4}c}{4a^{5}x^{4}}-\frac{f}{7a\,x^{7}}+\frac{be}{7a^{2}x^{7}}-\frac{b^{2}d}{7a^{3}x^{7}}+\frac{b^{3}c}{7a^{4}x^{7}}-\frac{e}{10a\,x^{10}}+\frac{bd}{10a^{2}x^{10}}-\frac{b^{2}c}{10a^{3}x^{10}}-\frac{d}{13a\,x^{13}}+\frac{bc}{13a^{2}x^{13}}-\frac{c}{16a\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x)
```

```
[Out] -1/10/a^3/x^10*b^2*c+1/7/a^2/x^7*b*e-1/7/a^3/x^7*b^2*d+1/7/a^4/x^7*b^3*c-1/
a^3*b^2/x*f+1/a^4*b^3/x*e-1/a^5*b^4/x*d+1/a^6*b^5/x*c+1/4/a^2*b/x^4*f-1/4/a
^3*b^2/x^4*e+1/4/a^4*b^3/x^4*d-1/4/a^5*b^4/x^4*c+1/13/a^2/x^13*b*c+1/10/a^2
/x^10*b*d-1/16*c/a/x^16-1/3/a^3*b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))*f+1/3/a^4*b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))*e-1/3/a^5*b^4*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*d+1/3/a^6*b^5*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))*c-1/7/a/x^7*f-1/13/a/x^13*d-1/3/a^4*b^3/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))*e+1/3/a^5*b^4/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-1/10/a/x^10*e+1/
6/a^6*b^5/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c-1/3/a^6*b^5/(a/b)
^(1/3)*ln(x+(a/b)^(1/3))*c-1/6/a^3*b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))*f+1/6/a^4*b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-1/6
```

```
/a^5*b^4/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/3/a^3*b^2/(a/b)^
(1/3)*ln(x+(a/b)^(1/3))*f
```

maxima [A] time = 3.03, size = 353, normalized size = 1.01

$$\frac{\sqrt{3}\left(b^5c - ab^4d + a^2b^3e - a^3b^2f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(b^5c - ab^4d + a^2b^3e - a^3b^2f\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(b^5c - ab^4d + a^2b^3e - a^3b^2f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{7280\left(b^5c - ab^4d + a^2b^3e - a^3b^2f\right)x^{15} - 1820\left(ab^4c - a^2b^3d + a^3b^2e - a^4bf\right)x^{12} + 1040\left(a^2b^3c - a^3b^2d + a^4be - a^5f\right)x^9 - 728\left(a^3b^2c - a^4bd + a^5e\right)x^6 - 455a^5c + 560\left(a^4bc - a^5d\right)x^3}{7280a^6x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^6*(a/b)^(1/3)) + 1/6*(b^5*c - a*b^4*d + a
^2*b^3*e - a^3*b^2*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/
3)) - 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(x + (a/b)^(1/3))/(a
^6*(a/b)^(1/3)) + 1/7280*(7280*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^
15 - 1820*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 1040*(a^2*b^3*
c - a^3*b^2*d + a^4*b*e - a^5*f)*x^9 - 728*(a^3*b^2*c - a^4*b*d + a^5*e)*x^
6 - 455*a^5*c + 560*(a^4*b*c - a^5*d)*x^3)/(a^6*x^16)
```

mupad [B] time = 5.16, size = 323, normalized size = 0.92

$$-\frac{\frac{c}{16a} - \frac{x^9\left(-fa^3+ea^2b-dab^2+cb^3\right)}{7a^4} + \frac{x^3\left(ad-bc\right)}{13a^2} + \frac{x^6\left(ea^2-dab+cb^2\right)}{10a^3} + \frac{bx^{12}\left(-fa^3+ea^2b-dab^2+cb^3\right)}{4a^5} - \frac{b^2x^{15}\left(-fa^3+ea^2b-dab^2+cb^3\right)}{a^6}}{x^{16}} - \frac{b^{7/3}\ln\left(b^{1/3}x + a^{1/3}\right)\left(-fa^3 + ea^2b - dab^2 + cb^3\right)}{3a^{19/3}} + \frac{b^{7/3}\ln\left(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-fa^3 + ea^2b - dab^2 + cb^3\right)}{3a^{19/3}} - \frac{b^{7/3}\ln\left(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-fa^3 + ea^2b - dab^2 + cb^3\right)}{3a^{19/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x)
```

```
[Out] (b^(7/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3)) - (b^(7/3)*log(b^(1/
3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3)) - (c/(16*
a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(7*a^4) + (x^3*(a*d - b*c))/
(13*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(10*a^3) + (b*x^12*(b^3*c - a^3*f
- a*b^2*d + a^2*b*e))/(4*a^5) - (b^2*x^15*(b^3*c - a^3*f - a*b^2*d + a^2*b*
e))/a^6)/x^16 - (b^(7/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**17/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.198 $\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=220

$$\frac{x^9\left(3a^2f-2abe+b^2d\right)}{9b^4}+\frac{a^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a^2\log\left(a+bx^3\right)\left(-6a^3f+5a^2be-4ab^2d+3b^3c\right)}{3b^7}-$$

Rubi [A] time = 0.34, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^6\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{6b^5}-\frac{ax^3\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a^2\log\left(a+bx^3\right)\left(5a^2be-6a^3f-4ab^2d+3b^3c\right)}{3b^7}+\frac{x^9\left(3a^2f-2abe+b^2d\right)}{9b^4}+\frac{x^{12}(be-2af)}{12b^3}+\frac{fx^{15}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] -(a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x^3)/(3*b^6) + ((b^3*c - 2*
a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6)/(6*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*
x^9)/(9*b^4) + ((b*e - 2*a*f)*x^12)/(12*b^3) + (f*x^15)/(15*b^2) + (a^3*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^7*(a + b*x^3)) + (a^2*(3*b^3*c - 4*a
*b^2*d + 5*a^2*b*e - 6*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{(a+bx)^2}dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a\left(-2b^3c+3ab^2d-4a^2be+5a^3f\right)}{b^6}+\frac{\left(b^3c-2ab^2d+3a^2be-\right.}{b^5}\right.\right.\\
&= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x^3}{3b^6}+\frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^6}{6b^5}+
\end{aligned}$$

Mathematica [A] time = 0.21, size = 205, normalized size = 0.93

$$\frac{20b^3x^9\left(3a^2f-2abe+b^2d\right)+30b^2x^6\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)+60abx^3\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)-\frac{60a^3\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}+60a^2\log\left(a+bx^3\right)\left(-6a^3f+5a^2be-4ab^2d+3b^3c\right)+15b^4x^{12}(be-2af)+12b^5fx^{15}}{180b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (60*a*b*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x^3 + 30*b^2*(b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6 + 20*b^3*(b^2*d - 2*a*b*e + 3*a^2*f)*x
^9 + 15*b^4*(b*e - 2*a*f)*x^12 + 12*b^5*f*x^15 - (60*a^3*(-(b^3*c) + a*b^2*
d - a^2*b*e + a^3*f))/(a + b*x^3) + 60*a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e
 - 6*a^3*f)*Log[a + b*x^3])/(180*b^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]
```

fricas [A] time = 0.41, size = 303, normalized size = 1.38

$$\frac{12\,b^6fx^{18}+3\left(5\,b^6e-6\,ab^5f\right)x^{15}+5\left(4\,b^6d-5\,ab^5e+6\,a^2b^4f\right)x^{12}+10\left(3\,b^6c-4\,ab^5d+5\,a^2b^4e-6\,a^3b^3f\right)x^9+60\,a^3b^3c-60\,a^4b^2d+60\,a^5be-60\,a^6f-30\left(3\,ab^5c-4\,a^2b^4d+5\,a^3b^3e-6\,a^4b^2f\right)x^6-60\left(2\,a^2b^4c-3\,a^3b^3d+4\,a^4b^2e-5\,a^5bf\right)x^3+60\left(3\,a^3b^3c-4\,a^4b^2d+5\,a^5be-6\,a^6f+\left(3\,a^2b^4c-4\,a^3b^3d+5\,a^4b^2e-6\,a^5bf\right)x^3\right)\log\left(bx^3+a\right)}{180\left(b^8x^3+ab^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^6*f*x^18 + 3*(5*b^6*e - 6*a*b^5*f)*x^15 + 5*(4*b^6*d - 5*a*b^5*
e + 6*a^2*b^4*f)*x^12 + 10*(3*b^6*c - 4*a*b^5*d + 5*a^2*b^4*e - 6*a^3*b^3*f
)*x^9 + 60*a^3*b^3*c - 60*a^4*b^2*d + 60*a^5*b*e - 60*a^6*f - 30*(3*a*b^5*c
 - 4*a^2*b^4*d + 5*a^3*b^3*e - 6*a^4*b^2*f)*x^6 - 60*(2*a^2*b^4*c - 3*a^3*b
^3*d + 4*a^4*b^2*e - 5*a^5*b*f)*x^3 + 60*(3*a^3*b^3*c - 4*a^4*b^2*d + 5*a^5
*b*e - 6*a^6*f + (3*a^2*b^4*c - 4*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)
*log(b*x^3 + a))/(b^8*x^3 + a*b^7)
```

giac [A] time = 0.25, size = 300, normalized size = 1.36

$$\frac{\left(3\,a^2b^3c-4\,a^3b^2d-6\,a^5f+5\,a^4be\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^7}-\frac{3\,a^2b^4cx^3-4\,a^3b^3dx^3-6\,a^5bfx^3+5\,a^4b^2x^3e+2\,a^3b^3c-3\,a^4b^2d-5\,a^6f+4\,a^5be}{3\left(bx^3+a\right)b^7}+\frac{12\,b^8fx^{15}-30\,ab^7fx^{12}+15\,b^8x^{12}e+20\,b^8dx^9+60\,a^2b^6fx^9-40\,ab^7x^9e+30\,b^8cx^6-60\,ab^7dx^6-120\,a^3b^5fx^6+90\,a^2b^6x^6e-120\,ab^7cx^3+180\,a^2b^6dx^3+300\,a^4b^4fx^3-240\,a^3b^5x^3e}{180\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*b^3*c - 4*a^3*b^2*d - 6*a^5*f + 5*a^4*b*e)*log(abs(b*x^3 + a))/b
^7 - 1/3*(3*a^2*b^4*c*x^3 - 4*a^3*b^3*d*x^3 - 6*a^5*b*f*x^3 + 5*a^4*b^2*x^3
*e + 2*a^3*b^3*c - 3*a^4*b^2*d - 5*a^6*f + 4*a^5*b*e)/((b*x^3 + a)*b^7) + 1
/180*(12*b^8*f*x^15 - 30*a*b^7*f*x^12 + 15*b^8*x^12*e + 20*b^8*d*x^9 + 60*a
^2*b^6*f*x^9 - 40*a*b^7*x^9*e + 30*b^8*c*x^6 - 60*a*b^7*d*x^6 - 120*a^3*b^5
*f*x^6 + 90*a^2*b^6*x^6*e - 120*a*b^7*c*x^3 + 180*a^2*b^6*d*x^3 + 300*a^4*b
^4*f*x^3 - 240*a^3*b^5*x^3*e)/b^10
```

maple [A] time = 0.06, size = 288, normalized size = 1.31

$$\frac{fx^{15}}{15b^2}-\frac{afx^{12}}{6b^3}+\frac{ex^{12}}{12b^2}+\frac{a^2fx^9}{3b^4}-\frac{2aex^9}{9b^3}+\frac{dx^9}{9b^2}-\frac{2a^3fx^6}{3b^5}+\frac{a^2ex^6}{2b^4}-\frac{adx^6}{3b^3}+\frac{cx^6}{6b^2}+\frac{5a^4fx^3}{3b^6}-\frac{4a^3ex^3}{3b^5}+\frac{a^2dx^3}{b^4}-\frac{2acx^3}{3b^3}-\frac{a^6f}{3\left(bx^3+a\right)b^7}+\frac{a^5e}{3\left(bx^3+a\right)b^6}-\frac{2a^5f\ln\left(bx^3+a\right)}{b^7}-\frac{a^4d}{3\left(bx^3+a\right)b^5}+\frac{5a^4e\ln\left(bx^3+a\right)}{3b^6}+\frac{a^3c}{3\left(bx^3+a\right)b^4}-\frac{4a^3d\ln\left(bx^3+a\right)}{3b^5}+\frac{a^2c\ln\left(bx^3+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/15*f*x^15/b^2-1/6/b^3*x^12*a*f+1/12/b^2*x^12*e+1/3/b^4*x^9*a^2*f-2/9/b^3*
x^9*a*e+1/9/b^2*x^9*d-2/3/b^5*x^6*a^3*f+1/2/b^4*x^6*a^2*e-1/3/b^3*x^6*a*d+1
/6/b^2*x^6*c+5/3/b^6*x^3*a^4*f-4/3/b^5*x^3*a^3*e+1/b^4*x^3*a^2*d-2/3/b^3*x^
3*a*c-2*a^5/b^7*ln(b*x^3+a)*f+5/3*a^4/b^6*ln(b*x^3+a)*e-4/3*a^3/b^5*ln(b*x^
```

```
3+a)*d+a^2/b^4*ln(b*x^3+a)*c-1/3*a^6/b^7/(b*x^3+a)*f+1/3*a^5/b^6/(b*x^3+a)*
e-1/3*a^4/b^5/(b*x^3+a)*d+1/3*a^3/b^4/(b*x^3+a)*c
```

maxima [A] time = 1.30, size = 222, normalized size = 1.01

$$\frac{a^3b^3c - a^4b^2d + a^5be - a^6f}{3\left(b^8x^3 + ab^7\right)} + \frac{12\,b^4fx^{15} + 15\left(b^4e - 2\,ab^3f\right)x^{12} + 20\left(b^4d - 2\,ab^3e + 3\,a^2b^2f\right)x^9 + 30\left(b^4c - 2\,ab^3d + 3\,a^2b^2e - 4\,a^3bf\right)x^6 - 60\left(2\,ab^3c - 3\,a^2b^2d + 4\,a^3be - 5\,a^4f\right)x^3}{180\,b^6} + \frac{\left(3\,a^2b^3c - 4\,a^3b^2d + 5\,a^4be - 6\,a^5f\right)\log\left(bx^3 + a\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)/(b^8*x^3 + a*b^7) + 1/180*(12
*b^4*f*x^15 + 15*(b^4*e - 2*a*b^3*f)*x^12 + 20*(b^4*d - 2*a*b^3*e + 3*a^2*b
^2*f)*x^9 + 30*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^6 - 60*(2*a*
b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x^3)/b^6 + 1/3*(3*a^2*b^3*c - 4*
a^3*b^2*d + 5*a^4*b*e - 6*a^5*f)*log(b*x^3 + a)/b^7
```

mupad [B] time = 4.99, size = 356, normalized size = 1.62

$$x^{12}\left(\frac{e}{12\,b^2} - \frac{a\,f}{6\,b^3}\right) - x^3\left(\frac{2\,a\left(\frac{c}{b^2} - \frac{a^2\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right)}{b^2} + \frac{2\,a\left(\frac{a^2f}{b^4} - \frac{d}{b^2} + \frac{2\,a\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)}{3\,b} - \frac{a^2\left(\frac{a^2f}{b^4} - \frac{d}{b^2} + \frac{2\,a\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b^2}\right) - x^9\left(\frac{a^2f}{9\,b^4} - \frac{d}{9\,b^2} + \frac{2\,a\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right)}{9\,b}\right) + x^6\left(\frac{c}{6\,b^2} - \frac{a^2\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right)}{6\,b^2} + \frac{a\left(\frac{a^2f}{b^4} - \frac{d}{b^2} + \frac{2\,a\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right) - \frac{\ln\left(b\,x^3 + a\right)\left(6\,f\,a^5 - 5\,e\,a^4\,b + 4\,d\,a^3\,b^2 - 3\,c\,a^2\,b^3\right)}{3\,b^7} + \frac{f\,x^{15}}{15\,b^2} - \frac{f\,a^6 - e\,a^5\,b + d\,a^4\,b^2 - c\,a^3\,b^3}{3\,b\left(b^7\,x^3 + a\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^12*(e/(12*b^2) - (a*f)/(6*b^3)) - x^3*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f
)/b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b
))/(3*b) - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b
^2)) - x^9*((a^2*f)/(9*b^4) - d/(9*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(9*b)
) + x^6*(c/(6*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(6*b^2) + (a*((a^2*f)/b^4
- d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b)) - (log(a + b*x^3)*(6*a^5*f
 - 3*a^2*b^3*c + 4*a^3*b^2*d - 5*a^4*b*e))/(3*b^7) + (f*x^15)/(15*b^2) - (a
^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e)/(3*b*(a*b^6 + b^7*x^3))
```

sympy [A] time = 14.42, size = 236, normalized size = 1.07

$$-\frac{a^2\left(6a^3f - 5a^2be + 4ab^2d - 3b^3c\right)\log\left(a + bx^3\right)}{3b^7} + x^{12}\left(-\frac{af}{6b^3} + \frac{e}{12b^2}\right) + x^9\left(\frac{a^2f}{3b^4} - \frac{2ae}{9b^3} + \frac{d}{9b^2}\right) + x^6\left(-\frac{2a^3f}{3b^5} + \frac{a^2e}{2b^4} - \frac{ad}{3b^3} + \frac{c}{6b^2}\right) + x^3\left(\frac{5a^4f}{3b^6} - \frac{4a^3e}{3b^5} + \frac{a^2d}{b^4} - \frac{2ac}{3b^3}\right) + \frac{-a^6f + a^5be - a^4b^2d + a^3b^3c}{3ab^7 + 3b^8x^3} + \frac{fx^{15}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -a**2*(6*a**3*f - 5*a**2*b*e + 4*a*b**2*d - 3*b**3*c)*log(a + b*x**3)/(3*b*
*7) + x**12*(-a*f/(6*b**3) + e/(12*b**2)) + x**9*(a**2*f/(3*b**4) - 2*a*e/(
9*b**3) + d/(9*b**2)) + x**6*(-2*a**3*f/(3*b**5) + a**2*e/(2*b**4) - a*d/(3
*b**3) + c/(6*b**2)) + x**3*(5*a**4*f/(3*b**6) - 4*a**3*e/(3*b**5) + a**2*d
/b**4 - 2*a*c/(3*b**3)) + (-a**6*f + a**5*b*e - a**4*b**2*d + a**3*b**3*c)/
(3*a*b**7 + 3*b**8*x**3) + f*x**15/(15*b**2)
```

3.199 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$

Optimal. Leaf size=180

$$\frac{x^6(3a^2f-2abe+b^2d)}{6b^4}-\frac{a^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6(a+bx^3)}-\frac{a\log(a+bx^3)(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6}+\frac{x^3(\cdot}{}$$

Rubi [A] time = 0.26, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{x^3(3a^2be-4a^3f-2ab^2d+b^3c)}{3b^5}-\frac{a^2(a^2be+a^3(-f)-ab^2d+b^3c)}{3b^6(a+bx^3)}-\frac{a\log(a+bx^3)(4a^2be-5a^3f-3ab^2d+2b^3c)}{3b^6}+\frac{x^6(3a^2f-2abe+b^2d)}{6b^4}+\frac{x^9(be-2af)}{9b^3}+\frac{fx^{12}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e
 + 3*a^2*f)*x^6)/(6*b^4) + ((b*e - 2*a*f)*x^9)/(9*b^3) + (f*x^12)/(12*b^2)
- (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^6*(a + b*x^3)) - (a*(2*b^3
*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx)^2} dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^3c-2ab^2d+3a^2be-4a^3f}{b^5}+\frac{(b^2d-2abe+3a^2f)x}{b^4}+\frac{(be-2af}{b^3}\right.\right. \\
&= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)x^3}{3b^5}+\frac{(b^2d-2abe+3a^2f)x^6}{6b^4}+\frac{(be-2af)x^9}{9b^3}+\frac{f}{1}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 167, normalized size = 0.93

$$\frac{6b^2x^6(3a^2f-2abe+b^2d)+12bx^3(-4a^3f+3a^2be-2ab^2d+b^3c)+\frac{12a^2(a^3f-a^2be+ab^2d-b^3c)}{a+bx^3}+12a\log(a+bx^3)(5a^3f-4a^2be+3ab^2d-2b^3c)+4b^3x^9(be-2af)+3b^4fx^{12}}{36b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (12*b*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3 + 6*b^2*(b^2*d - 2*a*b*
e + 3*a^2*f)*x^6 + 4*b^3*(b*e - 2*a*f)*x^9 + 3*b^4*f*x^12 + (12*a^2*(-(b^3*
c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 12*a*(-2*b^3*c + 3*a*b^2*d -
 4*a^2*b*e + 5*a^3*f)*Log[a + b*x^3])/(36*b^6)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]
```

fricas [A]   time = 0.40, size = 257, normalized size = 1.43

$$\frac{3\,b^5fx^{15}+\left(4\,b^5e-5\,ab^4f\right)x^{12}+2\left(3\,b^5d-4\,ab^4e+5\,a^2b^3f\right)x^9+6\left(2\,b^5c-3\,ab^4d+4\,a^2b^3e-5\,a^3b^2f\right)x^6-12\,a^2b^3c+12\,a^3b^2d-12\,a^4be+12\,a^5f+12\left(ab^4c-2\,a^2b^3d+3\,a^3b^2e-4\,a^4bf\right)x^3-12\left(2\,a^2b^3c-3\,a^3b^2d+4\,a^4be-5\,a^5f+\left(2\,ab^4c-3\,a^2b^3d+4\,a^3b^2e-5\,a^4bf\right)x^3\right)\log\left(bx^3+a\right)}{36\left(b^7x^3+ab^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*f*x^15 + (4*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 4*a*b^4*e +
5*a^2*b^3*f)*x^9 + 6*(2*b^5*c - 3*a*b^4*d + 4*a^2*b^3*e - 5*a^3*b^2*f)*x^6
- 12*a^2*b^3*c + 12*a^3*b^2*d - 12*a^4*b*e + 12*a^5*f + 12*(a*b^4*c - 2*a^2
*b^3*d + 3*a^3*b^2*e - 4*a^4*b*f)*x^3 - 12*(2*a^2*b^3*c - 3*a^3*b^2*d + 4*a
^4*b*e - 5*a^5*f + (2*a*b^4*c - 3*a^2*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)
*log(b*x^3 + a))/(b^7*x^3 + a*b^6)
```

giac [A]   time = 0.18, size = 248, normalized size = 1.38

$$-\frac{\left(2\,ab^3c-3\,a^2b^2d-5\,a^4f+4\,a^3be\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^6}+\frac{2\,ab^4cx^3-3\,a^2b^3dx^3-5\,a^4bfx^3+4\,a^3b^2x^3e+a^2b^3c-2\,a^3b^2d-4\,a^5f+3\,a^4be}{3\left(bx^3+a\right)b^6}+\frac{3\,b^6fx^{12}-8\,ab^5fx^9+4\,b^6x^9e+6\,b^6dx^6+18\,a^2b^4fx^6-12\,ab^5x^6e+12\,b^6cx^3-24\,ab^5dx^3-48\,a^3b^3fx^3+36\,a^2b^4x^3e}{36\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*a*b^3*c - 3*a^2*b^2*d - 5*a^4*f + 4*a^3*b*e)*log(abs(b*x^3 + a))/b^
6 + 1/3*(2*a*b^4*c*x^3 - 3*a^2*b^3*d*x^3 - 5*a^4*b*f*x^3 + 4*a^3*b^2*x^3*e
+ a^2*b^3*c - 2*a^3*b^2*d - 4*a^5*f + 3*a^4*b*e)/((b*x^3 + a)*b^6) + 1/36*(
3*b^6*f*x^12 - 8*a*b^5*f*x^9 + 4*b^6*x^9*e + 6*b^6*d*x^6 + 18*a^2*b^4*f*x^6
 - 12*a*b^5*x^6*e + 12*b^6*c*x^3 - 24*a*b^5*d*x^3 - 48*a^3*b^3*f*x^3 + 36*a
^2*b^4*x^3*e)/b^8
```

maple [A]   time = 0.06, size = 240, normalized size = 1.33

$$\frac{f\,x^{12}}{12b^2}-\frac{2af\,x^9}{9b^3}+\frac{e\,x^9}{9b^2}+\frac{a^2f\,x^6}{2b^4}-\frac{ae\,x^6}{3b^3}+\frac{d\,x^6}{6b^2}-\frac{4a^3f\,x^3}{3b^5}+\frac{a^2e\,x^3}{b^4}-\frac{2ad\,x^3}{3b^3}+\frac{c\,x^3}{3b^2}+\frac{a^5f}{3\left(b\,x^3+a\right)b^6}-\frac{a^4e}{3\left(b\,x^3+a\right)b^5}+\frac{5a^4f\ln\left(b\,x^3+a\right)}{3b^6}+\frac{a^3d}{3\left(b\,x^3+a\right)b^4}-\frac{4a^3e\ln\left(b\,x^3+a\right)}{3b^5}-\frac{a^2c}{3\left(b\,x^3+a\right)b^3}+\frac{a^2d\ln\left(b\,x^3+a\right)}{b^4}-\frac{2ac\ln\left(b\,x^3+a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/12*f*x^12/b^2-2/9/b^3*x^9*a*f+1/9/b^2*x^9*e+1/2/b^4*x^6*a^2*f-1/3/b^3*x^6
*a*e+1/6/b^2*x^6*d-4/3*a^3/b^5*f*x^3+a^2/b^4*e*x^3-2/3*a/b^3*d*x^3+1/3/b^2*
c*x^3+5/3*a^4/b^6*ln(b*x^3+a)*f-4/3*a^3/b^5*ln(b*x^3+a)*e+a^2/b^4*ln(b*x^3+
a)*d-2/3*a/b^3*ln(b*x^3+a)*c+1/3*a^5/b^6/(b*x^3+a)*f-1/3*a^4/b^5/(b*x^3+a)*
e+1/3*a^3/b^4/(b*x^3+a)*d-1/3*a^2/b^3/(b*x^3+a)*c
```

maxima [A]   time = 1.38, size = 180, normalized size = 1.00

$$-\frac{a^2b^3c-a^3b^2d+a^4be-a^5f}{3\left(b^7x^3+ab^6\right)}+\frac{3\,b^3fx^{12}+4\left(b^3e-2\,ab^2f\right)x^9+6\left(b^3d-2\,ab^2e+3\,a^2bf\right)x^6+12\left(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f\right)x^3}{36\,b^5}-\frac{\left(2\,ab^3c-3\,a^2b^2d+4\,a^3be-5\,a^4f\right)\log\left(bx^3+a\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)/(b^7*x^3 + a*b^6) + 1/36*(3*
b^3*f*x^12 + 4*(b^3*e - 2*a*b^2*f)*x^9 + 6*(b^3*d - 2*a*b^2*e + 3*a^2*b*f)*
x^6 + 12*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/b^5 - 1/3*(2*a*b^3*
c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*log(b*x^3 + a)/b^6
```

mupad [B] time = 5.00, size = 233, normalized size = 1.29

$$x^9\left(\frac{e}{9\,b^2}-\frac{2\,a\,f}{9\,b^3}\right)-x^6\left(\frac{a^2\,f}{6\,b^4}-\frac{d}{6\,b^2}+\frac{a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right)+x^3\left(\frac{c}{3\,b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^{12}}{12\,b^2}+\frac{f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3}{3\,b\left(b^6\,x^3+a\,b^5\right)}+\frac{\ln\left(b\,x^3+a\right)\left(5\,f\,a^4-4\,e\,a^3\,b+3\,d\,a^2\,b^2-2\,c\,a\,b^3\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^9*(e/(9*b^2) - (2*a*f)/(9*b^3)) - x^6*((a^2*f)/(6*b^4) - d/(6*b^2) + (a*(
e/b^2 - (2*a*f)/b^3))/(3*b)) + x^3*(c/(3*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))
/(3*b^2) + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b
)) + (f*x^12)/(12*b^2) + (a^5*f - a^2*b^3*c + a^3*b^2*d - a^4*b*e)/(3*b*(a*
b^5 + b^6*x^3)) + (log(a + b*x^3)*(5*a^4*f + 3*a^2*b^2*d - 2*a*b^3*c - 4*a^
3*b*e))/(3*b^6)
```

sympy [A] time = 12.38, size = 189, normalized size = 1.05

$$\frac{a\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)\log\left(a+bx^3\right)}{3b^6}+x^9\left(-\frac{2af}{9b^3}+\frac{e}{9b^2}\right)+x^6\left(\frac{a^2f}{2b^4}-\frac{ae}{3b^3}+\frac{d}{6b^2}\right)+x^3\left(-\frac{4a^3f}{3b^5}+\frac{a^2e}{b^4}-\frac{2ad}{3b^3}+\frac{c}{3b^2}\right)+\frac{a^5f-a^4be+a^3b^2d-a^2b^3c}{3ab^6+3b^7x^3}+\frac{fx^{12}}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] a*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d - 2*b**3*c)*log(a + b*x**3)/(3*b**6)
+ x**9*(-2*a*f/(9*b**3) + e/(9*b**2)) + x**6*(a**2*f/(2*b**4) - a*e/(3*b**3
) + d/(6*b**2)) + x**3*(-4*a**3*f/(3*b**5) + a**2*e/b**4 - 2*a*d/(3*b**3) +
 c/(3*b**2)) + (a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 +
3*b**7*x**3) + f*x**12/(12*b**2)
```

3.200 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$

Optimal. Leaf size=140

$$\frac{x^3(3a^2f-2abe+b^2d)}{3b^4}+\frac{a(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5(a+bx^3)}+\frac{\log(a+bx^3)(-4a^3f+3a^2be-2ab^2d+b^3c)}{3b^5}+\frac{x^6(b$$

Rubi [A] time = 0.20, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{a(a^2be+a^3(-f)-ab^2d+b^3c)}{3b^5(a+bx^3)}+\frac{\log(a+bx^3)(3a^2be-4a^3f-2ab^2d+b^3c)}{3b^5}+\frac{x^3(3a^2f-2abe+b^2d)}{3b^4}+\frac{x^6(be-2af)}{6b^3}+\frac{fx^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^6)/(6*b^3) + (
f*x^9)/(9*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^5*(a + b*x^3)
) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(3*b^5)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx)^2} dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2d-2abe+3a^2f}{b^4}+\frac{(be-2af)x}{b^3}+\frac{fx^2}{b^2}+\frac{a(-b^3c+ab^2d-a^2\right.\right.}{b^4(a+bx)}\\ &= \frac{(b^2d-2abe+3a^2f)x^3}{3b^4}+\frac{(be-2af)x^6}{6b^3}+\frac{fx^9}{9b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)}{3b^5(a+bx^3)}\end{aligned}$$

Mathematica [A] time = 0.12, size = 129, normalized size = 0.92

$$\frac{6bx^3(3a^2f-2abe+b^2d)+\frac{6a(a^3(-f)+a^2be-ab^2d+b^3c)}{a+bx^3}+6\log(a+bx^3)(-4a^3f+3a^2be-2ab^2d+b^3c)+3b^2x^6(be-2af)+2b^3fx^9}{18b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (6*b*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3 + 3*b^2*(b*e - 2*a*f)*x^6 + 2*b^3*f*x^
9 + (6*a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3) + 6*(b^3*c - 2*a*
b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(18*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(c + dx^3 + ex^6 + fx^9\right)}{\left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]
```

fricas [A] time = 0.40, size = 202, normalized size = 1.44

$$\frac{2\,b^4fx^{12} + \left(3\,b^4e - 4\,ab^3f\right)x^9 + 3\left(2\,b^4d - 3\,ab^3e + 4\,a^2b^2f\right)x^6 + 6\,ab^3c - 6\,a^2b^2d + 6\,a^3be - 6\,a^4f + 6\left(ab^3d - 2\,a^2b^2e + 3\,a^3bf\right)x^3 + 6\left(ab^3c - 2\,a^2b^2d + 3\,a^3be - 4\,a^4f + \left(b^4c - 2\,ab^3d + 3\,a^2b^2e - 4\,a^3bf\right)x^3\right)\log\left(bx^3 + a\right)}{18\left(b^6x^3 + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^4*f*x^12 + (3*b^4*e - 4*a*b^3*f)*x^9 + 3*(2*b^4*d - 3*a*b^3*e + 4
*a^2*b^2*f)*x^6 + 6*a*b^3*c - 6*a^2*b^2*d + 6*a^3*b*e - 6*a^4*f + 6*(a*b^3*
d - 2*a^2*b^2*e + 3*a^3*b*f)*x^3 + 6*(a*b^3*c - 2*a^2*b^2*d + 3*a^3*b*e - 4
*a^4*f + (b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^3)*log(b*x^3 + a))
/(b^6*x^3 + a*b^5)
```

giac [A] time = 0.20, size = 217, normalized size = 1.55

$$\frac{\frac{\left(bx^3+a\right)^3\left(2\,f-\frac{3\left(4\,abf-b^2e\right)}{\left(bx^3+a\right)b}+\frac{6\left(b^4d+6\,a^2b^2f-3\,ab^3e\right)}{\left(bx^3+a\right)^2b^2}\right)}{b^4}-\frac{6\left(b^3c-2\,ab^2d-4\,a^3f+3\,a^2be\right)\log\left(\frac{\left|bx^3+a\right|}{\left(bx^3+a\right)^2\left|b\right|}\right)}{b^4}+\frac{6\left(\frac{ab^6c}{bx^3+a}-\frac{a^2b^5d}{bx^3+a}-\frac{a^4b^3f}{bx^3+a}+\frac{a^3b^4e}{bx^3+a}\right)}{b^7}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/18*((b*x^3 + a)^3*(2*f - 3*(4*a*b*f - b^2*e)/((b*x^3 + a)*b) + 6*(b^4*d +
 6*a^2*b^2*f - 3*a*b^3*e)/((b*x^3 + a)^2*b^2))/b^4 - 6*(b^3*c - 2*a*b^2*d -
 4*a^3*f + 3*a^2*b*e)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^4 + 6*(a
*b^6*c/(b*x^3 + a) - a^2*b^5*d/(b*x^3 + a) - a^4*b^3*f/(b*x^3 + a) + a^3*b^
4*e/(b*x^3 + a))/b^7)/b
```

maple [A] time = 0.06, size = 192, normalized size = 1.37

$$\frac{f\,x^9}{9b^2}-\frac{af\,x^6}{3b^3}+\frac{e\,x^6}{6b^2}+\frac{a^2f\,x^3}{b^4}-\frac{2ae\,x^3}{3b^3}+\frac{d\,x^3}{3b^2}-\frac{a^4f}{3\left(b\,x^3+a\right)b^5}+\frac{a^3e}{3\left(b\,x^3+a\right)b^4}-\frac{4a^3f\ln\left(b\,x^3+a\right)}{3b^5}-\frac{a^2d}{3\left(b\,x^3+a\right)b^3}+\frac{a^2e\ln\left(b\,x^3+a\right)}{b^4}+\frac{ac}{3\left(b\,x^3+a\right)b^2}-\frac{2ad\ln\left(b\,x^3+a\right)}{3b^3}+\frac{c\ln\left(b\,x^3+a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/9/b^2*f*x^9-1/3/b^3*x^6*a*f+1/6/b^2*x^6*e+1/b^4*x^3*a^2*f-2/3/b^3*x^3*a*e
+1/3/b^2*x^3*d-4/3/b^5*ln(b*x^3+a)*a^3*f+1/b^4*ln(b*x^3+a)*a^2*e-2/3/b^3*ln
(b*x^3+a)*a*d+1/3/b^2*ln(b*x^3+a)*c-1/3/b^5*a^4/(b*x^3+a)*f+1/3/b^4*a^3/(b*
x^3+a)*e-1/3/b^3*a^2/(b*x^3+a)*d+1/3/b^2*a/(b*x^3+a)*c
```

maxima [A] time = 1.40, size = 138, normalized size = 0.99

$$\frac{ab^3c - a^2b^2d + a^3be - a^4f}{3\left(b^6x^3 + ab^5\right)} + \frac{2\,b^2fx^9 + 3\left(b^2e - 2\,abf\right)x^6 + 6\left(b^2d - 2\,abe + 3\,a^2f\right)x^3}{18\,b^4} + \frac{\left(b^3c - 2\,ab^2d + 3\,a^2be - 4\,a^3f\right)\log\left(bx^3 + a\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)/(b^6*x^3 + a*b^5) + 1/18*(2*b^2
*f*x^9 + 3*(b^2*e - 2*a*b*f)*x^6 + 6*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/b^4 +
 1/3*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*log(b*x^3 + a)/b^5
```

mupad [B] time = 4.93, size = 155, normalized size = 1.11

$$x^6\left(\frac{e}{6\,b^2}-\frac{a\,f}{3\,b^3}\right)-x^3\left(\frac{a^2\,f}{3\,b^4}-\frac{d}{3\,b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(-4\,f\,a^3+3\,e\,a^2\,b-2\,d\,a\,b^2+c\,b^3\right)}{3\,b^5}-\frac{f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3}{3\,b\,\left(b^5\,x^3+a\,b^4\right)}+\frac{f\,x^9}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^6*(e/(6*b^2) - (a*f)/(3*b^3)) - x^3*((a^2*f)/(3*b^4) - d/(3*b^2) + (2*a*(
e/b^2 - (2*a*f)/b^3))/(3*b)) + (log(a + b*x^3)*(b^3*c - 4*a^3*f - 2*a*b^2*d
 + 3*a^2*b*e))/(3*b^5) - (a^4*f + a^2*b^2*d - a*b^3*c - a^3*b*e)/(3*b*(a*b^
4 + b^5*x^3)) + (f*x^9)/(9*b^2)
```

sympy [A] time = 12.81, size = 141, normalized size = 1.01

$$x^6\left(-\frac{af}{3b^3}+\frac{e}{6b^2}\right)+x^3\left(\frac{a^2f}{b^4}-\frac{2ae}{3b^3}+\frac{d}{3b^2}\right)+\frac{-a^4f+a^3be-a^2b^2d+ab^3c}{3ab^5+3b^6x^3}+\frac{fx^9}{9b^2}-\frac{\left(4a^3f-3a^2be+2ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**6*(-a*f/(3*b**3) + e/(6*b**2)) + x**3*(a**2*f/b**4 - 2*a*e/(3*b**3) + d/
(3*b**2)) + (-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(3*a*b**5 + 3*b**
6*x**3) + f*x**9/(9*b**2) - (4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)*l
og(a + b*x**3)/(3*b**5)
```

3.201 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=103

$$\frac{\log\left(a+bx^3\right)\left(3a^2f-2abe+b^2d\right)}{3b^4}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3b^4\left(a+bx^3\right)}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^6}{6b^2}$$

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1819, 1850}

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3b^4\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^6)/(6*b^2) - (b^3*c - a*b^2*d + a^2*b*e
- a^3*f)/(3*b^4*(a + b*x^3)) + ((b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])
/(3*b^4)
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{be-2af}{b^3}+\frac{fx}{b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)^2}+\frac{b^2d-2abe+3a^2f}{b^3(a+bx)}\right)\right.\\ &= \frac{(be-2af)x^3}{3b^3}+\frac{fx^6}{6b^2}-\frac{b^3c-ab^2d+a^2be-a^3f}{3b^4\left(a+bx^3\right)}+\frac{\left(b^2d-2abe+3a^2f\right)\log\left(a+\right.}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.90

$$\frac{2\log\left(a+bx^3\right)\left(3a^2f-2abe+b^2d\right)+\frac{2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}+2bx^3(be-2af)+b^2fx^6}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (2*b*(b*e - 2*a*f)*x^3 + b^2*f*x^6 + (2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3
*f))/(a + b*x^3) + 2*(b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])/(6*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(c + dx^3 + ex^6 + fx^9\right)}{\left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]
```

fricas [A] time = 0.39, size = 143, normalized size = 1.39

$$\frac{b^3 f x^9 + \left(2 b^3 e - 3 a b^2 f\right) x^6 - 2 b^3 c + 2 a b^2 d - 2 a^2 b e + 2 a^3 f + 2 \left(a b^2 e - 2 a^2 b f\right) x^3 + 2 \left(a b^2 d - 2 a^2 b e + 3 a^3 f + \left(b^3 d - 2 a b^2 e + 3 a^2 b f\right) x^3\right) \log\left(b x^3 + a\right)}{6 \left(b^5 x^3 + a b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*f*x^9 + (2*b^3*e - 3*a*b^2*f)*x^6 - 2*b^3*c + 2*a*b^2*d - 2*a^2*b*
e + 2*a^3*f + 2*(a*b^2*e - 2*a^2*b*f)*x^3 + 2*(a*b^2*d - 2*a^2*b*e + 3*a^3*
f + (b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

giac [B] time = 0.19, size = 206, normalized size = 2.00

$$-\frac{1}{6} f \left(\frac{\left(bx^3 + a\right)^2 \left(\frac{6a}{bx^3+a} - 1\right)}{b^4} + \frac{6a^2 \log\left(\frac{\left|bx^3+a\right|}{\left(bx^3+a\right)^2 |b|}\right)}{b^4} - \frac{2a^3}{\left(bx^3 + a\right) b^4}\right) + \frac{1}{3} \left(\frac{2a \log\left(\frac{\left|bx^3+a\right|}{\left(bx^3+a\right)^2 |b|}\right)}{b^3} + \frac{bx^3 + a}{b^3} - \frac{a^2}{\left(bx^3 + a\right) b^3}\right) e - \frac{d \left(\frac{\log\left(\frac{\left|bx^3+a\right|}{\left(bx^3+a\right)^2 |b|}\right)}{b} - \frac{a}{\left(bx^3+a\right) b}\right)}{3b} - \frac{c}{3 \left(bx^3 + a\right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*f*((b*x^3 + a)^2*(6*a/(b*x^3 + a) - 1)/b^4 + 6*a^2*log(abs(b*x^3 + a)/
((b*x^3 + a)^2*abs(b)))/b^4 - 2*a^3/((b*x^3 + a)*b^4)) + 1/3*(2*a*log(abs(b
*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^3 + (b*x^3 + a)/b^3 - a^2/((b*x^3 + a)*
b^3))*e - 1/3*d*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 +
 a)*b))/b - 1/3*c/((b*x^3 + a)*b)
```

maple [A] time = 0.07, size = 142, normalized size = 1.38

$$\frac{f x^6}{6b^2} - \frac{2af x^3}{3b^3} + \frac{e x^3}{3b^2} + \frac{a^3 f}{3 \left(b x^3 + a\right) b^4} - \frac{a^2 e}{3 \left(b x^3 + a\right) b^3} + \frac{a^2 f \ln\left(b x^3 + a\right)}{b^4} + \frac{ad}{3 \left(b x^3 + a\right) b^2} - \frac{2ae \ln\left(b x^3 + a\right)}{3b^3} - \frac{c}{3 \left(b x^3 + a\right) b} + \frac{d \ln\left(b x^3 + a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/6*f*x^6/b^2-2/3/b^3*x^3*a*f+1/3/b^2*x^3*e+1/b^4*ln(b*x^3+a)*a^2*f-2/3/b^3
*ln(b*x^3+a)*a*e+1/3/b^2*ln(b*x^3+a)*d+1/3/b^4/(b*x^3+a)*a^3*f-1/3/b^3/(b*x
^3+a)*a^2*e+1/3/b^2/(b*x^3+a)*a*d-1/3/b/(b*x^3+a)*c
```

maxima [A] time = 1.35, size = 98, normalized size = 0.95

$$-\frac{b^3 c - a b^2 d + a^2 b e - a^3 f}{3 \left(b^5 x^3 + a b^4\right)} + \frac{b f x^6 + 2 \left(b e - 2 a f\right) x^3}{6 b^3} + \frac{\left(b^2 d - 2 a b e + 3 a^2 f\right) \log\left(b x^3 + a\right)}{3 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(b^5*x^3 + a*b^4) + 1/6*(b*f*x^6 +
 2*(b*e - 2*a*f)*x^3)/b^3 + 1/3*(b^2*d - 2*a*b*e + 3*a^2*f)*log(b*x^3 + a)/
b^4
```

mupad [B] time = 0.09, size = 103, normalized size = 1.00

$$x^3\left(\frac{e}{3\,b^2}-\frac{2\,a\,f}{3\,b^3}\right)+\frac{f\,x^6}{6\,b^2}-\frac{-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3}{3\,b\,\left(b^4\,x^3+a\,b^3\right)}+\frac{\ln\left(b\,x^3+a\right)\,\left(3\,f\,a^2-2\,e\,a\,b+d\,b^2\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^3*(e/(3*b^2) - (2*a*f)/(3*b^3)) + (f*x^6)/(6*b^2) - (b^3*c - a^3*f - a*b^
2*d + a^2*b*e)/(3*b*(a*b^3 + b^4*x^3)) + (log(a + b*x^3)*(b^2*d + 3*a^2*f -
 2*a*b*e))/(3*b^4)
```

sympy [A] time = 11.61, size = 100, normalized size = 0.97

$$x^3\left(-\frac{2af}{3b^3}+\frac{e}{3b^2}\right)+\frac{a^3f-a^2be+ab^2d-b^3c}{3ab^4+3b^5x^3}+\frac{fx^6}{6b^2}+\frac{\left(3a^2f-2abe+b^2d\right)\log\left(a+bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**3*(-2*a*f/(3*b**3) + e/(3*b**2)) + (a**3*f - a**2*b*e + a*b**2*d - b**3*
c)/(3*a*b**4 + 3*b**5*x**3) + f*x**6/(6*b**2) + (3*a**2*f - 2*a*b*e + b**2*
d)*log(a + b*x**3)/(3*b**4)
```

3.202 $\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=100

$$\frac{c\log(x)}{a^2}-\frac{\log\left(a+bx^3\right)\left(2a^3f-a^2be+b^3c\right)}{3a^2b^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3ab^3\left(a+bx^3\right)}+\frac{fx^3}{3b^2}$$

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3ab^3\left(a+bx^3\right)}-\frac{\log\left(a+bx^3\right)\left(-a^2be+2a^3f+b^3c\right)}{3a^2b^3}+\frac{c\log(x)}{a^2}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

```
[Out] (f*x^3)/(3*b^2) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a*b^3*(a + b*x^3))
 + (c*Log[x])/a^2 - ((b^3*c - a^2*b*e + 2*a^3*f)*Log[a + b*x^3])/(3*a^2*b^3
)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)^2}\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int \left(\frac{f}{b^2}+\frac{c}{a^2x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)^2}+\frac{-b^3c+a^2be-2a^3f}{a^2b^2(a+bx)}\right)dx,x\right.\\&=\frac{fx^3}{3b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{3ab^3\left(a+bx^3\right)}+\frac{c\log(x)}{a^2}-\frac{\left(b^3c-a^2be+2a^3f\right)\log\left(a+bx^3\right)}{3a^2b^3}\end{aligned}$$

Mathematica [A] time = 0.18, size = 95, normalized size = 0.95

$$\frac{\frac{\log\left(a+bx^3\right)\left(-2a^3f+a^2be-b^3c\right)+\frac{a\left(a^3(-f)+a^2b\left(e+fx^3\right)+ab^2\left(fx^6-d\right)+b^3c\right)}{a+bx^3}}{b^3}+3c\log(x)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

```
[Out] (3*c*Log[x] + ((a*(b^3*c - a^3*f + a^2*b*(e + f*x^3) + a*b^2*(-d + f*x^6)))
/(a + b*x^3) + (-(b^3*c) + a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/b^3)/(3*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x\left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2), x]
```

fricas [A] time = 0.44, size = 145, normalized size = 1.45

$$\frac{a^2b^2fx^6 + a^3bfx^3 + ab^3c - a^2b^2d + a^3be - a^4f - \left(ab^3c - a^3be + 2\,a^4f + \left(b^4c - a^2b^2e + 2\,a^3bf\right)x^3\right)\log\left(bx^3 + a\right) + 3\left(b^4cx^3 + ab^3c\right)\log(x)}{3\left(a^2b^4x^3 + a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*b^2*f*x^6 + a^3*b*f*x^3 + a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f -
(a*b^3*c - a^3*b*e + 2*a^4*f + (b^4*c - a^2*b^2*e + 2*a^3*b*f)*x^3)*log(b*x
^3 + a) + 3*(b^4*c*x^3 + a*b^3*c)*log(x))/(a^2*b^4*x^3 + a^3*b^3)
```

giac [A] time = 0.17, size = 125, normalized size = 1.25

$$\frac{fx^3}{3\,b^2} + \frac{c\log\left(|x|\right)}{a^2} - \frac{\left(b^3c + 2\,a^3f - a^2be\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^2b^3} + \frac{b^4cx^3 + 2\,a^3bfx^3 - a^2b^2x^3e + 2\,ab^3c - a^2b^2d + a^4f}{3\left(bx^3 + a\right)a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b^2 + c*log(abs(x))/a^2 - 1/3*(b^3*c + 2*a^3*f - a^2*b*e)*log(abs
(b*x^3 + a))/(a^2*b^3) + 1/3*(b^4*c*x^3 + 2*a^3*b*f*x^3 - a^2*b^2*x^3*e + 2
*a*b^3*c - a^2*b^2*d + a^4*f)/((b*x^3 + a)*a^2*b^3)
```

maple [A] time = 0.06, size = 125, normalized size = 1.25

$$\frac{fx^3}{3b^2} - \frac{a^2f}{3\left(bx^3+a\right)b^3} + \frac{ae}{3\left(bx^3+a\right)b^2} - \frac{2af\ln\left(bx^3+a\right)}{3b^3} + \frac{c}{3\left(bx^3+a\right)a} + \frac{c\ln(x)}{a^2} - \frac{c\ln\left(bx^3+a\right)}{3a^2} - \frac{d}{3\left(bx^3+a\right)b} + \frac{e\ln\left(bx^3+a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x)
```

```
[Out] 1/3/b^2*f*x^3-2/3*a/b^3*ln(b*x^3+a)*f+1/3/b^2*ln(b*x^3+a)*e-1/3*c*ln(b*x^3+
a)/a^2-1/3*a^2/b^3/(b*x^3+a)*f+1/3*a/b^2/(b*x^3+a)*e-1/3/b/(b*x^3+a)*d+1/3/
a/(b*x^3+a)*c+1/a^2*c*ln(x)
```

maxima [A] time = 1.32, size = 100, normalized size = 1.00

$$\frac{fx^3}{3\,b^2} + \frac{b^3c - ab^2d + a^2be - a^3f}{3\left(ab^4x^3 + a^2b^3\right)} + \frac{c\log\left(x^3\right)}{3\,a^2} - \frac{\left(b^3c - a^2be + 2\,a^3f\right)\log\left(bx^3 + a\right)}{3\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a*b^4*x^3 + a^2*b^
3) + 1/3*c*log(x^3)/a^2 - 1/3*(b^3*c - a^2*b*e + 2*a^3*f)*log(b*x^3 + a)/(a
^2*b^3)
```

mupad [B] time = 5.03, size = 100, normalized size = 1.00

$$\frac{f\,x^3}{3\,b^2}+\frac{c\,\ln(x)}{a^2}+\frac{-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3}{3\,a\,b\,\left(b^3\,x^3+a\,b^2\right)}-\frac{\ln\left(b\,x^3+a\right)\,\left(2\,f\,a^3-e\,a^2\,b+c\,b^3\right)}{3\,a^2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x)
```

```
[Out] (f*x^3)/(3*b^2) + (c*log(x))/a^2 + (b^3*c - a^3*f - a*b^2*d + a^2*b*e)/(3*a
*b*(a*b^2 + b^3*x^3)) - (log(a + b*x^3)*(b^3*c + 2*a^3*f - a^2*b*e))/(3*a^2
*b^3)
```

sympy [A] time = 41.96, size = 95, normalized size = 0.95

$$\frac{-a^3f+a^2be-ab^2d+b^3c}{3a^2b^3+3ab^4x^3}+\frac{fx^3}{3b^2}+\frac{c\log(x)}{a^2}-\frac{\left(2a^3f-a^2be+b^3c\right)\log\left(\frac{a}{b}+x^3\right)}{3a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**2,x)
```

```
[Out] (-a**3*f + a**2*b*e - a*b**2*d + b**3*c)/(3*a**2*b**3 + 3*a*b**4*x**3) + f*
x**3/(3*b**2) + c*log(x)/a**2 - (2*a**3*f - a**2*b*e + b**3*c)*log(a/b + x*
*3)/(3*a**2*b**3)
```

3.203 $\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=109

$$\frac{\log\left(a+bx^3\right)\left(a^3f-ab^2d+2b^3c\right)}{3a^3b^2}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{3a^2x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^2b^2\left(a+bx^3\right)}$$

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(a^3f-ab^2d+2b^3c\right)}{3a^3b^2}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -c/(3*a^2*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^2*b^2*(a + b*x^3)
) - ((2*b*c - a*d)*Log[x])/a^3 + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x^3
])/(3*a^3*b^2)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^2}+\frac{-2bc+ad}{a^3x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)^2}+\frac{2b^3c-ab^2d+a^3f}{a^3b(a+bx)}\right)d\right.\\ &= -\frac{c}{3a^2x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{3a^2b^2\left(a+bx^3\right)}-\frac{(2bc-ad)\log(x)}{a^3}+\frac{\left(2b^3c-ab^2d+a^3f\right)\log\left(a\right.}{3a^3b^2}\end{aligned}$$

Mathematica [A] time = 0.15, size = 97, normalized size = 0.89

$$\frac{\frac{\log\left(a+bx^3\right)\left(a^3f-ab^2d+2b^3c\right)}{b^2}+\frac{a\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^2\left(a+bx^3\right)}+3\log(x)(ad-2bc)-\frac{ac}{x^3}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((a*c)/x^3) + (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^3)
) + 3*(-2*b*c + a*d)*Log[x] + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x^3])/
b^2)/(3*a^3)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^4 \left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2), x]
```

fricas [A]   time = 0.44, size = 172, normalized size = 1.58

$$-\frac{a^{2} b^{2} c + \left(2 \, a b^{3} c - a^{2} b^{2} d + a^{3} b e - a^{4} f\right)x^{3} - \left(\left(2 \, b^{4} c - a b^{3} d + a^{3} b f\right)x^{6} + \left(2 \, a b^{3} c - a^{2} b^{2} d + a^{4} f\right)x^{3}\right)\log\left(bx^{3} + a\right) + 3 \, \left(\left(2 \, b^{4} c - a b^{3} d\right)x^{6} + \left(2 \, a b^{3} c - a^{2} b^{2} d\right)x^{3}\right)\log(x)}{3 \, \left(a^{3} b^{3} x^{6} + a^{4} b^{2} x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*b^2*c + (2*a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^3 - ((2*b^4*c
 - a*b^3*d + a^3*b*f)*x^6 + (2*a*b^3*c - a^2*b^2*d + a^4*f)*x^3)*log(b*x^3
+ a) + 3*((2*b^4*c - a*b^3*d)*x^6 + (2*a*b^3*c - a^2*b^2*d)*x^3)*log(x))/(a
^3*b^3*x^6 + a^4*b^2*x^3)
```

giac [A]   time = 0.21, size = 131, normalized size = 1.20

$$-\frac{(2 \, bc - ad) \log\left(|x|\right)}{a^{3}} + \frac{\left(2 \, b^{3} c - ab^{2} d + a^{3} f\right) \log\left(|bx^{3} + a|\right)}{3 \, a^{3} b^{2}} - \frac{a^{2} bfx^{6} + 4 \, b^{3} cx^{3} - 2 \, ab^{2} dx^{3} - a^{3} fx^{3} + 2 \, a^{2} bx^{3} e + 2 \, ab^{2} c}{6 \, \left(bx^{6} + ax^{3}\right)a^{2} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(2*b*c - a*d)*log(abs(x))/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3*f)*log(abs(b*
x^3 + a))/(a^3*b^2) - 1/6*(a^2*b*f*x^6 + 4*b^3*c*x^3 - 2*a*b^2*d*x^3 - a^3*
f*x^3 + 2*a^2*b*x^3*e + 2*a*b^2*c)/((b*x^6 + a*x^3)*a^2*b^2)
```

maple [A]   time = 0.06, size = 132, normalized size = 1.21

$$\frac{af}{3 \left(b\,x^{3}+a\right) b^{2}}+\frac{d}{3 \left(b\,x^{3}+a\right) a}-\frac{bc}{3 \left(b\,x^{3}+a\right) a^{2}}+\frac{d \ln(x)}{a^{2}}-\frac{d \ln\left(b\,x^{3}+a\right)}{3a^{2}}-\frac{2bc \ln(x)}{a^{3}}+\frac{2bc \ln\left(b\,x^{3}+a\right)}{3a^{3}}-\frac{e}{3 \left(b\,x^{3}+a\right) b}+\frac{f \ln\left(b\,x^{3}+a\right)}{3b^{2}}-\frac{c}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 1/3*f*ln(b*x^3+a)/b^2-1/3*d*ln(b*x^3+a)/a^2+2/3*b*c*ln(b*x^3+a)/a^3+1/3*a/b
^2/(b*x^3+a)*f-1/3/b/(b*x^3+a)*e+1/3/a/(b*x^3+a)*d-1/3/a^2*b/(b*x^3+a)*c-1/
3/a^2*c/x^3+d*ln(x)/a^2-2*b*c*ln(x)/a^3
```

maxima [A]   time = 1.43, size = 116, normalized size = 1.06

$$-\frac{ab^{2} c + \left(2 \, b^{3} c - ab^{2} d + a^{2} be - a^{3} f\right)x^{3}}{3 \, \left(a^{2} b^{3} x^{6} + a^{3} b^{2} x^{3}\right)} - \frac{(2 \, bc - ad) \log\left(x^{3}\right)}{3 \, a^{3}} + \frac{\left(2 \, b^{3} c - ab^{2} d + a^{3} f\right) \log\left(bx^{3} + a\right)}{3 \, a^{3} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a*b^2*c + (2*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(a^2*b^3*x^6 + a
^3*b^2*x^3) - 1/3*(2*b*c - a*d)*log(x^3)/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3
*f)*log(b*x^3 + a)/(a^3*b^2)
```

mupad [**B**] time = 5.05, size = 109, normalized size = 1.00

$$\frac{\ln(x)\ (a\,d-2\,b\,c)}{a^3}-\frac{\frac{c}{3\,a}+\frac{x^3\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+2\,c\,b^3\right)}{3\,a^2\,b^2}}{b\,x^6+a\,x^3}+\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^3-d\,a\,b^2+2\,c\,b^3\right)}{3\,a^3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (log(x)*(a*d - 2*b*c))/a^3 - (c/(3*a) + (x^3*(2*b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(3*a^2*b^2))/(a*x^3 + b*x^6) + (log(a + b*x^3)*(2*b^3*c + a^3*f -
a*b^2*d))/(3*a^3*b^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.204 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^2}\,dx$

Optimal. Leaf size=130

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{6a^2x^6}-\frac{\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)}{3a^4}+\frac{\log(x)\left(a^2e-2abd+3b^2c\right)}{a^4}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^3b\left(a+bx^3\right)}$$

Rubi [A] time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^3b\left(a+bx^3\right)}-\frac{\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)}{3a^4}+\frac{\log(x)\left(a^2e-2abd+3b^2c\right)}{a^4}+\frac{2bc-ad}{3a^3x^3}-\frac{c}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -c/(6*a^2*x^6) + (2*b*c - a*d)/(3*a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a
^3*f)/(3*a^3*b*(a + b*x^3)) + ((3*b^2*c - 2*a*b*d + a^2*e)*Log[x])/a^4 - ((
3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/(3*a^4)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^3}+\frac{-2bc+ad}{a^3x^2}+\frac{3b^2c-2abd+a^2e}{a^4x}+\frac{-b^3c+ab^2d-a^2be+a^3}{a^3(a+bx)^2}\right.\right.\\ &= -\frac{c}{6a^2x^6}+\frac{2bc-ad}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^3b\left(a+bx^3\right)}+\frac{\left(3b^2c-2abd+a^2e\right)\log(x)}{a^4}-\frac{(}{}\end{aligned}$$

Mathematica [A] time = 0.14, size = 118, normalized size = 0.91

$$-\frac{2\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)-6\log(x)\left(a^2e-2abd+3b^2c\right)+\frac{a^2c}{x^6}+\frac{2a\left(a^3f-a^2be+ab^2d-b^3c\right)}{b\left(a+bx^3\right)}+\frac{2a(ad-2bc)}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*((a^2*c)/x^6 + (2*a*(-2*b*c + a*d))/x^3 + (2*a*(-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f))/(b*(a + b*x^3)) - 6*(3*b^2*c - 2*a*b*d + a^2*e)*Log[x] + 2
*(3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/a^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^7 \left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2), x]
```

fricas [A] time = 0.43, size = 208, normalized size = 1.60

$$\frac{2\left(3\,ab^3c - 2\,a^2b^2d + a^3be - a^4f\right)x^6 - a^3bc + \left(3\,a^2b^2c - 2\,a^3bd\right)x^3 - 2\left(\left(3\,b^4c - 2\,ab^3d + a^2b^2e\right)x^9 + \left(3\,ab^3c - 2\,a^2b^2d + a^3be\right)x^6\right)\log\left(bx^3 + a\right) + 6\left(\left(3\,b^4c - 2\,ab^3d + a^2b^2e\right)x^9 + \left(3\,ab^3c - 2\,a^2b^2d + a^3be\right)x^6\right)\log(x)}{6\left(a^4b^2x^9 + a^5bx^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e - a^4*f)*x^6 - a^3*b*c + (3*a^2*b
^2*c - 2*a^3*b*d)*x^3 - 2*((3*b^4*c - 2*a*b^3*d + a^2*b^2*e)*x^9 + (3*a*b^3
*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(b*x^3 + a) + 6*((3*b^4*c - 2*a*b^3*d +
 a^2*b^2*e)*x^9 + (3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(x))/(a^4*b^2
*x^9 + a^5*b*x^6)
```

giac [A] time = 0.17, size = 201, normalized size = 1.55

$$\frac{\left(3\,b^2c - 2\,abd + a^2e\right)\log\left(|x|\right)}{a^4} - \frac{\left(3\,b^3c - 2\,ab^2d + a^2be\right)\log\left(|bx^3 + a|\right)}{3\,a^4b} + \frac{3\,b^4cx^3 - 2\,ab^3dx^3 + a^2b^2x^3e + 4\,ab^3c - 3\,a^2b^2d - a^4f + 2\,a^3be}{3\left(bx^3 + a\right)a^4b} - \frac{9\,b^2cx^6 - 6\,abdx^6 + 3\,a^2x^6e - 4\,abcx^3 + 2\,a^2dx^3 + a^2c}{6\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (3*b^2*c - 2*a*b*d + a^2*e)*log(abs(x))/a^4 - 1/3*(3*b^3*c - 2*a*b^2*d + a^
2*b*e)*log(abs(b*x^3 + a))/(a^4*b) + 1/3*(3*b^4*c*x^3 - 2*a*b^3*d*x^3 + a^2
*b^2*x^3*e + 4*a*b^3*c - 3*a^2*b^2*d - a^4*f + 2*a^3*b*e)/((b*x^3 + a)*a^4*
b) - 1/6*(9*b^2*c*x^6 - 6*a*b*d*x^6 + 3*a^2*x^6*e - 4*a*b*c*x^3 + 2*a^2*d*x
^3 + a^2*c)/(a^4*x^6)
```

maple [A] time = 0.07, size = 167, normalized size = 1.28

$$\frac{e}{3\left(b\,x^3 + a\right)a} - \frac{bd}{3\left(b\,x^3 + a\right)a^2} + \frac{e\ln(x)}{a^2} - \frac{e\ln\left(b\,x^3 + a\right)}{3a^2} + \frac{b^2c}{3\left(b\,x^3 + a\right)a^3} - \frac{2bd\ln(x)}{a^3} + \frac{2bd\ln\left(b\,x^3 + a\right)}{3a^3} + \frac{3b^2c\ln(x)}{a^4} - \frac{b^2c\ln\left(b\,x^3 + a\right)}{a^4} - \frac{f}{3\left(b\,x^3 + a\right)b} - \frac{d}{3a^2x^3} + \frac{2bc}{3a^3x^3} - \frac{c}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x)
```

```
[Out] -1/3*e*ln(b*x^3+a)/a^2+2/3/a^3*ln(b*x^3+a)*b*d-1/a^4*ln(b*x^3+a)*b^2*c-1/3/
b/(b*x^3+a)*f+1/3/a/(b*x^3+a)*e-1/3/a^2*b/(b*x^3+a)*d+1/3/a^3*b^2/(b*x^3+a)
*c-1/6*c/a^2/x^6-1/3/a^2/x^3*d+2/3/a^3/x^3*b*c+e*ln(x)/a^2-2/a^3*ln(x)*b*d+
3/a^4*ln(x)*b^2*c
```

maxima [A] time = 1.35, size = 138, normalized size = 1.06

$$\frac{2\left(3\,b^3c - 2\,ab^2d + a^2be - a^3f\right)x^6 - a^2bc + \left(3\,ab^2c - 2\,a^2bd\right)x^3}{6\left(a^3b^2x^9 + a^4bx^6\right)} - \frac{\left(3\,b^2c - 2\,abd + a^2e\right)\log\left(bx^3 + a\right)}{3\,a^4} + \frac{\left(3\,b^2c - 2\,abd + a^2e\right)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*b^3*c - 2*a*b^2*d + a^2*b*e - a^3*f)*x^6 - a^2*b*c + (3*a*b^2*c -
 2*a^2*b*d)*x^3)/(a^3*b^2*x^9 + a^4*b*x^6) - 1/3*(3*b^2*c - 2*a*b*d + a^2*e
)*log(b*x^3 + a)/a^4 + 1/3*(3*b^2*c - 2*a*b*d + a^2*e)*log(x^3)/a^4
```

mupad [B] time = 5.01, size = 130, normalized size = 1.00

$$\frac{\ln(x)\,\left(e\,a^2-2\,d\,a\,b+3\,c\,b^2\right)}{a^4}-\frac{\ln\left(b\,x^3+a\right)\,\left(e\,a^2-2\,d\,a\,b+3\,c\,b^2\right)}{3\,a^4}-\frac{\frac{c}{6\,a}+\frac{x^3\,(2\,a\,d-3\,b\,c)}{6\,a^2}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-2\,d\,a\,b^2+3\,c\,b^3\right)}{3\,a^3\,b}}{b\,x^9+a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x)
```

```
[Out] (log(x)*(3*b^2*c + a^2*e - 2*a*b*d))/a^4 - (log(a + b*x^3)*(3*b^2*c + a^2*e
 - 2*a*b*d))/(3*a^4) - (c/(6*a) + (x^3*(2*a*d - 3*b*c))/(6*a^2) - (x^6*(3*b
^3*c - a^3*f - 2*a*b^2*d + a^2*b*e))/(3*a^3*b))/(a*x^6 + b*x^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.205 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2}\,dx$

Optimal. Leaf size=175

$$\frac{2bc-ad}{6a^3x^6}-\frac{c}{9a^2x^9}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)}{3a^5}-\frac{\log(x)\left(a^3(-f)+2a^2be-3\right.}{a^5}$$

Rubi [A] time = 0.20, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(2a^2be+a^3(-f)-3ab^2d+4b^3c\right)}{3a^5}-\frac{\log(x)\left(2a^2be+a^3(-f)-3ab^2d+4b^3c\right)}{a^5}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{2bc-ad}{6a^3x^6}-\frac{c}{9a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(6*a^3*x^6) - (3*b^2*c - 2*a*b*d + a^2*e)/(3
*a^4*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*(a + b*x^3)) - ((4*b
^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[x])/a^5 + ((4*b^3*c - 3*a*b^2*d +
 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^5)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^4}+\frac{-2bc+ad}{a^3x^3}+\frac{3b^2c-2abd+a^2e}{a^4x^2}+\frac{-4b^3c+3ab^2d-2a^2be+a^3\cdots}{a^5x}\right.\right.\\ &= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{6a^3x^6}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4\left(a+bx^3\right)}-\frac{\left(4b^3c-3ab^2d\cdots\right.}{}\end{aligned}$$

Mathematica [A] time = 0.14, size = 160, normalized size = 0.91

$$\frac{-\frac{2a^3c}{x^9}-\frac{6a\left(a^2e-2abd+3b^2c\right)}{x^3}-\frac{3a^2(ad-2bc)}{x^6}+\frac{6a\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}+6\log\left(a+bx^3\right)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)+18\log(x)\left(a^3f-2a^2be+3ab^2d-4b^3c\right)}{18a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

```
[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-2*b*c + a*d))/x^6 - (6*a*(3*b^2*c - 2*a*b*d + a^
2*e))/x^3 + (6*a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 18*(
-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f)*Log[x] + 6*(4*b^3*c - 3*a*b^2*d +
 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2), x]
```

fricas [A] time = 0.47, size = 261, normalized size = 1.49

$$-\frac{6\left(4\,ab^3c-3\,a^2b^2d+2\,a^3be-a^4f\right)x^9+3\left(4\,a^2b^2c-3\,a^3bd+2\,a^4e\right)x^6+2\,a^4c-\left(4\,a^3bc-3\,a^4d\right)x^3-6\left(\left(4\,b^4c-3\,ab^3d+2\,a^2b^2e-a^3bf\right)x^{12}+\left(4\,ab^3c-3\,a^2b^2d+2\,a^3be-a^4f\right)x^9\right)\log\left(bx^3+a\right)+18\left(\left(4\,b^4c-3\,ab^3d+2\,a^2b^2e-a^3bf\right)x^{12}+\left(4\,ab^3c-3\,a^2b^2d+2\,a^3be-a^4f\right)x^9\right)\log(x)}{18\left(a^5bx^{12}+a^6x^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(6*(4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9 + 3*(4*a^2*b^2*c
 - 3*a^3*b*d + 2*a^4*e)*x^6 + 2*a^4*c - (4*a^3*b*c - 3*a^4*d)*x^3 - 6*((4*b
^4*c - 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^12 + (4*a*b^3*c - 3*a^2*b^2*d +
 2*a^3*b*e - a^4*f)*x^9)*log(b*x^3 + a) + 18*((4*b^4*c - 3*a*b^3*d + 2*a^2*
b^2*e - a^3*b*f)*x^12 + (4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9)*
log(x))/(a^5*b*x^12 + a^6*x^9)
```

giac [A] time = 0.20, size = 275, normalized size = 1.57

$$-\frac{\left(4\,b^3c-3\,ab^2d-a^3f+2\,a^2be\right)\log\left(|x|\right)}{a^5}+\frac{\left(4\,b^4c-3\,ab^3d-a^3bf+2\,a^2b^2e\right)\log\left(|bx^3+a|\right)}{3\,a^5b}-\frac{4\,b^4cx^3-3\,ab^3dx^3-a^3bfx^3+2\,a^2b^2x^3e+5\,ab^3c-4\,a^2b^2d-2\,a^4f+3\,a^3be}{3\left(bx^3+a\right)a^5}+\frac{44\,b^3cx^9-33\,ab^2dx^9-11\,a^3fx^9+22\,a^2bx^9e-18\,ab^2cx^6+12\,a^2bdx^6-6\,a^3x^6e+6\,a^2bcx^3-3\,a^3dx^3-2\,a^3c}{18\,a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(4*b^3*c - 3*a*b^2*d - a^3*f + 2*a^2*b*e)*log(abs(x))/a^5 + 1/3*(4*b^4*c -
 3*a*b^3*d - a^3*b*f + 2*a^2*b^2*e)*log(abs(b*x^3 + a))/(a^5*b) - 1/3*(4*b^
4*c*x^3 - 3*a*b^3*d*x^3 - a^3*b*f*x^3 + 2*a^2*b^2*x^3*e + 5*a*b^3*c - 4*a^2
*b^2*d - 2*a^4*f + 3*a^3*b*e)/((b*x^3 + a)*a^5) + 1/18*(44*b^3*c*x^9 - 33*a
*b^2*d*x^9 - 11*a^3*f*x^9 + 22*a^2*b*x^9*e - 18*a*b^2*c*x^6 + 12*a^2*b*d*x^
6 - 6*a^3*x^6*e + 6*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^5*x^9)
```

maple [A] time = 0.06, size = 229, normalized size = 1.31

$$\frac{f}{3\left(b\,x^3+a\right)a}-\frac{be}{3\left(b\,x^3+a\right)a^2}+\frac{f\ln(x)}{a^2}-\frac{f\ln\left(b\,x^3+a\right)}{3a^2}+\frac{b^2d}{3\left(b\,x^3+a\right)a^3}-\frac{2be\ln(x)}{a^3}+\frac{2be\ln\left(b\,x^3+a\right)}{3a^3}-\frac{b^3c}{3\left(b\,x^3+a\right)a^4}+\frac{3b^2d\ln(x)}{a^4}-\frac{b^2d\ln\left(b\,x^3+a\right)}{a^4}-\frac{4b^3c\ln(x)}{a^5}+\frac{4b^3c\ln\left(b\,x^3+a\right)}{3a^5}-\frac{e}{3a^2x^3}+\frac{2bd}{3a^3x^3}-\frac{b^2c}{a^4x^3}-\frac{d}{6a^2x^6}+\frac{bc}{3a^3x^6}-\frac{c}{9a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*ln(b*x^3+a)*f+2/3*b/a^3*ln(b*x^3+a)*e-b^2/a^4*ln(b*x^3+a)*d+4/3*b^
3/a^5*ln(b*x^3+a)*c+1/3/a/(b*x^3+a)*f-1/3*b/a^2/(b*x^3+a)*e+1/3*b^2/a^3/(b*
x^3+a)*d-1/3*b^3/a^4/(b*x^3+a)*c-1/9/a^2*c/x^9-1/6/a^2/x^6*d+1/3/a^3/x^6*b*
c-1/3/a^2/x^3*e+2/3/a^3/x^3*b*d-1/a^4/x^3*b^2*c+1/a^2*ln(x)*f-2/a^3*ln(x)*b
*e+3/a^4*ln(x)*b^2*d-4/a^5*ln(x)*b^3*c
```

maxima [A] time = 1.43, size = 181, normalized size = 1.03

$$-\frac{6\left(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f\right)x^9+3\left(4\,ab^2c-3\,a^2bd+2\,a^3e\right)x^6+2\,a^3c-\left(4\,a^2bc-3\,a^3d\right)x^3}{18\left(a^4bx^{12}+a^5x^9\right)}+\frac{\left(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,a^5}-\frac{\left(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f\right)\log\left(x^3\right)}{3\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/18*(6*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*x^9 + 3*(4*a*b^2*c - 3*a
^2*b*d + 2*a^3*e)*x^6 + 2*a^3*c - (4*a^2*b*c - 3*a^3*d)*x^3)/(a^4*b*x^12 +
a^5*x^9) + 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*log(b*x^3 + a)/a^5
 - 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*log(x^3)/a^5
```

mupad [**B**] time = 5.08, size = 175, normalized size = 1.00

$$\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{3\,a^5}-\frac{\frac{c}{9\,a}+\frac{x^9\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{3\,a^4}+\frac{x^3\,\left(3\,a\,d-4\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(2\,e\,a^2-3\,d\,a\,b+4\,c\,b^2\right)}{6\,a^3}}{b\,x^{12}+a\,x^9}-\frac{\ln\left(x\right)\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x)
```

```
[Out] (log(a + b*x^3)*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/(3*a^5) - (c/(9*
a) + (x^9*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/(3*a^4) + (x^3*(3*a*d
- 4*b*c))/(18*a^2) + (x^6*(4*b^2*c + 2*a^2*e - 3*a*b*d))/(6*a^3))/(a*x^9 +
b*x^12) - (log(x)*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/a^5
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.206 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=214

$$\frac{2bc-ad}{9a^3x^9}-\frac{c}{12a^2x^{12}}-\frac{a^2e-2abd+3b^2c}{6a^4x^6}-\frac{b\log\left(a+bx^3\right)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{3a^6}+\frac{b\log(x)\left(-2a^3f+3\right.}{}$$

Rubi [A] time = 0.23, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5x^3}-\frac{b\log\left(a+bx^3\right)\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{3a^6}+\frac{b\log(x)\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{a^6}-\frac{a^2e-2abd+3b^2c}{6a^4x^6}+\frac{2bc-ad}{9a^3x^9}-\frac{c}{12a^2x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

```
[Out] -c/(12*a^2*x^12) + (2*b*c - a*d)/(9*a^3*x^9) - (3*b^2*c - 2*a*b*d + a^2*e)/
(6*a^4*x^6) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*(a + b*x^3)) + (b*(5*b^3*c - 4*a*
b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x])/a^6 - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*
b*e - 2*a^3*f)*Log[a + b*x^3])/(3*a^6)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)^2}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{c}{a^2x^5}+\frac{-2bc+ad}{a^3x^4}+\frac{3b^2c-2abd+a^2e}{a^4x^3}+\frac{-4b^3c+3ab^2d-2a^2be+}{a^5x^2}\right.\right.\\
&= -\frac{c}{12a^2x^{12}}+\frac{2bc-ad}{9a^3x^9}-\frac{3b^2c-2abd+a^2e}{6a^4x^6}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}+\frac{b\left(b^3\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 198, normalized size = 0.93

$$-\frac{\frac{3a^4c}{x^{12}}+\frac{4a^3(ad-2bc)}{x^9}+\frac{6a^2\left(a^2e-2abd+3b^2c\right)}{x^6}+\frac{12ab\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}+\frac{12a\left(a^3f-2a^2be+3ab^2d-4b^3c\right)}{x^3}+12b\log\left(a+bx^3\right)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)-36b\log(x)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{36a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

```
[Out] -1/36*((3*a^4*c)/x^12 + (4*a^3*(-2*b*c + a*d))/x^9 + (6*a^2*(3*b^2*c - 2*a*
b*d + a^2*e))/x^6 + (12*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/x^3 +
 (12*a*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) - 36*b*(5*b^3*
c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x] + 12*b*(5*b^3*c - 4*a*b^2*d + 3
*a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/a^6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{13} \left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2), x]
```

fricas [A] time = 0.48, size = 310, normalized size = 1.45

$$\frac{12\left(5ab^4c - 4a^2b^3d + 3a^3b^2e - 2a^4bf\right)x^{12} + 6\left(5a^2b^3c - 4a^3b^2d + 3a^4be - 2a^5f\right)x^9 - 2\left(5a^3b^2c - 4a^4bd + 3a^5e\right)x^6 - 3a^5c + \left(5a^4bc - 4a^5d\right)x^3 - 12\left(\left(5b^5c - 4ab^4d + 3a^2b^3e - 2a^3b^2f\right)x^{15} + \left(5ab^4c - 4a^2b^3d + 3a^3b^2e - 2a^4bf\right)x^{12}\right)\log\left(bx^3 + a\right) + 36\left(\left(5b^5c - 4ab^4d + 3a^2b^3e - 2a^3b^2f\right)x^{15} + \left(5ab^4c - 4a^2b^3d + 3a^3b^2e - 2a^4bf\right)x^{12}\right)\log(x)}{36\left(a^6bx^{15} + a^7x^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*(5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^4*b*f)*x^12 + 6*(5*a^
2*b^3*c - 4*a^3*b^2*d + 3*a^4*b*e - 2*a^5*f)*x^9 - 2*(5*a^3*b^2*c - 4*a^4*b
*d + 3*a^5*e)*x^6 - 3*a^5*c + (5*a^4*b*c - 4*a^5*d)*x^3 - 12*((5*b^5*c - 4*
a*b^4*d + 3*a^2*b^3*e - 2*a^3*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^
3*b^2*e - 2*a^4*b*f)*x^12)*log(b*x^3 + a) + 36*((5*b^5*c - 4*a*b^4*d + 3*a^
2*b^3*e - 2*a^3*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^
4*b*f)*x^12)*log(x))/(a^6*b*x^15 + a^7*x^12)
```

giac [A] time = 0.17, size = 331, normalized size = 1.55

$$\frac{\left(5b^4c - 4ab^3d - 2a^3bf + 3a^2b^2e\right)\log\left(|x|\right)}{a^6} - \frac{\left(5b^5c - 4ab^4d - 2a^3b^2f + 3a^2b^3e\right)\log\left(|bx^3 + a|\right)}{3a^6b} + \frac{5b^5cx^3 - 4ab^4dx^3 - 2a^3b^2fx^3 + 3a^2b^3x^3e + 6ab^4c - 5a^2b^3d - 3a^4bf + 4a^3b^2e}{3\left(bx^3 + a\right)a^6} - \frac{125b^4cx^{12} - 100ab^3dx^{12} - 50a^3bfx^{12} + 75a^2b^2x^{12}e - 48ab^3cx^9 + 36a^2b^2dx^9 + 12a^4fx^9 - 24a^3bx^9e + 18a^2b^2cx^6 - 12a^3bdx^6 + 6a^4x^6e - 8a^3bcx^3 + 4a^4dx^3 + 3a^4c}{36a^6x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (5*b^4*c - 4*a*b^3*d - 2*a^3*b*f + 3*a^2*b^2*e)*log(abs(x))/a^6 - 1/3*(5*b^
5*c - 4*a*b^4*d - 2*a^3*b^2*f + 3*a^2*b^3*e)*log(abs(b*x^3 + a))/(a^6*b) +
1/3*(5*b^5*c*x^3 - 4*a*b^4*d*x^3 - 2*a^3*b^2*f*x^3 + 3*a^2*b^3*x^3*e + 6*a*
b^4*c - 5*a^2*b^3*d - 3*a^4*b*f + 4*a^3*b^2*e)/((b*x^3 + a)*a^6) - 1/36*(12
5*b^4*c*x^12 - 100*a*b^3*d*x^12 - 50*a^3*b*f*x^12 + 75*a^2*b^2*x^12*e - 48*
a*b^3*c*x^9 + 36*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 24*a^3*b*x^9*e + 18*a^2*b^2
*c*x^6 - 12*a^3*b*d*x^6 + 6*a^4*x^6*e - 8*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4
*c)/(a^6*x^12)
```

maple [A] time = 0.07, size = 282, normalized size = 1.32

$$-\frac{bf}{3\left(bx^3+a\right)a^2}+\frac{b^2e}{3\left(bx^3+a\right)a^3}-\frac{2bf\ln(x)}{a^3}+\frac{2bf\ln\left(bx^3+a\right)}{3a^3}-\frac{b^3d}{3\left(bx^3+a\right)a^4}+\frac{3b^2e\ln(x)}{a^4}-\frac{b^2e\ln\left(bx^3+a\right)}{a^4}+\frac{b^4c}{3\left(bx^3+a\right)a^5}-\frac{4b^3d\ln(x)}{a^5}+\frac{4b^3d\ln\left(bx^3+a\right)}{3a^5}+\frac{5b^4c\ln(x)}{a^6}-\frac{5b^4c\ln\left(bx^3+a\right)}{3a^6}-\frac{f}{3a^2x^3}+\frac{2be}{3a^3x^3}-\frac{b^2d}{a^4x^3}+\frac{4b^3c}{3a^5x^3}-\frac{e}{6a^2x^6}+\frac{bd}{3a^3x^6}-\frac{b^2c}{2a^4x^6}-\frac{d}{9a^2x^9}+\frac{2bc}{9a^3x^9}-\frac{c}{12a^2x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x)
```

```
[Out] 2/3/a^3*b*ln(b*x^3+a)*f-1/a^4*b^2*ln(b*x^3+a)*e+4/3/a^5*b^3*ln(b*x^3+a)*d-5
/3/a^6*b^4*ln(b*x^3+a)*c-1/3/a^2*b/(b*x^3+a)*f+1/3/a^3*b^2/(b*x^3+a)*e-1/3/
a^4*b^3/(b*x^3+a)*d+1/3/a^5*b^4/(b*x^3+a)*c-1/12*c/a^2/x^12-1/9/a^2/x^9*d+2
/9/a^3/x^9*b*c-1/6/a^2/x^6*e+1/3/a^3/x^6*b*d-1/2/a^4/x^6*b^2*c-1/3/a^2/x^3*
```

```
f+2/3/a^3/x^3*b*e-1/a^4/x^3*b^2*d+4/3/a^5/x^3*b^3*c-2*b/a^3*ln(x)*f+3*b^2/a
^4*ln(x)*e-4*b^3/a^5*ln(x)*d+5*b^4/a^6*ln(x)*c
```

maxima [A] time = 1.44, size = 226, normalized size = 1.06

$$\frac{12\left(5b^4c-4ab^3d+3a^2b^2e-2a^3bf\right)x^{12}+6\left(5ab^3c-4a^2b^2d+3a^3be-2a^4f\right)x^9-2\left(5a^2b^2c-4a^3bd+3a^4e\right)x^6-3a^4c+\left(5a^3bc-4a^4d\right)x^3}{36\left(a^5bx^{15}+a^6x^{12}\right)}-\frac{\left(5b^4c-4ab^3d+3a^2b^2e-2a^3bf\right)\log\left(bx^3+a\right)}{3a^6}+\frac{\left(5b^4c-4ab^3d+3a^2b^2e-2a^3bf\right)\log\left(x^3\right)}{3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/36*(12*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*x^12 + 6*(5*a*b^3*
c - 4*a^2*b^2*d + 3*a^3*b*e - 2*a^4*f)*x^9 - 2*(5*a^2*b^2*c - 4*a^3*b*d + 3
*a^4*e)*x^6 - 3*a^4*c + (5*a^3*b*c - 4*a^4*d)*x^3)/(a^5*b*x^15 + a^6*x^12)
- 1/3*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*log(b*x^3 + a)/a^6 +
1/3*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*log(x^3)/a^6
```

mupad [B] time = 5.09, size = 216, normalized size = 1.01

$$\frac{\ln(x)\left(-2fa^3b+3ea^2b^2-4dab^3+5cb^4\right)}{a^6}-\frac{\ln\left(bx^3+a\right)\left(-2fa^3b+3ea^2b^2-4dab^3+5cb^4\right)}{3a^6}-\frac{\frac{c}{12a}-\frac{x^9\left(-2fa^3+3ea^2b-4dab^2+5cb^3\right)}{6a^4}+\frac{x^3\left(4ad-5bc\right)}{36a^2}+\frac{x^6\left(3ea^2-4dab+5cb^2\right)}{18a^3}-\frac{bx^{12}\left(-2fa^3+3ea^2b-4dab^2+5cb^3\right)}{3a^5}}{bx^{15}+ax^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x)
```

```
[Out] (log(x)*(5*b^4*c + 3*a^2*b^2*e - 4*a*b^3*d - 2*a^3*b*f))/a^6 - (log(a + b*x
^3)*(5*b^4*c + 3*a^2*b^2*e - 4*a*b^3*d - 2*a^3*b*f))/(3*a^6) - (c/(12*a) -
(x^9*(5*b^3*c - 2*a^3*f - 4*a*b^2*d + 3*a^2*b*e))/(6*a^4) + (x^3*(4*a*d - 5
*b*c))/(36*a^2) + (x^6*(5*b^2*c + 3*a^2*e - 4*a*b*d))/(18*a^3) - (b*x^12*(5
*b^3*c - 2*a^3*f - 4*a*b^2*d + 3*a^2*b*e))/(3*a^5))/(a*x^12 + b*x^15)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.207 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=369

$$\frac{x^7\left(3a^2f-2abe+b^2d\right)}{7b^4}-\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{ax\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{b^6}+\frac{x^4\left(-4a^3f+3a\right.}{}$$

Rubi [A] time = 0.47, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1828, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{x^4\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{4b^5}-\frac{a^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(13a^2be-16a^3f-10ab^2d+7b^3c\right)}{18b^{19/3}}-\frac{ax\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{b^6}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(13a^2be-16a^3f-10ab^2d+7b^3c\right)}{9b^{19/3}}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(13a^2be-16a^3f-10ab^2d+7b^3c\right)}{3\sqrt{3}b^{19/3}}+\frac{x^7\left(3a^2f-2abe+b^2d\right)}{7b^4}+\frac{x^{10}(be-2af)}{10b^3}+\frac{fx^{13}}{13b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] -((a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x)/b^6) + ((b^3*c - 2*a*b^
2*d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)
/(7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) - (a^2*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^6*(a + b*x^3)) - (a^(4/3)*(7*b^3*c - 1
0*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(3*Sqrt[3]*b^(19/3)) + (a^(4/3)*(7*b^3*c - 10*a*b^2*d + 13*a^2*b
*e - 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) - (a^(4/3)*(7*b^3*c -
 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*b^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^6\left(a+bx^3\right)} - \frac{\int \frac{-a^3\left(b^3c-ab^2d+a^2be-a^3f\right)+3a^2b\left(b^3c-ab^2d+a^2be-a^3\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^6\left(a+bx^3\right)} - \frac{\int\left(3a^2\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)-3\cdots\right.}{\cdots}$$

$$= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5} + \frac{(b\cdots}{\cdots}$$

$$= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5} + \frac{(b\cdots}{\cdots}$$

$$= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5} + \frac{(b\cdots}{\cdots}$$

$$= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5} + \frac{(b\cdots}{\cdots}$$

$$= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5} + \frac{(b\cdots}{\cdots}$$

Mathematica [A]   time = 0.44, size = 364, normalized size = 0.99

$$\frac{x^7\left(3a^2f-2abe+b^2d\right)}{7b^4}+\frac{a^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^6\left(a+bx^3\right)}+\frac{ax\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)}{b^6}+\frac{x^4\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{4b^5}+\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(16a^3f-13a^2be+10ab^2d-7b^3c\right)}{18b^{19/3}}-\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(16a^3f-13a^2be+10ab^2d-7b^3c\right)}{9b^{19/3}}+\frac{a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(16a^3f-13a^2be+10ab^2d-7b^3c\right)}{3\sqrt{3}b^{19/3}}+\frac{x^{10}(be-2af)}{10b^3}+\frac{fx^{13}}{13b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2*
d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)/(
7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) + (a^2*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x)/(3*b^6*(a + b*x^3)) + (a^(4/3)*(-7*b^3*c +
 10*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/(3*Sqrt[3]*b^(19/3)) - (a^(4/3)*(-7*b^3*c + 10*a*b^2*d - 13*a^2*b*e
+ 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) + (a^(4/3)*(-7*b^3*c + 1
0*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*b^(19/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 \left(c + dx^3 + ex^6 + fx^9\right)}{\left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]
```

fricas [A] time = 0.43, size = 488, normalized size = 1.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16380*(1260*b^5*f*x^16 + 126*(13*b^5*e - 16*a*b^4*f)*x^13 + 234*(10*b^5*d
 - 13*a*b^4*e + 16*a^2*b^3*f)*x^10 + 585*(7*b^5*c - 10*a*b^4*d + 13*a^2*b^3
*e - 16*a^3*b^2*f)*x^7 - 4095*(7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16
*a^4*b*f)*x^4 - 1820*sqrt(3)*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*
a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^
(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(7*a^2*b
^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 1
3*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3)) - 1820*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f + (7*a
*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1/3)*log(x
- (-a/b)^(1/3)) - 5460*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f)
*x)/(b^7*x^3 + a*b^6)
```

giac [A] time = 0.20, size = 451, normalized size = 1.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d - 16*(-
a*b^2)^(1/3)*a^4*f + 13*(-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/b^7 - 1/9*(7*a^2*b^3*c - 10*a^3*b^2*d - 16*a^5*f
 + 13*a^4*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) + 1/18*(7*(-
a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d - 16*(-a*b^2)^(1/3)*a^4*
f + 13*(-a*b^2)^(1/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7
 - 1/3*(a^2*b^3*c*x - a^3*b^2*d*x - a^5*f*x + a^4*b*x*e)/((b*x^3 + a)*b^6)
+ 1/1820*(140*b^24*f*x^13 - 364*a*b^23*f*x^10 + 182*b^24*x^10*e + 260*b^24*
d*x^7 + 780*a^2*b^22*f*x^7 - 520*a*b^23*x^7*e + 455*b^24*c*x^4 - 910*a*b^23
```

*d*x^4 - 1820*a^3*b^21*f*x^4 + 1365*a^2*b^22*x^4*e - 3640*a*b^23*c*x + 5460
*a^2*b^22*d*x + 9100*a^4*b^20*f*x - 7280*a^3*b^21*x*e)/b^26

maple [A] time = 0.05, size = 622, normalized size = 1.69

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] -10/9*a^3/b^5*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
+7/9*a^2/b^4*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-
16/9*a^5/b^7*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+
13/9*a^4/b^6*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+
3/7/b^4*x^7*a^2*f-1/5/b^3*x^10*a*f-1/2/b^3*x^4*a*d+3/4/b^4*x^4*a^2*e-2/7/b^
3*x^7*a*e-1/b^5*x^4*a^3*f+5*a^4/b^6*f*x-4*a^3/b^5*e*x+3*a^2/b^4*d*x-2*a/b^3
*c*x-16/9*a^5/b^7*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+8/9*a^5/b^7*f/(a/b)^(2/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+13/9*a^4/b^6*e/(a/b)^(2/3)*ln(x+(a/b)^(1
/3))-13/18*a^4/b^6*e/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-10/9*a^3
/b^5*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-7/18*a^2/b^4*c/(a/b)^(2/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))+1/10/b^2*x^10*e+1/7/b^2*x^7*d+1/4/b^2*x^4*c+7/9*a^2/
b^4*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/3*a^3/b^4*x/(b*x^3+a)*d-1/3*a^2/b^3*x
/(b*x^3+a)*c+1/3*a^5/b^6*x/(b*x^3+a)*f-1/3*a^4/b^5*x/(b*x^3+a)*e+5/9*a^3/b^
5*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/13*f*x^13/b^2

maxima [A] time = 2.98, size = 369, normalized size = 1.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] -1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x/(b^7*x^3 + a*b^6) + 1/1820
*(140*b^4*f*x^13 + 182*(b^4*e - 2*a*b^3*f)*x^10 + 260*(b^4*d - 2*a*b^3*e +
3*a^2*b^2*f)*x^7 + 455*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^4 -
1820*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x)/b^6 + 1/9*sqrt(3)*(
7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f)*arctan(1/3*sqrt(3)*(2*x
 - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(2/3)) - 1/18*(7*a^2*b^3*c - 10*a^3
*b^2*d + 13*a^4*b*e - 16*a^5*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^7
(a/b)^(2/3)) + 1/9(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f)*lo
g(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))

mupad [B] time = 0.35, size = 481, normalized size = 1.30

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)

[Out] x^10*(e/(10*b^2) - (a*f)/(5*b^3)) - x*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/
b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b))
/b - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b^2) - x^7
*((a^2*f)/(7*b^4) - d/(7*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(7*b)) + x^4*(c
/(4*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(4*b^2) + (a*((a^2*f)/b^4 - d/b^2 +
(2*a*(e/b^2 - (2*a*f)/b^3))/b))/(2*b)) + (f*x^13)/(13*b^2) + (x*((a^5*f)/3
- (a^2*b^3*c)/3 + (a^3*b^2*d)/3 - (a^4*b*e)/3))/(a*b^6 + b^7*x^3) + (a^(4/3
)*log(b^(1/3)*x + a^(1/3))*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e))/

```
(9*b^(19/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3
^(1/2)*1i)/2 - 1/2)*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e))/(9*b^(1
9/3)) - (a^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e))/(9*b^(19/3))
```

sympy [A] time = 15.90, size = 500, normalized size = 1.36

$$x^{10}\left(-\frac{af}{5b^3}+\frac{e}{10b^2}\right)+x^7\left(\frac{3a^2f}{7b^4}-\frac{2ae}{7b^3}+\frac{d}{7b^2}\right)+x^4\left(-\frac{a^3f}{b^5}+\frac{3a^2e}{4b^4}-\frac{ad}{2b^3}+\frac{c}{4b^2}\right)+x\left(\frac{5a^4f}{b^6}-\frac{4a^3e}{b^5}+\frac{3a^2d}{b^4}-\frac{2ac}{b^3}\right)+\frac{x\left(a^5f-a^4be+a^3b^2d-a^2b^3c\right)}{3ab^6+3b^7x^3}+\operatorname{RootSum}\left(729t^3b^{19}+4096a^{13}f^3-9984a^{12}bef^2+7680a^{11}b^2df^2+8112a^{11}b^2e^2f-5376a^{10}b^3cf^2-12480a^{10}b^3def-2197a^{10}b^3e^3+8736a^9b^4cef+4800a^9b^4d^2f+5070a^9b^4de^2-6720a^8b^5cdf-3549a^8b^5ce^2-3900a^8b^5d^2e+2352a^7b^6c^2f+5460a^7b^6cde+1000a^7b^6d^3-1911a^6b^7c^2e-2100a^6b^7cd^2+1470a^5b^8c^2d-343a^4b^9c^3,\left(t\mapsto t\log\left(-\frac{9tb^6}{16a^4f-13a^3be+10a^2b^2d-7ab^3c}+x\right)\right)\right)+\frac{fx^{13}}{13b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**10*(-a*f/(5*b**3) + e/(10*b**2)) + x**7*(3*a**2*f/(7*b**4) - 2*a*e/(7*b*
*3) + d/(7*b**2)) + x**4*(-a**3*f/b**5 + 3*a**2*e/(4*b**4) - a*d/(2*b**3) +
 c/(4*b**2)) + x*(5*a**4*f/b**6 - 4*a**3*e/b**5 + 3*a**2*d/b**4 - 2*a*c/b**
3) + x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 + 3*b**7*x
**3) + RootSum(729*_t**3*b**19 + 4096*a**13*f**3 - 9984*a**12*b*e*f**2 + 76
80*a**11*b**2*d*f**2 + 8112*a**11*b**2*e**2*f - 5376*a**10*b**3*c*f**2 - 12
480*a**10*b**3*d*e*f - 2197*a**10*b**3*e**3 + 8736*a**9*b**4*c*e*f + 4800*a
**9*b**4*d**2*f + 5070*a**9*b**4*d*e**2 - 6720*a**8*b**5*c*d*f - 3549*a**8*
b**5*c*e**2 - 3900*a**8*b**5*d**2*e + 2352*a**7*b**6*c**2*f + 5460*a**7*b**
6*c*d*e + 1000*a**7*b**6*d**3 - 1911*a**6*b**7*c**2*e - 2100*a**6*b**7*c*d*
*2 + 1470*a**5*b**8*c**2*d - 343*a**4*b**9*c**3, Lambda(_t, _t*log(-9*_t*b*
*6/(16*a**4*f - 13*a**3*b*e + 10*a**2*b**2*d - 7*a*b**3*c) + x))) + f*x**13
/(13*b**2)
```

3.208 $\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{x^5\left(3a^2f-2abe+b^2d\right)}{5b^4}+\frac{x^2\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{2b^5}+\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{a^{2/3}\log\left(a^{2/3}\right.}{}$$

Rubi [A] time = 0.71, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{2b^5}+\frac{ax^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(11a^2be-14a^3f-8ab^2d+5b^3c\right)}{18b^{17/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(11a^2be-14a^3f-8ab^2d+5b^3c\right)}{9b^{17/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(11a^2be-14a^3f-8ab^2d+5b^3c\right)}{3\sqrt{3}b^{17/3}}+\frac{x^5\left(3a^2f-2abe+b^2d\right)}{5b^4}+\frac{x^8(be-2af)}{8b^3}+\frac{fx^{11}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/(2*b^5) + ((b^2*d - 2*a*b*e
 + 3*a^2*f)*x^5)/(5*b^4) + ((b*e - 2*a*f)*x^8)/(8*b^3) + (f*x^11)/(11*b^2)
+ (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^5*(a + b*x^3)) + (a^(2/3
)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(17/3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*
d + 11*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(17/3)) - (a^(2/3
)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(18*b^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}dx = \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^5\left(a+bx^3\right)} - \frac{\int \frac{2a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)x-3ab^2\left(b^3c-ab^2d+a^2be-a^3\right.}{a}}{3}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^5\left(a+bx^3\right)} - \frac{\int \frac{x\left(2a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)-3ab^2\left(b^3c-ab^2d+a^2be-a\right.\right.}{a}}{3}$$

$$= \frac{fx^{11}}{11b^2} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^5\left(a+bx^3\right)} - \frac{\int x\left(22a^2b^2\left(b^3c-ab^2d+a^2be-a^3f\right)-33ab^3\left(b^3c\right.\right.}{}$$

$$= \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^5\left(a+bx^3\right)} - \frac{\int x\left(176a^2b^3\left(b^3c-ab^2d+\right.\right.}{}$$

$$= \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^5\left(a+bx^3\right)} - \frac{\int\left(-264ab^3\left(b^3c-\right.\right.}{}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^2}{2b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} +$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^2}{2b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} +$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^2}{2b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} +$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^2}{2b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} +$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^2}{2b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} +$$

Mathematica [A] time = 0.20, size = 319, normalized size = 0.95

$$\frac{792b^{5/3}x^5\left(3a^2f-2abe+b^2d\right)+1980b^{2/3}x^2\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)+\frac{1320ab^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a+bx^3}-440a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(14a^3f-11a^2be+8ab^2d-5b^3c\right)-440\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(14a^3f-11a^2be+8ab^2d-5b^3c\right)+220a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(14a^3f-11a^2be+8ab^2d-5b^3c\right)+495b^{8/3}x^8(be-2af)+360b^{11/3}fx^{11}}{3960b^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (1980*b^(2/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2 + 792*b^(5/3)*(
b^2*d - 2*a*b*e + 3*a^2*f)*x^5 + 495*b^(8/3)*(b*e - 2*a*f)*x^8 + 360*b^(11/
3)*f*x^11 + (1320*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b
*x^3) - 440*Sqrt[3]*a^(2/3)*(-5*b^3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-5*b^3*c + 8*a*b
^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-5*b^
3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(3960*b^(17/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]
```

fricas [A] time = 0.42, size = 455, normalized size = 1.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3960*(360*b^4*f*x^14 + 45*(11*b^4*e - 14*a*b^3*f)*x^11 + 99*(8*b^4*d - 11
*a*b^3*e + 14*a^2*b^2*f)*x^8 + 396*(5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14
*a^3*b*f)*x^5 + 660*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*x^2 -
 440*sqrt(3)*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*c -
8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*
sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(5*a*b^3*c - 8*a^2*b^2*d
 + 11*a^3*b*e - 14*a^4*f + (5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f
)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/
3)) - 440*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*c - 8*a
*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2
/b^2)^(2/3)))/(b^6*x^3 + a*b^5)
```

giac [A] time = 0.20, size = 442, normalized size = 1.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(5*a*b^3*c*(-a/b)^(1/3) - 8*a^2*b^2*d*(-a/b)^(1/3) - 14*a^4*f*(-a/b)^(1
/3) + 11*a^3*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b
^5) + 1/9*sqrt(3)*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a*b^2)^(2/3)*a*b^2*d - 14*(
-a*b^2)^(2/3)*a^3*f + 11*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/3*(a*b^3*c*x^2 - a^2*b^2*d*x^2 - a^4*f*
x^2 + a^3*b*x^2*e)/((b*x^3 + a)*b^5) - 1/18*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a
*b^2)^(2/3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 11*(-a*b^2)^(2/3)*a^2*b*e)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 + 1/440*(40*b^20*f*x^11 - 110*
a*b^19*f*x^8 + 55*b^20*x^8*e + 88*b^20*d*x^5 + 264*a^2*b^18*f*x^5 - 176*a*b
^19*x^5*e + 220*b^20*c*x^2 - 440*a*b^19*d*x^2 - 880*a^3*b^17*f*x^2 + 660*a^
2*b^18*x^2*e)/b^22
```

maple [B] time = 0.05, size = 584, normalized size = 1.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*a^3/b^4*x^2/(b*x^3+a)*e-1/3*a^2/b^3*x^2/(b*x^3+a)*d+1/3*a/b^2*x^2/(b*x^
3+a)*c+4/9*a^2/b^4*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+14/9*a^4
/b^6*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-5/9*a/b^
3*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-11/9*a^3/b^
5*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+8/9*a^2/b^4
*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/4/b^3*x^8*
a*f+3/2/b^4*x^2*a^2*e-1/b^3*x^2*a*d-2/b^5*x^2*a^3*f+3/5*a^2/b^4*f*x^5-2/5*a
/b^3*e*x^5+1/5/b^2*d*x^5-8/9*a^2/b^4*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-14/9*a
```

```
^4/b^6*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7/9*a^4/b^6*f/(a/b)^(1/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))+11/9*a^3/b^5*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-11/18*
a^3/b^5*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*a^4/b^5*x^2/(b*
x^3+a)*f+1/2/b^2*x^2*c+5/9*a/b^3*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18*a/b^3
*c/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/8/b^2*x^8*e+1/11*f*x^11/
b^2
```

maxima [A] time = 3.02, size = 325, normalized size = 0.97

$$\frac{\left(ab^3c - a^2b^2d + a^3be - a^4f\right)x^2}{3\left(b^6x^3 + ab^5\right)} - \frac{\sqrt{3}\left(5\,ab^3c - 8\,a^2b^2d + 11\,a^3be - 14\,a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{40\,b^3fx^{11} + 55\left(b^3e - 2\,ab^2f\right)x^8 + 88\left(b^3d - 2\,ab^2e + 3\,a^2bf\right)x^5 + 220\left(b^3c - 2\,ab^2d + 3\,a^2be - 4\,a^3f\right)x^2}{440\,b^5} - \frac{\left(5\,ab^3c - 8\,a^2b^2d + 11\,a^3be - 14\,a^4f\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(5\,ab^3c - 8\,a^2b^2d + 11\,a^3be - 14\,a^4f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^2/(b^6*x^3 + a*b^5) - 1/9*sqr
t(3)*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/440*(40*b^3*f*x^11 +
55*(b^3*e - 2*a*b^2*f)*x^8 + 88*(b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^5 + 220*(
b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/b^5 - 1/18*(5*a*b^3*c - 8*a^2
*b^2*d + 11*a^3*b*e - 14*a^4*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6
*(a/b)^(1/3)) + 1/9*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*log(x
 + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

mupad [B] time = 5.28, size = 362, normalized size = 1.08

$$x^8\left(\frac{e}{8\,b^2} - \frac{a\,f}{4\,b^3}\right) - x^5\left(\frac{a^2\,f}{5\,b^4} - \frac{d}{5\,b^2} + \frac{2\,a\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right)}{5\,b}\right) + x^2\left(\frac{c}{2\,b^2} - \frac{a^2\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right)}{2\,b^2} + \frac{a\left(\frac{a^2\,f}{b^4} - \frac{d}{b^2} + \frac{2\,a\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right) + \frac{f\,x^{11}}{11\,b^2} - \frac{x^2\left(\frac{f\,a^4}{3} - \frac{e\,a^3\,b}{3} + \frac{d\,a^2\,b^2}{3} - \frac{c\,a\,b^3}{3}\right)}{b^6\,x^3 + a\,b^5} + \frac{a^{2/3}\ln\left(b^{1/3}\,x + a^{1/3}\right)\left(-14\,f\,a^3 + 11\,e\,a^2\,b - 8\,d\,a\,b^2 + 5\,c\,b^3\right)}{9\,b^{17/3}} - \frac{a^{2/3}\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-14\,f\,a^3 + 11\,e\,a^2\,b - 8\,d\,a\,b^2 + 5\,c\,b^3\right)}{9\,b^{17/3}} + \frac{a^{2/3}\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-14\,f\,a^3 + 11\,e\,a^2\,b - 8\,d\,a\,b^2 + 5\,c\,b^3\right)}{9\,b^{17/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^8*(e/(8*b^2) - (a*f)/(4*b^3)) - x^5*((a^2*f)/(5*b^4) - d/(5*b^2) + (2*a*(
e/b^2 - (2*a*f)/b^3))/(5*b)) + x^2*(c/(2*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))
/(2*b^2) + (a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) + (
f*x^11)/(11*b^2) - (x^2*((a^4*f)/3 + (a^2*b^2*d)/3 - (a*b^3*c)/3 - (a^3*b*e
)/3))/(a*b^5 + b^6*x^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 14*a
^3*f - 8*a*b^2*d + 11*a^2*b*e))/(9*b^(17/3)) - (a^(2/3)*log(3^(1/2)*a^(1/3)
*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 14*a^3*f - 8
*a*b^2*d + 11*a^2*b*e))/(9*b^(17/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*
b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 14*a^3*f - 8*a*b^2*d
 + 11*a^2*b*e))/(9*b^(17/3))
```

sympy [A] time = 58.23, size = 539, normalized size = 1.61

$$x^8\left(-\frac{af}{4b^3} + \frac{e}{8b^2}\right) + x^5\left(\frac{3a^2f}{5b^4} - \frac{2ae}{5b^3} + \frac{d}{5b^2}\right) + x^2\left(-\frac{2a^3f}{b^5} + \frac{3a^2e}{2b^4} - \frac{ad}{b^3} + \frac{c}{2b^2}\right) + \frac{x^2\left(-a^4f + a^3be - a^2b^2d + ab^3c\right)}{3ab^5 + 3b^6x^3} + \operatorname{RootSum}\left(729t^3b^{17} + 2744a^{11}f^3 - 6468a^{10}bef^2 + 4704a^9b^2df^2 + 5082a^9b^2e^2f - 2940a^8b^3cf^2 - 7392a^8b^3def - 1331a^8b^3e^3 + 4620a^7b^4cef + 2688a^7b^4d^2f + 2904a^7b^4de^2 - 3360a^6b^5cdf - 1815a^6b^5ce^2 - 2112a^6b^5d^2e + 1050a^5b^6c^2f + 2640a^5b^6cde + 512a^5b^6d^3 - 825a^4b^7c^2e - 960a^4b^7cd^2 + 600a^3b^8c^2d - 125a^2b^9c^3, \left(t \mapsto t\log\left(\frac{81t^2b^{11}}{196a^7f^2 - 308a^6bef + 224a^5b^2df + 121a^5b^2e^2 - 140a^4b^3cf - 176a^4b^3de + 110a^3b^4ce + 64a^3b^4d^2 - 80a^2b^5cd + 25ab^6c^2} + x\right)\right)\right) + \frac{fx^{11}}{11b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**8*(-a*f/(4*b**3) + e/(8*b**2)) + x**5*(3*a**2*f/(5*b**4) - 2*a*e/(5*b**3
) + d/(5*b**2)) + x**2*(-2*a**3*f/b**5 + 3*a**2*e/(2*b**4) - a*d/b**3 + c/(
2*b**2)) + x**2*(-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(3*a*b**5 + 3
*b**6*x**3) + RootSum(729*_t**3*b**17 + 2744*a**11*f**3 - 6468*a**10*b*e*f*
*2 + 4704*a**9*b**2*d*f**2 + 5082*a**9*b**2*e**2*f - 2940*a**8*b**3*c*f**2
- 7392*a**8*b**3*d*e*f - 1331*a**8*b**3*e**3 + 4620*a**7*b**4*c*e*f + 2688*
a**7*b**4*d**2*f + 2904*a**7*b**4*d*e**2 - 3360*a**6*b**5*c*d*f - 1815*a**6
*b**5*c*e**2 - 2112*a**6*b**5*d**2*e + 1050*a**5*b**6*c**2*f + 2640*a**5*b*
*6*c*d*e + 512*a**5*b**6*d**3 - 825*a**4*b**7*c**2*e - 960*a**4*b**7*c*d**2
 + 600*a**3*b**8*c**2*d - 125*a**2*b**9*c**3, Lambda(_t, _t*log(81*_t**2*b*
```

```
*11/(196*a**7*f**2 - 308*a**6*b*e*f + 224*a**5*b**2*d*f + 121*a**5*b**2*e**
2 - 140*a**4*b**3*c*f - 176*a**4*b**3*d*e + 110*a**3*b**4*c*e + 64*a**3*b**
4*d**2 - 80*a**2*b**5*c*d + 25*a*b**6*c**2) + x))) + f*x**11/(11*b**2)
```

3.209 $\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=328

$$\frac{x^4\left(3a^2f-2abe+b^2d\right)}{4b^4}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-13a^3f+10a^2be-7ab^2d+4b^3c\right)}{9b^{16/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-13\right.}{3\sqrt{}}$$

Rubi [A] time = 0.37, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1828, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(10a^2be-13a^3f-7ab^2d+4b^3c\right)}{18b^{16/3}}+\frac{x\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{b^5}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(10a^2be-13a^3f-7ab^2d+4b^3c\right)}{9b^{16/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(10a^2be-13a^3f-7ab^2d+4b^3c\right)}{3\sqrt{3}b^{16/3}}+\frac{x^4\left(3a^2f-2abe+b^2d\right)}{4b^4}+\frac{x^7(be-2af)}{7b^3}+\frac{fx^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a
^2*f)*x^4)/(4*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^10)/(10*b^2) + (a*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^5*(a + b*x^3)) + (a^(1/3)*(4*b^3
*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(3*Sqrt[3]*b^(16/3)) - (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a
^2*b*e - 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(16/3)) + (a^(1/3)*(4*b^3
*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(18*b^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx = \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^5\left(a+bx^3\right)} - \frac{\int \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)-3ab\left(b^3c-ab^2d+a^2be-a^3f\right)x^3-3ab \ldots}{a+bx^3}}{3ab^5}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^5\left(a+bx^3\right)} - \frac{\int \left(-3a\left(b^3c-2ab^2d+3a^2be-4a^3f\right)-3ab\left(b^2 \ldots\right.\right.}{\ldots}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^4}{4b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx \ldots}{10 \ldots}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^4}{4b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx \ldots}{10 \ldots}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^4}{4b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx \ldots}{10 \ldots}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^4}{4b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx \ldots}{10 \ldots}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^4}{4b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx \ldots}{10 \ldots}$$

Mathematica [A] time = 0.30, size = 315, normalized size = 0.96

$$\frac{315b^{4/3}x^4\left(3a^2f-2abe+b^2d\right)+\frac{420a\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a+bx^3}+1260\sqrt[3]{b}x\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)+140\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(13a^3f-10a^2be+7ab^2d-4b^3c\right)-140\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(13a^3f-10a^2be+7ab^2d-4b^3c\right)-70\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(13a^3f-10a^2be+7ab^2d-4b^3c\right)+180b^{7/3}x^7(be-2af)+126b^{10/3}fx^{10}}{1260b^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x + 315*b^(4/3)*(b^
2*d - 2*a*b*e + 3*a^2*f)*x^4 + 180*b^(7/3)*(b*e - 2*a*f)*x^7 + 126*b^(10/3)
*f*x^10 + (420*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)
 - 140*Sqrt[3]*a^(1/3)*(-4*b^3*c + 7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-4*b^3*c + 7*a*b^2*d
- 10*a^2*b*e + 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-4*b^3*c +
7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(1260*b^(16/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]
```

fricas [A] time = 0.42, size = 423, normalized size = 1.29

$$\frac{126\,b^4fx^{13}+18\left(10\,b^4e-13\,ab^3f\right)x^{10}+45\left(7\,b^4d-10\,ab^3e+13\,a^2b^2f\right)x^7+315\left(4\,b^4c-7\,ab^3d+10\,a^2b^2e-13\,a^3bf\right)x^4-140\,\sqrt{3}\left(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f+\left(4\,b^4c-7\,ab^3d+10\,a^2b^2e-13\,a^3bf\right)x^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+70\left(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f+\left(4\,b^4c-7\,ab^3d+10\,a^2b^2e-13\,a^3bf\right)x^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-140\left(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f+\left(4\,b^4c-7\,ab^3d+10\,a^2b^2e-13\,a^3bf\right)x^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+420\left(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f\right)x}{1260\left(b^6x^3+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(126*b^4*f*x^13 + 18*(10*b^4*e - 13*a*b^3*f)*x^10 + 45*(7*b^4*d - 10
*a*b^3*e + 13*a^2*b^2*f)*x^7 + 315*(4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13
*a^3*b*f)*x^4 - 140*sqrt(3)*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*
f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*arct
an(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(4*a*b^3*c - 7*a^2*b
^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3
*b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*(4*a*b^
3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b
^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(4*a*b^3*c -
 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*x)/(b^6*x^3 + a*b^5)
```

giac [A] time = 0.18, size = 394, normalized size = 1.20

$$-\frac{\sqrt{3}\left(4\,\left(-ab^2\right)^{\frac{1}{3}}b^3c-7\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d-13\,\left(-ab^2\right)^{\frac{1}{3}}a^3f+10\,\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^6}+\frac{\left(4\,ab^3c-7\,a^2b^2d-13\,a^4f+10\,a^3be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}-\frac{\left(4\,\left(-ab^2\right)^{\frac{1}{3}}b^3c-7\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d-13\,\left(-ab^2\right)^{\frac{1}{3}}a^3f+10\,\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^6}+\frac{ab^3cx-a^2b^2dx-a^4fx+a^3bxe}{3\left(bx^3+a\right)b^5}+\frac{14\,b^{18}fx^{10}-40\,ab^{17}fx^7+20\,b^{18}x^7e+35\,b^{18}dx^4+105\,a^2b^{16}fx^4-70\,ab^{17}x^4e+140\,b^{18}cx-280\,ab^{17}dx-560\,a^3b^{15}fx+420\,a^2b^{16}xe}{140\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(4*(-a*b^2)^(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d - 13*(-a*b^
2)^(1/3)*a^3*f + 10*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/b^6 + 1/9*(4*a*b^3*c - 7*a^2*b^2*d - 13*a^4*f + 10*a
^3*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(4*(-a*b^2)^
(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d - 13*(-a*b^2)^(1/3)*a^3*f + 10*(-a*b
^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^6 + 1/3*(a*b^
3*c*x - a^2*b^2*d*x - a^4*f*x + a^3*b*x*e)/((b*x^3 + a)*b^5) + 1/140*(14*b^
18*f*x^10 - 40*a*b^17*f*x^7 + 20*b^18*x^7*e + 35*b^18*d*x^4 + 105*a^2*b^16*
f*x^4 - 70*a*b^17*x^4*e + 140*b^18*c*x - 280*a*b^17*d*x - 560*a^3*b^15*f*x
+ 420*a^2*b^16*x*e)/b^20
```

maple [B] time = 0.06, size = 567, normalized size = 1.73

$$\frac{fx^{10}}{10b^2}-\frac{2afx^7}{7b^3}+\frac{ex^7}{7b^2}+\frac{3a^2fx^4}{4b^4}-\frac{aex^4}{2b^3}+\frac{dx^4}{4b^2}-\frac{a^4fx}{3\left(bx^3+a\right)b^5}+\frac{a^3ex}{3\left(bx^3+a\right)b^4}-\frac{a^2dx}{3\left(bx^3+a\right)b^3}+\frac{acx}{3\left(bx^3+a\right)b^2}+\frac{13\sqrt{3}\,a^4f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^6}+\frac{13a^4f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^6}-\frac{13a^4f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^6}-\frac{10\sqrt{3}\,a^3e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}-\frac{10a^3e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}+\frac{5a^3e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}-\frac{4a^3fx}{b^5}+\frac{7\sqrt{3}\,a^2d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{7a^2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{7a^2d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{3a^2ex}{b^4}-\frac{4\sqrt{3}\,ac\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{4ac\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2ac\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{2adx}{b^3}+\frac{cx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] -4/9*a/b^3*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+13
/9*a^4/b^6*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-10
/9*a^3/b^5*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+7/
9*a^2/b^4*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/2
/b^3*x^4*a*e-4/b^5*a^3*f*x+3/b^4*a^2*e*x-2/b^3*a*d*x+3/4/b^4*x^4*a^2*f-2/7*
a/b^3*f*x^7+2/9*a/b^3*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9*a
/b^3*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/7/b^2*e*x^7-1/3*a^4/b^5*x/(b*x^3+a)*
f+1/3*a^3/b^4*x/(b*x^3+a)*e-1/3*a^2/b^3*x/(b*x^3+a)*d-13/18*a^4/b^6*f/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*a/b^2*x/(b*x^3+a)*c+13/9*a^4/b
^6*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/4/b^2*x^4*d+1/b^2*c*x-10/9*a^3/b^5*e/(
a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/9*a^3/b^5*e/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+7/9*a^2/b^4*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-7/18*a^2/b^4*d/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/10*f*x^10/b^2
```

maxima [A] time = 3.02, size = 321, normalized size = 0.98

$$\frac{\left(ab^{3}c - a^{2}b^{2}d + a^{3}be - a^{4}f\right)x}{3\left(b^{6}x^{3} + ab^{5}\right)} + \frac{14\,b^{3}fx^{10} + 20\left(b^{3}e - 2\,ab^{2}f\right)x^{7} + 35\left(b^{3}d - 2\,ab^{2}e + 3\,a^{2}bf\right)x^{4} + 140\left(b^{3}c - 2\,ab^{2}d + 3\,a^{2}be - 4\,a^{3}f\right)x}{140\,b^{5}} - \frac{\sqrt{3}\left(4\,ab^{3}c - 7\,a^{2}b^{2}d + 10\,a^{3}be - 13\,a^{4}f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(4\,ab^{3}c - 7\,a^{2}b^{2}d + 10\,a^{3}be - 13\,a^{4}f\right)\log\left(x^{2} - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(4\,ab^{3}c - 7\,a^{2}b^{2}d + 10\,a^{3}be - 13\,a^{4}f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x/(b^6*x^3 + a*b^5) + 1/140*(14
*b^3*f*x^10 + 20*(b^3*e - 2*a*b^2*f)*x^7 + 35*(b^3*d - 2*a*b^2*e + 3*a^2*b*
f)*x^4 + 140*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 - 1/9*sqrt(3)
*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(2/3)) + 1/18*(4*a*b^3*c - 7*a^2*b^2
*d + 10*a^3*b*e - 13*a^4*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/
b)^(2/3)) - 1/9*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*log(x + (
a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

mupad [B] time = 5.20, size = 358, normalized size = 1.09

$$x^{7}\left(\frac{e}{7\,b^{2}} - \frac{2\,a\,f}{7\,b^{3}}\right) + x\left(\frac{c}{b^{2}} - \frac{a^{2}\left(\frac{e}{b^{2}} - \frac{2\,a\,f}{b^{3}}\right)}{b^{2}} + \frac{2\,a\left(\frac{a^{2}f}{b^{4}} - \frac{d}{b^{2}} + \frac{2\,a\left(\frac{e}{b^{2}} - \frac{2\,a\,f}{b^{3}}\right)}{b}\right)}{b}\right) - x^{4}\left(\frac{a^{2}f}{4\,b^{4}} - \frac{d}{4\,b^{2}} + \frac{a\left(\frac{e}{b^{2}} - \frac{2\,a\,f}{b^{3}}\right)}{2\,b}\right) - \frac{x\left(\frac{f\,a^{4}}{3} - \frac{e\,a^{3}\,b}{3} + \frac{d\,a^{2}\,b^{2}}{3} - \frac{c\,a\,b^{3}}{3}\right)}{b^{6}\,x^{3} + a\,b^{5}} + \frac{f\,x^{10}}{10\,b^{2}} - \frac{a^{1/3}\ln\left(b^{1/3}\,x + a^{1/3}\right)\left(-13\,f\,a^{3} + 10\,e\,a^{2}\,b - 7\,d\,a\,b^{2} + 4\,c\,b^{3}\right)}{9\,b^{16/3}} - \frac{a^{1/3}\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-13\,f\,a^{3} + 10\,e\,a^{2}\,b - 7\,d\,a\,b^{2} + 4\,c\,b^{3}\right)}{9\,b^{16/3}} + \frac{a^{1/3}\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-13\,f\,a^{3} + 10\,e\,a^{2}\,b - 7\,d\,a\,b^{2} + 4\,c\,b^{3}\right)}{9\,b^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^7*(e/(7*b^2) - (2*a*f)/(7*b^3)) + x*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/b^3))/
b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) - x^4*
((a^2*f)/(4*b^4) - d/(4*b^2) + (a*(e/b^2 - (2*a*f)/b^3))/(2*b)) - (x*((a^4*
f)/3 + (a^2*b^2*d)/3 - (a*b^3*c)/3 - (a^3*b*e)/3))/(a*b^5 + b^6*x^3) + (f*x
^10)/(10*b^2) - (a^(1/3)*log(b^(1/3)*x + a^(1/3))*(4*b^3*c - 13*a^3*f - 7*a
*b^2*d + 10*a^2*b*e))/(9*b^(16/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*b^3*c - 13*a^3*f - 7*a*b^2*d +
 10*a^2*b*e))/(9*b^(16/3)) + (a^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
+ a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*b^3*c - 13*a^3*f - 7*a*b^2*d + 10*a^2*
b*e))/(9*b^(16/3))
```

sympy [A] time = 14.98, size = 449, normalized size = 1.37

$$x^{7}\left(-\frac{2af}{7b^{3}} + \frac{e}{7b^{2}}\right) + x^{4}\left(\frac{3a^{2}f}{4b^{4}} - \frac{ae}{2b^{3}} + \frac{d}{4b^{2}}\right) + x\left(-\frac{4a^{3}f}{b^{5}} + \frac{3a^{2}e}{b^{4}} - \frac{2ad}{b^{3}} + \frac{c}{b^{2}}\right) + \frac{x\left(-a^{4}f + a^{3}be - a^{2}b^{2}d + ab^{3}c\right)}{3ab^{5} + 3b^{6}x^{3}} + \operatorname{RootSum}\left(729t^{3}b^{16} - 2197a^{10}f^{3} + 5070a^{9}bef^{2} - 3549a^{8}b^{2}df^{2} - 3900a^{8}b^{2}e^{2}f + 2028a^{7}b^{3}cf^{2} + 5460a^{7}b^{3}def + 1000a^{7}b^{3}e^{3} - 3120a^{6}b^{4}cef - 1911a^{6}b^{4}d^{2}f - 2100a^{6}b^{4}de^{2} + 2184a^{5}b^{5}cdf + 1200a^{5}b^{5}ce^{2} + 1470a^{5}b^{5}d^{2}e - 624a^{4}b^{6}c^{2}f - 1680a^{4}b^{6}cde - 343a^{4}b^{6}d^{3} + 480a^{3}b^{7}c^{2}e + 588a^{3}b^{7}cd^{2} - 336a^{2}b^{8}c^{2}d + 64ab^{9}c^{3}, \left(t \mapsto t\log\left(\frac{9tb^{5}}{13a^{3}f - 10a^{2}be + 7ab^{2}d - 4b^{3}c} + x\right)\right)\right) + \frac{fx^{10}}{10b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**7*(-2*a*f/(7*b**3) + e/(7*b**2)) + x**4*(3*a**2*f/(4*b**4) - a*e/(2*b**3
) + d/(4*b**2)) + x*(-4*a**3*f/b**5 + 3*a**2*e/b**4 - 2*a*d/b**3 + c/b**2)
+ x*(-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(3*a*b**5 + 3*b**6*x**3)
+ RootSum(729*_t**3*b**16 - 2197*a**10*f**3 + 5070*a**9*b*e*f**2 - 3549*a**
8*b**2*d*f**2 - 3900*a**8*b**2*e**2*f + 2028*a**7*b**3*c*f**2 + 5460*a**7*b
**3*d*e*f + 1000*a**7*b**3*e**3 - 3120*a**6*b**4*c*e*f - 1911*a**6*b**4*d**
2*f - 2100*a**6*b**4*d*e**2 + 2184*a**5*b**5*c*d*f + 1200*a**5*b**5*c*e**2
+ 1470*a**5*b**5*d**2*e - 624*a**4*b**6*c**2*f - 1680*a**4*b**6*c*d*e - 343
*a**4*b**6*d**3 + 480*a**3*b**7*c**2*e + 588*a**3*b**7*c*d**2 - 336*a**2*b*
*8*c**2*d + 64*a*b**9*c**3, Lambda(_t, _t*log(9*_t*b**5/(13*a**3*f - 10*a**
2*b*e + 7*a*b**2*d - 4*b**3*c) + x))) + f*x**10/(10*b**2)
```

3.210 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=298

$$\frac{x^2\left(3a^2f-2abe+b^2d\right)}{2b^4}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-11a^3f+8a^2be-5ab^2d+2b^3c\right)}{9\sqrt[3]{a}\,b^{14/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-11a^3f+8a^2be\right.}{3\sqrt{3}\sqrt[3]{a}\,b^{14/3}}$$

Rubi [A] time = 0.46, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(8a^2be-11a^3f-5ab^2d+2b^3c\right)}{18\sqrt[3]{a}\,b^{14/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(8a^2be-11a^3f-5ab^2d+2b^3c\right)}{9\sqrt[3]{a}\,b^{14/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(8a^2be-11a^3f-5ab^2d+2b^3c\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{14/3}}+\frac{x^2\left(3a^2f-2abe+b^2d\right)}{2b^4}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/(2*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (
f*x^8)/(8*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^4*(a + b*x^
3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(14/3)) - ((2*b^3*c - 5*a*
b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(14/3)
) + ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{\int \frac{-2ab\left(b^3c-ab^2d+a^2be-a^3f\right)x-3ab^2\left(b^2d-abe+a^2f\right)x^4-3ab^3}{a+bx^3}}{3ab^5}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{\int \frac{x\left(-2ab\left(b^3c-ab^2d+a^2be-a^3f\right)-3ab^2\left(b^2d-abe+a^2f\right)x^3-3ab}{a+bx^3}}{3ab^5}$$

$$= \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{\int \frac{x\left(-16ab^2\left(b^3c-ab^2d+a^2be-a^3f\right)-24ab^3\left(b^2d-abe+a\right.\right.}{a+bx^3}}{24ab^6}$$

$$= \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{\int \left(-24ab^2\left(b^2d-2abe+3a^2f\right)x-24ab\right.}{}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} +$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} -$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} -$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} -$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} -$$

Mathematica [A] time = 0.30, size = 282, normalized size = 0.95

$$\frac{180b^{2/3}x^2\left(3a^2f-2abe+b^2d\right)+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(11a^3f-8a^2be+5ab^2d-2b^3c\right)}{\sqrt[3]{a}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(11a^3f-8a^2be+5ab^2d-2b^3c\right)}{\sqrt[3]{a}}-\frac{120b^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a+bx^3}+\frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-11a^3f+8a^2be-5ab^2d+2b^3c\right)}{\sqrt[3]{a}}+72b^{5/3}x^5(be-2af)+45b^{8/3}fx^8}{360b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (180*b^(2/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2 + 72*b^(5/3)*(b*e - 2*a*f)*x^5
 + 45*b^(8/3)*f*x^8 - (120*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)
/(a + b*x^3) + (40*Sqrt[3]*(-2*b^3*c + 5*a*b^2*d - 8*a^2*b*e + 11*a^3*f)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-2*b^3*c + 5*a*b^
2*d - 8*a^2*b*e + 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(2*b^3*
c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(1/3))/(360*b^(14/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]
```

fricas [A] time = 0.45, size = 920, normalized size = 3.09

$$\left[\frac{45ab^5fx^{11} + 9\left(8ab^5e - 11a^2b^4f\right)x^8 + 36\left(5ab^5d - 8a^2b^4e + 11a^3b^3f\right)x^5 - 60\left(2ab^5c - 5a^2b^4d + 8a^3b^3e - 11a^4b^2f\right)x^2 - 60\sqrt{\frac{1}{3}}\left(2a^2b^4c - 5a^3b^3d + 8a^4b^2e - 11a^5bf + \left(2ab^5c - 5a^2b^4d + 8a^3b^3e - 11a^4b^2f\right)x^3\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3 - ab - 3\sqrt{\frac{1}{3}}\left(abx + 2(ab^2)^{\frac{2}{3}}x^2 - (ab^2)^{\frac{1}{3}}a\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}} - 3(ab^2)^{\frac{2}{3}}x}{bx^3 + a}\right) + 20\left(2ab^3c - 5a^2b^2d + 8a^3be - 11a^4f + \left(2b^4c - 5ab^3d + 8a^2b^2e - 11a^3bf\right)x^3\right)(ab^2)^{\frac{2}{3}}\log\left(b^2x^2 - (ab^2)^{\frac{1}{3}}bx + (ab^2)^{\frac{2}{3}}\right) - 40\left(2ab^3c - 5a^2b^2d + 8a^3be - 11a^4f + \left(2b^4c - 5ab^3d + 8a^2b^2e - 11a^3bf\right)x^3\right)(ab^2)^{\frac{2}{3}}\log\left(bx + (ab^2)^{\frac{1}{3}}\right)}{360\left(ab^7x^3 + a^2b^6\right)}, \frac{45ab^5fx^{11} + 9\left(8ab^5e - 11a^2b^4f\right)x^8 + 36\left(5ab^5d - 8a^2b^4e + 11a^3b^3f\right)x^5 - 60\left(2ab^5c - 5a^2b^4d + 8a^3b^3e - 11a^4b^2f\right)x^2 - 120\sqrt{\frac{1}{3}}\left(2a^2b^4c - 5a^3b^3d + 8a^4b^2e - 11a^5bf + \left(2ab^5c - 5a^2b^4d + 8a^3b^3e - 11a^4b^2f\right)x^3\right)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2bx - (ab^2)^{\frac{1}{3}}\right)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{b}\right) + 20\left(2ab^3c - 5a^2b^2d + 8a^3be - 11a^4f + \left(2b^4c - 5ab^3d + 8a^2b^2e - 11a^3bf\right)x^3\right)(ab^2)^{\frac{2}{3}}\log\left(b^2x^2 - (ab^2)^{\frac{1}{3}}bx + (ab^2)^{\frac{2}{3}}\right) - 40\left(2ab^3c - 5a^2b^2d + 8a^3be - 11a^4f + \left(2b^4c - 5ab^3d + 8a^2b^2e - 11a^3bf\right)x^3\right)(ab^2)^{\frac{2}{3}}\log\left(bx + (ab^2)^{\frac{1}{3}}\right)}{360\left(ab^7x^3 + a^2b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/360*(45*a*b^5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d
- 8*a^2*b^4*e + 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3
*e - 11*a^4*b^2*f)*x^2 - 60*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^
2*e - 11*a^5*b*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x
^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a
*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)
*x)/(b*x^3 + a)) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*
b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^
2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^
3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a
*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6), 1/360*(45*a*b^
5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d - 8*a^2*b^4*e +
 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2
*f)*x^2 - 120*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^2*e - 11*a^5*b
*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x^3)*sqrt((a*b^
2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)
/b) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b
^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(
1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^
4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*l
og(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6)]
```

giac [A] time = 0.19, size = 344, normalized size = 1.15

$$\frac{\sqrt{3}\left(2b^3c - 5ab^2d - 11a^3f + 8a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}b^4} - \frac{\left(2b^3c - 5ab^2d - 11a^3f + 8a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}b^4} - \frac{\left(2b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 11a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 8a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab^4} - \frac{b^3cx^2 - ab^2dx^2 - a^3fx^2 + a^2bx^2e}{3\left(bx^3 + a\right)b^4} + \frac{5b^{14}fx^8 - 16ab^{13}fx^5 + 8b^{14}x^5e + 20b^{14}dx^2 + 60a^2b^{12}fx^2 - 40ab^{13}x^2e}{40b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(2*b^3*c - 5*a*b^2*d - 11*a^3*f + 8*a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^4) - 1/18*(2*b^3*c -
5*a*b^2*d - 11*a^3*f + 8*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(1/3)*b^4) - 1/9*(2*b^3*c*(-a/b)^(1/3) - 5*a*b^2*d*(-a/b)^(1/3) -
 11*a^3*f*(-a/b)^(1/3) + 8*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a*b^4) - 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^
2*e)/((b*x^3 + a)*b^4) + 1/40*(5*b^14*f*x^8 - 16*a*b^13*f*x^5 + 8*b^14*x^5*
e + 20*b^14*d*x^2 + 60*a^2*b^12*f*x^2 - 40*a*b^13*x^2*e)/b^16
```

maple [B] time = 0.06, size = 529, normalized size = 1.78

$$\frac{fx^8}{8b^2} - \frac{2afx^5}{5b^3} + \frac{ex^5}{5b^2} + \frac{a^3fx^2}{3\left(bx^3 + a\right)b^4} - \frac{a^2ex^2}{3\left(bx^3 + a\right)b^3} + \frac{adx^2}{3\left(bx^3 + a\right)b^2} - \frac{cx^2}{3\left(bx^3 + a\right)b} + \frac{3a^2fx^2}{2b^4} - \frac{aex^2}{b^3} + \frac{dx^2}{2b^2} - \frac{11\sqrt{3}\,a^3f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^5} + \frac{11a^3f\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^5} - \frac{11a^3f\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^5} + \frac{8\sqrt{3}\,a^2e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} - \frac{8a^2e\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} + \frac{4a^2e\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} - \frac{5\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} + \frac{5ad\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} - \frac{5ad\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} + \frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{2c\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{c\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/8*f*x^8/b^2-2/5/b^3*x^5*a*f+1/5/b^2*x^5*e+3/2/b^4*x^2*a^2*f-1/b^3*x^2*a*e
+1/2/b^2*x^2*d+1/3/b^4*x^2/(b*x^3+a)*a^3*f-1/3/b^3*x^2/(b*x^3+a)*a^2*e+1/3/
```

```
b^2*x^2/(b*x^3+a)*a*d-1/3/b*x^2/(b*x^3+a)*c+11/9/b^5*a^3*f/(a/b)^(1/3)*ln(x
+(a/b)^(1/3))-11/18/b^5*a^3*f/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
-11/9/b^5*a^3*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-8/9/b^4*a^2*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+4/9/b^4*a^2*e/(a/b)^(1/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))+8/9/b^4*a^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))+5/9/b^3*a*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18
/b^3*a*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9/b^3*a*d*3^(1/2)/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/9/b^2*c/(a/b)^(1/3)*l
n(x+(a/b)^(1/3))+1/9/b^2*c/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/
9/b^2*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 3.08, size = 277, normalized size = 0.93

$$-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3\left(b^5x^3+ab^4\right)}+\frac{\sqrt{3}\left(2b^3c-5ab^2d+8a^2be-11a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5b^2fx^8+8\left(b^2e-2abf\right)x^5+20\left(b^2d-2abe+3a^2f\right)x^2}{40b^4}+\frac{\left(2b^3c-5ab^2d+8a^2be-11a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(2b^3c-5ab^2d+8a^2be-11a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2/(b^5*x^3 + a*b^4) + 1/9*sqrt(3
)*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/40*(5*b^2*f*x^8 + 8*(b^2*e -
2*a*b*f)*x^5 + 20*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/b^4 + 1/18*(2*b^3*c - 5*
a*b^2*d + 8*a^2*b*e - 11*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5
*(a/b)^(1/3)) - 1/9*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*log(x + (a
/b)^(1/3))/(b^5*(a/b)^(1/3))
```

mupad [B] time = 5.22, size = 287, normalized size = 0.96

$$x^5\left(\frac{e}{5b^2}-\frac{2af}{5b^3}\right)-x^2\left(\frac{a^2f}{2b^4}-\frac{d}{2b^2}+\frac{a\left(\frac{e}{b^2}-\frac{2af}{b^3}\right)}{b}\right)+\frac{fx^8}{8b^2}-\frac{x^2\left(-\frac{fa^3}{3}+\frac{ea^2b}{3}-\frac{dab^2}{3}+\frac{cb^3}{3}\right)}{b^5x^3+ab^4}-\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(-11fa^3+8ea^2b-5dab^2+2cb^3\right)}{9a^{1/3}b^{14/3}}+\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}1i\right)\left(\frac{1}{2}+\frac{\sqrt{3}1i}{2}\right)\left(-11fa^3+8ea^2b-5dab^2+2cb^3\right)}{9a^{1/3}b^{14/3}}-\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}1i\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1i}{2}\right)\left(-11fa^3+8ea^2b-5dab^2+2cb^3\right)}{9a^{1/3}b^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^5*(e/(5*b^2) - (2*a*f)/(5*b^3)) - x^2*((a^2*f)/(2*b^4) - d/(2*b^2) + (a*(
e/b^2 - (2*a*f)/b^3))/b) + (f*x^8)/(8*b^2) - (x^2*((b^3*c)/3 - (a^3*f)/3 -
(a*b^2*d)/3 + (a^2*b*e)/3))/(a*b^4 + b^5*x^3) - (log(b^(1/3)*x + a^(1/3))*(
2*b^3*c - 11*a^3*f - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3)) + (log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c -
 11*a^3*f - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3)) - (log(3^(1/2)*a^(
1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c - 11*a^3*f
 - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3))
```

sympy [A] time = 51.29, size = 490, normalized size = 1.64

$$x^5\left(-\frac{2af}{5b^3}+\frac{e}{5b^2}\right)+x^2\left(\frac{3a^2f}{2b^4}-\frac{ae}{b^3}+\frac{d}{2b^2}\right)+\frac{x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{3ab^4+3b^5x^3}+\operatorname{RootSum}\left(729t^3ab^{14}-1331a^9f^3+2904a^8bef^2-1815a^7b^2df^2-2112a^7b^2e^2f+726a^6b^3cf^2+2640a^6b^3def+512a^6b^3e^3-1056a^5b^4cef-825a^5b^4d^2f-960a^5b^4de^2+660a^4b^5cdf+384a^4b^5ce^2+600a^4b^5d^2e-132a^3b^6c^2f-480a^3b^6cde-125a^3b^6d^3+96a^2b^7c^2e+150a^2b^7cd^2-60ab^8c^2d+8b^9c^3,\left(t\mapsto t\log\left(\frac{81t^2ab^9}{121a^6f^2-176a^5bef+110a^4b^2df+64a^4b^2e^2-44a^3b^3cf-80a^3b^3de+32a^2b^4ce+25a^2b^4d^2-20ab^5cd+4b^6c^2}+x\right)\right)\right)+\frac{fx^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**5*(-2*a*f/(5*b**3) + e/(5*b**2)) + x**2*(3*a**2*f/(2*b**4) - a*e/b**3 +
d/(2*b**2)) + x**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b*
*5*x**3) + RootSum(729*_t**3*a*b**14 - 1331*a**9*f**3 + 2904*a**8*b*e*f**2
- 1815*a**7*b**2*d*f**2 - 2112*a**7*b**2*e**2*f + 726*a**6*b**3*c*f**2 + 26
40*a**6*b**3*d*e*f + 512*a**6*b**3*e**3 - 1056*a**5*b**4*c*e*f - 825*a**5*b
**4*d**2*f - 960*a**5*b**4*d*e**2 + 660*a**4*b**5*c*d*f + 384*a**4*b**5*c*e
**2 + 600*a**4*b**5*d**2*e - 132*a**3*b**6*c**2*f - 480*a**3*b**6*c*d*e - 1
25*a**3*b**6*d**3 + 96*a**2*b**7*c**2*e + 150*a**2*b**7*c*d**2 - 60*a*b**8*
c**2*d + 8*b**9*c**3, Lambda(_t, _t*log(81*_t**2*a*b**9/(121*a**6*f**2 - 17
```

```
6*a**5*b*e*f + 110*a**4*b**2*d*f + 64*a**4*b**2*e**2 - 44*a**3*b**3*c*f - 8
0*a**3*b**3*d*e + 32*a**2*b**4*c*e + 25*a**2*b**4*d**2 - 20*a*b**5*c*d + 4*
b**6*c**2) + x))) + f*x**8/(8*b**2)
```

3.211 $\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=288

$$\frac{x\left(3a^2f-2abe+b^2d\right)}{b^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-10a^3f+7a^2be-4ab^2d\right.}{18a^{2/3}b^{13/3}}$$

Rubi [A] time = 0.33, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1828, 1887, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(7a^2be-10a^3f-4ab^2d+b^3c\right)}{18a^{2/3}b^{13/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(7a^2be-10a^3f-4ab^2d+b^3c\right)}{9a^{2/3}b^{13/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7a^2be-10a^3f-4ab^2d+b^3c\right)}{3\sqrt{3}\,a^{2/3}b^{13/3}}+\frac{x\left(3a^2f-2abe+b^2d\right)}{b^4}+\frac{x^4(be-2af)}{4b^3}+\frac{fx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^4)/(4*b^3) + (f*x^7)
/(7*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^4*(a + b*x^3)) - ((
b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(13/3)) + ((b^3*c - 4*a*b^2*d + 7*a^
2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(13/3)) - ((b^3*c
- 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(2/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)} - \frac{\int \frac{-a\left(b^3c-ab^2d+a^2be-a^3f\right)-3ab\left(b^2d-abe+a^2f\right)x^3-3ab^2(b}{a+bx^3}}{3ab^4}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)} - \frac{\int\left(-3a\left(b^2d-2abe+3a^2f\right)-3ab(be-2af)\right.}{3a}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)}$$

Mathematica [A] time = 0.19, size = 277, normalized size = 0.96

$$\frac{252\sqrt[3]{b}\,x\left(3a^2f-2abe+b^2d\right)-\frac{84\sqrt[3]{b}\,x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a+bx^3}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{a^{2/3}}+\frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(10a^3f-7a^2be+4ab^2d-b^3c\right)}{a^{2/3}}+\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(10a^3f-7a^2be+4ab^2d-b^3c\right)}{a^{2/3}}+63b^{4/3}x^4(be-2af)+36b^{7/3}fx^7}{252b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

[Out] (252*b^(1/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x + 63*b^(4/3)*(b*e - 2*a*f)*x^4 + 36*b^(7/3)*f*x^7 - (84*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3) + (28*Sqrt[3]*(-(b^3*c) + 4*a*b^2*d - 7*a^2*b*e + 10*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c) + 4*a*b^2*d - 7*a^2*b*e + 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(252*b^(13/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(c + dx^3 + ex^6 + fx^9\right)}{\left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]

fricas [A] time = 0.47, size = 946, normalized size = 3.28

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/252*(36*a^2*b^4*f*x^10 + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2*b^4*d - 7*a^3*b^3*e + 10*a^4*b^2*f)*x^4 - 42*sqrt(1/3)*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e - 10*a^4*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x)/(a^2*b^6*x^3 + a^3*b^5), 1/252*(36*a^2*b^4*f*x^10 + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2*b^4*d - 7*a^3*b^3*e + 10*a^4*b^2*f)*x^4 + 84*sqrt(1/3)*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e - 10*a^4*b^2*f)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x)/(a^2*b^6*x^3 + a^3*b^5)]

giac [A] time = 0.18, size = 295, normalized size = 1.02

$$-\frac{\sqrt{3}\left(b^3c - 4\,ab^2d - 10\,a^3f + 7\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}b^3} - \frac{\left(b^3c - 4\,ab^2d - 10\,a^3f + 7\,a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}b^3} - \frac{\left(b^3c - 4\,ab^2d - 10\,a^3f + 7\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4} - \frac{b^3cx - ab^2dx - a^3fx + a^2bxe}{3\left(bx^3 + a\right)b^4} + \frac{4\,b^{12}fx^7 - 14\,ab^{11}fx^4 + 7\,b^{12}x^4e + 28\,b^{12}dx + 84\,a^2b^{10}fx - 56\,ab^{11}xe}{28\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*sqrt(3)*(b^3*c - 4*a*b^2*d - 10*a^3*f + 7*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^3) - 1/18*(b^3*c - 4*a*b^2*d - 10*a^3*f + 7*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-

a*b^2)^(2/3)*b^3) - 1/9*(b^3*c - 4*a*b^2*d - 10*a^3*f + 7*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*b^4) + 1/28*(4*b^12*f*x^7 - 14*a*b^11*f*x^4 + 7*b^12*x^4*e + 28*b^12*d*x + 84*a^2*b^10*f*x - 56*a*b^11*x*e)/b^14

maple [B] time = 0.05, size = 514, normalized size = 1.78

$$\frac{fx^7}{7b^2}-\frac{afx^4}{2b^3}+\frac{ex^4}{4b^2}+\frac{a^3fx}{3(bx^3+a)b^4}-\frac{a^2ex}{3(bx^3+a)b^3}+\frac{adx}{3(bx^3+a)b^2}-\frac{cx}{3(bx^3+a)b}-\frac{10\sqrt{3}a^3f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{(\frac{a}{b})^{\frac{1}{3}}}-1\right)}{3}\right)}{9(\frac{a}{b})^{\frac{2}{3}}b^5}-\frac{10a^3f\ln\left(x+(\frac{a}{b})^{\frac{1}{3}}\right)}{9(\frac{a}{b})^{\frac{2}{3}}b^5}+\frac{5a^3f\ln\left(x^2-(\frac{a}{b})^{\frac{1}{3}}x+(\frac{a}{b})^{\frac{2}{3}}\right)}{9(\frac{a}{b})^{\frac{2}{3}}b^5}+\frac{7\sqrt{3}a^2e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{(\frac{a}{b})^{\frac{1}{3}}}-1\right)}{3}\right)}{9(\frac{a}{b})^{\frac{2}{3}}b^4}+\frac{7a^2e\ln\left(x+(\frac{a}{b})^{\frac{1}{3}}\right)}{9(\frac{a}{b})^{\frac{2}{3}}b^4}-\frac{7a^2e\ln\left(x^2-(\frac{a}{b})^{\frac{1}{3}}x+(\frac{a}{b})^{\frac{2}{3}}\right)}{18(\frac{a}{b})^{\frac{2}{3}}b^4}+\frac{3a^2fx}{b^4}-\frac{4\sqrt{3}ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{(\frac{a}{b})^{\frac{1}{3}}}-1\right)}{3}\right)}{9(\frac{a}{b})^{\frac{2}{3}}b^3}-\frac{4ad\ln\left(x+(\frac{a}{b})^{\frac{1}{3}}\right)}{9(\frac{a}{b})^{\frac{2}{3}}b^3}+\frac{2ad\ln\left(x^2-(\frac{a}{b})^{\frac{1}{3}}x+(\frac{a}{b})^{\frac{2}{3}}\right)}{9(\frac{a}{b})^{\frac{2}{3}}b^3}-\frac{2aex}{b^3}+\frac{\sqrt{3}c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{(\frac{a}{b})^{\frac{1}{3}}}-1\right)}{3}\right)}{9(\frac{a}{b})^{\frac{2}{3}}b^2}+\frac{c\ln\left(x+(\frac{a}{b})^{\frac{1}{3}}\right)}{9(\frac{a}{b})^{\frac{2}{3}}b^2}-\frac{c\ln\left(x^2-(\frac{a}{b})^{\frac{1}{3}}x+(\frac{a}{b})^{\frac{2}{3}}\right)}{18(\frac{a}{b})^{\frac{2}{3}}b^2}+\frac{dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] 1/7/b^2*f*x^7-1/2/b^3*x^4*a*f+1/4/b^2*x^4*e+3/b^4*a^2*f*x-2/b^3*a*e*x+1/b^2*d*x+1/3/b^4*x/(b*x^3+a)*a^3*f-1/3/b^3*x/(b*x^3+a)*a^2*e+1/3/b^2*x/(b*x^3+a)*a*d-1/3/b*x/(b*x^3+a)*c-10/9/b^5*a^3*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/9/b^5*a^3*f/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-10/9/b^5*a^3*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+7/9/b^4*a^2*e/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-7/18/b^4*a^2*e/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+7/9/b^4*a^2*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-4/9/b^3*a*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9/b^3*a*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/b^3*a*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9/b^2*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18/b^2*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/9/b^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))

maxima [A] time = 2.98, size = 270, normalized size = 0.94

$$-\frac{(b^3c-ab^2d+a^2be-a^3f)x}{3(b^5x^3+ab^4)}+\frac{4b^2fx^7+7(b^2e-2abf)x^4+28(b^2d-2abe+3a^2f)x}{28b^4}+\frac{\sqrt{3}(b^3c-4ab^2d+7a^2be-10a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-(\frac{a}{b})^{\frac{1}{3}}\right)}{3(\frac{a}{b})^{\frac{1}{3}}}\right)}{9b^5(\frac{a}{b})^{\frac{2}{3}}}-\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(x^2-x(\frac{a}{b})^{\frac{1}{3}}+(\frac{a}{b})^{\frac{2}{3}}\right)}{18b^5(\frac{a}{b})^{\frac{2}{3}}}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(x+(\frac{a}{b})^{\frac{1}{3}}\right)}{9b^5(\frac{a}{b})^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] -1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(b^5*x^3 + a*b^4) + 1/28*(4*b^2*f*x^7 + 7*(b^2*e - 2*a*b*f)*x^4 + 28*(b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + 1/9*sqrt(3)*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) - 1/18*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(2/3)) + 1/9*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3))

mupad [B] time = 0.31, size = 280, normalized size = 0.97

$$x^4\left(\frac{e}{4b^2}-\frac{af}{2b^3}\right)-x\left(\frac{a^2f}{b^4}-\frac{d}{b^2}+\frac{2a\left(\frac{e}{b^2}-\frac{2af}{b^3}\right)}{b}\right)-\frac{x\left(-\frac{fa^3}{3}+\frac{ea^2b}{3}-\frac{dab^2}{3}+\frac{cb^3}{3}\right)}{b^5x^3+ab^4}+\frac{fx^7}{7b^2}+\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(-10fa^3+7ea^2b-4dab^2+cb^3\right)}{9a^{2/3}b^{13/3}}+\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}1i\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1i}{2}\right)\left(-10fa^3+7ea^2b-4dab^2+cb^3\right)}{9a^{2/3}b^{13/3}}-\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}1i\right)\left(\frac{1}{2}+\frac{\sqrt{3}1i}{2}\right)\left(-10fa^3+7ea^2b-4dab^2+cb^3\right)}{9a^{2/3}b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)

[Out] x^4*(e/(4*b^2) - (a*f)/(2*b^3)) - x*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b) - (x*((b^3*c)/3 - (a^3*f)/3 - (a*b^2*d)/3 + (a^2*b*e)/3))/(a*b^4 + b^5*x^3) + (f*x^7)/(7*b^2) + (log(b^(1/3)*x + a^(1/3))*(b^3*c - 10*a^3*f - 4*a*b^2*d + 7*a^2*b*e))/(9*a^(2/3)*b^(13/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - 10*a^3*f - 4*a*b^2*d + 7*a^2*b*e))/(9*a^(2/3)*b^(13/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - 10*a^3*f - 4*a*b^2*d + 7*a^2*b*e))/(9*a^(2/3)*b^(13/3))

sympy [A] time = 12.90, size = 401, normalized size = 1.39

$$x^4\left(-\frac{af}{2b^3}+\frac{e}{4b^2}\right)+x\left(\frac{3a^2f}{b^4}-\frac{2ae}{b^3}+\frac{d}{b^2}\right)+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{3ab^4+3b^5x^3}+\operatorname{RootSum}\left(729t^3a^2b^{13}+1000a^9f^3-2100a^8bef^2+1200a^7b^2df^2+1470a^7b^2e^2f-300a^6b^3cf^2-1680a^6b^3def-343a^6b^3e^3+420a^5b^4cef+480a^5b^4d^2f+588a^5b^4de^2-240a^4b^5cdf-147a^4b^5ce^2-336a^4b^5d^2e+30a^3b^6c^2f+168a^3b^6cde+64a^3b^6d^3-21a^2b^7c^2e-48a^2b^7cd^2+12ab^8c^2d-b^9c^3,\left(t\mapsto t\log\left(-\frac{9tab^4}{10a^3f-7a^2be+4ab^2d-b^3c}+x\right)\right)\right)+\frac{fx^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**4*(-a*f/(2*b**3) + e/(4*b**2)) + x*(3*a**2*f/b**4 - 2*a*e/b**3 + d/b**2)
 + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3) + Roo
tSum(729*_t**3*a**2*b**13 + 1000*a**9*f**3 - 2100*a**8*b*e*f**2 + 1200*a**7
*b**2*d*f**2 + 1470*a**7*b**2*e**2*f - 300*a**6*b**3*c*f**2 - 1680*a**6*b**
3*d*e*f - 343*a**6*b**3*e**3 + 420*a**5*b**4*c*e*f + 480*a**5*b**4*d**2*f +
 588*a**5*b**4*d*e**2 - 240*a**4*b**5*c*d*f - 147*a**4*b**5*c*e**2 - 336*a*
*4*b**5*d**2*e + 30*a**3*b**6*c**2*f + 168*a**3*b**6*c*d*e + 64*a**3*b**6*d
**3 - 21*a**2*b**7*c**2*e - 48*a**2*b**7*c*d**2 + 12*a*b**8*c**2*d - b**9*c
**3, Lambda(_t, _t*log(-9*_t*a*b**4/(10*a**3*f - 7*a**2*b*e + 4*a*b**2*d -
b**3*c) + x))) + f*x**7/(7*b**2)
```

3.212 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$

Optimal. Leaf size=271

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{18a^{4/3}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Rubi [A] time = 0.29, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1828, 1594, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^2be+8a^3f+2ab^2d+b^3c\right)}{18a^{4/3}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^2be+8a^3f+2ab^2d+b^3c\right)}{9a^{4/3}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^2be+8a^3f+2ab^2d+b^3c\right)}{3\sqrt{3}a^{4/3}b^{11/3}}+\frac{x^2(be-2af)}{2b^3}+\frac{fx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x^2)/(2*b^3) + (f*x^5)/(5*b^2) + ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*x^2)/(3*a*b^3*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3
)*b^(11/3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(9*a^(4/3)*b^(11/3)) + ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\int \frac{-b\left(b^3c+2ab^2d-2a^2be+2a^3f\right)x-3ab^2(be-af)x^4-3ab^3fx^7}{a+bx^3}}{3ab^4}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\int \frac{x\left(-b\left(b^3c+2ab^2d-2a^2be+2a^3f\right)-3ab^2(be-af)x^3-3ab^3fx^6\right)}{a+bx^3}}{3ab^4}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\int \left(-3ab(be-2af)x-3ab^2fx^4+\frac{\left(-b^4c-2ab^3d+5\right.}{a+}\right.}{3ab^4}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} + \frac{\left(b^3c+2ab^2d-5a^2be\right.}{3ab^3}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\left(b^3c+2ab^2d-5a^2be+\right.}{9a^{4/3}b}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\left(b^3c+2ab^2d-5a^2be+\right.}{9a^{4/}}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\left(b^3c+2ab^2d-5a^2be+\right.}{9a^{4/}}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\left(b^3c+2ab^2d-5a^2be+\right.}{3\sqrt{3}\,a}$$

Mathematica [A] time = 0.19, size = 255, normalized size = 0.94

$$\frac{\frac{30b^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a\left(a+bx^3\right)} - \frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{a^{4/3}} - \frac{10\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{a^{4/3}} + \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(8a^3f-5a^2be+2ab^2d+b^3c\right)}{a^{4/3}} + 45b^{2/3}x^2(be-2af)+18b^{5/3}fx^5}{90b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (45*b^(2/3)*(b*e - 2*a*f)*x^2 + 18*b^(5/3)*f*x^5 + (30*b^(2/3)*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt[3]*(b^3*c + 2*a*b^2
*d - 5*a^2*b*e + 8*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4
/3) - (10*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/a^(4/3) + (5*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(90*b^(11/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]
```

fricas [A] time = 0.47, size = 874, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*c
 - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 15*sqrt(1/3)*(a^2*b^4*c +
 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*b^3
*e + 8*a^4*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)
/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*(a*b^3*c + 2*a^2*b^2*d - 5*a^3*b
*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^2)^
(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^3*c + 2*
a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*
b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^3 + a^3*b^5)
, 1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*
c - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 30*sqrt(1/3)*(a^2*b^4*c
+ 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*b^
3*e + 8*a^4*b^2*f)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(a*b^3*c + 2*a^2*b^2*d - 5*a^
3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^3*c +
 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a
^3*b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^3 + a^3*b
^5)]
```

giac [A] time = 0.22, size = 318, normalized size = 1.17

$$\frac{\sqrt{3}\left(b^{3}c+2\,ab^{2}d+8\,a^{3}f-5\,a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}}-\frac{\left(b^{3}c+2\,ab^{2}d+8\,a^{3}f-5\,a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}}-\frac{\left(b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+8\,a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}b^{3}}+\frac{b^{3}cx^{2}-ab^{2}dx^{2}-a^{3}fx^{2}+a^{2}bx^{2}e}{3\left(bx^{3}+a\right)ab^{3}}+\frac{2\,b^{8}fx^{5}-10\,ab^{7}fx^{2}+5\,b^{8}x^{2}e}{10\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(b^3*c + 2*a*b^2*d + 8*a^3*f - 5*a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^3) - 1/18*(b^3*c + 2*a
*b^2*d + 8*a^3*f - 5*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a
*b^2)^(1/3)*a*b^3) - 1/9*(b^3*c*(-a/b)^(1/3) + 2*a*b^2*d*(-a/b)^(1/3) + 8*a
^3*f*(-a/b)^(1/3) - 5*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^3) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e
)/((b*x^3 + a)*a*b^3) + 1/10*(2*b^8*f*x^5 - 10*a*b^7*f*x^2 + 5*b^8*x^2*e)/b
^10
```

maple [B] time = 0.05, size = 495, normalized size = 1.83

$$\frac{f\,x^{5}}{5b^{2}}-\frac{a^{2}f\,x^{2}}{3\left(b\,x^{3}+a\right)b^{3}}+\frac{ae\,x^{2}}{3\left(b\,x^{3}+a\right)b^{2}}+\frac{c\,x^{2}}{3\left(b\,x^{3}+a\right)a}-\frac{d\,x^{2}}{3\left(b\,x^{3}+a\right)b}-\frac{af\,x^{2}}{b^{3}}+\frac{e\,x^{2}}{2b^{2}}+\frac{8\sqrt{3}\,a^{2}f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{4}}-\frac{8a^{2}f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{4}}+\frac{4a^{2}f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{4}}-\frac{5\sqrt{3}\,ae\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{5ae\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{5ae\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{2\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/5/b^2*f*x^5-1/b^3*x^2*a*f+1/2/b^2*x^2*e-1/3/b^3*a^2*x^2/(b*x^3+a)*f+1/3/b
^2*a*x^2/(b*x^3+a)*e-1/3/b*x^2/(b*x^3+a)*d+1/3/a*x^2/(b*x^3+a)*c-8/9/b^4*a^
2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f+5/9/b^3*a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e
-2/9/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-1/9/b/a/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))*c+4/9/b^4*a^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-5/18/b^3*
a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/9/b^2/(a/b)^(1/3)*ln(x^
```

```
2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/18/b/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*c+8/9/b^4*a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))*f-5/9/b^3*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))*e+2/9/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)*d+1/9/b/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 3.12, size = 259, normalized size = 0.96

$$\frac{\left(b^{3} c - a b^{2} d + a^{2} b e - a^{3} f\right) x^{2}}{3 \left(a b^{4} x^{3} + a^{2} b^{3}\right)} + \frac{2 \, b f x^{5} + 5 \left(b e - 2 \, a f\right) x^{2}}{10 \, b^{3}} + \frac{\sqrt{3} \left(b^{3} c + 2 \, a b^{2} d - 5 \, a^{2} b e + 8 \, a^{3} f\right) \arctan\left(\frac{\sqrt{3} \left(2 \, x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \, a b^{4} \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(b^{3} c + 2 \, a b^{2} d - 5 \, a^{2} b e + 8 \, a^{3} f\right) \log\left(x^{2} - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \, a b^{4} \left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(b^{3} c + 2 \, a b^{2} d - 5 \, a^{2} b e + 8 \, a^{3} f\right) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \, a b^{4} \left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2/(a*b^4*x^3 + a^2*b^3) + 1/10*(2
*b*f*x^5 + 5*(b*e - 2*a*f)*x^2)/b^3 + 1/9*sqrt(3)*(b^3*c + 2*a*b^2*d - 5*a^
2*b*e + 8*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4
*(a/b)^(1/3)) + 1/18*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(a*b^4*(a/b)^(1/3)) - 1/9*(b^3*c + 2*a*b^2*d - 5
*a^2*b*e + 8*a^3*f)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(1/3))
```

mupad [B] time = 5.23, size = 246, normalized size = 0.91

$$x^2 \left(\frac{e}{2 b^2} - \frac{a f}{b^3}\right) + \frac{f x^5}{5 b^2} - \frac{\ln\left(b^{1/3} x + a^{1/3}\right) \left(8 f a^3 - 5 e a^2 b + 2 d a b^2 + c b^3\right)}{9 a^{4/3} b^{11/3}} + \frac{x^2 \left(-f a^3 + e a^2 b - d a b^2 + c b^3\right)}{3 a \left(b^4 x^3 + a b^3\right)} + \frac{\ln\left(2 b^{1/3} x - a^{1/3} + \sqrt{3} a^{1/3} 1i\right) \left(\frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(8 f a^3 - 5 e a^2 b + 2 d a b^2 + c b^3\right)}{9 a^{4/3} b^{11/3}} - \frac{\ln\left(a^{1/3} - 2 b^{1/3} x + \sqrt{3} a^{1/3} 1i\right) \left(-\frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(8 f a^3 - 5 e a^2 b + 2 d a b^2 + c b^3\right)}{9 a^{4/3} b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

```
[Out] x^2*(e/(2*b^2) - (a*f)/b^3) + (f*x^5)/(5*b^2) - (log(b^(1/3)*x + a^(1/3))*(
b^3*c + 8*a^3*f + 2*a*b^2*d - 5*a^2*b*e))/(9*a^(4/3)*b^(11/3)) + (x^2*(b^3*
c - a^3*f - a*b^2*d + a^2*b*e))/(3*a*(a*b^3 + b^4*x^3)) + (log(3^(1/2)*a^(1
/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c + 8*a^3*f + 2
*a*b^2*d - 5*a^2*b*e))/(9*a^(4/3)*b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b
^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c + 8*a^3*f + 2*a*b^2*d - 5
*a^2*b*e))/(9*a^(4/3)*b^(11/3))
```

sympy [A] time = 22.48, size = 461, normalized size = 1.70

$$x^{2} \left(- \frac{a f}{b^{3}} + \frac{e}{2 b^{2}}\right) + \frac{x^{2} \left(- a^{3} f + a^{2} b e - a b^{2} d + b^{3} c\right)}{3 a^{2} b^{3} + 3 a b^{4} x^{3}} + \operatorname{RootSum}{\left(729 t^{3} a^{4} b^{11} + 512 a^{9} f^{3} - 960 a^{8} b e f^{2} + 384 a^{7} b^{2} d f^{2} + 600 a^{7} b^{2} e^{2} f + 192 a^{6} b^{3} c f^{2} - 480 a^{6} b^{3} d e f - 125 a^{6} b^{3} e^{3} - 240 a^{5} b^{4} c e f + 96 a^{5} b^{4} d^{2} f + 150 a^{5} b^{4} d e^{2} + 96 a^{4} b^{5} c d f + 75 a^{4} b^{5} c e^{2} - 60 a^{4} b^{5} d^{2} e + 24 a^{3} b^{6} c^{2} f - 60 a^{3} b^{6} c d e + 8 a^{3} b^{6} d^{3} - 15 a^{2} b^{7} c^{2} e + 12 a^{2} b^{7} c d^{2} + 6 a b^{8} c^{2} d + b^{9} c^{3}, \left(t \mapsto t \log{\left(\frac{81 t^{2} a^{3} b^{7}}{64 a^{6} f^{2} - 80 a^{5} b e f + 32 a^{4} b^{2} d f + 25 a^{4} b^{2} e^{2} + 16 a^{3} b^{3} c f - 20 a^{3} b^{3} d e - 10 a^{2} b^{4} c e + 4 a^{2} b^{4} d^{2} + 4 a b^{5} c d + b^{6} c^{2}} + x \right)} \right)\right)} + \frac{f x^{5}}{5 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**2*(-a*f/b**3 + e/(2*b**2)) + x**2*(-a**3*f + a**2*b*e - a*b**2*d + b**3*
c)/(3*a**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*a**4*b**11 + 512*a**9*
f**3 - 960*a**8*b*e*f**2 + 384*a**7*b**2*d*f**2 + 600*a**7*b**2*e**2*f + 19
2*a**6*b**3*c*f**2 - 480*a**6*b**3*d*e*f - 125*a**6*b**3*e**3 - 240*a**5*b*
*4*c*e*f + 96*a**5*b**4*d**2*f + 150*a**5*b**4*d*e**2 + 96*a**4*b**5*c*d*f
+ 75*a**4*b**5*c*e**2 - 60*a**4*b**5*d**2*e + 24*a**3*b**6*c**2*f - 60*a**3
*b**6*c*d*e + 8*a**3*b**6*d**3 - 15*a**2*b**7*c**2*e + 12*a**2*b**7*c*d**2
+ 6*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(81*_t**2*a**3*b**7/(64*a**
6*f**2 - 80*a**5*b*e*f + 32*a**4*b**2*d*f + 25*a**4*b**2*e**2 + 16*a**3*b**
3*c*f - 20*a**3*b**3*d*e - 10*a**2*b**4*c*e + 4*a**2*b**4*d**2 + 4*a*b**5*c
*d + b**6*c**2) + x))) + f*x**5/(5*b**2)
```

3.213 $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=264

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{18a^{5/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\right.}{}$$

Rubi [A] time = 0.26, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1858, 1411, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{18a^{5/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{9a^{5/3}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{3\sqrt{3}\,a^{5/3}b^{10/3}}+\frac{x(be-2af)}{b^3}+\frac{fx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^4)/(4*b^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x)/(3*a*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/
3)) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(9*a^(5/3)*b^(10/3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^2}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} - \frac{\int \frac{-2b^3c-ab^2d+a^2be-a^3f-3ab(be-af)x^3-3ab^2fx^6}{a+bx^3}\,dx}{3ab^3}$$

$$= \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} - \frac{\int \frac{4b\left(-2b^3c-ab^2d+a^2be-a^3f\right)-\left(-12a^2b^2f+12ab^2(be-af)\cdots\right.}{a+bx^3}}{12ab^4}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\cdots\right.}{3ab^3}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\cdots\right.}{9a^{5/3}b^3}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\cdots\right.}{9a^{5/3}b^{10/3}}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\cdots\right.}{9a^{5/3}b^{10/3}}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} - \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\cdots\right.}{3\sqrt{3}\,a^{5/3}b^{10\cdots}}$$

Mathematica [A] time = 0.20, size = 251, normalized size = 0.95

$$\frac{\frac{12\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a\left(a+bx^3\right)}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{a^{5/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{a^{5/3}}-\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{a^{5/3}}+36\sqrt[3]{b}x(be-2af)+9b^{4/3}fx^4}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*(b*e - 2*a*f)*x + 9*b^(4/3)*f*x^4 + (12*b^(1/3)*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(2*b^3*c + a*b^2*d - 4
*a^2*b*e + 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) +
(4*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(5
/3) - (2*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2, x]
```

fricas [A] time = 0.45, size = 861, normalized size = 3.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/36*(9*a^3*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 6*sqrt(1/3)*(2
*a^2*b^4*c + a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d -
 4*a^3*b^3*e + 7*a^4*b^2*f)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*
(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b
)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*(2*a*b^3*c + a^2*b^2*d
- 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^3)*
(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3
*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7
*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c - a
^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3
*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c
+ a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d - 4*a^3*b^3*
e + 7*a^4*b^2*f)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/
3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(2*a*b^3*c + a^2*b^2
*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^
3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*
b^3*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e
+ 7*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c
- a^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4)]
```

giac [A] time = 0.21, size = 273, normalized size = 1.03

$$-\frac{\sqrt{3}\left(2b^3c+ab^2d+7a^3f-4a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^2}-\frac{\left(2b^3c+ab^2d+7a^3f-4a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab^2}-\frac{\left(2b^3c+ab^2d+7a^3f-4a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b^3}+\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{3\left(bx^3+a\right)ab^3}+\frac{b^6fx^4-8ab^5fx+4b^6xe}{4b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^3*c + a*b^2*d + 7*a^3*f - 4*a^2*b*e)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1/18*(2*b^3*c +
a*b^2*d + 7*a^3*f - 4*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(2/3)*a*b^2) - 1/9*(2*b^3*c + a*b^2*d + 7*a^3*f - 4*a^2*b*e)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3) + 1/3*(b^3*c*x - a*b^2*d*x - a^3
*f*x + a^2*b*x*e)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*f*x^4 - 8*a*b^5*f*x + 4*b^
6*x*e)/b^8
```

maple [B] time = 0.06, size = 482, normalized size = 1.83

$$\frac{f x^4}{4b^2}-\frac{a^2 f x}{3\left(b x^3+a\right)b^3}+\frac{aex}{3\left(b x^3+a\right)b^2}+\frac{cx}{3\left(b x^3+a\right)a}-\frac{dx}{3\left(b x^3+a\right)b}+\frac{7\sqrt{3}\,a^2 f \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{7a^2 f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{7a^2 f \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{4\sqrt{3}\,ae \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{4ae\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2ae\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{2afx}{b^3}+\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{ex}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/4*f*x^4/b^2-2/b^3*a*f*x+1/b^2*e*x-1/3/b^3*a^2*x/(b*x^3+a)*f+1/3/b^2*a*x/(
b*x^3+a)*e-1/3/b*x/(b*x^3+a)*d+1/3/a*x/(b*x^3+a)*c+7/9/b^4*a^2/(a/b)^(2/3)*
ln(x+(a/b)^(1/3))*f-4/9/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+1/9/b^2/(a/b)
^(2/3)*ln(x+(a/b)^(1/3))*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-7/18/b^4
*a^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f+2/9/b^3*a/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-1/18/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+7
/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-4/
9/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/9/b
^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+2/9/b/a/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 3.05, size = 254, normalized size = 0.96

$$\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3\left(ab^4x^3+a^2b^3\right)}+\frac{bfx^4+4\left(be-2af\right)x}{4b^3}+\frac{\sqrt{3}\left(2b^3c+ab^2d-4a^2be+7a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(a*b^4*x^3 + a^2*b^3) + 1/4*(b*f*
x^4 + 4*(b*e - 2*a*f)*x)/b^3 + 1/9*sqrt(3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e +
 7*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4*(a/b)^
(2/3)) - 1/18*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(a*b^4*(a/b)^(2/3)) + 1/9*(2*b^3*c + a*b^2*d - 4*a^2*b*
e + 7*a^3*f)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

mupad [B] time = 5.18, size = 241, normalized size = 0.91

$$x\left(\frac{e}{b^2}-\frac{2af}{b^3}\right)+\frac{fx^4}{4b^2}+\frac{x\left(-fa^3+ea^2b-dab^2+cb^3\right)}{3a\left(b^4x^3+ab^3\right)}+\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(7fa^3-4ea^2b+dab^2+2cb^3\right)}{9a^{5/3}b^{10/3}}+\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(7fa^3-4ea^2b+dab^2+2cb^3\right)}{9a^{5/3}b^{10/3}}-\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(7fa^3-4ea^2b+dab^2+2cb^3\right)}{9a^{5/3}b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x)
```

```
[Out] x*(e/b^2 - (2*a*f)/b^3) + (f*x^4)/(4*b^2) + (x*(b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(3*a*(a*b^3 + b^4*x^3)) + (log(b^(1/3)*x + a^(1/3))*(2*b^3*c + 7*a
^3*f + a*b^2*d - 4*a^2*b*e))/(9*a^(5/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i
 + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c + 7*a^3*f + a*b^2
*d - 4*a^2*b*e))/(9*a^(5/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c + 7*a^3*f + a*b^2*d - 4*a^2*b
*e))/(9*a^(5/3)*b^(10/3))
```

sympy [**A**] time = 7.02, size = 377, normalized size = 1.43

$$x\left(-\frac{2af}{b^3}+\frac{e}{b^2}\right)+\frac{x\left(-a^3f+a^2be-ab^2d+b^3c\right)}{3a^2b^3+3ab^4x^3}+\operatorname{RootSum}\left(729t^3a^5b^{10}-343a^9f^3+588a^8bef^2-147a^7b^2df^2-336a^7b^2e^2f-294a^6b^3cf^2+168a^6b^3def+64a^6b^3e^3+336a^5b^4cef-21a^5b^4d^2f-48a^5b^4de^2-84a^4b^5cdf-96a^4b^5ce^2+12a^4b^5d^2e-84a^3b^6c^2f+48a^3b^6cde-a^3b^6d^3+48a^2b^7c^2e-6a^2b^7cd^2-12ab^8c^2d-8b^9c^3,\left(t\mapsto t\log\left(\frac{9ta^2b^3}{7a^3f-4a^2be+ab^2d+2b^3c}+x\right)\right)\right)+\frac{fx^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(-2*a*f/b**3 + e/b**2) + x*(-a**3*f + a**2*b*e - a*b**2*d + b**3*c)/(3*a*
*2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*a**5*b**10 - 343*a**9*f**3 + 5
88*a**8*b*e*f**2 - 147*a**7*b**2*d*f**2 - 336*a**7*b**2*e**2*f - 294*a**6*b
**3*c*f**2 + 168*a**6*b**3*d*e*f + 64*a**6*b**3*e**3 + 336*a**5*b**4*c*e*f
- 21*a**5*b**4*d**2*f - 48*a**5*b**4*d*e**2 - 84*a**4*b**5*c*d*f - 96*a**4*
b**5*c*e**2 + 12*a**4*b**5*d**2*e - 84*a**3*b**6*c**2*f + 48*a**3*b**6*c*d*
e - a**3*b**6*d**3 + 48*a**2*b**7*c**2*e - 6*a**2*b**7*c*d**2 - 12*a*b**8*c
**2*d - 8*b**9*c**3, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*f - 4*a**2*b*
e + a*b**2*d + 2*b**3*c) + x))) + f*x**4/(4*b**2)
```

3.214 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=265

$$-\frac{c}{a^2x}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{18a^{7/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}\right.}{}$$

Rubi [A] time = 0.25, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1488, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^2be+5a^3f-ab^2d+4b^3c\right)}{18a^{7/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^2be+5a^3f-ab^2d+4b^3c\right)}{9a^{7/3}b^{8/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^2be+5a^3f-ab^2d+4b^3c\right)}{3\sqrt{3}a^{7/3}b^{8/3}}-\frac{c}{a^2x}+\frac{fx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x^2)/(2*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/
(3*a^2*b^2*(a + b*x^3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(8/3)) +
((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(
7/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}-\frac{\int \frac{-3b^3c+b\left(\frac{b^3c}{a}-b^2d-2abe+2a^2f\right)x^3-3ab^2fx^6}{x^2\left(a+bx^3\right)}\,dx}{3ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}-\frac{\int \left(-\frac{3b^3c}{ax^2}-3abfx+\frac{b\left(4b^3c-ab^2d-2a^2be+5a^3f\right)x}{a\left(a+bx^3\right)}\right)dx}{3ab^3}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\int \frac{x}{a+bx^3}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\int \frac{1}{\sqrt[3]{a}+}}{9a^{7/3}b^{7/3}}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\log\left(\sqrt[3]{}\right.}{9a^{7/3}b^{8/3}}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\log\left(\sqrt[3]{}\right.}{9a^{7/3}b^{8/3}}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\tan^{-1}\left(\right.}{3\sqrt{3}\,a^{7/3}b^{8/3}}$$

Mathematica [A] time = 0.21, size = 255, normalized size = 0.96

$$\frac{1}{18}\left(-\frac{18c}{a^2x}+\frac{6x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^2b^2\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{a^{7/3}b^{8/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{a^{7/3}b^{8/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^3f-2a^2be-ab^2d+4b^3c\right)}{a^{7/3}b^{8/3}}+\frac{9fx^2}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*c)/(a^2*x) + (9*f*x^2)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*x^2)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(4*b^3*c - a*b^2*d - 2*a^2*b*e +
5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(7/3)*b^(8/3)) + (
2*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(7
/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(8/3)))/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2), x]
```

fricas [A] time = 0.45, size = 860, normalized size = 3.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(9*a^3*b^3*f*x^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*
b^3*e - 5*a^4*b^2*f)*x^3 + 3*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*
e + 5*a^4*b^2*f)*x^4 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*
x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*
b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*
x)/(b*x^3 + a)) - ((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a
*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a
*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*
a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/
3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x), 1/18*(9*a^3*b^3*f*x
^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*b^3*e - 5*a^4*b^2*f)
*x^3 + 6*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*e + 5*a^4*b^2*f)*x^4
 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*x)*sqrt((a*b^2)^(1/3
)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (
(4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d
- 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (
a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a
*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b*x + (a*b^2
)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x)]
```

giac [A] time = 0.19, size = 305, normalized size = 1.15

$$\frac{fx^2}{2b^2} - \frac{\sqrt{3}\left(4b^3c - ab^2d + 5a^3f - 2a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^2b^2} + \frac{\left(4b^3c - ab^2d + 5a^3f - 2a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^2b^2} + \frac{\left(4b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^3b^2} - \frac{4b^3cx^3 - ab^2dx^3 - a^3fx^3 + a^2bx^3e + 3ab^2c}{3\left(bx^4 + ax\right)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^2 - 1/9*sqrt(3)*(4*b^3*c - a*b^2*d + 5*a^3*f - 2*a^2*b*e)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b^2) +
 1/18*(4*b^3*c - a*b^2*d + 5*a^3*f - 2*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) +
```

```
(-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b^2) + 1/9*(4*b^3*c*(-a/b)^(1/3) - a*b^2*
d*(-a/b)^(1/3) + 5*a^3*f*(-a/b)^(1/3) - 2*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3
)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/3*(4*b^3*c*x^3 - a*b^2*d*x^3 - a
^3*f*x^3 + a^2*b*x^3*e + 3*a*b^2*c)/((b*x^4 + a*x)*a^2*b^2)
```

maple [B] time = 0.06, size = 474, normalized size = 1.79

$$\frac{af\,x^2}{3\left(b\,x^3+a\right)b^2}+\frac{d\,x^2}{3\left(b\,x^3+a\right)a}-\frac{bc\,x^2}{3\left(b\,x^3+a\right)a^2}-\frac{e\,x^2}{3\left(b\,x^3+a\right)b}+\frac{f\,x^2}{2b^2}-\frac{5\sqrt{3}\,af\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{5af\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{5af\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{4\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{4c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{2c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{2\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] 1/2*f*x^2/b^2+1/3*a/b^2*x^2/(b*x^3+a)*f-1/3/b*x^2/(b*x^3+a)*e+1/3/a*x^2/(b*
x^3+a)*d-1/3/a^2*b*x^2/(b*x^3+a)*c+5/9*a/b^3*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3)
)-5/18*a/b^3*f/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9*a/b^3*f*3^
(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/a/b*d/(a/b)^(
1/3)*ln(x+(a/b)^(1/3))+1/18/a/b*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+1/9/a/b*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+
4/9/a^2*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2*c/(a/b)^(1/3)*ln(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3))-4/9/a^2*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))-2/9/b^2*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2*e/(a/b)^(1/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/b^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/a^2*c/x
```

maxima [A] time = 3.01, size = 258, normalized size = 0.97

$$\frac{fx^2}{2b^2}-\frac{3ab^2c+\left(4b^3c-ab^2d+a^2be-a^3f\right)x^3}{3\left(a^2b^3x^4+a^3b^2x\right)}-\frac{\sqrt{3}\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*f*x^2/b^2 - 1/3*(3*a*b^2*c + (4*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)
/(a^2*b^3*x^4 + a^3*b^2*x) - 1/9*sqrt(3)*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5
*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^
(1/3)) - 1/18*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(1/3)) + 1/9*(4*b^3*c - a*b^2*d - 2*a^2*
b*e + 5*a^3*f)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(1/3))
```

mupad [B] time = 5.39, size = 244, normalized size = 0.92

$$\frac{f\,x^2}{2\,b^2}-\frac{\frac{x^3\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{3\,a^2}+\frac{b^2\,c}{a}}{b^3\,x^4+a\,b^2\,x}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] (f*x^2)/(2*b^2) - ((x^3*(4*b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^2) + (b
^2*c)/a)/(b^3*x^4 + a*b^2*x) + (log(b^(1/3)*x + a^(1/3))*(4*b^3*c + 5*a^3*f
 - a*b^2*d - 2*a^2*b*e))/(9*a^(7/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*
b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*b^3*c + 5*a^3*f - a*b^2*d -
2*a^2*b*e))/(9*a^(7/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a
^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*b^3*c + 5*a^3*f - a*b^2*d - 2*a^2*b*e))/(
9*a^(7/3)*b^(8/3))
```

sympy [A] time = 32.22, size = 457, normalized size = 1.72

$$\frac{-3ab^2c+x^3\left(a^3f-a^2be+ab^2d-4b^3c\right)}{3a^3b^2x+3a^2b^3x^4}+\operatorname{RootSum}\left(729t^3a^7b^8-125a^9f^3+150a^8bef^2+75a^7b^2df^2-60a^7b^2e^2f-300a^6b^3cf^2-60a^6b^3def+8a^6b^3e^3+240a^5b^4cef-15a^5b^4d^2f+12a^5b^4de^2+120a^4b^5cdf-48a^4b^5ce^2+6a^4b^5d^2e-240a^3b^6c^2f-48a^3b^6cde+a^3b^6d^3+96a^2b^7c^2e-12a^2b^7cd^2+48ab^8c^2d-64b^9c^3,\left(t\mapsto t\log\left(\frac{81t^2a^5b^5}{25a^6f^2-20a^5bef-10a^4b^2df+4a^4b^2e^2+40a^3b^3cf+4a^3b^3de-16a^2b^4ce+a^2b^4d^2-8ab^5cd+16b^6c^2}+x\right)\right)\right)+\frac{fx^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*a*b**2*c + x**3*(a**3*f - a**2*b*e + a*b**2*d - 4*b**3*c))/(3*a**3*b**2
*x + 3*a**2*b**3*x**4) + RootSum(729*_t**3*a**7*b**8 - 125*a**9*f**3 + 150*
a**8*b*e*f**2 + 75*a**7*b**2*d*f**2 - 60*a**7*b**2*e**2*f - 300*a**6*b**3*c
*f**2 - 60*a**6*b**3*d*e*f + 8*a**6*b**3*e**3 + 240*a**5*b**4*c*e*f - 15*a*
*5*b**4*d**2*f + 12*a**5*b**4*d*e**2 + 120*a**4*b**5*c*d*f - 48*a**4*b**5*c
*e**2 + 6*a**4*b**5*d**2*e - 240*a**3*b**6*c**2*f - 48*a**3*b**6*c*d*e + a*
*3*b**6*d**3 + 96*a**2*b**7*c**2*e - 12*a**2*b**7*c*d**2 + 48*a*b**8*c**2*d
 - 64*b**9*c**3, Lambda(_t, _t*log(81*_t**2*a**5*b**5/(25*a**6*f**2 - 20*a*
*5*b*e*f - 10*a**4*b**2*d*f + 4*a**4*b**2*e**2 + 40*a**3*b**3*c*f + 4*a**3*
b**3*d*e - 16*a**2*b**4*c*e + a**2*b**4*d**2 - 8*a*b**5*c*d + 16*b**6*c**2)
 + x))) + f*x**2/(2*b**2)
```

3.215 $\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=260

$$-\frac{c}{2a^2x^2}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{18a^{8/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}\right.}{}$$

Rubi [A] time = 0.25, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1488, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-a^2be+4a^3f-2ab^2d+5b^3c\right)}{18a^{8/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(-a^2be+4a^3f-2ab^2d+5b^3c\right)}{9a^{8/3}b^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^2be+4a^3f-2ab^2d+5b^3c\right)}{3\sqrt{3}\,a^{8/3}b^{7/3}}-\frac{c}{2a^2x^2}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -c/(2*a^2*x^2) + (f*x)/b^2 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^2
*b^2*(a + b*x^3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(7/3)) - ((5*b^
3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b
^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+b\left(\frac{2b^3c}{a}-2b^2d-abe+a^2f\right)x^3-3ab^2fx^6}{x^3\left(a+bx^3\right)}\,dx}{3ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)} - \frac{\int \left(-3abf-\frac{3b^3c}{ax^3}+\frac{b\left(5b^3c-2ab^2d-a^2be+4a^3f\right)}{a\left(a+bx^3\right)}\right)dx}{3ab^3}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\int \frac{}{a\cdots}}{3a^2b^2}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\int \frac{}{\sqrt[3]{\cdots}}}{9a^{8/3}b^2}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\log}{9a^{8/3}b^{7/3}}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\log}{9a^{8/3}b^{7/3}}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\tan}{3\sqrt{3}\,a^{8/3}b^{7/3}}$$

Mathematica [A] time = 0.22, size = 250, normalized size = 0.96

$$\frac{1}{18}\left(-\frac{9c}{a^2x^2}+\frac{6x\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{a^{8/3}b^{7/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{a^{8/3}b^{7/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{a^{8/3}b^{7/3}}+\frac{18fx}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*c)/(a^2*x^2) + (18*f*x)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*x)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*
a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(8/3)*b^(7/3)) - (2*
(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(8/3
)*b^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(8/3)*b^(7/3)))/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^3 \left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2), x]
```

fricas [A] time = 0.45, size = 902, normalized size = 3.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a^
4*b^2*e - 8*a^5*b*f)*x^3 + 3*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^3*
e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*
x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3
*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^
(1/3)/b))/(b*x^3 + a)) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5
 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*
b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c - 2*a*b^3*d - a^
2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^
2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^5 + a^5*b^3*x^2),
 1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a^
4*b^2*e - 8*a^5*b*f)*x^3 - 6*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^3*
e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*
x^2)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b
)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e
 + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^
2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c
 - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*
b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^
5 + a^5*b^3*x^2)]
```

giac [A] time = 0.18, size = 261, normalized size = 1.00

$$\frac{fx}{b^2} + \frac{\sqrt{3}\left(5\,b^3c - 2\,ab^2d + 4\,a^3f - a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a^2b} + \frac{\left(5\,b^3c - 2\,ab^2d + 4\,a^3f - a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a^2b} + \frac{\left(5\,b^3c - 2\,ab^2d + 4\,a^3f - a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^2} - \frac{c}{2\,a^2x^2} - \frac{b^3cx - ab^2dx - a^3fx + a^2bxe}{3\left(bx^3 + a\right)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] f*x/b^2 + 1/9*sqrt(3)*(5*b^3*c - 2*a*b^2*d + 4*a^3*f - a^2*b*e)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) + 1/18*(5
```

```
*b^3*c - 2*a*b^2*d + 4*a^3*f - a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(2/3)*a^2*b) + 1/9*(5*b^3*c - 2*a*b^2*d + 4*a^3*f - a^2*b*e
)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/2*c/(a^2*x^2) - 1/3
*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a^2*b^2)
```

maple [B] time = 0.06, size = 463, normalized size = 1.78

$$\frac{afx}{3\left(bx^3+a\right)b^2}+\frac{dx}{3\left(bx^3+a\right)a}-\frac{bcx}{3\left(bx^3+a\right)a^2}-\frac{ex}{3\left(bx^3+a\right)b}-\frac{4\sqrt{3}\,af\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{4af\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2af\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{5\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{5c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{5c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{fx}{b^2}-\frac{c}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] 1/b^2*f*x+1/3*a/b^2*x/(b*x^3+a)*f-1/3/b*x/(b*x^3+a)*e+1/3/a*x/(b*x^3+a)*d-1
/3/a^2*b*x/(b*x^3+a)*c-4/9*a/b^3*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9*a/b^3*
f/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9*a/b^3*f/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/9/a/b*d/(a/b)^(2/3)*ln(x+(a/
b)^(1/3))-1/9/a/b*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/a/b*d
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-5/9/a^2*c/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))+5/18/a^2*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))-5/9/a^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))+1/9/b^2*e/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18/b^2*e/(a/b)^(2/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/9/b^2*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))-1/2/a^2*c/x^2
```

maxima [A] time = 2.97, size = 258, normalized size = 0.99

$$-\frac{3ab^2c+\left(5b^3c-2ab^2d+2a^2be-2a^3f\right)x^3}{6\left(a^2b^3x^5+a^3b^2x^2\right)}+\frac{fx}{b^2}-\frac{\sqrt{3}\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a*b^2*c + (5*b^3*c - 2*a*b^2*d + 2*a^2*b*e - 2*a^3*f)*x^3)/(a^2*b^3
*x^5 + a^3*b^2*x^2) + f*x/b^2 - 1/9*sqrt(3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e
+ 4*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/
b)^(2/3)) + 1/18*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) - 1/9*(5*b^3*c - 2*a*b^2*d - a
^2*b*e + 4*a^3*f)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

mupad [B] time = 5.22, size = 245, normalized size = 0.94

$$\frac{fx}{b^2}-\frac{\frac{x^3\left(-2fa^3+2ea^2b-2dab^2+5cb^3\right)}{6a^2}+\frac{b^2c}{2a}}{b^3x^5+ab^2x^2}-\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(4fa^3-ea^2b-2dab^2+5cb^3\right)}{9a^{8/3}b^{7/3}}-\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(4fa^3-ea^2b-2dab^2+5cb^3\right)}{9a^{8/3}b^{7/3}}+\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(4fa^3-ea^2b-2dab^2+5cb^3\right)}{9a^{8/3}b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] (f*x)/b^2 - ((x^3*(5*b^3*c - 2*a^3*f - 2*a*b^2*d + 2*a^2*b*e))/(6*a^2) + (b
^2*c)/(2*a))/(b^3*x^5 + a*b^2*x^2) - (log(b^(1/3)*x + a^(1/3))*(5*b^3*c + 4
*a^3*f - 2*a*b^2*d - a^2*b*e))/(9*a^(8/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1
i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c + 4*a^3*f - 2*a*
b^2*d - a^2*b*e))/(9*a^(8/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c + 4*a^3*f - 2*a*b^2*d - a^2*b
*e))/(9*a^(8/3)*b^(7/3))
```

sympy [A] time = 77.38, size = 381, normalized size = 1.47

$$\frac{-3ab^2c+x^3\left(2a^3f-2a^2be+2ab^2d-5b^3c\right)}{6a^3b^2x^2+6a^2b^3x^5}+\operatorname{RootSum}\left(729t^3a^8b^7+64a^9f^3-48a^8bef^2-96a^7b^2df^2+12a^7b^2e^2f+240a^6b^3cf^2+48a^6b^3def-a^6b^3e^3-120a^5b^4cef+48a^5b^4d^2f-6a^5b^4de^2-240a^4b^5cdf+15a^4b^5ce^2-12a^4b^5d^2e+300a^3b^6c^2f+60a^3b^6cde-8a^3b^6d^3-75a^2b^7c^2e+60a^2b^7cd^2-150ab^8c^2d+125b^9c^3,\left(t\mapsto t\log\left(-\frac{9ta^3b^2}{4a^3f-a^2be-2ab^2d+5b^3c}+x\right)\right)\right)+\frac{fx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] (-3*a*b**2*c + x**3*(2*a**3*f - 2*a**2*b*e + 2*a*b**2*d - 5*b**3*c))/(6*a**
3*b**2*x**2 + 6*a**2*b**3*x**5) + RootSum(729*_t**3*a**8*b**7 + 64*a**9*f**
3 - 48*a**8*b*e*f**2 - 96*a**7*b**2*d*f**2 + 12*a**7*b**2*e**2*f + 240*a**6
*b**3*c*f**2 + 48*a**6*b**3*d*e*f - a**6*b**3*e**3 - 120*a**5*b**4*c*e*f +
48*a**5*b**4*d**2*f - 6*a**5*b**4*d*e**2 - 240*a**4*b**5*c*d*f + 15*a**4*b*
*5*c*e**2 - 12*a**4*b**5*d**2*e + 300*a**3*b**6*c**2*f + 60*a**3*b**6*c*d*e
 - 8*a**3*b**6*d**3 - 75*a**2*b**7*c**2*e + 60*a**2*b**7*c*d**2 - 150*a*b**
8*c**2*d + 125*b**9*c**3, Lambda(_t, _t*log(-9*_t*a**3*b**2/(4*a**3*f - a**
2*b*e - 2*a*b**2*d + 5*b**3*c) + x))) + f*x/b**2
```

3.216 $\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=269

$$\frac{2bc-ad}{a^3x}-\frac{c}{4a^2x^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(2a^3f+a^2be-4ab^2d+7b^3\right.}{18a^{10/3}b^{5/3}}$$

Rubi [A] time = 0.29, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^2be+2a^3f-4ab^2d+7b^3c\right)}{18a^{10/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^2be+2a^3f-4ab^2d+7b^3c\right)}{9a^{10/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^2be+2a^3f-4ab^2d+7b^3c\right)}{3\sqrt{3}\,a^{10/3}b^{5/3}}+\frac{2bc-ad}{a^3x}-\frac{c}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] -c/(4*a^2*x^4) + (2*b*c - a*d)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*x^2)/(3*a^3*b*(a + b*x^3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)*b^(5
/3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])
/(9*a^(10/3)*b^(5/3)) + ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{b^3c}{a^2}-\frac{b^2d}{a}+be+2af\right)x^6}{x^5\left(a+bx^3\right)}\,dx}{3ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^5}-\frac{3b^3(-2bc+ad)}{a^2x^2}-\frac{b^2\left(7b^3c-4ab^2d+a^2be+2a^3f\right)x}{a^2\left(a+bx^3\right)}\right) dx}{3ab^3}$$

$$= -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)}+\frac{\left(7b^3c-4ab^2d+a^2be+2a^3f\right)}{3a^3b}$$

$$= -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)}-\frac{\left(7b^3c-4ab^2d+a^2be+2a^3f\right)}{9a^{10/3}b^{4/3}}$$

$$= -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)}-\frac{\left(7b^3c-4ab^2d+a^2be+2a^3f\right)}{9a^{10/3}b^{5/3}}$$

$$= -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)}-\frac{\left(7b^3c-4ab^2d+a^2be+2a^3f\right)}{9a^{10/3}b^{5/3}}$$

$$= -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)}-\frac{\left(7b^3c-4ab^2d+a^2be+2a^3f\right)}{3\sqrt{3}\,a^{10/3}b^{5/3}}$$

Mathematica [A] time = 0.27, size = 255, normalized size = 0.95

$$\frac{-\frac{9a^{4/3}c}{x^4}-\frac{12\sqrt[3]{a}x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{b\left(a+bx^3\right)}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{b^{5/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{b^{5/3}}+\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^3f+a^2be-4ab^2d+7b^3c\right)}{b^{5/3}}-\frac{36\sqrt[3]{a}(ad-2bc)}{x}}{36a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(4/3)*c)/x^4 - (36*a^(1/3)*(-2*b*c + a*d))/x - (12*a^(1/3)*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*Sqrt[3]*(7*b^3*c -
4*a*b^2*d + a^2*b*e + 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/b^(5/3) - (4*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/b^(5/3) + (2*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(36*a^(10/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^5\left(a + bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2), x]
```

fricas [A] time = 0.45, size = 902, normalized size = 3.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*
x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 6*sqrt(1/3)*((7*a*b^5*c - 4*a^2*b
^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^3*d + a^4*b^2*
e + 2*a^5*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/
3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
 - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 2*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e +
 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 4*((7*b^4*c -
 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*
e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^4*x^7 +
a^5*b^3*x^4), -1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e
- a^4*b^2*f)*x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 12*sqrt(1/3)*((7*a*b
^5*c - 4*a^2*b^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^
3*d + a^4*b^2*e + 2*a^5*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*
(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 2*((7*b^4*c - 4*a*b^3
*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^
4*f)*x^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
 + 4*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^
2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))
/(a^4*b^4*x^7 + a^5*b^3*x^4)]
```

giac [A] time = 0.20, size = 310, normalized size = 1.15

$$\frac{\sqrt{3}\left(7\,b^3c - 4\,ab^2d + 2\,a^3f + a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^3b} - \frac{\left(7\,b^3c - 4\,ab^2d + 2\,a^3f + a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^3b} - \frac{\left(7\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4b} + \frac{b^3cx^2 - ab^2dx^2 - a^3fx^2 + a^2bx^2e}{3\left(bx^3 + a\right)a^3b} + \frac{8\,bcx^3 - 4\,adx^3 - ac}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*b^3*c - 4*a*b^2*d + 2*a^3*f + a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3*b) - 1/18*(7*b^3*c - 4
```

```
*a*b^2*d + 2*a^3*f + a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a
*b^2)^(1/3)*a^3*b) - 1/9*(7*b^3*c*(-a/b)^(1/3) - 4*a*b^2*d*(-a/b)^(1/3) + 2
*a^3*f*(-a/b)^(1/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^4*b) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/
((b*x^3 + a)*a^3*b) + 1/4*(8*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^3*x^4)
```

maple [**B**] time = 0.07, size = 486, normalized size = 1.81

$$\frac{e x^2}{3\left(b x^3+a\right)a}-\frac{bd x^2}{3\left(b x^3+a\right)a^2}+\frac{b^2c x^2}{3\left(b x^3+a\right)a^3}-\frac{f x^2}{3\left(b x^3+a\right)b}+\frac{\sqrt{3}\, e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{4\sqrt{3}\, d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{4d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{2d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{7\sqrt{3}\, bc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{7bc\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{7bc\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{2\sqrt{3}\, f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{2f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{d}{a^2x}+\frac{2bc}{a^3x}-\frac{c}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x)
```

```
[Out] -1/3/b*x^2/(b*x^3+a)*f+1/3/a*x^2/(b*x^3+a)*e-1/3/a^2*b*x^2/(b*x^3+a)*d+1/3/
a^3*b^2*x^2/(b*x^3+a)*c-2/9/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-1/9/a/b/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))*e+4/9/a^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-7/9/a^
3*b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/9/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))*f+1/18/a/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-2
/9/a^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+7/18/a^3*b/(a/b)^(1/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+2/9/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+1/9/a/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))*e-4/9/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*d+7/9/a^3*b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*c-1/4*c/a^2/x^4-d/a^2/x+2/a^3/x*b*c
```

maxima [**A**] time = 2.93, size = 267, normalized size = 0.99

$$\frac{4\left(7 b^3c-4 ab^2d+a^2be-a^3f\right)x^6-3 a^2bc+3\left(7 ab^2c-4 a^2bd\right)x^3}{12\left(a^3b^2x^7+a^4bx^4\right)}+\frac{\sqrt{3}\left(7 b^3c-4 ab^2d+a^2be+2 a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(7 b^3c-4 ab^2d+a^2be+2 a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(7 b^3c-4 ab^2d+a^2be+2 a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(7*b^3*c - 4*a*b^2*d + a^2*b*e - a^3*f)*x^6 - 3*a^2*b*c + 3*(7*a*b^
2*c - 4*a^2*b*d)*x^3)/(a^3*b^2*x^7 + a^4*b*x^4) + 1/9*sqrt(3)*(7*b^3*c - 4*
a*b^2*d + a^2*b*e + 2*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(a^3*b^2*(a/b)^(1/3)) + 1/18*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3)) - 1/9*(7*b^3
*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(1/
3))
```

mupad [**B**] time = 5.18, size = 247, normalized size = 0.92

$$-\frac{\frac{c}{4a}+\frac{x^3\left(4ad-7bc\right)}{4a^2}-\frac{x^6\left(-fa^3+ea^2b-4dab^2+7cb^3\right)}{3a^3b}}{bx^7+ax^4}-\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(2fa^3+ea^2b-4dab^2+7cb^3\right)}{9a^{10/3}b^{5/3}}+\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(2fa^3+ea^2b-4dab^2+7cb^3\right)}{9a^{10/3}b^{5/3}}-\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(2fa^3+ea^2b-4dab^2+7cb^3\right)}{9a^{10/3}b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7*
b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*e))/(9*a^(10/3)*b^(5/3)) - (log(b^(1/3)
*x + a^(1/3))*(7*b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*e))/(9*a^(10/3)*b^(5/3
)) - (c/(4*a) + (x^3*(4*a*d - 7*b*c))/(4*a^2) - (x^6*(7*b^3*c - a^3*f - 4*a
*b^2*d + a^2*b*e))/(3*a^3*b))/(a*x^4 + b*x^7) - (log(3^(1/2)*a^(1/3)*1i - 2
*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*b^3*c + 2*a^3*f - 4*a*b^2*d
 + a^2*b*e))/(9*a^(10/3)*b^(5/3))
```

sympy [**A**] time = 177.03, size = 473, normalized size = 1.76

$$\operatorname{RootSum}\left(729t^3a^{10}b^5+8a^9f^3+12a^8bef^2-48a^7b^2df^2+6a^7b^2e^2f+84a^6b^3cf^2-48a^6b^3def+a^6b^3e^3+84a^5b^4cef+96a^5b^4d^2f-12a^5b^4de^2-336a^4b^5cdf+21a^4b^5ce^2+48a^4b^5d^2e+294a^3b^6c^2f-168a^3b^6cde-64a^3b^6d^3+147a^2b^7c^2e+336a^2b^7cd^2-588ab^8c^2d+343b^9c^3,\left(t\mapsto t\log\left(\frac{81t^2a^7b^3}{4a^6f^2+4a^5bef-16a^4b^2df+a^4b^2e^2+28a^3b^3cf-8a^3b^3de+14a^2b^4ce+16a^2b^4d^2-56ab^5cd+49b^6c^2}+x\right)\right)\right)+\frac{-3a^2bc+x^6\left(-4a^3f+4a^2be-16ab^2d+28b^3c\right)+x^3\left(-12a^2bd+21ab^2c\right)}{12a^4bx^4+12a^3b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**10*b**5 + 8*a**9*f**3 + 12*a**8*b*e*f**2 - 48*a**7*b**
2*d*f**2 + 6*a**7*b**2*e**2*f + 84*a**6*b**3*c*f**2 - 48*a**6*b**3*d*e*f +
a**6*b**3*e**3 + 84*a**5*b**4*c*e*f + 96*a**5*b**4*d**2*f - 12*a**5*b**4*d*
e**2 - 336*a**4*b**5*c*d*f + 21*a**4*b**5*c*e**2 + 48*a**4*b**5*d**2*e + 29
4*a**3*b**6*c**2*f - 168*a**3*b**6*c*d*e - 64*a**3*b**6*d**3 + 147*a**2*b**
7*c**2*e + 336*a**2*b**7*c*d**2 - 588*a*b**8*c**2*d + 343*b**9*c**3, Lambda
(_t, _t*log(81*_t**2*a**7*b**3/(4*a**6*f**2 + 4*a**5*b*e*f - 16*a**4*b**2*d
*f + a**4*b**2*e**2 + 28*a**3*b**3*c*f - 8*a**3*b**3*d*e + 14*a**2*b**4*c*e
 + 16*a**2*b**4*d**2 - 56*a*b**5*c*d + 49*b**6*c**2) + x))) + (-3*a**2*b*c
+ x**6*(-4*a**3*f + 4*a**2*b*e - 16*a*b**2*d + 28*b**3*c) + x**3*(-12*a**2*
b*d + 21*a*b**2*c))/(12*a**4*b*x**4 + 12*a**3*b**2*x**7)
```

3.217 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}\,dx$

Optimal. Leaf size=270

$$\frac{2bc-ad}{2a^3x^2}-\frac{c}{5a^2x^5}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{18a^{11/3}b^{4/3}}+$$

Rubi [A] time = 0.27, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^2be+a^3f-5ab^2d+8b^3c\right)}{18a^{11/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^2be+a^3f-5ab^2d+8b^3c\right)}{9a^{11/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^2be+a^3f-5ab^2d+8b^3c\right)}{3\sqrt{3}a^{11/3}b^{4/3}}+\frac{2bc-ad}{2a^3x^2}-\frac{c}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(2*a^3*x^2) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*x)/(3*a^3*b*(a + b*x^3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)*b^
(4/3)) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/(9*a^(11/3)*b^(4/3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{2b^3c}{a^2}-\frac{2b^2d}{a}+2be+af\right)x^6}{x^6\left(a+bx^3\right)}\,dx}{3ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^6}-\frac{3b^3(-2bc+ad)}{a^2x^3}-\frac{b^2\left(8b^3c-5ab^2d+2a^2be+a^3f\right)}{a^2\left(a+bx^3\right)}\right)dx}{3ab^3}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)}+\frac{\left(8b^3c-5ab^2d+2a^2be+a^3\right.}{3a^3b}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)}+\frac{\left(8b^3c-5ab^2d+2a^2be+a^3\right.}{9a^{11/3}b}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)}+\frac{\left(8b^3c-5ab^2d+2a^2be+a^3\right.}{9a^{11/3}b^{4/3}}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)}+\frac{\left(8b^3c-5ab^2d+2a^2be+a^3\right.}{9a^{11/3}b^{4/3}}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^3b\left(a+bx^3\right)}-\frac{\left(8b^3c-5ab^2d+2a^2be+a^3\right.}{3\sqrt{3}\,a^{11/3}b}$$

Mathematica [A] time = 0.19, size = 253, normalized size = 0.94

$$\frac{-\frac{45a^{2/3}(ad-2bc)}{x^2}-\frac{18a^{5/3}c}{x^5}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{b^{4/3}}-\frac{10\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{b^{4/3}}-\frac{30a^{2/3}x\left(a^3f-a^2be+ab^2d-b^3c\right)}{b\left(a+bx^3\right)}-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{b^{4/3}}}{90a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(5/3)*c)/x^5 - (45*a^(2/3)*(-2*b*c + a*d))/x^2 - (30*a^(2/3)*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (10*Sqrt[3]*(8*b^3*c
- 5*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/b^(4/3) + (10*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/b^(4/3) - (5*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(90*a^(11/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^6 \left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2), x]
```

fricas [A] time = 0.45, size = 897, normalized size = 3.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^4*b^2*e - 2*a^5*
b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 15*sqrt(1/3)*((8*a*b^5*c - 5
*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c - 5*a^3*b^3*d + 2*
a^4*b^2*e + a^5*b*f)*x^5)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)
^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)
*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 5*((8*b^4*c - 5*a*b^3*d + 2*a^2*
b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(
a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 10*((8*b^4*
c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a
^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^3*x^8
 + a^6*b^2*x^5), -1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^
4*b^2*e - 2*a^5*b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 30*sqrt(1/3)
*((8*a*b^5*c - 5*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c -
5*a^3*b^3*d + 2*a^4*b^2*e + a^5*b*f)*x^5)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt
(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 5*
((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2
*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x +
(a^2*b)^(1/3)*a) - 10*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 +
(8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^5*b^3*x^8 + a^6*b^2*x^5)]
```

giac [A] time = 0.18, size = 264, normalized size = 0.98

$$-\frac{\sqrt{3}\left(8 b^{3} c - 5 a b^{2} d + a^{3} f + 2 a^{2} b e\right) \arctan\left(\frac{\sqrt{3}\left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \left(-a b^{2}\right)^{\frac{2}{3}} a^{3}} - \frac{\left(8 b^{3} c - 5 a b^{2} d + a^{3} f + 2 a^{2} b e\right) \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(-a b^{2}\right)^{\frac{2}{3}} a^{3}} - \frac{\left(8 b^{3} c - 5 a b^{2} d + a^{3} f + 2 a^{2} b e\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9 a^{4} b} + \frac{b^{3} c x - a b^{2} d x - a^{3} f x + a^{2} b x e}{3 \left(b x^{3} + a\right) a^{3} b} + \frac{10 b c x^{3} - 5 a d x^{3} - 2 a c}{10 a^{3} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(8*b^3*c - 5*a*b^2*d + a^3*f + 2*a^2*b*e)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/18*(8*b^3*c - 5*
```

```
a*b^2*d + a^3*f + 2*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*
b^2)^(2/3)*a^3) - 1/9*(8*b^3*c - 5*a*b^2*d + a^3*f + 2*a^2*b*e)*(-a/b)^(1/3
)*log(abs(x - (-a/b)^(1/3)))/(a^4*b) + 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x +
 a^2*b*x*e)/((b*x^3 + a)*a^3*b) + 1/10*(10*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/(a^
3*x^5)
```

maple [B] time = 0.06, size = 477, normalized size = 1.77

$$\frac{ex}{3\left(b\,x^{3}+a\right)a}-\frac{bdx}{3\left(b\,x^{3}+a\right)a^{2}}+\frac{b^{2}cx}{3\left(b\,x^{3}+a\right)a^{3}}-\frac{fx}{3\left(b\,x^{3}+a\right)b}+\frac{2\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{5\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{5d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{5d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{8\sqrt{3}\,bc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{8bc\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{4bc\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{d}{2a^{2}x^{2}}+\frac{bc}{a^{3}x^{2}}-\frac{c}{5a^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x)
```

```
[Out] -1/3/b*x/(b*x^3+a)*f+1/3/a*x/(b*x^3+a)*e-1/3/a^2*b*x/(b*x^3+a)*d+1/3/a^3*b^
2*x/(b*x^3+a)*c+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f+2/9/a/b/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))*e-5/9/a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+8/9/a^3*b/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))*c-1/18/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))*f-1/9/a/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+5/18/a^2/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d-4/9/a^3*b/(a/b)^(2/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))*f+2/9/a/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*e-5/9/a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*d+8/9/a^3*b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*c-1/5/a^2*c/x^5-1/2*d/a^2/x^2+1/a^3/x^2*b*c
```

maxima [A] time = 2.93, size = 268, normalized size = 0.99

$$\frac{5\left(8\,b^{3}c-5\,ab^{2}d+2\,a^{2}be-2\,a^{3}f\right)x^{6}-6\,a^{2}bc+3\left(8\,ab^{2}c-5\,a^{2}bd\right)x^{3}}{30\left(a^{3}b^{2}x^{8}+a^{4}bx^{5}\right)}+\frac{\sqrt{3}\left(8\,b^{3}c-5\,ab^{2}d+2\,a^{2}be+a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(8\,b^{3}c-5\,ab^{2}d+2\,a^{2}be+a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{3}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(8\,b^{3}c-5\,ab^{2}d+2\,a^{2}be+a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{3}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(5*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e - 2*a^3*f)*x^6 - 6*a^2*b*c + 3*(8*
a*b^2*c - 5*a^2*b*d)*x^3)/(a^3*b^2*x^8 + a^4*b*x^5) + 1/9*sqrt(3)*(8*b^3*c
- 5*a*b^2*d + 2*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) - 1/18*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a
^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 1/9*(8
*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)
^(2/3))
```

mupad [B] time = 5.13, size = 248, normalized size = 0.92

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}}-\frac{\frac{c}{5\,a}+\frac{x^3\,\left(5\,a\,d-8\,b\,c\right)}{10\,a^2}-\frac{x^6\,\left(-2\,f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{6\,a^3\,b}}{b\,x^8+a\,x^5}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(8*b^3*c + a^3*f - 5*a*b^2*d + 2*a^2*b*e))/(9*a^(
11/3)*b^(4/3)) - (c/(5*a) + (x^3*(5*a*d - 8*b*c))/(10*a^2) - (x^6*(8*b^3*c
- 2*a^3*f - 5*a*b^2*d + 2*a^2*b*e))/(6*a^3*b))/(a*x^5 + b*x^8) + (log(3^(1/
2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(8*b^3*c + a^
3*f - 5*a*b^2*d + 2*a^2*b*e))/(9*a^(11/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)*1
i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(8*b^3*c + a^3*f - 5*a*b^
2*d + 2*a^2*b*e))/(9*a^(11/3)*b^(4/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.218 $\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=297

$$\frac{2bc-ad}{4a^3x^4}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{a^4x}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3(-f)+4a^2be-7ab^2d+10b^3c\right)}{18a^{13/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{\right.}$$

Rubi [A] time = 0.38, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(4a^2be+a^3(-f)-7ab^2d+10b^3c\right)}{18a^{13/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(4a^2be+a^3(-f)-7ab^2d+10b^3c\right)}{9a^{13/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(4a^2be+a^3(-f)-7ab^2d+10b^3c\right)}{3\sqrt{3}\,a^{13/3}b^{2/3}}-\frac{a^2e-2abd+3b^2c}{a^4x}+\frac{2bc-ad}{4a^3x^4}-\frac{c}{7a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(4*a^3*x^4) - (3*b^2*c - 2*a*b*d + a^2*e)/(a
^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^4*(a + b*x^3)) + ((1
0*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(3*Sqrt[3]*a^(13/3)*b^(2/3)) + ((10*b^3*c - 7*a*b^2*d + 4*
a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(13/3)*b^(2/3)) - ((10*b^3*
c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(13/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x \cdots}{a^3}}{x^8\left(a+bx^3\right)}}{3ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)} - \frac{\int\left(-\frac{3b^3c}{ax^8}-\frac{3b^3(-2bc+ad)}{a^2x^5}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^2}-\frac{b^3\left(-10b^3 \cdots\right.}{\cdots}\right.}{3ab^3}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)}-\frac{\left(10b^3c- \cdots\right.}{\cdots}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)}+\frac{\left(10b^3c- \cdots\right.}{\cdots}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)}+\frac{\left(10b^3c- \cdots\right.}{\cdots}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)}+\frac{\left(10b^3c- \cdots\right.}{\cdots}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^4\left(a+bx^3\right)}+\frac{\left(10b^3c- \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.26, size = 281, normalized size = 0.95

$$\frac{-\frac{63a^{4/3}(ad-2bc)}{x^4}-\frac{36a^{7/3}c}{x^7}-\frac{252\sqrt[3]{a}\left(a^2e-2abd+3b^2c\right)}{x}+\frac{84\sqrt[3]{a}x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+4a^2be-7ab^2d+10b^3c\right)}{b^{2/3}}+\frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3(-f)+4a^2be-7ab^2d+10b^3c\right)}{b^{2/3}}+\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f-4a^2be+7ab^2d-10b^3c\right)}{b^{2/3}}}{252a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

```
[Out] ((-36*a^(7/3)*c)/x^7 - (63*a^(4/3)*(-2*b*c + a*d))/x^4 - (252*a^(1/3)*(3*b^
2*c - 2*a*b*d + a^2*e))/x + (84*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3
*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*
f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(10*b^3*c - 7
*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/b^(2/3) + (14*(-10*
b^3*c + 7*a*b^2*d - 4*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2]])/b^(2/3))/(252*a^(13/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^8 \left(a + bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2), x]
```

fricas [A] time = 0.45, size = 982, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^9 + 36*a
^4*b^2*c + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(10*a^3*b^
3*c - 7*a^4*b^2*d)*x^3 + 42*sqrt(1/3)*((10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^
3*e - a^4*b^2*f)*x^10 + (10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e - a^5*b*f
)*x^7)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2
*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)
^(2/3)*x)/(b*x^3 + a)) + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)
*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((10*b^4*c - 7*a*b^
3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e -
 a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^3*x^10 + a^6*
b^2*x^7), -1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x
^9 + 36*a^4*b^2*c + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(
10*a^3*b^3*c - 7*a^4*b^2*d)*x^3 + 84*sqrt(1/3)*((10*a*b^5*c - 7*a^2*b^4*d +
 4*a^3*b^3*e - a^4*b^2*f)*x^10 + (10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e
- a^5*b*f)*x^7)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^
(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e
 - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((10*b^4
*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d +
4*a^3*b*e - a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^3*
x^10 + a^6*b^2*x^7)]
```

giac [A] time = 0.23, size = 333, normalized size = 1.12

$$-\frac{\sqrt{3} \left(10 \, b^{3} c - 7 \, a b^{2} d - a^{3} f + 4 \, a^{2} b e\right) \arctan\left(\frac{\sqrt{3} \left(2 \, x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \, \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \, \left(-a b^{2}\right)^{\frac{1}{3}} a^{4}} + \frac{\left(10 \, b^{3} c - 7 \, a b^{2} d - a^{3} f + 4 \, a^{2} b e\right) \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \, \left(-a b^{2}\right)^{\frac{1}{3}} a^{4}} + \frac{\left(10 \, b^{3} c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 7 \, a b^{2} d \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^{3} f \left(-\frac{a}{b}\right)^{\frac{1}{3}} + 4 \, a^{2} b \left(-\frac{a}{b}\right)^{\frac{1}{3}} e\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left({\left| x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right|}\right)}{9 \, a^{5}} - \frac{b^{3} c x^{2} - a b^{2} d x^{2} - a^{3} f x^{2} + a^{2} b x^{2} e}{3 \, {\left(b x^{3} + a\right)} a^{4}} - \frac{84 \, b^{2} c x^{6} - 56 \, a b d x^{6} + 28 \, a^{2} x^{6} e - 14 \, a b c x^{3} + 7 \, a^{2} d x^{3} + 4 \, a^{2} c}{28 \, a^{4} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(10*b^3*c - 7*a*b^2*d - a^3*f + 4*a^2*b*e)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^4) + 1/18*(10*b^3*c -
```

```
7*a*b^2*d - a^3*f + 4*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(1/3)*a^4) + 1/9*(10*b^3*c*(-a/b)^(1/3) - 7*a*b^2*d*(-a/b)^(1/3) - a
^3*f*(-a/b)^(1/3) + 4*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/a^5 - 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/((b*
x^3 + a)*a^4) - 1/28*(84*b^2*c*x^6 - 56*a*b*d*x^6 + 28*a^2*x^6*e - 14*a*b*c
*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^4*x^7)
```

maple [B] time = 0.07, size = 529, normalized size = 1.78

$$\frac{f x^{2}}{3\left(b x^{3}+a\right) a}-\frac{b e x^{2}}{3\left(b x^{3}+a\right) a^{2}}+\frac{b^{2} d x^{2}}{3\left(b x^{3}+a\right) a^{3}}-\frac{b^{3} c x^{2}}{3\left(b x^{3}+a\right) a^{4}}+\frac{\sqrt{3}\, f \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a b}-\frac{f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a b}+\frac{f \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}} a b}-\frac{4\sqrt{3}\, e \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{2}}+\frac{4 e \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{2}}-\frac{2 e \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{2}}+\frac{7\sqrt{3}\, b d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}}-\frac{7 b d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}}+\frac{7 b d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}}-\frac{10\sqrt{3}\, b^{2} c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{4}}+\frac{10 b^{2} c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{4}}-\frac{5 b^{2} c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{4}}-\frac{e}{a^{2} x}+\frac{2 b d}{a^{3} x}-\frac{3 b^{2} c}{a^{4} x}-\frac{d}{4 a^{2} x^{4}}+\frac{b c}{2 a^{3} x^{4}}-\frac{c}{7 a^{2} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x^2/(b*x^3+a)*f-1/3/a^2*x^2/(b*x^3+a)*b*e+1/3/a^3*x^2/(b*x^3+a)*b^2*d
-1/3/a^4*x^2/(b*x^3+a)*b^3*c+4/9/a^2*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^
2*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/a^2*e*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-7/9/a^3*b*d/(a/b)^(1/3)*ln(x+(
a/b)^(1/3))+7/18/a^3*b*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+7/9/
a^3*b*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+10/9/a^
4*b^2*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/9/a^4*b^2*c/(a/b)^(1/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))-10/9/a^4*b^2*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))-1/9/a*f/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/18/a*f/b/(
a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/9/a*f*3^(1/2)/b/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/7/a^2*c/x^7-1/4/a^2/x^4*d+1/2/a^3
/x^4*b*c-e/a^2/x+2/a^3/x*b*d-3/a^4/x*b^2*c
```

maxima [A] time = 3.05, size = 292, normalized size = 0.98

$$-\frac{28\left(10 b^{3} c-7 a b^{2} d+4 a^{2} b e-a^{3} f\right) x^{9}+21\left(10 a b^{2} c-7 a^{2} b d+4 a^{3} e\right) x^{6}+12 a^{3} c-3\left(10 a^{2} b c-7 a^{3} d\right) x^{3}}{84\left(a^{4} b x^{10}+a^{5} x^{7}\right)}-\frac{\sqrt{3}\left(10 b^{3} c-7 a b^{2} d+4 a^{2} b e-a^{3} f\right) \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 a^{4} b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(10 b^{3} c-7 a b^{2} d+4 a^{2} b e-a^{3} f\right) \log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 a^{4} b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(10 b^{3} c-7 a b^{2} d+4 a^{2} b e-a^{3} f\right) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 a^{4} b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/84*(28*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*x^9 + 21*(10*a*b^2*c -
 7*a^2*b*d + 4*a^3*e)*x^6 + 12*a^3*c - 3*(10*a^2*b*c - 7*a^3*d)*x^3)/(a^4*b
*x^10 + a^5*x^7) - 1/9*sqrt(3)*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3)) - 1/
18*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/
b)^(2/3))/(a^4*b*(a/b)^(1/3)) + 1/9*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3
*f)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
```

mupad [B] time = 5.18, size = 274, normalized size = 0.92

$$\frac{\ln\left(b^{1/3} x+a^{1/3}\right)\left(-f a^{3}+4 e a^{2} b-7 d a b^{2}+10 c b^{3}\right)}{9 a^{13/3} b^{2/3}}-\frac{\frac{c}{7 a}+\frac{x^{9}\left(-f a^{3}+4 e a^{2} b-7 d a b^{2}+10 c b^{3}\right)}{3 a^{4}}+\frac{x^{3}\left(7 a d-10 b c\right)}{28 a^{2}}+\frac{x^{6}\left(4 e a^{2}-7 d a b+10 c b^{2}\right)}{4 a^{3}}}{b x^{10}+a x^{7}}-\frac{\ln\left(2 b^{1/3} x-a^{1/3}+\sqrt{3}\, a^{1/3}\, 1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(-f a^{3}+4 e a^{2} b-7 d a b^{2}+10 c b^{3}\right)}{9 a^{13/3} b^{2/3}}+\frac{\ln\left(a^{1/3}-2 b^{1/3} x+\sqrt{3}\, a^{1/3}\, 1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(-f a^{3}+4 e a^{2} b-7 d a b^{2}+10 c b^{3}\right)}{9 a^{13/3} b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e))/(9*a^
(13/3)*b^(2/3)) - (c/(7*a) + (x^9*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e
))/(3*a^4) + (x^3*(7*a*d - 10*b*c))/(28*a^2) + (x^6*(10*b^2*c + 4*a^2*e - 7
*a*b*d))/(4*a^3))/(a*x^7 + b*x^10) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
- a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e
))/(9*a^(13/3)*b^(2/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*
((3^(1/2)*1i)/2 - 1/2)*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e))/(9*a^(13
/3)*b^(2/3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**2,x)
[Out] Timed out
```

3.219 $\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=297

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{8a^2x^8}-\frac{a^2e-2abd+3b^2c}{2a^4x^2}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{9a^{14/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2\right.}{3\sqrt{}}$$

Rubi [A] time = 0.37, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^2be-2a^3f-8ab^2d+11b^3c\right)}{18a^{14/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^2be-2a^3f-8ab^2d+11b^3c\right)}{9a^{14/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^2be-2a^3f-8ab^2d+11b^3c\right)}{3\sqrt{3}a^{14/3}\sqrt[3]{b}}-\frac{a^2e-2abd+3b^2c}{2a^4x^2}+\frac{2bc-ad}{5a^3x^5}-\frac{c}{8a^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

```
[Out] -c/(8*a^2*x^8) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/(2
*a^4*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^4*(a + b*x^3)) + (
(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(14/3)*b^(1/3)) - ((11*b^3*c - 8*a*b^2*d
+ 5*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(14/3)*b^(1/3)) + ((1
1*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(18*a^(14/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{2b^3\left(b^3c-ab^2d+a^2be-a\right.}{a^3}}{x^9\left(a+bx^3\right)}}{3ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)} - \frac{\int\left(-\frac{3b^3c}{ax^9}-\frac{3b^3(-2bc+ad)}{a^2x^6}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^3}-\frac{b^3(-1}{}\right.}{3ab^3}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{(11b^3c}{}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{(11b^3c}{}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{(11b^3c}{}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{(11b^3c}{}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}+\frac{(11b^3c}{}$$

Mathematica [A] time = 0.24, size = 280, normalized size = 0.94

$$\frac{-\frac{72a^{5/3}(ad-2bc)}{x^5}-\frac{45a^{8/3}c}{x^8}-\frac{180a^{2/3}\left(a^2e-2abd+3b^2c\right)}{x^2}+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^3f-5a^2be+8ab^2d-11b^3c\right)}{\sqrt[3]{b}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{\sqrt[3]{b}}+\frac{120a^{2/3}x\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}+\frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{\sqrt[3]{b}}}{360a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

```
[Out] ((-45*a^(8/3)*c)/x^8 - (72*a^(5/3)*(-2*b*c + a*d))/x^5 - (180*a^(2/3)*(3*b^
2*c - 2*a*b*d + a^2*e))/x^2 + (120*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a
^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-11*b^3*c
 + 8*a*b^2*d - 5*a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20
*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/b^(1/3))/(360*a^(14/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2), x]
```

fricas [A] time = 0.44, size = 959, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f)*x^9 + 45
*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(11*a^4*b^2*
c - 8*a^5*b*d)*x^3 + 60*sqrt(1/3)*((11*a*b^5*c - 8*a^2*b^4*d + 5*a^3*b^3*e
- 2*a^4*b^2*f)*x^11 + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f
)*x^8)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(
1/3)/b))/(b*x^3 + a)) - 20*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f
)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^(2/3
)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*((11*b^4*c - 8*a*b^
3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e
 - 2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^2*x^11 +
a^7*b*x^8), -1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*
f)*x^9 + 45*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(
11*a^4*b^2*c - 8*a^5*b*d)*x^3 + 120*sqrt(1/3)*((11*a*b^5*c - 8*a^2*b^4*d +
5*a^3*b^3*e - 2*a^4*b^2*f)*x^11 + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e
 - 2*a^5*b*f)*x^8)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*
x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*((11*b^4*c - 8*a*b^3*d
 + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e -
2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a
) + 40*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c
 - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b
)^(2/3)))/(a^6*b^2*x^11 + a^7*b*x^8)]
```

giac [A] time = 0.20, size = 347, normalized size = 1.17

$$\frac{\left(11\,b^3c-8\,ab^2d-2\,a^3f+5\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5}-\frac{\sqrt{3}\left(11\,\left(-ab^2\right)^{\frac{1}{3}}b^3c-8\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d-2\,\left(-ab^2\right)^{\frac{1}{3}}a^3f+5\,\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5b}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{3\left(bx^3+a\right)a^4}-\frac{\left(11\,\left(-ab^2\right)^{\frac{1}{3}}b^3c-8\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d-2\,\left(-ab^2\right)^{\frac{1}{3}}a^3f+5\,\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5b}-\frac{60\,b^2cx^6-40\,abdx^6+20\,a^2x^6e-16\,abcx^3+8\,a^2dx^3+5\,a^2c}{40\,a^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(11*b^3*c - 8*a*b^2*d - 2*a^3*f + 5*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/a^5 - 1/9*sqrt(3)*(11*(-a*b^2)^(1/3)*b^3*c - 8*(-a*b^2)^(1/3)
*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 1/3*(b^3*c*x - a*b^2*d*x
 - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a^4) - 1/18*(11*(-a*b^2)^(1/3)*b^3*c -
 8*(-a*b^2)^(1/3)*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2)^(1/3)*a^2*b
*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/40*(60*b^2*c*x^6 -
 40*a*b*d*x^6 + 20*a^2*x^6*e - 16*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^4*x
^8)
```

maple [B] time = 0.06, size = 520, normalized size = 1.75

$$\frac{fx}{3\left(bx^{3}+a\right)a}-\frac{bex}{3\left(bx^{3}+a\right)a^{2}}+\frac{b^{2}dx}{3\left(bx^{3}+a\right)a^{3}}-\frac{b^{3}cx}{3\left(bx^{3}+a\right)a^{4}}+\frac{2\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{5\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{5e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{5e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{8\sqrt{3}\,bd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{8bd\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{4bd\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{11\sqrt{3}\,b^{2}c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}-\frac{11b^{2}c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}+\frac{11b^{2}c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}-\frac{e}{2a^{2}x^{2}}+\frac{bd}{a^{3}x^{2}}-\frac{3b^{2}c}{2a^{4}x^{2}}-\frac{d}{5a^{2}x^{5}}+\frac{2bc}{5a^{3}x^{5}}-\frac{c}{8a^{2}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x/(b*x^3+a)*f-1/3/a^2*x/(b*x^3+a)*b*e+1/3/a^3*x/(b*x^3+a)*b^2*d-1/3/a
^4*x/(b*x^3+a)*b^3*c-5/9/a^2*e/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/18/a^2*e/(a/
b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9/a^2*e/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+8/9/a^3*b*d/(a/b)^(2/3)*ln(x+(a/b)^(1
/3))-4/9/a^3*b*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+8/9/a^3*b*d/
(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-11/9/a^4*b^2*c/
(a/b)^(2/3)*ln(x+(a/b)^(1/3))+11/18/a^4*b^2*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))-11/9/a^4*b^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))+2/9/a*f/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/9/a*f/b/(a/b)^(2
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/a*f/b/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/8*c/a^2/x^8-1/5/a^2/x^5*d+2/5/a^3/x^5*b*
c-1/2/a^2/x^2*e+1/a^3/x^2*b*d-3/2/a^4/x^2*b^2*c
```

maxima [A] time = 3.03, size = 292, normalized size = 0.98

$$-\frac{20\left(11\,b^{3}c-8\,ab^{2}d+5\,a^{2}be-2\,a^{3}f\right)x^{9}+12\left(11\,ab^{2}c-8\,a^{2}bd+5\,a^{3}e\right)x^{6}+15\,a^{3}c-3\left(11\,a^{2}bc-8\,a^{3}d\right)x^{3}}{120\left(a^{4}bx^{11}+a^{5}x^{8}\right)}-\frac{\sqrt{3}\left(11\,b^{3}c-8\,ab^{2}d+5\,a^{2}be-2\,a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{4}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(11\,b^{3}c-8\,ab^{2}d+5\,a^{2}be-2\,a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{4}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(11\,b^{3}c-8\,ab^{2}d+5\,a^{2}be-2\,a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{4}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/120*(20*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^9 + 12*(11*a*b^2*
c - 8*a^2*b*d + 5*a^3*e)*x^6 + 15*a^3*c - 3*(11*a^2*b*c - 8*a^3*d)*x^3)/(a^
4*b*x^11 + a^5*x^8) - 1/9*sqrt(3)*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3
*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
 + 1/18*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) - 1/9*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*
e - 2*a^3*f)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

mupad [B] time = 5.20, size = 274, normalized size = 0.92

$$-\frac{\frac{c}{8\,a}+\frac{x^{9}\left(-2\,f\,a^{3}+5\,e\,a^{2}\,b-8\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{6\,a^{4}}+\frac{x^{3}\left(8\,a\,d-11\,b\,c\right)}{40\,a^{2}}+\frac{x^{6}\left(5\,e\,a^{2}-8\,d\,a\,b+11\,c\,b^{2}\right)}{10\,a^{3}}}{b\,x^{11}+a\,x^{8}}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-2\,f\,a^{3}+5\,e\,a^{2}\,b-8\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{9\,a^{14/3}\,b^{1/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-2\,f\,a^{3}+5\,e\,a^{2}\,b-8\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{9\,a^{14/3}\,b^{1/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-2\,f\,a^{3}+5\,e\,a^{2}\,b-8\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{9\,a^{14/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(11
*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*b^(1/3)) - (log(b^(1
/3)*x + a^(1/3))*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*
b^(1/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2
 - 1/2)*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*b^(1/3))
```

```
- (c/(8*a) + (x^9*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(6*a^4) + (
x^3*(8*a*d - 11*b*c))/(40*a^2) + (x^6*(11*b^2*c + 5*a^2*e - 8*a*b*d))/(10*a
^3))/(a*x^8 + b*x^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.220 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=334

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{10a^2x^{10}}-\frac{a^2e-2abd+3b^2c}{4a^4x^4}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{9a^{16/3}}-\frac{\sqrt[3]{b}\tan^{-1}\Big(}{}$$

Rubi [A] time = 0.46, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$\frac{bx^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(7a^2be-4a^3f-10ab^2d+13b^3c\right)}{18a^{16/3}}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{a^5x}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(7a^2be-4a^3f-10ab^2d+13b^3c\right)}{9a^{16/3}}-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7a^2be-4a^3f-10ab^2d+13b^3c\right)}{3\sqrt{3}\,a^{16/3}}-\frac{a^2e-2abd+3b^2c}{4a^4x^4}+\frac{2bc-ad}{7a^3x^7}-\frac{c}{10a^2x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

```
[Out] -c/(10*a^2*x^10) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/
(4*a^4*x^4) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^5*(a + b*x^3)) - (b^(1/3)*(13*b^3*c
 - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(3*Sqrt[3]*a^(16/3)) - (b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2
*b*e - 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(16/3)) + (b^(1/3)*(13*b^3*c
 - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*a^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^5\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c-ab^2d+a^2be-a^3f\right)}{a^3}}{x^{11}\left(a+bx^3\right)}}{3ab^3}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^5\left(a+bx^3\right)} - \int\left(-\frac{3b^3c}{ax^{11}}-\frac{3b^3(-2bc+ad)}{a^2x^8}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^5}-\frac{3b^3\left(-4b^3\right)}{}\right)$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-\right)}{}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-\right)}{}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-\right)}{}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-\right)}{}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-\right)}{}$$

Mathematica [A] time = 0.21, size = 319, normalized size = 0.96

$$\frac{-\frac{180a^{7/3}(ad-2bc)}{x^7}-\frac{126a^{10/3}c}{x^{10}}-\frac{315a^{4/3}\left(a^2e-2abd+3b^2c\right)}{x^4}-\frac{420\sqrt[3]{a}\,bx^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}-\frac{1260\sqrt[3]{a}\left(a^3f-2a^2be+3ab^2d-4b^3c\right)}{x}+140\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(4a^3f-7a^2be+10ab^2d-13b^3c\right)-140\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)+70\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-4a^3f+7a^2be-10ab^2d+13b^3c\right)}{1260a^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

```
[Out] ((-126*a^(10/3)*c)/x^10 - (180*a^(7/3)*(-2*b*c + a*d))/x^7 - (315*a^(4/3)*(
3*b^2*c - 2*a*b*d + a^2*e))/x^4 - (1260*a^(1/3)*(-4*b^3*c + 3*a*b^2*d - 2*a
^2*b*e + a^3*f))/x - (420*a^(1/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*
x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e -
 4*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-13*b^
3*c + 10*a*b^2*d - 7*a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/
3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(1260*a^(16/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{11}\left(a + bx^3\right)^2}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2), x]
```

fricas [A] time = 0.42, size = 442, normalized size = 1.32

$$\frac{420\left(13\,b^4c - 10\,ab^3d + 7\,a^2b^2e - 4\,a^3bf\right)x^{12} + 315\left(13\,ab^3c - 10\,a^2b^2d + 7\,a^3be - 4\,a^4f\right)x^9 - 45\left(13\,a^2b^2c - 10\,a^3bd + 7\,a^4e\right)x^6 - 126\,a^4c + 18\left(13\,a^3bc - 10\,a^4d\right)x^3 + 140\,\sqrt{3}\left(\left(13\,b^4c - 10\,ab^3d + 7\,a^2b^2e - 4\,a^3bf\right)x^{13} + \left(13\,ab^3c - 10\,a^2b^2d + 7\,a^3be - 4\,a^4f\right)x^{10}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\,\sqrt{3}\right) + 70\left(\left(13\,b^4c - 10\,ab^3d + 7\,a^2b^2e - 4\,a^3bf\right)x^{13} + \left(13\,ab^3c - 10\,a^2b^2d + 7\,a^3be - 4\,a^4f\right)x^{10}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2 - ax\left(\frac{b}{a}\right)^{\frac{2}{3}} + a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right) - 140\left(\left(13\,b^4c - 10\,ab^3d + 7\,a^2b^2e - 4\,a^3bf\right)x^{13} + \left(13\,ab^3c - 10\,a^2b^2d + 7\,a^3be - 4\,a^4f\right)x^{10}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx + a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{1260\left(a^5bx^{13} + a^6x^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(420*(13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^12 + 315*(1
3*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^9 - 45*(13*a^2*b^2*c - 10
*a^3*b*d + 7*a^4*e)*x^6 - 126*a^4*c + 18*(13*a^3*b*c - 10*a^4*d)*x^3 + 140*
sqrt(3)*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3
*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*arctan(2/3*sqrt(
3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e
- 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)
*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 140*((13*b^4*c
- 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d +
 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)))/(a^5*b*x^
13 + a^6*x^10)
```

giac [A] time = 0.37, size = 437, normalized size = 1.31

$$-\frac{\left(13\,b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 10\,ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4\,a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 7\,a^2b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^6} - \frac{\sqrt{3}\left(13\left(-ab^2\right)^{\frac{2}{3}}b^3c - 10\left(-ab^2\right)^{\frac{2}{3}}ab^2d - 4\left(-ab^2\right)^{\frac{2}{3}}a^3f + 7\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^6b} + \frac{b^4cx^2 - ab^3dx^2 - a^3bfx^2 + a^2b^2x^2e}{3\left(bx^3 + a\right)a^5} + \frac{\left(13\left(-ab^2\right)^{\frac{2}{3}}b^3c - 10\left(-ab^2\right)^{\frac{2}{3}}ab^2d - 4\left(-ab^2\right)^{\frac{2}{3}}a^3f + 7\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^6b} + \frac{560\,b^3cx^9 - 420\,ab^2dx^9 - 140\,a^3fx^9 + 280\,a^2bx^9e - 105\,ab^2cx^6 + 70\,a^2bdx^6 - 35\,a^3x^6e + 40\,a^2bcx^3 - 20\,a^3dx^3 - 14\,a^3c}{140\,a^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(13*b^4*c*(-a/b)^(1/3) - 10*a*b^3*d*(-a/b)^(1/3) - 4*a^3*b*f*(-a/b)^(1
/3) + 7*a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6
 - 1/9*sqrt(3)*(13*(-a*b^2)^(2/3)*b^3*c - 10*(-a*b^2)^(2/3)*a*b^2*d - 4*(-a
*b^2)^(2/3)*a^3*f + 7*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/(a^6*b) + 1/3*(b^4*c*x^2 - a*b^3*d*x^2 - a^3*b*f*x
^2 + a^2*b^2*x^2*e)/((b*x^3 + a)*a^5) + 1/18*(13*(-a*b^2)^(2/3)*b^3*c - 10*
(-a*b^2)^(2/3)*a*b^2*d - 4*(-a*b^2)^(2/3)*a^3*f + 7*(-a*b^2)^(2/3)*a^2*b*e)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) + 1/140*(560*b^3*c*x^9 -
420*a*b^2*d*x^9 - 140*a^3*f*x^9 + 280*a^2*b*x^9*e - 105*a*b^2*c*x^6 + 70*a^
```

```
2*b*d*x^6 - 35*a^3*x^6*e + 40*a^2*b*c*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^5*x
^10)
```

maple [A] time = 0.06, size = 575, normalized size = 1.72

$$-\frac{bf\,x^{2}}{3\left(b\,x^{3}+a\right)a^{2}}+\frac{b^{2}e\,x^{2}}{3\left(b\,x^{3}+a\right)a^{3}}-\frac{b^{3}d\,x^{2}}{3\left(b\,x^{3}+a\right)a^{4}}+\frac{b^{4}c\,x^{2}}{3\left(b\,x^{3}+a\right)a^{5}}-\frac{4\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}+\frac{4f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}-\frac{2f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}+\frac{7\sqrt{3}\,be\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}-\frac{7be\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{7be\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}-\frac{10\sqrt{3}\,b^{2}d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}}+\frac{10b^{2}d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}}-\frac{5b^{2}d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}}+\frac{13\sqrt{3}\,b^{3}c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{5}}-\frac{13b^{3}c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{5}}+\frac{13b^{3}c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{5}}-\frac{f}{a^{2}x}+\frac{2be}{a^{3}x}-\frac{3b^{2}d}{a^{4}x}+\frac{4b^{3}c}{a^{5}x}-\frac{e}{4a^{2}x^{4}}+\frac{bd}{2a^{3}x^{4}}-\frac{3b^{2}c}{4a^{4}x^{4}}-\frac{d}{7a^{2}x^{7}}+\frac{2bc}{7a^{3}x^{7}}-\frac{c}{10a^{2}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x)
```

```
[Out] 4/a^5/x*b^3*c+4/9/a^2*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2*f/(a/b)^(1/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/7/a^3/x^7*b*c+1/2/a^3/x^4*b*d-3/4/a^4/
x^4*b^2*c+2/a^3/x*b*e-3/a^4/x*b^2*d-1/3*b/a^2*x^2/(b*x^3+a)*f+1/3*b^2/a^3*x
^2/(b*x^3+a)*e-10/9*b^2/a^4*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))+13/9*b^3/a^5*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))+7/9*b/a^3*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))-1/a^2/x*f-1/4/a^2/x^4*e-1/7/a^2/x^7*d-1/10*c/a^2/x^10-1/3*b^3/a
^4*x^2/(b*x^3+a)*d+1/3*b^4/a^5*x^2/(b*x^3+a)*c-4/9/a^2*f*3^(1/2)/(a/b)^(1/3
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-7/9*b/a^3*e/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))+7/18*b/a^3*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+10/9*b^
2/a^4*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/9*b^2/a^4*d/(a/b)^(1/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))-13/9*b^3/a^5*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+13/18*b
^3/a^5*c/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
```

maxima [A] time = 3.13, size = 323, normalized size = 0.97

$$\frac{140\left(13\,b^{4}c-10\,ab^{3}d+7\,a^{2}b^{2}e-4\,a^{3}bf\right)x^{12}+105\left(13\,ab^{3}c-10\,a^{2}b^{2}d+7\,a^{3}be-4\,a^{4}f\right)x^{9}-15\left(13\,a^{2}b^{2}c-10\,a^{3}bd+7\,a^{4}e\right)x^{6}-42\,a^{4}c+6\left(13\,a^{3}bc-10\,a^{4}d\right)x^{3}}{420\left(a^{5}bx^{13}+a^{6}x^{10}\right)}+\frac{\sqrt{3}\left(13\,b^{3}c-10\,ab^{2}d+7\,a^{2}be-4\,a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(13\,b^{3}c-10\,ab^{2}d+7\,a^{2}be-4\,a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(13\,b^{3}c-10\,ab^{2}d+7\,a^{2}be-4\,a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/420*(140*(13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^12 + 105*(13
*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^9 - 15*(13*a^2*b^2*c - 10*
a^3*b*d + 7*a^4*e)*x^6 - 42*a^4*c + 6*(13*a^3*b*c - 10*a^4*d)*x^3)/(a^5*b*x
^13 + a^6*x^10) + 1/9*sqrt(3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(1/3)) + 1/
18*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(a^5*(a/b)^(1/3)) - 1/9*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4
*a^3*f)*log(x + (a/b)^(1/3))/(a^5*(a/b)^(1/3))
```

mupad [B] time = 5.41, size = 310, normalized size = 0.93

$$-\frac{\frac{c}{10\,a}-\frac{x^{9}\left(-4\,f\,a^{3}+7\,e\,a^{2}\,b-10\,d\,a\,b^{2}+13\,c\,b^{3}\right)}{4\,a^{4}}+\frac{x^{3}\left(10\,a\,d-13\,b\,c\right)}{70\,a^{2}}+\frac{x^{6}\left(7\,e\,a^{2}-10\,d\,a\,b+13\,c\,b^{2}\right)}{28\,a^{3}}-\frac{b\,x^{12}\left(-4\,f\,a^{3}+7\,e\,a^{2}\,b-10\,d\,a\,b^{2}+13\,c\,b^{3}\right)}{3\,a^{5}}}{b\,x^{13}+a\,x^{10}}-\frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-4\,f\,a^{3}+7\,e\,a^{2}\,b-10\,d\,a\,b^{2}+13\,c\,b^{3}\right)}{9\,a^{16/3}}+\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-4\,f\,a^{3}+7\,e\,a^{2}\,b-10\,d\,a\,b^{2}+13\,c\,b^{3}\right)}{9\,a^{16/3}}-\frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-4\,f\,a^{3}+7\,e\,a^{2}\,b-10\,d\,a\,b^{2}+13\,c\,b^{3}\right)}{9\,a^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x)
```

```
[Out] (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))/(9*a^(16/3)) - (b^(1/3)
*log(b^(1/3)*x + a^(1/3))*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))/(9
*a^(16/3)) - (c/(10*a) - (x^9*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e)
)/(4*a^4) + (x^3*(10*a*d - 13*b*c))/(70*a^2) + (x^6*(13*b^2*c + 7*a^2*e - 1
0*a*b*d))/(28*a^3) - (b*x^12*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))
/(3*a^5))/(a*x^10 + b*x^13) - (b^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2
*b*e))/(9*a^(16/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.221 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{2bc-ad}{8a^3x^8}-\frac{c}{11a^2x^{11}}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{18a^{17/3}}+\frac{b^{2/}}{}$$

Rubi [A] time = 0.43, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{2a^5x^2}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(8a^2be-5a^3f-11ab^2d+14b^3c\right)}{18a^{17/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(8a^2be-5a^3f-11ab^2d+14b^3c\right)}{9a^{17/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(8a^2be-5a^3f-11ab^2d+14b^3c\right)}{3\sqrt{3}\,a^{17/3}}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}+\frac{2bc-ad}{8a^3x^8}-\frac{c}{11a^2x^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```

```
[Out] -c/(11*a^2*x^11) + (2*b*c - a*d)/(8*a^3*x^8) - (3*b^2*c - 2*a*b*d + a^2*e)/
(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(2*a^5*x^2) + (b*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^5*(a + b*x^3)) - (b^(2/3)*(14*b^3
*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(3*Sqrt[3]*a^(17/3)) + (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a
^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(17/3)) - (b^(2/3)*(14*b^3
*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(18*a^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^5\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c-ab^2d+a^2be-a\right.}{a^3}}{x^{12}\left(a+bx^3\right)}}{3ab^3}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^5\left(a+bx^3\right)} - \frac{\int\left(-\frac{3b^3c}{ax^{12}}-\frac{3b^3(-2bc+ad)}{a^2x^9}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^6}-\frac{3b^3\left(-\right.}{}\right.}{3}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b\left(b^3\right.}{}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b\left(b^3\right.}{}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b\left(b^3\right.}{}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b\left(b^3\right.}{}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b\left(b^3\right.}{}$$

Mathematica [A] time = 0.33, size = 317, normalized size = 0.95

$$\frac{-\frac{495a^{8/3}(ad-2bc)}{x^8}-\frac{360a^{11/3}c}{x^{11}}-\frac{792a^{5/3}\left(a^2e-2abd+3b^2c\right)}{x^5}+440b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)-440\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)-\frac{1320a^{2/3}bx\left(a^3f-a^2be+ab^2d-b^3c\right)}{a+bx^3}-\frac{1980a^{2/3}\left(a^3f-2a^2be+3ab^2d-4b^3c\right)}{x^2}+220b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^3f-8a^2be+11ab^2d-14b^3c\right)}{3960a^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```

```
[Out] ((-360*a^(11/3)*c)/x^11 - (495*a^(8/3)*(-2*b*c + a*d))/x^8 - (792*a^(5/3)*(
3*b^2*c - 2*a*b*d + a^2*e))/x^5 - (1980*a^(2/3)*(-4*b^3*c + 3*a*b^2*d - 2*a
^2*b*e + a^3*f))/x^2 - (1320*a^(2/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*x)/(a + b*x^3) - 440*Sqrt[3]*b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e
- 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(14*b^
3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2
/3)*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(3960*a^(17/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2), x]
```

fricas [A] time = 0.43, size = 475, normalized size = 1.42

$$\frac{660\left(14b^4c-11ab^3d+8a^2b^2e-5a^3bf\right)x^{12}+396\left(14ab^3c-11a^2b^2d+8a^3be-5a^4f\right)x^9-99\left(14a^2b^2c-11a^3bd+8a^4e\right)x^6-360a^4c+45\left(14a^3bc-11a^4d\right)x^3-440\sqrt{3}\left(\left(14b^4c-11ab^3d+8a^2b^2e-5a^3bf\right)x^{14}+\left(14ab^3c-11a^2b^2d+8a^3be-5a^4f\right)x^{11}\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+220\left(\left(14b^4c-11ab^3d+8a^2b^2e-5a^3bf\right)x^{14}+\left(14ab^3c-11a^2b^2d+8a^3be-5a^4f\right)x^{11}\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-440\left(\left(14b^4c-11ab^3d+8a^2b^2e-5a^3bf\right)x^{14}+\left(14ab^3c-11a^2b^2d+8a^3be-5a^4f\right)x^{11}\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)}{3960\left(a^5bx^{14}+a^6x^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3960*(660*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^12 + 396*(1
4*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^9 - 99*(14*a^2*b^2*c - 11
*a^3*b*d + 8*a^4*e)*x^6 - 360*a^4*c + 45*(14*a^3*b*c - 11*a^4*d)*x^3 - 440*
sqrt(3)*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3
*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*arctan(1/3*
(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((14*b^4*c - 11*a*b^3
*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e
 - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a
^2*(-b^2/a^2)^(2/3)) - 440*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*
f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2
)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b*x^14 + a^6*x^11)
```

giac [A] time = 0.18, size = 391, normalized size = 1.17

$$\frac{\sqrt{3}\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-11\left(-ab^2\right)^{\frac{1}{3}}ab^2d-5\left(-ab^2\right)^{\frac{1}{3}}a^3f+8\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^6}-\frac{\left(14b^4c-11ab^3d-5a^3bf+8a^2b^2e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^6}+\frac{\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-11\left(-ab^2\right)^{\frac{1}{3}}ab^2d-5\left(-ab^2\right)^{\frac{1}{3}}a^3f+8\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^6}+\frac{b^4cx-ab^3dx-a^3bfx+a^2b^2xe}{3\left(bx^3+a\right)a^5}+\frac{880b^3cx^9-660ab^2dx^9-220a^3fx^9+440a^2bx^9e-264ab^2cx^6+176a^2bdx^6-88a^3x^6e+110a^2bcx^3-55a^3dx^3-40a^3c}{440a^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d - 5*(-a*b^
2)^(1/3)*a^3*f + 8*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/a^6 - 1/9*(14*b^4*c - 11*a*b^3*d - 5*a^3*b*f + 8*a^2*
b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 + 1/18*(14*(-a*b^2)^(1/3
)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 8*(-a*b^2)^(
1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 + 1/3*(b^4*c*x -
 a*b^3*d*x - a^3*b*f*x + a^2*b^2*x*e)/((b*x^3 + a)*a^5) + 1/440*(880*b^3*c*
x^9 - 660*a*b^2*d*x^9 - 220*a^3*f*x^9 + 440*a^2*b*x^9*e - 264*a*b^2*c*x^6 +
 176*a^2*b*d*x^6 - 88*a^3*x^6*e + 110*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c
)/(a^5*x^11)
```

maple [A]   time = 0.06, size = 566, normalized size = 1.69

$$-\frac{bfx}{3\left(bx^{3}+a\right)a^{2}}+\frac{b^{2}ex}{3\left(bx^{3}+a\right)a^{3}}-\frac{b^{3}dx}{3\left(bx^{3}+a\right)a^{4}}+\frac{b^{4}cx}{3\left(bx^{3}+a\right)a^{5}}-\frac{5\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{5f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{5f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{8\sqrt{3}\,be\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{8be\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{4be\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{11\sqrt{3}\,b^{2}d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}-\frac{11b^{2}d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}+\frac{11b^{2}d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}+\frac{14\sqrt{3}\,b^{3}c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{5}}+\frac{14b^{3}c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{5}}-\frac{7b^{3}c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{5}}-\frac{f}{2a^{2}x^{2}}+\frac{be}{a^{3}x^{2}}-\frac{3b^{2}d}{2a^{4}x^{2}}+\frac{2b^{3}c}{a^{5}x^{2}}-\frac{e}{5a^{2}x^{5}}+\frac{2bd}{5a^{3}x^{5}}-\frac{3b^{2}c}{5a^{4}x^{5}}-\frac{d}{8a^{2}x^{8}}+\frac{bc}{4a^{3}x^{8}}-\frac{c}{11a^{2}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x)
```

```
[Out] 5/18/a^2*f/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/a^3/x^2*b*e-3/2/
a^4/x^2*b^2*d+2/a^5/x^2*b^3*c+1/4/a^3/x^8*b*c+2/5/a^3/x^5*b*d-3/5/a^4/x^5*b
^2*c-5/9/a^2*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/3*b/a^2*x/(b*x^3+a)*f+1/3*b^
2/a^3*x/(b*x^3+a)*e-1/3*b^3/a^4*x/(b*x^3+a)*d+11/18*b^2/a^4*d/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+14/9*b^3/a^5*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))-7/9*b^3/a^5*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*b^4/a^5*
x/(b*x^3+a)*c-1/5/a^2/x^5*e-1/2/a^2/x^2*f-1/8/a^2/x^8*d-5/9/a^2*f/(a/b)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+8/9*b/a^3*e/(a/b)^(2/3)*
ln(x+(a/b)^(1/3))-4/9*b/a^3*e/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
-11/9*b^2/a^4*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+8/9*b/a^3*e/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-11/9*b^2/a^4*d/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+14/9*b^3/a^5*c/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/11*c/a^2/x^11
```

maxima [A]   time = 3.07, size = 323, normalized size = 0.96

$$\frac{220\left(14b^{4}c-11ab^{3}d+8a^{2}b^{2}e-5a^{3}bf\right)x^{12}+132\left(14ab^{3}c-11a^{2}b^{2}d+8a^{3}be-5a^{4}f\right)x^{9}-33\left(14a^{2}b^{2}c-11a^{3}bd+8a^{4}e\right)x^{6}-120a^{4}c+15\left(14a^{3}bc-11a^{4}d\right)x^{3}}{1320\left(a^{5}bx^{14}+a^{6}x^{11}\right)}+\frac{\sqrt{3}\left(14b^{3}c-11ab^{2}d+8a^{2}be-5a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(14b^{3}c-11ab^{2}d+8a^{2}be-5a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(14b^{3}c-11ab^{2}d+8a^{2}be-5a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1320*(220*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^12 + 132*(1
4*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^9 - 33*(14*a^2*b^2*c - 11
*a^3*b*d + 8*a^4*e)*x^6 - 120*a^4*c + 15*(14*a^3*b*c - 11*a^4*d)*x^3)/(a^5*
b*x^14 + a^6*x^11) + 1/9*sqrt(3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3
*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(2/3)) -
 1/18*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*log(x^2 - x*(a/b)^(1/3)
 + (a/b)^(2/3))/(a^5*(a/b)^(2/3)) + 1/9*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e
- 5*a^3*f)*log(x + (a/b)^(1/3))/(a^5*(a/b)^(2/3))
```

mupad [B]   time = 5.12, size = 310, normalized size = 0.93

$$\frac{b^{2/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-5fa^{3}+8ea^{2}b-11dab^{2}+14cb^{3}\right)}{9a^{17/3}}-\frac{\frac{c}{11a}-\frac{x^{9}\left(-5fa^{3}+8ea^{2}b-11dab^{2}+14cb^{3}\right)}{10a^{4}}+\frac{x^{3}\left(11ad-14bc\right)}{88a^{2}}+\frac{x^{6}\left(8ea^{2}-11dab+14cb^{2}\right)}{40a^{3}}-\frac{bx^{12}\left(-5fa^{3}+8ea^{2}b-11dab^{2}+14cb^{3}\right)}{6a^{5}}}{bx^{14}+ax^{11}}+\frac{b^{2/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-5fa^{3}+8ea^{2}b-11dab^{2}+14cb^{3}\right)}{9a^{17/3}}-\frac{b^{2/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-5fa^{3}+8ea^{2}b-11dab^{2}+14cb^{3}\right)}{9a^{17/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*a^2*
b*e))/(9*a^(17/3)) - (c/(11*a) - (x^9*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*
a^2*b*e))/(10*a^4) + (x^3*(11*a*d - 14*b*c))/(88*a^2) + (x^6*(14*b^2*c + 8*
a^2*e - 11*a*b*d))/(40*a^3) - (b*x^12*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*
a^2*b*e))/(6*a^5))/(a*x^11 + b*x^14) + (b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*
b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(14*b^3*c - 5*a^3*f - 11*a*b^2*
d + 8*a^2*b*e))/(9*a^(17/3)) - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*
x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*a^
2*b*e))/(9*a^(17/3))
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.222 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^2}\,dx$

Optimal. Leaf size=375

$$\frac{2bc-ad}{10a^3x^{10}}-\frac{c}{13a^2x^{13}}-\frac{a^2e-2abd+3b^2c}{7a^4x^7}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{18a^{19/3}}$$

Rubi [A] time = 0.53, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$-\frac{b^2x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{4a^5x^4}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(10a^2be-7a^3f-13ab^2d+16b^3c\right)}{18a^{19/3}}-\frac{b\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{a^6x}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(10a^2be-7a^3f-13ab^2d+16b^3c\right)}{9a^{19/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(10a^2be-7a^3f-13ab^2d+16b^3c\right)}{3\sqrt{3}\,a^{19/3}}-\frac{a^2e-2abd+3b^2c}{7a^4x^7}+\frac{2bc-ad}{10a^3x^{10}}-\frac{c}{13a^2x^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

```
[Out] -c/(13*a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b*d + a^2*e
)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) - (b*
(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^(4/3)*(16*b^3*c - 13*a*b
^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(3*Sqrt[3]*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e -
7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) - (b^(4/3)*(16*b^3*c - 13*a
*b^2*d + 10*a^2*b*e - 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(18*a^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^6\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c-ab^2d+a^2be-\right.}{a^3}}{x^{14}\left(a+\right.}}{3a}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^6\left(a+bx^3\right)} - \frac{\int\left(-\frac{3b^3c}{ax^{14}}-\frac{3b^3(-2bc+ad)}{a^2x^{11}}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^8}-\frac{3b^3\left(-\right.}{}\right.}{}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b\left(5b^3c\right.}{}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b\left(5b^3c\right.}{}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b\left(5b^3c\right.}{}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b\left(5b^3c\right.}{}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b\left(5b^3c\right.}{}$$

Mathematica [A] time = 0.41, size = 370, normalized size = 0.99

$$\frac{2bc-ad}{10a^3x^{10}}-\frac{c}{13a^2x^{13}}-\frac{a^2e-2abd+3b^2c}{7a^4x^7}+\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(7a^3f-10a^2be+13ab^2d-16b^3c\right)}{18a^{19/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{9a^{19/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-7a^3f+10a^2be-13ab^2d+16b^3c\right)}{3\sqrt{3}a^{19/3}}+\frac{b^2x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{b\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{4a^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

```
[Out] -1/13*c/(a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b*d + a^2
*e)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) + (
b*(-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^(4/3)*(16*b^3*c -
13*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/(3*Sqrt[3]*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e -
7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) + (b^(4/3)*(-16*b^3*c + 13*
a*b^2*d - 10*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(19/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2), x]
```

fricas [A] time = 0.42, size = 507, normalized size = 1.35

$$-\frac{5460\left(16b^5c-13ab^4d+10a^2b^3e-7a^3b^2f\right)x^{15}+4095\left(16ab^4c-13a^2b^3d+10a^3b^2e-7a^4bf\right)x^{12}-585\left(16a^2b^3c-13a^3b^2d+10a^4be-7a^5f\right)x^9+234\left(16a^3b^2c-13a^4bd+10a^5e\right)x^6+1260a^5c-126\left(16a^4bc-13a^5d\right)x^3+1820\sqrt{3}\left(\left(16b^5c-13ab^4d+10a^2b^3e-7a^3b^2f\right)x^{16}+\left(16ab^4c-13a^2b^3d+10a^3b^2e-7a^4bf\right)x^{13}\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-910\left(\left(16b^5c-13ab^4d+10a^2b^3e-7a^3b^2f\right)x^{16}+\left(16ab^4c-13a^2b^3d+10a^3b^2e-7a^4bf\right)x^{13}\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+1820\left(\left(16b^5c-13ab^4d+10a^2b^3e-7a^3b^2f\right)x^{16}+\left(16ab^4c-13a^2b^3d+10a^3b^2e-7a^4bf\right)x^{13}\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{16380\left(a^6bx^{16}+a^7x^{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16380*(5460*(16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^15 +
4095*(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^12 - 585*(16*
a^2*b^3*c - 13*a^3*b^2*d + 10*a^4*b*e - 7*a^5*f)*x^9 + 234*(16*a^3*b^2*c -
13*a^4*b*d + 10*a^5*e)*x^6 + 1260*a^5*c - 126*(16*a^4*b*c - 13*a^5*d)*x^3 +
 1820*sqrt(3)*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^16 +
(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*a
rctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*((16*b^5*c - 13*a*b^4
*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^16 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3
*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b
/a)^(1/3)) + 1820*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^1
6 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/
3)*log(b*x + a*(-b/a)^(2/3)))/(a^6*b*x^16 + a^7*x^13)
```

giac [A] time = 0.21, size = 482, normalized size = 1.29

$$\frac{\sqrt{3}\left(16\left(-ab^2\right)^{\frac{2}{3}}b^3c-13\left(-ab^2\right)^{\frac{2}{3}}ab^2d-7\left(-ab^2\right)^{\frac{2}{3}}a^3f+10\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^7}+\frac{\left(16b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-13ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10a^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^7}-\frac{\left(16\left(-ab^2\right)^{\frac{2}{3}}b^3c-13\left(-ab^2\right)^{\frac{2}{3}}ab^2d-7\left(-ab^2\right)^{\frac{2}{3}}a^3f+10\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^7}-\frac{b^5cx^2-ab^4dx^2-a^3b^2fx^2+a^2b^3x^2e}{3\left(bx^3+a\right)a^6}-\frac{9100b^4cx^{12}-7280ab^3dx^{12}-3640a^3bfx^{12}+5460a^2b^2x^{12}e-1820ab^3cx^9+1365a^2b^2dx^9+455a^4fx^9-910a^3bx^9e+780a^2b^2cx^6-520a^3bdx^6+260a^4x^6e-364a^3bcx^3+182a^4dx^3+140a^4c}{1820a^6x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^2)^(2/3)*a*b^2*d - 7*(-a*b^
2)^(2/3)*a^3*f + 10*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/a^7 + 1/9*(16*b^5*c*(-a/b)^(1/3) - 13*a*b^4*d*(-a/b)
^(1/3) - 7*a^3*b^2*f*(-a/b)^(1/3) + 10*a^2*b^3*(-a/b)^(1/3)*e)*(-a/b)^(1/3)
*log(abs(x - (-a/b)^(1/3)))/a^7 - 1/18*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^
2)^(2/3)*a*b^2*d - 7*(-a*b^2)^(2/3)*a^3*f + 10*(-a*b^2)^(2/3)*a^2*b*e)*log(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 - 1/3*(b^5*c*x^2 - a*b^4*d*x^2 - a
^3*b^2*f*x^2 + a^2*b^3*x^2*e)/((b*x^3 + a)*a^6) - 1/1820*(9100*b^4*c*x^12 -
```

```
 7280*a*b^3*d*x^12 - 3640*a^3*b*f*x^12 + 5460*a^2*b^2*x^12*e - 1820*a*b^3*c
*x^9 + 1365*a^2*b^2*d*x^9 + 455*a^4*f*x^9 - 910*a^3*b*x^9*e + 780*a^2*b^2*c
*x^6 - 520*a^3*b*d*x^6 + 260*a^4*x^6*e - 364*a^3*b*c*x^3 + 182*a^4*d*x^3 +
140*a^4*c)/(a^6*x^13)
```

maple [A] time = 0.06, size = 631, normalized size = 1.68

$$\frac{b^{2} f x^{2}}{3 \left(b x^{3}+a \right) a^{3}}-\frac{b^{3} e x^{2}}{3 \left(b x^{3}+a \right) a^{4}}+\frac{b^{4} d x^{2}}{3 \left(b x^{3}+a \right) a^{5}}-\frac{b^{5} c x^{2}}{3 \left(b x^{3}+a \right) a^{6}}+\frac{7 \sqrt{3}\, b f \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}}-\frac{7 b f \ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}}+\frac{7 b f \ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}}-\frac{10 \sqrt{3}\, b^{2} e \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{4}}+\frac{10 b^{2} e \ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{4}}-\frac{5 b^{2} e \ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{4}}+\frac{13 \sqrt{3}\, b^{3} d \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{5}}-\frac{13 b^{3} d \ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{5}}+\frac{13 b^{3} d \ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{5}}-\frac{16 \sqrt{3}\, b^{4} c \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{6}}+\frac{16 b^{4} c \ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{6}}-\frac{8 b^{4} c \ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{6}}+\frac{2 b f}{a^{3} x}-\frac{3 b^{2} e}{a^{4} x}+\frac{4 b^{3} d}{a^{5} x}-\frac{5 b^{4} c}{a^{6} x}-\frac{f}{4 a^{2} x^{4}}+\frac{b e}{2 a^{3} x^{4}}-\frac{3 b^{2} d}{4 a^{4} x^{4}}+\frac{b^{3} c}{a^{5} x^{4}}-\frac{e}{7 a^{2} x^{7}}+\frac{2 b d}{7 a^{3} x^{7}}-\frac{3 b^{2} c}{7 a^{4} x^{7}}-\frac{d}{10 a^{2} x^{10}}+\frac{b c}{5 a^{3} x^{10}}-\frac{c}{13 a^{2} x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x)
```

```
[Out] -3/4/a^4/x^4*b^2*d+1/a^5/x^4*b^3*c+2*b/a^3/x*f-3*b^2/a^4/x*e+4*b^3/a^5/x*d-
5*b^4/a^6/x*c+1/5/a^3/x^10*b*c+2/7/a^3/x^7*b*d-3/7/a^4/x^7*b^2*c+1/2/a^3/x^
4*b*e-1/10/a^2/x^10*d+1/3*b^2/a^3*x^2/(b*x^3+a)*f-1/3*b^3/a^4*x^2/(b*x^3+a)
*e+7/9*b/a^3*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-
10/9*b^2/a^4*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+
13/9*b^3/a^5*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-
16/9*b^4/a^6*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-
1/7/a^2/x^7*e-1/4/a^2/x^4*f+1/3*b^4/a^5*x^2/(b*x^3+a)*d-1/3*b^5/a^6*x^2/(b*
x^3+a)*c+10/9*b^2/a^4*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/9*b^2/a^4*e/(a/b)^(
1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-13/9*b^3/a^5*d/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))+13/18*b^3/a^5*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+16/9
*b^4/a^6*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-8/9*b^4/a^6*c/(a/b)^(1/3)*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))-7/9*b/a^3*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7/18*b/
a^3*f/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/13*c/a^2/x^13
```

maxima [A] time = 2.97, size = 374, normalized size = 1.00

$$-\frac{1820 \, {\left(16 \, b^{5} c - 13 \, a b^{4} d + 10 \, a^{2} b^{3} e - 7 \, a^{3} b^{2} f\right)} x^{15} + 1365 \, {\left(16 \, a b^{4} c - 13 \, a^{2} b^{3} d + 10 \, a^{3} b^{2} e - 7 \, a^{4} b f\right)} x^{12} - 195 \, {\left(16 \, a^{2} b^{3} c - 13 \, a^{3} b^{2} d + 10 \, a^{4} b e - 7 \, a^{5} f\right)} x^{9} + 78 \, {\left(16 \, a^{3} b^{2} c - 13 \, a^{4} b d + 10 \, a^{5} e\right)} x^{6} + 420 \, a^{5} c - 42 \, {\left(16 \, a^{4} b c - 13 \, a^{5} d\right)} x^{3}}{5460 \, {\left(a^{6} b x^{16} + a^{7} x^{13}\right)}} - \frac{\sqrt{3} {\left(16 \, b^{4} c - 13 \, a b^{3} d + 10 \, a^{2} b^{2} e - 7 \, a^{3} b f\right)} \arctan\left(\frac{\sqrt{3} {\left(2 \, x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}}{3 \, \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \, a^{6} \left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{{\left(16 \, b^{4} c - 13 \, a b^{3} d + 10 \, a^{2} b^{2} e - 7 \, a^{3} b f\right)} \log\left(x^{2} - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \, a^{6} \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{{\left(16 \, b^{4} c - 13 \, a b^{3} d + 10 \, a^{2} b^{2} e - 7 \, a^{3} b f\right)} \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \, a^{6} \left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/5460*(1820*(16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^15 + 1
365*(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^12 - 195*(16*a
^2*b^3*c - 13*a^3*b^2*d + 10*a^4*b*e - 7*a^5*f)*x^9 + 78*(16*a^3*b^2*c - 13
*a^4*b*d + 10*a^5*e)*x^6 + 420*a^5*c - 42*(16*a^4*b*c - 13*a^5*d)*x^3)/(a^6
*b*x^16 + a^7*x^13) - 1/9*sqrt(3)*(16*b^4*c - 13*a*b^3*d + 10*a^2*b^2*e - 7
*a^3*b*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^6*(a/b)^(1
/3)) - 1/18*(16*b^4*c - 13*a*b^3*d + 10*a^2*b^2*e - 7*a^3*b*f)*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/3)) + 1/9*(16*b^4*c - 13*a*b^3*d +
 10*a^2*b^2*e - 7*a^3*b*f)*log(x + (a/b)^(1/3))/(a^6*(a/b)^(1/3))
```

mupad [B] time = 5.12, size = 348, normalized size = 0.93

$$\frac{b^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}}-\frac{\frac{c}{13\,a}-\frac{x^9\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{28\,a^4}+\frac{x^3\,\left(13\,a\,d-16\,b\,c\right)}{130\,a^2}+\frac{x^6\,\left(10\,e\,a^2-13\,d\,a\,b+16\,c\,b^2\right)}{70\,a^3}+\frac{b\,x^{12}\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{4\,a^5}+\frac{b^2\,x^{15}\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{3\,a^6}}{b\,x^{16}+a\,x^{13}}-\frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}}+\frac{b^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 10*a^2
*b*e))/(9*a^(19/3)) - (c/(13*a) - (x^9*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 1
0*a^2*b*e))/(28*a^4) + (x^3*(13*a*d - 16*b*c))/(130*a^2) + (x^6*(16*b^2*c +
 10*a^2*e - 13*a*b*d))/(70*a^3) + (b*x^12*(16*b^3*c - 7*a^3*f - 13*a*b^2*d
+ 10*a^2*b*e))/(4*a^5) + (b^2*x^15*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 10*a^
2*b*e))/(3*a^6))/(a*x^13 + b*x^16) - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(16*b^3*c - 7*a^3*f - 13*a*b^2*d
```

```
+ 10*a^2*b*e))/(9*a^(19/3)) + (b^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 10*a^
2*b*e))/(9*a^(19/3))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.223 $\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=266

$$\frac{x^9\left(6a^2f-3abe+b^2d\right)}{9b^5}+\frac{a^3\left(-7a^3f+6a^2be-5ab^2d+4b^3c\right)}{3b^8\left(a+bx^3\right)}+\frac{a^2\log\left(a+bx^3\right)\left(-21a^3f+15a^2be-10ab^2d+6b^3\right.}{3b^8}$$

Rubi [A] time = 0.44, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^6\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{6b^6}-\frac{ax^3\left(10a^2be-15a^3f-6ab^2d+3b^3c\right)}{3b^7}+\frac{a^3\left(6a^2be-7a^3f-5ab^2d+4b^3c\right)}{3b^8\left(a+bx^3\right)}-\frac{a^4\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^8\left(a+bx^3\right)^2}+\frac{a^2\log\left(a+bx^3\right)\left(15a^2be-21a^3f-10ab^2d+6b^3c\right)}{3b^8}+\frac{x^9\left(6a^2f-3abe+b^2d\right)}{9b^5}+\frac{x^{12}(be-3af)}{12b^4}+\frac{fx^{15}}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] -(a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x^3)/(3*b^7) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6)/(6*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*
f)*x^9)/(9*b^5) + ((b*e - 3*a*f)*x^12)/(12*b^4) + (f*x^15)/(15*b^3) - (a^4*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^8*(a + b*x^3)^2) + (a^3*(4*b^3*c
- 5*a*b^2*d + 6*a^2*b*e - 7*a^3*f))/(3*b^8*(a + b*x^3)) + (a^2*(6*b^3*c - 1
0*a*b^2*d + 15*a^2*b*e - 21*a^3*f)*Log[a + b*x^3])/(3*b^8)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{14}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^4\left(c+dx+ex^2+fx^3\right)}{(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{a\left(-3b^3c+6ab^2d-10a^2be+15a^3f\right)}{b^7}+\frac{\left(b^3c-3ab^2d+6a^2be-1\right.}{b^6}\right.\right.\\ &= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x^3}{3b^7}+\frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^6}{6b^6}+\end{aligned}$$

Mathematica [A] time = 0.19, size = 246, normalized size = 0.92

$$\frac{20b^3x^9\left(6a^2f-3abe+b^2d\right)+30b^2x^6\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)+60abx^3\left(15a^3f-10a^2be+6ab^2d-3b^3c\right)-\frac{60a^3\left(7a^3f-6a^2be+5ab^2d-4b^3c\right)}{a+bx^3}+60a^2\log\left(a+bx^3\right)\left(-21a^3f+15a^2be-10ab^2d+6b^3c\right)+\frac{30a^4\left(a^3f-a^2be+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}+15b^4x^{12}(be-3af)+12b^5fx^{15}}{180b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (60*a*b*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x^3 + 30*b^2*(b^3*c
- 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6 + 20*b^3*(b^2*d - 3*a*b*e + 6*a^2*f
)*x^9 + 15*b^4*(b*e - 3*a*f)*x^12 + 12*b^5*f*x^15 + (30*a^4*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (60*a^3*(-4*b^3*c + 5*a*b^2*d - 6*
a^2*b*e + 7*a^3*f))/(a + b*x^3) + 60*a^2*(6*b^3*c - 10*a*b^2*d + 15*a^2*b*e
 - 21*a^3*f)*Log[a + b*x^3])/(180*b^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [A] time = 0.39, size = 396, normalized size = 1.49

$$\frac{12\,b^{7}fx^{21}+3\left(5\,b^{7}e-7\,ab^{6}f\right)x^{18}+2\left(10\,b^{7}d-15\,ab^{6}e+21\,a^{2}b^{5}f\right)x^{15}+5\left(6\,b^{7}c-10\,ab^{6}d+15\,a^{2}b^{5}e-21\,a^{3}b^{4}f\right)x^{12}-20\left(6\,ab^{6}c-10\,a^{2}b^{5}d+15\,a^{3}b^{4}e-21\,a^{4}b^{3}f\right)x^{9}+210\,a^{4}b^{3}c-270\,a^{5}b^{2}d+330\,a^{6}be-390\,a^{7}f-30\left(11\,a^{2}b^{5}c-21\,a^{3}b^{4}d+34\,a^{4}b^{3}e-50\,a^{5}b^{2}f\right)x^{6}+60\left(a^{3}b^{4}c+a^{4}b^{3}d-4\,a^{5}b^{2}e+8\,a^{6}bf\right)x^{3}+60\left(6\,a^{4}b^{3}c-10\,a^{5}b^{2}d+15\,a^{6}be-21\,a^{7}f+\left(6\,a^{2}b^{5}c-10\,a^{3}b^{4}d+15\,a^{4}b^{3}e-21\,a^{5}b^{2}f\right)x^{6}+2\left(6\,a^{3}b^{4}c-10\,a^{4}b^{3}d+15\,a^{5}b^{2}e-21\,a^{6}bf\right)x^{3}\right)\log\left(bx^{3}+a\right)}{180\left(b^{10}x^{6}+2\,ab^{9}x^{3}+a^{2}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^7*f*x^21 + 3*(5*b^7*e - 7*a*b^6*f)*x^18 + 2*(10*b^7*d - 15*a*b^
6*e + 21*a^2*b^5*f)*x^15 + 5*(6*b^7*c - 10*a*b^6*d + 15*a^2*b^5*e - 21*a^3*
b^4*f)*x^12 - 20*(6*a*b^6*c - 10*a^2*b^5*d + 15*a^3*b^4*e - 21*a^4*b^3*f)*x
^9 + 210*a^4*b^3*c - 270*a^5*b^2*d + 330*a^6*b*e - 390*a^7*f - 30*(11*a^2*b
^5*c - 21*a^3*b^4*d + 34*a^4*b^3*e - 50*a^5*b^2*f)*x^6 + 60*(a^3*b^4*c + a^
4*b^3*d - 4*a^5*b^2*e + 8*a^6*b*f)*x^3 + 60*(6*a^4*b^3*c - 10*a^5*b^2*d + 1
5*a^6*b*e - 21*a^7*f + (6*a^2*b^5*c - 10*a^3*b^4*d + 15*a^4*b^3*e - 21*a^5*
b^2*f)*x^6 + 2*(6*a^3*b^4*c - 10*a^4*b^3*d + 15*a^5*b^2*e - 21*a^6*b*f)*x^3
)*log(b*x^3 + a))/(b^10*x^6 + 2*a*b^9*x^3 + a^2*b^8)
```

giac [A] time = 0.18, size = 349, normalized size = 1.31

$$\frac{\left(6\,a^{2}b^{3}c-10\,a^{3}b^{2}d-21\,a^{5}f+15\,a^{4}be\right)\log\left(\left|bx^{3}+a\right|\right)}{3\,b^{8}}-\frac{18\,a^{2}b^{5}cx^{6}-30\,a^{3}b^{4}dx^{6}-63\,a^{5}b^{2}fx^{6}+45\,a^{4}b^{3}x^{6}e+28\,a^{3}b^{4}cx^{3}-50\,a^{4}b^{3}dx^{3}-112\,a^{6}bfx^{3}+78\,a^{5}b^{2}x^{3}e+11\,a^{4}b^{3}c-21\,a^{5}b^{2}d-50\,a^{7}f+34\,a^{6}be}{6\left(bx^{3}+a\right)^{2}b^{8}}+\frac{12\,b^{12}fx^{15}-45\,ab^{11}fx^{12}+15\,b^{12}x^{12}e+20\,b^{12}dx^{9}+120\,a^{2}b^{10}fx^{9}-60\,ab^{11}x^{9}e+30\,b^{12}cx^{6}-90\,ab^{11}dx^{6}-300\,a^{3}b^{9}fx^{6}+180\,a^{2}b^{10}x^{6}e-180\,ab^{11}cx^{3}+360\,a^{2}b^{10}dx^{3}+900\,a^{4}b^{8}fx^{3}-600\,a^{3}b^{9}x^{3}e}{180\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*b^3*c - 10*a^3*b^2*d - 21*a^5*f + 15*a^4*b*e)*log(abs(b*x^3 + a)
)/b^8 - 1/6*(18*a^2*b^5*c*x^6 - 30*a^3*b^4*d*x^6 - 63*a^5*b^2*f*x^6 + 45*a^
4*b^3*x^6*e + 28*a^3*b^4*c*x^3 - 50*a^4*b^3*d*x^3 - 112*a^6*b*f*x^3 + 78*a^
5*b^2*x^3*e + 11*a^4*b^3*c - 21*a^5*b^2*d - 50*a^7*f + 34*a^6*b*e)/((b*x^3
+ a)^2*b^8) + 1/180*(12*b^12*f*x^15 - 45*a*b^11*f*x^12 + 15*b^12*x^12*e + 2
0*b^12*d*x^9 + 120*a^2*b^10*f*x^9 - 60*a*b^11*x^9*e + 30*b^12*c*x^6 - 90*a*
b^11*d*x^6 - 300*a^3*b^9*f*x^6 + 180*a^2*b^10*x^6*e - 180*a*b^11*c*x^3 + 36
0*a^2*b^10*d*x^3 + 900*a^4*b^8*f*x^3 - 600*a^3*b^9*x^3*e)/b^15
```

maple [A] time = 0.06, size = 361, normalized size = 1.36

$$\frac{fx^{15}}{15b^{3}}-\frac{afx^{12}}{4b^{4}}+\frac{ex^{12}}{12b^{3}}+\frac{2a^{2}fx^{9}}{3b^{5}}-\frac{aex^{9}}{3b^{4}}+\frac{dx^{9}}{9b^{3}}-\frac{5a^{3}fx^{6}}{3b^{6}}+\frac{a^{2}ex^{6}}{b^{5}}-\frac{adx^{6}}{2b^{4}}+\frac{cx^{6}}{6b^{3}}+\frac{5a^{4}fx^{3}}{b^{7}}-\frac{10a^{3}ex^{3}}{3b^{6}}+\frac{2a^{2}dx^{3}}{b^{5}}-\frac{acx^{3}}{b^{4}}+\frac{a^{7}f}{6\left(bx^{3}+a\right)^{2}b^{8}}-\frac{a^{6}e}{6\left(bx^{3}+a\right)^{2}b^{7}}+\frac{a^{5}d}{6\left(bx^{3}+a\right)^{2}b^{6}}-\frac{a^{4}c}{6\left(bx^{3}+a\right)^{2}b^{5}}-\frac{7a^{6}f}{3\left(bx^{3}+a\right)b^{8}}+\frac{2a^{5}e}{\left(bx^{3}+a\right)b^{7}}-\frac{7a^{5}f\ln\left(bx^{3}+a\right)}{b^{8}}-\frac{5a^{4}d}{3\left(bx^{3}+a\right)b^{6}}+\frac{5a^{4}e\ln\left(bx^{3}+a\right)}{b^{7}}+\frac{4a^{3}c}{3\left(bx^{3}+a\right)b^{5}}-\frac{10a^{3}d\ln\left(bx^{3}+a\right)}{3b^{6}}+\frac{2a^{2}c\ln\left(bx^{3}+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*a^7/b^8/(b*x^3+a)^2*f-1/6*a^6/b^7/(b*x^3+a)^2*e+1/6*a^5/b^6/(b*x^3+a)^2
*d+5/b^7*x^3*a^4*f-10/3/b^6*x^3*a^3*e+2/b^5*x^3*a^2*d-1/b^4*x^3*a*c-1/4/b^4
*x^12*a*f+2/3/b^5*x^9*a^2*f-1/3/b^4*x^9*a*e-5/3/b^6*x^6*a^3*f+1/b^5*x^6*a^2
*e-1/2/b^4*x^6*a*d+5*a^4/b^7*ln(b*x^3+a)*e-10/3*a^3/b^6*ln(b*x^3+a)*d+2*a^2
/b^5*ln(b*x^3+a)*c-1/6*a^4/b^5/(b*x^3+a)^2*c-7/3*a^6/b^8/(b*x^3+a)*f+2*a^5/
b^7/(b*x^3+a)*e-5/3*a^4/b^6/(b*x^3+a)*d+4/3*a^3/b^5/(b*x^3+a)*c-7*a^5/b^8*l
n(b*x^3+a)*f+1/12/b^3*x^12*e+1/9/b^3*x^9*d+1/6/b^3*x^6*c+1/15*f*x^15/b^3
```

maxima [A] time = 1.54, size = 275, normalized size = 1.03

$$\frac{7\,a^4b^3c-9\,a^5b^2d+11\,a^6be-13\,a^7f+2\left(4\,a^3b^4c-5\,a^4b^3d+6\,a^5b^2e-7\,a^6bf\right)x^3}{6\left(b^{10}x^6+2\,ab^9x^3+a^2b^8\right)}+\frac{12\,b^4fx^{15}+15\left(b^4e-3\,ab^3f\right)x^{12}+20\left(b^4d-3\,ab^3e+6\,a^2b^2f\right)x^9+30\left(b^4c-3\,ab^3d+6\,a^2b^2e-10\,a^3bf\right)x^6-60\left(3\,ab^3c-6\,a^2b^2d+10\,a^3be-15\,a^4f\right)x^3}{180\,b^7}+\frac{\left(6\,a^2b^3c-10\,a^3b^2d+15\,a^4be-21\,a^5f\right)\log\left(bx^3+a\right)}{3\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(7*a^4*b^3*c - 9*a^5*b^2*d + 11*a^6*b*e - 13*a^7*f + 2*(4*a^3*b^4*c - 5
*a^4*b^3*d + 6*a^5*b^2*e - 7*a^6*b*f)*x^3)/(b^10*x^6 + 2*a*b^9*x^3 + a^2*b^
8) + 1/180*(12*b^4*f*x^15 + 15*(b^4*e - 3*a*b^3*f)*x^12 + 20*(b^4*d - 3*a*b
^3*e + 6*a^2*b^2*f)*x^9 + 30*(b^4*c - 3*a*b^3*d + 6*a^2*b^2*e - 10*a^3*b*f)
*x^6 - 60*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*x^3)/b^7 + 1/3*
(6*a^2*b^3*c - 10*a^3*b^2*d + 15*a^4*b*e - 21*a^5*f)*log(b*x^3 + a)/b^8
```

mupad [B] time = 4.96, size = 449, normalized size = 1.69

$$x^{12}\left(\frac{e}{12\,b^3}-\frac{a\,f}{4\,b^4}\right)+x^6\left(\frac{c}{6\,b^3}-\frac{a^3\,f}{6\,b^6}-\frac{a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b^2}+\frac{a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{2\,b}\right)-x^9\left(\frac{a^2\,f}{3\,b^5}-\frac{d}{9\,b^3}+\frac{a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{3\,b}\right)-\frac{\frac{13\,f\,a^7-11\,e\,a^6\,b+9\,d\,a^5\,b^2-7\,c\,a^4\,b^3}{6\,b}+x^3\left(\frac{7\,f\,a^6}{3}-2\,e\,a^5\,b+\frac{5\,d\,a^4\,b^2}{3}-\frac{4\,c\,a^3\,b^3}{3}\right)}{a^2\,b^7+2\,a\,b^8\,x^3+b^9\,x^6}-x^3\left(\frac{a\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)}{b}-\frac{a^2\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b^2}+\frac{a^3\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{3\,b^3}\right)-\frac{\ln\left(b\,x^3+a\right)\left(21\,f\,a^5-15\,e\,a^4\,b+10\,d\,a^3\,b^2-6\,c\,a^2\,b^3\right)}{3\,b^8}+\frac{f\,x^{15}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^12*(e/(12*b^3) - (a*f)/(4*b^4)) + x^6*(c/(6*b^3) - (a^3*f)/(6*b^6) - (a^2
*(e/b^3 - (3*a*f)/b^4))/(2*b^2) + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 -
 (3*a*f)/b^4))/b))/(2*b)) - x^9*((a^2*f)/(3*b^5) - d/(9*b^3) + (a*(e/b^3 -
(3*a*f)/b^4))/(3*b)) - ((13*a^7*f - 7*a^4*b^3*c + 9*a^5*b^2*d - 11*a^6*b*e)
/(6*b) + x^3*((7*a^6*f)/3 - (4*a^3*b^3*c)/3 + (5*a^4*b^2*d)/3 - 2*a^5*b*e))
/(a^2*b^7 + b^9*x^6 + 2*a*b^8*x^3) - x^3*((a*(c/b^3 - (a^3*f)/b^6 - (3*a^2*
(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3
*a*f)/b^4))/b))/b))/b - (a^2*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)
/b^4))/b))/b^2 + (a^3*(e/b^3 - (3*a*f)/b^4))/(3*b^3)) - (log(a + b*x^3)*(21
*a^5*f - 6*a^2*b^3*c + 10*a^3*b^2*d - 15*a^4*b*e))/(3*b^8) + (f*x^15)/(15*b
^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.224 $\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=226

$$\frac{x^6\left(6a^2f-3abe+b^2d\right)}{6b^5}-\frac{a^2\left(-6a^3f+5a^2be-4ab^2d+3b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{a\log\left(a+bx^3\right.}{}$$

Rubi [A] time = 0.33, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{x^3\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{3b^6}-\frac{a^2\left(5a^2be-6a^3f-4ab^2d+3b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{a\log\left(a+bx^3\right)\left(10a^2be-15a^3f-6ab^2d+3b^3c\right)}{3b^7}+\frac{x^6\left(6a^2f-3abe+b^2d\right)}{6b^5}+\frac{x^9(be-3af)}{9b^4}+\frac{fx^{12}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^6)/(6*b^5) + ((b*e - 3*a*f)*x^9)/(9*b^4) + (f*x^12)/(12*b^3)
 + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^7*(a + b*x^3)^2) - (a^2*(
3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f))/(3*b^7*(a + b*x^3)) - (a*(3*b^3
*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{(a+bx)^3}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{b^3c-3ab^2d+6a^2be-10a^3f}{b^6}+\frac{\left(b^2d-3abe+6a^2f\right)x}{b^5}+\frac{(be-\right.\right.\right.\\
&= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^3}{3b^6}+\frac{\left(b^2d-3abe+6a^2f\right)x^6}{6b^5}+\frac{(be-3af)x^9}{9b^4}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 208, normalized size = 0.92

$$\frac{6b^2x^6\left(6a^2f-3abe+b^2d\right)+12bx^3\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)+\frac{12a^2\left(6a^3f-5a^2be+4ab^2d-3b^3c\right)}{a+bx^3}+\frac{6a^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2}+12a\log\left(a+bx^3\right)\left(15a^3f-10a^2be+6ab^2d-3b^3c\right)+4b^3x^9(be-3af)+3b^4fx^{12}}{36b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (12*b*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3 + 6*b^2*(b^2*d - 3*a*b
*e + 6*a^2*f)*x^6 + 4*b^3*(b*e - 3*a*f)*x^9 + 3*b^4*f*x^12 + (6*a^3*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3)^2 + (12*a^2*(-3*b^3*c + 4*a*b^2*d
 - 5*a^2*b*e + 6*a^3*f))/(a + b*x^3) + 12*a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*
b*e + 15*a^3*f)*Log[a + b*x^3])/(36*b^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11} \left(c + dx^3 + ex^6 + fx^9\right)}{\left(a + bx^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [A] time = 0.40, size = 353, normalized size = 1.56

$$\frac{3 b^6 f x^{18} + 2 \left(2 b^6 e - 3 a b^5 f\right) x^{15} + \left(6 b^6 d - 10 a b^5 e + 15 a^2 b^4 f\right) x^{12} + 4 \left(3 b^6 c - 6 a b^5 d + 10 a^2 b^4 e - 15 a^3 b^3 f\right) x^9 - 30 a^3 b^3 c + 42 a^4 b^2 d - 54 a^5 b e + 66 a^6 f + 6 \left(4 a b^5 c - 11 a^2 b^4 d + 21 a^3 b^3 e - 34 a^4 b^2 f\right) x^6 - 12 \left(2 a^2 b^4 c - a^3 b^3 d - a^4 b^2 e + 4 a^5 b f\right) x^3 - 12 \left(3 a^3 b^3 c - 6 a^4 b^2 d + 10 a^5 b e - 15 a^6 f + \left(3 a b^5 c - 6 a^2 b^4 d + 10 a^3 b^3 e - 15 a^4 b^2 f\right) x^6 + 2 \left(3 a^2 b^4 c - 6 a^3 b^3 d + 10 a^4 b^2 e - 15 a^5 b f\right) x^3\right) \log\left(b x^3 + a\right)}{36 \left(b^9 x^6 + 2 a b^8 x^3 + a^2 b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^6*f*x^18 + 2*(2*b^6*e - 3*a*b^5*f)*x^15 + (6*b^6*d - 10*a*b^5*e +
 15*a^2*b^4*f)*x^12 + 4*(3*b^6*c - 6*a*b^5*d + 10*a^2*b^4*e - 15*a^3*b^3*f)
*x^9 - 30*a^3*b^3*c + 42*a^4*b^2*d - 54*a^5*b*e + 66*a^6*f + 6*(4*a*b^5*c -
 11*a^2*b^4*d + 21*a^3*b^3*e - 34*a^4*b^2*f)*x^6 - 12*(2*a^2*b^4*c - a^3*b^
3*d - a^4*b^2*e + 4*a^5*b*f)*x^3 - 12*(3*a^3*b^3*c - 6*a^4*b^2*d + 10*a^5*b
*e - 15*a^6*f + (3*a*b^5*c - 6*a^2*b^4*d + 10*a^3*b^3*e - 15*a^4*b^2*f)*x^6
 + 2*(3*a^2*b^4*c - 6*a^3*b^3*d + 10*a^4*b^2*e - 15*a^5*b*f)*x^3)*log(b*x^3
 + a))/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

giac [A] time = 0.25, size = 298, normalized size = 1.32

$$-\frac{\left(3 a b^3 c - 6 a^2 b^2 d - 15 a^4 f + 10 a^3 b e\right) \log\left(\left|b x^3 + a\right|\right)}{3 b^7} + \frac{9 a b^5 c x^6 - 18 a^2 b^4 d x^6 - 45 a^4 b^2 f x^6 + 30 a^3 b^3 x^6 e + 12 a^2 b^4 c x^3 - 28 a^3 b^3 d x^3 - 78 a^5 b f x^3 + 50 a^4 b^2 x^3 e + 4 a^3 b^3 c - 11 a^4 b^2 d - 34 a^6 f + 21 a^5 b e}{6 \left(b x^3 + a\right)^2 b^7} + \frac{3 b^9 f x^{12} - 12 a b^8 f x^9 + 4 b^9 x^9 e + 6 b^9 d x^6 + 36 a^2 b^7 f x^6 - 18 a b^8 x^6 e + 12 b^9 c x^3 - 36 a b^8 d x^3 - 120 a^3 b^6 f x^3 + 72 a^2 b^7 x^3 e}{36 b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*a*b^3*c - 6*a^2*b^2*d - 15*a^4*f + 10*a^3*b*e)*log(abs(b*x^3 + a))/
b^7 + 1/6*(9*a*b^5*c*x^6 - 18*a^2*b^4*d*x^6 - 45*a^4*b^2*f*x^6 + 30*a^3*b^3
*x^6*e + 12*a^2*b^4*c*x^3 - 28*a^3*b^3*d*x^3 - 78*a^5*b*f*x^3 + 50*a^4*b^2*
x^3*e + 4*a^3*b^3*c - 11*a^4*b^2*d - 34*a^6*f + 21*a^5*b*e)/((b*x^3 + a)^2*
b^7) + 1/36*(3*b^9*f*x^12 - 12*a*b^8*f*x^9 + 4*b^9*x^9*e + 6*b^9*d*x^6 + 36
*a^2*b^7*f*x^6 - 18*a*b^8*x^6*e + 12*b^9*c*x^3 - 36*a*b^8*d*x^3 - 120*a^3*b
^6*f*x^3 + 72*a^2*b^7*x^3*e)/b^12
```

maple [A] time = 0.06, size = 313, normalized size = 1.38

$$\frac{f x^{12}}{12 b^3} - \frac{a f x^9}{3 b^4} + \frac{e x^9}{9 b^3} + \frac{a^2 f x^6}{b^5} - \frac{a e x^6}{2 b^4} + \frac{d x^6}{6 b^3} - \frac{10 a^3 f x^3}{3 b^6} + \frac{2 a^2 e x^3}{b^5} - \frac{a d x^3}{b^4} + \frac{c x^3}{3 b^3} - \frac{a^6 f}{6 \left(b x^3 + a\right)^2 b^7} + \frac{a^5 e}{6 \left(b x^3 + a\right)^2 b^6} - \frac{a^4 d}{6 \left(b x^3 + a\right)^2 b^5} + \frac{a^3 c}{6 \left(b x^3 + a\right)^2 b^4} + \frac{2 a^5 f}{\left(b x^3 + a\right) b^7} - \frac{5 a^4 e}{3 \left(b x^3 + a\right) b^6} + \frac{5 a^4 f \ln\left(b x^3 + a\right)}{b^7} + \frac{4 a^3 d}{3 \left(b x^3 + a\right) b^5} - \frac{10 a^3 e \ln\left(b x^3 + a\right)}{3 b^6} - \frac{a^2 c}{\left(b x^3 + a\right) b^4} + \frac{2 a^2 d \ln\left(b x^3 + a\right)}{b^5} - \frac{a c \ln\left(b x^3 + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/12*f*x^12/b^3-1/3/b^4*x^9*a*f+1/9/b^3*x^9*e+1/b^5*x^6*a^2*f-1/2/b^4*x^6*a
*e+1/6/b^3*x^6*d-10/3/b^6*x^3*a^3*f+2/b^5*x^3*a^2*e-1/b^4*x^3*a*d+1/3/b^3*x
```

```
^3*c-1/6*a^6/b^7/(b*x^3+a)^2*f+1/6*a^5/b^6/(b*x^3+a)^2*e-1/6*a^4/b^5/(b*x^3
+a)^2*d+1/6*a^3/b^4/(b*x^3+a)^2*c+5*a^4/b^7*ln(b*x^3+a)*f-10/3*a^3/b^6*ln(b
*x^3+a)*e+2*a^2/b^5*ln(b*x^3+a)*d-a/b^4*ln(b*x^3+a)*c+2*a^5/b^7/(b*x^3+a)*f
-5/3*a^4/b^6/(b*x^3+a)*e+4/3*a^3/b^5/(b*x^3+a)*d-a^2/b^4/(b*x^3+a)*c
```

maxima [A] time = 1.42, size = 233, normalized size = 1.03

$$-\frac{5\,a^{3}b^{3}c-7\,a^{4}b^{2}d+9\,a^{5}be-11\,a^{6}f+2\left(3\,a^{2}b^{4}c-4\,a^{3}b^{3}d+5\,a^{4}b^{2}e-6\,a^{5}bf\right)x^{3}}{6\left(b^{9}x^{6}+2\,ab^{8}x^{3}+a^{2}b^{7}\right)}+\frac{3\,b^{3}fx^{12}+4\left(b^{3}e-3\,ab^{2}f\right)x^{9}+6\left(b^{3}d-3\,ab^{2}e+6\,a^{2}bf\right)x^{6}+12\left(b^{3}c-3\,ab^{2}d+6\,a^{2}be-10\,a^{3}f\right)x^{3}}{36\,b^{6}}-\frac{\left(3\,ab^{3}c-6\,a^{2}b^{2}d+10\,a^{3}be-15\,a^{4}f\right)\log\left(bx^{3}+a\right)}{3\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(5*a^3*b^3*c - 7*a^4*b^2*d + 9*a^5*b*e - 11*a^6*f + 2*(3*a^2*b^4*c - 4
*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7
) + 1/36*(3*b^3*f*x^12 + 4*(b^3*e - 3*a*b^2*f)*x^9 + 6*(b^3*d - 3*a*b^2*e +
 6*a^2*b*f)*x^6 + 12*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/b^6 -
1/3*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*log(b*x^3 + a)/b^7
```

mupad [B] time = 4.97, size = 293, normalized size = 1.30

$$x^9\left(\frac{e}{9\,b^3}-\frac{a\,f}{3\,b^4}\right)+x^3\left(\frac{c}{3\,b^3}-\frac{a^3\,f}{3\,b^6}-\frac{a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)-x^6\left(\frac{a^2\,f}{2\,b^5}-\frac{d}{6\,b^3}+\frac{a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b}\right)+\frac{\frac{11\,f\,a^6-9\,e\,a^5\,b+7\,d\,a^4\,b^2-5\,c\,a^3\,b^3}{6\,b}+x^3\left(2\,f\,a^5-\frac{5\,e\,a^4\,b}{3}+\frac{4\,d\,a^3\,b^2}{3}-c\,a^2\,b^3\right)}{a^2\,b^6+2\,a\,b^7\,x^3+b^8\,x^6}+\frac{f\,x^{12}}{12\,b^3}+\frac{\ln\left(b\,x^3+a\right)\left(15\,f\,a^4-10\,e\,a^3\,b+6\,d\,a^2\,b^2-3\,c\,a\,b^3\right)}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^9*(e/(9*b^3) - (a*f)/(3*b^4)) + x^3*(c/(3*b^3) - (a^3*f)/(3*b^6) - (a^2*(
e/b^3 - (3*a*f)/b^4))/b^2 + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*
f)/b^4))/b))/b) - x^6*((a^2*f)/(2*b^5) - d/(6*b^3) + (a*(e/b^3 - (3*a*f)/b^
4))/(2*b)) + ((11*a^6*f - 5*a^3*b^3*c + 7*a^4*b^2*d - 9*a^5*b*e)/(6*b) + x^
3*(2*a^5*f - a^2*b^3*c + (4*a^3*b^2*d)/3 - (5*a^4*b*e)/3))/(a^2*b^6 + b^8*x
^6 + 2*a*b^7*x^3) + (f*x^12)/(12*b^3) + (log(a + b*x^3)*(15*a^4*f + 6*a^2*b
^2*d - 3*a*b^3*c - 10*a^3*b*e))/(3*b^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.225 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=186

$$\frac{x^3\left(6a^2f-3abe+b^2d\right)}{3b^5}+\frac{a\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{a^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}+\frac{\log\left(a+bx^3\right)\left(-1\right.}{}$$

Rubi [A] time = 0.27, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{a^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}+\frac{\log\left(a+bx^3\right)\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{3b^6}+\frac{x^3\left(6a^2f-3abe+b^2d\right)}{3b^5}+\frac{x^6(be-3af)}{6b^4}+\frac{fx^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^6)/(6*b^4) + (
f*x^9)/(9*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^6*(a + b*x^
3)^2) + (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f))/(3*b^6*(a + b*x^3))
 + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{x^2\left(c+dx+ex^2+fx^3\right)}{(a+bx)^3}\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int\left(\frac{b^2d-3abe+6a^2f}{b^5}+\frac{(be-3af)x}{b^4}+\frac{fx^2}{b^3}-\frac{a^2\left(-b^3c+ab^2d-a^2b\right.}{b^5(a+bx)^3}\right.\right.\\&=\frac{\left(b^2d-3abe+6a^2f\right)x^3}{3b^5}+\frac{(be-3af)x^6}{6b^4}+\frac{fx^9}{9b^3}-\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)}{6b^6\left(a+bx^3\right)^2}+\end{aligned}$$

Mathematica [A] time = 0.17, size = 170, normalized size = 0.91

$$\frac{6bx^3\left(6a^2f-3abe+b^2d\right)-\frac{6a\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)}{a+bx^3}+\frac{3a^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}+6\log\left(a+bx^3\right)\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)+3b^2x^6(be-3af)+2b^3fx^9}{18b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (6*b*(b^2*d - 3*a*b*e + 6*a^2*f)*x^3 + 3*b^2*(b*e - 3*a*f)*x^6 + 2*b^3*f*x^
9 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (6*a*(-2
*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f))/(a + b*x^3) + 6*(b^3*c - 3*a*b^2
*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(18*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 \left(c + dx^3 + ex^6 + fx^9\right)}{\left(a + bx^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [A] time = 0.40, size = 295, normalized size = 1.59

$$\frac{2\,b^5 f x^{15} + \left(3\,b^5 e - 5\,ab^4 f\right)x^{12} + 2\left(3\,b^5 d - 6\,ab^4 e + 10\,a^2 b^3 f\right)x^9 + 3\left(4\,ab^4 d - 11\,a^2 b^3 e + 21\,a^3 b^2 f\right)x^6 + 9\,a^2 b^3 c - 15\,a^3 b^2 d + 21\,a^4 be - 27\,a^5 f + 6\left(2\,ab^4 c - 2\,a^2 b^3 d + a^3 b^2 e + a^4 bf\right)x^3 + 6\left(\left(b^5 c - 3\,ab^4 d + 6\,a^2 b^3 e - 10\,a^3 b^2 f\right)x^6 + a^2 b^3 c - 3\,a^3 b^2 d + 6\,a^4 be - 10\,a^5 f + 2\left(ab^4 c - 3\,a^2 b^3 d + 6\,a^3 b^2 e - 10\,a^4 bf\right)x^3\right)\log\left(bx^3 + a\right)}{18\left(b^8 x^6 + 2\,ab^7 x^3 + a^2 b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^5*f*x^15 + (3*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 6*a*b^4*e +
10*a^2*b^3*f)*x^9 + 3*(4*a*b^4*d - 11*a^2*b^3*e + 21*a^3*b^2*f)*x^6 + 9*a^2
*b^3*c - 15*a^3*b^2*d + 21*a^4*b*e - 27*a^5*f + 6*(2*a*b^4*c - 2*a^2*b^3*d
+ a^3*b^2*e + a^4*b*f)*x^3 + 6*((b^5*c - 3*a*b^4*d + 6*a^2*b^3*e - 10*a^3*b
^2*f)*x^6 + a^2*b^3*c - 3*a^3*b^2*d + 6*a^4*b*e - 10*a^5*f + 2*(a*b^4*c - 3
*a^2*b^3*d + 6*a^3*b^2*e - 10*a^4*b*f)*x^3)*log(b*x^3 + a))/(b^8*x^6 + 2*a*
b^7*x^3 + a^2*b^6)
```

giac [A] time = 0.22, size = 236, normalized size = 1.27

$$\frac{\left(b^3 c - 3\,ab^2 d - 10\,a^3 f + 6\,a^2 be\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^6} - \frac{3\,b^5 cx^6 - 9\,ab^4 dx^6 - 30\,a^3 b^2 f x^6 + 18\,a^2 b^3 x^6 e + 2\,ab^4 cx^3 - 12\,a^2 b^3 dx^3 - 50\,a^4 bf x^3 + 28\,a^3 b^2 x^3 e - 4\,a^3 b^2 d - 21\,a^5 f + 11\,a^4 be}{6\left(bx^3 + a\right)^2 b^6} + \frac{2\,b^6 f x^9 - 9\,ab^5 f x^6 + 3\,b^6 x^6 e + 6\,b^6 dx^3 + 36\,a^2 b^4 f x^3 - 18\,ab^5 x^3 e}{18\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - 3*a*b^2*d - 10*a^3*f + 6*a^2*b*e)*log(abs(b*x^3 + a))/b^6 - 1/
6*(3*b^5*c*x^6 - 9*a*b^4*d*x^6 - 30*a^3*b^2*f*x^6 + 18*a^2*b^3*x^6*e + 2*a*
b^4*c*x^3 - 12*a^2*b^3*d*x^3 - 50*a^4*b*f*x^3 + 28*a^3*b^2*x^3*e - 4*a^3*b^
2*d - 21*a^5*f + 11*a^4*b*e)/((b*x^3 + a)^2*b^6) + 1/18*(2*b^6*f*x^9 - 9*a*
b^5*f*x^6 + 3*b^6*x^6*e + 6*b^6*d*x^3 + 36*a^2*b^4*f*x^3 - 18*a*b^5*x^3*e)/
b^9
```

maple [A] time = 0.06, size = 266, normalized size = 1.43

$$\frac{f x^9}{9b^3} - \frac{af x^6}{2b^4} + \frac{e x^6}{6b^3} + \frac{2a^2 f x^3}{b^5} - \frac{ae x^3}{b^4} + \frac{d x^3}{3b^3} + \frac{a^5 f}{6\left(b x^3 + a\right)^2 b^6} - \frac{a^4 e}{6\left(b x^3 + a\right)^2 b^5} + \frac{a^3 d}{6\left(b x^3 + a\right)^2 b^4} - \frac{a^2 c}{6\left(b x^3 + a\right)^2 b^3} - \frac{5a^4 f}{3\left(b x^3 + a\right) b^6} + \frac{4a^3 e}{3\left(b x^3 + a\right) b^5} - \frac{10a^3 f \ln\left(b x^3 + a\right)}{3b^6} - \frac{a^2 d}{\left(b x^3 + a\right) b^4} + \frac{2a^2 e \ln\left(b x^3 + a\right)}{b^5} + \frac{2ac}{3\left(b x^3 + a\right) b^3} - \frac{ad \ln\left(b x^3 + a\right)}{b^4} + \frac{c \ln\left(b x^3 + a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/9/b^3*f*x^9-1/2/b^4*x^6*a*f+1/6/b^3*x^6*e+2/b^5*x^3*a^2*f-1/b^4*x^3*a*e+1
/3/b^3*x^3*d+1/6/b^6*a^5/(b*x^3+a)^2*f-1/6/b^5*a^4/(b*x^3+a)^2*e+1/6/b^4*a^
3/(b*x^3+a)^2*d-1/6/b^3*a^2/(b*x^3+a)^2*c-10/3/b^6*ln(b*x^3+a)*a^3*f+2/b^5*
ln(b*x^3+a)*a^2*e-1/b^4*ln(b*x^3+a)*a*d+1/3/b^3*ln(b*x^3+a)*c-5/3/b^6*a^4/(
```

```
b*x^3+a)*f+4/3/b^5*a^3/(b*x^3+a)*e-1/b^4*a^2/(b*x^3+a)*d+2/3/b^3*a/(b*x^3+a
)*c
```

maxima [A] time = 1.35, size = 191, normalized size = 1.03

$$\frac{3\,a^2b^3c-5\,a^3b^2d+7\,a^4be-9\,a^5f+2\left(2\,ab^4c-3\,a^2b^3d+4\,a^3b^2e-5\,a^4bf\right)x^3}{6\left(b^8x^6+2\,ab^7x^3+a^2b^6\right)}+\frac{2\,b^2fx^9+3\left(b^2e-3\,abf\right)x^6+6\left(b^2d-3\,abe+6\,a^2f\right)x^3}{18\,b^5}+\frac{\left(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f\right)\log\left(bx^3+a\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*b^3*c - 5*a^3*b^2*d + 7*a^4*b*e - 9*a^5*f + 2*(2*a*b^4*c - 3*a^2
*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6) +
1/18*(2*b^2*f*x^9 + 3*(b^2*e - 3*a*b*f)*x^6 + 6*(b^2*d - 3*a*b*e + 6*a^2*f)
*x^3)/b^5 + 1/3*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*log(b*x^3 + a)/b
^6
```

mupad [B] time = 4.92, size = 204, normalized size = 1.10

$$x^6\left(\frac{e}{6\,b^3}-\frac{a\,f}{2\,b^4}\right)-\frac{x^3\left(\frac{5\,f\,a^4}{3}-\frac{4\,e\,a^3\,b}{3}+d\,a^2\,b^2-\frac{2\,c\,a\,b^3}{3}\right)+\frac{9\,f\,a^5-7\,e\,a^4\,b+5\,d\,a^3\,b^2-3\,c\,a^2\,b^3}{6\,b}}{a^2\,b^5+2\,a\,b^6\,x^3+b^7\,x^6}-x^3\left(\frac{a^2\,f}{b^5}-\frac{d}{3\,b^3}+\frac{a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)+\frac{\ln\left(b\,x^3+a\right)\left(-10\,f\,a^3+6\,e\,a^2\,b-3\,d\,a\,b^2+c\,b^3\right)}{3\,b^6}+\frac{f\,x^9}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^6*(e/(6*b^3) - (a*f)/(2*b^4)) - (x^3*((5*a^4*f)/3 + a^2*b^2*d - (2*a*b^3*
c)/3 - (4*a^3*b*e)/3) + (9*a^5*f - 3*a^2*b^3*c + 5*a^3*b^2*d - 7*a^4*b*e)/(
6*b))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) - x^3*((a^2*f)/b^5 - d/(3*b^3) + (a
*(e/b^3 - (3*a*f)/b^4))/b) + (log(a + b*x^3)*(b^3*c - 10*a^3*f - 3*a*b^2*d
+ 6*a^2*b*e))/(3*b^6) + (f*x^9)/(9*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.226 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=146

$$\frac{\log\left(a+bx^3\right)\left(6a^2f-3abe+b^2d\right)}{3b^5}-\frac{-4a^3f+3a^2be-2ab^2d+b^3c}{3b^5\left(a+bx^3\right)}+\frac{a\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}+\frac{x^3(be-3a}{3b^4}$$

Rubi [A] time = 0.20, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{3a^2be-4a^3f-2ab^2d+b^3c}{3b^5\left(a+bx^3\right)}+\frac{a\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}+\frac{\log\left(a+bx^3\right)\left(6a^2f-3abe+b^2d\right)}{3b^5}+\frac{x^3(be-3af)}{3b^4}+\frac{fx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x^3)/(3*b^4) + (f*x^6)/(6*b^3) + (a*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f))/(6*b^5*(a + b*x^3)^2) - (b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3
*f)/(3*b^5*(a + b*x^3)) + ((b^2*d - 3*a*b*e + 6*a^2*f)*Log[a + b*x^3])/(3*b
^5)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x\left(c+dx+ex^2+fx^3\right)}{(a+bx)^3}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{be-3af}{b^4}+\frac{fx}{b^3}+\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^4(a+bx)^3}+\frac{b^3c-2ab^2d}{b^4(a}\right.\right.\\
&= \frac{(be-3af)x^3}{3b^4}+\frac{fx^6}{6b^3}+\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{6b^5\left(a+bx^3\right)^2}-\frac{b^3c-2ab^2d+3a^2be-}{3b^5\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 145, normalized size = 0.99

$$\frac{7a^4f+a^3b\left(2fx^3-5e\right)+2\left(a+bx^3\right)^2\log\left(a+bx^3\right)\left(6a^2f-3abe+b^2d\right)+a^2b^2\left(3d-4ex^3-11fx^6\right)-ab^3\left(c-4x^3\left(d+ex^3-fx^6\right)\right)+b^4x^3\left(-2c+2ex^6+fx^9\right)}{6b^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (7*a^4*f + a^3*b*(-5*e + 2*f*x^3) + a^2*b^2*(3*d - 4*e*x^3 - 11*f*x^6) + b^
4*x^3*(-2*c + 2*e*x^6 + f*x^9) - a*b^3*(c - 4*x^3*(d + e*x^3 - f*x^6)) + 2*
(b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^5*(a + b*x^3
)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(c + dx^3 + ex^6 + fx^9\right)}{\left(a + bx^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [A] time = 0.41, size = 225, normalized size = 1.54

$$\frac{b^4 f x^{12} + 2\left(b^4 e - 2\,ab^3 f\right)x^9 + \left(4\,ab^3 e - 11\,a^2 b^2 f\right)x^6 - ab^3 c + 3\,a^2 b^2 d - 5\,a^3 be + 7\,a^4 f - 2\left(b^4 c - 2\,ab^3 d + 2\,a^2 b^2 e - a^3 bf\right)x^3 + 2\left(\left(b^4 d - 3\,ab^3 e + 6\,a^2 b^2 f\right)x^6 + a^2 b^2 d - 3\,a^3 be + 6\,a^4 f + 2\left(ab^3 d - 3\,a^2 b^2 e + 6\,a^3 bf\right)x^3\right)\log\left(bx^3 + a\right)}{6\left(b^7 x^6 + 2\,ab^6 x^3 + a^2 b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*f*x^12 + 2*(b^4*e - 2*a*b^3*f)*x^9 + (4*a*b^3*e - 11*a^2*b^2*f)*x^
6 - a*b^3*c + 3*a^2*b^2*d - 5*a^3*b*e + 7*a^4*f - 2*(b^4*c - 2*a*b^3*d + 2*
a^2*b^2*e - a^3*b*f)*x^3 + 2*((b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^6 + a^2*b
^2*d - 3*a^3*b*e + 6*a^4*f + 2*(a*b^3*d - 3*a^2*b^2*e + 6*a^3*b*f)*x^3)*log
(b*x^3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

giac [A] time = 0.18, size = 146, normalized size = 1.00

$$\frac{\left(b^2 d + 6\,a^2 f - 3\,abe\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^5} + \frac{b^3 f x^6 - 6\,ab^2 f x^3 + 2\,b^3 x^3 e}{6\,b^6} - \frac{ab^3 c - 3\,a^2 b^2 d - 7\,a^4 f + 5\,a^3 be + 2\left(b^4 c - 2\,ab^3 d - 4\,a^3 bf + 3\,a^2 b^2 e\right)x^3}{6\left(bx^3 + a\right)^2 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*d + 6*a^2*f - 3*a*b*e)*log(abs(b*x^3 + a))/b^5 + 1/6*(b^3*f*x^6 -
6*a*b^2*f*x^3 + 2*b^3*x^3*e)/b^6 - 1/6*(a*b^3*c - 3*a^2*b^2*d - 7*a^4*f + 5
*a^3*b*e + 2*(b^4*c - 2*a*b^3*d - 4*a^3*b*f + 3*a^2*b^2*e)*x^3)/((b*x^3 + a
)^2*b^5)
```

maple [A] time = 0.07, size = 213, normalized size = 1.46

$$\frac{f x^6}{6b^3} - \frac{af x^3}{b^4} + \frac{e x^3}{3b^3} - \frac{a^4 f}{6\left(b x^3 + a\right)^2 b^5} + \frac{a^3 e}{6\left(b x^3 + a\right)^2 b^4} - \frac{a^2 d}{6\left(b x^3 + a\right)^2 b^3} + \frac{ac}{6\left(b x^3 + a\right)^2 b^2} + \frac{4a^3 f}{3\left(b x^3 + a\right) b^5} - \frac{a^2 e}{\left(b x^3 + a\right) b^4} + \frac{2a^2 f \ln\left(b x^3 + a\right)}{b^5} + \frac{2ad}{3\left(b x^3 + a\right) b^3} - \frac{ae \ln\left(b x^3 + a\right)}{b^4} - \frac{c}{3\left(b x^3 + a\right) b^2} + \frac{d \ln\left(b x^3 + a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*f*x^6/b^3-1/b^4*x^3*a*f+1/3/b^3*x^3*e-1/6/b^5*a^4/(b*x^3+a)^2*f+1/6/b^4
*a^3/(b*x^3+a)^2*e-1/6/b^3*a^2/(b*x^3+a)^2*d+1/6/b^2*a/(b*x^3+a)^2*c+2/b^5*
ln(b*x^3+a)*a^2*f-1/b^4*ln(b*x^3+a)*a*e+1/3/b^3*ln(b*x^3+a)*d+4/3/b^5/(b*x^
3+a)*a^3*f-1/b^4/(b*x^3+a)*a^2*e+2/3/b^3/(b*x^3+a)*a*d-1/3/b^2/(b*x^3+a)*c
```

maxima [A] time = 1.39, size = 147, normalized size = 1.01

$$-\frac{ab^3 c - 3\,a^2 b^2 d + 5\,a^3 be - 7\,a^4 f + 2\left(b^4 c - 2\,ab^3 d + 3\,a^2 b^2 e - 4\,a^3 bf\right)x^3}{6\left(b^7 x^6 + 2\,ab^6 x^3 + a^2 b^5\right)} + \frac{bf x^6 + 2\left(be - 3\,af\right)x^3}{6\,b^4} + \frac{\left(b^2 d - 3\,abe + 6\,a^2 f\right)\log\left(bx^3 + a\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + 2*(b^4*c - 2*a*b^3*d +
3*a^2*b^2*e - 4*a^3*b*f)*x^3)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5) + 1/6*(b*f*
x^6 + 2*(b*e - 3*a*f)*x^3)/b^4 + 1/3*(b^2*d - 3*a*b*e + 6*a^2*f)*log(b*x^3
+ a)/b^5
```

mupad [B] time = 0.10, size = 152, normalized size = 1.04

$$x^3\left(\frac{e}{3b^3}-\frac{af}{b^4}\right)+\frac{\frac{7fa^4-5ea^3b+3da^2b^2-cab^3}{6b}-x^3\left(-\frac{4fa^3}{3}+ea^2b-\frac{2dab^2}{3}+\frac{cb^3}{3}\right)}{a^2b^4+2ab^5x^3+b^6x^6}+\frac{fx^6}{6b^3}+\frac{\ln\left(bx^3+a\right)\left(6fa^2-3eab+db^2\right)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^3*(e/(3*b^3) - (a*f)/b^4) + ((7*a^4*f + 3*a^2*b^2*d - a*b^3*c - 5*a^3*b*e
)/(6*b) - x^3*((b^3*c)/3 - (4*a^3*f)/3 - (2*a*b^2*d)/3 + a^2*b*e))/(a^2*b^4
 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^6)/(6*b^3) + (log(a + b*x^3)*(b^2*d + 6*a^
2*f - 3*a*b*e))/(3*b^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.227 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=109

$$-\frac{3a^2f-2abe+b^2d}{3b^4\left(a+bx^3\right)}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6b^4\left(a+bx^3\right)^2}+\frac{(be-3af)\log\left(a+bx^3\right)}{3b^4}+\frac{fx^3}{3b^3}$$

Rubi [A] time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1819, 1850}

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6b^4\left(a+bx^3\right)^2}-\frac{3a^2f-2abe+b^2d}{3b^4\left(a+bx^3\right)}+\frac{(be-3af)\log\left(a+bx^3\right)}{3b^4}+\frac{fx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (f*x^3)/(3*b^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*b^4*(a + b*x^3)^2)
 - (b^2*d - 2*a*b*e + 3*a^2*f)/(3*b^4*(a + b*x^3)) + ((b*e - 3*a*f)*Log[a +
 b*x^3])/(3*b^4)
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{f}{b^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)^3}+\frac{b^2d-2abe+3a^2f}{b^3(a+bx)^2}+\frac{be-3af}{b^3(a+bx)}\right)\right.\\ &= \frac{fx^3}{3b^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6b^4\left(a+bx^3\right)^2}-\frac{b^2d-2abe+3a^2f}{3b^4\left(a+bx^3\right)}+\frac{(be-3af)\log\left(a+bx^3\right)}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 105, normalized size = 0.96

$$\frac{-5a^3f+a^2b\left(3e-4fx^3\right)+ab^2\left(-d+4ex^3+4fx^6\right)+2\left(a+bx^3\right)^2(be-3af)\log\left(a+bx^3\right)-b^3\left(c+2dx^3-2fx^9\right)}{6b^4\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (-5*a^3*f + a^2*b*(3*e - 4*f*x^3) + a*b^2*(-d + 4*e*x^3 + 4*f*x^6) - b^3*(c
 + 2*d*x^3 - 2*f*x^9) + 2*(b*e - 3*a*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^
4*(a + b*x^3)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [A] time = 0.40, size = 158, normalized size = 1.45

$$\frac{2\,b^3fx^9+4\,ab^2fx^6-b^3c-ab^2d+3\,a^2be-5\,a^3f-2\left(b^3d-2\,ab^2e+2\,a^2bf\right)x^3+2\left(\left(b^3e-3\,ab^2f\right)x^6+a^2be-3\,a^3f+2\left(ab^2e-3\,a^2bf\right)x^3\right)\log\left(bx^3+a\right)}{6\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*f*x^9 + 4*a*b^2*f*x^6 - b^3*c - a*b^2*d + 3*a^2*b*e - 5*a^3*f -
2*(b^3*d - 2*a*b^2*e + 2*a^2*b*f)*x^3 + 2*((b^3*e - 3*a*b^2*f)*x^6 + a^2*b*
e - 3*a^3*f + 2*(a*b^2*e - 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b
^5*x^3 + a^2*b^4)
```

giac [A] time = 0.20, size = 100, normalized size = 0.92

$$\frac{fx^3}{3\,b^3}-\frac{\left(3\,af-be\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^4}-\frac{b^3c+ab^2d+5\,a^3f+2\left(b^3d+3\,a^2bf-2\,ab^2e\right)x^3-3\,a^2be}{6\left(bx^3+a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b^3 - 1/3*(3*a*f - b*e)*log(abs(b*x^3 + a))/b^4 - 1/6*(b^3*c + a*
b^2*d + 5*a^3*f + 2*(b^3*d + 3*a^2*b*f - 2*a*b^2*e)*x^3 - 3*a^2*b*e)/((b*x^
3 + a)^2*b^4)
```

maple [A] time = 0.06, size = 156, normalized size = 1.43

$$\frac{f\,x^3}{3b^3}+\frac{a^3f}{6\left(b\,x^3+a\right)^2b^4}-\frac{a^2e}{6\left(b\,x^3+a\right)^2b^3}+\frac{ad}{6\left(b\,x^3+a\right)^2b^2}-\frac{c}{6\left(b\,x^3+a\right)^2b}-\frac{a^2f}{\left(b\,x^3+a\right)b^4}+\frac{2ae}{3\left(b\,x^3+a\right)b^3}-\frac{af\ln\left(b\,x^3+a\right)}{b^4}-\frac{d}{3\left(b\,x^3+a\right)b^2}+\frac{e\ln\left(b\,x^3+a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/3/b^3*f*x^3+1/6/b^4/(b*x^3+a)^2*a^3*f-1/6/b^3/(b*x^3+a)^2*a^2*e+1/6/b^2/(
b*x^3+a)^2*a*d-1/6/b/(b*x^3+a)^2*c-1/b^4*ln(b*x^3+a)*a*f+1/3/b^3*ln(b*x^3+a
)*e-1/b^4/(b*x^3+a)*a^2*f+2/3/b^3/(b*x^3+a)*a*e-1/3/b^2/(b*x^3+a)*d
```

maxima [A] time = 1.38, size = 109, normalized size = 1.00

$$\frac{fx^3}{3\,b^3}-\frac{b^3c+ab^2d-3\,a^2be+5\,a^3f+2\left(b^3d-2\,ab^2e+3\,a^2bf\right)x^3}{6\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}+\frac{\left(be-3\,af\right)\log\left(bx^3+a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b^3 - 1/6*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f + 2*(b^3*d - 2*a
*b^2*e + 3*a^2*b*f)*x^3)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) + 1/3*(b*e - 3*a
*f)*log(b*x^3 + a)/b^4
```

mupad [B] time = 4.94, size = 112, normalized size = 1.03

$$\frac{f\,x^3}{3\,b^3}-\frac{x^3\,\left(f\,a^2-\frac{2\,e\,a\,b}{3}+\frac{d\,b^2}{3}\right)+\frac{5\,f\,a^3-3\,e\,a^2\,b+d\,a\,b^2+c\,b^3}{6\,b}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}-\frac{\ln\left(b\,x^3+a\right)\,\left(3\,a\,f-b\,e\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] (f*x^3)/(3*b^3) - (x^3*((b^2*d)/3 + a^2*f - (2*a*b*e)/3) + (b^3*c + 5*a^3*f
 + a*b^2*d - 3*a^2*b*e)/(6*b))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) - (log(a +
 b*x^3)*(3*a*f - b*e))/(3*b^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.228 $\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=114

$$-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)+\frac{c\log(x)}{a^3}+\frac{2a^3f-a^2be+b^3c}{3a^2b^3\left(a+bx^3\right)}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6ab^3\left(a+bx^3\right)^2}$$

Rubi [A] time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6ab^3\left(a+bx^3\right)^2}+\frac{-a^2be+2a^3f+b^3c}{3a^2b^3\left(a+bx^3\right)}-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)+\frac{c\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a*b^3*(a + b*x^3)^2) + (b^3*c - a^2*
b*e + 2*a^3*f)/(3*a^2*b^3*(a + b*x^3)) + (c*Log[x])/a^3 - ((c/a^3 - f/b^3)*
Log[a + b*x^3])/3
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)^3}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)^3}+\frac{-b^3c+a^2be-2a^3f}{a^2b^2(a+bx)^2}+\frac{-b^3c+a}{a^3b^2(a+}\right.\right.$$

$$=\frac{b^3c-ab^2d+a^2be-a^3f}{6ab^3\left(a+bx^3\right)^2}+\frac{b^3c-a^2be+2a^3f}{3a^2b^3\left(a+bx^3\right)}+\frac{c\log(x)}{a^3}-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+\right.$$

Mathematica [A] time = 0.13, size = 104, normalized size = 0.91

$$\frac{\frac{2\left(a^3f-b^3c\right)\log\left(a+bx^3\right)+\frac{a\left(3a^4f-a^3b\left(e-4fx^3\right)-a^2b^2\left(d+2ex^3\right)+3ab^3c+2b^4cx^3\right)}{\left(a+bx^3\right)^2}}{b^3}+6c\log(x)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] (6*c*Log[x] + ((a*(3*a*b^3*c + 3*a^4*f + 2*b^4*c*x^3 - a^2*b^2*(d + 2*e*x^3
) - a^3*b*(e - 4*f*x^3)))/(a + b*x^3)^2 + 2*(-(b^3*c) + a^3*f)*Log[a + b*x^
3])/b^3)/(6*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x\left(a + bx^3\right)^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3), x]
```

fricas [A] time = 0.44, size = 187, normalized size = 1.64

$$\frac{3\,a^2b^3c - a^3b^2d - a^4be + 3\,a^5f + 2\left(ab^4c - a^3b^2e + 2\,a^4bf\right)x^3 - 2\left(\left(b^5c - a^3b^2f\right)x^6 + a^2b^3c - a^5f + 2\left(ab^4c - a^4bf\right)x^3\right)\log\left(bx^3 + a\right) + 6\left(b^5cx^6 + 2\,ab^4cx^3 + a^2b^3c\right)\log(x)}{6\left(a^3b^5x^6 + 2\,a^4b^4x^3 + a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*b^3*c - a^3*b^2*d - a^4*b*e + 3*a^5*f + 2*(a*b^4*c - a^3*b^2*e +
 2*a^4*b*f)*x^3 - 2*((b^5*c - a^3*b^2*f)*x^6 + a^2*b^3*c - a^5*f + 2*(a*b^4
*c - a^4*b*f)*x^3)*log(b*x^3 + a) + 6*(b^5*c*x^6 + 2*a*b^4*c*x^3 + a^2*b^3*
c)*log(x))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)
```

giac [A] time = 0.24, size = 128, normalized size = 1.12

$$\frac{c\log\left(|x|\right)}{a^3} - \frac{\left(b^3c - a^3f\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^3b^3} + \frac{3\,b^4cx^6 - 3\,a^3bfx^6 + 8\,ab^3cx^3 - 2\,a^4fx^3 - 2\,a^3bx^3e + 6\,a^2b^2c - a^3bd - a^4e}{6\left(bx^3 + a\right)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a^3 - 1/3*(b^3*c - a^3*f)*log(abs(b*x^3 + a))/(a^3*b^3) + 1/6
*(3*b^4*c*x^6 - 3*a^3*b*f*x^6 + 8*a*b^3*c*x^3 - 2*a^4*f*x^3 - 2*a^3*b*x^3*e
 + 6*a^2*b^2*c - a^3*b*d - a^4*e)/((b*x^3 + a)^2*a^3*b^2)
```

maple [A] time = 0.06, size = 147, normalized size = 1.29

$$-\frac{a^2f}{6\left(bx^3+a\right)^2b^3} + \frac{ae}{6\left(bx^3+a\right)^2b^2} + \frac{c}{6\left(bx^3+a\right)^2a} - \frac{d}{6\left(bx^3+a\right)^2b} + \frac{2af}{3\left(bx^3+a\right)b^3} + \frac{c}{3\left(bx^3+a\right)a^2} + \frac{c\ln(x)}{a^3} - \frac{c\ln\left(bx^3+a\right)}{3a^3} - \frac{e}{3\left(bx^3+a\right)b^2} + \frac{f\ln\left(bx^3+a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x)
```

```
[Out] -1/6*a^2/b^3/(b*x^3+a)^2*f+1/6*a/b^2/(b*x^3+a)^2*e-1/6/b/(b*x^3+a)^2*d+1/6/
a/(b*x^3+a)^2*c+1/3/b^3*ln(b*x^3+a)*f-1/3*c*ln(b*x^3+a)/a^3+2/3*a/b^3/(b*x^
3+a)*f-1/3/b^2/(b*x^3+a)*e+1/3/a^2/(b*x^3+a)*c+c*ln(x)/a^3
```

maxima [A] time = 1.37, size = 129, normalized size = 1.13

$$\frac{3\,ab^3c - a^2b^2d - a^3be + 3\,a^4f + 2\left(b^4c - a^2b^2e + 2\,a^3bf\right)x^3}{6\left(a^2b^5x^6 + 2\,a^3b^4x^3 + a^4b^3\right)} + \frac{c\log\left(x^3\right)}{3\,a^3} - \frac{\left(b^3c - a^3f\right)\log\left(bx^3 + a\right)}{3\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f + 2*(b^4*c - a^2*b^2*e + 2*a
^3*b*f)*x^3)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3) + 1/3*c*log(x^3)/a^3 -
 1/3*(b^3*c - a^3*f)*log(b*x^3 + a)/(a^3*b^3)
```

mupad [B]   time = 0.18, size = 123, normalized size = 1.08

$$\frac{\frac{3\,f\,a^3-e\,a^2\,b-d\,a\,b^2+3\,c\,b^3}{6\,a\,b^3}+\frac{x^3\,\left(2\,f\,a^3-e\,a^2\,b+c\,b^3\right)}{3\,a^2\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{c\,\ln(x)}{a^3}-\frac{\ln\left(b\,x^3+a\right)\,\left(b^3\,c-a^3\,f\right)}{3\,a^3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x)
```

```
[Out] ((3*b^3*c + 3*a^3*f - a*b^2*d - a^2*b*e)/(6*a*b^3) + (x^3*(b^3*c + 2*a^3*f
- a^2*b*e))/(3*a^2*b^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (c*log(x))/a^3 - (lo
g(a + b*x^3)*(b^3*c - a^3*f))/(3*a^3*b^3)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.229 $\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=134

$$\frac{(3bc-ad)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3bc-ad)}{a^4}-\frac{a^3f-ab^2d+2b^3c}{3a^3b^2\left(a+bx^3\right)}-\frac{c}{3a^3x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{6a^2b^2\left(a+bx^3\right)^2}$$

Rubi [A] time = 0.17, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^2b^2\left(a+bx^3\right)^2}-\frac{a^3f-ab^2d+2b^3c}{3a^3b^2\left(a+bx^3\right)}+\frac{(3bc-ad)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3bc-ad)}{a^4}-\frac{c}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -c/(3*a^3*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^2*b^2*(a + b*x^3)
^2) - (2*b^3*c - a*b^2*d + a^3*f)/(3*a^3*b^2*(a + b*x^3)) - ((3*b*c - a*d)*
Log[x])/a^4 + ((3*b*c - a*d)*Log[a + b*x^3])/(3*a^4)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^2}+\frac{-3bc+ad}{a^4x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)^3}+\frac{2b^3c-ab^2d+a^3f}{a^3b(a+bx)^2}-\right.\right.\\ &= -\frac{c}{3a^3x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6a^2b^2\left(a+bx^3\right)^2}-\frac{2b^3c-ab^2d+a^3f}{3a^3b^2\left(a+bx^3\right)}-\frac{(3bc-ad)\log(x)}{a^4}+\frac{(3bc}{}\end{aligned}$$

Mathematica [A] time = 0.11, size = 121, normalized size = 0.90

$$\frac{-\frac{2a\left(a^3f-ab^2d+2b^3c\right)}{b^2\left(a+bx^3\right)}+\frac{a^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^2\left(a+bx^3\right)^2}+2(3bc-ad)\log\left(a+bx^3\right)+6\log(x)(ad-3bc)-\frac{2ac}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a*c)/x^3 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^
3)^2) - (2*a*(2*b^3*c - a*b^2*d + a^3*f))/(b^2*(a + b*x^3)) + 6*(-3*b*c + a
*d)*Log[x] + 2*(3*b*c - a*d)*Log[a + b*x^3])/(6*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3), x]
```

fricas [A] time = 0.43, size = 250, normalized size = 1.87

$$-\frac{2\left(3\,ab^4c-a^2b^3d+a^4bf\right)x^6+2\,a^3b^2c+\left(9\,a^2b^3c-3\,a^3b^2d+a^4be+a^5f\right)x^3-2\left(\left(3\,b^5c-ab^4d\right)x^9+2\left(3\,ab^4c-a^2b^3d\right)x^6+\left(3\,a^2b^3c-a^3b^2d\right)x^3\right)\log\left(bx^3+a\right)+6\left(\left(3\,b^5c-ab^4d\right)x^9+2\left(3\,ab^4c-a^2b^3d\right)x^6+\left(3\,a^2b^3c-a^3b^2d\right)x^3\right)\log(x)}{6\left(a^4b^4x^9+2\,a^5b^3x^6+a^6b^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*a*b^4*c - a^2*b^3*d + a^4*b*f)*x^6 + 2*a^3*b^2*c + (9*a^2*b^3*c
- 3*a^3*b^2*d + a^4*b*e + a^5*f)*x^3 - 2*((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*
b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a^3*b^2*d)*x^3)*log(b*x^3 + a) + 6*
((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a
^3*b^2*d)*x^3)*log(x))/(a^4*b^4*x^9 + 2*a^5*b^3*x^6 + a^6*b^2*x^3)
```

giac [A] time = 0.18, size = 173, normalized size = 1.29

$$-\frac{(3\,bc-ad)\log\left(|x|\right)}{a^4}+\frac{\left(3\,b^2c-abd\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^4b}+\frac{3\,bcx^3-adx^3-ac}{3\,a^4x^3}-\frac{9\,b^5cx^6-3\,ab^4dx^6+22\,ab^4cx^3-8\,a^2b^3dx^3+2\,a^4bfx^3+14\,a^2b^3c-6\,a^3b^2d+a^5f+a^4be}{6\left(bx^3+a\right)^2a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -(3*b*c - a*d)*log(abs(x))/a^4 + 1/3*(3*b^2*c - a*b*d)*log(abs(b*x^3 + a))/
(a^4*b) + 1/3*(3*b*c*x^3 - a*d*x^3 - a*c)/(a^4*x^3) - 1/6*(9*b^5*c*x^6 - 3*
a*b^4*d*x^6 + 22*a*b^4*c*x^3 - 8*a^2*b^3*d*x^3 + 2*a^4*b*f*x^3 + 14*a^2*b^3
*c - 6*a^3*b^2*d + a^5*f + a^4*b*e)/((b*x^3 + a)^2*a^4*b^2)
```

maple [A] time = 0.06, size = 163, normalized size = 1.22

$$\frac{af}{6\left(b\,x^3+a\right)^2b^2}+\frac{d}{6\left(b\,x^3+a\right)^2a}-\frac{bc}{6\left(b\,x^3+a\right)^2a^2}-\frac{e}{6\left(b\,x^3+a\right)^2b}+\frac{d}{3\left(b\,x^3+a\right)a^2}-\frac{2bc}{3\left(b\,x^3+a\right)a^3}+\frac{d\ln(x)}{a^3}-\frac{d\ln\left(b\,x^3+a\right)}{3a^3}-\frac{3bc\ln(x)}{a^4}+\frac{bc\ln\left(b\,x^3+a\right)}{a^4}-\frac{f}{3\left(b\,x^3+a\right)b^2}-\frac{c}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x)
```

```
[Out] 1/6*a/b^2/(b*x^3+a)^2*f-1/6/b/(b*x^3+a)^2*e+1/6/a/(b*x^3+a)^2*d-1/6/a^2*b/(
b*x^3+a)^2*c-1/3*d*ln(b*x^3+a)/a^3+b*c*ln(b*x^3+a)/a^4-1/3/b^2/(b*x^3+a)*f+
1/3/a^2/(b*x^3+a)*d-2/3/a^3*b/(b*x^3+a)*c-1/3/a^3*c/x^3+d*ln(x)/a^3-3*b*c*l
n(x)/a^4
```

maxima [A] time = 1.36, size = 144, normalized size = 1.07

$$-\frac{2\left(3\,b^4c-ab^3d+a^3bf\right)x^6+2\,a^2b^2c+\left(9\,ab^3c-3\,a^2b^2d+a^3be+a^4f\right)x^3}{6\left(a^3b^4x^9+2\,a^4b^3x^6+a^5b^2x^3\right)}+\frac{(3\,bc-ad)\log\left(bx^3+a\right)}{3\,a^4}-\frac{(3\,bc-ad)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(3*b^4*c - a*b^3*d + a^3*b*f)*x^6 + 2*a^2*b^2*c + (9*a*b^3*c - 3*a^
2*b^2*d + a^3*b*e + a^4*f)*x^3)/(a^3*b^4*x^9 + 2*a^4*b^3*x^6 + a^5*b^2*x^3)
 + 1/3*(3*b*c - a*d)*log(b*x^3 + a)/a^4 - 1/3*(3*b*c - a*d)*log(x^3)/a^4
```

mupad [B] time = 5.07, size = 135, normalized size = 1.01

$$\frac{\ln(x)\,(a\,d-3\,b\,c)}{a^4}-\frac{\ln\left(b\,x^3+a\right)\,(a\,d-3\,b\,c)}{3\,a^4}-\frac{\frac{c}{3\,a}+\frac{x^6\,\left(f\,a^3-d\,a\,b^2+3\,c\,b^3\right)}{3\,a^3\,b}+\frac{x^3\,\left(f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+9\,c\,b^3\right)}{6\,a^2\,b^2}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] (log(x)*(a*d - 3*b*c))/a^4 - (log(a + b*x^3)*(a*d - 3*b*c))/(3*a^4) - (c/(3
*a) + (x^6*(3*b^3*c + a^3*f - a*b^2*d))/(3*a^3*b) + (x^3*(9*b^3*c + a^3*f -
 3*a*b^2*d + a^2*b*e))/(6*a^2*b^2))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.230 $\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=163

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{6a^3x^6}-\frac{\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)}{3a^5}+\frac{\log(x)\left(a^2e-3abd+6b^2c\right)}{a^5}+\frac{a^2e-2abd+3b^2c}{3a^4\left(a+bx^3\right)}+\frac{a^3(-f)}{6}$$

Rubi [A] time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^3b\left(a+bx^3\right)^2}+\frac{a^2e-2abd+3b^2c}{3a^4\left(a+bx^3\right)}-\frac{\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)}{3a^5}+\frac{\log(x)\left(a^2e-3abd+6b^2c\right)}{a^5}+\frac{3bc-ad}{3a^4x^3}-\frac{c}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] -c/(6*a^3*x^6) + (3*b*c - a*d)/(3*a^4*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a
^3*f)/(6*a^3*b*(a + b*x^3)^2) + (3*b^2*c - 2*a*b*d + a^2*e)/(3*a^4*(a + b*x
^3)) + ((6*b^2*c - 3*a*b*d + a^2*e)*Log[x])/a^5 - ((6*b^2*c - 3*a*b*d + a^2
*e)*Log[a + b*x^3])/(3*a^5)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^3}+\frac{-3bc+ad}{a^4x^2}+\frac{6b^2c-3abd+a^2e}{a^5x}+\frac{-b^3c+ab^2d-a^2be+a^3}{a^3(a+bx)^3}\right.\right.\\ &= -\frac{c}{6a^3x^6}+\frac{3bc-ad}{3a^4x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^3b\left(a+bx^3\right)^2}+\frac{3b^2c-2abd+a^2e}{3a^4\left(a+bx^3\right)}+\frac{\left(6b^2c-3a\right.}{}\end{aligned}$$

Mathematica [A] time = 0.13, size = 149, normalized size = 0.91

$$\frac{\frac{2a\left(a^2e-2abd+3b^2c\right)}{a+bx^3}-2\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)+6\log(x)\left(a^2e-3abd+6b^2c\right)-\frac{a^2c}{x^6}+\frac{a^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b\left(a+bx^3\right)^2}-\frac{2a(ad-3bc)}{x^3}}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] (-((a^2*c)/x^6) - (2*a*(-3*b*c + a*d))/x^3 + (a^2*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f))/(b*(a + b*x^3)^2) + (2*a*(3*b^2*c - 2*a*b*d + a^2*e))/(a + b*x^
3) + 6*(6*b^2*c - 3*a*b*d + a^2*e)*Log[x] - 2*(6*b^2*c - 3*a*b*d + a^2*e)*L
og[a + b*x^3])/(6*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^7 \left(a + bx^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3), x]
```

fricas [B] time = 0.43, size = 316, normalized size = 1.94

$$\frac{2 \left(6 \, ab^{4}c - 3 \, a^{2}b^{3}d + a^{3}b^{2}e\right)x^{9} + \left(18 \, a^{2}b^{3}c - 9 \, a^{3}b^{2}d + 3 \, a^{4}be - a^{5}f\right)x^{6} - a^{4}bc + 2 \left(2 \, a^{3}b^{2}c - a^{4}bd\right)x^{3} - 2 \left(\left(6 \, b^{5}c - 3 \, ab^{4}d + a^{2}b^{3}e\right)x^{12} + 2 \left(6 \, ab^{4}c - 3 \, a^{2}b^{3}d + a^{3}b^{2}e\right)x^{9} + \left(6 \, a^{2}b^{3}c - 3 \, a^{3}b^{2}d + a^{4}be\right)x^{6}\right) \log \left(bx^{3} + a\right) + 6 \left(\left(6 \, b^{5}c - 3 \, ab^{4}d + a^{2}b^{3}e\right)x^{12} + 2 \left(6 \, ab^{4}c - 3 \, a^{2}b^{3}d + a^{3}b^{2}e\right)x^{9} + \left(6 \, a^{2}b^{3}c - 3 \, a^{3}b^{2}d + a^{4}be\right)x^{6}\right) \log\left(x\right)}{6 \left(a^{5}b^{3}x^{12} + 2 \, a^{6}b^{2}x^{9} + a^{7}bx^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (18*a^2*b^3*c - 9*a^3*b^
2*d + 3*a^4*b*e - a^5*f)*x^6 - a^4*b*c + 2*(2*a^3*b^2*c - a^4*b*d)*x^3 - 2*
((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*
b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(b*x^3 + a) + 6*
((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*
b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(x))/(a^5*b^3*x^
12 + 2*a^6*b^2*x^9 + a^7*b*x^6)
```

giac [A] time = 0.19, size = 189, normalized size = 1.16

$$\frac{\left(6 \, b^{2}c - 3 \, abd + a^{2}e\right) \log \left(|x|\right)}{a^{5}} - \frac{\left(6 \, b^{3}c - 3 \, ab^{2}d + a^{2}be\right) \log \left(|bx^{3} + a|\right)}{3 \, a^{5}b} + \frac{12 \, b^{4}cx^{9} - 6 \, ab^{3}dx^{9} + 2 \, a^{2}b^{2}x^{9}e + 18 \, ab^{3}cx^{6} - 9 \, a^{2}b^{2}dx^{6} - a^{4}fx^{6} + 3 \, a^{3}bx^{6}e + 4 \, a^{2}b^{2}cx^{3} - 2 \, a^{3}bdx^{3} - a^{3}bc}{6 \left(bx^{6} + ax^{3}\right)^{2} a^{4}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (6*b^2*c - 3*a*b*d + a^2*e)*log(abs(x))/a^5 - 1/3*(6*b^3*c - 3*a*b^2*d + a^
2*b*e)*log(abs(b*x^3 + a))/(a^5*b) + 1/6*(12*b^4*c*x^9 - 6*a*b^3*d*x^9 + 2*
a^2*b^2*x^9*e + 18*a*b^3*c*x^6 - 9*a^2*b^2*d*x^6 - a^4*f*x^6 + 3*a^3*b*x^6*
e + 4*a^2*b^2*c*x^3 - 2*a^3*b*d*x^3 - a^3*b*c)/((b*x^6 + a*x^3)^2*a^4*b)
```

maple [A] time = 0.06, size = 213, normalized size = 1.31

$$\frac{e}{6 \left(b \, x^{3}+a \right)^{2} a}-\frac{b d}{6 \left(b \, x^{3}+a \right)^{2} a^{2}}+\frac{b^{2} c}{6 \left(b \, x^{3}+a \right)^{2} a^{3}}-\frac{f}{6 \left(b \, x^{3}+a \right)^{2} b}+\frac{e}{3 \left(b \, x^{3}+a \right) a^{2}}-\frac{2 b d}{3 \left(b \, x^{3}+a \right) a^{3}}+\frac{e \ln \left(x \right)}{a^{3}}-\frac{e \ln \left(b \, x^{3}+a \right)}{3 a^{3}}+\frac{b^{2} c}{\left(b \, x^{3}+a \right) a^{4}}-\frac{3 b d \ln \left(x \right)}{a^{4}}+\frac{b d \ln \left(b \, x^{3}+a \right)}{a^{4}}+\frac{6 b^{2} c \ln \left(x \right)}{a^{5}}-\frac{2 b^{2} c \ln \left(b \, x^{3}+a \right)}{a^{5}}-\frac{d}{3 a^{3} x^{3}}+\frac{b c}{a^{4} x^{3}}-\frac{c}{6 a^{3} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x)
```

```
[Out] -1/6/b/(b*x^3+a)^2*f+1/6/a/(b*x^3+a)^2*e-1/6/a^2*b/(b*x^3+a)^2*d+1/6/a^3*b^
2/(b*x^3+a)^2*c-1/3*e*ln(b*x^3+a)/a^3+1/a^4*ln(b*x^3+a)*b*d-2/a^5*ln(b*x^3+
a)*b^2*c+1/3/a^2/(b*x^3+a)*e-2/3/a^3*b/(b*x^3+a)*d+1/a^4*b^2/(b*x^3+a)*c-1/
6*c/a^3/x^6-1/3/a^3/x^3*d+1/a^4/x^3*b*c+e*ln(x)/a^3-3/a^4*ln(x)*b*d+6/a^5*l
n(x)*b^2*c
```

maxima [A] time = 1.40, size = 182, normalized size = 1.12

$$\frac{2\left(6\,b^{4}c-3\,ab^{3}d+a^{2}b^{2}e\right)x^{9}+\left(18\,ab^{3}c-9\,a^{2}b^{2}d+3\,a^{3}be-a^{4}f\right)x^{6}-a^{3}bc+2\left(2\,a^{2}b^{2}c-a^{3}bd\right)x^{3}}{6\left(a^{4}b^{3}x^{12}+2\,a^{5}b^{2}x^{9}+a^{6}bx^{6}\right)}-\frac{\left(6\,b^{2}c-3\,abd+a^{2}e\right)\log\left(bx^{3}+a\right)}{3\,a^{5}}+\frac{\left(6\,b^{2}c-3\,abd+a^{2}e\right)\log\left(x^{3}\right)}{3\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(6*b^4*c - 3*a*b^3*d + a^2*b^2*e)*x^9 + (18*a*b^3*c - 9*a^2*b^2*d +
3*a^3*b*e - a^4*f)*x^6 - a^3*b*c + 2*(2*a^2*b^2*c - a^3*b*d)*x^3)/(a^4*b^3*
x^12 + 2*a^5*b^2*x^9 + a^6*b*x^6) - 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*log(b*x
^3 + a)/a^5 + 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*log(x^3)/a^5
```

mupad [B] time = 5.10, size = 167, normalized size = 1.02

$$\frac{\ln(x)\,\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{a^5}-\frac{\ln\left(b\,x^3+a\right)\,\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{3\,a^5}-\frac{\frac{c}{6\,a}+\frac{x^3\,\left(a\,d-2\,b\,c\right)}{3\,a^2}-\frac{b\,x^9\,\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{3\,a^4}-\frac{x^6\,\left(-f\,a^3+3\,e\,a^2\,b-9\,d\,a\,b^2+18\,c\,b^3\right)}{6\,a^3\,b}}{a^2\,x^6+2\,a\,b\,x^9+b^2\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x)
```

```
[Out] (log(x)*(6*b^2*c + a^2*e - 3*a*b*d))/a^5 - (log(a + b*x^3)*(6*b^2*c + a^2*e
 - 3*a*b*d))/(3*a^5) - (c/(6*a) + (x^3*(a*d - 2*b*c))/(3*a^2) - (b*x^9*(6*b
^2*c + a^2*e - 3*a*b*d))/(3*a^4) - (x^6*(18*b^3*c - a^3*f - 9*a*b^2*d + 3*a
^2*b*e))/(6*a^3*b))/(a^2*x^6 + b^2*x^12 + 2*a*b*x^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.231 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=218

$$\frac{3bc-ad}{6a^4x^6}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)}{3a^6}-\frac{\log(x)\left(a^3(-f)+3a^2be-\right.}{a^6}$$

Rubi [A] time = 0.26, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5\left(a+bx^3\right)}-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^4\left(a+bx^3\right)^2}+\frac{\log\left(a+bx^3\right)\left(3a^2be+a^3(-f)-6ab^2d+10b^3c\right)}{3a^6}-\frac{\log(x)\left(3a^2be+a^3(-f)-6ab^2d+10b^3c\right)}{a^6}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{3bc-ad}{6a^4x^6}-\frac{c}{9a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(6*a^4*x^6) - (6*b^2*c - 3*a*b*d + a^2*e)/(3
*a^5*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*(a + b*x^3)^2) - (4*
b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*(a + b*x^3)) - ((10*b^3*c - 6
*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[x])/a^6 + ((10*b^3*c - 6*a*b^2*d + 3*a^2*
b*e - a^3*f)*Log[a + b*x^3])/(3*a^6)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)^3}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^4}+\frac{-3bc+ad}{a^4x^3}+\frac{6b^2c-3abd+a^2e}{a^5x^2}+\frac{-10b^3c+6ab^2d-3a^2be+a}{a^6x}\right.\right.$$

$$=-\frac{c}{9a^3x^9}+\frac{3bc-ad}{6a^4x^6}-\frac{6b^2c-3abd+a^2e}{3a^5x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6a^4\left(a+bx^3\right)^2}-\frac{4b^3c-3ab^2d}{3a^5\left(a\right.}$$

Mathematica [A] time = 0.17, size = 200, normalized size = 0.92

$$\frac{-\frac{2a^3c}{x^9}-\frac{6a\left(a^2e-3abd+6b^2c\right)}{x^3}-\frac{3a^2(ad-3bc)}{x^6}+\frac{3a^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}+\frac{6a\left(a^3f-2a^2be+3ab^2d-4b^3c\right)}{a+bx^3}+6\log\left(a+bx^3\right)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)+18\log(x)\left(a^3f-3a^2be+6ab^2d-10b^3c\right)}{18a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-3*b*c + a*d))/x^6 - (6*a*(6*b^2*c - 3*a*b*d + a^
2*e))/x^3 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 +
(6*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/(a + b*x^3) + 18*(-10*b^3*
c + 6*a*b^2*d - 3*a^2*b*e + a^3*f)*Log[x] + 6*(10*b^3*c - 6*a*b^2*d + 3*a^2
*b*e - a^3*f)*Log[a + b*x^3])/(18*a^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3), x]
```

fricas [A] time = 0.47, size = 396, normalized size = 1.82

$$-\frac{6\left(10ab^4c-6a^2b^3d+3a^3b^2e-a^4bf\right)x^{12}+9\left(10a^2b^3c-6a^3b^2d+3a^4be-a^5f\right)x^9+2\left(10a^3b^2c-6a^4bd+3a^5e\right)x^6+2a^5c-\left(5a^4bc-3a^5d\right)x^3-6\left(\left(10b^5c-6ab^4d+3a^2b^3e-a^3b^2f\right)x^{15}+2\left(10ab^4c-6a^2b^3d+3a^3b^2e-a^4bf\right)x^{12}+\left(10a^2b^3c-6a^3b^2d+3a^4be-a^5f\right)x^9\right)\log\left(bx^3+a\right)+18\left(\left(10b^5c-6ab^4d+3a^2b^3e-a^3b^2f\right)x^{15}+2\left(10ab^4c-6a^2b^3d+3a^3b^2e-a^4bf\right)x^{12}+\left(10a^2b^3c-6a^3b^2d+3a^4be-a^5f\right)x^9\right)\log(x)}{18\left(a^6b^2x^{15}+2a^7bx^{12}+a^8x^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/18*(6*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + 9*(10*a^
2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9 + 2*(10*a^3*b^2*c - 6*a^4*b*
d + 3*a^5*e)*x^6 + 2*a^5*c - (5*a^4*b*c - 3*a^5*d)*x^3 - 6*((10*b^5*c - 6*a
*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*x^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^
3*b^2*e - a^4*b*f)*x^12 + (10*a^2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*
x^9)*log(b*x^3 + a) + 18*((10*b^5*c - 6*a*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*
x^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + (10*a^2*
b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9)*log(x))/(a^6*b^2*x^15 + 2*a^7
*b*x^12 + a^8*x^9)
```

giac [A] time = 0.19, size = 324, normalized size = 1.49

$$-\frac{\left(10b^3c-6ab^2d-a^3f+3a^2be\right)\log\left(|x|\right)}{a^6}+\frac{\left(10b^4c-6ab^3d-a^3bf+3a^2b^2e\right)\log\left(\left|bx^3+a\right|\right)}{3a^6b}-\frac{30b^5cx^6-18ab^4dx^6-3a^3b^2fx^6+9a^2b^3x^6e+68ab^4cx^3-42a^2b^3dx^3-8a^4bfx^3+22a^3b^2x^3e+39a^2b^3c-25a^3b^2d-6a^5f+14a^4be}{6\left(bx^3+a\right)^2a^6}+\frac{110b^3cx^9-66ab^2dx^9-11a^3fx^9+33a^2bx^9e-36ab^2cx^6+18a^2bdx^6-6a^3x^6e+9a^2bcx^3-3a^3dx^3-2a^3c}{18a^6x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -(10*b^3*c - 6*a*b^2*d - a^3*f + 3*a^2*b*e)*log(abs(x))/a^6 + 1/3*(10*b^4*c
 - 6*a*b^3*d - a^3*b*f + 3*a^2*b^2*e)*log(abs(b*x^3 + a))/(a^6*b) - 1/6*(30
*b^5*c*x^6 - 18*a*b^4*d*x^6 - 3*a^3*b^2*f*x^6 + 9*a^2*b^3*x^6*e + 68*a*b^4*
c*x^3 - 42*a^2*b^3*d*x^3 - 8*a^4*b*f*x^3 + 22*a^3*b^2*x^3*e + 39*a^2*b^3*c
- 25*a^3*b^2*d - 6*a^5*f + 14*a^4*b*e)/((b*x^3 + a)^2*a^6) + 1/18*(110*b^3*
c*x^9 - 66*a*b^2*d*x^9 - 11*a^3*f*x^9 + 33*a^2*b*x^9*e - 36*a*b^2*c*x^6 + 1
8*a^2*b*d*x^6 - 6*a^3*x^6*e + 9*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^6*x
^9)
```

maple [A] time = 0.07, size = 293, normalized size = 1.34

$$\frac{f}{6\left(bx^3+a\right)^2a}-\frac{be}{6\left(bx^3+a\right)^2a^2}+\frac{b^2d}{6\left(bx^3+a\right)^2a^3}-\frac{b^3c}{6\left(bx^3+a\right)^2a^4}+\frac{f}{3\left(bx^3+a\right)a^2}-\frac{2be}{3\left(bx^3+a\right)a^3}+\frac{f\ln(x)}{a^3}-\frac{f\ln\left(bx^3+a\right)}{3a^3}+\frac{b^2d}{\left(bx^3+a\right)a^4}-\frac{3be\ln(x)}{a^4}+\frac{be\ln\left(bx^3+a\right)}{a^4}-\frac{4b^3c}{3\left(bx^3+a\right)a^5}+\frac{6b^2d\ln(x)}{a^5}-\frac{2b^2d\ln\left(bx^3+a\right)}{a^5}-\frac{10b^3c\ln(x)}{a^6}+\frac{10b^3c\ln\left(bx^3+a\right)}{3a^6}-\frac{e}{3a^3x^3}+\frac{bd}{a^4x^3}-\frac{2b^2c}{a^5x^3}-\frac{d}{6a^3x^6}+\frac{bc}{2a^4x^6}-\frac{c}{9a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x)
```

```
[Out] 1/6/a/(b*x^3+a)^2*f-1/6/a^2*b/(b*x^3+a)^2*e+1/6/a^3*b^2/(b*x^3+a)^2*d-1/6/a
^4*b^3/(b*x^3+a)^2*c-1/3/a^3*ln(b*x^3+a)*f+1/a^4*b*ln(b*x^3+a)*e-2/a^5*b^2*
ln(b*x^3+a)*d+10/3/a^6*b^3*ln(b*x^3+a)*c+1/3/a^2/(b*x^3+a)*f-2/3/a^3*b/(b*x
^3+a)*e+1/a^4*b^2/(b*x^3+a)*d-4/3/a^5*b^3/(b*x^3+a)*c-1/9/a^3*c/x^9-1/6/a^3
/x^6*d+1/2/a^4/x^6*b*c-1/3/a^3/x^3*e+1/a^4/x^3*b*d-2/a^5/x^3*b^2*c+1/a^3*ln
(x)*f-3/a^4*ln(x)*b*e+6/a^5*ln(x)*b^2*d-10/a^6*ln(x)*b^3*c
```

maxima [A] time = 1.46, size = 232, normalized size = 1.06

$$-\frac{6\left(10\,b^4c-6\,ab^3d+3\,a^2b^2e-a^3bf\right)x^{12}+9\left(10\,ab^3c-6\,a^2b^2d+3\,a^3be-a^4f\right)x^9+2\left(10\,a^2b^2c-6\,a^3bd+3\,a^4e\right)x^6+2\,a^4c-\left(5\,a^3bc-3\,a^4d\right)x^3}{18\left(a^5b^2x^{15}+2\,a^6bx^{12}+a^7x^9\right)}+\frac{\left(10\,b^3c-6\,ab^2d+3\,a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,a^6}-\frac{\left(10\,b^3c-6\,ab^2d+3\,a^2be-a^3f\right)\log\left(x^3\right)}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(6*(10*b^4*c - 6*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^12 + 9*(10*a*b^3*
c - 6*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^9 + 2*(10*a^2*b^2*c - 6*a^3*b*d + 3*
a^4*e)*x^6 + 2*a^4*c - (5*a^3*b*c - 3*a^4*d)*x^3)/(a^5*b^2*x^15 + 2*a^6*b*x
^12 + a^7*x^9) + 1/3*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(b*x^3 +
 a)/a^6 - 1/3*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(x^3)/a^6
```

mupad [B] time = 5.17, size = 222, normalized size = 1.02

$$\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^6}-\frac{\frac{c}{9\,a}+\frac{x^9\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{2\,a^4}+\frac{x^3\,\left(3\,a\,d-5\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(3\,e\,a^2-6\,d\,a\,b+10\,c\,b^2\right)}{9\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^5}}{a^2\,x^9+2\,a\,b\,x^{12}+b^2\,x^{15}}-\frac{\ln(x)\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x)
```

```
[Out] (log(a + b*x^3)*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(3*a^6) - (c/(9
*a) + (x^9*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(2*a^4) + (x^3*(3*a*
d - 5*b*c))/(18*a^2) + (x^6*(10*b^2*c + 3*a^2*e - 6*a*b*d))/(9*a^3) + (b*x^
12*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(3*a^5))/(a^2*x^9 + b^2*x^15
 + 2*a*b*x^12) - (log(x)*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/a^6
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.232 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=258

$$\frac{3bc-ad}{9a^4x^9}-\frac{c}{12a^3x^{12}}-\frac{a^2e-3abd+6b^2c}{6a^5x^6}-\frac{b\log\left(a+bx^3\right)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{3a^7}+\frac{b\log(x)\left(-3a^3f+\right.}{}$$

Rubi [A] time = 0.30, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{b\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{3a^6x^3}-\frac{b\log\left(a+bx^3\right)\left(6a^2be-3a^3f-10ab^2d+15b^3c\right)}{3a^7}+\frac{b\log(x)\left(6a^2be-3a^3f-10ab^2d+15b^3c\right)}{a^7}-\frac{a^2e-3abd+6b^2c}{6a^5x^6}+\frac{3bc-ad}{9a^4x^9}-\frac{c}{12a^3x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

```
[Out] -c/(12*a^3*x^12) + (3*b*c - a*d)/(9*a^4*x^9) - (6*b^2*c - 3*a*b*d + a^2*e)/
(6*a^5*x^6) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*a^5*(a + b*x^3)^2) + (b*(5*b^3*c - 4
*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(3*a^6*(a + b*x^3)) + (b*(15*b^3*c - 10*a*
b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[x])/a^7 - (b*(15*b^3*c - 10*a*b^2*d + 6*a^
2*b*e - 3*a^3*f)*Log[a + b*x^3])/(3*a^7)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{c}{a^3x^5}+\frac{-3bc+ad}{a^4x^4}+\frac{6b^2c-3abd+a^2e}{a^5x^3}+\frac{-10b^3c+6ab^2d-3a^2be}{a^6x^2}\right.\right.\\ &= -\frac{c}{12a^3x^{12}}+\frac{3bc-ad}{9a^4x^9}-\frac{6b^2c-3abd+a^2e}{6a^5x^6}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}+\frac{b(b}{}\end{aligned}$$

Mathematica [A] time = 0.27, size = 238, normalized size = 0.92

$$\frac{12b\log\left(a+bx^3\right)\left(3a^3f-6a^2be+10ab^2d-15b^3c\right)+36b\log(x)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)-\frac{a\left(a^5\left(3c+4dx^3+6ex^6+12fx^9\right)-2a^4bx^3\left(3c+5dx^3+12ex^6-27fx^9\right)+a^3b^2x^6\left(15c+40dx^3-108ex^6+36fx^9\right)-12a^2b^3x^9\left(5c-15dx^3+6ex^6\right)+30ab^4x^{12}\left(4dx^3-9c\right)-180b^5cx^{15}\right)}{x^{12}\left(a+bx^3\right)^2}}{36a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

```
[Out] (-((a*(-180*b^5*c*x^15 + 30*a*b^4*x^12*(-9*c + 4*d*x^3) - 12*a^2*b^3*x^9*(5
*c - 15*d*x^3 + 6*e*x^6) - 2*a^4*b*x^3*(3*c + 5*d*x^3 + 12*e*x^6 - 27*f*x^9
) + a^5*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + a^3*b^2*x^6*(15*c + 40*d*x^3
 - 108*e*x^6 + 36*f*x^9)))/(x^12*(a + b*x^3)^2)) + 36*b*(15*b^3*c - 10*a*b^
2*d + 6*a^2*b*e - 3*a^3*f)*Log[x] + 12*b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*
e + 3*a^3*f)*Log[a + b*x^3])/(36*a^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{13}\left(a + bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3), x]
```

fricas [A] time = 0.50, size = 448, normalized size = 1.74

$$\frac{12\left(15ab^5c - 10a^2b^4d + 6a^3b^3e - 3a^4b^2f\right)x^{15} + 18\left(15a^2b^4c - 10a^3b^3d + 6a^4b^2e - 3a^5bf\right)x^{12} + 4\left(15a^3b^3c - 10a^4b^2d + 6a^5be - 3a^6f\right)x^9 - 3a^6c - \left(15a^4b^2c - 10a^5bd + 6a^6e\right)x^6 + 2\left(3a^5bc - 2a^6d\right)x^3 - 12\left(\left(15b^6c - 10ab^5d + 6a^2b^4e - 3a^3b^3f\right)x^{18} + 2\left(15ab^5c - 10a^2b^4d + 6a^3b^3e - 3a^4b^2f\right)x^{15} + \left(15a^2b^4c - 10a^3b^3d + 6a^4b^2e - 3a^5bf\right)x^{12}\right)\log\left(bx^3 + a\right) + 36\left(\left(15b^6c - 10ab^5d + 6a^2b^4e - 3a^3b^3f\right)x^{18} + 2\left(15ab^5c - 10a^2b^4d + 6a^3b^3e - 3a^4b^2f\right)x^{15} + \left(15a^2b^4c - 10a^3b^3d + 6a^4b^2e - 3a^5bf\right)x^{12}\right)\log(x)}{36\left(a^7b^2x^{18} + 2a^8bx^{15} + a^9x^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(12*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + 18*
(15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^12 + 4*(15*a^3*b^
3*c - 10*a^4*b^2*d + 6*a^5*b*e - 3*a^6*f)*x^9 - 3*a^6*c - (15*a^4*b^2*c - 1
0*a^5*b*d + 6*a^6*e)*x^6 + 2*(3*a^5*b*c - 2*a^6*d)*x^3 - 12*((15*b^6*c - 10
*a*b^5*d + 6*a^2*b^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d +
 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2
*e - 3*a^5*b*f)*x^12)*log(b*x^3 + a) + 36*((15*b^6*c - 10*a*b^5*d + 6*a^2*b
^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a
^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^
12)*log(x))/(a^7*b^2*x^18 + 2*a^8*b*x^15 + a^9*x^12)
```

giac [A] time = 0.20, size = 380, normalized size = 1.47

$$\frac{\left(15b^4c - 10ab^3d - 3a^3bf + 6a^2b^2e\right)\log\left(|x|\right)}{a^7} - \frac{\left(15b^5c - 10ab^4d - 3a^3b^2f + 6a^2b^3e\right)\log\left(\left|bx^3 + a\right|\right)}{3a^7b} + \frac{45b^6cx^6 - 30ab^5dx^6 - 9a^3b^3fx^6 + 18a^2b^4x^6e + 100ab^5cx^3 - 68a^2b^4dx^3 - 22a^4b^2fx^3 + 42a^3b^3x^3e + 56a^2b^4c - 39a^3b^3d - 14a^5bf + 25a^4b^2e}{6\left(bx^3 + a\right)^2a^7} - \frac{375b^4cx^{12} - 250ab^3dx^{12} - 75a^3bfx^{12} + 150a^2b^2x^{12}e - 120ab^3cx^9 + 72a^2b^2dx^9 + 12a^4fx^9 - 36a^3bx^9e + 36a^2b^2cx^6 - 18a^3bdx^6 + 6a^4x^6e - 12a^3bcx^3 + 4a^4dx^3 + 3a^4c}{36a^7x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (15*b^4*c - 10*a*b^3*d - 3*a^3*b*f + 6*a^2*b^2*e)*log(abs(x))/a^7 - 1/3*(15
*b^5*c - 10*a*b^4*d - 3*a^3*b^2*f + 6*a^2*b^3*e)*log(abs(b*x^3 + a))/(a^7*b
) + 1/6*(45*b^6*c*x^6 - 30*a*b^5*d*x^6 - 9*a^3*b^3*f*x^6 + 18*a^2*b^4*x^6*e
 + 100*a*b^5*c*x^3 - 68*a^2*b^4*d*x^3 - 22*a^4*b^2*f*x^3 + 42*a^3*b^3*x^3*e
 + 56*a^2*b^4*c - 39*a^3*b^3*d - 14*a^5*b*f + 25*a^4*b^2*e)/((b*x^3 + a)^2*
a^7) - 1/36*(375*b^4*c*x^12 - 250*a*b^3*d*x^12 - 75*a^3*b*f*x^12 + 150*a^2*
b^2*x^12*e - 120*a*b^3*c*x^9 + 72*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 36*a^3*b*x
^9*e + 36*a^2*b^2*c*x^6 - 18*a^3*b*d*x^6 + 6*a^4*x^6*e - 12*a^3*b*c*x^3 + 4
*a^4*d*x^3 + 3*a^4*c)/(a^7*x^12)
```

maple [A] time = 0.06, size = 349, normalized size = 1.35

$$-\frac{bf}{6\left(bx^3+a\right)^2a^2} + \frac{b^2e}{6\left(bx^3+a\right)^2a^3} - \frac{b^3d}{6\left(bx^3+a\right)^2a^4} + \frac{b^4c}{6\left(bx^3+a\right)^2a^5} - \frac{2bf}{3\left(bx^3+a\right)a^3} + \frac{b^2e}{\left(bx^3+a\right)a^4} - \frac{3bf\ln(x)}{a^4} + \frac{bf\ln\left(bx^3+a\right)}{a^4} - \frac{4b^3d}{3\left(bx^3+a\right)a^5} + \frac{6b^2e\ln(x)}{a^5} - \frac{2b^2e\ln\left(bx^3+a\right)}{a^5} + \frac{5b^4c}{3\left(bx^3+a\right)a^6} - \frac{10b^3d\ln(x)}{a^6} + \frac{10b^3d\ln\left(bx^3+a\right)}{3a^6} + \frac{15b^4c\ln(x)}{a^7} - \frac{5b^4c\ln\left(bx^3+a\right)}{a^7} - \frac{f}{3a^3x^3} + \frac{be}{a^4x^3} - \frac{2b^2d}{a^5x^3} + \frac{10b^3c}{3a^6x^3} - \frac{e}{6a^3x^6} + \frac{bd}{2a^4x^6} - \frac{b^2c}{a^5x^6} - \frac{d}{9a^3x^9} + \frac{bc}{3a^4x^9} - \frac{c}{12a^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x)
```

```
[Out] -1/9/a^3/x^9*d-1/6/a^3/x^6*e-1/3/a^3/x^3*f-10*b^3/a^6*ln(x)*d+15*b^4/a^7*ln
(x)*c+1/a^4*b*ln(b*x^3+a)*f-2/a^5*b^2*ln(b*x^3+a)*e+10/3/a^6*b^3*ln(b*x^3+a
)*d-5/a^7*b^4*ln(b*x^3+a)*c+1/3/a^4/x^9*b*c+1/2/a^4/x^6*b*d-1/a^5/x^6*b^2*c
+1/a^4/x^3*b*e-2/a^5/x^3*b^2*d+10/3/a^6/x^3*b^3*c-1/6/a^2*b/(b*x^3+a)^2*f+1
/6/a^3*b^2/(b*x^3+a)^2*e-1/6/a^4*b^3/(b*x^3+a)^2*d+1/6/a^5*b^4/(b*x^3+a)^2*
c-2/3/a^3*b/(b*x^3+a)*f+1/a^4*b^2/(b*x^3+a)*e-4/3/a^5*b^3/(b*x^3+a)*d+5/3/a
^6*b^4/(b*x^3+a)*c-3*b/a^4*ln(x)*f+6*b^2/a^5*ln(x)*e-1/12*c/a^3/x^12
```

maxima [A] time = 1.47, size = 280, normalized size = 1.09

$$\frac{12\left(15\,b^5c-10\,ab^4d+6\,a^2b^3e-3\,a^3b^2f\right)x^{15}+18\left(15\,ab^4c-10\,a^2b^3d+6\,a^3b^2e-3\,a^4bf\right)x^{12}+4\left(15\,a^2b^3c-10\,a^3b^2d+6\,a^4be-3\,a^5f\right)x^9-\left(15\,a^3b^2c-10\,a^4bd+6\,a^5e\right)x^6-3\,a^5c+2\left(3\,a^4bc-2\,a^5d\right)x^3}{36\left(a^6b^2x^{18}+2\,a^7bx^{15}+a^8x^{12}\right)}-\frac{\left(15\,b^4c-10\,ab^3d+6\,a^2b^2e-3\,a^3bf\right)\log\left(bx^3+a\right)}{3\,a^7}+\frac{\left(15\,b^4c-10\,ab^3d+6\,a^2b^2e-3\,a^3bf\right)\log\left(x^3\right)}{3\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(12*(15*b^5*c - 10*a*b^4*d + 6*a^2*b^3*e - 3*a^3*b^2*f)*x^15 + 18*(15*
a*b^4*c - 10*a^2*b^3*d + 6*a^3*b^2*e - 3*a^4*b*f)*x^12 + 4*(15*a^2*b^3*c -
10*a^3*b^2*d + 6*a^4*b*e - 3*a^5*f)*x^9 - (15*a^3*b^2*c - 10*a^4*b*d + 6*a^
5*e)*x^6 - 3*a^5*c + 2*(3*a^4*b*c - 2*a^5*d)*x^3)/(a^6*b^2*x^18 + 2*a^7*b*x
^15 + a^8*x^12) - 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*log
(b*x^3 + a)/a^7 + 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*log
(x^3)/a^7
```

mupad [B] time = 0.31, size = 265, normalized size = 1.03

$$\frac{\ln(x)\,\left(-3\,f\,a^3\,b+6\,e\,a^2\,b^2-10\,d\,a\,b^3+15\,c\,b^4\right)}{a^7}-\frac{\ln\left(b\,x^3+a\right)\,\left(-3\,f\,a^3\,b+6\,e\,a^2\,b^2-10\,d\,a\,b^3+15\,c\,b^4\right)}{3\,a^7}-\frac{\frac{c}{12\,a}-\frac{x^9\,\left(-3\,f\,a^3+6\,e\,a^2\,b-10\,d\,a\,b^2+15\,c\,b^3\right)}{9\,a^4}+\frac{x^3\,\left(2\,a\,d-3\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(6\,e\,a^2-10\,d\,a\,b+15\,c\,b^2\right)}{36\,a^3}-\frac{b\,x^{12}\,\left(-3\,f\,a^3+6\,e\,a^2\,b-10\,d\,a\,b^2+15\,c\,b^3\right)}{2\,a^5}-\frac{b^2\,x^{15}\,\left(-3\,f\,a^3+6\,e\,a^2\,b-10\,d\,a\,b^2+15\,c\,b^3\right)}{3\,a^6}}{a^2\,x^{12}+2\,a\,b\,x^{15}+b^2\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x)
```

```
[Out] (log(x)*(15*b^4*c + 6*a^2*b^2*e - 10*a*b^3*d - 3*a^3*b*f))/a^7 - (log(a + b
*x^3)*(15*b^4*c + 6*a^2*b^2*e - 10*a*b^3*d - 3*a^3*b*f))/(3*a^7) - (c/(12*a
) - (x^9*(15*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(9*a^4) + (x^3*(2*a
*d - 3*b*c))/(18*a^2) + (x^6*(15*b^2*c + 6*a^2*e - 10*a*b*d))/(36*a^3) - (b
*x^12*(15*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(2*a^5) - (b^2*x^15*(1
5*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(3*a^6))/(a^2*x^12 + b^2*x^18
+ 2*a*b*x^15)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.233 $\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=416

$$\frac{x^7\left(6a^2f-3abe+b^2d\right)}{7b^5}-\frac{a^2x\left(-37a^3f+31a^2be-25ab^2d+19b^3c\right)}{18b^7\left(a+bx^3\right)}+\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{ax\left(-15a^3\right.}{}$$

Rubi [A] time = 0.74, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1828, 1858, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{x^4\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{4b^6}-\frac{a^2x\left(31a^2be-37a^3f-25ab^2d+19b^3c\right)}{18b^7\left(a+bx^3\right)}+\frac{a^3x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(104a^2be-152a^3f-65ab^2d+35b^3c\right)}{54b^{22/3}}-\frac{ax\left(10a^2be-15a^3f-6ab^2d+3b^3c\right)}{b^7}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(104a^2be-152a^3f-65ab^2d+35b^3c\right)}{27b^{22/3}}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(104a^2be-152a^3f-65ab^2d+35b^3c\right)}{9\sqrt{3}b^{22/3}}+\frac{x^7\left(6a^2f-3abe+b^2d\right)}{7b^5}+\frac{x^{10}(be-3af)}{10b^4}+\frac{fx^{13}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] -((a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x)/b^7) + ((b^3*c - 3*a*
b^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x
^7)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) - (a^2*(19*b^3*c -
 25*a*b^2*d + 31*a^2*b*e - 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) - (a^(4/3)*(35
*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(22/3)) + (a^(4/3)*(35*b^3*c - 65*a*b^2
*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(22/3)) - (a^
(4/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^{12}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = \frac{a^3\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^7\left(a+bx^3\right)^2} - \frac{\int \frac{a^4\left(b^3c-ab^2d+a^2be-a^3f\right)-6a^3b\left(b^3c-ab^2d+a^2be-a^3f\right)x^3+}{}}{}$$

$$= \frac{a^3\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^7\left(a+bx^3\right)^2} - \frac{a^2\left(19b^3c-25ab^2d+31a^2be-37a^3f\right)x}{18b^7\left(a+bx^3\right)} + \int$$

$$= \frac{a^3\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^7\left(a+bx^3\right)^2} - \frac{a^2\left(19b^3c-25ab^2d+31a^2be-37a^3f\right)x}{18b^7\left(a+bx^3\right)} + \frac{\int}{}$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^4}{4b^6} +$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^4}{4b^6} +$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^4}{4b^6} +$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^4}{4b^6} +$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^4}{4b^6} +$$

Mathematica [A] time = 0.69, size = 411, normalized size = 0.99

$$\frac{x^7\left(6a^2f-3abe+b^2d\right)}{7b^5}+\frac{a^2x\left(37a^3f-31a^2be+25ab^2d-19b^3c\right)}{18b^7\left(a+bx^3\right)}+\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}+\frac{ax\left(15a^3f-10a^2be+6ab^2d-3b^3c\right)}{b^7}+\frac{x^4\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{4b^6}+\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(152a^3f-104a^2be+65ab^2d-35b^3c\right)}{54b^{22/3}}-\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(152a^3f-104a^2be+65ab^2d-35b^3c\right)}{27b^{22/3}}+\frac{a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(152a^3f-104a^2be+65ab^2d-35b^3c\right)}{9\sqrt{3}b^{22/3}}+\frac{x^{10}(be-3af)}{10b^4}+\frac{fx^{13}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b^
2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^7
)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) + (a^2*(-19*b^3*c +
25*a*b^2*d - 31*a^2*b*e + 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) + (a^(4/3)*(-35
*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(22/3)) - (a^(4/3)*(-35*b^3*c + 65*a*b^2*d -
104*a^2*b*e + 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/(27*b^(22/3)) + (a^(4/3)
*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2]])/(54*b^(22/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
[Out] IntegrateAlgebraic[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [A] time = 0.44, size = 667, normalized size = 1.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
[Out] 1/49140*(3780*b^6*f*x^19 + 378*(13*b^6*e - 19*a*b^5*f)*x^16 + 108*(65*b^6*d
 - 104*a*b^5*e + 152*a^2*b^4*f)*x^13 + 351*(35*b^6*c - 65*a*b^5*d + 104*a^2
*b^4*e - 152*a^3*b^3*f)*x^10 - 3510*(35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^
3*e - 152*a^4*b^2*f)*x^7 - 9555*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*
e - 152*a^5*b*f)*x^4 - 1820*sqrt(3)*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*
b*e - 152*a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*
f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)
*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*
(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*a^6*f + (35*a*b^5*c - 65*a
^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^
3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3)) - 1820*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*
a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2
*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1
/3)*log(x - (-a/b)^(1/3)) - 5460*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e
 - 152*a^6*f)*x)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

giac [A] time = 0.20, size = 500, normalized size = 1.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/27*sqrt(3)*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d - 152
*(-a*b^2)^(1/3)*a^4*f + 104*(-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/27*(35*a^2*b^3*c - 65*a^3*b^2*d - 15
2*a^5*f + 104*a^4*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/
54*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d - 152*(-a*b^2)^
(1/3)*a^4*f + 104*(-a*b^2)^(1/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3))/b^8 - 1/18*(19*a^2*b^4*c*x^4 - 25*a^3*b^3*d*x^4 - 37*a^5*b*f*x^4 +
31*a^4*b^2*x^4*e + 16*a^3*b^3*c*x - 22*a^4*b^2*d*x - 34*a^6*f*x + 28*a^5*b*
x*e)/((b*x^3 + a)^2*b^7) + 1/1820*(140*b^36*f*x^13 - 546*a*b^35*f*x^10 + 18
2*b^36*x^10*e + 260*b^36*d*x^7 + 1560*a^2*b^34*f*x^7 - 780*a*b^35*x^7*e + 4
55*b^36*c*x^4 - 1365*a*b^35*d*x^4 - 4550*a^3*b^33*f*x^4 + 2730*a^2*b^34*x^4
*e - 5460*a*b^35*c*x + 10920*a^2*b^34*d*x + 27300*a^4*b^32*f*x - 18200*a^3*
b^33*x*e)/b^39
```

maple [A] time = 0.07, size = 706, normalized size = 1.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
[Out] 35/27*a^2/b^5*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-65/27*a^3/b^6*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)-8/9*a^3/b^4/(b*x^3+a)^2*c*x+76/27*a^5/b^8*f/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
```

```
)*x+(a/b)^(2/3))+104/27*a^4/b^7*e/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+3/2/b^5*x^4
*a^2*e-3/4/b^4*x^4*a*d-3/10/b^4*x^10*a*f+6/7/b^5*x^7*a^2*f-3/7/b^4*x^7*a*e-
5/2/b^6*x^4*a^3*f+15/b^7*a^4*f*x-10/b^6*a^3*e*x+6/b^5*a^2*d*x-3/b^4*a*c*x-1
52/27*a^5/b^8*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
+104/27*a^4/b^7*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))+1/10/b^3*x^10*e+1/7/b^3*x^7*d+1/4/b^3*x^4*c+35/27*a^2/b^5*c/(a/b)^(2/3)*
ln(x+(a/b)^(1/3))-35/54*a^2/b^5*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))-65/27*a^3/b^6*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+65/54*a^3/b^6*d/(a/b)^(2
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-52/27*a^4/b^7*e/(a/b)^(2/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+37/18*a^5/b^6/(b*x^3+a)^2*x^4*f-31/18*a^4/b^5/(b*x
^3+a)^2*x^4*e+25/18*a^3/b^4/(b*x^3+a)^2*x^4*d-19/18*a^2/b^3/(b*x^3+a)^2*x^4
*c+17/9*a^6/b^7/(b*x^3+a)^2*f*x-14/9*a^5/b^6/(b*x^3+a)^2*e*x+11/9*a^4/b^5/(
b*x^3+a)^2*d*x-152/27*a^5/b^8*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/13*f*x^13/b
^3
```

maxima [A] time = 3.06, size = 424, normalized size = 1.02

$$-\frac{\left(19\,a^{2}b^{4}c-25\,a^{3}b^{3}d+31\,a^{4}b^{2}e-37\,a^{5}bf\right)x^{4}+2\left(8\,a^{3}b^{3}c-11\,a^{4}b^{2}d+14\,a^{5}be-17\,a^{6}f\right)x}{18\left(b^{9}x^{6}+2\,ab^{8}x^{3}+a^{2}b^{7}\right)}+\frac{140\,b^{4}fx^{13}+182\left(b^{4}e-3\,ab^{3}f\right)x^{10}+260\left(b^{4}d-3\,ab^{3}e+6\,a^{2}b^{2}f\right)x^{7}+455\left(b^{4}c-3\,ab^{3}d+6\,a^{2}b^{2}e-10\,a^{3}bf\right)x^{4}-1820\left(3\,ab^{3}c-6\,a^{2}b^{2}d+10\,a^{3}be-15\,a^{4}f\right)x}{1820\,b^{7}}+\frac{\sqrt{3}\left(35\,a^{2}b^{3}c-65\,a^{3}b^{2}d+104\,a^{4}be-152\,a^{5}f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^{8}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(35\,a^{2}b^{3}c-65\,a^{3}b^{2}d+104\,a^{4}be-152\,a^{5}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^{8}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(35\,a^{2}b^{3}c-65\,a^{3}b^{2}d+104\,a^{4}be-152\,a^{5}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^{8}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((19*a^2*b^4*c - 25*a^3*b^3*d + 31*a^4*b^2*e - 37*a^5*b*f)*x^4 + 2*(8
*a^3*b^3*c - 11*a^4*b^2*d + 14*a^5*b*e - 17*a^6*f)*x)/(b^9*x^6 + 2*a*b^8*x^
3 + a^2*b^7) + 1/1820*(140*b^4*f*x^13 + 182*(b^4*e - 3*a*b^3*f)*x^10 + 260*
(b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^7 + 455*(b^4*c - 3*a*b^3*d + 6*a^2*b^2*
e - 10*a^3*b*f)*x^4 - 1820*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f
)*x)/b^7 + 1/27*sqrt(3)*(35*a^2*b^3*c - 65*a^3*b^2*d + 104*a^4*b*e - 152*a^
5*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^8*(a/b)^(2/3))
- 1/54*(35*a^2*b^3*c - 65*a^3*b^2*d + 104*a^4*b*e - 152*a^5*f)*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(b^8*(a/b)^(2/3)) + 1/27*(35*a^2*b^3*c - 65*a^3*
b^2*d + 104*a^4*b*e - 152*a^5*f)*log(x + (a/b)^(1/3))/(b^8*(a/b)^(2/3))
```

mupad [B] time = 5.24, size = 575, normalized size = 1.38

$$x^{10}\left(\frac{e}{10\,b^{3}}-\frac{3\,a\,f}{10\,b^{4}}\right)+x^{4}\left(\frac{c}{4\,b^{3}}-\frac{a^{3}f}{4\,b^{6}}-\frac{3\,a^{2}\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{4\,b^{2}}+\frac{3\,a\left(\frac{3\,a^{2}f}{b^{5}}-\frac{d}{b^{3}}+\frac{3\,a\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b}\right)}{4\,b}\right)+\frac{x\left(\frac{17\,f\,a^{6}}{9}-\frac{14\,e\,a^{5}b}{9}+\frac{11\,d\,a^{4}b^{2}}{9}-\frac{8\,c\,a^{3}b^{3}}{9}\right)-x^{4}\left(-\frac{37\,f\,a^{5}b}{18}+\frac{31\,e\,a^{4}b^{2}}{18}-\frac{25\,d\,a^{3}b^{3}}{18}+\frac{19\,c\,a^{2}b^{4}}{18}\right)}{a^{2}b^{7}+2\,a\,b^{8}x^{3}+b^{9}x^{6}}-x\left(\frac{3\,a\left(\frac{c}{b^{3}}-\frac{a^{3}f}{b^{6}}-\frac{3\,a^{2}\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b^{2}}+\frac{3\,a\left(\frac{3\,a^{2}f}{b^{5}}-\frac{d}{b^{3}}+\frac{3\,a\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b}\right)}{b}\right)}{b}-\frac{3\,a^{2}\left(\frac{3\,a^{2}f}{b^{5}}-\frac{d}{b^{3}}+\frac{3\,a\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b}\right)}{b^{2}}+\frac{a^{3}\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b^{3}}\right)-x^{7}\left(\frac{3\,a^{2}f}{7\,b^{5}}-\frac{d}{7\,b^{3}}+\frac{3\,a\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{7\,b}\right)+\frac{f\,x^{13}}{13\,b^{3}}+\frac{a^{4/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(-152\,f\,a^{3}+104\,e\,a^{2}b-65\,d\,a\,b^{2}+35\,c\,b^{3}\right)}{27\,b^{22/3}}+\frac{a^{4/3}\ln\left(2\,b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-152\,f\,a^{3}+104\,e\,a^{2}b-65\,d\,a\,b^{2}+35\,c\,b^{3}\right)}{27\,b^{22/3}}-\frac{a^{4/3}\ln\left(a^{1/3}-2\,b^{1/3}x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-152\,f\,a^{3}+104\,e\,a^{2}b-65\,d\,a\,b^{2}+35\,c\,b^{3}\right)}{27\,b^{22/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^10*(e/(10*b^3) - (3*a*f)/(10*b^4)) + x^4*(c/(4*b^3) - (a^3*f)/(4*b^6) - (
3*a^2*(e/b^3 - (3*a*f)/b^4))/(4*b^2) + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(
e/b^3 - (3*a*f)/b^4))/b))/(4*b)) + (x*((17*a^6*f)/9 - (8*a^3*b^3*c)/9 + (11
*a^4*b^2*d)/9 - (14*a^5*b*e)/9) - x^4*((19*a^2*b^4*c)/18 - (25*a^3*b^3*d)/1
8 + (31*a^4*b^2*e)/18 - (37*a^5*b*f)/18))/(a^2*b^7 + b^9*x^6 + 2*a*b^8*x^3)
 - x*((3*a*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*
((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b))/b - (3*a^2*((3
*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b^2 + (a^3*(e/b^3 - (
3*a*f)/b^4))/b^3) - x^7*((3*a^2*f)/(7*b^5) - d/(7*b^3) + (3*a*(e/b^3 - (3*a
*f)/b^4))/(7*b)) + (f*x^13)/(13*b^3) + (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(3
5*b^3*c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3)) + (a^(4/3)*l
og(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(35*b
^3*c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3)) - (a^(4/3)*log(
3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(35*b^3*
c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.234 $\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=384

$$\frac{x^5\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{x^2\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{2b^6}+\frac{ax^2\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{9b^6\left(a+bx^3\right)}-\frac{a^2x^2\left(a^3(\right.}{}$$

Rubi [A] time = 1.05, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{2b^6}+\frac{ax^2\left(13a^2be-16a^3f-10ab^2d+7b^3c\right)}{9b^6\left(a+bx^3\right)}-\frac{a^2x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(77a^2be-119a^3f-44ab^2d+20b^3c\right)}{54b^{20/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(77a^2be-119a^3f-44ab^2d+20b^3c\right)}{27b^{20/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(77a^2be-119a^3f-44ab^2d+20b^3c\right)}{9\sqrt{3}b^{20/3}}+\frac{x^5\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{x^8(be-3af)}{8b^4}+\frac{fx^{11}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3)
 - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) + (a
*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) +
(a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(20/3)) + (a^(2/3)*(20*b^3*c
 - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20
/3)) - (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\int \frac{x^{10}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} - \frac{\int \frac{-2a^3b\left(b^3c-ab^2d+a^2be-a^3f\right)x+6a^2b^2\left(b^3c-ab^2d+a^2be\right.}{}}{}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} - \frac{\int \frac{x\left(-2a^3b\left(b^3c-ab^2d+a^2be-a^3f\right)+6a^2b^2\left(b^3c-ab^2d+a^2b\right.\right.}{}}{}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(7b^3c-10ab^2d+13a^2be-16a^3f\right)x^2}{9b^6\left(a+bx^3\right)} +$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(7b^3c-10ab^2d+13a^2be-16a^3f\right)x^2}{9b^6\left(a+bx^3\right)} +$$

$$= \frac{fx^{11}}{11b^3} - \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(7b^3c-10ab^2d+13a^2be-16a^3f\right)}{9b^6\left(a+bx^3\right)}$$

$$= \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} - \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(7b^3c-10ab^2d+1\right.}{9b^6\left(a+\right.}$$

$$= \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} - \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(7b^3c-10ab^2d+1\right.}{9b^6\left(a+\right.}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^2}{2b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} +$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^2}{2b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} +$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^2}{2b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} +$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^2}{2b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} +$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^2}{2b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} +$$

Mathematica [A] time = 0.55, size = 380, normalized size = 0.99

$$\frac{x^5\left(6a^2f-3abe+b^2d\right)}{5b^5} + \frac{x^2\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{2b^6} + \frac{ax^2\left(-16a^3f+13a^2be-10ab^2d+7b^3c\right)}{9b^6\left(a+bx^3\right)} + \frac{a^2x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{6b^6\left(a+bx^3\right)^2} + \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(119a^3f-77a^2be+44ab^2d-20b^3c\right)}{54b^{20/3}} - \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(119a^3f-77a^2be+44ab^2d-20b^3c\right)}{27b^{20/3}} - \frac{a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(119a^3f-77a^2be+44ab^2d-20b^3c\right)}{9\sqrt{3}b^{20/3}} + \frac{x^8(be-3af)}{8b^4} + \frac{fx^{11}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3)
```

```
 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) +
 (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3))
 - (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(20/3)) - (a^(2/3)*(-20*b^3*c
+ 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/(27*b^(20/
3)) + (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/(54*b^(20/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
[Out] IntegrateAlgebraic[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [A] time = 0.43, size = 634, normalized size = 1.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
[Out] 1/11880*(1080*b^5*f*x^17 + 135*(11*b^5*e - 17*a*b^4*f)*x^14 + 54*(44*b^5*d
- 77*a*b^4*e + 119*a^2*b^3*f)*x^11 + 297*(20*b^5*c - 44*a*b^4*d + 77*a^2*b^
3*e - 119*a^3*b^2*f)*x^8 + 1056*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e -
 119*a^4*b*f)*x^5 + 660*(20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5
*f)*x^2 - 440*sqrt(3)*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*
f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4
*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*arcta
n(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*((20*b^5*c - 44
*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d
+ 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 11
9*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2
/b^2)^(1/3)) - 440*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*f)*
x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4*c
- 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x
+ b*(-a^2/b^2)^(2/3)))/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)
```

giac [A] time = 0.20, size = 491, normalized size = 1.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/27*(20*a*b^3*c*(-a/b)^(1/3) - 44*a^2*b^2*d*(-a/b)^(1/3) - 119*a^4*f*(-a/b
)^(1/3) + 77*a^3*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^6) + 1/27*sqrt(3)*(20*(-a*b^2)^(2/3)*b^3*c - 44*(-a*b^2)^(2/3)*a*b^2*d
 - 119*(-a*b^2)^(2/3)*a^3*f + 77*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/54*(20*(-a*b^2)^(2/3)*b^3*c - 4
4*(-a*b^2)^(2/3)*a*b^2*d - 119*(-a*b^2)^(2/3)*a^3*f + 77*(-a*b^2)^(2/3)*a^2
*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^8 + 1/18*(14*a*b^4*c*x^5 -
 20*a^2*b^3*d*x^5 - 32*a^4*b*f*x^5 + 26*a^3*b^2*x^5*e + 11*a^2*b^3*c*x^2 -
17*a^3*b^2*d*x^2 - 29*a^5*f*x^2 + 23*a^4*b*x^2*e)/((b*x^3 + a)^2*b^6) + 1/4
40*(40*b^30*f*x^11 - 165*a*b^29*f*x^8 + 55*b^30*x^8*e + 88*b^30*d*x^5 + 528
```

```
*a^2*b^28*f*x^5 - 264*a*b^29*x^5*e + 220*b^30*c*x^2 - 660*a*b^29*d*x^2 - 22
00*a^3*b^27*f*x^2 + 1320*a^2*b^28*x^2*e)/b^33
```

maple [**B**] time = 0.07, size = 668, normalized size = 1.74

$$\frac{f x^{11}}{11 b^{3}}-\frac{3 a f x^{8}}{8 b^{4}}+\frac{e x^{8}}{8 b^{3}}-\frac{16 a^{4} f x^{5}}{9 \left(b x^{3}+a\right)^{2} b^{5}}+\frac{13 a^{3} e x^{5}}{9 \left(b x^{3}+a\right)^{2} b^{4}}-\frac{10 a^{2} d x^{5}}{9 \left(b x^{3}+a\right)^{2} b^{3}}+\frac{7 a c x^{5}}{9 \left(b x^{3}+a\right)^{2} b^{2}}+\frac{6 a^{2} f x^{5}}{5 b^{5}}-\frac{3 a e x^{5}}{5 b^{4}}+\frac{d x^{5}}{5 b^{3}}-\frac{29 a^{5} f x^{2}}{18 \left(b x^{3}+a\right)^{2} b^{6}}+\frac{23 a^{4} e x^{2}}{18 \left(b x^{3}+a\right)^{2} b^{5}}-\frac{17 a^{3} d x^{2}}{18 \left(b x^{3}+a\right)^{2} b^{4}}+\frac{11 a^{2} c x^{2}}{18 \left(b x^{3}+a\right)^{2} b^{3}}-\frac{5 a^{3} f x^{2}}{b^{6}}+\frac{3 a^{2} e x^{2}}{b^{5}}-\frac{3 a d x^{2}}{2 b^{4}}+\frac{c x^{2}}{2 b^{3}}+\frac{119 \sqrt{3}\, a^{4} f \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{7}}-\frac{119 a^{4} f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{7}}+\frac{119 a^{4} f \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{7}}-\frac{77 \sqrt{3}\, a^{3} e \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{6}}+\frac{77 a^{3} e \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{6}}-\frac{77 a^{3} e \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{6}}+\frac{44 \sqrt{3}\, a^{2} d \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{5}}-\frac{44 a^{2} d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{5}}+\frac{22 a^{2} d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{5}}-\frac{20 \sqrt{3}\, a c \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{4}}+\frac{20 a c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{4}}-\frac{10 a c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 119/27*a^4/b^7*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)-77/27*a^3/b^6*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))-20/27*a/b^4*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)+44/27*a^2/b^5*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))-3/8/b^4*x^8*a*f+6/5/b^5*x^5*a^2*f+3/b^5*x^2*a^2*e-3/2/b^4*x^2*a*d-5/b^6*
x^2*a^3*f-3/5/b^4*x^5*a*e+1/11*f*x^11/b^3-119/27*a^4/b^7*f/(a/b)^(1/3)*ln(x
+(a/b)^(1/3))+11/18*a^2/b^3/(b*x^3+a)^2*x^2*c-17/18*a^3/b^4/(b*x^3+a)^2*x^2
*d+22/27*a^2/b^5*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/8/b^3*x^
8*e+1/5/b^3*x^5*d+1/2/b^3*x^2*c+20/27*a/b^4*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
-10/9*a^2/b^3/(b*x^3+a)^2*x^5*d-10/27*a/b^4*c/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))-77/54*a^3/b^6*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))-44/27*a^2/b^5*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+13/9*a^3/b^4/(b*x^3+a)^2*
x^5*e-16/9*a^4/b^5/(b*x^3+a)^2*x^5*f+119/54*a^4/b^7*f/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+77/27*a^3/b^6*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7/9*
a/b^2/(b*x^3+a)^2*x^5*c-29/18*a^5/b^6/(b*x^3+a)^2*x^2*f+23/18*a^4/b^5/(b*x^
3+a)^2*x^2*e
```

maxima [**A**] time = 3.00, size = 380, normalized size = 0.99

$$\frac{2 \left(7 a b^{4} c-10 a^{2} b^{3} d+13 a^{3} b^{2} e-16 a^{4} b f\right) x^{5}+\left(11 a^{2} b^{3} c-17 a^{3} b^{2} d+23 a^{4} b e-29 a^{5} f\right) x^{2}}{18 \left(b^{8} x^{6}+2 a b^{7} x^{3}+a^{2} b^{6}\right)}-\frac{\sqrt{3}\left(20 a b^{3} c-44 a^{2} b^{2} d+77 a^{3} b e-119 a^{4} f\right) \arctan\left(\frac{\sqrt{3}\left(2 x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 b^{7}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{40 b^{3} f x^{11}+55\left(b^{3} e-3 a b^{2} f\right) x^{8}+88\left(b^{3} d-3 a b^{2} e+6 a^{2} b f\right) x^{5}+220\left(b^{3} c-3 a b^{2} d+6 a^{2} b e-10 a^{3} f\right) x^{2}}{440 b^{6}}-\frac{\left(20 a b^{3} c-44 a^{2} b^{2} d+77 a^{3} b e-119 a^{4} f\right) \log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 b^{7}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(20 a b^{3} c-44 a^{2} b^{2} d+77 a^{3} b e-119 a^{4} f\right) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 b^{7}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^5 + (11*a^
2*b^3*c - 17*a^3*b^2*d + 23*a^4*b*e - 29*a^5*f)*x^2)/(b^8*x^6 + 2*a*b^7*x^3
 + a^2*b^6) - 1/27*sqrt(3)*(20*a*b^3*c - 44*a^2*b^2*d + 77*a^3*b*e - 119*a^
4*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(1/3))
+ 1/440*(40*b^3*f*x^11 + 55*(b^3*e - 3*a*b^2*f)*x^8 + 88*(b^3*d - 3*a*b^2*e
 + 6*a^2*b*f)*x^5 + 220*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/b^6
 - 1/54*(20*a*b^3*c - 44*a^2*b^2*d + 77*a^3*b*e - 119*a^4*f)*log(x^2 - x*(a
/b)^(1/3) + (a/b)^(2/3))/(b^7*(a/b)^(1/3)) + 1/27*(20*a*b^3*c - 44*a^2*b^2*
d + 77*a^3*b*e - 119*a^4*f)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(1/3))
```

mupad [**B**] time = 5.34, size = 425, normalized size = 1.11

$$x^{8}\left(\frac{e}{8 b^{3}}-\frac{3 a f}{8 b^{4}}\right)+x^{2}\left(\frac{c}{2 b^{3}}-\frac{a^{3} f}{2 b^{6}}-\frac{3 a^{2}\left(\frac{e}{b^{3}}-\frac{3 a f}{b^{4}}\right)}{2 b^{2}}+\frac{3 a\left(\frac{3 a^{2} f}{b^{5}}-\frac{d}{b^{3}}+\frac{3 a\left(\frac{e}{b^{3}}-\frac{3 a f}{b^{4}}\right)}{b}\right)}{2 b}\right)-\frac{\left(\frac{16 f a^{4} b}{9}-\frac{13 e a^{3} b^{2}}{9}+\frac{10 d a^{2} b^{3}}{9}-\frac{7 c a b^{4}}{9}\right) x^{5}+\left(\frac{29 f a^{5}}{18}-\frac{23 e a^{4} b}{18}+\frac{17 d a^{3} b^{2}}{18}-\frac{11 c a^{2} b^{3}}{18}\right) x^{2}}{a^{2} b^{6}+2 a b^{7} x^{3}+b^{8} x^{6}}-x^{5}\left(\frac{3 a^{2} f}{5 b^{5}}-\frac{d}{5 b^{3}}+\frac{3 a\left(\frac{e}{b^{3}}-\frac{3 a f}{b^{4}}\right)}{5 b}\right)+\frac{f x^{11}}{11 b^{3}}+\frac{a^{2/3} \ln\left(b^{1/3} x+a^{1/3}\right)\left(-119 f a^{3}+77 e a^{2} b-44 d a b^{2}+20 c b^{3}\right)}{27 b^{20/3}}-\frac{a^{2/3} \ln\left(2 b^{1/3} x-a^{1/3}+\sqrt{3}\, a^{1/3} 1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(-119 f a^{3}+77 e a^{2} b-44 d a b^{2}+20 c b^{3}\right)}{27 b^{20/3}}+\frac{a^{2/3} \ln\left(a^{1/3}-2 b^{1/3} x+\sqrt{3}\, a^{1/3} 1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(-119 f a^{3}+77 e a^{2} b-44 d a b^{2}+20 c b^{3}\right)}{27 b^{20/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^8*(e/(8*b^3) - (3*a*f)/(8*b^4)) + x^2*(c/(2*b^3) - (a^3*f)/(2*b^6) - (3*a
^2*(e/b^3 - (3*a*f)/b^4))/(2*b^2) + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b
^3 - (3*a*f)/b^4))/b))/(2*b)) - (x^2*((29*a^5*f)/18 - (11*a^2*b^3*c)/18 + (
17*a^3*b^2*d)/18 - (23*a^4*b*e)/18) + x^5*((10*a^2*b^3*d)/9 - (13*a^3*b^2*e
)/9 - (7*a*b^4*c)/9 + (16*a^4*b*f)/9))/(a^2*b^6 + b^8*x^6 + 2*a*b^7*x^3) -
x^5*((3*a^2*f)/(5*b^5) - d/(5*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(5*b)) + (
f*x^11)/(11*b^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(20*b^3*c - 119*a^3*f
```

```
- 44*a*b^2*d + 77*a^2*b*e))/(27*b^(20/3)) - (a^(2/3)*log(3^(1/2)*a^(1/3)*1i
 + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(20*b^3*c - 119*a^3*f - 44
*a*b^2*d + 77*a^2*b*e))/(27*b^(20/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2
*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(20*b^3*c - 119*a^3*f - 44*a*b
^2*d + 77*a^2*b*e))/(27*b^(20/3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.235 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=375

$$\frac{x^4\left(6a^2f-3abe+b^2d\right)}{4b^5}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-104a^3f+65a^2be-35ab^2d+14b^3c\right)}{27b^{19/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-10\right.}{9}$$

Rubi [A] time = 0.61, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1828, 1858, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{ax\left(25a^2be-31a^3f-19ab^2d+13b^3c\right)}{18b^6\left(a+bx^3\right)}-\frac{a^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(65a^2be-104a^3f-35ab^2d+14b^3c\right)}{54b^{19/3}}+\frac{x\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{b^6}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(65a^2be-104a^3f-35ab^2d+14b^3c\right)}{27b^{19/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(65a^2be-104a^3f-35ab^2d+14b^3c\right)}{9\sqrt{3}b^{19/3}}+\frac{x^4\left(6a^2f-3abe+b^2d\right)}{4b^5}+\frac{x^7(be-3af)}{7b^4}+\frac{fx^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*
a^2*f)*x^4)/(4*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^10)/(10*b^3) - (a^
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^6*(a + b*x^3)^2) + (a*(13*b^3
*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x)/(18*b^6*(a + b*x^3)) + (a^(1/3)
*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(19/3)) - (a^(1/3)*(14*b^3*c - 35*a*
b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(19/3)) + (
a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^6\left(a+bx^3\right)^2} - \frac{\int \frac{-a^3\left(b^3c-ab^2d+a^2be-a^3f\right)+6a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{}}{}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(13b^3c-19ab^2d+25a^2be-31a^3f\right)x}{18b^6\left(a+bx^3\right)} + \frac{\int}{}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(13b^3c-19ab^2d+25a^2be-31a^3f\right)x}{18b^6\left(a+bx^3\right)} + \frac{\int}{}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{f}{1}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{f}{1}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{f}{1}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{f}{1}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{f}{1}$$

Mathematica [A] time = 0.47, size = 362, normalized size = 0.97

$$\frac{945b^{4/3}x^4\left(6a^2f-3abe+b^2d\right)+\frac{210a\sqrt[3]{b}x\left(-31a^3f+25a^2be-19ab^2d+13b^3c\right)}{a+bx^3}+\frac{630a^2\sqrt[3]{b}x\left(a^3f-a^2be+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}+3780\sqrt[3]{b}x\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)+140\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(104a^3f-65a^2be+35ab^2d-14b^3c\right)-140\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(104a^3f-65a^2be+35ab^2d-14b^3c\right)-70\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(104a^3f-65a^2be+35ab^2d-14b^3c\right)+540b^{7/3}x^7(be-3af)+378b^{10/3}fx^{10}}{3780b^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (3780*b^(1/3)*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x + 945*b^(4/3)*(b
^2*d - 3*a*b*e + 6*a^2*f)*x^4 + 540*b^(7/3)*(b*e - 3*a*f)*x^7 + 378*b^(10/3
)*f*x^10 + (630*a^2*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x))/(a +
b*x^3)^2 + (210*a*b^(1/3)*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x
)/(a + b*x^3) - 140*Sqrt[3]*a^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e +
104*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-14*b
^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a
^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*b^(19/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
[Out] IntegrateAlgebraic[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [A] time = 0.44, size = 602, normalized size = 1.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
[Out] 1/3780*(378*b^5*f*x^16 + 108*(5*b^5*e - 8*a*b^4*f)*x^13 + 27*(35*b^5*d - 65
*a*b^4*e + 104*a^2*b^3*f)*x^10 + 270*(14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e
- 104*a^3*b^2*f)*x^7 + 735*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*
a^4*b*f)*x^4 - 140*sqrt(3)*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3
*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f + 2*(14*
a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)*arcta
n(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*((14*b^5*c - 35*a*b^4
*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a
^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*
b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*((14*b^5
*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3
*b^2*d + 65*a^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2
*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(14*a^2*b^3*c
 - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f)*x)/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b
^6)
```

giac [A] time = 0.20, size = 443, normalized size = 1.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] -1/27*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*d - 104*(-
a*b^2)^(1/3)*a^3*f + 65*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/27*(14*a*b^3*c - 35*a^2*b^2*d - 104*a^4*
f + 65*a^3*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) - 1/54*(14*
(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*d - 104*(-a*b^2)^(1/3)*a^3*f
 + 65*(-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7
+ 1/18*(13*a*b^4*c*x^4 - 19*a^2*b^3*d*x^4 - 31*a^4*b*f*x^4 + 25*a^3*b^2*x^4
*e + 10*a^2*b^3*c*x - 16*a^3*b^2*d*x - 28*a^5*f*x + 22*a^4*b*x*e)/((b*x^3 +
 a)^2*b^6) + 1/140*(14*b^27*f*x^10 - 60*a*b^26*f*x^7 + 20*b^27*x^7*e + 35*b
^27*d*x^4 + 210*a^2*b^25*f*x^4 - 105*a*b^26*x^4*e + 140*b^27*c*x - 420*a*b^
26*d*x - 1400*a^3*b^24*f*x + 840*a^2*b^25*x*e)/b^30
```

maple [A] time = 0.06, size = 651, normalized size = 1.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
[Out] -14/27*a/b^4*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+
104/27*a^4/b^7*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)-65/27*a^3/b^6*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))+35/27*a^2/b^5*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))-14/27*a/b^4*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+7/27*a/b^4*c/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-31/18*a^4/b^5/(b*x^3+a)^2*x^4*f-10/b^6*a^3*
```

```
f*x+6/b^5*a^2*e*x-3/b^4*a*d*x-3/4/b^4*x^4*a*e-3/7/b^4*x^7*a*f+3/2/b^5*x^4*a
^2*f+1/10*f*x^10/b^3+104/27*a^4/b^7*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-52/27*a
^4/b^7*f/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-65/27*a^3/b^6*e/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))+65/54*a^3/b^6*e/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))+1/7/b^3*x^7*e+1/4/b^3*x^4*d+1/b^3*c*x+35/27*a^2/b^5*d/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))-35/54*a^2/b^5*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))-19/18*a^2/b^3/(b*x^3+a)^2*x^4*d+13/18*a/b^2/(b*x^3+a)^2*x^4*c+11/9
*a^4/b^5/(b*x^3+a)^2*e*x-8/9*a^3/b^4/(b*x^3+a)^2*d*x+5/9*a^2/b^3/(b*x^3+a)^
2*c*x+25/18*a^3/b^4/(b*x^3+a)^2*x^4*e-14/9*a^5/b^6/(b*x^3+a)^2*f*x
```

maxima [A] time = 3.03, size = 376, normalized size = 1.00

$$\frac{\left(13\,ab^4c - 19\,a^2b^3d + 25\,a^3b^2e - 31\,a^4bf\right)x^4 + 2\left(5\,a^2b^3c - 8\,a^3b^2d + 11\,a^4be - 14\,a^5f\right)x}{18\left(b^8x^6 + 2\,ab^7x^3 + a^2b^6\right)} + \frac{14\,b^3fx^{10} + 20\left(b^3e - 3\,ab^2f\right)x^7 + 35\left(b^3d - 3\,ab^2e + 6\,a^2bf\right)x^4 + 140\left(b^3c - 3\,ab^2d + 6\,a^2be - 10\,a^3f\right)x}{140\,b^6} - \frac{\sqrt{3}\left(14\,ab^3c - 35\,a^2b^2d + 65\,a^3be - 104\,a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(14\,ab^3c - 35\,a^2b^2d + 65\,a^3be - 104\,a^4f\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(14\,ab^3c - 35\,a^2b^2d + 65\,a^3be - 104\,a^4f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((13*a*b^4*c - 19*a^2*b^3*d + 25*a^3*b^2*e - 31*a^4*b*f)*x^4 + 2*(5*a^
2*b^3*c - 8*a^3*b^2*d + 11*a^4*b*e - 14*a^5*f)*x)/(b^8*x^6 + 2*a*b^7*x^3 +
a^2*b^6) + 1/140*(14*b^3*f*x^10 + 20*(b^3*e - 3*a*b^2*f)*x^7 + 35*(b^3*d -
3*a*b^2*e + 6*a^2*b*f)*x^4 + 140*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)
*x)/b^6 - 1/27*sqrt(3)*(14*a*b^3*c - 35*a^2*b^2*d + 65*a^3*b*e - 104*a^4*f)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(2/3)) + 1/
54*(14*a*b^3*c - 35*a^2*b^2*d + 65*a^3*b*e - 104*a^4*f)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(b^7*(a/b)^(2/3)) - 1/27*(14*a*b^3*c - 35*a^2*b^2*d + 6
5*a^3*b*e - 104*a^4*f)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

mupad [B] time = 5.35, size = 420, normalized size = 1.12

$$x^7\left(\frac{e}{7\,b^3} - \frac{3\,a\,f}{7\,b^4}\right) + x\left(\frac{c}{b^3} - \frac{a^3\,f}{b^6} - \frac{3\,a^2\left(\frac{e}{b^3} - \frac{3\,a\,f}{b^4}\right)}{b^2} + \frac{3\,a\left(\frac{3\,a^2\,f}{b^5} - \frac{d}{b^3} + \frac{3\,a\left(\frac{e}{b^3} - \frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right) - x^4\left(\frac{3\,a^2\,f}{4\,b^5} - \frac{d}{4\,b^3} + \frac{3\,a\left(\frac{e}{b^3} - \frac{3\,a\,f}{b^4}\right)}{4\,b}\right) - \frac{\left(\frac{31\,f\,a^4\,b}{18} - \frac{25\,e\,a^3\,b^2}{18} + \frac{19\,d\,a^2\,b^3}{18} - \frac{13\,c\,a\,b^4}{18}\right)x^4 + \left(\frac{14\,f\,a^5}{9} - \frac{11\,e\,a^4\,b}{9} + \frac{8\,d\,a^3\,b^2}{9} - \frac{5\,c\,a^2\,b^3}{9}\right)x}{a^2\,b^6 + 2\,a\,b^7\,x^3 + b^8\,x^6} + \frac{f\,x^{10}}{10\,b^3} - \frac{a^{1/3}\,\ln\left(b^{1/3}\,x + a^{1/3}\right)\left(-104\,f\,a^3 + 65\,e\,a^2\,b - 35\,d\,a\,b^2 + 14\,c\,b^3\right)}{27\,b^{19/3}} - \frac{a^{1/3}\,\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-104\,f\,a^3 + 65\,e\,a^2\,b - 35\,d\,a\,b^2 + 14\,c\,b^3\right)}{27\,b^{19/3}} + \frac{a^{1/3}\,\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-104\,f\,a^3 + 65\,e\,a^2\,b - 35\,d\,a\,b^2 + 14\,c\,b^3\right)}{27\,b^{19/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^7*(e/(7*b^3) - (3*a*f)/(7*b^4)) + x*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3
- (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b
^4))/b))/b) - x^4*((3*a^2*f)/(4*b^5) - d/(4*b^3) + (3*a*(e/b^3 - (3*a*f)/b^
4))/(4*b)) - (x*((14*a^5*f)/9 - (5*a^2*b^3*c)/9 + (8*a^3*b^2*d)/9 - (11*a^4
*b*e)/9) + x^4*((19*a^2*b^3*d)/18 - (25*a^3*b^2*e)/18 - (13*a*b^4*c)/18 + (
31*a^4*b*f)/18))/(a^2*b^6 + b^8*x^6 + 2*a*b^7*x^3) + (f*x^10)/(10*b^3) - (a
^(1/3)*log(b^(1/3)*x + a^(1/3))*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 65*a^2
*b*e))/(27*b^(19/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 65*a^2*b*e
))/(27*b^(19/3)) + (a^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))
*((3^(1/2)*1i)/2 + 1/2)*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 65*a^2*b*e))/(
27*b^(19/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.236 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=345

$$\frac{x^2\left(6a^2f-3abe+b^2d\right)}{2b^5}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-77a^3f+44a^2be-20ab^2d+5b^3c\right)}{27\sqrt[3]{a}\,b^{17/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-77a^3f+\right.}{9\sqrt{3}\sqrt[3]{a}}$$

Rubi [A] time = 0.76, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(10a^2be-13a^3f-7ab^2d+4b^3c\right)}{9b^5\left(a+bx^3\right)}+\frac{ax^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(44a^2be-77a^3f-20ab^2d+5b^3c\right)}{54\sqrt[3]{a}\,b^{17/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(44a^2be-77a^3f-20ab^2d+5b^3c\right)}{27\sqrt[3]{a}\,b^{17/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(44a^2be-77a^3f-20ab^2d+5b^3c\right)}{9\sqrt{3}\sqrt[3]{a}\,b^{17/3}}+\frac{x^2\left(6a^2f-3abe+b^2d\right)}{2b^5}+\frac{x^5(be-3af)}{5b^4}+\frac{fx^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^2)/(2*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (
f*x^8)/(8*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^5*(a + b*
x^3)^2) - ((4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*x^2)/(9*b^5*(a + b
*x^3)) - ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(17/3)) - ((5*b^3*c -
 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*
b^(17/3)) + ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(17/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\int \frac{2a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)x-6ab^2\left(b^3c-ab^2d+a^2be-a^3\right.}{\left(a+\right.}}{6}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\int \frac{x\left(2a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)-6ab^2\left(b^3c-ab^2d+a^2be-a\right.\right.}{\left(a\right.}}{6}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3c-7ab^2d+10a^2be-13a^3f\right)x^2}{9b^5\left(a+bx^3\right)} + \frac{\int \frac{2a}{}}{}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3c-7ab^2d+10a^2be-13a^3f\right)x^2}{9b^5\left(a+bx^3\right)} + \frac{\int \frac{x(}{}}{}$$

$$= \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3c-7ab^2d+10a^2be-13a^3f\right)x^2}{9b^5\left(a+bx^3\right)}$$

$$= \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3c-7ab^2d+10a^2be-13a^3f\right)x^2}{9b^5\left(a+bx^3\right)}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right.}{6b^5\left(a+bx^3\right)^2}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right.}{6b^5\left(a+bx^3\right)^2}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right.}{6b^5\left(a+bx^3\right)^2}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right.}{6b^5\left(a+bx^3\right)^2}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right.}{6b^5\left(a+bx^3\right)^2}$$

Mathematica [A] time = 0.37, size = 329, normalized size = 0.95

$$\frac{540b^{2/3}x^2\left(6a^2f-3abe+b^2d\right)+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(77a^3f-44a^2be+20ab^2d-5b^3c\right)}{\sqrt[3]{a}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(77a^3f-44a^2be+20ab^2d-5b^3c\right)}{\sqrt[3]{a}}-\frac{120b^{2/3}x^2\left(-13a^3f+10a^2be-7ab^2d+4b^3c\right)}{a+bx^3}+\frac{180ab^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2}+\frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-77a^3f+44a^2be-20ab^2d+5b^3c\right)}{\sqrt[3]{a}}+216b^{5/3}x^5(be-3af)+135b^{8/3}fx^8}{1080b^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (540*b^(2/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2 + 216*b^(5/3)*(b*e - 3*a*f)*x^
5 + 135*b^(8/3)*f*x^8 + (180*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^2)/(a + b*x^3)^2 - (120*b^(2/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^
3*f)*x^2)/(a + b*x^3) + (40*Sqrt[3]*(-5*b^3*c + 20*a*b^2*d - 44*a^2*b*e + 7
7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-5*b^3
```

```
*c + 20*a*b^2*d - 44*a^2*b*e + 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3)
+ (20*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(1080*b^(17/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [B] time = 0.45, size = 1278, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1080*(135*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b^
6*d - 44*a^2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 44*
a^3*b^4*e - 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3
*e - 77*a^5*b^2*f)*x^2 - 60*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*a^5*
b^2*e - 77*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*
f)*x^6 + 2*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^3)*
sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2
)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/
(b*x^3 + a)) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6
 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a
^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b
^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e -
 77*a^3*b^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2
*(5*a*b^4*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*
log(b*x + (a*b^2)^(1/3)))/(a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b^7), 1/1080*(13
5*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b^6*d - 44*a^
2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e -
 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5
*b^2*f)*x^2 - 120*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*a^5*b^2*e - 77
*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*f)*x^6 + 2
*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^3)*sqrt((a*b^
2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)
/b) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6 + 5*a^2*
b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a^2*b^3*d
+ 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)
*b*x + (a*b^2)^(2/3)) - 40*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b
^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4
*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b*x +
 (a*b^2)^(1/3)))/(a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b^7)]
```

giac [A] time = 0.20, size = 391, normalized size = 1.13

$$\frac{\sqrt{3}\left(5\,b^3c-20\,ab^2d-77\,a^3f+44\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}b^5}-\frac{\left(5\,b^3c-20\,ab^2d-77\,a^3f+44\,a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}b^5}-\frac{\left(5\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-77\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^5}-\frac{8\,b^4cx^5-14\,ab^3dx^5-26\,a^3bfx^5+20\,a^2b^2x^5e+5\,ab^3cx^2-11\,a^2b^2dx^2-23\,a^4fx^2+17\,a^3bx^2e}{18\left(bx^3+a\right)^2b^5}+\frac{5\,b^{21}fx^8-24\,ab^{20}fx^5+8\,b^{21}x^5e+20\,b^{21}dx^2+120\,a^2b^{19}fx^2-60\,ab^{20}x^2e}{40\,b^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(5*b^3*c - 20*a*b^2*d - 77*a^3*f + 44*a^2*b*e)*arctan(1/3*sqrt
(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^5) - 1/54*(5*b^3*c
```

```
 - 20*a*b^2*d - 77*a^3*f + 44*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/((-a*b^2)^(1/3)*b^5) - 1/27*(5*b^3*c*(-a/b)^(1/3) - 20*a*b^2*d*(-a/b)^
(1/3) - 77*a^3*f*(-a/b)^(1/3) + 44*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(8*b^4*c*x^5 - 14*a*b^3*d*x^5 - 26*a^3
*b*f*x^5 + 20*a^2*b^2*x^5*e + 5*a*b^3*c*x^2 - 11*a^2*b^2*d*x^2 - 23*a^4*f*x
^2 + 17*a^3*b*x^2*e)/((b*x^3 + a)^2*b^5) + 1/40*(5*b^21*f*x^8 - 24*a*b^20*f
*x^5 + 8*b^21*x^5*e + 20*b^21*d*x^2 + 120*a^2*b^19*f*x^2 - 60*a*b^20*x^2*e)
/b^24
```

maple [B] time = 0.06, size = 611, normalized size = 1.77

$$\frac{fx^8}{8b^3}+\frac{13a^3fx^5}{9(bx^3+a)^2b^4}-\frac{10a^2ex^5}{9(bx^3+a)^2b^3}+\frac{7adx^5}{9(bx^3+a)^2b^2}-\frac{4cx^5}{9(bx^3+a)^2b}-\frac{3afx^5}{5b^4}+\frac{ex^5}{5b^3}+\frac{23a^4fx^2}{18(bx^3+a)^2b^5}-\frac{17a^3ex^2}{18(bx^3+a)^2b^4}+\frac{11a^2dx^2}{18(bx^3+a)^2b^3}-\frac{5acx^2}{18(bx^3+a)^2b^2}+\frac{3a^2fx^2}{b^5}-\frac{3aex^2}{2b^4}+\frac{dx^2}{2b^3}-\frac{77\sqrt{3}\,a^3f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{(\frac{a}{b})^{\frac{1}{3}}}-1\right)}{3}\right)}{27(\frac{a}{b})^{\frac{1}{3}}b^6}+\frac{77a^3f\ln\left(x+(\frac{a}{b})^{\frac{1}{3}}\right)}{27(\frac{a}{b})^{\frac{1}{3}}b^6}-\frac{77a^3f\ln\left(x^2-(\frac{a}{b})^{\frac{1}{3}}x+(\frac{a}{b})^{\frac{2}{3}}\right)}{54(\frac{a}{b})^{\frac{1}{3}}b^6}+\frac{44\sqrt{3}\,a^2e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{(\frac{a}{b})^{\frac{1}{3}}}-1\right)}{3}\right)}{27(\frac{a}{b})^{\frac{1}{3}}b^5}-\frac{44a^2e\ln\left(x+(\frac{a}{b})^{\frac{1}{3}}\right)}{27(\frac{a}{b})^{\frac{1}{3}}b^5}+\frac{22a^2e\ln\left(x^2-(\frac{a}{b})^{\frac{1}{3}}x+(\frac{a}{b})^{\frac{2}{3}}\right)}{27(\frac{a}{b})^{\frac{1}{3}}b^5}-\frac{20\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{(\frac{a}{b})^{\frac{1}{3}}}-1\right)}{3}\right)}{27(\frac{a}{b})^{\frac{1}{3}}b^4}+\frac{20ad\ln\left(x+(\frac{a}{b})^{\frac{1}{3}}\right)}{27(\frac{a}{b})^{\frac{1}{3}}b^4}-\frac{10ad\ln\left(x^2-(\frac{a}{b})^{\frac{1}{3}}x+(\frac{a}{b})^{\frac{2}{3}}\right)}{27(\frac{a}{b})^{\frac{1}{3}}b^4}+\frac{5\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{(\frac{a}{b})^{\frac{1}{3}}}-1\right)}{3}\right)}{27(\frac{a}{b})^{\frac{1}{3}}b^3}-\frac{5c\ln\left(x+(\frac{a}{b})^{\frac{1}{3}}\right)}{27(\frac{a}{b})^{\frac{1}{3}}b^3}+\frac{5c\ln\left(x^2-(\frac{a}{b})^{\frac{1}{3}}x+(\frac{a}{b})^{\frac{2}{3}}\right)}{54(\frac{a}{b})^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 44/27/b^5*a^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-20/27/b^4*a*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-
77/27/b^6*a^3*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-3/2/b^4*x^2*a*e-3/5/b^4*x^5*a*f+3/b^5*x^2*a^2*f-4/9/b/(b*x^3+a)^2*x^5*c-5/
27/b^3*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+5/54/b^3*c/(a/b)^(1/3)*ln(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3))+1/5/b^3*x^5*e+1/2/b^3*x^2*d+1/8*f*x^8/b^3+13/9/b^4/(b*
x^3+a)^2*x^5*a^3*f-10/9/b^3/(b*x^3+a)^2*x^5*a^2*e+7/9/b^2/(b*x^3+a)^2*x^5*a
*d+23/18/b^5/(b*x^3+a)^2*x^2*a^4*f-17/18/b^4/(b*x^3+a)^2*x^2*a^3*e-77/54/b^
6*a^3*f/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-44/27/b^5*a^2*e/(a/b)
^(1/3)*ln(x+(a/b)^(1/3))+22/27/b^5*a^2*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+20/27/b^4*a*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-10/27/b^4*a*d/(a/b)
^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+5/27/b^3*c*3^(1/2)/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+11/18/b^3/(b*x^3+a)^2*x^2*a^2*d-5/18/b
^2/(b*x^3+a)^2*x^2*a*c+77/27/b^6*a^3*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
```

maxima [A] time = 3.08, size = 330, normalized size = 0.96

$$-\frac{2\left(4b^4c-7ab^3d+10a^2b^2e-13a^3bf\right)x^5+\left(5ab^3c-11a^2b^2d+17a^3be-23a^4f\right)x^2}{18\left(b^7x^6+2ab^6x^3+a^2b^5\right)}+\frac{\sqrt{3}\left(5b^3c-20ab^2d+44a^2be-77a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-(\frac{a}{b})^{\frac{1}{3}}\right)}{3(\frac{a}{b})^{\frac{1}{3}}}\right)}{27b^6(\frac{a}{b})^{\frac{1}{3}}}+\frac{5b^2fx^8+8\left(b^2e-3abf\right)x^5+20\left(b^2d-3abe+6a^2f\right)x^2}{40b^5}+\frac{\left(5b^3c-20ab^2d+44a^2be-77a^3f\right)\log\left(x^2-x(\frac{a}{b})^{\frac{1}{3}}+(\frac{a}{b})^{\frac{2}{3}}\right)}{54b^6(\frac{a}{b})^{\frac{1}{3}}}-\frac{\left(5b^3c-20ab^2d+44a^2be-77a^3f\right)\log\left(x+(\frac{a}{b})^{\frac{1}{3}}\right)}{27b^6(\frac{a}{b})^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^5 + (5*a*b^3*c
 - 11*a^2*b^2*d + 17*a^3*b*e - 23*a^4*f)*x^2)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*
b^5) + 1/27*sqrt(3)*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/40*(5*b^2
*f*x^8 + 8*(b^2*e - 3*a*b*f)*x^5 + 20*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2)/b^5
+ 1/54*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(b^6*(a/b)^(1/3)) - 1/27*(5*b^3*c - 20*a*b^2*d + 44*a^2*b
*e - 77*a^3*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

mupad [B] time = 5.53, size = 338, normalized size = 0.98

$$x^5\left(\frac{e}{5b^3}-\frac{3af}{5b^4}\right)+\frac{x^2\left(\frac{23fa^4}{18}-\frac{17ea^3b}{18}+\frac{11da^2b^2}{18}-\frac{5cab^3}{18}\right)-x^5\left(-\frac{13fa^3b}{9}+\frac{10ea^2b^2}{9}-\frac{7dab^3}{9}+\frac{4cb^4}{9}\right)}{a^2b^5+2ab^6x^3+b^7x^6}-x^2\left(\frac{3a^2f}{2b^5}-\frac{d}{2b^3}+\frac{3a\left(\frac{e}{b^3}-\frac{3af}{b^4}\right)}{2b}\right)+\frac{fx^8}{8b^3}-\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(-77fa^3+44ea^2b-20dab^2+5cb^3\right)}{27a^{1/3}b^{17/3}}+\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-77fa^3+44ea^2b-20dab^2+5cb^3\right)}{27a^{1/3}b^{17/3}}-\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-77fa^3+44ea^2b-20dab^2+5cb^3\right)}{27a^{1/3}b^{17/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^5*(e/(5*b^3) - (3*a*f)/(5*b^4)) + (x^2*((23*a^4*f)/18 + (11*a^2*b^2*d)/18
 - (5*a*b^3*c)/18 - (17*a^3*b*e)/18) - x^5*((4*b^4*c)/9 + (10*a^2*b^2*e)/9
- (7*a*b^3*d)/9 - (13*a^3*b*f)/9))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) - x^2*
((3*a^2*f)/(2*b^5) - d/(2*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(2*b)) + (f*x^
```

```
8)/(8*b^3) - (log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 77*a^3*f - 20*a*b^2*d + 4
4*a^2*b*e))/(27*a^(1/3)*b^(17/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
 a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 77*a^3*f - 20*a*b^2*d + 44*a^2*
b*e))/(27*a^(1/3)*b^(17/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/
3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 77*a^3*f - 20*a*b^2*d + 44*a^2*b*e))/
(27*a^(1/3)*b^(17/3))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.237 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=336

$$\frac{x\left(6a^2f-3abe+b^2d\right)}{b^5}-\frac{x\left(-25a^3f+19a^2be-13ab^2d+7b^3c\right)}{18b^5\left(a+bx^3\right)}+\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\right.}{}$$

Rubi [A] time = 0.51, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1828, 1858, 1887, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(19a^2be-25a^3f-13ab^2d+7b^3c\right)}{18b^5\left(a+bx^3\right)}+\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^2be-65a^3f-14ab^2d+2b^3c\right)}{54a^{2/3}b^{16/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(35a^2be-65a^3f-14ab^2d+2b^3c\right)}{27a^{2/3}b^{16/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(35a^2be-65a^3f-14ab^2d+2b^3c\right)}{9\sqrt{3}a^{2/3}b^{16/3}}+\frac{x\left(6a^2f-3abe+b^2d\right)}{b^5}+\frac{x^4(be-3af)}{4b^4}+\frac{fx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x)/b^5 + ((b*e - 3*a*f)*x^4)/(4*b^4) + (f*x^7)
/(7*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^5*(a + b*x^3)^2)
- ((7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)*x)/(18*b^5*(a + b*x^3)) -
 ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3
)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(16/3)) + ((2*b^3*c - 14*a*b^
2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(16/3)
) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(2/3)*b^(16/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx = \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\int \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)-6ab\left(b^3c-ab^2d+a^2be-a^3f\right)x^3-}{\left(a+bx^3\right.}}{6ab^{5}}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\left(7b^3c-13ab^2d+19a^2be-25a^3f\right)x}{18b^5\left(a+bx^3\right)} + \int \frac{2a^{2}}{}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\left(7b^3c-13ab^2d+19a^2be-25a^3f\right)x}{18b^5\left(a+bx^3\right)} + \frac{\int \left(1\right.}{}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{6b^5\left(a+bx^3\right)^2}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{6b^5\left(a+bx^3\right)^2}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{6b^5\left(a+bx^3\right)^2}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{6b^5\left(a+bx^3\right)^2}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{6b^5\left(a+bx^3\right)^2}$$

Mathematica [A] time = 0.37, size = 323, normalized size = 0.96

$$\frac{756\sqrt[3]{b}\,x\left(6a^2f-3abe+b^2d\right)-\frac{42\sqrt[3]{b}\,x\left(-25a^3f+19a^2be-13ab^2d+7b^3c\right)}{a+bx^3}+\frac{126a\sqrt[3]{b}\,x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(-65a^3f+35a^2be-14ab^2d+2b^3c\right)}{a^{2/3}}+\frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(65a^3f-35a^2be+14ab^2d-2b^3c\right)}{a^{2/3}}+\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(65a^3f-35a^2be+14ab^2d-2b^3c\right)}{a^{2/3}}+189b^{4/3}x^4(be-3af)+108b^{7/3}fx^7}{756b^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (756*b^(1/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x + 189*b^(4/3)*(b*e - 3*a*f)*x^4
+ 108*b^(7/3)*f*x^7 + (126*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)
/(a + b*x^3)^2 - (42*b^(1/3)*(7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)
*x)/(a + b*x^3) + (28*Sqrt[3]*(-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*
f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(2*b^3*c - 14
*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(
-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/a^(2/3))/(756*b^(16/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [B] time = 0.45, size = 1318, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 + 54*(14*
a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 14*a^3*b^
4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 - 42*sqrt(1/3)*(2*a^3*b^4*c - 14*a^4
*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35*a^3*b^4
*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3*e - 65*
a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*
x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqr
t((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*
e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f
+ 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2
/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*((2*b^5*c - 14*
a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 3
5*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*
b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(2*a^3*b^4*c - 14
*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a
^4*b^6), 1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 +
 54*(14*a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 1
4*a^3*b^4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 + 84*sqrt(1/3)*(2*a^3*b^4*c
- 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35
*a^3*b^4*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3
*e - 65*a^5*b^2*f)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b
)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*((2*b^5*c -
 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d
 + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*
a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3
)*a) + 28*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2
*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d
 + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3
)) - 84*(2*a^3*b^4*c - 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^
8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6)]
```

giac [A] time = 0.20, size = 345, normalized size = 1.03

$$-\frac{\sqrt{3}\left(2\,b^{3}c-14\,ab^{2}d-65\,a^{3}f+35\,a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{2}{3}}b^{4}}-\frac{\left(2\,b^{3}c-14\,ab^{2}d-65\,a^{3}f+35\,a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^{2}\right)^{\frac{2}{3}}b^{4}}-\frac{\left(2\,b^{3}c-14\,ab^{2}d-65\,a^{3}f+35\,a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^{5}}-\frac{7\,b^{4}cx^{4}-13\,ab^{3}dx^{4}-25\,a^{3}bfx^{4}+19\,a^{2}b^{2}x^{4}e+4\,ab^{3}cx-10\,a^{2}b^{2}dx-22\,a^{4}fx+16\,a^{3}bxe}{18\left(bx^{3}+a\right)^{2}b^{5}}+\frac{4\,b^{18}fx^{7}-21\,ab^{17}fx^{4}+7\,b^{18}x^{4}e+28\,b^{18}dx+168\,a^{2}b^{16}fx-84\,ab^{17}xe}{28\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*b^3*c - 14*a*b^2*d - 65*a^3*f + 35*a^2*b*e)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^4) - 1/54*(2*b^3*
c - 14*a*b^2*d - 65*a^3*f + 35*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(2/3)*b^4) - 1/27*(2*b^3*c - 14*a*b^2*d - 65*a^3*f + 35*a^2
*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(7*b^4*c*x^4 -
 13*a*b^3*d*x^4 - 25*a^3*b*f*x^4 + 19*a^2*b^2*x^4*e + 4*a*b^3*c*x - 10*a^2*
b^2*d*x - 22*a^4*f*x + 16*a^3*b*x*e)/((b*x^3 + a)^2*b^5) + 1/28*(4*b^18*f*x
^7 - 21*a*b^17*f*x^4 + 7*b^18*x^4*e + 28*b^18*d*x + 168*a^2*b^16*f*x - 84*a
*b^17*x*e)/b^21
```

maple [B] time = 0.05, size = 596, normalized size = 1.77

$$\frac{f x^7}{7b^3}+\frac{25a^3 f x^4}{18\left(b x^3+a\right)^2 b^4}-\frac{19a^2 e x^4}{18\left(b x^3+a\right)^2 b^3}+\frac{13ad x^4}{18\left(b x^3+a\right)^2 b^2}-\frac{7c x^4}{18\left(b x^3+a\right)^2 b}-\frac{3af x^4}{4b^4}+\frac{e x^4}{4b^3}+\frac{11a^4 f x}{9\left(b x^3+a\right)^2 b^5}-\frac{8a^3 e x}{9\left(b x^3+a\right)^2 b^4}+\frac{5a^2 d x}{9\left(b x^3+a\right)^2 b^3}-\frac{2acx}{9\left(b x^3+a\right)^2 b^2}-\frac{65\sqrt{3}\,a^3 f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^6}-\frac{65a^3 f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^6}+\frac{65a^3 f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}} b^6}+\frac{35\sqrt{3}\,a^2 e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^5}+\frac{35a^2 e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^5}-\frac{35a^2 e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}} b^5}+\frac{6a^2 f x}{b^5}-\frac{14\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^4}-\frac{14ad\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^4}+\frac{7ad\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^4}-\frac{3aex}{b^4}+\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3}+\frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3}-\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3}+\frac{dx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 65/54/b^6*a^3*f/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+35/27/b^5*a^2
*e/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-14/27/b^4*a*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-3/b^4*a*e*x-3/4/b^4*x^4*a*f+2/27/b^3*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-7/18
/b/(b*x^3+a)^2*x^4*c+6/b^5*a^2*f*x-1/27/b^3*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))-8/9/b^4/(b*x^3+a)^2*a^3*e*x+1/7/b^3*f*x^7+13/18/b^2/(b*x^3
+a)^2*x^4*a*d-35/54/b^5*a^2*e/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
-2/9/b^2/(b*x^3+a)^2*a*c*x+11/9/b^5/(b*x^3+a)^2*a^4*f*x+1/4/b^3*x^4*e+7/27/
b^4*a*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/b^3*d*x+2/27/b^3*c/
(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-65/27/b^6*a^3*f
/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-65/27/b^6*a^3*f/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+35/27/b^5*a^2*e/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-14/27/b^4*a*d/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+25/18/b^4/(b*x^3+a)^2*x^4*a^3*f-19/18/b^3/(
b*x^3+a)^2*x^4*a^2*e+5/9/b^3/(b*x^3+a)^2*a^2*d*x
```

maxima [A] time = 3.04, size = 326, normalized size = 0.97

$$-\frac{\left(7 b^4 c-13 a b^3 d+19 a^2 b^2 e-25 a^3 b f\right) x^4+2\left(2 a b^3 c-5 a^2 b^2 d+8 a^3 b e-11 a^4 f\right) x}{18\left(b^7 x^6+2 a b^6 x^3+a^2 b^5\right)}+\frac{4 b^2 f x^7+7\left(b^2 e-3 a b f\right) x^4+28\left(b^2 d-3 a b e+6 a^2 f\right) x}{28 b^5}+\frac{\sqrt{3}\left(2 b^3 c-14 a b^2 d+35 a^2 b e-65 a^3 f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2 b^3 c-14 a b^2 d+35 a^2 b e-65 a^3 f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2 b^3 c-14 a b^2 d+35 a^2 b e-65 a^3 f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*b^4*c - 13*a*b^3*d + 19*a^2*b^2*e - 25*a^3*b*f)*x^4 + 2*(2*a*b^3*
c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f)*x)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5
) + 1/28*(4*b^2*f*x^7 + 7*(b^2*e - 3*a*b*f)*x^4 + 28*(b^2*d - 3*a*b*e + 6*a
^2*f)*x)/b^5 + 1/27*sqrt(3)*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(2/3)) - 1/5
4*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^6*(a/b)^(2/3)) + 1/27*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e -
65*a^3*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

mupad [B] time = 5.30, size = 335, normalized size = 1.00

$$x^4\left(\frac{e}{4b^3}-\frac{3af}{4b^4}\right)-x\left(\frac{3a^2 f}{b^5}-\frac{d}{b^3}+\frac{3a\left(\frac{e}{b^3}-\frac{3af}{b^4}\right)}{b}\right)-\frac{x^4\left(-\frac{25 f a^3 b}{18}+\frac{19 e a^2 b^2}{18}-\frac{13 d a b^3}{18}+\frac{7 c b^4}{18}\right)-x\left(\frac{11 f a^4}{9}-\frac{8 e a^3 b}{9}+\frac{5 d a^2 b^2}{9}-\frac{2 c a b^3}{9}\right)}{a^2 b^5+2 a b^6 x^3+b^7 x^6}+\frac{f x^7}{7b^3}+\frac{\ln\left(b^{1/3} x+a^{1/3}\right)\left(-65 f a^3+35 e a^2 b-14 d a b^2+2 c b^3\right)}{27 a^{2/3} b^{16/3}}+\frac{\ln\left(2 b^{1/3} x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-65 f a^3+35 e a^2 b-14 d a b^2+2 c b^3\right)}{27 a^{2/3} b^{16/3}}-\frac{\ln\left(a^{1/3}-2 b^{1/3} x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-65 f a^3+35 e a^2 b-14 d a b^2+2 c b^3\right)}{27 a^{2/3} b^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^4*(e/(4*b^3) - (3*a*f)/(4*b^4)) - x*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3
- (3*a*f)/b^4))/b) - (x^4*((7*b^4*c)/18 + (19*a^2*b^2*e)/18 - (13*a*b^3*d)/
18 - (25*a^3*b*f)/18) - x*((11*a^4*f)/9 + (5*a^2*b^2*d)/9 - (2*a*b^3*c)/9 -
 (8*a^3*b*e)/9))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) + (f*x^7)/(7*b^3) + (log
(b^(1/3)*x + a^(1/3))*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*a
^(2/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/
2)*1i)/2 - 1/2)*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*a^(2/3)
*b^(16/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)
/2 + 1/2)*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*a^(2/3)*b^(16
/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.238 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=316

$$\frac{x^2\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{9ab^4\left(a+bx^3\right)}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(44a^3f-\right.}{54a^{4/3}b^{14/3}}$$

Rubi [A] time = 0.50, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1851, 1594, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(7a^2be-10a^3f-4ab^2d+b^3c\right)}{9ab^4\left(a+bx^3\right)}-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-20a^2be+44a^3f+5ab^2d+b^3c\right)}{54a^{4/3}b^{14/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(-20a^2be+44a^3f+5ab^2d+b^3c\right)}{27a^{4/3}b^{14/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-20a^2be+44a^3f+5ab^2d+b^3c\right)}{9\sqrt{3}\,a^{4/3}b^{14/3}}+\frac{x^2(be-3af)}{2b^4}+\frac{fx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x^2)/(2*b^4) + (f*x^5)/(5*b^3) - ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*x^2)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10
*a^3*f)*x^2)/(9*a*b^4*(a + b*x^3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3
)*b^(14/3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/(27*a^(4/3)*b^(14/3)) + ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3
*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} - \frac{\int \frac{-2ab\left(b^3c-ab^2d+a^2be-a^3f\right)x-6ab^2\left(b^2d-abe+a^2f\right)x^4-}{\left(a+bx^3\right)^2}}{6ab^5}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} - \frac{\int \frac{x\left(-2ab\left(b^3c-ab^2d+a^2be-a^3f\right)-6ab^2\left(b^2d-abe+a^2f\right)x^3-\right.}{\left(a+bx^3\right)^2}}{6ab^5}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\right)x^2}{9ab^4\left(a+bx^3\right)} + \int \frac{2ab^5}{}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\right)x^2}{9ab^4\left(a+bx^3\right)} + \int \frac{x(2a}{}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\right)x^2}{9ab^4\left(a+bx^3\right)} + \int (18a$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be\right.}{9ab^4\left(a+bx\right.}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be\right.}{9ab^4\left(a+bx\right.}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be\right.}{9ab^4\left(a+bx\right.}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be\right.}{9ab^4\left(a+bx\right.}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be\right.}{9ab^4\left(a+bx\right.}$$

Mathematica [A] time = 0.29, size = 300, normalized size = 0.95

$$\frac{\frac{30b^{2/3}x^2\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{a\left(a+bx^3\right)} - \frac{45b^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2} - \frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(44a^3f-20a^2be+5ab^2d+b^3c\right)}{a^{4/3}} - \frac{10\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(44a^3f-20a^2be+5ab^2d+b^3c\right)}{a^{4/3}} + \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(44a^3f-20a^2be+5ab^2d+b^3c\right)}{a^{4/3}} + 135b^{2/3}x^2(be-3af) + 54b^{5/3}fx^5}{270b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (135*b^(2/3)*(b*e - 3*a*f)*x^2 + 54*b^(5/3)*f*x^5 - (45*b^(2/3)*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b*x^3)^2 + (30*b^(2/3)*(b^3*c - 4*a*b^2*
d + 7*a^2*b*e - 10*a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt[3]*(b^3*c + 5*a*b
^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/
a^(4/3) - (10*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/a^(4/3) + (5*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(270*b^(14/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [B] time = 0.45, size = 1224, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/270*(54*a^2*b^5*f*x^11 + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b^
6*c - 20*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c + 5*
a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 15*sqrt(1/3)*(a^3*b^4*c + 5*
a^4*b^3*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3*b^4
*e + 44*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5
*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^
3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c
 + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2
 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*((b^5*c + 5*a*b^4*d - 20*a^2*b
^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f
+ 2*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)
*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6), 1/270*
(54*a^2*b^5*f*x^11 + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b^6*c - 2
0*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c + 5*a^3*b^4
*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 30*sqrt(1/3)*(a^3*b^4*c + 5*a^4*b^3
*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3*b^4*e + 44
*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)
*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqr
t(-(-a*b^2)^(1/3)/a)/b) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^3*b^2
*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c + 5*
a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-
a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e
+ 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(
a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(
b*x - (-a*b^2)^(1/3)))/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6)]
```

giac [A] time = 0.20, size = 365, normalized size = 1.16

$$\frac{\sqrt{3}\left(b^3c+5\,ab^2d+44\,a^3f-20\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}ab^4}-\frac{\left(b^3c+5\,ab^2d+44\,a^3f-20\,a^2be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}ab^4}-\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4}+\frac{2\,b^4cx^5-8\,ab^3dx^5-20\,a^3bfx^5+14\,a^2b^2x^5e-ab^3cx^2-5\,a^2b^2dx^2-17\,a^4fx^2+11\,a^3bx^2e}{18\left(bx^3+a\right)^2ab^4}+\frac{2\,b^{12}fx^5-15\,ab^{11}fx^2+5\,b^{12}x^2e}{10\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(b^3*c + 5*a*b^2*d + 44*a^3*f - 20*a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^4) - 1/54*(b^3*c +
5*a*b^2*d + 44*a^3*f - 20*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(1/3)*a*b^4) - 1/27*(b^3*c*(-a/b)^(1/3) + 5*a*b^2*d*(-a/b)^(1/3)
 + 44*a^3*f*(-a/b)^(1/3) - 20*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^2*b^4) + 1/18*(2*b^4*c*x^5 - 8*a*b^3*d*x^5 - 20*a^3*b*f
```

```
*x^5 + 14*a^2*b^2*x^5*e - a*b^3*c*x^2 - 5*a^2*b^2*d*x^2 - 17*a^4*f*x^2 + 11
*a^3*b*x^2*e)/((b*x^3 + a)^2*a*b^4) + 1/10*(2*b^12*f*x^5 - 15*a*b^11*f*x^2
+ 5*b^12*x^2*e)/b^15
```

maple [B] time = 0.06, size = 574, normalized size = 1.82

$$-\frac{10a^2 f x^5}{9\left(b x^3+a\right)^2 b^3}+\frac{7ae x^5}{9\left(b x^3+a\right)^2 b^2}+\frac{c x^5}{9\left(b x^3+a\right)^2 a}-\frac{4d x^5}{9\left(b x^3+a\right)^2 b}+\frac{f x^5}{5b^3}-\frac{17a^3 f x^2}{18\left(b x^3+a\right)^2 b^4}+\frac{11a^2 e x^2}{18\left(b x^3+a\right)^2 b^3}-\frac{5ad x^2}{18\left(b x^3+a\right)^2 b^2}-\frac{c x^2}{18\left(b x^3+a\right)^2 b}-\frac{3af x^2}{2b^4}+\frac{e x^2}{2b^3}+\frac{44\sqrt{3}\, a^2 f \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^5}-\frac{44a^2 f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^5}+\frac{22a^2 f \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^5}-\frac{20\sqrt{3}\, ae \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^4}+\frac{20ae \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^4}-\frac{10ae \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^4}+\frac{\sqrt{3}\, c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^2}-\frac{c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^2}+\frac{c \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^2}+\frac{5\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3}-\frac{5d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3}+\frac{5d \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/5/b^3*f*x^5-3/2/b^4*x^2*a*f+1/2/b^3*x^2*e-10/9/b^3/(b*x^3+a)^2*a^2*x^5*f+
7/9/b^2/(b*x^3+a)^2*a*x^5*e-4/9/b/(b*x^3+a)^2*x^5*d+1/9/(b*x^3+a)^2/a*x^5*c
-17/18/b^4/(b*x^3+a)^2*x^2*a^3*f+11/18/b^3/(b*x^3+a)^2*x^2*a^2*e-5/18/b^2/(
b*x^3+a)^2*x^2*a*d-1/18/b/(b*x^3+a)^2*x^2*c-44/27/b^5*a^2/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))*f+20/27/b^4*a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-5/27/b^3/(a/b)^(
1/3)*ln(x+(a/b)^(1/3))*d-1/27/b^2/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+22/27/b
^5*a^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-10/27/b^4*a/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+5/54/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))*d+1/54/b^2/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*c+44/27/b^5*a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*f-20/27/b^4*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))*e+5/27/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*
d+1/27/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 3.10, size = 311, normalized size = 0.98

$$\frac{2\left(b^4c-4ab^3d+7a^2b^2e-10a^3bf\right)x^5-\left(ab^3c+5a^2b^2d-11a^3be+17a^4f\right)x^2}{18\left(ab^6x^6+2a^2b^5x^3+a^3b^4\right)}+\frac{2bfx^5+5\left(be-3af\right)x^2}{10b^4}+\frac{\sqrt{3}\left(b^3c+5ab^2d-20a^2be+44a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(b^3c+5ab^2d-20a^2be+44a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(b^3c+5ab^2d-20a^2be+44a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^5 - (a*b^3*c + 5*a
^2*b^2*d - 11*a^3*b*e + 17*a^4*f)*x^2)/(a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4
) + 1/10*(2*b*f*x^5 + 5*(b*e - 3*a*f)*x^2)/b^4 + 1/27*sqrt(3)*(b^3*c + 5*a*
b^2*d - 20*a^2*b*e + 44*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(a*b^5*(a/b)^(1/3)) + 1/54*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3
*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(1/3)) - 1/27*(b^3*
c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*log(x + (a/b)^(1/3))/(a*b^5*(a/b)^(1
/3))
```

mupad [B] time = 5.27, size = 295, normalized size = 0.93

$$x^2\left(\frac{e}{2b^3}-\frac{3af}{2b^4}\right)-\frac{x^2\left(\frac{17fa^3}{18}-\frac{11ea^2b}{18}+\frac{5dab^2}{18}+\frac{cb^3}{18}\right)-\frac{x^5\left(-10fa^3b+7ea^2b^2-4dab^3+cb^4\right)}{9a}}{a^2b^4+2ab^5x^3+b^6x^6}+\frac{fx^5}{5b^3}-\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(44fa^3-20ea^2b+5dab^2+cb^3\right)}{27a^{4/3}b^{14/3}}+\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(44fa^3-20ea^2b+5dab^2+cb^3\right)}{27a^{4/3}b^{14/3}}-\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(44fa^3-20ea^2b+5dab^2+cb^3\right)}{27a^{4/3}b^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x^2*(e/(2*b^3) - (3*a*f)/(2*b^4)) - (x^2*((b^3*c)/18 + (17*a^3*f)/18 + (5*a
*b^2*d)/18 - (11*a^2*b*e)/18) - (x^5*(b^4*c + 7*a^2*b^2*e - 4*a*b^3*d - 10*
a^3*b*f))/(9*a))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^5)/(5*b^3) - (log
(b^(1/3)*x + a^(1/3))*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4
/3)*b^(14/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*
1i)/2 + 1/2)*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4/3)*b^(14
/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4/3)*b^(14/3))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

3.239 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

Optimal. Leaf size=307

$$\frac{x\left(-19a^3f+13a^2be-7ab^2d+b^3c\right)}{18ab^4\left(a+bx^3\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^3f-1\right.}{54a^{5/3}b^{13/3}}$$

Rubi [A] time = 0.41, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1858, 1411, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(13a^2be-19a^3f-7ab^2d+b^3c\right)}{18ab^4\left(a+bx^3\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^2be+35a^3f+2ab^2d+b^3c\right)}{54a^{5/3}b^{13/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-14a^2be+35a^3f+2ab^2d+b^3c\right)}{27a^{5/3}b^{13/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-14a^2be+35a^3f+2ab^2d+b^3c\right)}{9\sqrt{3}a^{5/3}b^{13/3}}+\frac{x(be-3af)}{b^4}+\frac{fx^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^4)/(4*b^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 7*a*b^2*d + 13*a^2*b*e - 19*a^3*f)
*x)/(18*a*b^4*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(13/
3)) + ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/(27*a^(5/3)*b^(13/3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} - \frac{\int \frac{-a\left(b^3c-ab^2d+a^2be-a^3f\right)-6ab\left(b^2d-abe+a^2f\right)x^3-6ab^2(b\cdots}{\left(a+bx^3\right)^2}}{6ab^4}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-19a^3f\right)x}{18ab^4\left(a+bx^3\right)} + \frac{\int 2ab^3(\cdots}{\cdots}$$

$$= \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-19a^3f\right)x}{18ab^4\left(a+bx^3\right)} + \frac{\int \cdots}{\cdots}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-\cdots\right.}{18ab^4\left(a+bx^3\cdots\right.}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-\cdots\right.}{18ab^4\left(a+bx^3\cdots\right.}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-\cdots\right.}{18ab^4\left(a+bx^3\cdots\right.}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-\cdots\right.}{18ab^4\left(a+bx^3\cdots\right.}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-\cdots\right.}{18ab^4\left(a+bx^3\cdots\right.}$$

Mathematica [A] time = 0.31, size = 294, normalized size = 0.96

$$\frac{\frac{6\sqrt[3]{b}x\left(-19a^3f+13a^2be-7ab^2d+b^3c\right)}{a\left(a+bx^3\right)} - \frac{18\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2} + \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(35a^3f-14a^2be+2ab^2d+b^3c\right)}{a^{5/3}} - \frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(35a^3f-14a^2be+2ab^2d+b^3c\right)}{a^{5/3}} - \frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^3f-14a^2be+2ab^2d+b^3c\right)}{a^{5/3}} + 108\sqrt[3]{b}x(be-3af) + 27b^{4/3}fx^4}{108b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (108*b^(1/3)*(b*e - 3*a*f)*x + 27*b^(4/3)*f*x^4 - (18*b^(1/3)*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)^2 + (6*b^(1/3)*(b^3*c - 7*a*b^2*d + 1
3*a^2*b*e - 19*a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(b^3*c + 2*a*b^2*d -
14*a^2*b*e + 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3)
 + (4*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])
/a^(5/3) - (2*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(108*b^(13/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [B] time = 0.47, size = 1213, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/108*(27*a^3*b^4*f*x^10 + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b
^5*c - 14*a^3*b^4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 6*sqrt(1/3)*(a^3*
b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d -
14*a^3*b^4*e + 35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b^3*
e + 35*a^5*b^2*f)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1
/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)
*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*
e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2
*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*((b^5*c + 2*a*b^4*d - 14*
a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a
^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(
2/3)*log(a*b*x + (a^2*b)^(2/3)) - 12*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*
e + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5), 1/108*(27*a^3*b
^4*f*x^10 + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b^5*c - 14*a^3*b^
4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 12*sqrt(1/3)*(a^3*b^4*c + 2*a^4*b
^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d - 14*a^3*b^4*e +
35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b^3*e + 35*a^5*b^2*
f)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)
^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*e
 + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2*
(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*((b^5*c + 2*a*b^4*d - 14*a
^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^
5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) - 12*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e
 + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5)]
```

giac [A] time = 0.22, size = 319, normalized size = 1.04

$$-\frac{\sqrt{3}\left(b^{3} c + 2 a b^{2} d + 35 a^{3} f - 14 a^{2} b e\right) \arctan\left(\frac{\sqrt{3}\left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 \left(-a b^{2}\right)^{\frac{2}{3}} a b^{3}} - \frac{\left(b^{3} c + 2 a b^{2} d + 35 a^{3} f - 14 a^{2} b e\right) \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 \left(-a b^{2}\right)^{\frac{2}{3}} a b^{3}} - \frac{\left(b^{3} c + 2 a b^{2} d + 35 a^{3} f - 14 a^{2} b e\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27 a^{2} b^{4}} + \frac{b^{4} c x^{4} - 7 a b^{3} d x^{4} - 19 a^{3} b f x^{4} + 13 a^{2} b^{2} x^{4} e - 2 a b^{3} c x - 4 a^{2} b^{2} d x - 16 a^{4} f x + 10 a^{3} b x e}{18 \left(b x^{3} + a\right)^{2} a b^{4}} + \frac{b^{9} f x^{4} - 12 a b^{8} f x + 4 b^{9} x e}{4 b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(b^3*c + 2*a*b^2*d + 35*a^3*f - 14*a^2*b*e)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^3) - 1/54*(b^3*c +
 2*a*b^2*d + 35*a^3*f - 14*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/((-a*b^2)^(2/3)*a*b^3) - 1/27*(b^3*c + 2*a*b^2*d + 35*a^3*f - 14*a^2*b*e)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/18*(b^4*c*x^4 - 7*a*
b^3*d*x^4 - 19*a^3*b*f*x^4 + 13*a^2*b^2*x^4*e - 2*a*b^3*c*x - 4*a^2*b^2*d*x
 - 16*a^4*f*x + 10*a^3*b*x*e)/((b*x^3 + a)^2*a*b^4) + 1/4*(b^9*f*x^4 - 12*a
*b^8*f*x + 4*b^9*x*e)/b^12
```

maple [B] time = 0.06, size = 561, normalized size = 1.83

$$-\frac{19 a^{2} f x^{4}}{18 \left(b x^{3}+a\right)^{2} b^{3}}+\frac{13 a e x^{4}}{18 \left(b x^{3}+a\right)^{2} b^{2}}+\frac{c x^{4}}{18 \left(b x^{3}+a\right)^{2} a}-\frac{7 d x^{4}}{18 \left(b x^{3}+a\right)^{2} b}+\frac{f x^{4}}{4 b^{3}}-\frac{8 a^{3} f x}{9 \left(b x^{3}+a\right)^{2} b^{4}}+\frac{5 a^{2} e x}{9 \left(b x^{3}+a\right)^{2} b^{3}}-\frac{2 a d x}{9 \left(b x^{3}+a\right)^{2} b^{2}}-\frac{c x}{9 \left(b x^{3}+a\right)^{2} b}+\frac{35 \sqrt{3}\, a^{2} f \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{5}}+\frac{35 a^{2} f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{5}}-\frac{35 a^{2} f \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{5}}-\frac{14 \sqrt{3}\, a e \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}}-\frac{14 a e \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}}+\frac{7 a e \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}}-\frac{3 a f x}{b^{4}}+\frac{\sqrt{3}\, c \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a b^{2}}+\frac{c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a b^{2}}-\frac{c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 \left(\frac{a}{b}\right)^{\frac{2}{3}} a b^{2}}+\frac{2 \sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}+\frac{2 d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}-\frac{d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}+\frac{e x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/4*f*x^4/b^3-3/b^4*a*f*x+1/b^3*e*x-19/18/b^3/(b*x^3+a)^2*x^4*a^2*f+13/18/b
^2/(b*x^3+a)^2*x^4*a*e-7/18/b/(b*x^3+a)^2*x^4*d+1/18/(b*x^3+a)^2/a*x^4*c-8/
9/b^4/(b*x^3+a)^2*a^3*f*x+5/9/b^3/(b*x^3+a)^2*a^2*e*x-2/9/b^2/(b*x^3+a)^2*a
*d*x-1/9/b/(b*x^3+a)^2*c*x+35/27/b^5*a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-14
/27/b^4*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+2/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(
1/3))*d+1/27/b^2/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-35/54/b^5*a^2/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f+7/27/b^4*a/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))*e-1/27/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)*d-1/54/b^2/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+35/27/b^5*a^
2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-14/27/b^4*a
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e+2/27/b^3/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/27/b^2/a/(a/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 3.06, size = 305, normalized size = 0.99

$$\frac{\left(b^4c-7\,ab^3d+13\,a^2b^2e-19\,a^3bf\right)x^4-2\left(ab^3c+2\,a^2b^2d-5\,a^3be+8\,a^4f\right)x}{18\left(ab^6x^6+2\,a^2b^5x^3+a^3b^4\right)}+\frac{bfx^4+4\left(be-3\,af\right)x}{4\,b^4}+\frac{\sqrt{3}\left(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((b^4*c - 7*a*b^3*d + 13*a^2*b^2*e - 19*a^3*b*f)*x^4 - 2*(a*b^3*c + 2*
a^2*b^2*d - 5*a^3*b*e + 8*a^4*f)*x)/(a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4) +
 1/4*(b*f*x^4 + 4*(b*e - 3*a*f)*x)/b^4 + 1/27*sqrt(3)*(b^3*c + 2*a*b^2*d -
14*a^2*b*e + 35*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/
(a*b^5*(a/b)^(2/3)) - 1/54*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(2/3)) + 1/27*(b^3*c + 2*a*
b^2*d - 14*a^2*b*e + 35*a^3*f)*log(x + (a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
```

mupad [B] time = 5.14, size = 290, normalized size = 0.94

$$x\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)-\frac{x\left(\frac{8\,f\,a^3}{9}-\frac{5\,e\,a^2\,b}{9}+\frac{2\,d\,a\,b^2}{9}+\frac{c\,b^3}{9}\right)-\frac{x^4\left(-19\,f\,a^3\,b+13\,e\,a^2\,b^2-7\,d\,a\,b^3+c\,b^4\right)}{18\,a}}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+\frac{f\,x^4}{4\,b^3}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] x*(e/b^3 - (3*a*f)/b^4) - (x*((b^3*c)/9 + (8*a^3*f)/9 + (2*a*b^2*d)/9 - (5*
a^2*b*e)/9) - (x^4*(b^4*c + 13*a^2*b^2*e - 7*a*b^3*d - 19*a^3*b*f))/(18*a))
/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^4)/(4*b^3) + (log(b^(1/3)*x + a^(
1/3))*(b^3*c + 35*a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3)) +
(log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^
3*c + 35*a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3)) - (log(3^(1
/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c + 35*
a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.240 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=301

$$\frac{x^2\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{9a^2b^3\left(a+bx^3\right)}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-20a^3f+5a^2\right.}{54a^{7/3}b^{11/3}}$$

Rubi [A] time = 0.37, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1828, 1594, 1482, 459, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{9a^2b^3\left(a+bx^3\right)}+\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(5a^2be-20a^3f+ab^2d+2b^3c\right)}{54a^{7/3}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(5a^2be-20a^3f+ab^2d+2b^3c\right)}{27a^{7/3}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^2be-20a^3f+ab^2d+2b^3c\right)}{9\sqrt{3}\,a^{7/3}b^{11/3}}+\frac{fx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (f*x^2)/(2*b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a*b^3*(a + b
*x^3)^2) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/(9*a^2*b^3*(a +
b*x^3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(11/3)) - ((2*b^3*c + a*
b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(11/3
)) + ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^
2 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m
- Mod[m, n])/n)*(q + 1)), x] + Dist[1/(n*e^(2*p + (m - Mod[m, n])/n)*(q + 1
)), Int[x^Mod[m, n]*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*e^(2*p +
 (m - Mod[m, n])/n)*(q + 1)*x^(m - Mod[m, n])*(a + b*x^n + c*x^(2*n))^p - (
-d)^((m - Mod[m, n])/n - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d*(Mod[m, n] + 1) +
e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGt
Q[p, 0] && ILtQ[q, -1] && IGtQ[m, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} - \frac{\int \frac{-2b\left(2b^3c+ab^2d-a^2be+a^3f\right)x-6ab^2(be-af)x^4-6ab^3fx^7}{\left(a+bx^3\right)^2}\,dx}{6ab^4}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} - \frac{\int \frac{x\left(-2b\left(2b^3c+ab^2d-a^2be+a^3f\right)-6ab^2(be-af)x^3-6ab^3fx^6\right)}{\left(a+bx^3\right)^2}\,dx}{6ab^4}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} + \int \frac{x\left(2b^3\left(\frac{2b^3c}{a}+\cdots\right.\right.}{\cdots}$$

$$= \frac{fx^2}{2b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} + \frac{\left(2b^3c\cdots\right.}{\cdots}$$

$$= \frac{fx^2}{2b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} - \frac{\left(2b^3c\cdots\right.}{\cdots}$$

$$= \frac{fx^2}{2b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} - \frac{\left(2b^3c\cdots\right.}{\cdots}$$

$$= \frac{fx^2}{2b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} - \frac{\left(2b^3c\cdots\right.}{\cdots}$$

$$= \frac{fx^2}{2b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} - \frac{\left(2b^3c\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.32, size = 284, normalized size = 0.94

$$\frac{\frac{6b^{2/3}x^2\left(7a^3f-4a^2be+ab^2d+2b^3c\right)}{a^2\left(a+bx^3\right)} + \frac{9b^{2/3}x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a\left(a+bx^3\right)^2} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-20a^3f+5a^2be+ab^2d+2b^3c\right)}{a^{7/3}} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-20a^3f+5a^2be+ab^2d+2b^3c\right)}{a^{7/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-20a^3f+5a^2be+ab^2d+2b^3c\right)}{a^{7/3}} + 27b^{2/3}fx^2}{54b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (27*b^(2/3)*f*x^2 + (9*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a*
(a + b*x^3)^2) + (6*b^(2/3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/
(a^2*(a + b*x^3)) - (2*Sqrt[3]*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (2*(2*b^3*c + a*b^2*d
 + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) + ((2*b^3*c + a*
b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/a^(7/3))/(54*b^(11/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]
```

fricas [**B**] time = 0.44, size = 1158, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*b
^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2 -
3*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b^6*
c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*b^4*
d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3
- a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(
a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((2*b^5*c + a*b^4*d + 5
*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*a
^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(2
/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*c + a*b^4*
d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e -
 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^
2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5),
 1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*b
^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2 -
6*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b^6*
c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*b^4*
d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3
)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + ((2*b^5*c + a*b^4*d +
5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*
a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(
2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*c + a*b^4
*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e
- 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b
^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5)
]
```

giac [**A**] time = 0.21, size = 339, normalized size = 1.13

$$\frac{fx^2}{2b^3} + \frac{\sqrt{3}\left(2b^3c + ab^2d - 20a^3f + 5a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^2b^3} - \frac{\left(2b^3c + ab^2d - 20a^3f + 5a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}a^2b^3} - \frac{\left(2b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 20a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3b^3} + \frac{4b^4cx^5 + 2ab^3dx^5 + 14a^3bfx^5 - 8a^2b^2x^5e + 7ab^3cx^2 - a^2b^2dx^2 + 11a^4fx^2 - 5a^3bx^2e}{18\left(bx^3 + a\right)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^3 + 1/27*sqrt(3)*(2*b^3*c + a*b^2*d - 20*a^3*f + 5*a^2*b*e)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b^3)
 - 1/54*(2*b^3*c + a*b^2*d - 20*a^3*f + 5*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b^3) - 1/27*(2*b^3*c*(-a/b)^(1/3) + a*
b^2*d*(-a/b)^(1/3) - 20*a^3*f*(-a/b)^(1/3) + 5*a^2*b*(-a/b)^(1/3)*e)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/18*(4*b^4*c*x^5 + 2*a*b^3*d
*x^5 + 14*a^3*b*f*x^5 - 8*a^2*b^2*x^5*e + 7*a*b^3*c*x^2 - a^2*b^2*d*x^2 + 1
1*a^4*f*x^2 - 5*a^3*b*x^2*e)/((b*x^3 + a)^2*a^2*b^3)
```

maple [**B**] time = 0.06, size = 550, normalized size = 1.83

$$\frac{7afx^5}{9\left(bx^3+a\right)^2b^2} + \frac{dx^5}{9\left(bx^3+a\right)^2a} + \frac{2bcx^5}{9\left(bx^3+a\right)^2a^2} - \frac{4ex^5}{9\left(bx^3+a\right)^2b} + \frac{11a^2fx^2}{18\left(bx^3+a\right)^2b^3} - \frac{5aex^2}{18\left(bx^3+a\right)^2b^2} + \frac{7cx^2}{18\left(bx^3+a\right)^2a} - \frac{dx^2}{18\left(bx^3+a\right)^2b} + \frac{fx^2}{2b^3} - \frac{20\sqrt{3}\,af\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} + \frac{20af\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} - \frac{10af\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} + \frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2} - \frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2} + \frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2} + \frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b} - \frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b} + \frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b} + \frac{5\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} - \frac{5e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} + \frac{5e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/2*f*x^2/b^3+7/9/b^2/(b*x^3+a)^2*a*x^5*f-4/9/b/(b*x^3+a)^2*x^5*e+1/9/(b*x^
3+a)^2/a*x^5*d+2/9*b/(b*x^3+a)^2/a^2*x^5*c+11/18/b^3/(b*x^3+a)^2*a^2*x^2*f-
5/18/b^2/(b*x^3+a)^2*a*x^2*e-1/18/b/(b*x^3+a)^2*x^2*d+7/18/(b*x^3+a)^2/a*x^
2*c+20/27/b^4*a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-5/27/b^3/(a/b)^(1/3)*ln(x+(
a/b)^(1/3))*e-1/27/b^2/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-2/27/b/a^2/(a/b)^(
1/3)*ln(x+(a/b)^(1/3))*c-10/27/b^4*a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))*f+5/54/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/54/b^2
/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/27/b/a^2/(a/b)^(1/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c-20/27/b^4*a*3^(1/2)/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+5/27/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*e+1/27/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*d+2/27/b/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 2.94, size = 296, normalized size = 0.98

$$\frac{2\left(2\,b^4c + ab^3d - 4\,a^2b^2e + 7\,a^3bf\right)x^5 + \left(7\,ab^3c - a^2b^2d - 5\,a^3be + 11\,a^4f\right)x^2}{18\left(a^2b^5x^6 + 2\,a^3b^4x^3 + a^4b^3\right)} + \frac{fx^2}{2\,b^3} + \frac{\sqrt{3}\left(2\,b^3c + ab^2d + 5\,a^2be - 20\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(2\,b^3c + ab^2d + 5\,a^2be - 20\,a^3f\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(2\,b^3c + ab^2d + 5\,a^2be - 20\,a^3f\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^5 + (7*a*b^3*c - a^
2*b^2*d - 5*a^3*b*e + 11*a^4*f)*x^2)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3
) + 1/2*f*x^2/b^3 + 1/27*sqrt(3)*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4*(a/b)^(1/3))
+ 1/54*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*log(x^2 - x*(a/b)^(1/3) +
 (a/b)^(2/3))/(a^2*b^4*(a/b)^(1/3)) - 1/27*(2*b^3*c + a*b^2*d + 5*a^2*b*e -
 20*a^3*f)*log(x + (a/b)^(1/3))/(a^2*b^4*(a/b)^(1/3))
```

mupad [B] time = 5.27, size = 280, normalized size = 0.93

$$\frac{\frac{x^2\left(11\,f\,a^3-5\,e\,a^2\,b-d\,a\,b^2+7\,c\,b^3\right)}{18\,a}+\frac{x^5\left(7\,f\,a^3\,b-4\,e\,a^2\,b^2+d\,a\,b^3+2\,c\,b^4\right)}{9\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} + \frac{f\,x^2}{2\,b^3} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

```
[Out] ((x^2*(7*b^3*c + 11*a^3*f - a*b^2*d - 5*a^2*b*e))/(18*a) + (x^5*(2*b^4*c -
4*a^2*b^2*e + a*b^3*d + 7*a^3*b*f))/(9*a^2))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x
^3) + (f*x^2)/(2*b^3) - (log(b^(1/3)*x + a^(1/3))*(2*b^3*c - 20*a^3*f + a*b
^2*d + 5*a^2*b*e))/(27*a^(7/3)*b^(11/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1
/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c - 20*a^3*f + a*b^2*d + 5*a
^2*b*e))/(27*a^(7/3)*b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^
(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c - 20*a^3*f + a*b^2*d + 5*a^2*b*e))/(
27*a^(7/3)*b^(11/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.241 $\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=292

$$\frac{x\left(13a^3f-7a^2be+ab^2d+5b^3c\right)}{18a^2b^3\left(a+bx^3\right)}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^3f+2\right.}{54a^{8/3}b^{10/3}}$$

Rubi [A] time = 0.31, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1858, 1409, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(-7a^2be+13a^3f+ab^2d+5b^3c\right)}{18a^2b^3\left(a+bx^3\right)}+\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^2be-14a^3f+ab^2d+5b^3c\right)}{54a^{8/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^2be-14a^3f+ab^2d+5b^3c\right)}{27a^{8/3}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^2be-14a^3f+ab^2d+5b^3c\right)}{9\sqrt{3}a^{8/3}b^{10/3}}+\frac{fx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

```
[Out] (f*x)/b^3 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a*b^3*(a + b*x^3)^2)
 + ((5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(18*a^2*b^3*(a + b*x^3))
- ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(10/3)) + ((5*b^3*c + a*b^2*d +
2*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(10/3)) - ((5
*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(54*a^(8/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^3}dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} - \frac{\int \frac{-5b^3c-ab^2d+a^2be-a^3f-6ab(be-af)x^3-6ab^2fx^6}{\left(a+bx^3\right)^2}dx}{6ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} + \int \frac{2b^2\left(5b^3c+ab^2d\cdots\right.}{\cdots}$$

$$= \frac{fx}{b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} + \frac{\left(5b^3c+\cdots\right.}{\cdots}$$

$$= \frac{fx}{b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} + \frac{\left(5b^3c+\cdots\right.}{\cdots}$$

$$= \frac{fx}{b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} + \frac{\left(5b^3c+\cdots\right.}{\cdots}$$

$$= \frac{fx}{b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} + \frac{\left(5b^3c+\cdots\right.}{\cdots}$$

$$= \frac{fx}{b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} - \frac{\left(5b^3c+\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.24, size = 279, normalized size = 0.96

$$\frac{\frac{3\sqrt[3]{b}x\left(13a^3f-7a^2be+ab^2d+5b^3c\right)}{a^2\left(a+bx^3\right)}+\frac{9\sqrt[3]{b}x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a\left(a+bx^3\right)^2}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)}{a^{8/3}}-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)}{a^{8/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)}{a^{8/3}}+54\sqrt[3]{b}fx}{54b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(1/3)*f*x + (9*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a*(a +
 b*x^3)^2) + (3*b^(1/3)*(5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(a^2*
(a + b*x^3)) - (2*Sqrt[3]*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) + (2*(5*b^3*c + a*b^2*d + 2*
a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - ((5*b^3*c + a*b^2*d
 + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^
(8/3))/(54*b^(10/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^3}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3, x]
```

fricas [B] time = 0.45, size = 1184, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*e + 49*a^5
*b^2*f)*x^4 - 3*sqrt(1/3)*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e - 14*a^6*b
*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2*(5*a^2*b^
5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*l
og((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2
*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - ((5*
b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d
 + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b
*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a)
+ 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a
^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e -
14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 6*(4*a^3*b^4*
c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^4*b^6*x^6 + 2*a^5*b^5*x^3 +
 a^6*b^4), 1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*
e + 49*a^5*b^2*f)*x^4 + 6*sqrt(1/3)*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e
- 14*a^6*b*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2
*(5*a^2*b^5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*sqrt(-(-a^2*b)
^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-
a^2*b)^(1/3)/b)/a^2) - ((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^
6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3
*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(
2/3)*x - (-a^2*b)^(1/3)*a) + 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b
^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c +
 a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^
2*b)^(2/3)) + 6*(4*a^3*b^4*c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^
4*b^6*x^6 + 2*a^5*b^5*x^3 + a^6*b^4)]
```

giac [A] time = 0.20, size = 295, normalized size = 1.01

$$\frac{fx}{b^3} - \frac{\sqrt{3}\left(5\,b^3c + ab^2d - 14\,a^3f + 2\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2b^2} - \frac{\left(5\,b^3c + ab^2d - 14\,a^3f + 2\,a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2b^2} - \frac{\left(5\,b^3c + ab^2d - 14\,a^3f + 2\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3} + \frac{5\,b^4cx^4 + ab^3dx^4 + 13\,a^3bfx^4 - 7\,a^2b^2x^4e + 8\,ab^3cx - 2\,a^2b^2dx + 10\,a^4fx - 4\,a^3bxe}{18\left(bx^3 + a\right)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] f*x/b^3 - 1/27*sqrt(3)*(5*b^3*c + a*b^2*d - 14*a^3*f + 2*a^2*b*e)*arctan(1/
3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b^2) - 1/5
4*(5*b^3*c + a*b^2*d - 14*a^3*f + 2*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b^2) - 1/27*(5*b^3*c + a*b^2*d - 14*a^3*f +
2*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/18*(5*b^4*
c*x^4 + a*b^3*d*x^4 + 13*a^3*b*f*x^4 - 7*a^2*b^2*x^4*e + 8*a*b^3*c*x - 2*a^
2*b^2*d*x + 10*a^4*f*x - 4*a^3*b*x*e)/((b*x^3 + a)^2*a^2*b^3)
```

maple [B] time = 0.06, size = 539, normalized size = 1.85

$$\frac{13af\,x^4}{18\left(b\,x^3+a\right)^2b^2} + \frac{d\,x^4}{18\left(b\,x^3+a\right)^2a} + \frac{5bc\,x^4}{18\left(b\,x^3+a\right)^2a^2} - \frac{7e\,x^4}{18\left(b\,x^3+a\right)^2b} + \frac{5a^2fx}{9\left(b\,x^3+a\right)^2b^3} - \frac{2aex}{9\left(b\,x^3+a\right)^2b^2} + \frac{4cx}{9\left(b\,x^3+a\right)^2a} - \frac{dx}{9\left(b\,x^3+a\right)^2b} - \frac{14\sqrt{3}\,af\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} - \frac{14af\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} + \frac{7af\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} + \frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2} + \frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2} - \frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2} + \frac{5\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b} + \frac{5c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b} - \frac{5c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b} + \frac{2\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} + \frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} - \frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} + \frac{fx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/b^3*f*x+13/18/b^2/(b*x^3+a)^2*x^4*a*f-7/18/b/(b*x^3+a)^2*x^4*e+1/18/(b*x^
3+a)^2/a*x^4*d+5/18*b/(b*x^3+a)^2/a^2*x^4*c+5/9/b^3/(b*x^3+a)^2*a^2*f*x-2/9
/b^2/(b*x^3+a)^2*a*e*x-1/9/b/(b*x^3+a)^2*d*x+4/9/(b*x^3+a)^2/a*x*c-14/27/b^
4*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f+2/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*
```

```
e+1/27/b^2/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+5/27/b/a^2/(a/b)^(2/3)*ln(x+(a
/b)^(1/3))*c+7/27/b^4*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-1/2
7/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-1/54/b^2/a/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d-5/54/b/a^2/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))*c-14/27/b^4*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*f+2/27/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))*e+1/27/b^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*d+5/27/b/a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*c
```

maxima [A] time = 3.07, size = 291, normalized size = 1.00

$$\frac{\left(5\,b^4c+ab^3d-7\,a^2b^2e+13\,a^3bf\right)x^4+2\left(4\,ab^3c-a^2b^2d-2\,a^3be+5\,a^4f\right)x}{18\left(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3\right)}+\frac{fx}{b^3}+\frac{\sqrt{3}\left(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((5*b^4*c + a*b^3*d - 7*a^2*b^2*e + 13*a^3*b*f)*x^4 + 2*(4*a*b^3*c - a
^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)
+ f*x/b^3 + 1/27*sqrt(3)*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4*(a/b)^(2/3)) - 1/54*(
5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3))/(a^2*b^4*(a/b)^(2/3)) + 1/27*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*
f)*log(x + (a/b)^(1/3))/(a^2*b^4*(a/b)^(2/3))
```

mupad [B] time = 5.20, size = 275, normalized size = 0.94

$$\frac{\frac{x\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a}+\frac{x^4\left(13\,f\,a^3\,b-7\,e\,a^2\,b^2+d\,a\,b^3+5\,c\,b^4\right)}{18\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{f\,x}{b^3}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{8/3}\,b^{10/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{8/3}\,b^{10/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{8/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x)
```

```
[Out] ((x*(4*b^3*c + 5*a^3*f - a*b^2*d - 2*a^2*b*e))/(9*a) + (x^4*(5*b^4*c - 7*a^
2*b^2*e + a*b^3*d + 13*a^3*b*f))/(18*a^2))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3
) + (f*x)/b^3 + (log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 14*a^3*f + a*b^2*d + 2
*a^2*b*e))/(27*a^(8/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 14*a^3*f + a*b^2*d + 2*a^2*b*e))
/(27*a^(8/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*(
(3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 14*a^3*f + a*b^2*d + 2*a^2*b*e))/(27*a^(8/
3)*b^(10/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.242 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$

Optimal. Leaf size=303

$$-\frac{c}{a^3x}-\frac{x^2\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{9a^3b^2\left(a+bx^3\right)}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^3f\right.}{54a^{10/3}b^{8/3}}$$

Rubi [A] time = 0.34, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1829, 1484, 453, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(-a^2be+4a^3f-2ab^2d+5b^3c\right)}{9a^3b^2\left(a+bx^3\right)}-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be-5a^3f-2ab^2d+14b^3c\right)}{54a^{10/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-a^2be-5a^3f-2ab^2d+14b^3c\right)}{27a^{10/3}b^{8/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^2be-5a^3f-2ab^2d+14b^3c\right)}{9\sqrt{3}a^{10/3}b^{8/3}}-\frac{c}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^2*b^2*(a + b*
x^3)^2) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*x^2)/(9*a^3*b^2*(a + b
*x^3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(8/3)) + ((14*b^3*c - 2*
a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(8/3)
) - ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-((m
 - Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+2b\left(\frac{2b^3c}{a}-2b^2d-abe+a^2f\right)x^3-6ab^2fx^6}{x^2\left(a+bx^3\right)^2}\,dx}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)} + \frac{\int \frac{18ab^5c-2b^3\left(5b^3c\right.}{x^2} \cdots}{1 \cdots}$$

$$= -\frac{c}{a^3x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)} - \left(14b^3c - \cdots\right.$$

$$= -\frac{c}{a^3x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)} + \frac{\left(14b^3c - \cdots\right.}{\cdots}$$

$$= -\frac{c}{a^3x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)} + \frac{\left(14b^3c - \cdots\right.}{\cdots}$$

$$= -\frac{c}{a^3x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)} + \frac{\left(14b^3c - \cdots\right.}{\cdots}$$

$$= -\frac{c}{a^3x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)x^2}{9a^3b^2\left(a+bx^3\right)} + \frac{\left(14b^3c - \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.29, size = 286, normalized size = 0.94

$$\frac{-\frac{6\sqrt[3]{a}x^2\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{b^2\left(a+bx^3\right)} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^3f+a^2be+2ab^2d-14b^3c\right)}{b^{8/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-5a^3f-a^2be-2ab^2d+14b^3c\right)}{b^{8/3}} + \frac{9a^{4/3}x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^2\left(a+bx^3\right)^2} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^3f+a^2be+2ab^2d-14b^3c\right)}{b^{8/3}} - \frac{54\sqrt[3]{a}c}{x}}{54a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(1/3)*c)/x + (9*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)
/(b^2*(a + b*x^3)^2) - (6*a^(1/3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)
*x^2)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^
3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(8/3) - (2*(-14*b^3*c +
 2*a*b^2*d + a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(8/3) + ((-14*b
^3*c + 2*a*b^2*d + a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/b^(8/3))/(54*a^(10/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3), x]
```

fricas [B] time = 0.46, size = 1206, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e + 4*a^4*b^3*
f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a^5*b^2*f)*x^3 + 3*s
qrt(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 2*(14*
a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 + (14*a^3*b^4*c - 2*
a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3
 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt
((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((14*b^5*c - 2*a*b^
4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*d - a^3*b^2*
e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*a^5*f)*x)*(-
a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((14*b^
5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*
d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*
a^5*f)*x)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^6*x^7 + 2*a^5*b^
5*x^4 + a^6*b^4*x), -1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3
*b^4*e + 4*a^4*b^3*f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a
^5*b^2*f)*x^3 + 6*sqrt(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4*
b^3*f)*x^7 + 2*(14*a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 +
 (14*a^3*b^4*c - 2*a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*sqrt(-(-a*b^2)^(1/
3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
+ ((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2
*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4
*b*e - 5*a^5*f)*x)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^
2)^(2/3)) - 2*((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14
*a*b^4*c - 2*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3
*b^2*d - a^4*b*e - 5*a^5*f)*x)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a
^4*b^6*x^7 + 2*a^5*b^5*x^4 + a^6*b^4*x)]
```

giac [A] time = 0.21, size = 341, normalized size = 1.13

$$-\frac{\sqrt{3}\left(14\,b^{3}c-2\,ab^{2}d-5\,a^{3}f-a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}b^{2}}-\frac{c}{a^{3}x}+\frac{\left(14\,b^{3}c-2\,ab^{2}d-5\,a^{3}f-a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}b^{2}}+\frac{\left(14\,b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{4}b^{2}}-\frac{10\,b^{4}cx^{5}-4\,ab^{3}dx^{5}+8\,a^{3}bfx^{5}-2\,a^{2}b^{2}x^{5}e+13\,ab^{3}cx^{2}-7\,a^{2}b^{2}dx^{2}+5\,a^{4}fx^{2}+a^{3}bx^{2}e}{18\left(bx^{3}+a\right)^{2}a^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(14*b^3*c - 2*a*b^2*d - 5*a^3*f - a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3*b^2) - c/(a^3*x) +
1/54*(14*b^3*c - 2*a*b^2*d - 5*a^3*f - a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3*b^2) + 1/27*(14*b^3*c*(-a/b)^(1/3) - 2*a*
b^2*d*(-a/b)^(1/3) - 5*a^3*f*(-a/b)^(1/3) - a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b^2) - 1/18*(10*b^4*c*x^5 - 4*a*b^3*d*x
^5 + 8*a^3*b*f*x^5 - 2*a^2*b^2*x^5*e + 13*a*b^3*c*x^2 - 7*a^2*b^2*d*x^2 + 5
*a^4*f*x^2 + a^3*b*x^2*e)/((b*x^3 + a)^2*a^3*b^2)
```

maple [B] time = 0.12, size = 547, normalized size = 1.81

$$\frac{e\,x^{5}}{9\left(b\,x^{3}+a\right)^{2}a}+\frac{2bd\,x^{5}}{9\left(b\,x^{3}+a\right)^{2}a^{2}}-\frac{5b^{2}c\,x^{5}}{9\left(b\,x^{3}+a\right)^{2}a^{3}}-\frac{4f\,x^{5}}{9\left(b\,x^{3}+a\right)^{2}b}-\frac{5af\,x^{2}}{18\left(b\,x^{3}+a\right)^{2}b^{2}}+\frac{7d\,x^{2}}{18\left(b\,x^{3}+a\right)^{2}a}-\frac{13bc\,x^{2}}{18\left(b\,x^{3}+a\right)^{2}a^{2}}-\frac{e\,x^{2}}{18\left(b\,x^{3}+a\right)^{2}b}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}-\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{2\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{14\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{14c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}-\frac{7c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{5\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{5f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{5f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{c}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x)
```

```
[Out] -4/9/(b*x^3+a)^2/b*x^5*f+1/9/a/(b*x^3+a)^2*x^5*e+2/9/a^2/(b*x^3+a)^2*b*x^5*
d-5/9/a^3/(b*x^3+a)^2*b^2*x^5*c-5/18*a/(b*x^3+a)^2/b^2*x^2*f-1/18/(b*x^3+a)
```

```
^2/b*x^2*e+7/18/a/(b*x^3+a)^2*x^2*d-13/18/a^2/(b*x^3+a)^2*b*x^2*c-5/27/b^3/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-1/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e-
2/27/a^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d+14/27/a^3/(a/b)^(1/3)*ln(x+(a/b)
^(1/3))*c+5/54/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f+1/54/a/b
^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/27/a^2/b/(a/b)^(1/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d-7/27/a^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))*c+5/27/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*f+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))*e+2/27/a^2/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))*d-14/27/a^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))*c-1/a^3*c/x
```

maxima [A] time = 2.96, size = 300, normalized size = 0.99

$$-\frac{2\left(14\,b^4c-2\,ab^3d-a^2b^2e+4\,a^3bf\right)x^6+18\,a^2b^2c+\left(49\,ab^3c-7\,a^2b^2d+a^3be+5\,a^4f\right)x^3}{18\left(a^3b^4x^7+2\,a^4b^3x^4+a^5b^2x\right)}-\frac{\sqrt{3}\left(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(14*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^6 + 18*a^2*b^2*c
+ (49*a*b^3*c - 7*a^2*b^2*d + a^3*b*e + 5*a^4*f)*x^3)/(a^3*b^4*x^7 + 2*a^4*
b^3*x^4 + a^5*b^2*x) - 1/27*sqrt(3)*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3
*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^3*(a/b)^(1/3
)) - 1/54*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a^3*b^3*(a/b)^(1/3)) + 1/27*(14*b^3*c - 2*a*b^2*d - a^2*b
*e - 5*a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^3*(a/b)^(1/3))
```

mupad [B] time = 5.20, size = 276, normalized size = 0.91

$$-\frac{\frac{c}{a}+\frac{x^6\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+14\,c\,b^3\right)}{9\,a^3\,b}+\frac{x^3\left(5\,f\,a^3+e\,a^2\,b-7\,d\,a\,b^2+49\,c\,b^3\right)}{18\,a^2\,b^2}}{a^2\,x+2\,a\,b\,x^4+b^2\,x^7}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3\right)}{27\,a^{10/3}\,b^{8/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3\right)}{27\,a^{10/3}\,b^{8/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3\right)}{27\,a^{10/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*
a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)*b^(8/3)) - (log(b^(1/
3)*x + a^(1/3))*(5*a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)*b^
(8/3)) - (c/a + (x^6*(14*b^3*c + 4*a^3*f - 2*a*b^2*d - a^2*b*e))/(9*a^3*b)
+ (x^3*(49*b^3*c + 5*a^3*f - 7*a*b^2*d + a^2*b*e))/(18*a^2*b^2))/(a^2*x + b
^2*x^7 + 2*a*b*x^4) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(5*a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)
*b^(8/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.243 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3} \, dx$

Optimal. Leaf size=301

$$-\frac{c}{2a^3x^2}-\frac{x\left(7a^3f-a^2be-5ab^2d+11b^3c\right)}{18a^3b^2\left(a+bx^3\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-2\right.}{54a^{11/3}b}$$

Rubi [A] time = 0.33, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1829, 1484, 453, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(-a^2be+7a^3f-5ab^2d+11b^3c\right)}{18a^3b^2\left(a+bx^3\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-a^2be-2a^3f-5ab^2d+20b^3c\right)}{54a^{11/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(-a^2be-2a^3f-5ab^2d+20b^3c\right)}{27a^{11/3}b^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^2be-2a^3f-5ab^2d+20b^3c\right)}{9\sqrt{3}\,a^{11/3}b^{7/3}}-\frac{c}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -c/(2*a^3*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^2*b^2*(a + b*
x^3)^2) - ((11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)*x)/(18*a^3*b^2*(a + b
*x^3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(7/3)) - ((20*b^3*c - 5*
a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(7/3)
) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-((m
 - Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+b\left(\frac{5b^3c}{a}-5b^2d-abe+a^2f\right)x^3-6ab^2fx^6}{x^3\left(a+bx^3\right)^2}\,dx}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} + \frac{\int 18ab^5c-2b^3(1\cdots}{\cdots}$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \frac{(20b\cdots}{\cdots}$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \frac{(20b\cdots}{\cdots}$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \frac{(20b\cdots}{\cdots}$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \frac{(20b\cdots}{\cdots}$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} + \frac{(20b\cdots}{\cdots}$$

Mathematica [A] time = 0.30, size = 283, normalized size = 0.94

$$\frac{-\frac{27a^{2/3}c}{x^2} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^3f+a^2be+5ab^2d-20b^3c\right)}{b^{7/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)}{b^{7/3}} + \frac{9a^{5/3}x\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^2\left(a+bx^3\right)^2} - \frac{3a^{2/3}x\left(7a^3f-a^2be-5ab^2d+11b^3c\right)}{b^2\left(a+bx^3\right)} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^3f+a^2be+5ab^2d-20b^3c\right)}{b^{7/3}}}{54a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(2/3)*c)/x^2 + (9*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)
/(b^2*(a + b*x^3)^2) - (3*a^(2/3)*(11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f
)*x)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3
*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(7/3) + (2*(-20*b^3*c +
5*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(7/3) - ((-20*b^
3*c + 5*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/b^(7/3))/(54*a^(11/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3), x]
```

fricas [B] time = 0.46, size = 1217, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e + 7*a^5*b^
2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e + 2*a^6*b*f)*x^3 + 3*s
qrt(1/3)*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*a^4*b^3*f)*x^8 + 2*(20*
a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x^5 + (20*a^3*b^4*c - 5*
a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x
^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x -
 (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - ((20*b^5*c - 5*a*b
^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2
*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((20*b^
5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*
d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*
a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^5*x^8 + 2*a^6*
b^4*x^5 + a^7*b^3*x^2), -1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d
 - a^4*b^3*e + 7*a^5*b^2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e
 + 2*a^6*b*f)*x^3 + 6*sqrt(1/3)*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*
a^4*b^3*f)*x^8 + 2*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x
^5 + (20*a^3*b^4*c - 5*a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*sqrt((a^2*b)
^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*
b)^(1/3)/b)/a^2) - ((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 +
2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c -
5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(
2/3)*x + (a^2*b)^(1/3)*a) + 2*((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^
2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a
^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x +
(a^2*b)^(2/3)))/(a^5*b^5*x^8 + 2*a^6*b^4*x^5 + a^7*b^3*x^2)]
```

giac [A] time = 0.21, size = 312, normalized size = 1.04

$$\frac{\sqrt{3}\left(20\,b^{3}c-5\,ab^{2}d-2\,a^{3}f-a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}b}+\frac{\left(20\,b^{3}c-5\,ab^{2}d-2\,a^{3}f-a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}b}+\frac{\left(20\,b^{3}c-5\,ab^{2}d-2\,a^{3}f-a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{4}b^{2}}-\frac{20\,b^{4}cx^{6}-5\,ab^{3}dx^{6}+7\,a^{3}bfx^{6}-a^{2}b^{2}x^{6}e+32\,ab^{3}cx^{3}-8\,a^{2}b^{2}dx^{3}+4\,a^{4}fx^{3}+2\,a^{3}bx^{3}e+9\,a^{2}b^{2}c}{18\left(bx^{4}+ax\right)^{2}a^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(20*b^3*c - 5*a*b^2*d - 2*a^3*f - a^2*b*e)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3*b) + 1/54*(20*b^3*c
- 5*a*b^2*d - 2*a^3*f - a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(
(-a*b^2)^(2/3)*a^3*b) + 1/27*(20*b^3*c - 5*a*b^2*d - 2*a^3*f - a^2*b*e)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b^2) - 1/18*(20*b^4*c*x^6 - 5*a*b
^3*d*x^6 + 7*a^3*b*f*x^6 - a^2*b^2*x^6*e + 32*a*b^3*c*x^3 - 8*a^2*b^2*d*x^3
 + 4*a^4*f*x^3 + 2*a^3*b*x^3*e + 9*a^2*b^2*c)/((b*x^4 + a*x)^2*a^3*b^2)
```

maple [B] time = 0.06, size = 539, normalized size = 1.79

$$\frac{e\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}a}+\frac{5bd\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}a^{2}}-\frac{11b^{2}c\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}a^{3}}-\frac{7f\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}b}-\frac{2afx}{9\left(b\,x^{3}+a\right)^{2}b^{2}}+\frac{4dx}{9\left(b\,x^{3}+a\right)^{2}a}-\frac{7bcx}{9\left(b\,x^{3}+a\right)^{2}a^{2}}-\frac{ex}{9\left(b\,x^{3}+a\right)^{2}b}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}-\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{5\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{5d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{5d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{20\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{20c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{10c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{2\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{2f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{c}{2a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x)
```

```
[Out] -7/18/(b*x^3+a)^2/b*x^4*f+1/18/a/(b*x^3+a)^2*x^4*e+5/18/a^2/(b*x^3+a)^2*b*x
^4*d-11/18/a^3/(b*x^3+a)^2*b^2*x^4*c-2/9*a/(b*x^3+a)^2/b^2*x*f-1/9/(b*x^3+a
```

```
)^2/b*x*e+4/9/a/(b*x^3+a)^2*x*d-7/9/a^2/(b*x^3+a)^2*b*x*c+2/27/b^3/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))*f+1/27/a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+5/27/a^2
/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-20/27/a^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*
c-1/27/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-1/54/a/b^2/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-5/54/a^2/b/(a/b)^(2/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))*d+10/27/a^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))*c+2/27/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))*f+1/27/a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)*e+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*
d-20/27/a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1
/2*c/a^3/x^2
```

maxima [A] time = 3.07, size = 302, normalized size = 1.00

$$-\frac{\left(20\,b^{4}c-5\,ab^{3}d-a^{2}b^{2}e+7\,a^{3}bf\right)x^{6}+9\,a^{2}b^{2}c+2\left(16\,ab^{3}c-4\,a^{2}b^{2}d+a^{3}be+2\,a^{4}f\right)x^{3}}{18\left(a^{3}b^{4}x^{8}+2\,a^{4}b^{3}x^{5}+a^{5}b^{2}x^{2}\right)}-\frac{\sqrt{3}\left(20\,b^{3}c-5\,ab^{2}d-a^{2}be-2\,a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{3}b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(20\,b^{3}c-5\,ab^{2}d-a^{2}be-2\,a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{3}b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(20\,b^{3}c-5\,ab^{2}d-a^{2}be-2\,a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{3}b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((20*b^4*c - 5*a*b^3*d - a^2*b^2*e + 7*a^3*b*f)*x^6 + 9*a^2*b^2*c + 2
*(16*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^3)/(a^3*b^4*x^8 + 2*a^4*b
^3*x^5 + a^5*b^2*x^2) - 1/27*sqrt(3)*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^
3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^3*(a/b)^(2/
3)) + 1/54*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(a^3*b^3*(a/b)^(2/3)) - 1/27*(20*b^3*c - 5*a*b^2*d - a^2*
b*e - 2*a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^3*(a/b)^(2/3))
```

mupad [B] time = 5.16, size = 279, normalized size = 0.93

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}}-\frac{\frac{c}{2\,a}+\frac{x^3\,\left(2\,f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^2\,b^2}+\frac{x^6\,\left(7\,f\,a^3-e\,a^2\,b-5\,d\,a\,b^2+20\,c\,b^3\right)}{18\,a^3\,b}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*e))/(27*a
^(11/3)*b^(7/3)) - (c/(2*a) + (x^3*(16*b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*
e))/(9*a^2*b^2) + (x^6*(20*b^3*c + 7*a^3*f - 5*a*b^2*d - a^2*b*e))/(18*a^3*
b))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*
e))/(27*a^(11/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3)
)*((3^(1/2)*1i)/2 + 1/2)*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*e))/(27*a^
(11/3)*b^(7/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.244 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx$

Optimal. Leaf size=317

$$\frac{3bc-ad}{a^4x}-\frac{c}{4a^3x^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{54a^{13/3}b^{5/3}}$$

Rubi [A] time = 0.37, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1829, 1484, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(2a^2be+a^3f-5ab^2d+8b^3c\right)}{9a^4b\left(a+bx^3\right)}+\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(2a^2be+a^3f-14ab^2d+35b^3c\right)}{54a^{13/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(2a^2be+a^3f-14ab^2d+35b^3c\right)}{27a^{13/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^2be+a^3f-14ab^2d+35b^3c\right)}{9\sqrt{3}\,a^{13/3}b^{5/3}}+\frac{3bc-ad}{a^4x}-\frac{c}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] -c/(4*a^3*x^4) + (3*b*c - a*d)/(a^4*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*x^2)/(6*a^3*b*(a + b*x^3)^2) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f
)*x^2)/(9*a^4*b*(a + b*x^3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(13/3)*b^(
5/3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(27*a^(13/3)*b^(5/3)) + ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-((m
 - Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-2b^2\left(\frac{2b^3c}{a^2}-\frac{2b^2d}{a}+2be+af\right)x^6}{x^5\left(a+bx^3\right)^2}dx}{6ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} - \frac{\int \frac{-18a^2b^5c+18ab^5(2}{}}{}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} - \frac{\int \left(-\frac{18ab^5c}{x^5}+\frac{18b^5}{}\right.}{}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)}{9a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)}{9a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)}{9a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)}{9a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)}{9a^4b\left(a+bx^3\right)}$$

Mathematica [A] time = 0.32, size = 303, normalized size = 0.96

$$\frac{-\frac{27a^{4/3}c}{x^4}+\frac{12\sqrt[3]{a}x^2\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{b\left(a+bx^3\right)}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{b^{5/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{b^{5/3}}-\frac{18a^{4/3}x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{b\left(a+bx^3\right)^2}+\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{b^{5/3}}-\frac{108\sqrt[3]{a}(ad-3bc)}{x}}{108a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(4/3)*c)/x^4 - (108*a^(1/3)*(-3*b*c + a*d))/x - (18*a^(4/3)*(-(b^3*
c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)^2) + (12*a^(1/3)*(8*b^3
*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*Sqrt[3]*(35*b
^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]])/b^(5/3) - (4*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/
3) + b^(1/3)*x])/b^(5/3) + (2*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(108*a^(13/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3), x]
```

fricas [B] time = 0.46, size = 1254, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^9 - 27*a
^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^
6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 6*sqrt(1/3)*((35*a*b^6*c - 14*a^2*
b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^3*b^4*d + 2*
a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a^5*b^2*e + a
^6*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e + a^
3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b*f)*x^7 +
 (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b^2)^(2/3)*log(
b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c - 14*a*b^4*d
+ 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*
e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(
-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^6*b^4*x^7 + a^
7*b^3*x^4), 1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)
*x^9 - 27*a^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^
5*b^2*f)*x^6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 12*sqrt(1/3)*((35*a*b^6
*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^
3*b^4*d + 2*a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a
^5*b^2*e + a^6*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x +
(-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*((35*b^5*c - 14*a*b^4*d + 2*
a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e +
a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b
^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c
 - 14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*
d + 2*a^3*b^2*e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e +
 a^5*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^
6*b^4*x^7 + a^7*b^3*x^4)]
```

giac [A] time = 0.24, size = 357, normalized size = 1.13

$$\frac{\sqrt{3}\left(35\,b^{3}c-14\,ab^{2}d+a^{3}f+2\,a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{1}{3}}a^{4}b}-\frac{\left(35\,b^{3}c-14\,ab^{2}d+a^{3}f+2\,a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^{2}\right)^{\frac{1}{3}}a^{4}b}-\frac{\left(35\,b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{5}b}+\frac{16\,b^{4}cx^{5}-10\,ab^{3}dx^{5}+2\,a^{3}bfx^{5}+4\,a^{2}b^{2}x^{5}e+19\,ab^{3}cx^{2}-13\,a^{2}b^{2}dx^{2}-a^{4}fx^{2}+7\,a^{3}bx^{2}e}{18\left(bx^{3}+a\right)^{2}a^{4}b}+\frac{12\,bcx^{3}-4\,adx^{3}-ac}{4\,a^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(35*b^3*c - 14*a*b^2*d + a^3*f + 2*a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^4*b) - 1/54*(35*b^3*c
 - 14*a*b^2*d + a^3*f + 2*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(1/3)*a^4*b) - 1/27*(35*b^3*c*(-a/b)^(1/3) - 14*a*b^2*d*(-a/b)^(
1/3) + a^3*f*(-a/b)^(1/3) + 2*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^5*b) + 1/18*(16*b^4*c*x^5 - 10*a*b^3*d*x^5 + 2*a^3*b*f*
x^5 + 4*a^2*b^2*x^5*e + 19*a*b^3*c*x^2 - 13*a^2*b^2*d*x^2 - a^4*f*x^2 + 7*a
^3*b*x^2*e)/((b*x^3 + a)^2*a^4*b) + 1/4*(12*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^4
*x^4)
```

maple [B] time = 0.07, size = 574, normalized size = 1.81

$$\frac{fx^{5}}{9\left(bx^{3}+a\right)^{2}a}+\frac{2bex^{5}}{9\left(bx^{3}+a\right)^{2}a^{2}}-\frac{5b^{2}dx^{5}}{9\left(bx^{3}+a\right)^{2}a^{3}}+\frac{8b^{3}cx^{5}}{9\left(bx^{3}+a\right)^{2}a^{4}}+\frac{7ex^{2}}{18\left(bx^{3}+a\right)^{2}a}-\frac{13bdx^{2}}{18\left(bx^{3}+a\right)^{2}a^{2}}+\frac{19b^{2}cx^{2}}{18\left(bx^{3}+a\right)^{2}a^{3}}-\frac{fx^{2}}{18\left(bx^{3}+a\right)^{2}b}+\frac{\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^{2}}-\frac{f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^{2}}+\frac{f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^{2}}+\frac{2\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{14\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{14d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}-\frac{7d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{35\sqrt{3}\,bc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}}-\frac{35bc\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}}+\frac{35bc\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}}-\frac{d}{a^{3}x}+\frac{3bc}{a^{4}x}-\frac{c}{4a^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x)
```

```
[Out] 1/9/a/(b*x^3+a)^2*x^5*f+2/9/a^2/(b*x^3+a)^2*x^5*b*e-5/9/a^3/(b*x^3+a)^2*x^5
*b^2*d+8/9/a^4/(b*x^3+a)^2*x^5*b^3*c-1/18/(b*x^3+a)^2/b*x^2*f+7/18/a/(b*x^3
+a)^2*x^2*e-13/18/a^2/(b*x^3+a)^2*b*x^2*d+19/18/a^3/(b*x^3+a)^2*c*x^2*b^2-1
/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-2/27/a^2/b/(a/b)^(1/3)*ln(x+(a/b)
^(1/3))*e+14/27/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d-35/27/a^4*b/(a/b)^(1/3)
*ln(x+(a/b)^(1/3))*c+1/54/a/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))*f+1/27/a^2/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-7/27/a^3/(a
/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+35/54/a^4*b/(a/b)^(1/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))*f+2/27/a^2/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))*e-14/27/a^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*d+35/27/a^4*b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))*c-1/4*c/a^3/x^4-d/a^3/x+3/a^4/x*b*c
```

maxima [A] time = 3.03, size = 317, normalized size = 1.00

$$\frac{4\left(35\,b^4c-14\,ab^3d+2\,a^2b^2e+a^3bf\right)x^9+\left(245\,ab^3c-98\,a^2b^2d+14\,a^3be-2\,a^4f\right)x^6-9\,a^3bc+18\left(5\,a^2b^2c-2\,a^3bd\right)x^3}{36\left(a^4b^3x^{10}+2\,a^5b^2x^7+a^6bx^4\right)}+\frac{\sqrt{3}\left(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(4*(35*b^4*c - 14*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^9 + (245*a*b^3*c
- 98*a^2*b^2*d + 14*a^3*b*e - 2*a^4*f)*x^6 - 9*a^3*b*c + 18*(5*a^2*b^2*c -
2*a^3*b*d)*x^3)/(a^4*b^3*x^10 + 2*a^5*b^2*x^7 + a^6*b*x^4) + 1/27*sqrt(3)*(
35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(a^4*b^2*(a/b)^(1/3)) + 1/54*(35*b^3*c - 14*a*b^2*d + 2
*a^2*b*e + a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b^2*(a/b)^(1/
3)) - 1/27*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*log(x + (a/b)^(1/3))
/(a^4*b^2*(a/b)^(1/3))
```

mupad [B] time = 5.23, size = 293, normalized size = 0.92

$$-\frac{\frac{c}{4\,a}-\frac{x^9\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{9\,a^4}+\frac{x^3\left(2\,a\,d-5\,b\,c\right)}{2\,a^2}-\frac{x^6\left(-2\,f\,a^3+14\,e\,a^2\,b-98\,d\,a\,b^2+245\,c\,b^3\right)}{36\,a^3\,b}}{a^2\,x^4+2\,a\,b\,x^7+b^2\,x^{10}}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{27\,a^{13/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{27\,a^{13/3}\,b^{5/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3\right)}{27\,a^{13/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(35
*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*b^(5/3)) - (log(b^(1
/3)*x + a^(1/3))*(35*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*
b^(5/3)) - (c/(4*a) - (x^9*(35*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(9*
a^4) + (x^3*(2*a*d - 5*b*c))/(2*a^2) - (x^6*(245*b^3*c - 2*a^3*f - 98*a*b^2
*d + 14*a^2*b*e))/(36*a^3*b))/(a^2*x^4 + b^2*x^10 + 2*a*b*x^7) - (log(3^(1/
2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(35*b^3*c + a
^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*b^(5/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.245 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^3}\,dx$

Optimal. Leaf size=316

$$\frac{3bc-ad}{2a^4x^2}-\frac{c}{5a^3x^5}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(a^3f+5a^2be-20ab^2d+44b\right.}{54a^{14/3}b^{4/3}}$$

Rubi [A] time = 0.37, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1829, 1484, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(5a^2be+a^3f-11ab^2d+17b^3c\right)}{18a^4b\left(a+bx^3\right)}+\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(5a^2be+a^3f-20ab^2d+44b^3c\right)}{54a^{14/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(5a^2be+a^3f-20ab^2d+44b^3c\right)}{27a^{14/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^2be+a^3f-20ab^2d+44b^3c\right)}{9\sqrt{3}\,a^{14/3}b^{4/3}}+\frac{3bc-ad}{2a^4x^2}-\frac{c}{5a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(2*a^4*x^2) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*x)/(6*a^3*b*(a + b*x^3)^2) + ((17*b^3*c - 11*a*b^2*d + 5*a^2*b*e + a
^3*f)*x)/(18*a^4*b*(a + b*x^3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3
*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)*
b^(4/3)) + ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/(27*a^(14/3)*b^(4/3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(14/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-((m
 - Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{5b^3c}{a^2}-\frac{5b^2d}{a}+5be+af\right)x^6}{x^6\left(a+bx^3\right)^2}\,dx}{6ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} - \frac{\int \frac{-18a^2b^5c+18a}{}}{}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} - \frac{\int \left(-\frac{18ab^5c}{x^6}+\right.}{}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+\right.}{18a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+\right.}{18a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+\right.}{18a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+\right.}{18a^4b\left(a+bx^3\right)}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+\right.}{18a^4b\left(a+bx^3\right)}$$

Mathematica [A] time = 0.28, size = 299, normalized size = 0.95

$$\frac{-\frac{135a^{2/3}(ad-3bc)}{x^2}-\frac{54a^{5/3}c}{x^5}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{b^{4/3}}-\frac{10\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{b^{4/3}}-\frac{45a^{5/3}x\left(a^3f-a^2be+ab^2d-b^3c\right)}{b\left(a+bx^3\right)^2}+\frac{15a^{2/3}x\left(a^3f+5a^2be-11ab^2d+17b^3c\right)}{b\left(a+bx^3\right)}-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{b^{4/3}}}{270a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(5/3)*c)/x^5 - (135*a^(2/3)*(-3*b*c + a*d))/x^2 - (45*a^(5/3)*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)^2) + (15*a^(2/3)*(17*b^
3*c - 11*a*b^2*d + 5*a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (10*Sqrt[3]*(44*
b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(4/3) + (10*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(
1/3) + b^(1/3)*x])/b^(4/3) - (5*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(270*a^(14/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3), x]
```

fricas [B] time = 0.47, size = 1247, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^9 - 54
*a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^
6 + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 15*sqrt(1/3)*((44*a*b^6*c - 20*a^
2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^11 + 2*(44*a^2*b^5*c - 20*a^3*b^4*d +
5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d + 5*a^5*b^2*e +
 a^6*b*f)*x^5)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x
- a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(
a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a
^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8
+ (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*((44*b^5*c - 20*a*b^4*d +
 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e
 + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a
^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^4*x^11 + 2*a^7*b^3*x^8 + a^8
*b^2*x^5), 1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f
)*x^9 - 54*a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a
^6*b*f)*x^6 + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 30*sqrt(1/3)*((44*a*b^6
*c - 20*a^2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^11 + 2*(44*a^2*b^5*c - 20*a^
3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d + 5*a
^5*b^2*e + a^6*b*f)*x^5)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^
(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*((44*b^5*c - 20*a
*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a
^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)
*x^5)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(
(44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20
*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*
a^4*b*e + a^5*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^4*x^
11 + 2*a^7*b^3*x^8 + a^8*b^2*x^5)]
```

giac [A] time = 0.23, size = 310, normalized size = 0.98

$$-\frac{\sqrt{3}\left(44\,b^{3}c-20\,ab^{2}d+a^{3}f+5\,a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{2}{3}}a^{4}}-\frac{\left(44\,b^{3}c-20\,ab^{2}d+a^{3}f+5\,a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^{2}\right)^{\frac{2}{3}}a^{4}}-\frac{\left(44\,b^{3}c-20\,ab^{2}d+a^{3}f+5\,a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{5}b}+\frac{17\,b^{4}cx^{4}-11\,ab^{3}dx^{4}+a^{3}bfx^{4}+5\,a^{2}b^{2}x^{4}e+20\,ab^{3}cx-14\,a^{2}b^{2}dx-2\,a^{4}fx+8\,a^{3}bxe}{18\left(bx^{3}+a\right)^{2}a^{4}b}+\frac{15\,bcx^{3}-5\,adx^{3}-2\,ac}{10\,a^{4}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(44*b^3*c - 20*a*b^2*d + a^3*f + 5*a^2*b*e)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^4) - 1/54*(44*b^3*c
- 20*a*b^2*d + a^3*f + 5*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(2/3)*a^4) - 1/27*(44*b^3*c - 20*a*b^2*d + a^3*f + 5*a^2*b*e)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) + 1/18*(17*b^4*c*x^4 - 11*a*b^
3*d*x^4 + a^3*b*f*x^4 + 5*a^2*b^2*x^4*e + 20*a*b^3*c*x - 14*a^2*b^2*d*x - 2
*a^4*f*x + 8*a^3*b*x*e)/((b*x^3 + a)^2*a^4*b) + 1/10*(15*b*c*x^3 - 5*a*d*x^
3 - 2*a*c)/(a^4*x^5)
```

maple [B] time = 0.06, size = 566, normalized size = 1.79

$$\frac{f\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}a}+\frac{5be\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}a^{2}}-\frac{11b^{2}d\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}a^{3}}+\frac{17b^{3}c\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}a^{4}}+\frac{4ex}{9\left(b\,x^{3}+a\right)^{2}a}-\frac{7bdx}{9\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{10b^{2}cx}{9\left(b\,x^{3}+a\right)^{2}a^{3}}-\frac{fx}{9\left(b\,x^{3}+a\right)^{2}b}+\frac{\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}-\frac{f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{5\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{5e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{5e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{20\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{20d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{10d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{44\sqrt{3}\,bc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}+\frac{44bc\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}-\frac{22bc\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}-\frac{d}{2a^{3}x^{2}}+\frac{3bc}{2a^{4}x^{2}}-\frac{c}{5a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x)
```

```
[Out] 1/18/a/(b*x^3+a)^2*x^4*f+5/18/a^2/(b*x^3+a)^2*x^4*b*e-11/18/a^3/(b*x^3+a)^2
*x^4*b^2*d+17/18/a^4/(b*x^3+a)^2*x^4*b^3*c-1/9/(b*x^3+a)^2/b*x*f+4/9/a/(b*x
^3+a)^2*x*e-7/9/a^2/(b*x^3+a)^2*b*x*d+10/9/a^3/(b*x^3+a)^2*x*b^2*c+1/27/a/b
^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f+5/27/a^2/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))
*e-20/27/a^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+44/27/a^4*b/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*c-1/54/a/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-5/
54/a^2/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+10/27/a^3/(a/b)^(2
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d-22/27/a^4*b/(a/b)^(2/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))*c+1/27/a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))*f+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*e-20/27/a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))*d+44/27/a^4*b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))*c-1/5/a^3*c/x^5-1/2*d/a^3/x^2+3/2/a^4/x^2*b*c
```

maxima [A] time = 3.05, size = 318, normalized size = 1.01

$$\frac{5\left(44\,b^4c-20\,ab^3d+5\,a^2b^2e+a^3bf\right)x^9+2\left(176\,ab^3c-80\,a^2b^2d+20\,a^3be-5\,a^4f\right)x^6-18\,a^3bc+9\left(11\,a^2b^2c-5\,a^3bd\right)x^3}{90\left(a^4b^3x^{11}+2\,a^5b^2x^8+a^6bx^5\right)}+\frac{\sqrt{3}\left(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/90*(5*(44*b^4*c - 20*a*b^3*d + 5*a^2*b^2*e + a^3*b*f)*x^9 + 2*(176*a*b^3*
c - 80*a^2*b^2*d + 20*a^3*b*e - 5*a^4*f)*x^6 - 18*a^3*b*c + 9*(11*a^2*b^2*c
 - 5*a^3*b*d)*x^3)/(a^4*b^3*x^11 + 2*a^5*b^2*x^8 + a^6*b*x^5) + 1/27*sqrt(3
)*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a^4*b^2*(a/b)^(2/3)) - 1/54*(44*b^3*c - 20*a*b^2*d
+ 5*a^2*b*e + a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b^2*(a/b)^
(2/3)) + 1/27*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*log(x + (a/b)^(1/
3))/(a^4*b^2*(a/b)^(2/3))
```

mupad [B] time = 5.20, size = 293, normalized size = 0.93

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}-\frac{\frac{c}{5\,a}-\frac{x^9\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{18\,a^4}+\frac{x^3\,\left(5\,a\,d-11\,b\,c\right)}{10\,a^2}-\frac{x^6\,\left(-5\,f\,a^3+20\,e\,a^2\,b-80\,d\,a\,b^2+176\,c\,b^3\right)}{45\,a^3\,b}}{a^2\,x^5+2\,a\,b\,x^8+b^2\,x^{11}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*
a^(14/3)*b^(4/3)) - (c/(5*a) - (x^9*(44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*
b*e))/(18*a^4) + (x^3*(5*a*d - 11*b*c))/(10*a^2) - (x^6*(176*b^3*c - 5*a^3*
f - 80*a*b^2*d + 20*a^2*b*e))/(45*a^3*b))/(a^2*x^5 + b^2*x^11 + 2*a*b*x^8)
+ (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(
44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*a^(14/3)*b^(4/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(44*b^3*c
+ a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*a^(14/3)*b^(4/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.246 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3}\,dx$

Optimal. Leaf size=343

$$\frac{3bc-ad}{4a^4x^4}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{a^5x}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{54a^{16/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{\right.}}{}$$

Rubi [A] time = 0.57, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(5a^2be-2a^3f-8ab^2d+11b^3c\right)}{9a^5\left(a+bx^3\right)}-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(14a^2be-2a^3f-35ab^2d+65b^3c\right)}{54a^{16/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(14a^2be-2a^3f-35ab^2d+65b^3c\right)}{27a^{16/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(14a^2be-2a^3f-35ab^2d+65b^3c\right)}{9\sqrt{3}\,a^{16/3}b^{2/3}}-\frac{a^2e-3abd+6b^2c}{a^5x}+\frac{3bc-ad}{4a^4x^4}-\frac{c}{7a^3x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(4*a^4*x^4) - (6*b^2*c - 3*a*b*d + a^2*e)/(a
^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^4*(a + b*x^3)^2) - (
(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^2)/(9*a^5*(a + b*x^3)) + ((6
5*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(16/3)*b^(2/3)) + ((65*b^3*c - 35*a*b^2*d
 + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(16/3)*b^(2/3)) -
((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(16/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{4b^3\left(b^3c-ab^2d+a^2be-a^3f\right)}{a^3}}{x^8\left(a+bx^3\right)^2}}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)x^2}{9a^5\left(a+bx^3\right)} + \frac{\int \frac{18b^6c-18b^6(}{}}{}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)x^2}{9a^5\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^6c}{ax^8}+18\right.}{}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-\right.}{}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-\right.}{}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-\right.}{}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-\right.}{}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{4a^4x^4} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-\right.}{}$$

Mathematica [A] time = 0.31, size = 328, normalized size = 0.96

$$\frac{-\frac{189a^{4/3}(ad-3bc)}{x^4}-\frac{108a^{7/3}c}{x^7}-\frac{756\sqrt[3]{a}\left(a^2e-3abd+6b^2c\right)}{x}+\frac{84\sqrt[3]{a}x^2\left(2a^3f-5a^2be+8ab^2d-11b^3c\right)}{a+bx^3}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{b^{2/3}}+\frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{b^{2/3}}+\frac{126a^{4/3}x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}+\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^3f-14a^2be+35ab^2d-65b^3c\right)}{b^{2/3}}}{756a^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```

```
[Out] ((-108*a^(7/3)*c)/x^7 - (189*a^(4/3)*(-3*b*c + a*d))/x^4 - (756*a^(1/3)*(6*
b^2*c - 3*a*b*d + a^2*e))/x + (126*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x^2)/(a + b*x^3)^2 + (84*a^(1/3)*(-11*b^3*c + 8*a*b^2*d - 5*a^2*b*e
+ 2*a^3*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(65*b^3*c - 35*a*b^2*d + 14*a^2*b
*e - 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(6
5*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2
/3) + (14*(-65*b^3*c + 35*a*b^2*d - 14*a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(756*a^(16/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3), x]
```

fricas [B] time = 0.47, size = 1340, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="fricas")
[Out] [-1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^12 +
 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^9 + 108*a
^5*b^2*c + 54*(65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e)*x^6 - 27*(13*a^4
*b^3*c - 7*a^5*b^2*d)*x^3 + 42*sqrt(1/3)*((65*a*b^6*c - 35*a^2*b^5*d + 14*a
^3*b^4*e - 2*a^4*b^3*f)*x^13 + 2*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*
e - 2*a^5*b^2*f)*x^10 + (65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e - 2*a^6
*b*f)*x^7)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
 + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*
b^2)^(2/3)*x)/(b*x^3 + a)) + 14*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*
a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*
x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(
2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((65*b^5*c - 3
5*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d
 + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b
*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^6*b^4*x^13
+ 2*a^7*b^3*x^10 + a^8*b^2*x^7), -1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14
*a^3*b^4*e - 2*a^4*b^3*f)*x^12 + 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*
b^3*e - 2*a^5*b^2*f)*x^9 + 108*a^5*b^2*c + 54*(65*a^3*b^4*c - 35*a^4*b^3*d
+ 14*a^5*b^2*e)*x^6 - 27*(13*a^4*b^3*c - 7*a^5*b^2*d)*x^3 + 84*sqrt(1/3)*((
65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^13 + 2*(65*a^2*b^
5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^10 + (65*a^3*b^4*c - 35*
a^4*b^3*d + 14*a^5*b^2*e - 2*a^6*b*f)*x^7)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(s
qrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*((65*b^5*
c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*
b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*
a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (
-a*b^2)^(2/3)) - 28*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x
^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a
^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b*x
 - (-a*b^2)^(1/3)))/(a^6*b^4*x^13 + 2*a^7*b^3*x^10 + a^8*b^2*x^7)]
```

giac [A] time = 0.31, size = 380, normalized size = 1.11

$$-\frac{\sqrt{3}\left(65\,b^{3}c-35\,ab^{2}d-2\,a^{3}f+14\,a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{1}{3}}a^{5}}+\frac{\left(65\,b^{3}c-35\,ab^{2}d-2\,a^{3}f+14\,a^{2}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^{2}\right)^{\frac{1}{3}}a^{5}}+\frac{\left(65\,b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-35\,ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+14\,a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{6}}-\frac{22\,b^{4}cx^{5}-16\,ab^{3}dx^{5}-4\,a^{3}bfx^{5}+10\,a^{2}b^{2}x^{5}e+25\,ab^{3}cx^{2}-19\,a^{2}b^{2}dx^{2}-7\,a^{4}fx^{2}+13\,a^{3}bx^{2}e}{18\left(bx^{3}+a\right)^{2}a^{5}}-\frac{168\,b^{2}cx^{6}-84\,abdx^{6}+28\,a^{2}x^{6}e-21\,abcx^{3}+7\,a^{2}dx^{3}+4\,a^{2}c}{28\,a^{5}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="giac")
[Out] -1/27*sqrt(3)*(65*b^3*c - 35*a*b^2*d - 2*a^3*f + 14*a^2*b*e)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^5) + 1/54*(65*b^3
*c - 35*a*b^2*d - 2*a^3*f + 14*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(1/3)*a^5) + 1/27*(65*b^3*c*(-a/b)^(1/3) - 35*a*b^2*d*(-a/b
)^(1/3) - 2*a^3*f*(-a/b)^(1/3) + 14*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(
abs(x - (-a/b)^(1/3)))/a^6 - 1/18*(22*b^4*c*x^5 - 16*a*b^3*d*x^5 - 4*a^3*b*
f*x^5 + 10*a^2*b^2*x^5*e + 25*a*b^3*c*x^2 - 19*a^2*b^2*d*x^2 - 7*a^4*f*x^2
+ 13*a^3*b*x^2*e)/((b*x^3 + a)^2*a^5) - 1/28*(168*b^2*c*x^6 - 84*a*b*d*x^6
+ 28*a^2*x^6*e - 21*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^5*x^7)
```

maple [B] time = 0.07, size = 611, normalized size = 1.78

$$\frac{2bf x^5}{9(bx^3+a)^2 a^2}-\frac{5b^2e x^5}{9(bx^3+a)^2 a^3}+\frac{8b^3d x^5}{9(bx^3+a)^2 a^4}-\frac{11b^4c x^5}{9(bx^3+a)^2 a^5}+\frac{7f x^2}{18(bx^3+a)^2 a}-\frac{13be x^2}{18(bx^3+a)^2 a^2}+\frac{19b^2d x^2}{18(bx^3+a)^2 a^3}-\frac{25b^3c x^2}{18(bx^3+a)^2 a^4}+\frac{2\sqrt{3}\, f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2 b}-\frac{2f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2 b}+\frac{f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2 b}-\frac{14\sqrt{3}\, e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3}+\frac{14e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3}-\frac{7e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3}+\frac{35\sqrt{3}\, bd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^4}-\frac{35bd\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^4}+\frac{35bd\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} a^4}-\frac{65\sqrt{3}\, b^2c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^5}+\frac{65b^2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^5}-\frac{65b^2c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} a^5}-\frac{e}{a^3x}+\frac{3bd}{a^4x}-\frac{6b^2c}{a^5x}-\frac{d}{4a^3x^4}+\frac{3bc}{4a^4x^4}-\frac{c}{7a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x)
```

```
[Out] 2/9/a^2/(b*x^3+a)^2*x^5*b*f-5/9/a^3/(b*x^3+a)^2*x^5*e*b^2+8/9/a^4/(b*x^3+a)
^2*x^5*d*b^3-11/9/a^5/(b*x^3+a)^2*x^5*c*b^4-13/18/a^2/(b*x^3+a)^2*x^2*b*e+1
9/18/a^3/(b*x^3+a)^2*x^2*b^2*d-25/18/a^4/(b*x^3+a)^2*x^2*b^3*c-14/27/a^3*e*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-35/27/a^4*b*d/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))+35/54/a^4*b*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+65/27/a^5*b^2*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-65/54/a^5*b^2*c
/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+3/4/a^4/x^4*b*c+3/a^4/x*b*d-
6/a^5/x*b^2*c+7/18/a/(b*x^3+a)^2*x^2*f+14/27/a^3*e/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))-7/27/a^3*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+35/27/a^4*b*
d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-65/27/a^5*b^2
*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/27/a^2*f*3
^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/7/a^3*c/x^7-
1/4/a^3/x^4*d-e/a^3/x-2/27/a^2*f/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/27/a^2*f
/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
```

maxima [A] time = 3.05, size = 343, normalized size = 1.00

$$-\frac{28\left(65b^4c-35ab^3d+14a^2b^2e-2a^3bf\right)x^{12}+49\left(65ab^3c-35a^2b^2d+14a^3be-2a^4f\right)x^9+18\left(65a^2b^2c-35a^3bd+14a^4e\right)x^6+36a^4c-9\left(13a^3bc-7a^4d\right)x^3}{252\left(a^5b^2x^{13}+2a^6bx^{10}+a^7x^7\right)}-\frac{\sqrt{3}\left(65b^3c-35ab^2d+14a^2be-2a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(65b^3c-35ab^2d+14a^2be-2a^3f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(65b^3c-35ab^2d+14a^2be-2a^3f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/252*(28*(65*b^4*c - 35*a*b^3*d + 14*a^2*b^2*e - 2*a^3*b*f)*x^12 + 49*(65
*a*b^3*c - 35*a^2*b^2*d + 14*a^3*b*e - 2*a^4*f)*x^9 + 18*(65*a^2*b^2*c - 35
*a^3*b*d + 14*a^4*e)*x^6 + 36*a^4*c - 9*(13*a^3*b*c - 7*a^4*d)*x^3)/(a^5*b^
2*x^13 + 2*a^6*b*x^10 + a^7*x^7) - 1/27*sqrt(3)*(65*b^3*c - 35*a*b^2*d + 14
*a^2*b*e - 2*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^
5*b*(a/b)^(1/3)) - 1/54*(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*b*(a/b)^(1/3)) + 1/27*(65*b^3*c - 3
5*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*log(x + (a/b)^(1/3))/(a^5*b*(a/b)^(1/3))
```

mupad [B] time = 5.26, size = 321, normalized size = 0.94

$$\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(-2fa^3+14ea^2b-35dab^2+65cb^3\right)}{27a^{16/3}b^{2/3}}-\frac{\frac{c}{7a}+\frac{7x^9\left(-2fa^3+14ea^2b-35dab^2+65cb^3\right)}{36a^4}+\frac{x^3(7ad-13bc)}{28a^2}+\frac{x^6\left(14ea^2-35dab+65cb^2\right)}{14a^3}+\frac{bx^{12}\left(-2fa^3+14ea^2b-35dab^2+65cb^3\right)}{9a^5}}{a^2x^7+2abx^{10}+b^2x^{13}}-\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-2fa^3+14ea^2b-35dab^2+65cb^3\right)}{27a^{16/3}b^{2/3}}+\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-2fa^3+14ea^2b-35dab^2+65cb^3\right)}{27a^{16/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(65*b^3*c - 2*a^3*f - 35*a*b^2*d + 14*a^2*b*e))/(
27*a^(16/3)*b^(2/3)) - (c/(7*a) + (7*x^9*(65*b^3*c - 2*a^3*f - 35*a*b^2*d +
 14*a^2*b*e))/(36*a^4) + (x^3*(7*a*d - 13*b*c))/(28*a^2) + (x^6*(65*b^2*c +
 14*a^2*e - 35*a*b*d))/(14*a^3) + (b*x^12*(65*b^3*c - 2*a^3*f - 35*a*b^2*d
+ 14*a^2*b*e))/(9*a^5))/(a^2*x^7 + b^2*x^13 + 2*a*b*x^10) - (log(3^(1/2)*a^
(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(65*b^3*c - 2*a^3*
f - 35*a*b^2*d + 14*a^2*b*e))/(27*a^(16/3)*b^(2/3)) + (log(3^(1/2)*a^(1/3)*
1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(65*b^3*c - 2*a^3*f - 35
*a*b^2*d + 14*a^2*b*e))/(27*a^(16/3)*b^(2/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.247 $\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=341

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{8a^3x^8}-\frac{a^2e-3abd+6b^2c}{2a^5x^2}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(-5a^3f+20a^2be-44ab^2d+77b^3c\right)}{27a^{17/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\right.}{}$$

Rubi [A] time = 0.55, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(11a^2be-5a^3f-17ab^2d+23b^3c\right)}{18a^5\left(a+bx^3\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(20a^2be-5a^3f-44ab^2d+77b^3c\right)}{54a^{17/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(20a^2be-5a^3f-44ab^2d+77b^3c\right)}{27a^{17/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(20a^2be-5a^3f-44ab^2d+77b^3c\right)}{9\sqrt{3}\,a^{17/3}\sqrt[3]{b}}-\frac{a^2e-3abd+6b^2c}{2a^5x^2}+\frac{3bc-ad}{5a^4x^5}-\frac{c}{8a^3x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

```
[Out] -c/(8*a^3*x^8) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/(2
*a^5*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^4*(a + b*x^3)^2) -
 ((23*b^3*c - 17*a*b^2*d + 11*a^2*b*e - 5*a^3*f)*x)/(18*a^5*(a + b*x^3)) +
((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(17/3)*b^(1/3)) - ((77*b^3*c - 44*a*b^
2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(17/3)*b^(1/3))
 + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(17/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{5b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x}{a^3}}{x^9\left(a+bx^3\right)^2}}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+11a^2be-5a^3f\right)x}{18a^5\left(a+bx^3\right)} + \int \frac{18b^6c-18b^6\left(\right.}{}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+11a^2be-5a^3f\right)x}{18a^5\left(a+bx^3\right)} + \int\left(\frac{18b^6c}{ax^9}+18\right.$$

$$= -\frac{c}{8a^3x^8}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{2a^5x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2}-\frac{\left(23b^3c-1\right.}{}$$

$$= -\frac{c}{8a^3x^8}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{2a^5x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2}-\frac{\left(23b^3c-1\right.}{}$$

$$= -\frac{c}{8a^3x^8}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{2a^5x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2}-\frac{\left(23b^3c-1\right.}{}$$

$$= -\frac{c}{8a^3x^8}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{2a^5x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2}-\frac{\left(23b^3c-1\right.}{}$$

$$= -\frac{c}{8a^3x^8}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{2a^5x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2}-\frac{\left(23b^3c-1\right.}{}$$

Mathematica [A] time = 0.35, size = 324, normalized size = 0.95

$$\frac{-\frac{216a^{5/3}(ad-3bc)}{x^5}-\frac{135a^{8/3}c}{x^8}-\frac{540a^{2/3}\left(a^2e-3abd+6b^2c\right)}{x^2}+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^3f-20a^2be+44ab^2d-77b^3c\right)}{\sqrt[3]{b}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-5a^3f+20a^2be-44ab^2d+77b^3c\right)}{\sqrt[3]{b}}+\frac{180a^{5/3}x\left(a^3f-a^2be+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}+\frac{60a^{2/3}x\left(5a^3f-11a^2be+17ab^2d-23b^3c\right)}{a+bx^3}+\frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^3f+20a^2be-44ab^2d+77b^3c\right)}{\sqrt[3]{b}}}{1080a^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

```
[Out] ((-135*a^(8/3)*c)/x^8 - (216*a^(5/3)*(-3*b*c + a*d))/x^5 - (540*a^(2/3)*(6*
b^2*c - 3*a*b*d + a^2*e))/x^2 + (180*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e
+ a^3*f)*x)/(a + b*x^3)^2 + (60*a^(2/3)*(-23*b^3*c + 17*a*b^2*d - 11*a^2*b*
e + 5*a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(77*b^3*c - 44*a*b^2*d + 20*a^2*b
*e - 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-
77*b^3*c + 44*a*b^2*d - 20*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(
1/3) + (20*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(1080*a^(17/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3), x]
```

fricas [B] time = 0.46, size = 1317, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/1080*(60*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e - 5*a^5*b^2*f)*x^1
2 + 96*(77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^9 + 135*a
^6*b*c + 27*(77*a^4*b^3*c - 44*a^5*b^2*d + 20*a^6*b*e)*x^6 - 54*(7*a^5*b^2*
c - 4*a^6*b*d)*x^3 + 60*sqrt(1/3)*((77*a*b^6*c - 44*a^2*b^5*d + 20*a^3*b^4*
e - 5*a^4*b^3*f)*x^14 + 2*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e - 5*a
^5*b^2*f)*x^11 + (77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x
^8)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*s
qrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3
)/b))/(b*x^3 + a)) - 20*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*
f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (
77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*((77*b^5*c - 44*a*b^4*d +
 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b
^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*
f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^7*b^3*x^14 + 2*a^8*b^2
*x^11 + a^9*b*x^8), -1/1080*(60*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e
 - 5*a^5*b^2*f)*x^12 + 96*(77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a
^6*b*f)*x^9 + 135*a^6*b*c + 27*(77*a^4*b^3*c - 44*a^5*b^2*d + 20*a^6*b*e)*x
^6 - 54*(7*a^5*b^2*c - 4*a^6*b*d)*x^3 + 120*sqrt(1/3)*((77*a*b^6*c - 44*a^2
*b^5*d + 20*a^3*b^4*e - 5*a^4*b^3*f)*x^14 + 2*(77*a^2*b^5*c - 44*a^3*b^4*d
+ 20*a^4*b^3*e - 5*a^5*b^2*f)*x^11 + (77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*
b^2*e - 5*a^6*b*f)*x^8)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(
2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*((77*b^5*c - 44*a
*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d +
20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e
- 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 40*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77
*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c -
44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)
^(2/3)))/(a^7*b^3*x^14 + 2*a^8*b^2*x^11 + a^9*b*x^8)]
```

giac [A] time = 0.23, size = 394, normalized size = 1.16

$$\frac{\left(77 b^{3} c-44 a b^{2} d-5 a^{3} f+20 a^{2} b e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27 a^{6}}-\frac{\sqrt{3}\left(77\left(-a b^{2}\right)^{\frac{1}{3}} b^{3} c-44\left(-a b^{2}\right)^{\frac{1}{3}} a b^{2} d-5\left(-a b^{2}\right)^{\frac{1}{3}} a^{3} f+20\left(-a b^{2}\right)^{\frac{1}{3}} a^{2} b e\right) \arctan\left(\frac{\sqrt{3}\left(2 x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 a^{6} b}-\frac{\left(77\left(-a b^{2}\right)^{\frac{1}{3}} b^{3} c-44\left(-a b^{2}\right)^{\frac{1}{3}} a b^{2} d-5\left(-a b^{2}\right)^{\frac{1}{3}} a^{3} f+20\left(-a b^{2}\right)^{\frac{1}{3}} a^{2} b e\right) \log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 a^{6} b}-\frac{23 b^{4} c x^{4}-17 a b^{3} d x^{4}-5 a^{3} b f x^{4}+11 a^{2} b^{2} x^{4} e+26 a b^{3} c x-20 a^{2} b^{2} d x-8 a^{4} f x+14 a^{3} b x e}{18\left(b x^{3}+a\right)^{2} a^{5}}-\frac{120 b^{2} c x^{6}-60 a b d x^{6}+20 a^{2} x^{6} e-24 a b c x^{3}+8 a^{2} d x^{3}+5 a^{2} c}{40 a^{5} x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(77*b^3*c - 44*a*b^2*d - 5*a^3*f + 20*a^2*b*e)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/a^6 - 1/27*sqrt(3)*(77*(-a*b^2)^(1/3)*b^3*c - 44*(-a*b^2)^
(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 20*(-a*b^2)^(1/3)*a^2*b*e)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^6*b) - 1/54*(77*(-a*b^2)^
(1/3)*b^3*c - 44*(-a*b^2)^(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 20*(-a*b
^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) - 1/18*
(23*b^4*c*x^4 - 17*a*b^3*d*x^4 - 5*a^3*b*f*x^4 + 11*a^2*b^2*x^4*e + 26*a*b^
3*c*x - 20*a^2*b^2*d*x - 8*a^4*f*x + 14*a^3*b*x*e)/((b*x^3 + a)^2*a^5) - 1/
40*(120*b^2*c*x^6 - 60*a*b*d*x^6 + 20*a^2*x^6*e - 24*a*b*c*x^3 + 8*a^2*d*x^
3 + 5*a^2*c)/(a^5*x^8)
```

maple [B] time = 0.06, size = 603, normalized size = 1.77

$$\frac{5 b f x^{4}}{18 \left(b x^{3}+a\right)^{2} a^{2}}-\frac{11 b^{2} e x^{4}}{18 \left(b x^{3}+a\right)^{2} a^{3}}+\frac{17 b^{3} d x^{4}}{18 \left(b x^{3}+a\right)^{2} a^{4}}-\frac{23 b^{4} c x^{4}}{18 \left(b x^{3}+a\right)^{2} a^{5}}+\frac{4 f x}{9 \left(b x^{3}+a\right)^{2} a}-\frac{7 b e x}{9 \left(b x^{3}+a\right)^{2} a^{2}}+\frac{10 b^{2} d x}{9 \left(b x^{3}+a\right)^{2} a^{3}}-\frac{13 b^{3} c x}{9 \left(b x^{3}+a\right)^{2} a^{4}}+\frac{5 \sqrt{3}\, f \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{2} b}+\frac{5 f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{2} b}-\frac{5 f \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{2} b}-\frac{20 \sqrt{3}\, e \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{3}}-\frac{20 e \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{3}}+\frac{10 e \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{3}}+\frac{44 \sqrt{3}\, b d \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{4}}+\frac{44 b d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{4}}-\frac{22 b d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{4}}-\frac{77 \sqrt{3}\, b^{2} c \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{5}}-\frac{77 b^{2} c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{5}}+\frac{77 b^{2} c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^{5}}-\frac{e}{2 a^{3} x^{2}}+\frac{3 b d}{2 a^{4} x^{2}}-\frac{3 b^{2} c}{a^{5} x^{2}}-\frac{d}{5 a^{3} x^{5}}+\frac{3 b c}{5 a^{4} x^{5}}-\frac{c}{8 a^{3} x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x)
```

```
[Out] 10/27/a^3*e/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-3/a^5/x^2*b^2*c+3
/5/a^4/x^5*b*c+4/9/a/(b*x^3+a)^2*f*x-20/27/a^3*e/(a/b)^(2/3)*ln(x+(a/b)^(1/
3))+3/2/a^4/x^2*b*d-1/2/a^3/x^2*e+5/27/a^2*f/b/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+44/27/a^4*b*d/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-77/27/a^5*b^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/5/a^3/x^5*d+5/18/a^2/(b*x^3+a)^2*x^4*b*f-
11/18/a^3/(b*x^3+a)^2*x^4*b^2*e+17/18/a^4/(b*x^3+a)^2*x^4*b^3*d-23/18/a^5/(
b*x^3+a)^2*x^4*b^4*c-20/27/a^3*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))+44/27/a^4*b*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-22/27/a^4*b*d
/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-77/27/a^5*b^2*c/(a/b)^(2/3)*
ln(x+(a/b)^(1/3))+77/54/a^5*b^2*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+5/27/a^2*f/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-5/54/a^2*f/b/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+10/9/a^3/(b*x^3+a)^2*b^2*d*x-13/9/a^4/(b*x^
3+a)^2*b^3*c*x-7/9/a^2/(b*x^3+a)^2*b*e*x-1/8*c/a^3/x^8
```

maxima [A] time = 2.95, size = 343, normalized size = 1.01

$$-\frac{20 \left(77 b^{4} c-44 a b^{3} d+20 a^{2} b^{2} e-5 a^{3} b f\right) x^{12}+32 \left(77 a b^{3} c-44 a^{2} b^{2} d+20 a^{3} b e-5 a^{4} f\right) x^{9}+9 \left(77 a^{2} b^{2} c-44 a^{3} b d+20 a^{4} e\right) x^{6}+45 a^{4} c-18 \left(7 a^{3} b c-4 a^{4} d\right) x^{3}}{360 \left(a^{5} b^{2} x^{14}+2 a^{6} b x^{11}+a^{7} x^{8}\right)}-\frac{\sqrt{3} \left(77 b^{3} c-44 a b^{2} d+20 a^{2} b e-5 a^{3} f\right) \arctan\left(\frac{\sqrt{3}\left(2 x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 a^{5} b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(77 b^{3} c-44 a b^{2} d+20 a^{2} b e-5 a^{3} f\right) \log\left(x^{2}-x \left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 a^{5} b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(77 b^{3} c-44 a b^{2} d+20 a^{2} b e-5 a^{3} f\right) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 a^{5} b \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/360*(20*(77*b^4*c - 44*a*b^3*d + 20*a^2*b^2*e - 5*a^3*b*f)*x^12 + 32*(77
*a*b^3*c - 44*a^2*b^2*d + 20*a^3*b*e - 5*a^4*f)*x^9 + 9*(77*a^2*b^2*c - 44*
a^3*b*d + 20*a^4*e)*x^6 + 45*a^4*c - 18*(7*a^3*b*c - 4*a^4*d)*x^3)/(a^5*b^2
*x^14 + 2*a^6*b*x^11 + a^7*x^8) - 1/27*sqrt(3)*(77*b^3*c - 44*a*b^2*d + 20*
a^2*b*e - 5*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5
*b*(a/b)^(2/3)) + 1/54*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*b*(a/b)^(2/3)) - 1/27*(77*b^3*c - 44
*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*log(x + (a/b)^(1/3))/(a^5*b*(a/b)^(2/3))
```

mupad [B] time = 5.22, size = 321, normalized size = 0.94

$$-\frac{\frac{c}{8 a}+\frac{4 x^{9}\left(-5 f a^{3}+20 e a^{2} b-44 d a b^{2}+77 c b^{3}\right)}{45 a^{4}}+\frac{x^{3}\left(4 a d-7 b c\right)}{20 a^{2}}+\frac{x^{6}\left(20 e a^{2}-44 d a b+77 c b^{2}\right)}{40 a^{3}}+\frac{b x^{12}\left(-5 f a^{3}+20 e a^{2} b-44 d a b^{2}+77 c b^{3}\right)}{18 a^{5}}}{a^{2} x^{8}+2 a b x^{11}+b^{2} x^{14}}-\frac{\ln\left(b^{1/3} x+a^{1/3}\right)\left(-5 f a^{3}+20 e a^{2} b-44 d a b^{2}+77 c b^{3}\right)}{27 a^{17/3} b^{1/3}}-\frac{\ln\left(2 b^{1/3} x-a^{1/3}+\sqrt{3}\, a^{1/3}\, 1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(-5 f a^{3}+20 e a^{2} b-44 d a b^{2}+77 c b^{3}\right)}{27 a^{17/3} b^{1/3}}+\frac{\ln\left(a^{1/3}-2 b^{1/3} x+\sqrt{3}\, a^{1/3}\, 1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(-5 f a^{3}+20 e a^{2} b-44 d a b^{2}+77 c b^{3}\right)}{27 a^{17/3} b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(77
*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(17/3)*b^(1/3)) - (log(b
^(1/3)*x + a^(1/3))*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(
17/3)*b^(1/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)
*1i)/2 - 1/2)*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(17/3)*
b^(1/3)) - (c/(8*a) + (4*x^9*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e)
)/(45*a^4) + (x^3*(4*a*d - 7*b*c))/(20*a^2) + (x^6*(77*b^2*c + 20*a^2*e - 4
4*a*b*d))/(40*a^3) + (b*x^12*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e)
)/(18*a^5))/(a^2*x^8 + b^2*x^14 + 2*a*b*x^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.248 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=381

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{10a^3x^{10}}-\frac{a^2e-3abd+6b^2c}{4a^5x^4}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{27a^{19/3}}-\frac{\sqrt[3]{b}\tan^{-1}\left(\right.}{}$$

Rubi [A] time = 0.71, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$\frac{bx^2\left(8a^2be-5a^3f-11ab^2d+14b^3c\right)}{9a^6\left(a+bx^3\right)}+\frac{bx^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^2be-14a^3f-65ab^2d+104b^3c\right)}{54a^{19/3}}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{a^6x}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(35a^2be-14a^3f-65ab^2d+104b^3c\right)}{27a^{19/3}}-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(35a^2be-14a^3f-65ab^2d+104b^3c\right)}{9\sqrt{3}a^{19/3}}-\frac{a^2e-3abd+6b^2c}{4a^5x^4}+\frac{3bc-ad}{7a^4x^7}-\frac{c}{10a^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] -c/(10*a^3*x^10) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/
(4*a^5*x^4) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^5*(a + b*x^3)^2) + (b*(14*b^3*c -
11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*x^2)/(9*a^6*(a + b*x^3)) - (b^(1/3)*(104*
b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/
(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(19/3)) - (b^(1/3)*(104*b^3*c - 65*a*b^2*d
 + 35*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(19/3)) + (b^(1/3
)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(19/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^5\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab^2d+a^2be-a^3f\right)}{a^3}}{x^{11}\left(a+bx^3\right)^2}}{6ab^3}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^5\left(a+bx^3\right)^2} + \frac{b\left(14b^3c-11ab^2d+8a^2be-5a^3f\right)x^2}{9a^6\left(a+bx^3\right)} + \frac{\int \frac{18b^7c-18b}{}}{}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^5\left(a+bx^3\right)^2} + \frac{b\left(14b^3c-11ab^2d+8a^2be-5a^3f\right)x^2}{9a^6\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^7c}{ax^{11}} + \right.}{}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c\right.}{}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c\right.}{}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c\right.}{}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c\right.}{}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c\right.}{}$$

Mathematica [A] time = 0.59, size = 366, normalized size = 0.96

$$\frac{-\frac{540a^{7/3}(ad-3bc)}{x^7}-\frac{378a^{10/3}c}{x^{10}}-\frac{945a^{4/3}\left(a^2e-3abd+6b^2c\right)}{x^4}-\frac{420\sqrt[3]{a}bx^2\left(5a^3f-8a^2be+11ab^2d-14b^3c\right)}{a+bx^3}-\frac{3780\sqrt[3]{a}\left(a^3f-3a^2be+6ab^2d-10b^3c\right)}{x}+140\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(14a^3f-35a^2be+65ab^2d-104b^3c\right)-140\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)-\frac{630a^{4/3}bx^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}+70\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{3780a^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] ((-378*a^(10/3)*c)/x^10 - (540*a^(7/3)*(-3*b*c + a*d))/x^7 - (945*a^(4/3)*(
6*b^2*c - 3*a*b*d + a^2*e))/x^4 - (3780*a^(1/3)*(-10*b^3*c + 6*a*b^2*d - 3*
a^2*b*e + a^3*f))/x - (630*a^(4/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
*x^2)/(a + b*x^3)^2 - (420*a^(1/3)*b*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e +
5*a^3*f)*x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 3
5*a^2*b*e - 14*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(
1/3)*(-104*b^3*c + 65*a*b^2*d - 35*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3
)*x] + 70*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*a^(19/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3), x]
```

fricas [A] time = 0.44, size = 621, normalized size = 1.63

$$\frac{420\left(104 b^{5} c - 65 a b^{4} d + 35 a^{2} b^{3} e - 14 a^{3} b^{2} f\right) x^{15} + 735\left(104 a b^{4} c - 65 a^{2} b^{3} d + 35 a^{3} b^{2} e - 14 a^{4} b f\right) x^{12} + 270\left(104 a^{2} b^{3} c - 65 a^{3} b^{2} d + 35 a^{4} b e - 14 a^{5} f\right) x^{9} - 27\left(104 a^{3} b^{2} c - 65 a^{4} b d + 35 a^{5} e\right) x^{6} - 378 a^{5} c + 108\left(8 a^{4} b c - 5 a^{5} d\right) x^{3} + 140 \sqrt{3}\left(\left(104 b^{5} c - 65 a b^{4} d + 35 a^{2} b^{3} e - 14 a^{3} b^{2} f\right) x^{16} + 2\left(104 a b^{4} c - 65 a^{2} b^{3} d + 35 a^{3} b^{2} e - 14 a^{4} b f\right) x^{13} + \left(104 a^{2} b^{3} c - 65 a^{3} b^{2} d + 35 a^{4} b e - 14 a^{5} f\right) x^{10}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3} \sqrt{3} x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3} \sqrt{3}\right) + 70\left(\left(104 b^{5} c - 65 a b^{4} d + 35 a^{2} b^{3} e - 14 a^{3} b^{2} f\right) x^{16} + 2\left(104 a b^{4} c - 65 a^{2} b^{3} d + 35 a^{3} b^{2} e - 14 a^{4} b f\right) x^{13} + \left(104 a^{2} b^{3} c - 65 a^{3} b^{2} d + 35 a^{4} b e - 14 a^{5} f\right) x^{10}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}} \log\left(b x^{2} - a x \left(\frac{b}{a}\right)^{\frac{2}{3}} + a \left(\frac{b}{a}\right)^{\frac{1}{3}}\right) - 140\left(\left(104 b^{5} c - 65 a b^{4} d + 35 a^{2} b^{3} e - 14 a^{3} b^{2} f\right) x^{16} + 2\left(104 a b^{4} c - 65 a^{2} b^{3} d + 35 a^{3} b^{2} e - 14 a^{4} b f\right) x^{13} + \left(104 a^{2} b^{3} c - 65 a^{3} b^{2} d + 35 a^{4} b e - 14 a^{5} f\right) x^{10}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}} \log\left(b x + a \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{3780\left(a^{6} b^{2} x^{16} + 2 a^{7} b x^{13} + a^{8} x^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3780*(420*(104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^15 + 7
35*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^12 + 270*(104
*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^9 - 27*(104*a^3*b^2*c
- 65*a^4*b*d + 35*a^5*e)*x^6 - 378*a^5*c + 108*(8*a^4*b*c - 5*a^5*d)*x^3 +
140*sqrt(3)*((104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 +
2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*
b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*arctan(2/3*
sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*((104*b^5*c - 65*a*b^4*d + 35*a^2
*b^3*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e
- 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)
*x^10)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 140*((104
*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c -
65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^
2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)))/(a
^6*b^2*x^16 + 2*a^7*b*x^13 + a^8*x^10)
```

giac [A] time = 0.20, size = 486, normalized size = 1.28

$$-\frac{\left(104 b^{4} c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 65 a b^{3} d \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 14 a^{3} b f \left(-\frac{a}{b}\right)^{\frac{1}{3}} + 35 a^{2} b^{2} \left(-\frac{a}{b}\right)^{\frac{1}{3}} e\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27 a^{7}} - \frac{\sqrt{3}\left(104 \left(-a b^{2}\right)^{\frac{2}{3}} b^{3} c - 65 \left(-a b^{2}\right)^{\frac{2}{3}} a b^{2} d - 14 \left(-a b^{2}\right)^{\frac{2}{3}} a^{3} f + 35 \left(-a b^{2}\right)^{\frac{2}{3}} a^{2} b e\right) \arctan\left(\frac{\sqrt{3}\left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 a^{7} b} + \frac{\left(104 \left(-a b^{2}\right)^{\frac{2}{3}} b^{3} c - 65 \left(-a b^{2}\right)^{\frac{2}{3}} a b^{2} d - 14 \left(-a b^{2}\right)^{\frac{2}{3}} a^{3} f + 35 \left(-a b^{2}\right)^{\frac{2}{3}} a^{2} b e\right) \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 a^{7} b} + \frac{28 b^{5} c x^{5} - 22 a b^{4} d x^{5} - 10 a^{3} b^{2} f x^{5} + 16 a^{2} b^{3} x^{5} e + 31 a b^{4} c x^{2} - 25 a^{2} b^{3} d x^{2} - 13 a^{4} b f x^{2} + 19 a^{3} b^{2} x^{2} e}{18 \left(b x^{3} + a\right)^{2} a^{6}} + \frac{1400 b^{3} c x^{9} - 840 a b^{2} d x^{9} - 140 a^{3} f x^{9} + 420 a^{2} b x^{9} e - 210 a b^{2} c x^{6} + 105 a^{2} b d x^{6} - 35 a^{3} x^{6} e + 60 a^{2} b c x^{3} - 20 a^{3} d x^{3} - 14 a^{3} c}{140 a^{6} x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(104*b^4*c*(-a/b)^(1/3) - 65*a*b^3*d*(-a/b)^(1/3) - 14*a^3*b*f*(-a/b)
^(1/3) + 35*a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))
/a^7 - 1/27*sqrt(3)*(104*(-a*b^2)^(2/3)*b^3*c - 65*(-a*b^2)^(2/3)*a*b^2*d -
 14*(-a*b^2)^(2/3)*a^3*f + 35*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^7*b) + 1/54*(104*(-a*b^2)^(2/3)*b^3*c -
 65*(-a*b^2)^(2/3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 35*(-a*b^2)^(2/3)*a^
2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^7*b) + 1/18*(28*b^5*c*x^
5 - 22*a*b^4*d*x^5 - 10*a^3*b^2*f*x^5 + 16*a^2*b^3*x^5*e + 31*a*b^4*c*x^2 -
 25*a^2*b^3*d*x^2 - 13*a^4*b*f*x^2 + 19*a^3*b^2*x^2*e)/((b*x^3 + a)^2*a^6)
+ 1/140*(1400*b^3*c*x^9 - 840*a*b^2*d*x^9 - 140*a^3*f*x^9 + 420*a^2*b*x^9*e
 - 210*a*b^2*c*x^6 + 105*a^2*b*d*x^6 - 35*a^3*x^6*e + 60*a^2*b*c*x^3 - 20*a
^3*d*x^3 - 14*a^3*c)/(a^6*x^10)
```

maple [A] time = 0.08, size = 659, normalized size = 1.73

$$-\frac{5 b^{2} f x^{5}}{9 \left(b x^{3}+a\right)^{2} a^{3}}+\frac{8 b^{3} e x^{5}}{9 \left(b x^{3}+a\right)^{2} a^{4}}-\frac{11 b^{4} d x^{5}}{9 \left(b x^{3}+a\right)^{2} a^{5}}+\frac{14 b^{5} c x^{5}}{9 \left(b x^{3}+a\right)^{2} a^{6}}-\frac{13 b f x^{2}}{18 \left(b x^{3}+a\right)^{2} a^{2}}+\frac{19 b^{2} e x^{2}}{18 \left(b x^{3}+a\right)^{2} a^{3}}-\frac{25 b^{3} d x^{2}}{18 \left(b x^{3}+a\right)^{2} a^{4}}+\frac{31 b^{4} c x^{2}}{18 \left(b x^{3}+a\right)^{2} a^{5}}-\frac{14 \sqrt{3}\, f \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}}+\frac{14 f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}}-\frac{7 f \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}}+\frac{35 \sqrt{3}\, b e \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{4}}-\frac{35 b e \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{4}}+\frac{35 b e \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{4}}-\frac{65 \sqrt{3}\, b^{2} d \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{5}}+\frac{65 b^{2} d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{5}}-\frac{65 b^{2} d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{5}}+\frac{104 \sqrt{3}\, b^{3} c \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{6}}-\frac{104 b^{3} c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{6}}+\frac{52 b^{3} c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 \left(\frac{a}{b}\right)^{\frac{1}{3}} a^{6}}-\frac{f}{a^{3} x}+\frac{3 b e}{a^{4} x}-\frac{6 b^{2} d}{a^{5} x}+\frac{10 b^{3} c}{a^{6} x}-\frac{e}{4 a^{3} x^{4}}+\frac{3 b d}{4 a^{4} x^{4}}-\frac{3 b^{2} c}{2 a^{5} x^{4}}-\frac{d}{7 a^{3} x^{7}}+\frac{3 b c}{7 a^{4} x^{7}}-\frac{c}{10 a^{3} x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x)
```

```
[Out] -7/27/a^3*f/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+3/7/a^4/x^7*b*c+3
/4/a^4/x^4*b*d-3/2/a^5/x^4*b^2*c+3/a^4/x*b*e-6/a^5/x*b^2*d+10/a^6/x*b^3*c+1
4/27/a^3*f/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+35/27/a^4*b*e*3^(1/2)/(a/b)^(1/3)*
```

```
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-65/27/a^5*b^2*d*3^(1/2)/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+104/27/a^6*b^3*c*3^(1/2)/(a/b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/7/a^3/x^7*d-1/4/a^3/x^4*e-1/a^
3/x*f-1/10*c/a^3/x^10-5/9/a^3*b^2/(b*x^3+a)^2*x^5*f+8/9/a^4*b^3/(b*x^3+a)^2
*x^5*e-11/9/a^5*b^4/(b*x^3+a)^2*x^5*d+14/9/a^6*b^5/(b*x^3+a)^2*x^5*c-13/18/
a^2*b/(b*x^3+a)^2*x^2*f+19/18/a^3*b^2/(b*x^3+a)^2*x^2*e-25/18/a^4*b^3/(b*x^
3+a)^2*x^2*d+31/18/a^5*b^4/(b*x^3+a)^2*x^2*c-14/27/a^3*f*3^(1/2)/(a/b)^(1/3
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-35/27/a^4*b*e/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))+35/54/a^4*b*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+65/2
7/a^5*b^2*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-65/54/a^5*b^2*d/(a/b)^(1/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))-104/27/a^6*b^3*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
+52/27/a^6*b^3*c/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
```

maxima [A] time = 3.20, size = 376, normalized size = 0.99

$$\frac{140\left(104\,b^{5}c-65\,ab^{4}d+35\,a^{2}b^{3}e-14\,a^{3}b^{2}f\right)x^{15}+245\left(104\,ab^{4}c-65\,a^{2}b^{3}d+35\,a^{3}b^{2}e-14\,a^{4}bf\right)x^{12}+90\left(104\,a^{2}b^{3}c-65\,a^{3}b^{2}d+35\,a^{4}be-14\,a^{5}f\right)x^{9}-9\left(104\,a^{3}b^{2}c-65\,a^{4}bd+35\,a^{5}e\right)x^{6}-126\,a^{5}c+36\left(8\,a^{4}bc-5\,a^{5}d\right)x^{3}}{1260\left(a^{6}b^{2}x^{16}+2\,a^{7}bx^{13}+a^{8}x^{10}\right)}+\frac{\sqrt{3}\left(104\,b^{3}c-65\,ab^{2}d+35\,a^{2}be-14\,a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{6}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(104\,b^{3}c-65\,ab^{2}d+35\,a^{2}be-14\,a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{6}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(104\,b^{3}c-65\,ab^{2}d+35\,a^{2}be-14\,a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{6}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1260*(140*(104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^15 + 2
45*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^12 + 90*(104*
a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^9 - 9*(104*a^3*b^2*c -
65*a^4*b*d + 35*a^5*e)*x^6 - 126*a^5*c + 36*(8*a^4*b*c - 5*a^5*d)*x^3)/(a^6
*b^2*x^16 + 2*a^7*b*x^13 + a^8*x^10) + 1/27*sqrt(3)*(104*b^3*c - 65*a*b^2*d
 + 35*a^2*b*e - 14*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^6*(a/b)^(1/3)) + 1/54*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/3)) - 1/27*(104*b^3*
c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*log(x + (a/b)^(1/3))/(a^6*(a/b)^(1/
3))
```

mupad [B] time = 5.28, size = 359, normalized size = 0.94

$$-\frac{\frac{c}{10\,a}-\frac{x^9\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{14\,a^4}+\frac{x^3\,\left(5\,a\,d-8\,b\,c\right)}{35\,a^2}+\frac{x^6\,\left(35\,e\,a^2-65\,d\,a\,b+104\,c\,b^2\right)}{140\,a^3}-\frac{7\,b\,x^{12}\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{36\,a^5}-\frac{b^2\,x^{15}\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{9\,a^6}}{a^2\,x^{10}+2\,a\,b\,x^{13}+b^2\,x^{16}}-\frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}+\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}-\frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x)
```

```
[Out] (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
1/2)*(104*b^3*c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b*e))/(27*a^(19/3)) - (b^(
1/3)*log(b^(1/3)*x + a^(1/3))*(104*b^3*c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b
*e))/(27*a^(19/3)) - (c/(10*a) - (x^9*(104*b^3*c - 14*a^3*f - 65*a*b^2*d +
35*a^2*b*e))/(14*a^4) + (x^3*(5*a*d - 8*b*c))/(35*a^2) + (x^6*(104*b^2*c +
35*a^2*e - 65*a*b*d))/(140*a^3) - (7*b*x^12*(104*b^3*c - 14*a^3*f - 65*a*b^
2*d + 35*a^2*b*e))/(36*a^5) - (b^2*x^15*(104*b^3*c - 14*a^3*f - 65*a*b^2*d
+ 35*a^2*b*e))/(9*a^6))/(a^2*x^10 + b^2*x^16 + 2*a*b*x^13) - (b^(1/3)*log(3
^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(104*b^3*
c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b*e))/(27*a^(19/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.249 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3}\,dx$

Optimal. Leaf size=380

$$\frac{3bc-ad}{8a^4x^8}-\frac{c}{11a^3x^{11}}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3\right.}{54a^{20/3}}$$

Rubi [A] time = 0.67, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$\frac{bx\left(17a^2be-11a^3f-23ab^2d+29b^3c\right)}{18a^6\left(a+bx^3\right)}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{2a^6x^2}+\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(44a^2be-20a^3f-77ab^2d+119b^3c\right)}{54a^{20/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(44a^2be-20a^3f-77ab^2d+119b^3c\right)}{27a^{20/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(44a^2be-20a^3f-77ab^2d+119b^3c\right)}{9\sqrt{3}\,a^{20/3}}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}+\frac{3bc-ad}{8a^4x^8}-\frac{c}{11a^3x^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

```
[Out] -c/(11*a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e)/
(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*c
- 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) - (b^(2/3)*(1
19*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*b^
2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) - (b^(
2/3)*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^5\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab^2d+a^2be-a\right.}{a^3}}{x^{12}\left(a+bx^3\right)^2}}{6ab^3}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^5\left(a+bx^3\right)^2} + \frac{b\left(29b^3c-23ab^2d+17a^2be-11a^3f\right)x}{18a^6\left(a+bx^3\right)} + \frac{\int \frac{18b^7c}{}}{}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^5\left(a+bx^3\right)^2} + \frac{b\left(29b^3c-23ab^2d+17a^2be-11a^3f\right)x}{18a^6\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^7}{ax^1}\right.}{}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b\left(b\right.}{}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b\left(b\right.}{}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b\left(b\right.}{}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b\left(b\right.}{}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b\left(b\right.}{}$$

Mathematica [A] time = 0.58, size = 376, normalized size = 0.99

$$\frac{3bc-ad}{8a^4x^8} - \frac{c}{11a^3x^{11}} - \frac{a^2e-3abd+6b^2c}{5a^5x^5} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(20a^3f-44a^2be+77ab^2d-119b^3c\right)}{54a^{20/3}} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{27a^{20/3}} + \frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(20a^3f-44a^2be+77ab^2d-119b^3c\right)}{9\sqrt{3}a^{20/3}} + \frac{bx\left(-11a^3f+17a^2be-23ab^2d+29b^3c\right)}{18a^6\left(a+bx^3\right)} + \frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{2a^6x^2} + \frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

```
[Out] -1/11*c/(a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e
)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*
c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) + (b^(2/3)*
(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*b^2*d
 + 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) + (b^(2/3
)*(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3), x]
```

fricas [A] time = 0.45, size = 654, normalized size = 1.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11880*(660*(119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^15 +
1056*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^12 + 297*(1
19*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^9 - 54*(119*a^3*b^2*
c - 77*a^4*b*d + 44*a^5*e)*x^6 - 1080*a^5*c + 135*(17*a^4*b*c - 11*a^5*d)*x
^3 - 440*sqrt(3)*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^
17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119
*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*a
rctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((119*b^5*c
 - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2
*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d +
44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2
)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 440*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^3
*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20
*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^1
1)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^6*b^2*x^17 + 2*a^7*b*
x^14 + a^8*x^11)
```

giac [A] time = 0.35, size = 440, normalized size = 1.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(119*(-a*b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d - 20*(-a
*b^2)^(1/3)*a^3*f + 44*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-
a/b)^(1/3))/(-a/b)^(1/3))/a^7 - 1/27*(119*b^4*c - 77*a*b^3*d - 20*a^3*b*f +
 44*a^2*b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^7 + 1/54*(119*(-a*
b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d - 20*(-a*b^2)^(1/3)*a^3*f + 44
*(-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 + 1/1
8*(29*b^5*c*x^4 - 23*a*b^4*d*x^4 - 11*a^3*b^2*f*x^4 + 17*a^2*b^3*x^4*e + 32
*a*b^4*c*x - 26*a^2*b^3*d*x - 14*a^4*b*f*x + 20*a^3*b^2*x*e)/((b*x^3 + a)^2
*a^6) + 1/440*(2200*b^3*c*x^9 - 1320*a*b^2*d*x^9 - 220*a^3*f*x^9 + 660*a^2*
b*x^9*e - 528*a*b^2*c*x^6 + 264*a^2*b*d*x^6 - 88*a^3*x^6*e + 165*a^2*b*c*x^
3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^6*x^11)
```

maple [A] time = 0.07, size = 651, normalized size = 1.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x)
```

```
[Out] 3/5/a^4/x^5*b*d-6/5/a^5/x^5*b^2*c+3/2/a^4/x^2*b*e-3/a^5/x^2*b^2*d+5/a^6/x^2
*b^3*c-20/27/a^3*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+10/27/a^3*f/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+3/8/a^4/x^8*b*c-119/54/a^6*b^3*c/(a/b)^(2/3
```

```
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+29/18/a^6*b^5/(b*x^3+a)^2*x^4*c-7/9/a^2
*b/(b*x^3+a)^2*f*x+10/9/a^3*b^2/(b*x^3+a)^2*e*x-13/9/a^4*b^3/(b*x^3+a)^2*d*
x-11/18/a^3*b^2/(b*x^3+a)^2*x^4*f+17/18/a^4*b^3/(b*x^3+a)^2*x^4*e-23/18/a^5
*b^4/(b*x^3+a)^2*x^4*d-1/8/a^3/x^8*d-1/5/a^3/x^5*e-1/2/a^3/x^2*f+44/27/a^4*
b*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-77/27/a^5*b
^2*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+16/9/a^5*b
^4/(b*x^3+a)^2*c*x-20/27/a^3*f/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))+44/27/a^4*b*e/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-22/27/a^4*b*e/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+119/27/a^6*b^3*c/(a/b)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/11*c/a^3/x^11-77/27/a^5*b^
2*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+77/54/a^5*b^2*d/(a/b)^(2/3)*ln(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3))+119/27/a^6*b^3*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))
```

maxima [A] time = 3.29, size = 376, normalized size = 0.99

$$\frac{220\left(119\,b^{5}c-77\,ab^{4}d+44\,a^{2}b^{3}e-20\,a^{3}b^{2}f\right)x^{15}+352\left(119\,ab^{4}c-77\,a^{2}b^{3}d+44\,a^{3}b^{2}e-20\,a^{4}bf\right)x^{12}+99\left(119\,a^{2}b^{3}c-77\,a^{3}b^{2}d+44\,a^{4}be-20\,a^{5}f\right)x^{9}-18\left(119\,a^{3}b^{2}c-77\,a^{4}bd+44\,a^{5}e\right)x^{6}-360\,a^{5}c+45\left(17\,a^{4}bc-11\,a^{5}d\right)x^{3}}{3960\left(a^{6}b^{2}x^{17}+2\,a^{7}bx^{14}+a^{8}x^{11}\right)}+\frac{\sqrt{3}\left(119\,b^{3}c-77\,ab^{2}d+44\,a^{2}be-20\,a^{3}f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(119\,b^{3}c-77\,ab^{2}d+44\,a^{2}be-20\,a^{3}f\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(119\,b^{3}c-77\,ab^{2}d+44\,a^{2}be-20\,a^{3}f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3960*(220*(119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^15 + 3
52*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^12 + 99*(119*
a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^9 - 18*(119*a^3*b^2*c -
 77*a^4*b*d + 44*a^5*e)*x^6 - 360*a^5*c + 45*(17*a^4*b*c - 11*a^5*d)*x^3)/(
a^6*b^2*x^17 + 2*a^7*b*x^14 + a^8*x^11) + 1/27*sqrt(3)*(119*b^3*c - 77*a*b^
2*d + 44*a^2*b*e - 20*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(a^6*(a/b)^(2/3)) - 1/54*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^
3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(2/3)) + 1/27*(119*b
^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*log(x + (a/b)^(1/3))/(a^6*(a/b)^
(2/3))
```

mupad [B] time = 5.18, size = 359, normalized size = 0.94

$$\frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{27\,a^{20/3}}-\frac{\frac{c}{11\,a}-\frac{x^9\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{40\,a^4}+\frac{x^3\,\left(11\,a\,d-17\,b\,c\right)}{88\,a^2}+\frac{x^6\,\left(44\,e\,a^2-77\,d\,a\,b+119\,c\,b^2\right)}{220\,a^3}-\frac{4\,b\,x^{12}\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{45\,a^5}-\frac{b^2\,x^{15}\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{18\,a^6}}{a^2\,x^{11}+2\,a\,b\,x^{14}+b^2\,x^{17}}+\frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{27\,a^{20/3}}-\frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{27\,a^{20/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(119*b^3*c - 20*a^3*f - 77*a*b^2*d + 44*a
^2*b*e))/(27*a^(20/3)) - (c/(11*a) - (x^9*(119*b^3*c - 20*a^3*f - 77*a*b^2*
d + 44*a^2*b*e))/(40*a^4) + (x^3*(11*a*d - 17*b*c))/(88*a^2) + (x^6*(119*b^
2*c + 44*a^2*e - 77*a*b*d))/(220*a^3) - (4*b*x^12*(119*b^3*c - 20*a^3*f - 7
7*a*b^2*d + 44*a^2*b*e))/(45*a^5) - (b^2*x^15*(119*b^3*c - 20*a^3*f - 77*a*
b^2*d + 44*a^2*b*e))/(18*a^6))/(a^2*x^11 + b^2*x^17 + 2*a*b*x^14) + (b^(2/3
)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1
19*b^3*c - 20*a^3*f - 77*a*b^2*d + 44*a^2*b*e))/(27*a^(20/3)) - (b^(2/3)*lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(119*b
^3*c - 20*a^3*f - 77*a*b^2*d + 44*a^2*b*e))/(27*a^(20/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.250 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=424

$$\frac{3bc-ad}{10a^4x^{10}}-\frac{c}{13a^3x^{13}}-\frac{a^2e-3abd+6b^2c}{7a^5x^7}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{54a^{22/3}}$$

Rubi [A] time = 0.85, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$-\frac{b^2x^2\left(11a^2be-8a^3f-14ab^2d+17b^3c\right)}{9a^7\left(a+bx^3\right)}-\frac{b^2x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{4a^6x^4}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(65a^2be-35a^3f-104ab^2d+152b^3c\right)}{54a^{22/3}}-\frac{b\left(6a^2be-3a^3f-10ab^2d+15b^3c\right)}{a^7x}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(65a^2be-35a^3f-104ab^2d+152b^3c\right)}{27a^{22/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(65a^2be-35a^3f-104ab^2d+152b^3c\right)}{9\sqrt{3}\,a^{22/3}}-\frac{a^2e-3abd+6b^2c}{7a^5x^7}+\frac{3bc-ad}{10a^4x^{10}}-\frac{c}{13a^3x^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```

```
[Out] -c/(13*a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^2*e
)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) - (b
*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) - (b^2*(17*b^3*c - 14*a*
b^2*d + 11*a^2*b*e - 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(152*b^3*
c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d +
 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3)) - (b^(4/3)*
(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^6\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab^2d+a^2be-\cdots\right.}{a^3}}{x^{14}\left(a+\cdots\right.}}{6a\cdots}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^6\left(a+bx^3\right)^2} - \frac{b^2\left(17b^3c-14ab^2d+11a^2be-8a^3f\right)x^2}{9a^7\left(a+bx^3\right)} + \frac{\int \frac{18b^{\cdots}}{\cdots}}{\cdots}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^6\left(a+bx^3\right)^2} - \frac{b^2\left(17b^3c-14ab^2d+11a^2be-8a^3f\right)x^2}{9a^7\left(a+bx^3\right)} + \frac{\int\left(\frac{18\cdots}{ax\cdots}\right.}{\cdots}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b\left(15b^{\cdots}\right.}{\cdots}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b\left(15b^{\cdots}\right.}{\cdots}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b\left(15b^{\cdots}\right.}{\cdots}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b\left(15b^{\cdots}\right.}{\cdots}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b\left(15b^{\cdots}\right.}{\cdots}$$

Mathematica [A] time = 0.67, size = 419, normalized size = 0.99

$$\frac{3bc-ad}{10a^4x^{10}}-\frac{c}{13a^3x^{13}}-\frac{a^2e-3abd+6b^2c}{7a^5x^7}+\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^3f-65a^2be+104ab^2d-152b^3c\right)}{54a^{22/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{27a^{22/3}}+\frac{b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{9\sqrt{3}a^{22/3}}+\frac{b^2x^2\left(8a^3f-11a^2be+14ab^2d-17b^3c\right)}{9a^7\left(a+bx^3\right)}+\frac{b\left(3a^3f-6a^2be+10ab^2d-15b^3c\right)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{4a^6x^4}+\frac{b^2x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```

```
[Out] -1/13*c/(a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^2
*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) +
(b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) + (b^2*(-17*b^3*c
+ 14*a*b^2*d - 11*a^2*b*e + 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(1
52*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d
+ 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3)) + (b^(4/3)
*(-152*b^3*c + 104*a*b^2*d - 65*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
[Out] IntegrateAlgebraic[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3), x]
```

fricas [A] time = 0.44, size = 686, normalized size = 1.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/49140*(5460*(152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^18
 + 9555*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^15 +
3510*(152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^12 - 351
*(152*a^3*b^3*c - 104*a^4*b^2*d + 65*a^5*b*e - 35*a^6*f)*x^9 + 3780*a^6*c +
 108*(152*a^4*b^2*c - 104*a^5*b*d + 65*a^6*e)*x^6 - 378*(19*a^5*b*c - 13*a^
6*d)*x^3 + 1820*sqrt(3)*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b
^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*
x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-
b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*((152*b^6
*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104
*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b
^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)
^(2/3) - a*(-b/a)^(1/3)) + 1820*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e -
35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4
*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*
x^13)*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^7*b^2*x^19 + 2*a^8*b*x^16
+ a^9*x^13)
```

giac [A] time = 0.22, size = 531, normalized size = 1.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(152*(-a*b^2)^(2/3)*b^3*c - 104*(-a*b^2)^(2/3)*a*b^2*d - 35*(-
a*b^2)^(2/3)*a^3*f + 65*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/a^8 + 1/27*(152*b^5*c*(-a/b)^(1/3) - 104*a*b^4*d
*(-a/b)^(1/3) - 35*a^3*b^2*f*(-a/b)^(1/3) + 65*a^2*b^3*(-a/b)^(1/3)*e)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^8 - 1/54*(152*(-a*b^2)^(2/3)*b^3*c -
104*(-a*b^2)^(2/3)*a*b^2*d - 35*(-a*b^2)^(2/3)*a^3*f + 65*(-a*b^2)^(2/3)*a^
2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^8 - 1/18*(34*b^6*c*x^5 -
28*a*b^5*d*x^5 - 16*a^3*b^3*f*x^5 + 22*a^2*b^4*x^5*e + 37*a*b^5*c*x^2 - 31*
a^2*b^4*d*x^2 - 19*a^4*b^2*f*x^2 + 25*a^3*b^3*x^2*e)/((b*x^3 + a)^2*a^7) -
1/1820*(27300*b^4*c*x^12 - 18200*a*b^3*d*x^12 - 5460*a^3*b*f*x^12 + 10920*a
^2*b^2*x^12*e - 4550*a*b^3*c*x^9 + 2730*a^2*b^2*d*x^9 + 455*a^4*f*x^9 - 136
5*a^3*b*x^9*e + 1560*a^2*b^2*c*x^6 - 780*a^3*b*d*x^6 + 260*a^4*x^6*e - 546*
a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^7*x^13)
```

maple [A] time = 0.07, size = 716, normalized size = 1.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x)
```

```
[Out] -1/10/a^3/x^10*d+3/4/a^4/x^4*b*e-3/2/a^5/x^4*b^2*d+5/2/a^6/x^4*b^3*c+3/7/a^
4/x^7*b*d-6/7/a^5/x^7*b^2*c+3/10/a^4/x^10*b*c+3*b/a^4/x*f-6*b^2/a^5/x*e+10*
b^3/a^6/x*d-15*b^4/a^7/x*c-1/7/a^3/x^7*e-1/4/a^3/x^4*f-104/27/a^6*b^3*d/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))+52/27/a^6*b^3*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+152/27/a^7*b^4*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-76/27/a^7*b^4*
c/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+8/9/a^4*b^3/(b*x^3+a)^2*x^5
*f-11/9/a^5*b^4/(b*x^3+a)^2*x^5*e+14/9/a^6*b^5/(b*x^3+a)^2*x^5*d-17/9/a^7*b
^6/(b*x^3+a)^2*x^5*c+19/18/a^3*b^2/(b*x^3+a)^2*x^2*f-25/18/a^4*b^3/(b*x^3+a
)^2*x^2*e+31/18/a^5*b^4/(b*x^3+a)^2*x^2*d-37/18/a^6*b^5/(b*x^3+a)^2*x^2*c+3
5/27/a^4*b*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-65
/27/a^5*b^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1
04/27/a^6*b^3*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-152/27/a^7*b^4*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))-1/13*c/a^3/x^13+35/54/a^4*b*f/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))+65/27/a^5*b^2*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-65/54/a^5*b^2*e/(a/b)^(1/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-35/27/a^4*b*f/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))
```

maxima [A] time = 3.05, size = 427, normalized size = 1.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16380*(1820*(152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^18
 + 3185*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^15 +
1170*(152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^12 - 117
*(152*a^3*b^3*c - 104*a^4*b^2*d + 65*a^5*b*e - 35*a^6*f)*x^9 + 1260*a^6*c +
 36*(152*a^4*b^2*c - 104*a^5*b*d + 65*a^6*e)*x^6 - 126*(19*a^5*b*c - 13*a^6
*d)*x^3)/(a^7*b^2*x^19 + 2*a^8*b*x^16 + a^9*x^13) - 1/27*sqrt(3)*(152*b^4*c
 - 104*a*b^3*d + 65*a^2*b^2*e - 35*a^3*b*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)
^(1/3))/(a/b)^(1/3))/(a^7*(a/b)^(1/3)) - 1/54*(152*b^4*c - 104*a*b^3*d + 65
*a^2*b^2*e - 35*a^3*b*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^7*(a/b)^
(1/3)) + 1/27*(152*b^4*c - 104*a*b^3*d + 65*a^2*b^2*e - 35*a^3*b*f)*log(x +
 (a/b)^(1/3))/(a^7*(a/b)^(1/3))
```

mupad [B] time = 5.30, size = 397, normalized size = 0.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(152*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*
a^2*b*e))/(27*a^(22/3)) - (c/(13*a) - (x^9*(152*b^3*c - 35*a^3*f - 104*a*b^
2*d + 65*a^2*b*e))/(140*a^4) + (x^3*(13*a*d - 19*b*c))/(130*a^2) + (x^6*(15
2*b^2*c + 65*a^2*e - 104*a*b*d))/(455*a^3) + (b*x^12*(152*b^3*c - 35*a^3*f
- 104*a*b^2*d + 65*a^2*b*e))/(14*a^5) + (7*b^2*x^15*(152*b^3*c - 35*a^3*f -
 104*a*b^2*d + 65*a^2*b*e))/(36*a^6) + (b^3*x^18*(152*b^3*c - 35*a^3*f - 10
4*a*b^2*d + 65*a^2*b*e))/(9*a^7))/(a^2*x^13 + b^2*x^19 + 2*a*b*x^16) - (b^(
4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)
*(152*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*a^2*b*e))/(27*a^(22/3)) + (b^(4/3
)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1
52*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*a^2*b*e))/(27*a^(22/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.251 $\int \frac{(1-x)x^4}{1+x^3} \, dx$

Optimal. Leaf size=54

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1887, 1874, 31, 634, 618, 204, 628}

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x^4)/(1 + x^3),x]
```

```
[Out] x^2/2 - x^3/3 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[
1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

```
&& GtQ[a/b, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)x^4}{1+x^3}\,dx &= \int \left(x-x^2+\frac{(-1+x)x}{1+x^3}\right)dx\\
&= \frac{x^2}{2}-\frac{x^3}{3}+\int \frac{(-1+x)x}{1+x^3}\,dx\\
&= \frac{x^2}{2}-\frac{x^3}{3}+\frac{1}{3}\int \frac{-2+x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx\\
&= \frac{x^2}{2}-\frac{x^3}{3}+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= \frac{x^2}{2}-\frac{x^3}{3}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{x^2}{2}-\frac{x^3}{3}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.09

$$\frac{1}{6}\left(-2x^3+2\log\left(x^3+1\right)+3x^2-\log\left(x^2-x+1\right)+2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^4)/(1 + x^3),x]
```

```
[Out] (3*x^2 - 2*x^3 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[
1 - x + x^2] + 2*Log[1 + x^3])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1-x)x^4}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((1 - x)*x^4)/(1 + x^3),x]
```

```
[Out] IntegrateAlgebraic[((1 - x)*x^4)/(1 + x^3), x]
```

fricas [A] time = 0.40, size = 44, normalized size = 0.81

$$-\frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{6}\,\log\left(x^2-x+1\right)+\frac{2}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(x + 1)
```

giac [A] time = 0.16, size = 45, normalized size = 0.83

$$-\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(abs(x + 1))
```

maple [A] time = 0.05, size = 45, normalized size = 0.83

$$-\frac{x^3}{3}+\frac{x^2}{2}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{2\ln(x+1)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x^4/(x^3+1),x)
```

```
[Out] -1/3*x^3+1/2*x^2+2/3*ln(x+1)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)
*3^(1/2))
```

maxima [A] time = 2.90, size = 44, normalized size = 0.81

$$-\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(x + 1)
```

mupad [B] time = 0.10, size = 56, normalized size = 1.04

$$\frac{2\ln(x+1)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)+\frac{x^2}{2}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(x - 1))/(x^3 + 1),x)
```

```
[Out] (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - l
og(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) + x^2/2 - x^3/3
```

sympy [A] time = 0.18, size = 53, normalized size = 0.98

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{2\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x**4/(x**3+1),x)
```

```
[Out] -x**3/3 + x**2/2 + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sq
rt(3)*x/3 - sqrt(3)/3)/3
```

3.252 $\int \frac{(1-x)x^3}{1+x^3} \, dx$

Optimal. Leaf size=30

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1887, 1860, 31, 628}

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x^3)/(1 + x^3),x]
```

```
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)x^3}{1+x^3} \, dx &= \int \left(1-x-\frac{1-x}{1+x^3}\right) dx \\
&= x-\frac{x^2}{2}-\int \frac{1-x}{1+x^3} \, dx \\
&= x-\frac{x^2}{2}-\frac{1}{3}\int \frac{1-2x}{1-x+x^2} \, dx-\frac{2}{3}\int \frac{1}{1+x} \, dx \\
&= x-\frac{x^2}{2}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^3)/(1 + x^3),x]
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(1-x)x^3}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((1 - x)*x^3)/(1 + x^3),x]
[Out] IntegrateAlgebraic[((1 - x)*x^3)/(1 + x^3), x]
```

fricas [A] time = 0.40, size = 24, normalized size = 0.80

$$-\frac{1}{2}x^2+x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="fricas")
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

giac [A] time = 0.16, size = 25, normalized size = 0.83

$$-\frac{1}{2}x^2+x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="giac")
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

maple [A] time = 0.05, size = 25, normalized size = 0.83

$$-\frac{x^2}{2}+x-\frac{2\ln\left(x+1\right)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x^3/(x^3+1),x)
[Out] x-1/2*x^2-2/3*ln(x+1)+1/3*ln(x^2-x+1)
```

maxima [A] time = 3.01, size = 24, normalized size = 0.80

$$-\frac{1}{2}x^2+x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="maxima")
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

mupad [**B**] time = 0.03, size = 24, normalized size = 0.80

$$x-\frac{2\,\ln\left(x+1\right)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x - 1))/(x^3 + 1),x)
```

```
[Out] x - (2*log(x + 1))/3 + log(x^2 - x + 1)/3 - x^2/2
```

sympy [**A**] time = 0.12, size = 24, normalized size = 0.80

$$-\frac{x^2}{2}+x-\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x**3/(x**3+1),x)
```

```
[Out] -x**2/2 + x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3
```

3.253 $\int \frac{(1-x)x^2}{1+x^3} \, dx$

Optimal. Leaf size=44

$$\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1887, 1874, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] -x - ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[1 - x + x^2
]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

```
&& GtQ[a/b, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)x^2}{1+x^3}\,dx &= \int \left(-1+\frac{1+x^2}{1+x^3}\right)dx\\
&= -x+\int \frac{1+x^2}{1+x^3}\,dx\\
&= -x+\frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx\\
&= -x+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= -x+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -x+\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.20

$$\frac{1}{3}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] -x + ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x]/3 - Log[1 - x + x^2]/6
 + Log[1 + x^3]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1-x)x^2}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] IntegrateAlgebraic[((1 - x)*x^2)/(1 + x^3), x]
```

fricas [A] time = 0.41, size = 37, normalized size = 0.84

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(x + 1)
```

giac [A] time = 0.15, size = 38, normalized size = 0.86

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-x+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(abs(x + 1))
```

maple [A] time = 0.05, size = 38, normalized size = 0.86

$$-x+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{2\ln\left(x+1\right)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x^2/(x^3+1),x)
```

```
[Out] -x+2/3*ln(x+1)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.96, size = 37, normalized size = 0.84

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-x+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(x + 1)
```

mupad [B] time = 4.96, size = 49, normalized size = 1.11

$$\frac{2\ln\left(x+1\right)}{3}-x-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(x - 1))/(x^3 + 1),x)
```

```
[Out] (2*log(x + 1))/3 - x - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6)
 + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6)
```

sympy [A] time = 0.23, size = 44, normalized size = 1.00

$$-x+\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x**2/(x**3+1),x)
```

```
[Out] -x + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sq
rt(3)/3)/3
```

3.254 $\int \frac{(1-x)x}{1+x^3} dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1874, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x)/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - (2*Log[1 + x])/3 - Log[1 - x + x^2]/
6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

```
&& GtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)x}{1+x^3}\,dx &= \frac{1}{3}\int \frac{2-x}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1+x}\,dx\\ &= -\frac{2}{3}\log(1+x)-\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\ &= -\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.22

$$-\frac{1}{3}\log\left(x^3+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x)/(1 + x^3),x]
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6 - Lo
g[1 + x^3]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1-x)x}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((1 - x)*x)/(1 + x^3),x]
[Out] IntegrateAlgebraic[((1 - x)*x)/(1 + x^3), x]
```

fricas [A] time = 0.41, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
x + 1)
```

giac [A] time = 0.15, size = 35, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
abs(x + 1))
```

maple [A] time = 0.05, size = 35, normalized size = 0.85

$$\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{2\ln\left(x+1\right)}{3}-\frac{\ln\left(x^{2}-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x/(x^3+1),x)
```

```
[Out] -2/3*ln(x+1)-1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.87, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{1}{6}\log\left(x^{2}-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
x + 1)
```

mupad [B] time = 0.08, size = 63, normalized size = 1.54

$$-\frac{\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6}-\frac{\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6}-\frac{2\ \ln\left(x+1\right)}{3}-\frac{\sqrt{3}\ \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{6}+\frac{\sqrt{3}\ \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x - 1))/(x^3 + 1),x)
```

```
[Out] (3^(1/2)*log(x + (3^(1/2)*1i)/2 - 1/2)*1i)/6 - log(x + (3^(1/2)*1i)/2 - 1/2
)/6 - (2*log(x + 1))/3 - (3^(1/2)*log(x - (3^(1/2)*1i)/2 - 1/2)*1i)/6 - log
(x - (3^(1/2)*1i)/2 - 1/2)/6
```

sympy [A] time = 0.27, size = 42, normalized size = 1.02

$$-\frac{2\log\left(x+1\right)}{3}-\frac{\log\left(x^{2}-x+1\right)}{6}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x**3+1),x)
```

```
[Out] -2*log(x + 1)/3 - log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3
)/3)/3
```

3.255 $\int \frac{1-x}{x(1+x^3)}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1834, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - (2*Log[1 + x])/3 - Log[1 - x +
 x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x}{x\left(1+x^3\right)}\,dx &= \int \left(\frac{1}{x}-\frac{2}{3(1+x)}+\frac{-1-x}{3\left(1-x+x^2\right)}\right)dx \\
&= \log(x)-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1-x}{1-x+x^2}\,dx \\
&= \log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx \\
&= \log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.26

$$-\frac{1}{3}\log\left(x^3+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x]/3 + Log[1 - x + x^2]/6 - Log[1 + x^3]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x}{x\left(1+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] IntegrateAlgebraic[(1 - x)/(x*(1 + x^3)), x]
```

fricas [A] time = 0.42, size = 36, normalized size = 0.86

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(x + 1) + log(x)
```

giac [A] time = 0.16, size = 38, normalized size = 0.90

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(|x+1|)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(abs(x + 1)) + log(abs(x))
```

maple [A] time = 0.05, size = 37, normalized size = 0.88

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\ln(x)-\frac{2\ln(x+1)}{3}-\frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/x/(x^3+1),x)
```

```
[Out] -2/3*ln(x+1)+ln(x)-1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.88, size = 36, normalized size = 0.86

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(x + 1) + log(x)
```

mupad [B] time = 4.96, size = 48, normalized size = 1.14

$$\ln(x)-\frac{2\ \ln(x+1)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\ 1i}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\ 1i}{6}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\ 1i}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\ 1i}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x*(x^3 + 1)),x)
```

```
[Out] log(x) - (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 -
 1/6) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6)
```

sympy [A] time = 0.21, size = 46, normalized size = 1.10

$$\log(x)-\frac{2\log(x+1)}{3}-\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1)/3 - log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3
- sqrt(3)/3)/3
```

3.256 $\int \frac{1-x}{x^2(1+x^3)}\,dx$

Optimal. Leaf size=49

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{x}-\log(x)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1834, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{x}-\log(x)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(x^2*(1 + x^3)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + (2*Log[1 + x])/3 + L
og[1 - x + x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1-x}{x^2\left(1+x^3\right)}\,dx = \int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{2}{3(1+x)}+\frac{-2+x}{3\left(1-x+x^2\right)}\right)dx$$
$$= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-2+x}{1-x+x^2}\,dx$$
$$= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx$$
$$= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)$$
$$= -\frac{1}{x}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.22

$$\frac{1}{3}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{x}-\log(x)+\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x^2*(1 + x^3)),x]
```

```
[Out] -x^(-1) - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + Log[1 + x]/3 - Log[
1 - x + x^2]/6 + Log[1 + x^3]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x}{x^2\left(1+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x)/(x^2*(1 + x^3)),x]
```

```
[Out] IntegrateAlgebraic[(1 - x)/(x^2*(1 + x^3)), x]
```

fricas [A] time = 0.40, size = 48, normalized size = 0.98

$$-\frac{2\sqrt{3}\,x\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-x\log\left(x^2-x+1\right)-4\,x\log\left(x+1\right)+6\,x\log(x)+6}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) - x*log(x^2 - x + 1) - 4*x*
log(x + 1) + 6*x*log(x) + 6)/x
```

giac [A] time = 0.16, size = 45, normalized size = 0.92

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{x}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(|x+1|\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 - x + 1) + 2
/3*log(abs(x + 1)) - log(abs(x))
```

maple [A] time = 0.05, size = 44, normalized size = 0.90

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\ln(x)+\frac{2\ln(x+1)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/x^2/(x^3+1),x)
```

```
[Out] 2/3*ln(x+1)-1/x-ln(x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))
```

maxima [A] time = 3.03, size = 43, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 - x + 1) + 2
/3*log(x + 1) - log(x)
```

mupad [B] time = 0.08, size = 55, normalized size = 1.12

$$\frac{2\ \ln(x+1)}{3}-\ln(x)+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\ 1\mathrm{i}}{6}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\ 1\mathrm{i}}{6}\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x^2*(x^3 + 1)),x)
```

```
[Out] (2*log(x + 1))/3 - log(x) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 +
 1/6) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - 1/x
```

sympy [A] time = 0.22, size = 49, normalized size = 1.00

$$-\log(x)+\frac{2\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x**2/(x**3+1),x)
```

```
[Out] -log(x) + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3
 - sqrt(3)/3)/3 - 1/x
```

3.257 $\int \frac{1-x}{x^3(1+x^3)} dx$

Optimal. Leaf size=32

$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1834, 628}

$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(x^3*(1 + x^3)),x]
[Out] -1/(2*x^2) + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{x^3\left(1+x^3\right)}\,dx &= \int \left(\frac{1}{x^3}-\frac{1}{x^2}-\frac{2}{3(1+x)}+\frac{-1+2x}{3\left(1-x+x^2\right)}\right)dx\\ &= -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x^3*(1 + x^3)),x]
[Out] -1/2*1/x^2 + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x}{x^3\left(1+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x)/(x^3*(1 + x^3)),x]
[Out] IntegrateAlgebraic[(1 - x)/(x^3*(1 + x^3)), x]
```

fricas [A] time = 0.38, size = 33, normalized size = 1.03

$$\frac{2\,x^2\log\left(x^2-x+1\right)-4\,x^2\log\left(x+1\right)+6\,x-3}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="fricas")
[Out] 1/6*(2*x^2*log(x^2 - x + 1) - 4*x^2*log(x + 1) + 6*x - 3)/x^2
```

giac [A] time = 0.15, size = 29, normalized size = 0.91

$$\frac{2\,x-1}{2\,x^2}+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="giac")
[Out] 1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

maple [A] time = 0.05, size = 27, normalized size = 0.84

$$-\frac{2\ln\left(x+1\right)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}+\frac{1}{x}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/x^3/(x^3+1),x)
[Out] -1/2/x^2+1/x-2/3*ln(x+1)+1/3*ln(x^2-x+1)
```

maxima [A] time = 3.00, size = 28, normalized size = 0.88

$$\frac{2\,x-1}{2\,x^2}+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="maxima")
[Out] 1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

mupad [B] time = 0.07, size = 25, normalized size = 0.78

$$\frac{\ln\left(x^2-x+1\right)}{3}-\frac{2\,\ln\left(x+1\right)}{3}+\frac{x-\frac{1}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x^3*(x^3 + 1)),x)
[Out] log(x^2 - x + 1)/3 - (2*log(x + 1))/3 + (x - 1/2)/x^2
```

sympy [A] time = 0.13, size = 27, normalized size = 0.84

$$-\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{3}-\frac{1-2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x**3/(x**3+1),x)
```

```
[Out] -2*log(x + 1)/3 + log(x**2 - x + 1)/3 - (1 - 2*x)/(2*x**2)
```

3.258 $\int \frac{x(1+2x)}{1+x^3}\, dx$

Optimal. Leaf size=41

$$\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1874, 31, 634, 618, 204, 628}

$$\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + 2*x))/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x]/3 + (5*Log[1 - x + x^2])/
6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

```
&& GtQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(1+2x)}{1+x^3}\,dx &= \frac{1}{3}\int \frac{1}{1+x}\,dx+\frac{1}{3}\int \frac{-1+5x}{1-x+x^2}\,dx\\
&= \frac{1}{3}\log(1+x)+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx+\frac{5}{6}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= \frac{1}{3}\log(1+x)+\frac{5}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1+x)+\frac{5}{6}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.15

$$\frac{1}{6}\left(4\log\left(x^3+1\right)+\log\left(x^2-x+1\right)-2\log(x+1)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + 2*x))/(1 + x^3),x]
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + Log[1 - x + x^2] + 4
*Log[1 + x^3])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(1+2x)}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(1 + 2*x))/(1 + x^3),x]
[Out] IntegrateAlgebraic[(x*(1 + 2*x))/(1 + x^3), x]
```

fricas [A] time = 0.40, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

giac [A] time = 0.15, size = 35, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
abs(x + 1))
```

maple [A] time = 0.05, size = 35, normalized size = 0.85

$$\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(x+1\right)}{3}+\frac{5\ln\left(x^{2}-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x+1)/(x^3+1),x)
```

```
[Out] 1/3*ln(x+1)+5/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.99, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{5}{6}\log\left(x^{2}-x+1\right)+\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

mupad [B] time = 4.96, size = 63, normalized size = 1.54

$$\frac{5\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{6}+\frac{5\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{6}+\frac{\ln\left(x+1\right)}{3}-\frac{\sqrt{3}\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)1{}\mathrm{i}}{6}+\frac{\sqrt{3}\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)1{}\mathrm{i}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*x + 1))/(x^3 + 1),x)
```

```
[Out] (5*log(x - (3^(1/2)*1i)/2 - 1/2))/6 + (5*log(x + (3^(1/2)*1i)/2 - 1/2))/6 +
 log(x + 1)/3 - (3^(1/2)*log(x - (3^(1/2)*1i)/2 - 1/2)*1i)/6 + (3^(1/2)*log
(x + (3^(1/2)*1i)/2 - 1/2)*1i)/6
```

sympy [A] time = 0.18, size = 42, normalized size = 1.02

$$\frac{\log\left(x+1\right)}{3}+\frac{5\log\left(x^{2}-x+1\right)}{6}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x**3+1),x)
```

```
[Out] log(x + 1)/3 + 5*log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)
/3)/3
```

3.259 $\int \frac{x(1+2x)}{1-x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1875, 31, 634, 618, 204, 628}

$$-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + 2*x))/(1 - x^3),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x] - Log[1 + x + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(1+2x)}{1-x^3}\,dx &= \frac{1}{3}\int \frac{-3-3x}{1+x+x^2}\,dx + \int \frac{1}{1-x}\,dx\\ &= -\log(1-x) - \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx - \frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\ &= -\log(1-x) - \frac{1}{2}\log\left(1+x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x) - \frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.36

$$-\frac{2}{3}\log\left(1-x^3\right)+\frac{1}{6}\log\left(x^2+x+1\right)-\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + 2*x))/(1 - x^3),x]
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x]/3 + Log[1 + x + x^2]/6 -
(2*Log[1 - x^3])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(1+2x)}{1-x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(1 + 2*x))/(1 - x^3),x]
[Out] IntegrateAlgebraic[(x*(1 + 2*x))/(1 - x^3), x]
```

fricas [A] time = 0.40, size = 32, normalized size = 0.82

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(x -
1)
```

giac [A] time = 0.15, size = 33, normalized size = 0.85

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(abs
(x - 1))
```

maple [A] time = 0.06, size = 33, normalized size = 0.85

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}-\ln\left(x-1\right)-\frac{\ln\left(x^{2}+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x+1)/(-x^3+1),x)
```

```
[Out] -ln(x-1)-1/2*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.99, size = 32, normalized size = 0.82

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x+1\right)\right)-\frac{1}{2}\log\left(x^{2}+x+1\right)-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(x -
 1)
```

mupad [B] time = 0.09, size = 63, normalized size = 1.62

$$-\frac{\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}-\frac{\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}-\ln\left(x-1\right)+\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,1{}\mathrm{i}}{6}-\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,1{}\mathrm{i}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(2*x + 1))/(x^3 - 1),x)
```

```
[Out] (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/6 - log(x + (3^(1/2)*1i)/2 + 1/2
)/2 - log(x - 1) - log(x - (3^(1/2)*1i)/2 + 1/2)/2 - (3^(1/2)*log(x + (3^(1
/2)*1i)/2 + 1/2)*1i)/6
```

sympy [A] time = 0.16, size = 41, normalized size = 1.05

$$-\log\left(x-1\right)-\frac{\log\left(x^{2}+x+1\right)}{2}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x**3+1),x)
```

```
[Out] -log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)
/3
```

3.260 $\int x^2 \left(c+dx+ex^2\right) \left(a+bx^3\right) \, dx$

Optimal. Leaf size=55

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1628}

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(c+dx+ex^2\right) \left(a+bx^3\right) \, dx &= \int \left(acx^2+adx^3+aex^4+bcx^5+bdx^6+bex^7\right) \, dx\\ &= \frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.00

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c+dx+ex^2\right) \left(a+bx^3\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^2*(c + d*x + e*x^2)*(a + b*x^3), x]
```

fricas [A] time = 0.35, size = 43, normalized size = 0.78

$$\frac{1}{8}x^8eb+\frac{1}{7}x^7db+\frac{1}{6}x^6cb+\frac{1}{5}x^5ea+\frac{1}{4}x^4da+\frac{1}{3}x^3ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e*b + 1/7*x^7*d*b + 1/6*x^6*c*b + 1/5*x^5*e*a + 1/4*x^4*d*a + 1/3*x
^3*c*a
```

giac [A] time = 0.20, size = 45, normalized size = 0.82

$$\frac{1}{8}bx^8e + \frac{1}{7}bdx^7 + \frac{1}{6}bcx^6 + \frac{1}{5}ax^5e + \frac{1}{4}adx^4 + \frac{1}{3}acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/8*b*x^8*e + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*x^5*e + 1/4*a*d*x^4 + 1/3*a
*c*x^3
```

maple [A] time = 0.04, size = 44, normalized size = 0.80

$$\frac{1}{8}be\,x^8 + \frac{1}{7}bd\,x^7 + \frac{1}{6}bc\,x^6 + \frac{1}{5}ae\,x^5 + \frac{1}{4}ad\,x^4 + \frac{1}{3}ac\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a),x)
```

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*b*c*x^6+1/7*b*d*x^7+1/8*b*e*x^8
```

maxima [A] time = 1.33, size = 43, normalized size = 0.78

$$\frac{1}{8}bex^8 + \frac{1}{7}bdx^7 + \frac{1}{6}bcx^6 + \frac{1}{5}aex^5 + \frac{1}{4}adx^4 + \frac{1}{3}acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x^4 + 1/3*a
*c*x^3
```

mupad [B] time = 0.03, size = 43, normalized size = 0.78

$$\frac{b\,e\,x^8}{8} + \frac{b\,d\,x^7}{7} + \frac{b\,c\,x^6}{6} + \frac{a\,e\,x^5}{5} + \frac{a\,d\,x^4}{4} + \frac{a\,c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)*(c + d*x + e*x^2),x)
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (b*c*x^6)/6 + (a*e*x^5)/5 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

sympy [A] time = 0.08, size = 49, normalized size = 0.89

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{bcx^6}{6} + \frac{bdx^7}{7} + \frac{bex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*c*x**6/6 + b*d*x**7/7 + b*e*x**8/8
```

3.261 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1628}

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx&=\int\left(acx+adx^2+aex^3+bcx^4+bdx^5+bex^6\right)\,dx\\&=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7\end{aligned}$$

Mathematica [A] time = 0.00, size = 55, normalized size = 1.00

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[x*(c + d*x + e*x^2)*(a + b*x^3), x]
```

fricas [A] time = 0.35, size = 43, normalized size = 0.78

$$\frac{1}{7}x^7eb+\frac{1}{6}x^6db+\frac{1}{5}x^5cb+\frac{1}{4}x^4ea+\frac{1}{3}x^3da+\frac{1}{2}x^2ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e*b + 1/6*x^6*d*b + 1/5*x^5*c*b + 1/4*x^4*e*a + 1/3*x^3*d*a + 1/2*x
^2*c*a
```

giac [A] time = 0.18, size = 45, normalized size = 0.82

$$\frac{1}{7}bx^7e + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}ax^4e + \frac{1}{3}adx^3 + \frac{1}{2}acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*x^4*e + 1/3*a*d*x^3 + 1/2*a
*c*x^2
```

maple [A] time = 0.05, size = 44, normalized size = 0.80

$$\frac{1}{7}be\,x^7 + \frac{1}{6}bd\,x^6 + \frac{1}{5}bc\,x^5 + \frac{1}{4}ae\,x^4 + \frac{1}{3}ad\,x^3 + \frac{1}{2}ac\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a),x)
```

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7
```

maxima [A] time = 1.36, size = 43, normalized size = 0.78

$$\frac{1}{7}bex^7 + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}aex^4 + \frac{1}{3}adx^3 + \frac{1}{2}acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x^3 + 1/2*a
*c*x^2
```

mupad [B] time = 0.03, size = 43, normalized size = 0.78

$$\frac{b\,e\,x^7}{7} + \frac{b\,d\,x^6}{6} + \frac{b\,c\,x^5}{5} + \frac{a\,e\,x^4}{4} + \frac{a\,d\,x^3}{3} + \frac{a\,c\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)*(c + d*x + e*x^2),x)
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (b*c*x^5)/5 + (a*e*x^4)/4 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

sympy [A] time = 0.07, size = 49, normalized size = 0.89

$$\frac{acx^2}{2} + \frac{adx^3}{3} + \frac{aex^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*e*x**7/7
```

3.262 $\int \left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx$

Optimal. Leaf size=50

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1657}

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3),x]
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx &= \int \left(ac+adx+aex^2+bcx^3+bdx^4+bex^5\right)\,dx\\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6\end{aligned}$$

Mathematica [A] time = 0.00, size = 50, normalized size = 1.00

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3),x]
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)*(a + b*x^3),x]
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)*(a + b*x^3), x]
```

fricas [A] time = 0.37, size = 40, normalized size = 0.80

$$\frac{1}{6}x^6eb+\frac{1}{5}x^5db+\frac{1}{4}x^4cb+\frac{1}{3}x^3ea+\frac{1}{2}x^2da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*b + 1/5*x^5*d*b + 1/4*x^4*c*b + 1/3*x^3*e*a + 1/2*x^2*d*a + x*c*a
```

giac [A] time = 0.15, size = 42, normalized size = 0.84

$$\frac{1}{6}bx^6e + \frac{1}{5}bdx^5 + \frac{1}{4}bcx^4 + \frac{1}{3}ax^3e + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6*e + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*x^3*e + 1/2*a*d*x^2 + a*c*x
```

maple [A] time = 0.04, size = 41, normalized size = 0.82

$$\frac{1}{6}bex^6 + \frac{1}{5}bdx^5 + \frac{1}{4}bcx^4 + \frac{1}{3}aex^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a),x)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*b*c*x^4+1/5*b*d*x^5+1/6*b*e*x^6
```

maxima [A] time = 1.34, size = 40, normalized size = 0.80

$$\frac{1}{6}bex^6 + \frac{1}{5}bdx^5 + \frac{1}{4}bcx^4 + \frac{1}{3}aex^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

mupad [B] time = 0.02, size = 40, normalized size = 0.80

$$\frac{b\,e\,x^6}{6} + \frac{b\,d\,x^5}{5} + \frac{b\,c\,x^4}{4} + \frac{a\,e\,x^3}{3} + \frac{a\,d\,x^2}{2} + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x + e*x^2),x)
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^4)/4 + (a*e*x^3)/3 + (b*d*x^5)/5 + (b*e*x^6)/6
```

sympy [A] time = 0.07, size = 46, normalized size = 0.92

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{bcx^4}{4} + \frac{bdx^5}{5} + \frac{bex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*c*x**4/4 + b*d*x**5/5 + b*e*x**6/6
```

3.263 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx$

Optimal. Leaf size=46

$$ac\log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1628}

$$ac\log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx &= \int \left(ad + \frac{ac}{x} + aex + bcx^2 + bdx^3 + bex^4\right) dx\\ &= adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5 + ac\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$ac\log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3))/x, x]
```

fricas [A] time = 0.40, size = 38, normalized size = 0.83

$$\frac{1}{5}bex^5 + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}aex^2 + adx + ac\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="fricas")
```

```
[Out] 1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(x)
```

giac [A] time = 0.15, size = 41, normalized size = 0.89

$$\frac{1}{5}bx^5e + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}ax^2e + adx + ac\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*e + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*x^2*e + a*d*x + a*c*log(abs
(x))
```

maple [A] time = 0.05, size = 39, normalized size = 0.85

$$\frac{bex^5}{5} + \frac{bdx^4}{4} + \frac{bcx^3}{3} + \frac{aex^2}{2} + ac\ln(x) + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x,x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*b*c*x^3+1/4*b*d*x^4+1/5*b*e*x^5+a*c*ln(x)
```

maxima [A] time = 1.33, size = 38, normalized size = 0.83

$$\frac{1}{5}bex^5 + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}aex^2 + adx + ac\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="maxima")
```

```
[Out] 1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(x)
```

mupad [B] time = 0.03, size = 38, normalized size = 0.83

$$a\,c\,\ln(x) + a\,d\,x + \frac{b\,c\,x^3}{3} + \frac{a\,e\,x^2}{2} + \frac{b\,d\,x^4}{4} + \frac{b\,e\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2))/x,x)
```

```
[Out] a*c*log(x) + a*d*x + (b*c*x^3)/3 + (a*e*x^2)/2 + (b*d*x^4)/4 + (b*e*x^5)/5
```

sympy [A] time = 0.14, size = 44, normalized size = 0.96

$$ac\log(x) + adx + \frac{aex^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4} + \frac{bex^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x,x)
```

```
[Out] a*c*log(x) + a*d*x + a*e*x**2/2 + b*c*x**3/3 + b*d*x**4/4 + b*e*x**5/5
```

3.264 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx$

Optimal. Leaf size=44

$$-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1628}

$$-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx &= \int \left(ae+\frac{ac}{x^2}+\frac{ad}{x}+bcx+bdx^2+bex^3\right)dx\\ &= -\frac{ac}{x}+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+ad\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3))/x^2, x]
```

fricas [A] time = 0.41, size = 45, normalized size = 1.02

$$\frac{3\,bex^5+4\,bdx^4+6\,bcx^3+12\,aex^2+12\,adx\log(x)-12\,ac}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*e*x^5 + 4*b*d*x^4 + 6*b*c*x^3 + 12*a*e*x^2 + 12*a*d*x*log(x) - 12
*a*c)/x
```

giac [A] time = 0.15, size = 41, normalized size = 0.93

$$\frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{2}bcx^2 + axe + ad\log\left(|x|\right) - \frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*x*e + a*d*log(abs(x)) - a*c/x
```

maple [A] time = 0.06, size = 39, normalized size = 0.89

$$\frac{be\,x^4}{4} + \frac{bd\,x^3}{3} + \frac{bc\,x^2}{2} + ad\ln(x) + aex - \frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x^2,x)
```

```
[Out] -a*c/x+a*e*x+1/2*b*c*x^2+1/3*b*d*x^3+1/4*b*e*x^4+a*d*ln(x)
```

maxima [A] time = 1.34, size = 38, normalized size = 0.86

$$\frac{1}{4}bex^4 + \frac{1}{3}bdx^3 + \frac{1}{2}bcx^2 + aex + ad\log(x) - \frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*e*x + a*d*log(x) - a*c/x
```

mupad [B] time = 0.03, size = 38, normalized size = 0.86

$$a\,d\,\ln(x) + a\,e\,x - \frac{a\,c}{x} + \frac{b\,c\,x^2}{2} + \frac{b\,d\,x^3}{3} + \frac{b\,e\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2))/x^2,x)
```

```
[Out] a*d*log(x) + a*e*x - (a*c)/x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4
```

sympy [A] time = 0.16, size = 41, normalized size = 0.93

$$-\frac{ac}{x} + ad\log\left(x\right) + aex + \frac{bcx^2}{2} + \frac{bdx^3}{3} + \frac{bex^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x**2,x)
```

```
[Out] -a*c/x + a*d*log(x) + a*e*x + b*c*x**2/2 + b*d*x**3/3 + b*e*x**4/4
```

3.265 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx$

Optimal. Leaf size=44

$$-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1628}

$$-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
[Out] -(a*c)/(2*x^2) - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx &= \int \left(bc+\frac{ac}{x^3}+\frac{ad}{x^2}+\frac{ae}{x}+bdx+bex^2\right)dx\\ &= -\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3+ae\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
[Out] -1/2*(a*c)/x^2 - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3))/x^3, x]
```

fricas [A] time = 0.41, size = 45, normalized size = 1.02

$$\frac{2\,bex^5+3\,bdx^4+6\,bcx^3+6\,aex^2\log(x)-6\,adx-3\,ac}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*e*x^5 + 3*b*d*x^4 + 6*b*c*x^3 + 6*a*e*x^2*log(x) - 6*a*d*x - 3*a*c
)/x^2
```

giac [A] time = 0.16, size = 41, normalized size = 0.93

$$\frac{1}{3}\,bx^3e+\frac{1}{2}\,bdx^2+bcx+ae\log\left(|x|\right)-\frac{2\,adx+ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*e + 1/2*b*d*x^2 + b*c*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a*c)/x
^2
```

maple [A] time = 0.06, size = 39, normalized size = 0.89

$$\frac{be\,x^3}{3}+\frac{bd\,x^2}{2}+ae\ln(x)+bcx-\frac{ad}{x}-\frac{ac}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x^3,x)
```

```
[Out] -1/2*a*c/x^2-a*d/x+b*c*x+1/2*b*d*x^2+1/3*b*e*x^3+a*e*ln(x)
```

maxima [A] time = 1.35, size = 38, normalized size = 0.86

$$\frac{1}{3}\,bex^3+\frac{1}{2}\,bdx^2+bcx+ae\log(x)-\frac{2\,adx+ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*b*e*x^3 + 1/2*b*d*x^2 + b*c*x + a*e*log(x) - 1/2*(2*a*d*x + a*c)/x^2
```

mupad [B] time = 0.03, size = 38, normalized size = 0.86

$$a\,e\,\ln(x)-\frac{\frac{a\,c}{2}+a\,d\,x}{x^2}+b\,c\,x+\frac{b\,d\,x^2}{2}+\frac{b\,e\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2))/x^3,x)
```

```
[Out] a*e*log(x) - ((a*c)/2 + a*d*x)/x^2 + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3
```

sympy [A] time = 0.25, size = 44, normalized size = 1.00

$$ae\log\left(x\right)+bcx+\frac{bdx^2}{2}+\frac{bex^3}{3}+\frac{-ac-2adx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x**3,x)
```

```
[Out] a*e*log(x) + b*c*x + b*d*x**2/2 + b*e*x**3/3 + (-a*c - 2*a*d*x)/(2*x**2)
```

3.266 $\int x^2 \left(c+dx+ex^2\right) \left(a+bx^3\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{c\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1582, 1850}

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{c\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*d*x^
10)/10 + (b^2*e*x^11)/11 + (c*(a + b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx &= \frac{c\left(a+bx^3\right)^3}{9b}+\int \left(a+bx^3\right)^2\left(-cx^2+x^2\left(c+dx+ex^2\right)\right)dx\\ &= \frac{c\left(a+bx^3\right)^3}{9b}+\int \left(a^2dx^3+a^2ex^4+2abdx^6+2abex^7+b^2dx^9+b^2ex^{10}\right) d\\ &= \frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{c\left(a+b\right.}{9b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 97, normalized size = 1.18

$$\frac{1}{3}a^2cx^3+\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{3}abcx^6+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a*b*c*x^6)/3 + (2*a*b*d*x^
7)/7 + (a*b*e*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2, x]
```

fricas [A] time = 0.35, size = 79, normalized size = 0.96

$$\frac{1}{11}x^{11}eb^2+\frac{1}{10}x^{10}db^2+\frac{1}{9}x^9cb^2+\frac{1}{4}x^8eba+\frac{2}{7}x^7dba+\frac{1}{3}x^6cba+\frac{1}{5}x^5ea^2+\frac{1}{4}x^4da^2+\frac{1}{3}x^3ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b^2 + 1/4*x^8*e*b*a + 2/7*x^7
*d*b*a + 1/3*x^6*c*b*a + 1/5*x^5*e*a^2 + 1/4*x^4*d*a^2 + 1/3*x^3*c*a^2
```

giac [A] time = 0.16, size = 82, normalized size = 1.00

$$\frac{1}{11}\,b^2x^{11}e+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abx^8e+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2x^5e+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*x^8*e + 2/7*a*b
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*x^5*e + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

maple [A] time = 0.05, size = 80, normalized size = 0.98

$$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}abd\,x^7+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x)
```

```
[Out] 1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*e*x^8+2/7*a*b*d*x^7+1
/3*a*b*c*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1/3*a^2*c*x^3
```

maxima [A] time = 1.35, size = 79, normalized size = 0.96

$$\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abex^8+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2ex^5+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*e*x^8 + 2/7*a*b
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

mupad [B] time = 0.04, size = 79, normalized size = 0.96

$$\frac{e\,a^2\,x^5}{5}+\frac{d\,a^2\,x^4}{4}+\frac{c\,a^2\,x^3}{3}+\frac{e\,a\,b\,x^8}{4}+\frac{2\,d\,a\,b\,x^7}{7}+\frac{c\,a\,b\,x^6}{3}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^2*(c + d*x + e*x^2),x)
```

```
[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (b^2*c*x^9)/9 + (a^2*e*x^5)/5 + (b^2*d*x^10
)/10 + (b^2*e*x^11)/11 + (a*b*c*x^6)/3 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4
```

sympy [**A**] time = 0.09, size = 92, normalized size = 1.12

$$\frac{a^2cx^3}{3}+\frac{a^2dx^4}{4}+\frac{a^2ex^5}{5}+\frac{abcx^6}{3}+\frac{2abdx^7}{7}+\frac{abex^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + a*b*c*x**6/3 + 2*a*b*d*x**7
/7 + a*b*e*x**8/4 + b**2*c*x**9/9 + b**2*d*x**10/10 + b**2*e*x**11/11
```

3.267 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}$$

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1582, 1850}

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (2*a*b*e*x^7)/7 + (b^2*c*
x^8)/8 + (b^2*e*x^10)/10 + (d*(a + b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx&=\frac{d\left(a+bx^3\right)^3}{9b}+\int\left(a+bx^3\right)^2\left(-dx^2+x\left(c+dx+ex^2\right)\right)\,dx\\&=\frac{d\left(a+bx^3\right)^3}{9b}+\int\left(a^2cx+a^2ex^3+2abcx^4+2abex^6+b^2cx^7+b^2ex^9\right)\,dx\\&=\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}+\frac{d\left(a+bx^3\right)^3}{9b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 97, normalized size = 1.18

$$\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{9}b^2dx^9+\frac{1}{10}b^2ex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (a*b*d*x^
6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^8)/8 + (b^2*d*x^9)/9 + (b^2*e*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x*(c + d*x + e*x^2)*(a + b*x^3)^2, x]
```

fricas [A] time = 0.35, size = 79, normalized size = 0.96

$$\frac{1}{10}x^{10}eb^2+\frac{1}{9}x^9db^2+\frac{1}{8}x^8cb^2+\frac{2}{7}x^7eba+\frac{1}{3}x^6dba+\frac{2}{5}x^5cba+\frac{1}{4}x^4ea^2+\frac{1}{3}x^3da^2+\frac{1}{2}x^2ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*b^2 + 1/9*x^9*d*b^2 + 1/8*x^8*c*b^2 + 2/7*x^7*e*b*a + 1/3*x^6*d
*b*a + 2/5*x^5*c*b*a + 1/4*x^4*e*a^2 + 1/3*x^3*d*a^2 + 1/2*x^2*c*a^2
```

giac [A] time = 0.15, size = 82, normalized size = 1.00

$$\frac{1}{10}\,b^2x^{10}e+\frac{1}{9}\,b^2dx^9+\frac{1}{8}\,b^2cx^8+\frac{2}{7}\,abx^7e+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2x^4e+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*x^10*e + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d
*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

maple [A] time = 0.05, size = 80, normalized size = 0.98

$$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x)
```

```
[Out] 1/10*b^2*e*x^10+1/9*b^2*d*x^9+1/8*b^2*c*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5
*a*b*c*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2*a^2*c*x^2
```

maxima [A] time = 1.29, size = 79, normalized size = 0.96

$$\frac{1}{10}\,b^2ex^{10}+\frac{1}{9}\,b^2dx^9+\frac{1}{8}\,b^2cx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2ex^4+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*e*x^10 + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d
*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

mupad [B] time = 0.04, size = 79, normalized size = 0.96

$$\frac{e\,a^2\,x^4}{4}+\frac{d\,a^2\,x^3}{3}+\frac{c\,a^2\,x^2}{2}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{e\,b^2\,x^{10}}{10}+\frac{d\,b^2\,x^9}{9}+\frac{c\,b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^2*(c + d*x + e*x^2),x)
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (b^2*c*x^8)/8 + (a^2*e*x^4)/4 + (b^2*d*x^9)
/9 + (b^2*e*x^10)/10 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7
```

sympy [A] time = 0.09, size = 94, normalized size = 1.15

$$\frac{a^2cx^2}{2} + \frac{a^2dx^3}{3} + \frac{a^2ex^4}{4} + \frac{2abcx^5}{5} + \frac{abdx^6}{3} + \frac{2abex^7}{7} + \frac{b^2cx^8}{8} + \frac{b^2dx^9}{9} + \frac{b^2ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + 2*a*b*c*x**5/5 + a*b*d*x**6
/3 + 2*a*b*e*x**7/7 + b**2*c*x**8/8 + b**2*d*x**9/9 + b**2*e*x**10/10
```

3.268 $\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx$

Optimal. Leaf size=77

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{e\left(a+bx^3\right)^3}{9b}+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1582, 1850}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{e\left(a+bx^3\right)^3}{9b}+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8 + (e*(a + b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx &= \frac{e\left(a+bx^3\right)^3}{9b}+\int (c+dx)\left(a+bx^3\right)^2\,dx\\ &= \frac{e\left(a+bx^3\right)^3}{9b}+\int \left(a^2c+a^2dx+2abcx^3+2abdx^4+b^2cx^6+b^2dx^7\right)dx\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{e\left(a+bx^3\right)^3}{9b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 92, normalized size = 1.19

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{3}abex^6+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{1}{9}b^2ex^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 +
 (a*b*e*x^6)/3 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + (b^2*e*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)*(a + b*x^3)^2, x]
```

fricas [A] time = 0.36, size = 76, normalized size = 0.99

$$\frac{1}{9}x^9eb^2+\frac{1}{8}x^8db^2+\frac{1}{7}x^7cb^2+\frac{1}{3}x^6eba+\frac{2}{5}x^5dba+\frac{1}{2}x^4cba+\frac{1}{3}x^3ea^2+\frac{1}{2}x^2da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e*b^2 + 1/8*x^8*d*b^2 + 1/7*x^7*c*b^2 + 1/3*x^6*e*b*a + 2/5*x^5*d*b
*a + 1/2*x^4*c*b*a + 1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2
```

giac [A] time = 0.18, size = 79, normalized size = 1.03

$$\frac{1}{9}\,b^2x^9e+\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{1}{3}\,abx^6e+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{3}\,a^2x^3e+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9*e + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*x^6*e + 2/5*a*b*d*x
^5 + 1/2*a*b*c*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

maple [A] time = 0.04, size = 77, normalized size = 1.00

$$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}abe\,x^6+\frac{2}{5}abd\,x^5+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2,x)
```

```
[Out] 1/9*b^2*e*x^9+1/8*b^2*d*x^8+1/7*b^2*c*x^7+1/3*a*b*e*x^6+2/5*a*b*d*x^5+1/2*a
*b*c*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x
```

maxima [A] time = 1.40, size = 76, normalized size = 0.99

$$\frac{1}{9}\,b^2ex^9+\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{1}{3}\,abex^6+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*e*x^9 + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*e*x^6 + 2/5*a*b*d*x
^5 + 1/2*a*b*c*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

mupad [B] time = 0.04, size = 76, normalized size = 0.99

$$\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{e\,a\,b\,x^6}{3}+\frac{2\,d\,a\,b\,x^5}{5}+\frac{c\,a\,b\,x^4}{2}+\frac{e\,b^2\,x^9}{9}+\frac{d\,b^2\,x^8}{8}+\frac{c\,b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x + e*x^2),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^7)/7 + (a^2*e*x^3)/3 + (b^2*d*x^8)/8 + (b^2*e*x^9)
/9 + a^2*c*x + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (a*b*e*x^6)/3
```

sympy [A] time = 0.09, size = 88, normalized size = 1.14

$$a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{abcx^4}{2} + \frac{2abdx^5}{5} + \frac{abex^6}{3} + \frac{b^2cx^7}{7} + \frac{b^2dx^8}{8} + \frac{b^2ex^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 +
a*b*e*x**6/3 + b**2*c*x**7/7 + b**2*d*x**8/8 + b**2*e*x**9/9
```

3.269 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} dx$

Optimal. Leaf size=88

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8$$

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx+ex^2\right)\left(a+bx^3\right)^2}{x}\,dx &= \int \left(a^2d+\frac{a^2c}{x}+a^2ex+2abcx^2+2abdx^3+2abex^4+b^2cx^5+b^2dx^6+b^2ex^7\right) d\\ &= a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8+\end{aligned}$$

Mathematica [A] time = 0.01, size = 88, normalized size = 1.00

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx+ex^2\right)\left(a+bx^3\right)^2}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^2)/x, x]
```

fricas [A] time = 0.41, size = 74, normalized size = 0.84

$$\frac{1}{8}b^2ex^8+\frac{1}{7}b^2dx^7+\frac{1}{6}b^2cx^6+\frac{2}{5}abex^5+\frac{1}{2}abdx^4+\frac{2}{3}abcx^3+\frac{1}{2}a^2ex^2+a^2dx+a^2c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(x)
```

giac [A] time = 0.15, size = 78, normalized size = 0.89

$$\frac{1}{8}b^2x^8e+\frac{1}{7}b^2dx^7+\frac{1}{6}b^2cx^6+\frac{2}{5}abx^5e+\frac{1}{2}abdx^4+\frac{2}{3}abcx^3+\frac{1}{2}a^2x^2e+a^2dx+a^2c\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8*e + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*x^5*e + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*x^2*e + a^2*d*x + a^2*c*log(abs(x))
```

maple [A] time = 0.04, size = 75, normalized size = 0.85

$$\frac{b^2e\,x^8}{8}+\frac{b^2d\,x^7}{7}+\frac{b^2c\,x^6}{6}+\frac{2abe\,x^5}{5}+\frac{abd\,x^4}{2}+\frac{2abc\,x^3}{3}+\frac{a^2e\,x^2}{2}+a^2c\ln(x)+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x,x)
```

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/2*a*b*d*x^4+2/5*a*b*e*x^5+1/6*b^2*c*x
^6+1/7*b^2*d*x^7+1/8*b^2*e*x^8+a^2*c*ln(x)
```

maxima [A] time = 1.37, size = 74, normalized size = 0.84

$$\frac{1}{8}b^2ex^8+\frac{1}{7}b^2dx^7+\frac{1}{6}b^2cx^6+\frac{2}{5}abex^5+\frac{1}{2}abdx^4+\frac{2}{3}abcx^3+\frac{1}{2}a^2ex^2+a^2dx+a^2c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(x)
```

mupad [B] time = 0.04, size = 74, normalized size = 0.84

$$\frac{b^2\,c\,x^6}{6}+\frac{a^2\,e\,x^2}{2}+\frac{b^2\,d\,x^7}{7}+\frac{b^2\,e\,x^8}{8}+a^2\,c\,\ln(x)+a^2\,d\,x+\frac{2\,a\,b\,c\,x^3}{3}+\frac{a\,b\,d\,x^4}{2}+\frac{2\,a\,b\,e\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2))/x,x)
```

```
[Out] (b^2*c*x^6)/6 + (a^2*e*x^2)/2 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*log(x
) + a^2*d*x + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
```

sympy [A] time = 0.19, size = 88, normalized size = 1.00

$$a^2c\log(x)+a^2dx+\frac{a^2ex^2}{2}+\frac{2abcx^3}{3}+\frac{abdx^4}{2}+\frac{2abex^5}{5}+\frac{b^2cx^6}{6}+\frac{b^2dx^7}{7}+\frac{b^2ex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x,x)
```

```
[Out] a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + 2*a*b*c*x**3/3 + a*b*d*x**4/2 +
2*a*b*e*x**5/5 + b**2*c*x**6/6 + b**2*d*x**7/7 + b**2*e*x**8/8
```

3.270 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} \, dx$

Optimal. Leaf size=83

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+abcx^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}b^2cx^5+\frac{1}{6}b^2dx^6+\frac{1}{7}b^2ex^7$$

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+abcx^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}b^2cx^5+\frac{1}{6}b^2dx^6+\frac{1}{7}b^2ex^7$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^2
*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2}\,dx &= \int \left(a^2e+\frac{a^2c}{x^2}+\frac{a^2d}{x}+2abcx+2abdx^2+2abex^3+b^2cx^4+b^2dx^5+b^2ex^6\right)\\ &= -\frac{a^2c}{x}+a^2ex+abcx^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}b^2cx^5+\frac{1}{6}b^2dx^6+\frac{1}{7}b^2ex^7+a\end{aligned}$$

Mathematica [A] time = 0.01, size = 83, normalized size = 1.00

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+abcx^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}b^2cx^5+\frac{1}{6}b^2dx^6+\frac{1}{7}b^2ex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^2
*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2, x]
```

fricas [A] time = 0.41, size = 81, normalized size = 0.98

$$\frac{30\,b^2ex^8+35\,b^2dx^7+42\,b^2cx^6+105\,abex^5+140\,abdx^4+210\,abcx^3+210\,a^2ex^2+210\,a^2dx\log(x)-210\,a^2c}{210\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/210*(30*b^2*e*x^8 + 35*b^2*d*x^7 + 42*b^2*c*x^6 + 105*a*b*e*x^5 + 140*a*b
*d*x^4 + 210*a*b*c*x^3 + 210*a^2*e*x^2 + 210*a^2*d*x*log(x) - 210*a^2*c)/x
```

giac [A] time = 0.15, size = 77, normalized size = 0.93

$$\frac{1}{7}\,b^2x^7e+\frac{1}{6}\,b^2dx^6+\frac{1}{5}\,b^2cx^5+\frac{1}{2}\,abx^4e+\frac{2}{3}\,abdx^3+abcx^2+a^2xe+a^2d\log\left(|x|\right)-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7*e + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*x^4*e + 2/3*a*b*d*x
^3 + a*b*c*x^2 + a^2*x*e + a^2*d*log(abs(x)) - a^2*c/x
```

maple [A] time = 0.06, size = 74, normalized size = 0.89

$$\frac{b^2e\,x^7}{7}+\frac{b^2d\,x^6}{6}+\frac{b^2c\,x^5}{5}+\frac{abe\,x^4}{2}+\frac{2abd\,x^3}{3}+abc\,x^2+a^2d\ln(x)+a^2ex-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x)
```

```
[Out] -a^2*c/x+a^2*e*x+a*b*c*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*b^2*c*x^5+1/6*b^
2*d*x^6+1/7*b^2*e*x^7+a^2*d*ln(x)
```

maxima [A] time = 1.30, size = 73, normalized size = 0.88

$$\frac{1}{7}\,b^2ex^7+\frac{1}{6}\,b^2dx^6+\frac{1}{5}\,b^2cx^5+\frac{1}{2}\,abex^4+\frac{2}{3}\,abdx^3+abcx^2+a^2ex+a^2d\log(x)-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*e*x^7 + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*e*x^4 + 2/3*a*b*d*x
^3 + a*b*c*x^2 + a^2*e*x + a^2*d*log(x) - a^2*c/x
```

mupad [B] time = 0.04, size = 73, normalized size = 0.88

$$\frac{b^2\,c\,x^5}{5}-\frac{a^2\,c}{x}+\frac{b^2\,d\,x^6}{6}+\frac{b^2\,e\,x^7}{7}+a^2\,d\,\ln(x)+a^2\,e\,x+a\,b\,c\,x^2+\frac{2\,a\,b\,d\,x^3}{3}+\frac{a\,b\,e\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2))/x^2,x)
```

```
[Out] (b^2*c*x^5)/5 - (a^2*c)/x + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*log(x) +
a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2
```

sympy [A] time = 0.25, size = 82, normalized size = 0.99

$$-\frac{a^2c}{x}+a^2d\log\left(x\right)+a^2ex+abcx^2+\frac{2abdx^3}{3}+\frac{abex^4}{2}+\frac{b^2cx^5}{5}+\frac{b^2dx^6}{6}+\frac{b^2ex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**2,x)
```

```
[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + a*b*c*x**2 + 2*a*b*d*x**3/3 + a*b*e*
x**4/2 + b**2*c*x**5/5 + b**2*d*x**6/6 + b**2*e*x**7/7
```

3.271 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx$

Optimal. Leaf size=84

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
```

```
[Out] -(a^2*c)/(2*x^2) - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b
^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx &= \int \left(2abc+\frac{a^2c}{x^3}+\frac{a^2d}{x^2}+\frac{a^2e}{x}+2abdx+2abex^2+b^2cx^3+b^2dx^4+b^2ex^5\right)dx\\ &= -\frac{a^2c}{2x^2}-\frac{a^2d}{x}+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6+a^2e\log\end{aligned}$$

Mathematica [A] time = 0.01, size = 84, normalized size = 1.00

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
```

```
[Out] -1/2*(a^2*c)/x^2 - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b
^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3, x]
```

fricas [A] time = 0.39, size = 81, normalized size = 0.96

$$\frac{10\,b^2ex^8+12\,b^2dx^7+15\,b^2cx^6+40\,abex^5+60\,abdx^4+120\,abcx^3+60\,a^2ex^2\log(x)-60\,a^2dx-30\,a^2c}{60\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*e*x^8 + 12*b^2*d*x^7 + 15*b^2*c*x^6 + 40*a*b*e*x^5 + 60*a*b*d*
x^4 + 120*a*b*c*x^3 + 60*a^2*e*x^2*log(x) - 60*a^2*d*x - 30*a^2*c)/x^2
```

giac [A] time = 0.16, size = 78, normalized size = 0.93

$$\frac{1}{6}\,b^2x^6e+\frac{1}{5}\,b^2dx^5+\frac{1}{4}\,b^2cx^4+\frac{2}{3}\,abx^3e+abdx^2+2\,abcx+a^2e\log\left(|x|\right)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6*e + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*x^3*e + a*b*d*x^2 +
 2*a*b*c*x + a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

maple [A] time = 0.05, size = 75, normalized size = 0.89

$$\frac{b^2e\,x^6}{6}+\frac{b^2d\,x^5}{5}+\frac{b^2c\,x^4}{4}+\frac{2abe\,x^3}{3}+abd\,x^2+a^2e\ln(x)+2abcx-\frac{a^2d}{x}-\frac{a^2c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x)
```

```
[Out] -1/2*a^2*c/x^2-a^2*d/x+2*a*b*c*x+a*b*d*x^2+2/3*a*b*e*x^3+1/4*b^2*c*x^4+1/5*
b^2*d*x^5+1/6*b^2*e*x^6+a^2*e*ln(x)
```

maxima [A] time = 1.31, size = 74, normalized size = 0.88

$$\frac{1}{6}\,b^2ex^6+\frac{1}{5}\,b^2dx^5+\frac{1}{4}\,b^2cx^4+\frac{2}{3}\,abex^3+abdx^2+2\,abcx+a^2e\log(x)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*e*x^6 + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*e*x^3 + a*b*d*x^2 +
 2*a*b*c*x + a^2*e*log(x) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

mupad [B] time = 0.04, size = 74, normalized size = 0.88

$$\frac{b^2\,c\,x^4}{4}-\frac{\frac{a^2\,c}{2}+a^2\,d\,x}{x^2}+\frac{b^2\,d\,x^5}{5}+\frac{b^2\,e\,x^6}{6}+a^2\,e\,\ln(x)+a\,b\,d\,x^2+\frac{2\,a\,b\,e\,x^3}{3}+2\,a\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2))/x^3,x)
```

```
[Out] (b^2*c*x^4)/4 - ((a^2*c)/2 + a^2*d*x)/x^2 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 +
 a^2*e*log(x) + a*b*d*x^2 + (2*a*b*e*x^3)/3 + 2*a*b*c*x
```

sympy [A] time = 0.31, size = 87, normalized size = 1.04

$$a^2e\log\left(x\right)+2abcx+abdx^2+\frac{2abex^3}{3}+\frac{b^2cx^4}{4}+\frac{b^2dx^5}{5}+\frac{b^2ex^6}{6}+\frac{-a^2c-2a^2dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**3,x)
```

```
[Out] a**2*e*log(x) + 2*a*b*c*x + a*b*d*x**2 + 2*a*b*e*x**3/3 + b**2*c*x**4/4 + b
**2*d*x**5/5 + b**2*e*x**6/6 + (-a**2*c - 2*a**2*d*x)/(2*x**2)
```

3.272 $\int x^2 \left(c+dx+ex^2\right) \left(a+bx^3\right)^3 \, dx$

Optimal. Leaf size=110

$$\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{c\left(a+bx^3\right)^4}{12b}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1582, 1850}

$$\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{c\left(a+bx^3\right)^4}{12b}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (3*
a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14
+ (c*(a + b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx &= \frac{c\left(a+bx^3\right)^4}{12b}+\int \left(a+bx^3\right)^3\left(-cx^2+x^2\left(c+dx+ex^2\right)\right)dx\\ &= \frac{c\left(a+bx^3\right)^4}{12b}+\int \left(a^3dx^3+a^3ex^4+3a^2bdx^6+3a^2bex^7+3ab^2dx^9+3ab^2\right.\\ &= \frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b\end{aligned}$$

Mathematica [A] time = 0.00, size = 139, normalized size = 1.26

$$\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{2}a^2bcx^6+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{12}b^3cx^{12}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*b*c*x^6)/2 + (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^12)/12 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c + dx + ex^2\right) \left(a + bx^3\right)^3 \, dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3, x]

fricas [A] time = 0.35, size = 115, normalized size = 1.05

$$\frac{1}{14}x^{14}eb^3 + \frac{1}{13}x^{13}db^3 + \frac{1}{12}x^{12}cb^3 + \frac{3}{11}x^{11}eb^2a + \frac{3}{10}x^{10}db^2a + \frac{1}{3}x^9cb^2a + \frac{3}{8}x^8eba^2 + \frac{3}{7}x^7dba^2 + \frac{1}{2}x^6cba^2 + \frac{1}{5}x^5ea^3 + \frac{1}{4}x^4da^3 + \frac{1}{3}x^3ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/14*x^14*e*b^3 + 1/13*x^13*d*b^3 + 1/12*x^12*c*b^3 + 3/11*x^11*e*b^2*a + 3/10*x^10*d*b^2*a + 1/3*x^9*c*b^2*a + 3/8*x^8*e*b*a^2 + 3/7*x^7*d*b*a^2 + 1/2*x^6*c*b*a^2 + 1/5*x^5*e*a^3 + 1/4*x^4*d*a^3 + 1/3*x^3*c*a^3

giac [A] time = 0.16, size = 119, normalized size = 1.08

$$\frac{1}{14}b^3x^{14}e + \frac{1}{13}b^3dx^{13} + \frac{1}{12}b^3cx^{12} + \frac{3}{11}ab^2x^{11}e + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bx^8e + \frac{3}{7}a^2bdx^7 + \frac{1}{2}a^2bcx^6 + \frac{1}{5}a^3x^5e + \frac{1}{4}a^3dx^4 + \frac{1}{3}a^3cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*x^8*e + 3/7*a^2*b*d*x^7 + 1/2*a^2*b*c*x^6 + 1/5*a^3*x^5*e + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3

maple [A] time = 0.04, size = 116, normalized size = 1.05

$$\frac{1}{14}b^3e\,x^{14} + \frac{1}{13}b^3d\,x^{13} + \frac{1}{12}b^3c\,x^{12} + \frac{3}{11}a\,b^2e\,x^{11} + \frac{3}{10}a\,b^2d\,x^{10} + \frac{1}{3}a\,b^2c\,x^9 + \frac{3}{8}a^2be\,x^8 + \frac{3}{7}a^2bd\,x^7 + \frac{1}{2}a^2bc\,x^6 + \frac{1}{5}a^3e\,x^5 + \frac{1}{4}a^3d\,x^4 + \frac{1}{3}a^3c\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x)

[Out] 1/14*b^3*e*x^14+1/13*b^3*d*x^13+1/12*b^3*c*x^12+3/11*a*b^2*e*x^11+3/10*a*b^2*d*x^10+1/3*a*b^2*c*x^9+3/8*a^2*b*e*x^8+3/7*a^2*b*d*x^7+1/2*a^2*b*c*x^6+1/5*a^3*e*x^5+1/4*a^3*d*x^4+1/3*a^3*c*x^3

maxima [A] time = 1.33, size = 115, normalized size = 1.05

$$\frac{1}{14}b^3ex^{14} + \frac{1}{13}b^3dx^{13} + \frac{1}{12}b^3cx^{12} + \frac{3}{11}ab^2ex^{11} + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bex^8 + \frac{3}{7}a^2bdx^7 + \frac{1}{2}a^2bcx^6 + \frac{1}{5}a^3ex^5 + \frac{1}{4}a^3dx^4 + \frac{1}{3}a^3cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 + 3/7*a^2*b*d*x^7 + 1/2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3

mupad [B] time = 0.08, size = 115, normalized size = 1.05

$$\frac{e\,a^3\,x^5}{5}+\frac{d\,a^3\,x^4}{4}+\frac{c\,a^3\,x^3}{3}+\frac{3\,e\,a^2\,b\,x^8}{8}+\frac{3\,d\,a^2\,b\,x^7}{7}+\frac{c\,a^2\,b\,x^6}{2}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{e\,b^3\,x^{14}}{14}+\frac{d\,b^3\,x^{13}}{13}+\frac{c\,b^3\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^3*(c + d*x + e*x^2),x)
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (b^3*c*x^12)/12 + (a^3*e*x^5)/5 + (b^3*d*x^
13)/13 + (b^3*e*x^14)/14 + (a^2*b*c*x^6)/2 + (a*b^2*c*x^9)/3 + (3*a^2*b*d*x
^7)/7 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^8)/8 + (3*a*b^2*e*x^11)/11
```

sympy [A] time = 0.09, size = 138, normalized size = 1.25

$$\frac{a^3cx^3}{3}+\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{a^2bcx^6}{2}+\frac{3a^2bdx^7}{7}+\frac{3a^2bex^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{b^3cx^{12}}{12}+\frac{b^3dx^{13}}{13}+\frac{b^3ex^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + a**2*b*c*x**6/2 + 3*a**2*b*
d*x**7/7 + 3*a**2*b*e*x**8/8 + a*b**2*c*x**9/3 + 3*a*b**2*d*x**10/10 + 3*a*
b**2*e*x**11/11 + b**3*c*x**12/12 + b**3*d*x**13/13 + b**3*e*x**14/14
```

3.273 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx$

Optimal. Leaf size=110

$$\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{d\left(a+bx^3\right)^4}{12b}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3ex^{13}$$

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1582, 1850}

$$\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{d\left(a+bx^3\right)^4}{12b}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3ex^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (3*a^2*b*e*x^7)/7 + (3*
a*b^2*c*x^8)/8 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*e*x^13)/13 +
(d*(a + b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx&=\frac{d\left(a+bx^3\right)^4}{12b}+\int\left(a+bx^3\right)^3\left(-dx^2+x\left(c+dx+ex^2\right)\right)\,dx\\&=\frac{d\left(a+bx^3\right)^4}{12b}+\int\left(a^3cx+a^3ex^3+3a^2bcx^4+3a^2bex^6+3ab^2cx^7+3ab^2ex^9+\right.\\&=\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 139, normalized size = 1.26

$$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{1}{3}ab^2dx^9+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}+\frac{1}{12}b^3dx^{12}+\frac{1}{13}b^3ex^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (a*b^2*d*x^9)/3 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*d*x^12)/12 + (b^3*e*x^13)/13

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[x*(c + d*x + e*x^2)*(a + b*x^3)^3, x]

fricas [A] time = 0.37, size = 115, normalized size = 1.05

$$\frac{1}{13}x^{13}eb^3+\frac{1}{12}x^{12}db^3+\frac{1}{11}x^{11}cb^3+\frac{3}{10}x^{10}eb^2a+\frac{1}{3}x^9db^2a+\frac{3}{8}x^8cb^2a+\frac{3}{7}x^7eba^2+\frac{1}{2}x^6dba^2+\frac{3}{5}x^5cba^2+\frac{1}{4}x^4ea^3+\frac{1}{3}x^3da^3+\frac{1}{2}x^2ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/13*x^13*e*b^3 + 1/12*x^12*d*b^3 + 1/11*x^11*c*b^3 + 3/10*x^10*e*b^2*a + 1/3*x^9*d*b^2*a + 3/8*x^8*c*b^2*a + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*x^4*e*a^3 + 1/3*x^3*d*a^3 + 1/2*x^2*c*a^3

giac [A] time = 0.17, size = 119, normalized size = 1.08

$$\frac{1}{13}b^3x^{13}e+\frac{1}{12}b^3dx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{10}ab^2x^{10}e+\frac{1}{3}ab^2dx^9+\frac{3}{8}ab^2cx^8+\frac{3}{7}a^2bx^7e+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3x^4e+\frac{1}{3}a^3dx^3+\frac{1}{2}a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*x^10*e + 1/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2

maple [A] time = 0.04, size = 116, normalized size = 1.05

$$\frac{1}{13}b^3ex^{13}+\frac{1}{12}b^3dx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{10}ab^2ex^{10}+\frac{1}{3}ab^2dx^9+\frac{3}{8}ab^2cx^8+\frac{3}{7}a^2bex^7+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3ex^4+\frac{1}{3}a^3dx^3+\frac{1}{2}a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x)

[Out] 1/13*b^3*e*x^13+1/12*b^3*d*x^12+1/11*b^3*c*x^11+3/10*a*b^2*e*x^10+1/3*a*b^2*d*x^9+3/8*a*b^2*c*x^8+3/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*a^3*e*x^4+1/3*a^3*d*x^3+1/2*a^3*c*x^2

maxima [A] time = 1.37, size = 115, normalized size = 1.05

$$\frac{1}{13}b^3ex^{13}+\frac{1}{12}b^3dx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{10}ab^2ex^{10}+\frac{1}{3}ab^2dx^9+\frac{3}{8}ab^2cx^8+\frac{3}{7}a^2bex^7+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3ex^4+\frac{1}{3}a^3dx^3+\frac{1}{2}a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*e*x^10 + 1/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2

mupad [B] time = 0.07, size = 115, normalized size = 1.05

$$\frac{e\,a^3\,x^4}{4}+\frac{d\,a^3\,x^3}{3}+\frac{c\,a^3\,x^2}{2}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{3\,e\,a\,b^2\,x^{10}}{10}+\frac{d\,a\,b^2\,x^9}{3}+\frac{3\,c\,a\,b^2\,x^8}{8}+\frac{e\,b^3\,x^{13}}{13}+\frac{d\,b^3\,x^{12}}{12}+\frac{c\,b^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^3*(c + d*x + e*x^2),x)
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (b^3*c*x^11)/11 + (a^3*e*x^4)/4 + (b^3*d*x^
12)/12 + (b^3*e*x^13)/13 + (3*a^2*b*c*x^5)/5 + (3*a*b^2*c*x^8)/8 + (a^2*b*d
*x^6)/2 + (a*b^2*d*x^9)/3 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*e*x^10)/10
```

sympy [A] time = 0.09, size = 138, normalized size = 1.25

$$\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3ab^2cx^8}{8}+\frac{ab^2dx^9}{3}+\frac{3ab^2ex^{10}}{10}+\frac{b^3cx^{11}}{11}+\frac{b^3dx^{12}}{12}+\frac{b^3ex^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + 3*a**2*b*c*x**5/5 + a**2*b*
d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a*b**2*c*x**8/8 + a*b**2*d*x**9/3 + 3*a*b*
*2*e*x**10/10 + b**3*c*x**11/11 + b**3*d*x**12/12 + b**3*e*x**13/13
```

3.274 $\int (c+dx+ex^2)(a+bx^3)^3\,dx$

Optimal. Leaf size=105

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{e(a+bx^3)^4}{12b}+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Rubi [A] time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1582, 1850}

$$\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{e(a+bx^3)^4}{12b}+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (e*(a +
b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (c+dx+ex^2)(a+bx^3)^3\,dx &= \frac{e(a+bx^3)^4}{12b}+\int (c+dx)(a+bx^3)^3\,dx\\ &= \frac{e(a+bx^3)^4}{12b}+\int \left(a^3c+a^3dx+3a^2bcx^3+3a^2bdx^4+3ab^2cx^6+3ab^2dx^7+\right.\\ &= a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\end{aligned}$$

Mathematica [A] time = 0.04, size = 134, normalized size = 1.28

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{1}{2}a^2bex^6+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{3}ab^2ex^9+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}+\frac{1}{12}b^3ex^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^
5)/5 + (a^2*b*e*x^6)/2 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (a*b^2*e*x
^9)/3 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (b^3*e*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx + ex^2\right)\left(a + bx^3\right)^3\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)*(a + b*x^3)^3, x]
```

fricas [A] time = 0.37, size = 112, normalized size = 1.07

$$\frac{1}{12}x^{12}eb^3 + \frac{1}{11}x^{11}db^3 + \frac{1}{10}x^{10}cb^3 + \frac{1}{3}x^9eb^2a + \frac{3}{8}x^8db^2a + \frac{3}{7}x^7cb^2a + \frac{1}{2}x^6eba^2 + \frac{3}{5}x^5dba^2 + \frac{3}{4}x^4cba^2 + \frac{1}{3}x^3ea^3 + \frac{1}{2}x^2da^3 + xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e*b^3 + 1/11*x^11*d*b^3 + 1/10*x^10*c*b^3 + 1/3*x^9*e*b^2*a + 3/8
*x^8*d*b^2*a + 3/7*x^7*c*b^2*a + 1/2*x^6*e*b*a^2 + 3/5*x^5*d*b*a^2 + 3/4*x^
4*c*b*a^2 + 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3
```

giac [A] time = 0.15, size = 116, normalized size = 1.10

$$\frac{1}{12}\,b^3x^{12}e + \frac{1}{11}\,b^3dx^{11} + \frac{1}{10}\,b^3cx^{10} + \frac{1}{3}\,ab^2x^9e + \frac{3}{8}\,ab^2dx^8 + \frac{3}{7}\,ab^2cx^7 + \frac{1}{2}\,a^2bx^6e + \frac{3}{5}\,a^2bdx^5 + \frac{3}{4}\,a^2bcx^4 + \frac{1}{3}\,a^3x^3e + \frac{1}{2}\,a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12*e + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*x^9*e + 3/8
*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*x^6*e + 3/5*a^2*b*d*x^5 + 3/4*a^
2*b*c*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

maple [A] time = 0.05, size = 113, normalized size = 1.08

$$\frac{1}{12}b^3e\,x^{12} + \frac{1}{11}b^3d\,x^{11} + \frac{1}{10}b^3c\,x^{10} + \frac{1}{3}a\,b^2e\,x^9 + \frac{3}{8}a\,b^2d\,x^8 + \frac{3}{7}a\,b^2c\,x^7 + \frac{1}{2}a^2be\,x^6 + \frac{3}{5}a^2bd\,x^5 + \frac{3}{4}a^2bc\,x^4 + \frac{1}{3}a^3e\,x^3 + \frac{1}{2}a^3d\,x^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3,x)
```

```
[Out] 1/12*b^3*e*x^12+1/11*b^3*d*x^11+1/10*b^3*c*x^10+1/3*a*b^2*e*x^9+3/8*a*b^2*d
*x^8+3/7*a*b^2*c*x^7+1/2*a^2*b*e*x^6+3/5*a^2*b*d*x^5+3/4*a^2*b*c*x^4+1/3*a^
3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

maxima [A] time = 1.33, size = 112, normalized size = 1.07

$$\frac{1}{12}\,b^3ex^{12} + \frac{1}{11}\,b^3dx^{11} + \frac{1}{10}\,b^3cx^{10} + \frac{1}{3}\,ab^2ex^9 + \frac{3}{8}\,ab^2dx^8 + \frac{3}{7}\,ab^2cx^7 + \frac{1}{2}\,a^2bex^6 + \frac{3}{5}\,a^2bdx^5 + \frac{3}{4}\,a^2bcx^4 + \frac{1}{3}\,a^3ex^3 + \frac{1}{2}\,a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e*x^9 + 3/8
*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*a^2*b*d*x^5 + 3/4*a^
2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

mupad [B] time = 0.07, size = 112, normalized size = 1.07

$$\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{e\,a^2\,b\,x^6}{2}+\frac{3\,d\,a^2\,b\,x^5}{5}+\frac{3\,c\,a^2\,b\,x^4}{4}+\frac{e\,a\,b^2\,x^9}{3}+\frac{3\,d\,a\,b^2\,x^8}{8}+\frac{3\,c\,a\,b^2\,x^7}{7}+\frac{e\,b^3\,x^{12}}{12}+\frac{d\,b^3\,x^{11}}{11}+\frac{c\,b^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3*(c + d*x + e*x^2),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^10)/10 + (a^3*e*x^3)/3 + (b^3*d*x^11)/11 + (b^3*e*
x^12)/12 + a^3*c*x + (3*a^2*b*c*x^4)/4 + (3*a*b^2*c*x^7)/7 + (3*a^2*b*d*x^5
)/5 + (3*a*b^2*d*x^8)/8 + (a^2*b*e*x^6)/2 + (a*b^2*e*x^9)/3
```

sympy [A] time = 0.14, size = 134, normalized size = 1.28

$$a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{a^2bex^6}{2}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{ab^2ex^9}{3}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}+\frac{b^3ex^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x
**5/5 + a**2*b*e*x**6/2 + 3*a*b**2*c*x**7/7 + 3*a*b**2*d*x**8/8 + a*b**2*e*
x**9/3 + b**3*c*x**10/10 + b**3*d*x**11/11 + b**3*e*x**12/12
```

3.275 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x} \, dx$

Optimal. Leaf size=127

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex$$

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)
/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/
9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x} \, dx &= \int \left(a^3d+\frac{a^3c}{x}+a^3ex+3a^2bcx^2+3a^2bdx^3+3a^2bex^4+3ab^2cx^5+3ab^2dx^6+3\right.\\ &= a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2e\end{aligned}$$

Mathematica [A] time = 0.01, size = 127, normalized size = 1.00

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)
/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/
9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^3)/x, x]
```

fricas [A] time = 0.40, size = 109, normalized size = 0.86

$$\frac{1}{11}\,b^3ex^{11}+\frac{1}{10}\,b^3dx^{10}+\frac{1}{9}\,b^3cx^9+\frac{3}{8}\,ab^2ex^8+\frac{3}{7}\,ab^2dx^7+\frac{1}{2}\,ab^2cx^6+\frac{3}{5}\,a^2bex^5+\frac{3}{4}\,a^2bdx^4+a^2bcx^3+\frac{1}{2}\,a^3ex^2+a^3dx+a^3c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)
```

giac [A] time = 0.15, size = 114, normalized size = 0.90

$$\frac{1}{11}\,b^3x^{11}e+\frac{1}{10}\,b^3dx^{10}+\frac{1}{9}\,b^3cx^9+\frac{3}{8}\,ab^2x^8e+\frac{3}{7}\,ab^2dx^7+\frac{1}{2}\,ab^2cx^6+\frac{3}{5}\,a^2bx^5e+\frac{3}{4}\,a^2bdx^4+a^2bcx^3+\frac{1}{2}\,a^3x^2e+a^3dx+a^3c\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*e + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*x^8*e + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*x^5*e + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*x^2*e + a^3*d*x + a^3*c*log(abs(x))
```

maple [A] time = 0.05, size = 110, normalized size = 0.87

$$\frac{b^3e\,x^{11}}{11}+\frac{b^3d\,x^{10}}{10}+\frac{b^3c\,x^9}{9}+\frac{3a\,b^2e\,x^8}{8}+\frac{3a\,b^2d\,x^7}{7}+\frac{a\,b^2c\,x^6}{2}+\frac{3a^2be\,x^5}{5}+\frac{3a^2bd\,x^4}{4}+a^2bc\,x^3+\frac{a^3e\,x^2}{2}+a^3c\ln(x)+a^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x,x)
```

```
[Out] a^3*d*x+1/2*a^3*e*x^2+a^2*b*c*x^3+3/4*a^2*b*d*x^4+3/5*a^2*b*e*x^5+1/2*a*b^2
*c*x^6+3/7*a*b^2*d*x^7+3/8*a*b^2*e*x^8+1/9*b^3*c*x^9+1/10*b^3*d*x^10+1/11*b
^3*e*x^11+a^3*c*ln(x)
```

maxima [A] time = 1.29, size = 109, normalized size = 0.86

$$\frac{1}{11}\,b^3ex^{11}+\frac{1}{10}\,b^3dx^{10}+\frac{1}{9}\,b^3cx^9+\frac{3}{8}\,ab^2ex^8+\frac{3}{7}\,ab^2dx^7+\frac{1}{2}\,ab^2cx^6+\frac{3}{5}\,a^2bex^5+\frac{3}{4}\,a^2bdx^4+a^2bcx^3+\frac{1}{2}\,a^3ex^2+a^3dx+a^3c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)
```

mupad [B] time = 0.08, size = 109, normalized size = 0.86

$$\frac{b^3\,c\,x^9}{9}+\frac{a^3\,e\,x^2}{2}+\frac{b^3\,d\,x^{10}}{10}+\frac{b^3\,e\,x^{11}}{11}+a^3\,c\,\ln(x)+a^3\,d\,x+a^2\,b\,c\,x^3+\frac{a\,b^2\,c\,x^6}{2}+\frac{3\,a^2\,b\,d\,x^4}{4}+\frac{3\,a\,b^2\,d\,x^7}{7}+\frac{3\,a^2\,b\,e\,x^5}{5}+\frac{3\,a\,b^2\,e\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2))/x,x)
```

```
[Out] (b^3*c*x^9)/9 + (a^3*e*x^2)/2 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*l
og(x) + a^3*d*x + a^2*b*c*x^3 + (a*b^2*c*x^6)/2 + (3*a^2*b*d*x^4)/4 + (3*a*
b^2*d*x^7)/7 + (3*a^2*b*e*x^5)/5 + (3*a*b^2*e*x^8)/8
```

sympy [A] time = 0.29, size = 131, normalized size = 1.03

$$a^3c\log\left(x\right)+a^3dx+\frac{a^3ex^2}{2}+a^2bcx^3+\frac{3a^2bdx^4}{4}+\frac{3a^2bex^5}{5}+\frac{ab^2cx^6}{2}+\frac{3ab^2dx^7}{7}+\frac{3ab^2ex^8}{8}+\frac{b^3cx^9}{9}+\frac{b^3dx^{10}}{10}+\frac{b^3ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x,x)
```

```
[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + a**2*b*c*x**3 + 3*a**2*b*d*x**4/
4 + 3*a**2*b*e*x**5/5 + a*b**2*c*x**6/2 + 3*a*b**2*d*x**7/7 + 3*a*b**2*e*x*
*8/8 + b**3*c*x**9/9 + b**3*d*x**10/10 + b**3*e*x**11/11
```

3.276 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2} dx$

Optimal. Leaf size=125

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^1$$

Rubi [A] time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)/
4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)/8
 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx &= \int \left(a^3e+\frac{a^3c}{x^2}+\frac{a^3d}{x}+3a^2bcx+3a^2bdx^2+3a^2bex^3+3ab^2cx^4+3ab^2dx^5+\right.\\ &= -\frac{a^3c}{x}+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2e\end{aligned}$$

Mathematica [A] time = 0.02, size = 125, normalized size = 1.00

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)/
4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)/8
 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2, x]
```

fricas [A] time = 0.38, size = 117, normalized size = 0.94

$$\frac{252\,b^3ex^{11}+280\,b^3dx^{10}+315\,b^3cx^9+1080\,ab^2ex^8+1260\,ab^2dx^7+1512\,ab^2cx^6+1890\,a^2bex^5+2520\,a^2bdx^4+3780\,a^2bcx^3+2520\,a^3ex^2+2520\,a^3dx\log(x)-2520\,a^3c}{2520\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/2520*(252*b^3*e*x^11 + 280*b^3*d*x^10 + 315*b^3*c*x^9 + 1080*a*b^2*e*x^8
+ 1260*a*b^2*d*x^7 + 1512*a*b^2*c*x^6 + 1890*a^2*b*e*x^5 + 2520*a^2*b*d*x^4
 + 3780*a^2*b*c*x^3 + 2520*a^3*e*x^2 + 2520*a^3*d*x*log(x) - 2520*a^3*c)/x
```

giac [A] time = 0.18, size = 114, normalized size = 0.91

$$\frac{1}{10}\,b^3x^{10}e+\frac{1}{9}\,b^3dx^9+\frac{1}{8}\,b^3cx^8+\frac{3}{7}\,ab^2x^7e+\frac{1}{2}\,ab^2dx^6+\frac{3}{5}\,ab^2cx^5+\frac{3}{4}\,a^2bx^4e+a^2bdx^3+\frac{3}{2}\,a^2bcx^2+a^3xe+a^3d\log\left(|x|\right)-\frac{a^3c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10*e + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*x^7*e + 1/2*a*b
^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*x^4*e + a^2*b*d*x^3 + 3/2*a^2*b*c*x^
2 + a^3*x*e + a^3*d*log(abs(x)) - a^3*c/x
```

maple [A] time = 0.05, size = 110, normalized size = 0.88

$$\frac{b^3e\,x^{10}}{10}+\frac{b^3d\,x^9}{9}+\frac{b^3c\,x^8}{8}+\frac{3a\,b^2e\,x^7}{7}+\frac{a\,b^2d\,x^6}{2}+\frac{3a\,b^2c\,x^5}{5}+\frac{3a^2be\,x^4}{4}+a^2bd\,x^3+\frac{3a^2bc\,x^2}{2}+a^3d\ln(x)+a^3ex-\frac{a^3c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x)
```

```
[Out] -a^3*c/x+a^3*e*x+3/2*a^2*b*c*x^2+a^2*b*d*x^3+3/4*a^2*b*e*x^4+3/5*a*b^2*c*x^
5+1/2*a*b^2*d*x^6+3/7*a*b^2*e*x^7+1/8*b^3*c*x^8+1/9*b^3*d*x^9+1/10*b^3*e*x^
10+a^3*d*ln(x)
```

maxima [A] time = 1.35, size = 109, normalized size = 0.87

$$\frac{1}{10}\,b^3ex^{10}+\frac{1}{9}\,b^3dx^9+\frac{1}{8}\,b^3cx^8+\frac{3}{7}\,ab^2ex^7+\frac{1}{2}\,ab^2dx^6+\frac{3}{5}\,ab^2cx^5+\frac{3}{4}\,a^2bex^4+a^2bdx^3+\frac{3}{2}\,a^2bcx^2+a^3ex+a^3d\log(x)-\frac{a^3c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*e*x^10 + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*e*x^7 + 1/2*a*b
^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*e*x^4 + a^2*b*d*x^3 + 3/2*a^2*b*c*x^
2 + a^3*e*x + a^3*d*log(x) - a^3*c/x
```

mupad [B] time = 0.08, size = 109, normalized size = 0.87

$$\frac{b^3\,c\,x^8}{8}-\frac{a^3\,c}{x}+\frac{b^3\,d\,x^9}{9}+\frac{b^3\,e\,x^{10}}{10}+a^3\,d\,\ln(x)+a^3\,e\,x+\frac{3\,a^2\,b\,c\,x^2}{2}+\frac{3\,a\,b^2\,c\,x^5}{5}+a^2\,b\,d\,x^3+\frac{a\,b^2\,d\,x^6}{2}+\frac{3\,a^2\,b\,e\,x^4}{4}+\frac{3\,a\,b^2\,e\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2))/x^2,x)
```

```
[Out] (b^3*c*x^8)/8 - (a^3*c)/x + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*log(x)
+ a^3*e*x + (3*a^2*b*c*x^2)/2 + (3*a*b^2*c*x^5)/5 + a^2*b*d*x^3 + (a*b^2*d*
x^6)/2 + (3*a^2*b*e*x^4)/4 + (3*a*b^2*e*x^7)/7
```

sympy [A] time = 0.29, size = 128, normalized size = 1.02

$$-\frac{a^3c}{x}+a^3d\log\left(x\right)+a^3ex+\frac{3a^2bcx^2}{2}+a^2bdx^3+\frac{3a^2bex^4}{4}+\frac{3ab^2cx^5}{5}+\frac{ab^2dx^6}{2}+\frac{3ab^2ex^7}{7}+\frac{b^3cx^8}{8}+\frac{b^3dx^9}{9}+\frac{b^3ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**2,x)
```

```
[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + 3*a**2*b*c*x**2/2 + a**2*b*d*x**3 +
3*a**2*b*e*x**4/4 + 3*a*b**2*c*x**5/5 + a*b**2*d*x**6/2 + 3*a*b**2*e*x**7/7
 + b**3*c*x**8/8 + b**3*d*x**9/9 + b**3*e*x**10/10
```

3.277 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3} \, dx$

Optimal. Leaf size=126

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9$$

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

```
[Out] -(a^3*c)/(2*x^2) - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^
3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7
 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3} \, dx = \int \left(3a^2bc+\frac{a^3c}{x^3}+\frac{a^3d}{x^2}+\frac{a^3e}{x}+3a^2bdx+3a^2bex^2+3ab^2cx^3+3ab^2dx^4+3ab^2\right.$$

$$= -\frac{a^3c}{2x^2}-\frac{a^3d}{x}+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+$$

Mathematica [A] time = 0.01, size = 126, normalized size = 1.00

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*c)/x^2 - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^
3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7
 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3, x]
```

fricas [A] time = 0.41, size = 117, normalized size = 0.93

$$\frac{280\,b^3ex^{11}+315\,b^3dx^{10}+360\,b^3cx^9+1260\,ab^2ex^8+1512\,ab^2dx^7+1890\,ab^2cx^6+2520\,a^2bex^5+3780\,a^2bdx^4+7560\,a^2bcx^3+2520\,a^3ex^2\log(x)-2520\,a^3dx-1260\,a^3c}{2520\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/2520*(280*b^3*e*x^11 + 315*b^3*d*x^10 + 360*b^3*c*x^9 + 1260*a*b^2*e*x^8
+ 1512*a*b^2*d*x^7 + 1890*a*b^2*c*x^6 + 2520*a^2*b*e*x^5 + 3780*a^2*b*d*x^4
 + 7560*a^2*b*c*x^3 + 2520*a^3*e*x^2*log(x) - 2520*a^3*d*x - 1260*a^3*c)/x^
2
```

giac [A] time = 0.15, size = 115, normalized size = 0.91

$$\frac{1}{9}\,b^3x^9e+\frac{1}{8}\,b^3dx^8+\frac{1}{7}\,b^3cx^7+\frac{1}{2}\,ab^2x^6e+\frac{3}{5}\,ab^2dx^5+\frac{3}{4}\,ab^2cx^4+a^2bx^3e+\frac{3}{2}\,a^2bdx^2+3\,a^2bcx+a^3e\log\left(|x|\right)-\frac{2\,a^3dx+a^3c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*e + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*x^6*e + 3/5*a*b^2
*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*x^3*e + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x + a^
3*e*log(abs(x)) - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

maple [A] time = 0.05, size = 111, normalized size = 0.88

$$\frac{b^3e\,x^9}{9}+\frac{b^3d\,x^8}{8}+\frac{b^3c\,x^7}{7}+\frac{a\,b^2e\,x^6}{2}+\frac{3a\,b^2d\,x^5}{5}+\frac{3a\,b^2c\,x^4}{4}+a^2be\,x^3+\frac{3a^2bd\,x^2}{2}+a^3e\ln(x)+3a^2bcx-\frac{a^3d}{x}-\frac{a^3c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x)
```

```
[Out] -1/2*a^3*c/x^2-a^3*d/x+3*a^2*b*c*x+3/2*a^2*b*d*x^2+a^2*b*e*x^3+3/4*a*b^2*c*
x^4+3/5*a*b^2*d*x^5+1/2*a*b^2*e*x^6+1/7*b^3*c*x^7+1/8*b^3*d*x^8+1/9*b^3*e*x
^9+a^3*e*ln(x)
```

maxima [A] time = 1.32, size = 110, normalized size = 0.87

$$\frac{1}{9}\,b^3ex^9+\frac{1}{8}\,b^3dx^8+\frac{1}{7}\,b^3cx^7+\frac{1}{2}\,ab^2ex^6+\frac{3}{5}\,ab^2dx^5+\frac{3}{4}\,ab^2cx^4+a^2bex^3+\frac{3}{2}\,a^2bdx^2+3\,a^2bcx+a^3e\log(x)-\frac{2\,a^3dx+a^3c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^3*e*x^9 + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*e*x^6 + 3/5*a*b^2
*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*e*x^3 + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x + a^
3*e*log(x) - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

mupad [B] time = 4.90, size = 110, normalized size = 0.87

$$\frac{b^3\,c\,x^7}{7}-\frac{\frac{a^3\,c}{2}+a^3\,d\,x}{x^2}+\frac{b^3\,d\,x^8}{8}+\frac{b^3\,e\,x^9}{9}+a^3\,e\,\ln(x)+3\,a^2\,b\,c\,x+\frac{3\,a\,b^2\,c\,x^4}{4}+\frac{3\,a^2\,b\,d\,x^2}{2}+\frac{3\,a\,b^2\,d\,x^5}{5}+a^2\,b\,e\,x^3+\frac{a\,b^2\,e\,x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2))/x^3,x)
```

```
[Out] (b^3*c*x^7)/7 - ((a^3*c)/2 + a^3*d*x)/x^2 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 +
 a^3*e*log(x) + 3*a^2*b*c*x + (3*a*b^2*c*x^4)/4 + (3*a^2*b*d*x^2)/2 + (3*a*
b^2*d*x^5)/5 + a^2*b*e*x^3 + (a*b^2*e*x^6)/2
```

sympy [A] time = 0.36, size = 131, normalized size = 1.04

$$a^3 e \log(x) + 3a^2 bcx + \frac{3a^2 bdx^2}{2} + a^2 bex^3 + \frac{3ab^2 cx^4}{4} + \frac{3ab^2 dx^5}{5} + \frac{ab^2 ex^6}{2} + \frac{b^3 cx^7}{7} + \frac{b^3 dx^8}{8} + \frac{b^3 ex^9}{9} + \frac{-a^3 c - 2a^3 dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**3,x)
```

```
[Out] a**3*e*log(x) + 3*a**2*b*c*x + 3*a**2*b*d*x**2/2 + a**2*b*e*x**3 + 3*a*b**2
*c*x**4/4 + 3*a*b**2*d*x**5/5 + a*b**2*e*x**6/2 + b**3*c*x**7/7 + b**3*d*x*
*8/8 + b**3*e*x**9/9 + (-a**3*c - 2*a**3*d*x)/(2*x**2)
```

3.278 $\int x^2 \left(c+dx+ex^2\right)\left(a+bx^3\right)^4 \, dx$

Optimal. Leaf size=138

$$\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{c\left(a+bx^3\right)^5}{15b}+\frac{1}{16}b^4dx^{16}$$

Rubi [A] time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1582, 1850}

$$\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{c\left(a+bx^3\right)^5}{15b}+\frac{1}{16}b^4dx^{16}+\frac{1}{17}b^4ex^{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (3*a^
2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*e*
x^14)/7 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17 + (c*(a + b*x^3)^5)/(15*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx&=\frac{c\left(a+bx^3\right)^5}{15b}+\int\left(a+bx^3\right)^4\left(-cx^2+x^2\left(c+dx+ex^2\right)\right)dx\\&=\frac{c\left(a+bx^3\right)^5}{15b}+\int\left(a^4dx^3+a^4ex^4+4a^3bdx^6+4a^3bex^7+6a^2b^2dx^9+6a^2\right.\\&=\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 181, normalized size = 1.31

$$\frac{1}{3}a^4cx^3+\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{2}{3}a^3bcx^6+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{1}{3}ab^3cx^{12}+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{1}{15}b^4cx^{15}+\frac{1}{16}b^4dx^{16}+\frac{1}{17}b^4ex^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (2*a^3*b*c*x^6)/3 + (4*a^3*
b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 +
 (6*a^2*b^2*e*x^11)/11 + (a*b^3*c*x^12)/3 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*
e*x^14)/7 + (b^4*c*x^15)/15 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c + dx + ex^2\right) \left(a + bx^3\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] IntegrateAlgebraic[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4, x]
```

fricas [A] time = 0.36, size = 151, normalized size = 1.09

$$\frac{1}{17}x^{17}eb^4 + \frac{1}{16}x^{16}db^4 + \frac{1}{15}x^{15}cb^4 + \frac{2}{7}x^{14}eb^3a + \frac{4}{13}x^{13}db^3a + \frac{1}{3}x^{12}cb^3a + \frac{6}{11}x^{11}eb^2a^2 + \frac{3}{5}x^{10}db^2a^2 + \frac{2}{3}x^9cb^2a^2 + \frac{1}{2}x^8eba^3 + \frac{4}{7}x^7dba^3 + \frac{2}{3}x^6cba^3 + \frac{1}{5}x^5ea^4 + \frac{1}{4}x^4da^4 + \frac{1}{3}x^3ca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*e*b^4 + 1/16*x^16*d*b^4 + 1/15*x^15*c*b^4 + 2/7*x^14*e*b^3*a + 4/
13*x^13*d*b^3*a + 1/3*x^12*c*b^3*a + 6/11*x^11*e*b^2*a^2 + 3/5*x^10*d*b^2*a
^2 + 2/3*x^9*c*b^2*a^2 + 1/2*x^8*e*b*a^3 + 4/7*x^7*d*b*a^3 + 2/3*x^6*c*b*a^
3 + 1/5*x^5*e*a^4 + 1/4*x^4*d*a^4 + 1/3*x^3*c*a^4
```

giac [A] time = 0.16, size = 156, normalized size = 1.13

$$\frac{1}{17}b^4x^{17}e + \frac{1}{16}b^4dx^{16} + \frac{1}{15}b^4cx^{15} + \frac{2}{7}ab^3x^{14}e + \frac{4}{13}ab^3dx^{13} + \frac{1}{3}ab^3cx^{12} + \frac{6}{11}a^2b^2x^{11}e + \frac{3}{5}a^2b^2dx^{10} + \frac{2}{3}a^2b^2cx^9 + \frac{1}{2}a^3bx^8e + \frac{4}{7}a^3bdx^7 + \frac{2}{3}a^3bcx^6 + \frac{1}{5}a^4x^5e + \frac{1}{4}a^4dx^4 + \frac{1}{3}a^4cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/17*b^4*x^17*e + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*x^14*e + 4/
13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*x^11*e + 3/5*a^2*b^2*d*x^
10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*x^8*e + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^
6 + 1/5*a^4*x^5*e + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

maple [A] time = 0.04, size = 152, normalized size = 1.10

$$\frac{1}{17}b^4ex^{17} + \frac{1}{16}b^4dx^{16} + \frac{1}{15}b^4cx^{15} + \frac{2}{7}ab^3ex^{14} + \frac{4}{13}ab^3dx^{13} + \frac{1}{3}ab^3cx^{12} + \frac{6}{11}a^2b^2ex^{11} + \frac{3}{5}a^2b^2dx^{10} + \frac{2}{3}a^2b^2cx^9 + \frac{1}{2}a^3bex^8 + \frac{4}{7}a^3bdx^7 + \frac{2}{3}a^3bcx^6 + \frac{1}{5}a^4ex^5 + \frac{1}{4}a^4dx^4 + \frac{1}{3}a^4cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x)
```

```
[Out] 1/17*b^4*e*x^17+1/16*b^4*d*x^16+1/15*b^4*c*x^15+2/7*a*b^3*e*x^14+4/13*a*b^3
*d*x^13+1/3*a*b^3*c*x^12+6/11*a^2*b^2*e*x^11+3/5*a^2*b^2*d*x^10+2/3*a^2*b^2
*c*x^9+1/2*a^3*b*e*x^8+4/7*a^3*b*d*x^7+2/3*a^3*b*c*x^6+1/5*a^4*e*x^5+1/4*a^
4*d*x^4+1/3*a^4*c*x^3
```

maxima [A] time = 1.31, size = 151, normalized size = 1.09

$$\frac{1}{17}b^4ex^{17} + \frac{1}{16}b^4dx^{16} + \frac{1}{15}b^4cx^{15} + \frac{2}{7}ab^3ex^{14} + \frac{4}{13}ab^3dx^{13} + \frac{1}{3}ab^3cx^{12} + \frac{6}{11}a^2b^2ex^{11} + \frac{3}{5}a^2b^2dx^{10} + \frac{2}{3}a^2b^2cx^9 + \frac{1}{2}a^3bex^8 + \frac{4}{7}a^3bdx^7 + \frac{2}{3}a^3bcx^6 + \frac{1}{5}a^4ex^5 + \frac{1}{4}a^4dx^4 + \frac{1}{3}a^4cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e*x^14 + 4/
13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^
```

```
10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^
6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

mupad [B] time = 5.07, size = 151, normalized size = 1.09

$$\frac{e\,a^4\,x^5}{5}+\frac{d\,a^4\,x^4}{4}+\frac{c\,a^4\,x^3}{3}+\frac{e\,a^3\,b\,x^8}{2}+\frac{4\,d\,a^3\,b\,x^7}{7}+\frac{2\,c\,a^3\,b\,x^6}{3}+\frac{6\,e\,a^2\,b^2\,x^{11}}{11}+\frac{3\,d\,a^2\,b^2\,x^{10}}{5}+\frac{2\,c\,a^2\,b^2\,x^9}{3}+\frac{2\,e\,a\,b^3\,x^{14}}{7}+\frac{4\,d\,a\,b^3\,x^{13}}{13}+\frac{c\,a\,b^3\,x^{12}}{3}+\frac{e\,b^4\,x^{17}}{17}+\frac{d\,b^4\,x^{16}}{16}+\frac{c\,b^4\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^4*(c + d*x + e*x^2),x)
```

```
[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (b^4*c*x^15)/15 + (a^4*e*x^5)/5 + (b^4*d*x^
16)/16 + (b^4*e*x^17)/17 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*
a^2*b^2*e*x^11)/11 + (2*a^3*b*c*x^6)/3 + (a*b^3*c*x^12)/3 + (4*a^3*b*d*x^7)
/7 + (4*a*b^3*d*x^13)/13 + (a^3*b*e*x^8)/2 + (2*a*b^3*e*x^14)/7
```

sympy [A] time = 0.11, size = 184, normalized size = 1.33

$$\frac{a^4cx^3}{3}+\frac{a^4dx^4}{4}+\frac{a^4ex^5}{5}+\frac{2a^3bcx^6}{3}+\frac{4a^3bdx^7}{7}+\frac{a^3bex^8}{2}+\frac{2a^2b^2cx^9}{3}+\frac{3a^2b^2dx^{10}}{5}+\frac{6a^2b^2ex^{11}}{11}+\frac{ab^3cx^{12}}{3}+\frac{4ab^3dx^{13}}{13}+\frac{2ab^3ex^{14}}{7}+\frac{b^4cx^{15}}{15}+\frac{b^4dx^{16}}{16}+\frac{b^4ex^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x**3/3 + a**4*d*x**4/4 + a**4*e*x**5/5 + 2*a**3*b*c*x**6/3 + 4*a**3*
b*d*x**7/7 + a**3*b*e*x**8/2 + 2*a**2*b**2*c*x**9/3 + 3*a**2*b**2*d*x**10/5
 + 6*a**2*b**2*e*x**11/11 + a*b**3*c*x**12/3 + 4*a*b**3*d*x**13/13 + 2*a*b*
*3*e*x**14/7 + b**4*c*x**15/15 + b**4*d*x**16/16 + b**4*e*x**17/17
```

3.279 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx$

Optimal. Leaf size=138

$$\frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7+\frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}ab^3ex^{13}+\frac{d\left(a+bx^3\right)^5}{15b}+\frac{1}{14}b^4cx^{14}+\frac{1}{16}$$

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1582, 1850}

$$\frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7+\frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}ab^3ex^{13}+\frac{d\left(a+bx^3\right)^5}{15b}+\frac{1}{14}b^4cx^{14}+\frac{1}{16}b^4ex^{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^2)/2 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (4*a^3*b*e*x^7)/7 + (3*
a^2*b^2*c*x^8)/4 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (4*a*b^3*e*
x^13)/13 + (b^4*c*x^14)/14 + (b^4*e*x^16)/16 + (d*(a + b*x^3)^5)/(15*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx &= \frac{d\left(a+bx^3\right)^5}{15b}+\int\left(a+bx^3\right)^4\left(-dx^2+x\left(c+dx+ex^2\right)\right)dx\\ &= \frac{d\left(a+bx^3\right)^5}{15b}+\int\left(a^4cx+a^4ex^3+4a^3bcx^4+4a^3bex^6+6a^2b^2cx^7+6a^2b^2ex^9\right.\\ &= \frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7+\frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^1\end{aligned}$$

Mathematica [A] time = 0.01, size = 181, normalized size = 1.31

$$\frac{1}{2}a^4cx^2+\frac{1}{3}a^4dx^3+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{3}{4}a^2b^2cx^8+\frac{2}{3}a^2b^2dx^9+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{1}{3}ab^3dx^{12}+\frac{4}{13}ab^3ex^{13}+\frac{1}{14}b^4cx^{14}+\frac{1}{15}b^4dx^{15}+\frac{1}{16}b^4ex^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (2*a^3*
b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (3*a^2*b^2*c*x^8)/4 + (2*a^2*b^2*d*x^9)/3
+ (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (a*b^3*d*x^12)/3 + (4*a*b^3*
e*x^13)/13 + (b^4*c*x^14)/14 + (b^4*d*x^15)/15 + (b^4*e*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] IntegrateAlgebraic[x*(c + d*x + e*x^2)*(a + b*x^3)^4, x]
```

fricas [A] time = 0.35, size = 151, normalized size = 1.09

$$\frac{1}{16}x^{16}eb^4+\frac{1}{15}x^{15}db^4+\frac{1}{14}x^{14}cb^4+\frac{4}{13}x^{13}eb^3a+\frac{1}{3}x^{12}db^3a+\frac{4}{11}x^{11}cb^3a+\frac{3}{5}x^{10}eb^2a^2+\frac{2}{3}x^9db^2a^2+\frac{3}{4}x^8cb^2a^2+\frac{4}{7}x^7eba^3+\frac{2}{3}x^6dba^3+\frac{4}{5}x^5cba^3+\frac{1}{4}x^4ea^4+\frac{1}{3}x^3da^4+\frac{1}{2}x^2ca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e*b^4 + 1/15*x^15*d*b^4 + 1/14*x^14*c*b^4 + 4/13*x^13*e*b^3*a + 1
/3*x^12*d*b^3*a + 4/11*x^11*c*b^3*a + 3/5*x^10*e*b^2*a^2 + 2/3*x^9*d*b^2*a^
2 + 3/4*x^8*c*b^2*a^2 + 4/7*x^7*e*b*a^3 + 2/3*x^6*d*b*a^3 + 4/5*x^5*c*b*a^3
 + 1/4*x^4*e*a^4 + 1/3*x^3*d*a^4 + 1/2*x^2*c*a^4
```

giac [A] time = 0.16, size = 156, normalized size = 1.13

$$\frac{1}{16}b^4x^{16}e+\frac{1}{15}b^4dx^{15}+\frac{1}{14}b^4cx^{14}+\frac{4}{13}ab^3x^{13}e+\frac{1}{3}ab^3dx^{12}+\frac{4}{11}ab^3cx^{11}+\frac{3}{5}a^2b^2x^{10}e+\frac{2}{3}a^2b^2dx^9+\frac{3}{4}a^2b^2cx^8+\frac{4}{7}a^3bx^7e+\frac{2}{3}a^3bdx^6+\frac{4}{5}a^3bcx^5+\frac{1}{4}a^4x^4e+\frac{1}{3}a^4dx^3+\frac{1}{2}a^4cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*b^4*x^16*e + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*x^13*e + 1
/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*x^10*e + 2/3*a^2*b^2*d*x^
9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5
 + 1/4*a^4*x^4*e + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

maple [A] time = 0.04, size = 152, normalized size = 1.10

$$\frac{1}{16}b^4e\,x^{16}+\frac{1}{15}b^4d\,x^{15}+\frac{1}{14}b^4c\,x^{14}+\frac{4}{13}a\,b^3e\,x^{13}+\frac{1}{3}a\,b^3d\,x^{12}+\frac{4}{11}a\,b^3c\,x^{11}+\frac{3}{5}a^2b^2e\,x^{10}+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{4}a^2b^2c\,x^8+\frac{4}{7}a^3be\,x^7+\frac{2}{3}a^3bd\,x^6+\frac{4}{5}a^3bc\,x^5+\frac{1}{4}a^4e\,x^4+\frac{1}{3}a^4d\,x^3+\frac{1}{2}a^4c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x)
```

```
[Out] 1/16*b^4*e*x^16+1/15*b^4*d*x^15+1/14*b^4*c*x^14+4/13*a*b^3*e*x^13+1/3*a*b^3
*d*x^12+4/11*a*b^3*c*x^11+3/5*a^2*b^2*e*x^10+2/3*a^2*b^2*d*x^9+3/4*a^2*b^2*
c*x^8+4/7*a^3*b*e*x^7+2/3*a^3*b*d*x^6+4/5*a^3*b*c*x^5+1/4*a^4*e*x^4+1/3*a^4
*d*x^3+1/2*a^4*c*x^2
```

maxima [A] time = 1.34, size = 151, normalized size = 1.09

$$\frac{1}{16}b^4ex^{16}+\frac{1}{15}b^4dx^{15}+\frac{1}{14}b^4cx^{14}+\frac{4}{13}ab^3ex^{13}+\frac{1}{3}ab^3dx^{12}+\frac{4}{11}ab^3cx^{11}+\frac{3}{5}a^2b^2ex^{10}+\frac{2}{3}a^2b^2dx^9+\frac{3}{4}a^2b^2cx^8+\frac{4}{7}a^3bex^7+\frac{2}{3}a^3bdx^6+\frac{4}{5}a^3bcx^5+\frac{1}{4}a^4ex^4+\frac{1}{3}a^4dx^3+\frac{1}{2}a^4cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*e*x^13 + 1
/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^10 + 2/3*a^2*b^2*d*x^
```

```
9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5
 + 1/4*a^4*e*x^4 + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

mupad [B] time = 0.13, size = 151, normalized size = 1.09

$$\frac{e\,a^4\,x^4}{4}+\frac{d\,a^4\,x^3}{3}+\frac{c\,a^4\,x^2}{2}+\frac{4\,e\,a^3\,b\,x^7}{7}+\frac{2\,d\,a^3\,b\,x^6}{3}+\frac{4\,c\,a^3\,b\,x^5}{5}+\frac{3\,e\,a^2\,b^2\,x^{10}}{5}+\frac{2\,d\,a^2\,b^2\,x^9}{3}+\frac{3\,c\,a^2\,b^2\,x^8}{4}+\frac{4\,e\,a\,b^3\,x^{13}}{13}+\frac{d\,a\,b^3\,x^{12}}{3}+\frac{4\,c\,a\,b^3\,x^{11}}{11}+\frac{e\,b^4\,x^{16}}{16}+\frac{d\,b^4\,x^{15}}{15}+\frac{c\,b^4\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^4*(c + d*x + e*x^2),x)
```

```
[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (b^4*c*x^14)/14 + (a^4*e*x^4)/4 + (b^4*d*x^
15)/15 + (b^4*e*x^16)/16 + (3*a^2*b^2*c*x^8)/4 + (2*a^2*b^2*d*x^9)/3 + (3*a
^2*b^2*e*x^10)/5 + (4*a^3*b*c*x^5)/5 + (4*a*b^3*c*x^11)/11 + (2*a^3*b*d*x^6
)/3 + (a*b^3*d*x^12)/3 + (4*a^3*b*e*x^7)/7 + (4*a*b^3*e*x^13)/13
```

sympy [A] time = 0.11, size = 185, normalized size = 1.34

$$\frac{a^4cx^2}{2}+\frac{a^4dx^3}{3}+\frac{a^4ex^4}{4}+\frac{4a^3bcx^5}{5}+\frac{2a^3bdx^6}{3}+\frac{4a^3bex^7}{7}+\frac{3a^2b^2cx^8}{4}+\frac{2a^2b^2dx^9}{3}+\frac{3a^2b^2ex^{10}}{5}+\frac{4ab^3cx^{11}}{11}+\frac{ab^3dx^{12}}{3}+\frac{4ab^3ex^{13}}{13}+\frac{b^4cx^{14}}{14}+\frac{b^4dx^{15}}{15}+\frac{b^4ex^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x**2/2 + a**4*d*x**3/3 + a**4*e*x**4/4 + 4*a**3*b*c*x**5/5 + 2*a**3*
b*d*x**6/3 + 4*a**3*b*e*x**7/7 + 3*a**2*b**2*c*x**8/4 + 2*a**2*b**2*d*x**9/
3 + 3*a**2*b**2*e*x**10/5 + 4*a*b**3*c*x**11/11 + a*b**3*d*x**12/3 + 4*a*b*
*3*e*x**13/13 + b**4*c*x**14/14 + b**4*d*x**15/15 + b**4*e*x**16/16
```

3.280 $\int (c + dx + ex^2) (a + bx^3)^4 \, dx$

Optimal. Leaf size=130

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{e(a+bx^3)^5}{15b}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4d$$

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1582, 1850}

$$\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+a^3bcx^4+\frac{4}{5}a^3bdx^5+a^4cx+\frac{1}{2}a^4dx^2+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{e(a+bx^3)^5}{15b}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14 + (e*(a + b*x^3)^5)/(15*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (c+dx+ex^2)(a+bx^3)^4\,dx &= \frac{e(a+bx^3)^5}{15b}+\int (c+dx)(a+bx^3)^4\,dx\\ &= \frac{e(a+bx^3)^5}{15b}+\int (a^4c+a^4dx+4a^3bcx^3+4a^3bdx^4+6a^2b^2cx^6+6a^2b^2dx^7\\ &= a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\end{aligned}$$

Mathematica [A] time = 0.01, size = 173, normalized size = 1.33

$$a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{2}{3}a^3bex^6+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{3}a^2b^2ex^9+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{3}ab^3ex^{12}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}+\frac{1}{15}b^4ex^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 +
 (2*a^3*b*e*x^6)/3 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a^2*b^2
*e*x^9)/3 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (a*b^3*e*x^12)/3 + (
b^4*c*x^13)/13 + (b^4*d*x^14)/14 + (b^4*e*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx + ex^2\right)\left(a + bx^3\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)*(a + b*x^3)^4, x]
```

fricas [A] time = 0.36, size = 147, normalized size = 1.13

$$\frac{1}{15}x^{15}eb^4 + \frac{1}{14}x^{14}db^4 + \frac{1}{13}x^{13}cb^4 + \frac{1}{3}x^{12}eb^3a + \frac{4}{11}x^{11}db^3a + \frac{2}{5}x^{10}cb^3a + \frac{2}{3}x^9eb^2a^2 + \frac{3}{4}x^8db^2a^2 + \frac{6}{7}x^7cb^2a^2 + \frac{2}{3}x^6eba^3 + \frac{4}{5}x^5dba^3 + x^4cba^3 + \frac{1}{3}x^3ea^4 + \frac{1}{2}x^2da^4 + xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e*b^4 + 1/14*x^14*d*b^4 + 1/13*x^13*c*b^4 + 1/3*x^12*e*b^3*a + 4/
11*x^11*d*b^3*a + 2/5*x^10*c*b^3*a + 2/3*x^9*e*b^2*a^2 + 3/4*x^8*d*b^2*a^2
+ 6/7*x^7*c*b^2*a^2 + 2/3*x^6*e*b*a^3 + 4/5*x^5*d*b*a^3 + x^4*c*b*a^3 + 1/3
*x^3*e*a^4 + 1/2*x^2*d*a^4 + x*c*a^4
```

giac [A] time = 0.17, size = 152, normalized size = 1.17

$$\frac{1}{15}b^4x^{15}e + \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{1}{3}ab^3x^{12}e + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{2}{3}a^2b^2x^9e + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{2}{3}a^3bx^6e + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{3}a^4x^3e + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/15*b^4*x^15*e + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*x^12*e + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*x^9*e + 3/4*a^2*b^2*d*x^8
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*x^6*e + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x
```

maple [A] time = 0.04, size = 148, normalized size = 1.14

$$\frac{1}{15}b^4ex^{15} + \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{1}{3}ab^3ex^{12} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{2}{3}a^2b^2ex^9 + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{2}{3}a^3bex^6 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{3}a^4ex^3 + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4,x)
```

```
[Out] 1/15*b^4*e*x^15+1/14*b^4*d*x^14+1/13*b^4*c*x^13+1/3*a*b^3*e*x^12+4/11*a*b^3
*d*x^11+2/5*a*b^3*c*x^10+2/3*a^2*b^2*e*x^9+3/4*a^2*b^2*d*x^8+6/7*a^2*b^2*c*
x^7+2/3*a^3*b*e*x^6+4/5*a^3*b*d*x^5+a^3*b*c*x^4+1/3*a^4*e*x^3+1/2*a^4*d*x^2
+a^4*c*x
```

maxima [A] time = 1.31, size = 147, normalized size = 1.13

$$\frac{1}{15}b^4ex^{15} + \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{1}{3}ab^3ex^{12} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{2}{3}a^2b^2ex^9 + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{2}{3}a^3bex^6 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{3}a^4ex^3 + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e*x^12 + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9 + 3/4*a^2*b^2*d*x^8
```

```
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

mupad [B] time = 0.15, size = 147, normalized size = 1.13

$$\frac{e\,a^4\,x^3}{3}+\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{2\,e\,a^3\,b\,x^6}{3}+\frac{4\,d\,a^3\,b\,x^5}{5}+c\,a^3\,b\,x^4+\frac{2\,e\,a^2\,b^2\,x^9}{3}+\frac{3\,d\,a^2\,b^2\,x^8}{4}+\frac{6\,c\,a^2\,b^2\,x^7}{7}+\frac{e\,a\,b^3\,x^{12}}{3}+\frac{4\,d\,a\,b^3\,x^{11}}{11}+\frac{2\,c\,a\,b^3\,x^{10}}{5}+\frac{e\,b^4\,x^{15}}{15}+\frac{d\,b^4\,x^{14}}{14}+\frac{c\,b^4\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^4*(c + d*x + e*x^2),x)
```

```
[Out] (a^4*d*x^2)/2 + (b^4*c*x^13)/13 + (a^4*e*x^3)/3 + (b^4*d*x^14)/14 + (b^4*e*
x^15)/15 + a^4*c*x + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a^2*b^2
*e*x^9)/3 + a^3*b*c*x^4 + (2*a*b^3*c*x^10)/5 + (4*a^3*b*d*x^5)/5 + (4*a*b^3
*d*x^11)/11 + (2*a^3*b*e*x^6)/3 + (a*b^3*e*x^12)/3
```

sympy [A] time = 0.10, size = 178, normalized size = 1.37

$$a^4cx+\frac{a^4dx^2}{2}+\frac{a^4ex^3}{3}+a^3bcx^4+\frac{4a^3bdx^5}{5}+\frac{2a^3bex^6}{3}+\frac{6a^2b^2cx^7}{7}+\frac{3a^2b^2dx^8}{4}+\frac{2a^2b^2ex^9}{3}+\frac{2ab^3cx^{10}}{5}+\frac{4ab^3dx^{11}}{11}+\frac{ab^3ex^{12}}{3}+\frac{b^4cx^{13}}{13}+\frac{b^4dx^{14}}{14}+\frac{b^4ex^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/
5 + 2*a**3*b*e*x**6/3 + 6*a**2*b**2*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a**
2*b**2*e*x**9/3 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11 + a*b**3*e*x**12
/3 + b**4*c*x**13/13 + b**4*d*x**14/14 + b**4*e*x**15/15
```

3.281 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx$

Optimal. Leaf size=166

$$a^4c\log(x)+a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{3}a^3bcx^3+a^3bdx^4+\frac{4}{5}a^3bex^5+a^2b^2cx^6+\frac{6}{7}a^2b^2dx^7+\frac{3}{4}a^2b^2ex^8+\frac{4}{9}ab^3cx^9+\frac{2}{5}ab^3dx^{10}+\frac{4}{11}ab$$

Rubi [A] time = 0.11, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$a^2b^2cx^6+\frac{6}{7}a^2b^2dx^7+\frac{3}{4}a^2b^2ex^8+\frac{4}{3}a^3bcx^3+a^3bdx^4+\frac{4}{5}a^3bex^5+a^4c\log(x)+a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{9}ab^3cx^9+\frac{2}{5}ab^3dx^{10}+\frac{4}{11}ab^3ex^{11}+\frac{1}{12}b^4cx^{12}+\frac{1}{13}b^4dx^{13}+\frac{1}{14}b^4ex^{14}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)
/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c
*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (b^4
*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx &= \int \left(a^4d+\frac{a^4c}{x}+a^4ex+4a^3bcx^2+4a^3bdx^3+4a^3bex^4+6a^2b^2cx^5+6a^2b^2dx^6+\right.\\ &= a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{3}a^3bcx^3+a^3bdx^4+\frac{4}{5}a^3bex^5+a^2b^2cx^6+\frac{6}{7}a^2b^2dx^7+\frac{3}{4}a^2b^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 166, normalized size = 1.00

$$a^4c\log(x)+a^4dx+\frac{1}{2}a^4ex^2+\frac{4}{3}a^3bcx^3+a^3bdx^4+\frac{4}{5}a^3bex^5+a^2b^2cx^6+\frac{6}{7}a^2b^2dx^7+\frac{3}{4}a^2b^2ex^8+\frac{4}{9}ab^3cx^9+\frac{2}{5}ab^3dx^{10}+\frac{4}{11}ab^3ex^{11}+\frac{1}{12}b^4cx^{12}+\frac{1}{13}b^4dx^{13}+\frac{1}{14}b^4ex^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)
/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c
*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (b^4
*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^4)/x, x]
```

fricas [A] time = 0.41, size = 144, normalized size = 0.87

$$\frac{1}{14}b^4ex^{14}+\frac{1}{13}b^4dx^{13}+\frac{1}{12}b^4cx^{12}+\frac{4}{11}ab^3ex^{11}+\frac{2}{5}ab^3dx^{10}+\frac{4}{9}ab^3cx^9+\frac{3}{4}a^2b^2ex^8+\frac{6}{7}a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}a^3bex^5+a^3bdx^4+\frac{4}{3}a^3bcx^3+\frac{1}{2}a^4ex^2+a^4dx+a^4c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="fricas")
```

```
[Out] 1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
e*x^2 + a^4*d*x + a^4*c*log(x)
```

giac [A] time = 0.15, size = 150, normalized size = 0.90

$$\frac{1}{14}b^4x^{14}e+\frac{1}{13}b^4dx^{13}+\frac{1}{12}b^4cx^{12}+\frac{4}{11}ab^3x^{11}e+\frac{2}{5}ab^3dx^{10}+\frac{4}{9}ab^3cx^9+\frac{3}{4}a^2b^2x^8e+\frac{6}{7}a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}a^3bx^5e+a^3bdx^4+\frac{4}{3}a^3bcx^3+\frac{1}{2}a^4x^2e+a^4dx+a^4c\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="giac")
```

```
[Out] 1/14*b^4*x^14*e + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*x^11*e + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*x^8*e + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*x^5*e + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
x^2*e + a^4*d*x + a^4*c*log(abs(x))
```

maple [A] time = 0.04, size = 145, normalized size = 0.87

$$\frac{b^4ex^{14}}{14}+\frac{b^4dx^{13}}{13}+\frac{b^4cx^{12}}{12}+\frac{4ab^3ex^{11}}{11}+\frac{2ab^3dx^{10}}{5}+\frac{4ab^3cx^9}{9}+\frac{3a^2b^2ex^8}{4}+\frac{6a^2b^2dx^7}{7}+a^2b^2cx^6+\frac{4a^3bex^5}{5}+a^3bdx^4+\frac{4a^3bcx^3}{3}+\frac{a^4ex^2}{2}+a^4c\ln(x)+a^4dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x,x)
```

```
[Out] a^4*d*x+1/2*a^4*e*x^2+4/3*a^3*b*c*x^3+a^3*b*d*x^4+4/5*a^3*b*e*x^5+a^2*b^2*c
*x^6+6/7*a^2*b^2*d*x^7+3/4*a^2*b^2*e*x^8+4/9*a*b^3*c*x^9+2/5*a*b^3*d*x^10+4
/11*a*b^3*e*x^11+1/12*b^4*c*x^12+1/13*b^4*d*x^13+1/14*b^4*e*x^14+a^4*c*ln(x
)
```

maxima [A] time = 1.30, size = 144, normalized size = 0.87

$$\frac{1}{14}b^4ex^{14}+\frac{1}{13}b^4dx^{13}+\frac{1}{12}b^4cx^{12}+\frac{4}{11}ab^3ex^{11}+\frac{2}{5}ab^3dx^{10}+\frac{4}{9}ab^3cx^9+\frac{3}{4}a^2b^2ex^8+\frac{6}{7}a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}a^3bex^5+a^3bdx^4+\frac{4}{3}a^3bcx^3+\frac{1}{2}a^4ex^2+a^4dx+a^4c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="maxima")
```

```
[Out] 1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
e*x^2 + a^4*d*x + a^4*c*log(x)
```

mupad [B] time = 0.14, size = 144, normalized size = 0.87

$$\frac{b^4cx^{12}}{12}+\frac{a^4ex^2}{2}+\frac{b^4dx^{13}}{13}+\frac{b^4ex^{14}}{14}+a^4c\ln(x)+a^4dx+a^2b^2cx^6+\frac{6a^2b^2dx^7}{7}+\frac{3a^2b^2ex^8}{4}+\frac{4a^3bcx^3}{3}+\frac{4ab^3cx^9}{9}+a^3bdx^4+\frac{2ab^3dx^{10}}{5}+\frac{4a^3bex^5}{5}+\frac{4ab^3ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^4*(c + d*x + e*x^2))/x,x)
```

```
[Out] (b^4*c*x^12)/12 + (a^4*e*x^2)/2 + (b^4*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c
*log(x) + a^4*d*x + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)
```

```
/4 + (4*a^3*b*c*x^3)/3 + (4*a*b^3*c*x^9)/9 + a^3*b*d*x^4 + (2*a*b^3*d*x^10)
/5 + (4*a^3*b*e*x^5)/5 + (4*a*b^3*e*x^11)/11
```

sympy [A] time = 0.34, size = 175, normalized size = 1.05

$$a^4c\log(x) + a^4dx + \frac{a^4ex^2}{2} + \frac{4a^3bcx^3}{3} + a^3bdx^4 + \frac{4a^3bex^5}{5} + a^2b^2cx^6 + \frac{6a^2b^2dx^7}{7} + \frac{3a^2b^2ex^8}{4} + \frac{4ab^3cx^9}{9} + \frac{2ab^3dx^{10}}{5} + \frac{4ab^3ex^{11}}{11} + \frac{b^4cx^{12}}{12} + \frac{b^4dx^{13}}{13} + \frac{b^4ex^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x,x)
```

```
[Out] a**4*c*log(x) + a**4*d*x + a**4*e*x**2/2 + 4*a**3*b*c*x**3/3 + a**3*b*d*x**
4 + 4*a**3*b*e*x**5/5 + a**2*b**2*c*x**6 + 6*a**2*b**2*d*x**7/7 + 3*a**2*b*
*2*e*x**8/4 + 4*a*b**3*c*x**9/9 + 2*a*b**3*d*x**10/5 + 4*a*b**3*e*x**11/11
+ b**4*c*x**12/12 + b**4*d*x**13/13 + b**4*e*x**14/14
```

3.282 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx$

Optimal. Leaf size=162

$$-\frac{a^4c}{x}+a^4d\log(x)+a^4ex+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4+\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{6}{7}a^2b^2ex^7+\frac{1}{2}ab^3cx^8+\frac{4}{9}ab^3dx^9+\frac{2}{5}ab^3$$

Rubi [A] time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{6}{7}a^2b^2ex^7+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4-\frac{a^4c}{x}+a^4d\log(x)+a^4ex+\frac{1}{2}ab^3cx^8+\frac{4}{9}ab^3dx^9+\frac{2}{5}ab^3ex^{10}+\frac{1}{11}b^4cx^{11}+\frac{1}{12}b^4dx^{12}+\frac{1}{13}b^4ex^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

```
[Out] -((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
(6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2
 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/
12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx &= \int \left(a^4e+\frac{a^4c}{x^2}+\frac{a^4d}{x}+4a^3bcx+4a^3bdx^2+4a^3bex^3+6a^2b^2cx^4+6a^2b^2dx^5\right.\\ &= -\frac{a^4c}{x}+a^4ex+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4+\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{6}{7}a^2b^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 162, normalized size = 1.00

$$-\frac{a^4c}{x}+a^4d\log(x)+a^4ex+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4+\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{6}{7}a^2b^2ex^7+\frac{1}{2}ab^3cx^8+\frac{4}{9}ab^3dx^9+\frac{2}{5}ab^3ex^{10}+\frac{1}{11}b^4cx^{11}+\frac{1}{12}b^4dx^{12}+\frac{1}{13}b^4ex^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

```
[Out] -((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
(6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2
 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/
12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2, x]
```

fricas [A] time = 0.39, size = 153, normalized size = 0.94

$$\frac{13860\,b^4ex^{14}+15015\,b^4dx^{13}+16380\,b^4cx^{12}+72072\,ab^3ex^{11}+80080\,ab^3dx^{10}+90090\,ab^3cx^9+154440\,a^2b^2ex^8+180180\,a^2b^2dx^7+216216\,a^2b^2cx^6+180180\,a^3bex^5+240240\,a^3bdx^4+360360\,a^3bcx^3+180180\,a^4ex^2+180180\,a^4dx\log(x)-180180\,a^4c}{180180\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="fricas")
```

```
[Out] 1/180180*(13860*b^4*e*x^14 + 15015*b^4*d*x^13 + 16380*b^4*c*x^12 + 72072*a*
b^3*e*x^11 + 80080*a*b^3*d*x^10 + 90090*a*b^3*c*x^9 + 154440*a^2*b^2*e*x^8
+ 180180*a^2*b^2*d*x^7 + 216216*a^2*b^2*c*x^6 + 180180*a^3*b*e*x^5 + 240240
*a^3*b*d*x^4 + 360360*a^3*b*c*x^3 + 180180*a^4*e*x^2 + 180180*a^4*d*x*log(x
) - 180180*a^4*c)/x
```

giac [A] time = 0.16, size = 150, normalized size = 0.93

$$\frac{1}{13}\,b^4x^{13}e+\frac{1}{12}\,b^4dx^{12}+\frac{1}{11}\,b^4cx^{11}+\frac{2}{5}\,ab^3x^{10}e+\frac{4}{9}\,ab^3dx^9+\frac{1}{2}\,ab^3cx^8+\frac{6}{7}\,a^2b^2x^7e+a^2b^2dx^6+\frac{6}{5}\,a^2b^2cx^5+a^3bx^4e+\frac{4}{3}\,a^3bdx^3+2\,a^3bcx^2+a^4xe+a^4d\log\left(|x|\right)-\frac{a^4c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="giac")
```

```
[Out] 1/13*b^4*x^13*e + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*x^10*e + 4/
9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*x^7*e + a^2*b^2*d*x^6 + 6/5*a
^2*b^2*c*x^5 + a^3*b*x^4*e + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*x*e + a^
4*d*log(abs(x)) - a^4*c/x
```

maple [A] time = 0.05, size = 145, normalized size = 0.90

$$\frac{b^4e\,x^{13}}{13}+\frac{b^4d\,x^{12}}{12}+\frac{b^4c\,x^{11}}{11}+\frac{2a\,b^3e\,x^{10}}{5}+\frac{4a\,b^3d\,x^9}{9}+\frac{a\,b^3c\,x^8}{2}+\frac{6a^2b^2e\,x^7}{7}+a^2b^2d\,x^6+\frac{6a^2b^2c\,x^5}{5}+a^3be\,x^4+\frac{4a^3bd\,x^3}{3}+2a^3bc\,x^2+a^4d\ln(x)+a^4ex-\frac{a^4c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x)
```

```
[Out] -a^4*c/x+a^4*e*x+2*a^3*b*c*x^2+4/3*a^3*b*d*x^3+a^3*b*e*x^4+6/5*a^2*b^2*c*x^
5+a^2*b^2*d*x^6+6/7*a^2*b^2*e*x^7+1/2*a*b^3*c*x^8+4/9*a*b^3*d*x^9+2/5*a*b^3
*e*x^10+1/11*b^4*c*x^11+1/12*b^4*d*x^12+1/13*b^4*e*x^13+a^4*d*ln(x)
```

maxima [A] time = 1.31, size = 144, normalized size = 0.89

$$\frac{1}{13}\,b^4ex^{13}+\frac{1}{12}\,b^4dx^{12}+\frac{1}{11}\,b^4cx^{11}+\frac{2}{5}\,ab^3ex^{10}+\frac{4}{9}\,ab^3dx^9+\frac{1}{2}\,ab^3cx^8+\frac{6}{7}\,a^2b^2ex^7+a^2b^2dx^6+\frac{6}{5}\,a^2b^2cx^5+a^3bex^4+\frac{4}{3}\,a^3bdx^3+2\,a^3bcx^2+a^4ex+a^4d\log(x)-\frac{a^4c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^4*e*x^13 + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*e*x^10 + 4/
9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*e*x^7 + a^2*b^2*d*x^6 + 6/5*a
^2*b^2*c*x^5 + a^3*b*e*x^4 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*e*x + a^
4*d*log(x) - a^4*c/x
```

mupad [B] time = 4.99, size = 144, normalized size = 0.89

$$\frac{b^4\,c\,x^{11}}{11}-\frac{a^4\,c}{x}+\frac{b^4\,d\,x^{12}}{12}+\frac{b^4\,e\,x^{13}}{13}+a^4\,d\,\ln(x)+a^4\,e\,x+\frac{6\,a^2\,b^2\,c\,x^5}{5}+a^2\,b^2\,d\,x^6+\frac{6\,a^2\,b^2\,e\,x^7}{7}+2\,a^3\,b\,c\,x^2+\frac{a\,b^3\,c\,x^8}{2}+\frac{4\,a^3\,b\,d\,x^3}{3}+\frac{4\,a\,b^3\,d\,x^9}{9}+a^3\,b\,e\,x^4+\frac{2\,a\,b^3\,e\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^4*(c + d*x + e*x^2))/x^2,x)
```

```
[Out] (b^4*c*x^11)/11 - (a^4*c)/x + (b^4*d*x^12)/12 + (b^4*e*x^13)/13 + a^4*d*log
(x) + a^4*e*x + (6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 +
```

```
 2*a^3*b*c*x^2 + (a*b^3*c*x^8)/2 + (4*a^3*b*d*x^3)/3 + (4*a*b^3*d*x^9)/9 +
a^3*b*e*x^4 + (2*a*b^3*e*x^10)/5
```

sympy [A] time = 0.38, size = 168, normalized size = 1.04

$$-\frac{a^4c}{x}+a^4d\log(x)+a^4ex+2a^3bcx^2+\frac{4a^3bdx^3}{3}+a^3bex^4+\frac{6a^2b^2cx^5}{5}+a^2b^2dx^6+\frac{6a^2b^2ex^7}{7}+\frac{ab^3cx^8}{2}+\frac{4ab^3dx^9}{9}+\frac{2ab^3ex^{10}}{5}+\frac{b^4cx^{11}}{11}+\frac{b^4dx^{12}}{12}+\frac{b^4ex^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**2,x)
```

```
[Out] -a**4*c/x + a**4*d*log(x) + a**4*e*x + 2*a**3*b*c*x**2 + 4*a**3*b*d*x**3/3
+ a**3*b*e*x**4 + 6*a**2*b**2*c*x**5/5 + a**2*b**2*d*x**6 + 6*a**2*b**2*e*x
**7/7 + a*b**3*c*x**8/2 + 4*a*b**3*d*x**9/9 + 2*a*b**3*e*x**10/5 + b**4*c*x
**11/11 + b**4*d*x**12/12 + b**4*e*x**13/13
```

3.283 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx$

Optimal. Leaf size=166

$$-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+a^4e\log(x)+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9$$

Rubi [A] time = 0.12, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+a^4e\log(x)+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9+\frac{1}{10}b^4cx^{10}+\frac{1}{11}b^4dx^{11}+\frac{1}{12}b^4ex^{12}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

```
[Out] -(a^4*c)/(2*x^2) - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^3
)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^3*
c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*d*x
^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = \int \left(4a^3bc+\frac{a^4c}{x^3}+\frac{a^4d}{x^2}+\frac{a^4e}{x}+4a^3bdx+4a^3bex^2+6a^2b^2cx^3+6a^2b^2dx^4+6a\right.$$
$$= -\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6$$

Mathematica [A] time = 0.01, size = 166, normalized size = 1.00

$$-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+a^4e\log(x)+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9+\frac{1}{10}b^4cx^{10}+\frac{1}{11}b^4dx^{11}+\frac{1}{12}b^4ex^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

```
[Out] -1/2*(a^4*c)/x^2 - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^3
)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^3*
c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*d*x
^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3, x]
```

fricas [A] time = 0.41, size = 153, normalized size = 0.92

$$\frac{1155\,b^4ex^{14} + 1260\,b^4dx^{13} + 1386\,b^4cx^{12} + 6160\,ab^3ex^{11} + 6930\,ab^3dx^{10} + 7920\,ab^3cx^9 + 13860\,a^2b^2ex^8 + 16632\,a^2b^2dx^7 + 20790\,a^2b^2cx^6 + 18480\,a^3bex^5 + 27720\,a^3bdx^4 + 55440\,a^3bcx^3 + 13860\,a^4ex^2\log(x) - 13860\,a^4dx - 6930\,a^4c}{13860\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="fricas")
```

```
[Out] 1/13860*(1155*b^4*e*x^14 + 1260*b^4*d*x^13 + 1386*b^4*c*x^12 + 6160*a*b^3*e
*x^11 + 6930*a*b^3*d*x^10 + 7920*a*b^3*c*x^9 + 13860*a^2*b^2*e*x^8 + 16632*
a^2*b^2*d*x^7 + 20790*a^2*b^2*c*x^6 + 18480*a^3*b*e*x^5 + 27720*a^3*b*d*x^4
 + 55440*a^3*b*c*x^3 + 13860*a^4*e*x^2*log(x) - 13860*a^4*d*x - 6930*a^4*c)
/x^2
```

giac [A] time = 0.17, size = 152, normalized size = 0.92

$$\frac{1}{12}\,b^4x^{12}e + \frac{1}{11}\,b^4dx^{11} + \frac{1}{10}\,b^4cx^{10} + \frac{4}{9}\,ab^3x^9e + \frac{1}{2}\,ab^3dx^8 + \frac{4}{7}\,ab^3cx^7 + a^2b^2x^6e + \frac{6}{5}\,a^2b^2dx^5 + \frac{3}{2}\,a^2b^2cx^4 + \frac{4}{3}\,a^3bx^3e + 2\,a^3bdx^2 + 4\,a^3bcx + a^4e\log(|x|) - \frac{2\,a^4dx + a^4c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="giac")
```

```
[Out] 1/12*b^4*x^12*e + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*x^9*e + 1/2
*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*x^6*e + 6/5*a^2*b^2*d*x^5 + 3/2*a^
2*b^2*c*x^4 + 4/3*a^3*b*x^3*e + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(abs
(x)) - 1/2*(2*a^4*d*x + a^4*c)/x^2
```

maple [A] time = 0.05, size = 147, normalized size = 0.89

$$\frac{b^4e\,x^{12}}{12} + \frac{b^4d\,x^{11}}{11} + \frac{b^4c\,x^{10}}{10} + \frac{4a\,b^3e\,x^9}{9} + \frac{a\,b^3d\,x^8}{2} + \frac{4a\,b^3c\,x^7}{7} + a^2b^2e\,x^6 + \frac{6a^2b^2d\,x^5}{5} + \frac{3a^2b^2c\,x^4}{2} + \frac{4a^3be\,x^3}{3} + 2a^3bd\,x^2 + a^4e\ln(x) + 4a^3bcx - \frac{a^4d}{x} - \frac{a^4c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x)
```

```
[Out] -1/2*a^4*c/x^2-a^4*d/x+4*a^3*b*c*x+2*a^3*b*d*x^2+4/3*a^3*b*e*x^3+3/2*a^2*b^
2*c*x^4+6/5*a^2*b^2*d*x^5+a^2*b^2*e*x^6+4/7*a*b^3*c*x^7+1/2*a*b^3*d*x^8+4/9
*a*b^3*e*x^9+1/10*b^4*c*x^10+1/11*b^4*d*x^11+1/12*b^4*e*x^12+a^4*e*ln(x)
```

maxima [A] time = 1.33, size = 146, normalized size = 0.88

$$\frac{1}{12}\,b^4ex^{12} + \frac{1}{11}\,b^4dx^{11} + \frac{1}{10}\,b^4cx^{10} + \frac{4}{9}\,ab^3ex^9 + \frac{1}{2}\,ab^3dx^8 + \frac{4}{7}\,ab^3cx^7 + a^2b^2ex^6 + \frac{6}{5}\,a^2b^2dx^5 + \frac{3}{2}\,a^2b^2cx^4 + \frac{4}{3}\,a^3bex^3 + 2\,a^3bdx^2 + 4\,a^3bcx + a^4e\log(x) - \frac{2\,a^4dx + a^4c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*e*x^12 + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*e*x^9 + 1/2
*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*e*x^6 + 6/5*a^2*b^2*d*x^5 + 3/2*a^
2*b^2*c*x^4 + 4/3*a^3*b*e*x^3 + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(x)
- 1/2*(2*a^4*d*x + a^4*c)/x^2
```

mupad [B] time = 4.99, size = 146, normalized size = 0.88

$$\frac{b^4\,c\,x^{10}}{10} - \frac{\frac{a^4\,c}{2} + a^4\,d\,x}{x^2} + \frac{b^4\,d\,x^{11}}{11} + \frac{b^4\,e\,x^{12}}{12} + a^4\,e\,\ln(x) + \frac{3\,a^2\,b^2\,c\,x^4}{2} + \frac{6\,a^2\,b^2\,d\,x^5}{5} + a^2\,b^2\,e\,x^6 + 4\,a^3\,b\,c\,x + \frac{4\,a\,b^3\,c\,x^7}{7} + 2\,a^3\,b\,d\,x^2 + \frac{a\,b^3\,d\,x^8}{2} + \frac{4\,a^3\,b\,e\,x^3}{3} + \frac{4\,a\,b^3\,e\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^4*(c + d*x + e*x^2))/x^3,x)
```

```
[Out] (b^4*c*x^10)/10 - ((a^4*c)/2 + a^4*d*x)/x^2 + (b^4*d*x^11)/11 + (b^4*e*x^12
)/12 + a^4*e*log(x) + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e
```

```
*x^6 + 4*a^3*b*c*x + (4*a*b^3*c*x^7)/7 + 2*a^3*b*d*x^2 + (a*b^3*d*x^8)/2 +
(4*a^3*b*e*x^3)/3 + (4*a*b^3*e*x^9)/9
```

sympy [A] time = 0.44, size = 175, normalized size = 1.05

$$a^4e\log(x) + 4a^3bcx + 2a^3bdx^2 + \frac{4a^3bex^3}{3} + \frac{3a^2b^2cx^4}{2} + \frac{6a^2b^2dx^5}{5} + a^2b^2ex^6 + \frac{4ab^3cx^7}{7} + \frac{ab^3dx^8}{2} + \frac{4ab^3ex^9}{9} + \frac{b^4cx^{10}}{10} + \frac{b^4dx^{11}}{11} + \frac{b^4ex^{12}}{12} + \frac{-a^4c - 2a^4dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**3,x)
```

```
[Out] a**4*e*log(x) + 4*a**3*b*c*x + 2*a**3*b*d*x**2 + 4*a**3*b*e*x**3/3 + 3*a**2
*b**2*c*x**4/2 + 6*a**2*b**2*d*x**5/5 + a**2*b**2*e*x**6 + 4*a*b**3*c*x**7/
7 + a*b**3*d*x**8/2 + 4*a*b**3*e*x**9/9 + b**4*c*x**10/10 + b**4*d*x**11/11
 + b**4*e*x**12/12 + (-a**4*c - 2*a**4*d*x)/(2*x**2)
```

3.284 $\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\cdots\right.}{\sqrt{3}b^{5/3}}$$

Rubi [A] time = 0.26, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}+\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (c*x)/b + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (a^(1/3)*(b^(1/3)*c + a^(1/3)*d)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(
1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/
3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(4/3)) - (a*e*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(c+dx+ex^2\right)}{a+bx^3}\,dx &= \int\left(\frac{c}{b}+\frac{dx}{b}+\frac{ex^2}{b}-\frac{ac+adx+aex^2}{b\left(a+bx^3\right)}\right)dx\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\int\frac{ac+adx+aex^2}{a+bx^3}\,dx}{b}\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\int\frac{ac+adx}{a+bx^3}\,dx}{b}-\frac{(ae)\int\frac{x^2}{a+bx^3}\,dx}{b}\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{ae\log\left(a+bx^3\right)}{3b^2}-\frac{\int\frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}c+a^{4/3}d\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}c+a^{4/3}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}-\frac{\left(\sqrt[3]{a}\left(c-\right.\right.}{}\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}-\frac{\left(a^{2/3}\left(\sqrt[3]{b}c+\right.\right.}{}\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{6b^{4/3}}\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3b^{4/3}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 191, normalized size = 0.93

$$
\frac{\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(a^{2/3}d-\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2ae\log\left(a+bx^3\right)+6bcx+3bdx^2+2bex^3}{6b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (6*b*c*x + 3*b*d*x^2 + 2*b*e*x^3 + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a
^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(-(a^(1/3
```

```
)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(1/3)*b^(1/
3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*e*Lo
g[a + b*x^3]])/(6*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(c + dx + ex^2\right)}{a + bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2))/(a + b*x^3), x]
```

fricas [C] time = 1.27, size = 4798, normalized size = 23.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*e*x^3 + 18*b*d*x^2 - 2*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d
 + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*
b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*
b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)
*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^
3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2*log(1/36*((-I*sqrt(3) + 1)*
(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 +
a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3
 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6
 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*
b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d
+ 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 + 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sq
rt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/5
4*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^
3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*
a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^
2) + (b^2*c^3 + a*b*d^3)*x) + 36*b*c*x + (((-I*sqrt(3) + 1)*(a^2*e^2/b^4 -
(a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 +
1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d
*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3
+ a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e
^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2 + 3*sqrt(1/3)*b^2*s
qrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*
e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1
/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3)
 + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2
*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/
3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*
e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
 a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)
^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5
+ 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c
*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^
4) - 18*a*e)*log(-1/36*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)
/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2
*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/
3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/
18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e
```

)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*
e^2 - 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d
 + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*
b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*
b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)
a/b^5 + 1/18(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^
3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2) + 2*(b^2*c^3 + a*b*d^3)*x + 1/1
2*sqrt(1/3)*(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/
27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/
b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*
sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*
d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b
^6)^(1/3) + 6*a*e/b^2)*b^4*d - 6*b^3*c^2 - 6*a*b^2*d*e)*sqrt(-(((-I*sqrt(3)
 + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*
c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a
^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e
^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/
54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*
b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a
^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^
(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^4)) + (((-I*sqrt(3
) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b
*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 +
a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*
e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1
/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b
^2 - 3*sqrt(1/3)*b^2*sqrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2
*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d
 + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6
)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5
 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*
c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2
/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a
/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3
 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*
(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3
 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*
b*c*d + 36*a^2*e^2)/b^4) - 18*a*e)*log(-1/36*((-I*sqrt(3) + 1)*(a^2*e^2/b^4
 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5
 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*
c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c
^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^
3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d - 2*a*b*c*d^
2 + a*b*c^2*e - a^2*d*e^2 - 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sqrt(3) + 1)*(
a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a
*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3
 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6
 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b
^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2) + 2*(b^2*
c^3 + a*b*d^3)*x - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*
d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a
*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2
b)/b^6)^(1/3) + 9(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3
)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d
^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^4*d - 6*b^3*c^2 - 6*a*b^2*d*
e)*sqrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*

```
a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqr
t(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
 a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)
^(1/3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d +
a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c
*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/
b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/
b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 -
 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2
)/b^4)))/b^2
```

giac [A] time = 0.18, size = 208, normalized size = 1.01

$$-\frac{ae\log\left(|bx^3+a|\right)}{3\,b^2}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3}+\frac{2\,b^2x^3e+3\,b^2dx^2+6\,b^2cx}{6\,b^3}+\frac{\left(ab^6d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^6c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*a*e*log(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^
2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 - 1/6
*((-a*b^2)^(1/3)*b*c + (-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^
(2/3))/b^3 + 1/6*(2*b^2*x^3*e + 3*b^2*d*x^2 + 6*b^2*c*x)/b^3 + 1/3*(a*b^6*d
*(-a/b)^(1/3) + a*b^6*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7)
```

maple [A] time = 0.04, size = 231, normalized size = 1.13

$$\frac{e\,x^3}{3b}+\frac{d\,x^2}{2b}-\frac{\sqrt{3}\,ac\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{ac\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{ac\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{ad\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{ad\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{ae\ln\left(b\,x^3+a\right)}{3b^2}+\frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3/b*e*x^3+1/2/b*d*x^2+1/b*c*x-1/3/(a/b)^(2/3)*a/b^2*c*ln(x+(a/b)^(1/3))+1
/6*a/b^2*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*a/b^2*c/(a/b)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*a/b^2*d/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))-1/6*a/b^2*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))-1/3*a/b^2*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-1/3*a/b^2*e*ln(b*x^3+a)
```

maxima [A] time = 2.94, size = 190, normalized size = 0.93

$$\frac{2\,ex^3+3\,dx^2+6\,cx}{6\,b}-\frac{\sqrt{3}\left(abd\left(\frac{a}{b}\right)^{\frac{2}{3}}+abc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(2\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad\left(\frac{a}{b}\right)^{\frac{1}{3}}-ac\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}+ac\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*e*x^3 + 3*d*x^2 + 6*c*x)/b - 1/3*sqrt(3)*(a*b*d*(a/b)^(2/3) + a*b*c*
(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) -
1/6*(2*a*e*(a/b)^(2/3) + a*d*(a/b)^(1/3) - a*c)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(a*e*(a/b)^(2/3) - a*d*(a/b)^(1/3) + a*
c)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 5.07, size = 319, normalized size = 1.56

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,b^6\,z^3+27\,a\,b^4\,e\,z^2+9\,a\,b^3\,c\,d\,z+9\,a^2\,b^2\,e^2\,z+3\,a^2\,b\,c\,d\,e+a\,b^2\,c^3+a^3\,e^3-a^2\,b\,d^3,z,k\right)\left(6\,a^2\,e+\mathrm{root}\left(27\,b^6\,z^3+27\,a\,b^4\,e\,z^2+9\,a\,b^3\,c\,d\,z+9\,a^2\,b^2\,e^2\,z+3\,a^2\,b\,c\,d\,e+a\,b^2\,c^3+a^3\,e^3-a^2\,b\,d^3,z,k\right)\,a\,b^2\,9-3\,a\,b\,c\,x\right)+\frac{a^3\,e^2+b\,c\,d\,a^2}{b^2}+\frac{x\,\left(a^2\,d^2-a^2\,c\,e\right)}{b}\right)\,\mathrm{root}\left(27\,b^6\,z^3+27\,a\,b^4\,e\,z^2+9\,a\,b^3\,c\,d\,z+9\,a^2\,b^2\,e^2\,z+3\,a^2\,b\,c\,d\,e+a\,b^2\,c^3+a^3\,e^3-a^2\,b\,d^3,z,k\right)\right)+\frac{d\,x^2}{2\,b}+\frac{e\,x^3}{3\,b}+\frac{c\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2))/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*b^6*z^3 + 27*a*b^4*e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2
*z + 3*a^2*b*c*d*e + a*b^2*c^3 + a^3*e^3 - a^2*b*d^3, z, k)*(6*a^2*e + 9*ro
ot(27*b^6*z^3 + 27*a*b^4*e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2*z + 3*a^2*b*
c*d*e + a*b^2*c^3 + a^3*e^3 - a^2*b*d^3, z, k)*a*b^2 - 3*a*b*c*x) + (a^3*e^
2 + a^2*b*c*d)/b^2 + (x*(a^2*d^2 - a^2*c*e))/b)*root(27*b^6*z^3 + 27*a*b^4*
e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2*z + 3*a^2*b*c*d*e + a*b^2*c^3 + a^3*e
^3 - a^2*b*d^3, z, k), k, 1, 3) + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (c*x)/b
```

sympy [A] time = 1.64, size = 178, normalized size = 0.87

$$\operatorname{RootSum}{\left(27 t^{3} b^{6} + 27 t^{2} a b^{4} e + t \left(9 a^{2} b^{2} e^{2} + 9 a b^{3} c d\right) + a^{3} e^{3} + 3 a^{2} b c d e - a^{2} b d^{3} + a b^{2} c^{3}, \left(t \mapsto t \log{\left(x + \frac{9 t^{2} b^{4} d + 6 t a b^{2} d e - 3 t b^{3} c^{2} + a^{2} d e^{2} - a b c^{2} e + 2 a b c d^{2}}{a b d^{3} + b^{2} c^{3}} \right)} \right)\right)} + \frac{c x}{b} + \frac{d x^{2}}{2 b} + \frac{e x^{3}}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**6 + 27*_t**2*a*b**4*e + _t*(9*a**2*b**2*e**2 + 9*a*b**3
*c*d) + a**3*e**3 + 3*a**2*b*c*d*e - a**2*b*d**3 + a*b**2*c**3, Lambda(_t,
_t*log(x + (9*_t**2*b**4*d + 6*_t*a*b**2*d*e - 3*_t*b**3*c**2 + a**2*d*e**2
 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3)))) + c*x/b + d*x**2/(2
*b) + e*x**3/(3*b)
```

3.285 $\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\right.}{\sqrt{3}\,b^{5/3}}$$

Rubi [A] time = 0.25, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}+\frac{c\log\left(a+bx^3\right)}{3b}+\frac{dx}{b}+\frac{ex^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*d
- a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)*(d - (a^(1/3)
*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)) +
(c*Log[a + b*x^3])/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx+ex^2\right)}{a+bx^3}\,dx &= \int \left(\frac{d}{b}+\frac{ex}{b}-\frac{ad+aex-bcx^2}{b\left(a+bx^3\right)}\right)dx \\
&= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\int \frac{ad+aex-bcx^2}{a+bx^3}\,dx}{b} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\int \frac{ad+aex}{a+bx^3}\,dx}{b}+c\int \frac{x^2}{a+bx^3}\,dx \\
&= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{c\log\left(a+bx^3\right)}{3b}-\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}\,d+a^{4/3}e\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}\,d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}-\frac{\left(\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\right)}{3b} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\frac{c\log\left(a+bx^3\right)}{3b}-\frac{\left(a^{2/3}\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\right)\int}{2b^{4}} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\right.}{6b^{4/3}} \\
&= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\cdots
\end{aligned}$$

Mathematica [A] time = 0.10, size = 184, normalized size = 0.95

$$\frac{-\left(a^{2/3}e-\sqrt[3]{a}\sqrt[3]{b}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2\left(a^{2/3}e-\sqrt[3]{a}\sqrt[3]{b}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2b^{2/3}c\log\left(a+bx^3\right)+2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6b^{2/3}dx+3b^{2/3}ex^2}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*d*x + 3*b^(2/3)*e*x^2 + 2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-(a^(1/3)*b^(1/3)*d) + a
```

```
^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c*Log[a + b*x^3])/(
6*b^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(c + dx + ex^2\right)}{a + bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2))/(a + b*x^3), x]
```

fricas [C] time = 1.24, size = 4261, normalized size = 22.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*e*x^2 - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e
)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*
c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3
+ a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b*log(1/4*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 +
 a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d
*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/b^5)^(1/3) - 2*c/b)^2*b^3*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 + 1/2*(b^2*d
^2 + 2*b^2*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/
b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^
3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + (b*d^3 + a*e^3)*x) + 1
2*d*x + ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2
*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3
/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3
 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b
*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2
 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d
*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2
/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 +
 a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e
^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b
^2*c + 4*b*c^2 + 16*a*d*e)/b^3) + 6*c)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 +
 (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*
d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) -
 2*c/b)^2*b^3*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 - 1/2*(b^2*d^2 + 2*b^2*c*e)
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3
 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (
d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3
*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3
```

```
- 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + 2*(b*d^3 + a*e^3)*x + 3/4*sqrt(1/3)*(
(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3
- 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*
(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^3*e - 2*b^2*d^2 + 2*b^2*c*e)*sqrt(-((2
*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 -
3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3
 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b
*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d
^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 +
 a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/
b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3)) + ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^
2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3
+ a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*
e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*
b - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2
+ a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5
+ (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqr
t(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b
^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(
b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^
2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*
d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3) + 6*c)*log(-
1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/
b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3
- 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3*e - b*c*d^2 - b*c^2*e - 2*a*d*
e^2 - 1/2*(b^2*d^2 + 2*b^2*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 -
(b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)
*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)
*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b
^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + 2*(b*d
^3 + a*e^3)*x - 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (
b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*
a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^
5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^3*e - 2
*b^2*d^2 + 2*b^2*c*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*
c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/
b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5
+ (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(
2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 -
 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(
b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3)))/b
```

giac [A] time = 0.21, size = 195, normalized size = 1.01

$$\frac{c\log\left(\left|bx^{3}+a\right|\right)}{3\,b}-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{1}{3}}bd-\left(-ab^{2}\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{3}}+\frac{bx^{2}e+2\,bdx}{2\,b^{2}}-\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}bd+\left(-ab^{2}\right)^{\frac{2}{3}}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{3}}+\frac{\left(ab^{4}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+ab^{4}d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*c*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*d - (-a*b^2)^(2
/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/2*(b*x
^2*e + 2*b*d*x)/b^2 - 1/6*((-a*b^2)^(1/3)*b*d + (-a*b^2)^(2/3)*e)*log(x^2 +
 x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 1/3*(a*b^4*(-a/b)^(1/3)*e + a*b^4*d)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

maple [A] time = 0.05, size = 221, normalized size = 1.15

$$\frac{e x^{2}}{2 b}-\frac{\sqrt{3}\, a d \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}-\frac{a d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}+\frac{a d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}-\frac{\sqrt{3}\, a e \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}+\frac{a e \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}-\frac{a e \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}+\frac{c \ln\left(b x^{3}+a\right)}{3 b}+\frac{d x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/2/b*e*x^2+1/b*d*x-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*d+1/6/(a/b)^(2/
3)*a/b^2*d*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/b^2/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*d+1/3/b^2*a*e/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))-1/6/b^2*a*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/b^2*
a*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b*c*ln(
b*x^3+a)
```

maxima [A] time = 2.94, size = 181, normalized size = 0.94

$$-\frac{\sqrt{3}\left(a e\left(\frac{a}{b}\right)^{\frac{2}{3}}+a d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) \arctan\left(\frac{\sqrt{3}\left(2 x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 a b}+\frac{e x^{2}+2 d x}{2 b}+\frac{\left(2 b c\left(\frac{a}{b}\right)^{\frac{2}{3}}-a e\left(\frac{a}{b}\right)^{\frac{1}{3}}+a d\right) \log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b c\left(\frac{a}{b}\right)^{\frac{2}{3}}+a e\left(\frac{a}{b}\right)^{\frac{1}{3}}-a d\right) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(a*e*(a/b)^(2/3) + a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/2*(e*x^2 + 2*d*x)/b + 1/6*(2*b*c*(a/b)^
(2/3) - a*e*(a/b)^(1/3) + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(2/3)) + 1/3*(b*c*(a/b)^(2/3) + a*e*(a/b)^(1/3) - a*d)*log(x + (a/b)^
(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 5.13, size = 340, normalized size = 1.76

$$\left(\sum_{k=1}^{3} \ln\left(\frac{a\left(b c^{2}+\operatorname{root}\left(27 b^{5} z^{3}-27 b^{4} c z^{2}+9 a b^{2} d e z+9 b^{3} c^{2} z-3 a b c d e+a b d^{3}-a^{2} e^{3}-b^{2} c^{3}, z, k\right)^{2} b^{3} 9+a d e-\operatorname{root}\left(27 b^{5} z^{3}-27 b^{4} c z^{2}+9 a b^{2} d e z+9 b^{3} c^{2} z-3 a b c d e+a b d^{3}-a^{2} e^{3}-b^{2} c^{3}, z, k\right) b^{2} c 6+a e^{2} x+b c d x-\operatorname{root}\left(27 b^{5} z^{3}-27 b^{4} c z^{2}+9 a b^{2} d e z+9 b^{3} c^{2} z-3 a b c d e+a b d^{3}-a^{2} e^{3}-b^{2} c^{3}, z, k\right) b^{2} d x 3\right)}{b}\right) \operatorname{root}\left(27 b^{5} z^{3}-27 b^{4} c z^{2}+9 a b^{2} d e z+9 b^{3} c^{2} z-3 a b c d e+a b d^{3}-a^{2} e^{3}-b^{2} c^{3}, z, k\right)\right)+\frac{e x^{2}}{2 b}+\frac{d x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3),x)
```

```
[Out] symsum(log((a*(b*c^2 + 9*root(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9
*b^3*c^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)^2*b^3 + a*d*e
 - 6*root(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c
*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*b^2*c + a*e^2*x + b*c*d*x - 3*roo
t(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c*d*e + a
*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*b^2*d*x))/b)*root(27*b^5*z^3 - 27*b^4*c*z
^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^
3, z, k), k, 1, 3) + (e*x^2)/(2*b) + (d*x)/b
```

sympy [A] time = 1.49, size = 150, normalized size = 0.78

$$\operatorname{RootSum}\left(27t^3b^5 - 27t^2b^4c + t\left(9ab^2de + 9b^3c^2\right) - a^2e^3 - 3abcde + abd^3 - b^2c^3, \left(t \mapsto t\log\left(x + \frac{9t^2b^3e - 6tb^2ce - 3tb^2d^2 + 2ade^2 + bc^2e + bcd^2}{ae^3 + bd^3}\right)\right)\right) + \frac{dx}{b} + \frac{ex^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c + _t*(9*a*b**2*d*e + 9*b**3*c**2) -
 a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _t*log(x + (9*_
t**2*b**3*e - 6*_t*b**2*c*e - 3*_t*b**2*d**2 + 2*a*d*e**2 + b*c**2*e + b*c*
d**2)/(a*e**3 + b*d**3)))) + d*x/b + e*x**2/(2*b)
```

3.286 $\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=183

$$\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}}$$

Rubi [A] time = 0.23, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}}+\frac{d\log\left(a+bx^3\right)}{3b}+\frac{ex}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (e*x)/b - ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)) - ((b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(1/3)*b^(4/3)) + ((b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(4/3)) + (d*Log[a + b*x^3])
/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2\right)}{a+bx^3}\,dx = \int \left(\frac{e}{b}-\frac{ae-bcx-bdx^2}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{ex}{b}-\frac{\int \frac{ae-bcx-bdx^2}{a+bx^3}\,dx}{b}$$

$$= \frac{ex}{b}-\frac{\int \frac{ae-bcx}{a+bx^3}\,dx}{b}+d\int \frac{x^2}{a+bx^3}\,dx$$

$$= \frac{ex}{b}+\frac{d\log\left(a+bx^3\right)}{3b}-\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}\,bc+2a\sqrt[3]{b}\,e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}\,bc-a\sqrt[3]{b}\,e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\int \frac{1}{\sqrt[3]{a}+\cdots}}{3\sqrt[3]{a}\,b}$$

$$= \frac{ex}{b}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{4/3}}+\frac{d\log\left(a+bx^3\right)}{3b}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b\cdots}}{2b}$$

$$= \frac{ex}{b}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{4/3}}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{4/3}}+\cdots$$

$$= \frac{ex}{b}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{4/3}}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{4/3}}+\frac{\left(b^{2/3}c+a^{2/3}e\cdots\right.}{}$$

Mathematica [A] time = 0.06, size = 200, normalized size = 1.09

$$-\frac{\left(a^{4/3}\left(-\sqrt[3]{b}\right)e-a^{2/3}bc\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6ab^{5/3}}+\frac{\left(a^{4/3}\left(-\sqrt[3]{b}\right)e-a^{2/3}bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3ab^{5/3}}+\frac{\left(a^{2/3}bc-a^{4/3}\sqrt[3]{b}\,e\right)\tan^{-1}\left(\frac{2\sqrt[3]{b}\,x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,ab^{5/3}}+\frac{d\log\left(a+bx^3\right)}{3b}+\frac{ex}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (e*x)/b + ((a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a*b^(5/3)) + ((-(a^(2/3)*b*c) - a^(4/3)*b^(1
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a*b^(5/3)) - ((-(a^(2/3)*b*c) - a^(4/3)
*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(5/3)) +
 (d*Log[a + b*x^3])/(3*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c + dx + ex^2)}{a + bx^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x + e*x^2))/(a + b*x^3), x]
```

fricas [C] time = 1.23, size = 4628, normalized size = 25.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3
/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b
^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d
^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*log(-1/4*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*
d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*
e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e
)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^
2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c - a*b*c*d^2 + 2*a*b*c^2*e + a^2*d*
e^2 - 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^
2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^
3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d
/b) - (b^2*c^3 - a^2*e^3)*x) - 12*e*x - ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d
^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3)
- 2*d/b)*b - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2
- (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(
1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b
)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*
d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(
a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4*d^2 - 16*c*e
)/b^2) + 6*d)*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e
)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d
*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c
+ a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 + 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2
```

- c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b) - 2*(b^2*c^3 - a^2*e^3)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*a*b^3*c + 2*a*b^2*c*d + 2*a^2*b*e^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4*d^2 - 16*c*e)/b^2)) - ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4*d^2 - 16*c*e)/b^2) + 6*d)*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c + a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 + 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b) - 2*(b^2*c^3 - a^2*e^3)*x - 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*a*b^3*c + 2*a*b^2*c*d + 2*a^2*b*e^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4

```
*d^2 - 16*c*e)/b^2)))/b
```

giac [A]   time = 0.21, size = 178, normalized size = 0.97

$$\frac{\sqrt{3}\left(ae+\left(-ab^{2}\right)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}}+\frac{\left(ae-\left(-ab^{2}\right)^{\frac{1}{3}}c\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}}+\frac{xe}{b}+\frac{d\log\left(\left|bx^{3}+a\right|\right)}{3b}-\frac{\left(b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{2}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(a*e + (-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(-a*b^2)^(2/3) + 1/6*(a*e - (-a*b^2)^(1/3)*c)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) + x*e/b + 1/3*d*log(abs(b*x^3 +
a))/b - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b^3)
```

maple [A]   time = 0.05, size = 209, normalized size = 1.14

$$-\frac{\sqrt{3}\,ae\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{ae\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{ae\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{d\ln\left(bx^{3}+a\right)}{3b}+\frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/b*e*x-1/3/(a/b)^(2/3)*a/b^2*e*ln(x+(a/b)^(1/3))+1/6/(a/b)^(2/3)*a/b^2*e*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/(a/b)^(2/3)*3^(1/2)*a/b^2*e*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/b*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b*c
/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b*c*3^(1/2)/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b*d*ln(b*x^3+a)
```

maxima [A]   time = 2.94, size = 173, normalized size = 0.95

$$\frac{ex}{b}+\frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab}+\frac{\left(2bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] e*x/b + 1/3*sqrt(3)*(b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3))*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*b*d*(a/b)^(2/3) + b*c*(a/b)
^(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/
3*(b*d*(a/b)^(2/3) - b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(b^2*(a/b)
^(2/3))
```

mupad [B]   time = 5.16, size = 266, normalized size = 1.45

$$\left(\sum_{k=1}^{3}\ln\left(x\left(bc^{2}+ade\right)-\mathrm{root}\left(27ab^{4}z^{3}-27ab^{3}dz^{2}-9ab^{2}cez+9ab^{2}d^{2}z+3abcde-abd^{3}+a^{2}e^{3}+b^{2}c^{3},z,k\right)\left(6abd-\mathrm{root}\left(27ab^{4}z^{3}-27ab^{3}dz^{2}-9ab^{2}cez+9ab^{2}d^{2}z+3abcde-abd^{3}+a^{2}e^{3}+b^{2}c^{3},z,k\right)ab^{2}9+3abex\right)+ad^{2}-ace\right)\mathrm{root}\left(27ab^{4}z^{3}-27ab^{3}dz^{2}-9ab^{2}cez+9ab^{2}d^{2}z+3abcde-abd^{3}+a^{2}e^{3}+b^{2}c^{3},z,k\right)\right)+\frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3),x)
```

```
[Out] symsum(log(x*(b*c^2 + a*d*e) - root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2
*c*e*z + 9*a*b^2*d^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*(
```

```
6*a*b*d - 9*root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2*c*e*z + 9*a*b^2*d^
2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2 + 3*a*b*e*x) +
 a*d^2 - a*c*e)*root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2*c*e*z + 9*a*b^
2*d^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3) + (e*x
)/b
```

sympy [A] time = 1.43, size = 160, normalized size = 0.87

$$\mathrm{RootSum}\left(27t^3ab^4 - 27t^2ab^3d + t\left(-9ab^2ce + 9ab^2d^2\right) + a^2e^3 + 3abcde - abd^3 + b^2c^3, \left(t \mapsto t\log\left(x + \frac{-9t^2ab^3c - 3ta^2be^2 + 6tab^2cd + a^2de^2 + 2abc^2e - abcd^2}{a^2e^3 - b^2c^3}\right)\right)\right) + \frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**4 - 27*_t**2*a*b**3*d + _t*(-9*a*b**2*c*e + 9*a*b**2*
d**2) + a**2*e**3 + 3*a*b*c*d*e - a*b*d**3 + b**2*c**3, Lambda(_t, _t*log(x
 + (-9*_t**2*a*b**3*c - 3*_t*a**2*b*e**2 + 6*_t*a*b**2*c*d + a**2*d*e**2 +
2*a*b*c**2*e - a*b*c*d**2)/(a**2*e**3 - b**2*c**3)))) + e*x/b
```

3.287 $\int \frac{c+dx+ex^2}{a+bx^3} dx$

Optimal. Leaf size=177

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+$$

Rubi [A] time = 0.13, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) + (e*Log[a + b*x^3])/(3*b
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx+ex^2}{a+bx^3}\,dx &= e\int \frac{x^2}{a+bx^3}\,dx+\int \frac{c+dx}{a+bx^3}\,dx\\
&=\frac{e\log\left(a+bx^3\right)}{3b}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}\\
&=\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{e\log\left(a+bx^3\right)}{3b}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}}+\frac{1}{2}\Big(\\
&=\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}}+\frac{e\log\left(a+bx^3\right.}{3b}\\
&=-\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}\right.}{6a^{2/3}b^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 176, normalized size = 0.99

$$\frac{-\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2ae\log\left(a+bx^3\right)}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(1/3
) + b^(1/3)*x] - b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*e*Log[a + b*x^3])/(6*a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^3), x]
```

fricas [C] time = 1.20, size = 4671, normalized size = 26.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)
)/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b
^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2
*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)
*b*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^
2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) +
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a
^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/
b)^2*a^2*b^2*d + 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 - 1/2*(a*b^2*c^2 - 2*a
^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2
))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (
b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*
sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^
2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b
) + (b^2*c^3 + a*b*d^3)*x) - ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b
*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) +
 (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2
*b^3))^(1/3) - 2*e/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a
*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a
*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*
e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d
 + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3
 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b
^2)) + 6*e)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*
e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a
^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
 a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1
/3) - 2*e/b)^2*a^2*b^2*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 + 1/2*(a*b^2
*c^2 - 2*a^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e
^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^
2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/
3) - 2*e/b) + 2*(b^2*c^3 + a*b*d^3)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b
*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a^2*b^2*d + 2*a*b^2*c^2 + 2*
a^2*b*d*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2
```

```
)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*
b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(
1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*
d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3)
 - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a
*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(
a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3
+ a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(
1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b^2))) - ((2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b
*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*b - 3*sqrt(1/3)*b*sqrt(-((2*
(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3
 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*
(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2
*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a*b^2 + 4
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3
/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 +
 a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) +
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a*b*e +
 16*b*c*d + 4*a*e^2)/(a*b^2)) + 6*e)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*
b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b
)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*
e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*
c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a^2*b^2*d - 2*a*b*c*d^2 + a*b*c^2*e
 - a^2*d*e^2 + 1/2*(a*b^2*c^2 - 2*a^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1
)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b
^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b) + 2*(b^2*c^3 + a*b*d^3)*x - 3/4*sqrt(1
/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2
*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c
^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(
3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2
) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a^2
*b^2*d + 2*a*b^2*c^2 + 2*a^2*b*d*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3
) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(
a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2
)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d
*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a
*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a
*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b^2))
))/b
```

giac [A] time = 0.18, size = 163, normalized size = 0.92

$$-\frac{\sqrt{3}\left(bc-\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}}-\frac{\left(bc+\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}}+\frac{e\log\left(\left|bx^{3}+a\right|\right)}{3\,b}-\frac{\left(bd\left(-\frac{a}{b}\right)^{\frac{1}{3}}+bc\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) + 1/3*e*log(abs(b*x^3 + a))/b -
 1/3*(b*d*(-a/b)^(1/3) + b*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)
```

maple [A] time = 0.05, size = 200, normalized size = 1.13

$$\frac{\sqrt{3}\, c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b}-\frac{d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b}+\frac{d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b}+\frac{e \ln\left(b x^{3}+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))*c+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x-1))*c-1/3*d/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*d/b/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*d*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))+1/3/b*e*ln(b*x^3+a)
```

maxima [A] time = 3.01, size = 159, normalized size = 0.90

$$\frac{\sqrt{3}\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(2e\left(\frac{a}{b}\right)^{\frac{2}{3}}+d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(e\left(\frac{a}{b}\right)^{\frac{2}{3}}-d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(b*d*(a/b)^(2/3) + b*c*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*e*(a/b)^(2/3) + d*(a/b)^(1/3) - c)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(e*(a/b)^(2/3)
 - d*(a/b)^(1/3) + c)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 0.26, size = 274, normalized size = 1.55

$$\sum_{k=1}^{3}\ln\left(x\left(b\,d^{2}-b\,c\,e\right)+\operatorname{root}\left(27\,a^{2}\,b^{3}\,z^{3}-27\,a^{2}\,b^{2}\,e\,z^{2}+9\,a\,b^{2}\,c\,d\,z+9\,a^{2}\,b\,e^{2}\,z-3\,a\,b\,c\,d\,e+a\,b\,d^{3}-a^{2}\,e^{3}-b^{2}\,c^{3},z,k\right)\left(-6\,a\,b\,e+\operatorname{root}\left(27\,a^{2}\,b^{3}\,z^{3}-27\,a^{2}\,b^{2}\,e\,z^{2}+9\,a\,b^{2}\,c\,d\,z+9\,a^{2}\,b\,e^{2}\,z-3\,a\,b\,c\,d\,e+a\,b\,d^{3}-a^{2}\,e^{3}-b^{2}\,c^{3},z,k\right)\,a\,b^{2}\,9+3\,b^{2}\,c\,x\right)+a\,e^{2}+b\,c\,d\right)\operatorname{root}\left(27\,a^{2}\,b^{3}\,z^{3}-27\,a^{2}\,b^{2}\,e\,z^{2}+9\,a\,b^{2}\,c\,d\,z+9\,a^{2}\,b\,e^{2}\,z-3\,a\,b\,c\,d\,e+a\,b\,d^{3}-a^{2}\,e^{3}-b^{2}\,c^{3},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^3),x)
```

```
[Out] symsum(log(x*(b*d^2 - b*c*e) + root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a
*b^2*c*d*z + 9*a^2*b*e^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z,
k)*(9*root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a*b^2*c*d*z + 9*a^2*b*e^2*
z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a*b^2 - 6*a*b*e + 3*b^
2*c*x) + a*e^2 + b*c*d)*root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a*b^2*c*
d*z + 9*a^2*b*e^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k), k,
1, 3)
```

sympy [A] time = 1.42, size = 160, normalized size = 0.90

$$\operatorname{RootSum}\left(27t^{3}a^{2}b^{3}-27t^{2}a^{2}b^{2}e+t\left(9a^{2}be^{2}+9ab^{2}cd\right)-a^{2}e^{3}-3abcde+abd^{3}-b^{2}c^{3},\left(t\mapsto t\log\left(x+\frac{9t^{2}a^{2}b^{2}d-6ta^{2}bde+3tab^{2}c^{2}+a^{2}de^{2}-abc^{2}e+2abcd^{2}}{abd^{3}+b^{2}c^{3}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3 - 27*_t**2*a**2*b**2*e + _t*(9*a**2*b*e**2 + 9*a
*b**2*c*d) - a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b**2*d - 6*_t*a**2*b*d*e + 3*_t*a*b**2*c**2 + a**2*d*
e**2 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3))))
```

3.288 $\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=184

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}-$$

Rubi [A] time = 0.21, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{c\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] -(((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + (c*Log[x])/a + ((b^(1/3)*d - a^(1/3)*e)*Log
[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) - (c*Log[a
 + b*x^3])/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx &= \int \left(\frac{c}{ax}+\frac{ad+aex-bcx^2}{a\left(a+bx^3\right)}\right)dx \\
&= \frac{c\log(x)}{a}+\frac{\int \frac{ad+aex-bcx^2}{a+bx^3}\,dx}{a} \\
&= \frac{c\log(x)}{a}+\frac{\int \frac{ad+aex}{a+bx^3}\,dx}{a}-\frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a} \\
&= \frac{c\log(x)}{a}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}d+a^{4/3}e\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}
\end{aligned}$$

$$\begin{aligned}
&= \frac{c\log(x)}{a}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{1}{2}\left(\frac{d}{\sqrt[3]{a}}+\frac{e}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b} \\
&= \frac{c\log(x)}{a}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}-\frac{c\log(
\end{aligned}$$

$$= -\frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{c\log(x)}{a}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log$$

Mathematica [A] time = 0.10, size = 176, normalized size = 0.96

$$\frac{\left(a^{2/3}e-\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\left(\sqrt[3]{a}\sqrt[3]{b}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2b^{2/3}c\log\left(a+bx^3\right)-2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6b^{2/3}c\log(x)}{6ab^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 6*b^(2/3)*c*Log[x] + 2*(a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a
^(1/3) + b^(1/3)*x] + (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^(2/3)*c*Log[a + b*x^3])/(6*a*b^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2}{x\left(a + bx^3\right)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x*(a + b*x^3)), x]
```

fricas [C] time = 1.38, size = 4588, normalized size = 24.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a
^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3)
+ 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)
/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3
) + 6*c/a)*a*log(1/36*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))
/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^
2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b
*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
^3*b^2))^(1/3) + 6*c/a)^2*a^2*b*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 - 1/6*(a*
b*d^2 + 2*a*b*c*e)*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b
^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(
I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^
3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*
b^2))^(1/3) + 6*c/a) + (b*d^3 + a*e^3)*x) - (((-I*sqrt(3) + 1)*(c^2/a^2 - (
b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1
/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a
*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d
*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a + 3*sqrt(1/3)*a*sqrt(-(((-
I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b
*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*
c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)^2
*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^
3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1
/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(
3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e
^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(
1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)) - 18*c)*log(-1/36*((-I
*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*
c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c
^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)^2*
```

a^2*b*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 + 1/6*(a*b*d^2 + 2*a*b*c*e)*((-I*sq
rt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2
 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*
e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/
a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/5
4*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a) + 2*(
b*d^3 + a*e^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*
d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b
^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3
b) + 1/54(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a^2*b*e + 6*a*b*d^2 - 6*a*b*c*e)*sqrt(
-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/
18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c
^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b
^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c
/a)^2*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/
27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2
) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*
sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
+ a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^
2))^(1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b))) - (((-I*sqrt(3)
+ 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d
*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 +
1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2
*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a - 3*sqrt(
1/3)*a*sqrt(-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*
c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqr
t(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a
*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))
^(1/3) + 6*c/a)^2*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(
a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e
^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(
1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) +
1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)) - 18*
c)*log(-1/36*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c
^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*
e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^
(1/3) + 6*c/a)^2*a^2*b*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 + 1/6*(a*b*d^2 + 2
*a*b*c*e)*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/
a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/5
4*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3)
 + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3
)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/
3) + 6*c/a) + 2*(b*d^3 + a*e^3)*x - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(c^2/
a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3
b) + 1/54(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^
2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a^2*b*e + 6*a*b*d^2 -
 6*a*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b
^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(

```
I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^
3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*
b^2))^(1/3) + 6*c/a)^2*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d
*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
+ a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^
2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*
b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*
d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)))
 - 36*c*log(x))/a
```

giac [A] time = 0.18, size = 179, normalized size = 0.97

$$-\frac{\sqrt{3}\left(bd-\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}}-\frac{\left(bd+\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}}-\frac{c\log\left(\left|bx^{3}+a\right|\right)}{3a}+\frac{c\log\left(\left|x\right|\right)}{a}-\frac{\left(a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+a^{2}bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*d + (-a*b^2)^(1/3)*e)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*c*log(abs(b*x^3 + a))/a +
 c*log(abs(x))/a - 1/3*(a^2*b*(-a/b)^(1/3)*e + a^2*b*d)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a^3*b)
```

maple [A] time = 0.05, size = 207, normalized size = 1.12

$$\frac{c\ln(x)}{a}-\frac{c\ln\left(bx^{3}+a\right)}{3a}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a),x)
```

```
[Out] 1/3/(a/b)^(2/3)/b*d*ln(x+(a/b)^(1/3))-1/6/(a/b)^(2/3)/b*d*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/3/(a/b)^(2/3)*3^(1/2)/b*d*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-1/3/(a/b)^(1/3)/b*e*ln(x+(a/b)^(1/3))+1/6/(a/b)^(1/3)/b*e*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/(a/b)^(1/3)/b*e*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))-1/3/a*c*ln(b*x^3+a)+1/a*c*ln(x)
```

maxima [A] time = 3.02, size = 176, normalized size = 0.96

$$\frac{c\log(x)}{a}+\frac{\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^{2}}-\frac{\left(2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+ad\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] c*log(x)/a + 1/3*sqrt(3)*(a*e*(a/b)^(2/3) + a*d*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*b*c*(a/b)^(2/3) - a*e*(a
/b)^(1/3) + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) -
 1/3*(b*c*(a/b)^(2/3) + a*e*(a/b)^(1/3) - a*d)*log(x + (a/b)^(1/3))/(a*b*(a
/b)^(2/3))
```

mupad [B] time = 5.25, size = 716, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - 36*root(27*a^3*b^2*z^3 + 27*
a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2
*e^3 + b^2*c^3, z, k)^3*a^2*b^3*x - a*b*e^3*x - root(27*a^3*b^2*z^3 + 27*a^
2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e
^3 + b^2*c^3, z, k)*a*b^2*d^2 - 4*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 +
9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3,
z, k)*b^3*c^2*x + 3*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z
+ 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a^2*b^
2*e - 24*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c
^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a*b^3*c*x - 2*roo
t(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b
*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2*c*e - 2*b^2*c*d*e*x - 10*
root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*
a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2*d*e*x)*root(27*a^3*b^2
*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b
*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3) + (c*log(x))/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.289 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=192

$$-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}}+\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}a^{4/3}\sqrt[3]{b}}$$

Rubi [A] time = 0.21, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}}+\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a}-\frac{c}{ax}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(1/3)) + (d*Log[x])/a + ((b^(2/3)*c + a^(2
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(1/3)) - ((b^(2/3)*c + a^(2/3
)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(1/3)) -
(d*Log[a + b*x^3])/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)}\,dx = \int\left(\frac{c}{ax^2}+\frac{d}{ax}+\frac{ae-bcx-bdx^2}{a\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\int\frac{ae-bcx-bdx^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\int\frac{ae-bcx}{a+bx^3}\,dx}{a}-\frac{(bd)\int\frac{x^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{ax}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3\right)}{3a}+\frac{\int\frac{\sqrt[3]{a}\left(-\sqrt[3]{a}\,bc+2a\sqrt[3]{b}\,e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}\,bc-a\sqrt[3]{b}\,e\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{5/3}\sqrt[3]{b}}+\frac{\left(b^{2/3}c+a^{2/3}\right.}{}$$

$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\int\frac{}{a^{2/3}\ldots}}{2a}$$

$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b\ldots\right.}{6a^{4/3}\sqrt[3]{b}}$$

$$= -\frac{c}{ax}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}\sqrt[3]{b}}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{\left(b^{2\ldots\right.}{}$$

Mathematica [A] time = 0.25, size = 184, normalized size = 0.96

$$-\frac{\frac{\left(a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-\frac{2\left(a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}\,a^{2/3}\left(a^{2/3}e-b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+2ad\log\left(a+bx^3\right)+\frac{6ac}{x}-6ad\log(x)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -1/6*((6*a*c)/x + (2*Sqrt[3]*a^(2/3)*(-(b^(2/3)*c) + a^(2/3)*e)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 6*a*d*Log[x] - (2*(a^(2/3)*b^(2
/3)*c + a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((a^(2/3)*b^(2/3)*c
+ a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 2*a*
d*Log[a + b*x^3])/a^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2}{x^2\left(a + bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^2*(a + b*x^3)), x]
```

fricas [C] time = 1.43, size = 4524, normalized size = 23.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/1
8*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b
) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/
a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*x*log(-1/36
*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 -
 c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54
*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/
18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*
b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^3*b*c - a*b*c*d^2
 + 2*a*b*c^2*e + a^2*d*e^2 + 1/6*(2*a^2*b*c*d - a^3*e^2)*((-I*sqrt(3) + 1)*
(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3
)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^
3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3
- a^2*e^3)/(a^4*b))^(1/3) + 6*d/a) - (b^2*c^3 - a^2*e^3)*x) - 36*d*x*log(x)
 - (((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^
2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1
/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 +
 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*x - 3*sqrt(1/3)*
a*x*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1
/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4
*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^
3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*
((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 -
c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*
(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/1
8*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b
) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e
)/a^2) - 18*d*x)*log(1/36*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1
/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3)
 + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (
d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d
```

/a)^2*a^3*b*c + a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 - 1/6*(2*a^2*b*c*d - a^
3*e^2)*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*
(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
- 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^
3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a) - 2*(b^2*c^3 -
a^2*e^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/
(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) +
6*d/a)*a^3*b*c - 6*a^2*b*c*d - 6*a^3*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2
- (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3
+ a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b
))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^
3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c
*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))
^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e)/a^2)) - (((-I*sqrt(3) + 1)*(d^2/a^2
- (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3
+ a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b
))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^
3)/(a^4*b))^(1/3) + 6*d/a)*a*x + 3*sqrt(1/3)*a*x*sqrt(-(((-I*sqrt(3) + 1)*(
d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(
b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)
/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3
+ 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 -
a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*((-I*sqrt(3) + 1)*(d^2/a^2 - (
d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(
1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^
2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(
a^4*b))^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e)/a^2) - 18*d*x)*log(1/36*((-I
sqrt(3) + 1)(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)
d/a^3 + 1/54(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2
*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d
^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) -
1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^3*b*c + a*b*c*d^2 - 2*
a*b*c^2*e - a^2*d*e^2 - 1/6*(2*a^2*b*c*d - a^3*e^2)*((-I*sqrt(3) + 1)*(d^2/
a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*
c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^
4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1
/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2
*e^3)/(a^4*b))^(1/3) + 6*d/a) - 2*(b^2*c^3 - a^2*e^3)*x - 1/12*sqrt(1/3)*((
(-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c
*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(
b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18
*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
- 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a^3*b*c - 6*a^2*b*c*d -
6*a^3*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3
/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a
*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(
-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a
^2 - 12*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18

```
*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
 - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a
^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b
)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*d + 36*d^2 -
 144*c*e)/a^2)) + 36*c)/(a*x)
```

giac [A] time = 0.23, size = 201, normalized size = 1.05

$$-\frac{d\log\left(|bx^3+a|\right)}{3a}+\frac{d\log\left(|x|\right)}{a}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae+\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b}-\frac{c}{ax}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae-\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b}+\frac{\left(ab^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a + d*log(abs(x))/a + 1/3*sqrt(3)*((-a*b^2)^(1/3
)*a*e + (-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^2*b) - c/(a*x) + 1/6*((-a*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) + 1/3*(a*b^2*c*(-a/b)^(1/3) - a^
2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b)
```

maple [A] time = 0.05, size = 216, normalized size = 1.12

$$-\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{d\ln(x)}{a}-\frac{d\ln\left(bx^3+a\right)}{3a}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a),x)
```

```
[Out] 1/3*e/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*e/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/3*e/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))+1/3/a*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6/a*c/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/a*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))-1/3/a*d*ln(b*x^3+a)-1/a*c/x+1/a*d*ln(x)
```

maxima [A] time = 2.98, size = 186, normalized size = 0.97

$$\frac{d\log(x)}{a}-\frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2}-\frac{\left(2bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] d*log(x)/a - 1/3*sqrt(3)*(b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*b*d*(a/b)^(2/3) + b*c*(a
/b)^(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) -
 1/3*(b*d*(a/b)^(2/3) - b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(a*b*(a
/b)^(2/3)) - c/(a*x)
```

mupad [B] time = 5.06, size = 723, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)),x)
```

```
[Out] symsum(log((b^4*c^3*x + a^2*b^2*d*e^2 - 36*root(27*a^4*b*z^3 + 27*a^3*b*d*z
^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*
c^3, z, k)^3*a^4*b^3*x + a^2*b^2*e^3*x + a*b^3*c*d^2 - 3*root(27*a^4*b*z^3
+ 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 -
a^2*e^3 - b^2*c^3, z, k)^2*a^3*b^3*c - root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 -
 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3,
 z, k)*a^3*b^2*e^2 - 4*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z +
 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*d
^2*x - 24*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*
z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)^2*a^3*b^3*d*x + 2*root
(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*
e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*c*d + 2*a*b^3*c*d*e*x + 10*r
oot(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c
*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*c*e*x)/a^2)*root(27*a^4*b
*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d
^3 - a^2*e^3 - b^2*c^3, z, k), k, 1, 3) - c/(a*x) + (d*log(x))/a
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.290 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=203

$$\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\tan^{-1}\left(\cdots\right.}{\sqrt{3}\,a^{5/3}}$$

Rubi [A] time = 0.19, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{e\log\left(a+bx^3\right)}{3a}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -c/(2*a*x^2) - d/(a*x) + (b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (e*Log[x])/a - (b^(1/
3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^(2/3)
*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(6*a^(5/3)) - (e*Log[a + b*x^3])/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^3}+\frac{d}{ax^2}+\frac{e}{ax}-\frac{b\left(c+dx+ex^2\right)}{a\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b\int \frac{c+dx+ex^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b\int \frac{c+dx}{a+bx^3}\,dx}{a}-\frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{e\log\left(a+bx^3\right)}{3a}-\frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{5/3}}-\frac{\left(b\left(c\right.\right.}{}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}}-\frac{e\log\left(a+bx^3\right)}{3a}+\frac{\left(\sqrt[3]{b}\left(\sqrt[3]{b}\,c-\right.\right.}{}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}}+\frac{\sqrt[3]{b}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\right.}{6a^{5/3}}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\sqrt[3]{b}\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{e\log(x)}{a}-\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\right.}{3a^{5/3}}$$

Mathematica [A] time = 0.23, size = 192, normalized size = 0.95

$$\frac{\sqrt[3]{b}\left(\sqrt[3]{a}\,\sqrt[3]{b}\,c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(a^{2/3}d-\sqrt[3]{a}\,\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2ae\log\left(a+bx^3\right)-\frac{3ac}{x^2}-\frac{6ad}{x}+6ae\log(x)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a*c)/x^2 - (6*a*d)/x + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 6*a*e*Log[x] + 2*b^(1/3)*(
-(a^(1/3)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(1/
3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] -
2*a*e*Log[a + b*x^3])/(6*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^3*(a + b*x^3)), x]
```

fricas [C] time = 1.34, size = 4279, normalized size = 21.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 +
 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^
3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3
 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2*log(1/36*((-I*s
qrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d +
a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3
- 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d
 + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d + 2*a*b*c*d^2 - a*b*c^2*e +
a^2*d*e^2 + 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d
 + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 +
a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3
 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3)
 + 6*e/a) + (b^2*c^3 + a*b*d^3)*x) - 36*e*x^2*log(x) + 36*d*x - (((-I*sqrt(
3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^
2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*
c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a
*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2 + 3*sqrt(1/3)*a*x^2*sqrt(-(((-I*sq
rt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a
*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d
+ a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/a^2
 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*
(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)
^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/
54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a
^5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3) - 18*e*x^2)*log(-1/36
*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b
*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18
*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e
^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d - 2*a*b*c*d^2 + a*b*c
^2*e - a^2*d*e^2 - 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 -
```

```
(b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b
*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(
1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54
*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5
)^(1/3) + 6*e/a) + 2*(b^2*c^3 + a*b*d^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) +
 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e
/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*
e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2
)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a^4*d - 6*a^2*b*c^2 - 6*a^3*d*e)*sqrt(-(((-I
*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d
 + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c
*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/
a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/
54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a
^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 +
 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b
)/a^5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3)) - (((-I*sqrt(3) +
 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e
/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*
e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2
)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2 - 3*sqrt(1/3)*a*x^2*sqrt(-(((-I*sqrt(3
) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2
)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*
e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/a^2 - (
b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c
^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/
3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(
b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^
(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3) - 18*e*x^2)*log(-1/36*((-
I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d
 + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*
c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 -
 (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d - 2*a*b*c*d^2 + a*b*c^2*e
 - a^2*d*e^2 - 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*
c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3
 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3)
 + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*
c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1
/3) + 6*e/a) + 2*(b^2*c^3 + a*b*d^3)*x - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*
(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4
 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a
*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/
a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/a^5)^(1/3) + 6*e/a)*a^4*d - 6*a^2*b*c^2 - 6*a^3*d*e)*sqrt(-(((-I*sqr
t(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*
e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d +
 a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3
 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/a^2
- (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(
b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^
(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/5
```

```
4*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^
5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3)) + 18*c)/(a*x^2)
```

giac [A] time = 0.18, size = 204, normalized size = 1.00

$$-\frac{e\log\left(\left|bx^3+a\right|\right)}{3\,a}+\frac{e\log\left(\left|x\right|\right)}{a}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b}+\frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}-\frac{2\,dx+c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a + e*log(abs(x))/a - 1/3*sqrt(3)*((-a*b^2)^(1/3
)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^2*b) - 1/6*((-a*b^2)^(1/3)*b*c + (-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a^2*b) + 1/3*(a*b^2*d*(-a/b)^(1/3) + a*b^2*c)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) - 1/2*(2*d*x + c)/(a*x^2)
```

maple [A] time = 0.22, size = 225, normalized size = 1.11

$$-\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{e\ln(x)}{a}-\frac{e\ln\left(b\,x^3+a\right)}{3a}-\frac{d}{ax}-\frac{c}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a),x)
```

```
[Out] -1/3/(a/b)^(2/3)/a*c*ln(x+(a/b)^(1/3))+1/6/(a/b)^(2/3)/a*c*ln(x^2-(a/b)^(1/
3)*x+(a/b)^(2/3))-1/3/(a/b)^(2/3)*3^(1/2)/a*c*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))+1/3/(a/b)^(1/3)/a*d*ln(x+(a/b)^(1/3))-1/6/(a/b)^(1/3)/a*d*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/(a/b)^(1/3)/a*d*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))-1/3/a*e*ln(b*x^3+a)+1/a*e*ln(x)-1/2/a*c/x^2-1/a*d/x
```

maxima [A] time = 3.03, size = 177, normalized size = 0.87

$$\frac{e\log(x)}{a}-\frac{\sqrt{3}\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2}-\frac{\left(2\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(e\left(\frac{a}{b}\right)^{\frac{2}{3}}-d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,dx+c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] e*log(x)/a - 1/3*sqrt(3)*(b*d*(a/b)^(2/3) + b*c*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*e*(a/b)^(2/3) + d*(a/b)^
(1/3) - c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*(e*
(a/b)^(2/3) - d*(a/b)^(1/3) + c)*log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2
*(2*d*x + c)/(a*x^2)
```

mupad [B] time = 0.13, size = 701, normalized size = 3.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)),x)
```

```
[Out] symsum(log(-(b^5*c^3*x - a^2*b^3*d*e^2 + 36*root(27*a^5*z^3 + 27*a^4*e*z^2
+ 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3,
z, k)^3*a^5*b^3*x - a*b^4*c^2*e - a*b^4*d^3*x + root(27*a^5*z^3 + 27*a^4*e*
```

```
z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c
^3, z, k)*a^2*b^4*c^2 + 3*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z +
9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a^4*b^3*d
+ 4*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*
d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a^3*b^3*e^2*x + 24*root(27*a^5*z^3
 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2
*e^3 + b^2*c^3, z, k)^2*a^4*b^3*e*x - 2*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*
a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k
)*a^3*b^3*d*e + 2*a*b^4*c*d*e*x + 10*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2
*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a
^2*b^4*c*d*x)/a^3)*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e
^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3) - c/(2*a*
x^2) - d/(a*x) + (e*log(x))/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.291 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=190

$$-\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{5/3}}-\frac{\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}}$$

Rubi [A] time = 0.17, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1823, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{5/3}}-\frac{\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}}-\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] -(c + d*x + e*x^2)/(3*b*(a + b*x^3)) - ((b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(5/3)) + ((b
^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(5/3)) - ((d
 - (2*a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
18*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^2}\,dx &= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\int \frac{d+2ex}{a+bx^3}\,dx}{3b}\\
&= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d+2\sqrt[3]{a}e\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}d+2\sqrt[3]{a}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{2/3}b^{4/3}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{2/3}b}\\
&= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}}+\frac{\left(\frac{\sqrt[3]{b}d}{\sqrt[3]{a}}+2e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{4/3}}-\ldots\\
&= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}}-\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\ldots\right.}{18a^{2/3}b^{5/3}}\\
&= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}-\frac{\left(\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{5/3}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}}-\ldots
\end{aligned}
$$

Mathematica [A] time = 0.20, size = 174, normalized size = 0.92

$$
\frac{\frac{\left(2\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{2\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}-\frac{2\sqrt{3}\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}-\frac{6b^{2/3}(c+x(d+ex))}{a+bx^3}}{18b^{5/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*(b^(1/3)*d + 2*a^(
1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(b^(1/3)*
d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(b^(1/3)*d) + 2*a^(
1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(5/3
))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]

[Out] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2, x]

fricas [C] time = 1.21, size = 2077, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] -1/36*(12*e*x^2 + 2*(b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a^2*b^3*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a*b^2*d^2 + 8*a*d*e^2 + (b*d^3 + 8*a*e^3)*x) + 12*d*x - ((b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))) + 3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a^2*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a*b^2*d^2 - 8*a*d*e^2 + 2*(b*d^3 + 8*a*e^3)*x + 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a^2*b^3*e + a*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3))) - ((b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))) - 3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a^2*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a*b^2*d^2 - 8*a*d*e^2 + 2*(b*d^3 + 8*a*e^3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a^2*b^3*e + a*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^

```
5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3))) + 12*
c)/(b^2*x^3 + a*b)
```

giac [A] time = 0.20, size = 180, normalized size = 0.95

$$-\frac{\sqrt{3}\left(bd-2\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{2}{3}}b}-\frac{\left(bd+2\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{2}{3}}b}-\frac{\left(2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab}-\frac{x^{2}e+dx+c}{3\left(bx^{3}+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(b*d - 2*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/18*(b*d + 2*(-a*b^2)^(1/3)*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/9*(2*(-a/b)^(1
/3)*e + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 1/3*(x^2*e + d*x
 + c)/((b*x^3 + a)*b)
```

maple [A] time = 0.05, size = 219, normalized size = 1.15

$$\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{2\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{-\frac{ex^{2}}{3b}-\frac{dx}{3b}-\frac{c}{3b}}{bx^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (-1/3/b*e*x^2-1/3/b*d*x-1/3*c/b)/(b*x^3+a)+1/9/(a/b)^(2/3)/b^2*d*ln(x+(a/b)
^(1/3))-1/18/(a/b)^(2/3)/b^2*d*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/9/(a/b)^
(2/3)*3^(1/2)/b^2*d*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/9/b^2*e/(a/b)
^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+2/9/b^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 3.03, size = 163, normalized size = 0.86

$$-\frac{ex^{2}+dx+c}{3\left(b^{2}x^{3}+ab\right)}+\frac{\sqrt{3}\left(2e\left(\frac{a}{b}\right)^{\frac{1}{3}}+d\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^2 + d*x + c)/(b^2*x^3 + a*b) + 1/9*sqrt(3)*(2*e*(a/b)^(1/3) + d)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) + 1/1
8*(2*e*(a/b)^(1/3) - d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(
2/3)) - 1/9*(2*e*(a/b)^(1/3) - d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 0.22, size = 180, normalized size = 0.95

$$\left(\sum_{k=1}^{3}\ln\left(\frac{2de+4e^{2}x+\mathrm{root}\left(729a^{2}b^{5}z^{3}+54ab^{2}dez+8ae^{3}-bd^{3},z,k\right)^{2}ab^{3}81+\mathrm{root}\left(729a^{2}b^{5}z^{3}+54ab^{2}dez+8ae^{3}-bd^{3},z,k\right)b^{2}dx9}{b9}\right)\mathrm{root}\left(729a^{2}b^{5}z^{3}+54ab^{2}dez+8ae^{3}-bd^{3},z,k\right)\right)-\frac{\frac{c}{3b}+\frac{ex^{2}}{3b}+\frac{dx}{3b}}{bx^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((2*d*e + 4*e^2*x + 81*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z + 8*
a*e^3 - b*d^3, z, k)^2*a*b^3 + 9*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z + 8*
```

```
a*e^3 - b*d^3, z, k)*b^2*d*x)/(9*b))*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z
+ 8*a*e^3 - b*d^3, z, k), k, 1, 3) - (c/(3*b) + (e*x^2)/(3*b) + (d*x)/(3*b)
)/(a + b*x^3)
```

sympy [A] time = 2.33, size = 110, normalized size = 0.58

$$\operatorname{RootSum}\left(729t^3a^2b^5 + 54tab^2de + 8ae^3 - bd^3, \left(t \mapsto t\log\left(x + \frac{162t^2a^2b^3e + 9tab^2d^2 + 8ade^2}{8ae^3 + bd^3}\right)\right)\right) + \frac{-c - dx - ex^2}{3ab + 3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**2*b**5 + 54*_t*a*b**2*d*e + 8*a*e**3 - b*d**3, Lambda(
_t, _t*log(x + (162*_t**2*a**2*b**3*e + 9*_t*a*b**2*d**2 + 8*a*d*e**2)/(8*a
*e**3 + b*d**3)))) + (-c - d*x - e*x**2)/(3*a*b + 3*b**2*x**3)
```

3.292 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=200

$$\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{4/3}}-$$

Rubi [A] time = 0.15, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{4/3}}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(3*a*b*(a + b*x^3)) - ((b^(2/3)*c + a^(2/3)*e)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(4
/3)) - ((b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(4/3
)) + ((b^(2/3)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(4/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^2}\,dx &= -\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}-\frac{\int \frac{-ae-bcx}{a+bx^3}\,dx}{3ab}\\
&= -\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}-\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}\,bc-2a\sqrt[3]{b}\,e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}\,bc+a\sqrt[3]{b}\,e\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{5/3}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\int}{9a^{4/3}b}\\
&= -\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{4/3}}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}}}{18a^{4/3}b^{4/3}}\\
&= -\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{4/3}}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\right.}{18a^{4/3}b^{4/3}}\\
&= -\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\right.}{9a^{4/3}b^{4/3}}
\end{aligned}
$$

Mathematica [A] time = 0.20, size = 186, normalized size = 0.93

$$
\frac{-\left(a^{4/3}\sqrt[3]{b}\,e-a^{2/3}bc\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2\left(a^{4/3}\sqrt[3]{b}\,e-a^{2/3}bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt{3}\left(a^{2/3}bc+a^{4/3}\sqrt[3]{b}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{6ab^{2/3}\left(a(d+ex)-bcx^2\right)}{a+bx^3}}{18a^2b^{5/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a*b^(2/3)*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) - 2*Sqrt[3]*(a^(2/3)
*b*c + a^(4/3)*b^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(
-(a^(2/3)*b*c) + a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(2/3)*b
*c) + a^(4/3)*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1
8*a^2*b^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x + e*x^2))/(a + b*x^3)^2, x]
```

fricas [C] time = 1.22, size = 2358, normalized size = 11.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*c*x^2 - 12*a*e*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
 - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
 - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c - 1/2*((1/2)^(1/3)*
(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b
^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^
3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 + 2*a*b*c^2
*e + (b^2*c^3 + a^2*e^3)*x) - 12*a*d + ((a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4
))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)
/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3))) + 3*sqrt(1/3)*(a*b^2*x^
3 + a^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) +
 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4)
)^(1/3)))^2*a^2*b^2 + 16*c*e)/(a^2*b^2)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
- 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4
) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c + 1/2*((1/2)^(1/3)*(
I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^
4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3
)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 - 2*a*b*c^2*
e + 2*(b^2*c^3 + a^2*e^3)*x + 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1
/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b
^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b^3*c + 2*a^3*b*e^2)*sqrt(-(((1/2)
^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)
/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 +
 a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2 + 16
*c*e)/(a^2*b^2))) + ((a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2
*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^
(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c
^3 - a^2*e^3)/(a^4*b^4))^(1/3))) - 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-((
(1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2
*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*
c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2
 + 16*c*e)/(a^2*b^2)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^
2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e
*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*
e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^
2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)
^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*
c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 - 2*a*b*c^2*e + 2*(b^2*c^3 + a^
2*e^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)
/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*s
qrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(
```

```
a^4*b^4))^(1/3)))*a^3*b^3*c + 2*a^3*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) +
 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) -
 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4)
 - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2 + 16*c*e)/(a^2*b^2))))/
(a*b^2*x^3 + a^2*b)
```

giac [A] time = 0.18, size = 190, normalized size = 0.95

$$-\frac{\sqrt{3}\left(ae-\left(-ab^{2}\right)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{\left(ae+\left(-ab^{2}\right)^{\frac{1}{3}}c\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{\left(bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^{2}b}+\frac{bcx^{2}-axe-ad}{3\left(bx^{3}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(a*e - (-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(a*e + (-a*b^2)^(1/3)*c)*log(x^2
 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(b*c*(-a/b)^(1/3
) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/3*(b*c*x^2 - a
*x*e - a*d)/((b*x^3 + a)*a*b)
```

maple [A] time = 0.05, size = 228, normalized size = 1.14

$$\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{\frac{cx^{2}}{3a}-\frac{ex}{3b}-\frac{d}{3b}}{bx^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (1/3*c/a*x^2-1/3/b*e*x-1/3*d/b)/(b*x^3+a)+1/9/b^2*e/(a/b)^(2/3)*ln(x+(a/b)^
(1/3))-1/18/b^2*e/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/9/b^2*e/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/b/a*c/(a/b)^
(1/3)*ln(x+(a/b)^(1/3))+1/18/(a/b)^(1/3)/a/b*c*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+1/9/b/a*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.84, size = 185, normalized size = 0.92

$$\frac{bcx^{2}-aex-ad}{3\left(ab^{2}x^{3}+a^{2}b\right)}+\frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c*x^2 - a*e*x - a*d)/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(b*c*(a/b)^(1
/3) + a*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)
^(2/3)) + 1/18*(b*c*(a/b)^(1/3) - a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(a*b^2*(a/b)^(2/3)) - 1/9*(b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(
a*b^2*(a/b)^(2/3))
```

mupad [B] time = 5.17, size = 194, normalized size = 0.97

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(729a^{4}b^{4}z^{3}+27a^{2}b^{2}cez+b^{2}c^{3}-a^{2}e^{3},z,k\right)\left(bex+\mathrm{root}\left(729a^{4}b^{4}z^{3}+27a^{2}b^{2}cez+b^{2}c^{3}-a^{2}e^{3},z,k\right)ab^{2}9\right)+\frac{ce}{9a}+\frac{bc^{2}x}{9a^{2}}\right)\mathrm{root}\left(729a^{4}b^{4}z^{3}+27a^{2}b^{2}cez+b^{2}c^{3}-a^{2}e^{3},z,k\right)\right)-\frac{\frac{d}{3b}-\frac{cx^{2}}{3a}+\frac{ex}{3b}}{bx^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log(root(729*a^4*b^4*z^3 + 27*a^2*b^2*c*e*z + b^2*c^3 - a^2*e^3, z,
k)*(b*e*x + 9*root(729*a^4*b^4*z^3 + 27*a^2*b^2*c*e*z + b^2*c^3 - a^2*e^3,
z, k)*a*b^2) + (c*e)/(9*a) + (b*c^2*x)/(9*a^2))*root(729*a^4*b^4*z^3 + 27*a
^2*b^2*c*e*z + b^2*c^3 - a^2*e^3, z, k), k, 1, 3) - (d/(3*b) - (c*x^2)/(3*a
) + (e*x)/(3*b))/(a + b*x^3)
```

sympy [A] time = 1.85, size = 124, normalized size = 0.62

$$\mathrm{RootSum}\left(729t^3a^4b^4 + 27ta^2b^2ce - a^2e^3 + b^2c^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^3b^3c + 9ta^3be^2 + 2abc^2e}{a^2e^3 + b^2c^3}\right)\right)\right) + \frac{-ad - aex + bcx^2}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**4*b**4 + 27*_t*a**2*b**2*c*e - a**2*e**3 + b**2*c**3,
Lambda(_t, _t*log(x + (81*_t**2*a**3*b**3*c + 9*_t*a**3*b*e**2 + 2*a*b*c**2
*e)/(a**2*e**3 + b**2*c**3)))) + (-a*d - a*e*x + b*c*x**2)/(3*a**2*b + 3*a*
b**2*x**3)
```

3.293 $\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=199

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{3\sqrt{3}a^{5/3}b^{2/3}}$$

Rubi [A] time = 0.13, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1854, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

```
[Out] -(a*e - b*x*(c + d*x))/(3*a*b*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3))
 + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3))
 - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^2}\,dx = -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}-\frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$

$$= -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}-\frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}c-\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$= -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{18a^{5/3}b^{2/3}}$$

$$= -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{18a^{5/3}b^{2/3}}$$

$$= -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}-\frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-$$

Mathematica [A] time = 0.28, size = 189, normalized size = 0.95

$$\frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\left(4\sqrt[3]{a}b^{2/3}c-2a^{2/3}\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{6a(bx(c+dx)-ae)}{a+bx^3}-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{18a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

```
[Out] ((6*a*(-(a*e) + b*x*(c + d*x)))/(a + b*x^3) - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(2*
b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (4*a^(
1/3)*b^(2/3)*c - 2*a^(2/3)*b^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*b^
(1/3)*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^2*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^3)^2, x]
```

fricas [C]   time = 1.20, size = 2118, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*d*x^2 + 12*b*c*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4
*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b
*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b
*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3
)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^
3)/(a^5*b^2))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
+ a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d
*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a
^5*b^2))^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) - 12*a*e + ((
a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2
) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/
(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))
+ 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b
*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3
)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^
3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*
sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1
/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2)
 + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) +
 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8
*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d
^3)*x + 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5
*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) -
 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3
)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))) + ((a*b^2*x^3 + a^2*b)*((1/2)^(1
/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b
^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a
^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b^2*x^3 + a
^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8
*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b
*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*
b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d
^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*s
qrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^
2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(
a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3
) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(
1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2
)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a
^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*s
qrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
- a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*
```

```
c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*
c*d)/(a^3*b))))/(a*b^2*x^3 + a^2*b)
```

giac [A] time = 0.18, size = 184, normalized size = 0.92

$$-\frac{\sqrt{3}\left(2\,bc-\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{\left(2\,bc+\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}}+\frac{bdx^{2}+bcx-ae}{3\left(bx^{3}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)^(1
/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(b*d*x^2 + b*c
*x - a*e)/((b*x^3 + a)*a*b)
```

maple [A] time = 0.04, size = 253, normalized size = 1.27

$$\frac{d\,x^{2}}{3\left(b\,x^{3}+a\right)a}+\frac{cx}{3\left(b\,x^{3}+a\right)a}+\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{e}{3\left(b\,x^{3}+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3/(b*x^3+a)/a*c*x+2/9/(a/b)^(2/3)/a/b*c*ln(x+(a/b)^(1/3))-1/9/(a/b)^(2/3)
/a/b*c*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/(a/b)^(2/3)*3^(1/2)/a/b*c*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/(b*x^3+a)/a*d*x^2-1/9*d/a/b/(a/b)^(
1/3)*ln(x+(a/b)^(1/3))+1/18*d/a/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+1/9*d/a*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-
1/3*e/b/(b*x^3+a)
```

maxima [A] time = 3.02, size = 179, normalized size = 0.90

$$\frac{bdx^{2}+bcx-ae}{3\left(ab^{2}x^{3}+a^{2}b\right)}+\frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*d*x^2 + b*c*x - a*e)/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(d*(a/b)^(1/3
) + 2*c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/
3)) + 1/18*(d*(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*
b*(a/b)^(2/3)) - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^
(2/3))
```

mupad [B] time = 0.25, size = 175, normalized size = 0.88

$$\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d+d^{2}\,x+\mathrm{root}\left(729\,a^{5}\,b^{2}\,z^{3}+54\,a^{2}\,b\,c\,d\,z-8\,b\,c^{3}+a\,d^{3},z,k\right)^{2}a^{3}\,b\,81+\mathrm{root}\left(729\,a^{5}\,b^{2}\,z^{3}+54\,a^{2}\,b\,c\,d\,z-8\,b\,c^{3}+a\,d^{3},z,k\right)\,a\,b\,c\,x\,18\right)}{a^{2}\,9}\right)\mathrm{root}\left(729\,a^{5}\,b^{2}\,z^{3}+54\,a^{2}\,b\,c\,d\,z-8\,b\,c^{3}+a\,d^{3},z,k\right)\right)+\frac{\frac{d\,x^{2}}{3\,a}-\frac{e}{3\,b}+\frac{c\,x}{3\,a}}{b\,x^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z - 8
*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*b*c
*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) - e/(3*b) + (c*x)/
(3*a))/(a + b*x^3)
```

sympy [A] time = 1.38, size = 116, normalized size = 0.58

$$\operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{-ae + bcx + bdx^2}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (-a*e + b*c*x + b*d*x**2)/(3*a**2*b + 3*a*b**2*x**3)
```

3.294 $\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=222

$$-\frac{\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}\,e+2\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}$$

Rubi [A] time = 0.31, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}\,e+2\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{2/3}}+\frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^2}+\frac{c\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(3*a^2*(a + b*x^3)) - ((2*b^(1/3)*d + a^(1/3)*e
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(
2/3)) + (c*Log[x])/a^2 + ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x
])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx = \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-2bdx-bex^2}{x\left(a+bx^3\right)}\,dx}{3ab}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax} - \frac{b\left(2ad+aex-3bcx^2\right)}{a\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\int \frac{2ad+aex-3bcx^2}{a+bx^3}\,dx}{3a^2}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\int \frac{2ad+aex}{a+bx^3}\,dx}{3a^2} - \frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{c\log\left(a+bx^3\right)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}d+a^{4/3}e\right)+\sqrt[3]{b}\left(-2a\sqrt[3]{b}d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}\sqrt[3]{b}} - $$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{c\log\left(a+bx^3\right)}{3a^2} - \frac{\left(2\sqrt[3]{}\right.}{}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a\right.}{18a^5}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\right.}{9a^{5/3}b^{2/3}}$$

Mathematica [A] time = 0.24, size = 199, normalized size = 0.90

$$\frac{\frac{\left(a^{2/3}e-2\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} - \frac{2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{6a(c+x(d+ex))}{a+bx^3} - 6c\log\left(a+bx^3\right) + 18c\log(x)}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((6*a*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a^
(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 18*c*Log[x]
 + (2*(2*a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) +
 ((-2*a^(1/3)*b^(1/3)*d + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/b^(2/3) - 6*c*Log[a + b*x^3])/(18*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x*(a + b*x^3)^2), x]
```

fricas [C] time = 1.42, size = 5018, normalized size = 22.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/324*(108*a*e*x^2 + 108*a*d*x - 2*(a^2*b*x^3 + a^3)*((-I*sqrt(3) + 1)*(9*c
^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a
*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 +
 a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(
-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e
^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^
6*b^2))^(1/3) + 54*c/a^2)*log(1/324*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2
 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) +
 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d
^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1
/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1
/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 5
4*c/a^2)^2*a^4*b*e + 12*b*c*d^2 + 9*b*c^2*e + 4*a*d*e^2 - 1/9*(2*a^2*b*d^2
+ 3*a^2*b*c*e)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/
(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*
e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a
^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d
*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a
^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2) + (8*b*d^3 +
 a*e^3)*x) + 108*a*c - (162*b*c*x^3 - (a^2*b*x^3 + a^3)*((-I*sqrt(3) + 1)*(
9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 +
2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^
3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1
)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 +
a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/
(a^6*b^2))^(1/3) + 54*c/a^2) + 162*a*c - 3*sqrt(1/3)*(a^2*b*x^3 + a^3)*sqrt
(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a
^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^
2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/
3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b
) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(
4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)^2*a^4*b - 108*((-I*sqrt(
3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9
*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(
27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sq
rt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8
*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d
*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)*a^2*b*c + 2916*b*c^2 + 2592*a*d*e)/(a
^4*b)))*log(-1/324*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*
b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3
+ a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b
)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2
*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3
 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)^2*a^4*b*
e - 12*b*c*d^2 - 9*b*c^2*e - 4*a*d*e^2 + 1/9*(2*a^2*b*d^2 + 3*a^2*b*c*e)*((
-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 +
1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) -
1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) +
81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1
/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3
 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2) + 2*(8*b*d^3 + a*e^3)*x + 1/1
08*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(
-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e
```

```
^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^
6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*
e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^
2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)*a^4*b*e + 72*
a^2*b*d^2 - 54*a^2*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 +
2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/
1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3
- 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/16
2*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/14
58*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c
/a^2)^2*a^4*b - 108*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4
*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3
 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*
b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 +
2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^
3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)*a^2*b*c
 + 2916*b*c^2 + 2592*a*d*e)/(a^4*b))) - (162*b*c*x^3 - (a^2*b*x^3 + a^3)*((
-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 +
1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) -
1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) +
81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1
/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3
 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2) + 162*a*c + 3*sqrt(1/3)*(a^2*
b*x^3 + a^3)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4
*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3
 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*
b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 +
2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^
3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)^2*a^4*b
 - 108*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c
^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^
5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))
^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a
^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 -
 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)*a^2*b*c + 2916*b*c^2
 + 2592*a*d*e)/(a^4*b)))*log(-1/324*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2
 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) +
 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d
^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1
/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1
/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 5
4*c/a^2)^2*a^4*b*e - 12*b*c*d^2 - 9*b*c^2*e - 4*a*d*e^2 + 1/9*(2*a^2*b*d^2
+ 3*a^2*b*c*e)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/
(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*
e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a
^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d
*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a
^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2) + 2*(8*b*d^3
 + a*e^3)*x - 1/108*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*
a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/14
58*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 -
9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*
(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458
*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a
^2)*a^4*b*e + 72*a^2*b*d^2 - 54*a^2*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/
a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*
e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^
2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/
27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)
```

```
/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b
^2))^(1/3) + 54*c/a^2)^2*a^4*b - 108*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^
2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b)
+ 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*
d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 +
1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) -
1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) +
54*c/a^2)*a^2*b*c + 2916*b*c^2 + 2592*a*d*e)/(a^4*b))) + 324*(b*c*x^3 + a*c
)*log(x))/(a^2*b*x^3 + a^3)
```

giac [A] time = 0.18, size = 217, normalized size = 0.98

$$-\frac{\sqrt{3}\left(2\,bd-\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{\left(2\,bd+\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{c\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{2}}+\frac{c\log\left(\left|x\right|\right)}{a^{2}}+\frac{ax^{2}e+adx+ac}{3\left(bx^{3}+a\right)a^{2}}-\frac{\left(a^{3}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+2\,a^{3}bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{5}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*d + (-a*b^2)^(1/3)*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/3*c*log(abs(b*
x^3 + a))/a^2 + c*log(abs(x))/a^2 + 1/3*(a*x^2*e + a*d*x + a*c)/((b*x^3 + a
)*a^2) - 1/9*(a^3*b*(-a/b)^(1/3)*e + 2*a^3*b*d)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^5*b)
```

maple [A] time = 0.06, size = 274, normalized size = 1.23

$$\frac{e\,x^{2}}{3\left(b\,x^{3}+a\right)a}+\frac{dx}{3\left(b\,x^{3}+a\right)a}+\frac{2\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{c}{3\left(b\,x^{3}+a\right)a}+\frac{c\ln(x)}{a^{2}}-\frac{c\ln\left(b\,x^{3}+a\right)}{3a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^2,x)
```

```
[Out] 1/3/(b*x^3+a)/a*e*x^2+1/3/a*x/(b*x^3+a)*d+1/3/a/(b*x^3+a)*c+2/9/a/b*d/(a/b)
^(2/3)*ln(x+(a/b)^(1/3))-1/9/a/b*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+2/9/a/b*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-1/9/(a/b)^(1/3)/a/b*e*ln(x+(a/b)^(1/3))+1/18/(a/b)^(1/3)/a/b*e*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))+1/9*3^(1/2)/(a/b)^(1/3)/a/b*e*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))-1/3/a^2*c*ln(b*x^3+a)+1/a^2*c*ln(x)
```

maxima [A] time = 2.94, size = 203, normalized size = 0.91

$$\frac{ex^{2}+dx+c}{3\left(abx^{3}+a^{2}\right)}+\frac{c\log(x)}{a^{2}}+\frac{\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}}-\frac{\left(6\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ad\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(e*x^2 + d*x + c)/(a*b*x^3 + a^2) + c*log(x)/a^2 + 1/9*sqrt(3)*(a*e*(a/
b)^(2/3) + 2*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/a^3 - 1/18*(6*b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3) + 2*a*d)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/9*(3*b*c*(a/b)^(2/3) +
a*e*(a/b)^(1/3) - 2*a*d)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

mupad [B] time = 0.38, size = 490, normalized size = 2.21

$$\frac{\frac{c}{3a}+\frac{ex^{2}}{3a}+\frac{dx}{3a}}{bx^{3}+a}+\left(\sum_{k=1}^{3}\ln\left(\frac{4b^{2}cd^{2}-3b^{2}c^{2}e}{9a^{3}}-\mathrm{root}\left(729a^{6}b^{2}z^{3}+729a^{4}b^{2}cz^{2}+54a^{3}bdez+243a^{2}b^{2}c^{2}z+18abcde-8abd^{3}+27b^{2}c^{3}+a^{2}e^{3},z,k\right)\left(\mathrm{root}\left(729a^{6}b^{2}z^{3}+729a^{4}b^{2}cz^{2}+54a^{3}bdez+243a^{2}b^{2}c^{2}z+18abcde-8abd^{3}+27b^{2}c^{3}+a^{2}e^{3},z,k\right)\left(-ab^{2}e+24b^{3}cx+\mathrm{root}\left(729a^{6}b^{2}z^{3}+729a^{4}b^{2}cz^{2}+54a^{3}bdez+243a^{2}b^{2}c^{2}z+18abcde-8abd^{3}+27b^{2}c^{3}+a^{2}e^{3},z,k\right)a^{2}b^{3}x36\right)+\frac{4a^{2}b^{2}d^{2}+6cea^{2}b^{2}}{9a^{3}}+\frac{x\left(60dea^{2}b^{2}+108ab^{3}c^{2}\right)}{27a^{3}}\right)-\frac{x\left(-8b^{2}d^{3}+12cb^{2}de+abe^{3}\right)}{27a^{3}}\right)\mathrm{root}\left(729a^{6}b^{2}z^{3}+729a^{4}b^{2}cz^{2}+54a^{3}bdez+243a^{2}b^{2}c^{2}z+18abcde-8abd^{3}+27b^{2}c^{3}+a^{2}e^{3},z,k\right)\right)+\frac{c\ln(x)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c/(3*a) + (e*x^2)/(3*a) + (d*x)/(3*a))/(a + b*x^3) + symsum(log((4*b^2*c*d
^2 - 3*b^2*c^2*e)/(9*a^3) - root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a
^3*b*d*e*z + 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^
2*e^3, z, k)*(root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3*b*d*e*z + 2
43*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2*e^3, z, k)*(
24*b^3*c*x - a*b^2*e + 36*root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3
*b*d*e*z + 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2*
e^3, z, k)*a^2*b^3*x) + (4*a^2*b^2*d^2 + 6*a^2*b^2*c*e)/(9*a^3) + (x*(108*a
*b^3*c^2 + 60*a^2*b^2*d*e))/(27*a^3)) - (x*(a*b*e^3 - 8*b^2*d^3 + 12*b^2*c*
d*e))/(27*a^3))*root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3*b*d*e*z +
 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2*e^3, z, k)
, k, 1, 3) + (c*log(x))/a^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.295 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=231

$$-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(a^{2/3}e+2b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(2b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\right.}{3\sqrt{3}\,a^{7/3}\sqrt[3]{b}}$$

Rubi [A] time = 0.34, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(a^{2/3}e+2b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(2b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}\sqrt[3]{b}}+\frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^2}-\frac{c}{a^2x}+\frac{d\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(c/(a^2*x)) + (x*(a*e - b*c*x - b*d*x^2))/(3*a^2*(a + b*x^3)) + (2*(2*b^(2
/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sq
rt[3]*a^(7/3)*b^(1/3)) + (d*Log[x])/a^2 + (2*(2*b^(2/3)*c + a^(2/3)*e)*Log[
a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(1/3)) - ((2*b^(2/3)*c + a^(2/3)*e)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(7/3)*b^(1/3)) - (d*Log[a
+ b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^2}\,dx = \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-3bdx-2bex^2+\frac{b^2cx^3}{a}}{x^2\left(a+bx^3\right)}\,dx}{3ab}$$

$$= \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax^2}-\frac{3bd}{ax}-\frac{b\left(2ae-4bcx-3bdx^2\right)}{a\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{\int \frac{2ae-4bcx-3bdx^2}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{\int \frac{2ae-4bcx}{a+bx^3}\,dx}{3a^2} - \frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \frac{d\log\left(a+bx^3\right)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{a}\,bc+4a\sqrt[3]{b}\,e\right)+\sqrt[3]{b}\left(-4\cdots\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\cdots}}{9a^{8/3}\sqrt[3]{b}}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{2\left(2b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{d\log\left(a\cdots\right.}{3a\cdots}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{2\left(2b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{\left(2b^{2/3}c\cdots\right.}{\cdots}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{2\left(2b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}\sqrt[3]{b}} + \frac{d\log(x)}{a^2} + \frac{2\left(2b^{2/3}\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.34, size = 213, normalized size = 0.92

$$-\frac{\frac{\left(2a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}} - \frac{2\left(2a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} + \frac{2\sqrt{3}\,a^{2/3}\left(a^{2/3}e-2b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - \frac{3a\left(a(d+ex)-bcx^2\right)}{a+bx^3} + 3ad\log\left(a+bx^3\right) + \frac{9ac}{x} - 9ad\log(x)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -1/9*((9*a*c)/x - (3*a*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) + (2*Sqrt[3]
*a^(2/3)*(-2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/b^(1/3) - 9*a*d*Log[x] - (2*(2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(
1/3) + b^(1/3)*x])/b^(1/3) + ((2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 3*a*d*Log[a + b*x^3])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2), x]
```

fricas [C] time = 1.47, size = 4976, normalized size = 21.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/324*(432*b*c*x^3 - 108*a*e*x^2 - 108*a*d*x + 2*(a^2*b*x^4 + a^3*x)*((-I*
sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^
2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b
)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)
*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*
e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b
))^(1/3) + 54*d/a^2)*log(-1/324*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c
*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3
+ 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3
)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e
)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
 - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^5*b*c - 9*a*b
*c*d^2 + 16*a*b*c^2*e + 3*a^2*d*e^2 + 1/18*(6*a^3*b*c*d - a^4*e^2)*((-I*sqr
t(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 -
 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(
a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-
1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3
 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^
(1/3) + 54*d/a^2) - 2*(8*b^2*c^3 - a^2*e^3)*x) + 324*a*c + (162*b*d*x^4 + 1
62*a*d*x - (a^2*b*x^4 + a^3*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*
e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 +
 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)
/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)
*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
- 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) + 3*sqrt(1/3)*(a^2
*b*x^4 + a^3*x)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/
(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e
^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b)
)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 +
 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*
(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) +
 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e
)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
 - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d
^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(
3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3)
+ 54*d/a^2)*a^2*d + 2916*d^2 - 10368*c*e)/a^4))*log(1/324*((-I*sqrt(3) + 1)
*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d
/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) -
4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/
a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d
^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 5
4*d/a^2)^2*a^5*b*c + 9*a*b*c*d^2 - 16*a*b*c^2*e - 3*a^2*d*e^2 - 1/18*(6*a^3
*b*c*d - a^4*e^2)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/2
7*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 -
9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/
3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/14
58*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^
2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) - 4*(8*b^2*c^3 - a^2*e^3)*x + 1
/108*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*
d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*
(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3)
 + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458
```

```
*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*
c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*a^5*b*c - 54*a^3*b*c*d - 18*a^4*e
^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a
^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^
3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81
*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*
b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
 a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a
^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/
1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*
b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/1
62*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*
d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*
a^2*d + 2916*d^2 - 10368*c*e)/a^4)) + (162*b*d*x^4 + 162*a*d*x - (a^2*b*x^4
 + a^3*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^
6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3
 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*
(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b
^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) - 3*sqrt(1/3)*(a^2*b*x^4 + a^3*x)*sqrt(
-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/16
2*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d
*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(
3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 +
 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)
/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d
^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*
b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
 a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2
 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)
/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*a^2*d + 2
916*d^2 - 10368*c*e)/a^4))*log(1/324*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2
- 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2
*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^
2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 -
8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a
^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^5*b*c +
9*a*b*c*d^2 - 16*a*b*c^2*e - 3*a^2*d*e^2 - 1/18*(6*a^3*b*c*d - a^4*e^2)*((-
I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*
d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a
*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) +
1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^
2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7
*b))^(1/3) + 54*d/a^2) - 4*(8*b^2*c^3 - a^2*e^3)*x - 1/108*sqrt(1/3)*(((-I*
sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^
2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b
)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)
*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*
e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b
))^(1/3) + 54*d/a^2)*a^5*b*c - 54*a^3*b*c*d - 18*a^4*e^2)*sqrt(-(((-I*sqrt(
3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8
*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^
7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/
27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 -
 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1
/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/
a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*
a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a
^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/
```

```
a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4
/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*a^2*d + 2916*d^2 - 10
368*c*e)/a^4)) - 324*(b*d*x^4 + a*d*x)*log(x))/(a^2*b*x^4 + a^3*x)
```

giac [A] time = 0.18, size = 237, normalized size = 1.03

$$-\frac{d\log\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{d\log\left(\left|x\right|\right)}{a^2}+\frac{2\,\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae+2\,\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}-\frac{4\,bcx^3-ax^2e-adx+3\,ac}{3\left(bx^4+ax\right)a^2}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae-2\,\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b}+\frac{2\left(2\,a^2b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 + 2/9*sqrt(3)*((-a*b^2)^
(1/3)*a*e + 2*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/(a^3*b) - 1/3*(4*b*c*x^3 - a*x^2*e - a*d*x + 3*a*c)/((b*x^4 + a*
x)*a^2) + 1/9*((-a*b^2)^(1/3)*a*e - 2*(-a*b^2)^(2/3)*c)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/(a^3*b) + 2/9*(2*a^2*b^2*c*(-a/b)^(1/3) - a^3*b*e)*(-
a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b)
```

maple [A] time = 0.06, size = 275, normalized size = 1.19

$$-\frac{bc\,x^2}{3\left(b\,x^3+a\right)a^2}+\frac{ex}{3\left(b\,x^3+a\right)a}+\frac{2\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{d}{3\left(b\,x^3+a\right)a}-\frac{4\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{4c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{2c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{d\ln(x)}{a^2}-\frac{d\ln\left(b\,x^3+a\right)}{3a^2}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2/(b*x^3+a)*b*c*x^2+1/3/(b*x^3+a)/a*e*x+1/3/a/(b*x^3+a)*d+2/9/(a/b)^
(2/3)/a/b*e*ln(x+(a/b)^(1/3))-1/9/(a/b)^(2/3)/a/b*e*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))+2/9/(a/b)^(2/3)*3^(1/2)/a/b*e*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))+4/9/a^2*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2*c/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))-4/9*3^(1/2)/(a/b)^(1/3)/a^2*c*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))-1/3/a^2*d*ln(b*x^3+a)-1/a^2*c/x+1/a^2*d*ln(x)
```

maxima [A] time = 3.10, size = 222, normalized size = 0.96

$$-\frac{4\,bcx^3-aex^2-adx+3\,ac}{3\left(a^2bx^4+a^3x\right)}+\frac{d\log(x)}{a^2}-\frac{2\,\sqrt{3}\left(2\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3}-\frac{\left(3\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(4*b*c*x^3 - a*e*x^2 - a*d*x + 3*a*c)/(a^2*b*x^4 + a^3*x) + d*log(x)/a
^2 - 2/9*sqrt(3)*(2*b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/9*(3*b*d*(a/b)^(2/3) + 2*b*c*(a/b)^
(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1
/9*(3*b*d*(a/b)^(2/3) - 4*b*c*(a/b)^(1/3) - 2*a*e)*log(x + (a/b)^(1/3))/(a^
2*b*(a/b)^(2/3))
```

mupad [B] time = 5.47, size = 488, normalized size = 2.11

$$\left(\sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(729\,a^7\,b\,z^3+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b^2\,c^3,z,k\right)\left(\mathrm{root}\left(729\,a^7\,b\,z^3+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b^2\,c^3,z,k\right)\left(4\,b^3\,c+24\,b^3\,d\,x+\mathrm{root}\left(729\,a^7\,b\,z^3+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b^2\,c^3,z,k\right)\,a^2\,b^3\,x\,36\right)+\frac{4\,\left(a^3\,b^2\,e^2-6\,a^2\,b^3\,c\,d\right)}{9\,a^4}+\frac{4\,x\,\left(27\,a^3\,b^3\,d^2-60\,a^3\,b^3\,c\,e\right)}{27\,a^5}\right)+\frac{4\,\left(3\,c\,b^3\,d^2+a\,b^2\,d\,e^2\right)}{9\,a^4}+\frac{4\,x\,\left(2\,a^2\,b^2\,e^3+12\,d\,a\,b^3\,c\,e+16\,b^4\,c^3\right)}{27\,a^5}\right)\,\mathrm{root}\left(729\,a^7\,b\,z^3+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b^2\,c^3,z,k\right)\right)-\frac{\frac{c}{a}-\frac{e\,x^2}{3\,a}-\frac{d\,x}{3\,a}+\frac{4\,b\,c\,x^3}{3\,a^2}}{b\,x^4+a\,x}+\frac{d\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((4*(3*b^3*c*d^2 + a*b^2*d*e^2))/(9*a^4) - root(729*a^7*b*z^3 + 7
29*a^5*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*
d^3 - 8*a^2*e^3 - 64*b^2*c^3, z, k)*(root(729*a^7*b*z^3 + 729*a^5*b*d*z^2 -
 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*d^3 - 8*a^2*e^3
- 64*b^2*c^3, z, k)*(4*b^3*c + 24*b^3*d*x + 36*root(729*a^7*b*z^3 + 729*a^5
*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*d^3 -
8*a^2*e^3 - 64*b^2*c^3, z, k)*a^2*b^3*x) + (4*(a^3*b^2*e^2 - 6*a^2*b^3*c*d)
)/(9*a^4) + (4*x*(27*a^3*b^3*d^2 - 60*a^3*b^3*c*e))/(27*a^5)) + (4*x*(16*b^
4*c^3 + 2*a^2*b^2*e^3 + 12*a*b^3*c*d*e))/(27*a^5))*root(729*a^7*b*z^3 + 729
*a^5*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*d^
3 - 8*a^2*e^3 - 64*b^2*c^3, z, k), k, 1, 3) - (c/a - (e*x^2)/(3*a) - (d*x)/
(3*a) + (4*b*c*x^3)/(3*a^2))/(a*x + b*x^4) + (d*log(x))/a^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.296 $\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(4\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{3\sqrt{3}a}$$

Rubi [A] time = 0.35, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(4\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}-\frac{e\log\left(a+bx^3\right)}{3a^2}-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c + b*d*x + b*e*x^2))/(3*a^2*(a + b*x^3)
) + (b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (e*Log[x])/a^2 - (b^(1/3)*(5*b^(1/3)
*c - 4*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)) + (b^(1/3)*(5*b^(1/
3)*c - 4*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(
8/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-3bdx-3bex^2+\frac{2b^2cx^3}{a}+\frac{b^2dx^4}{a}}{x^3\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax^3}-\frac{3bd}{ax^2}-\frac{3be}{ax}+\frac{b^2\left(5c+4dx+3ex^2\right)}{a\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{b\int \frac{5c+4dx+3ex^2}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{b\int \frac{5c+4dx}{a+bx^3}\,dx}{3a^2} - \frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{e\log\left(a+bx^3\right)}{3a^2} - \frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(10\sqrt[3]{b}\,c+4\sqrt[3]{a}\,d\right)}{a^{2/3}-\sqrt[3]{a}}}{9a}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,c-4\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}} -$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,c-4\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}} +$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,c+4\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}} + \frac{e\log(x)}{a^2}$$

Mathematica [A] time = 0.22, size = 221, normalized size = 0.91

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}\,c-4a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(4a^{2/3}d-5\sqrt[3]{a}\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\frac{6a(ae-bx(c+dx))}{a+bx^3}+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(4\sqrt[3]{a}\,d+5\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6ae\log\left(a+bx^3\right)-\frac{9ac}{x^2}-\frac{18ad}{x}+18ae\log(x)}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a*c)/x^2 - (18*a*d)/x + (6*a*(a*e - b*x*(c + d*x)))/(a + b*x^3) + 2*Sq
rt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]] + 18*a*e*Log[x] + 2*b^(1/3)*(-5*a^(1/3)*b^(1/3)*c + 4*a^
(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 4*a^(2/3
)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 6*a*e*Log[a + b*x^3])
/(18*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2), x]
```

fricas [C] time = 1.41, size = 4774, normalized size = 19.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/324*(432*b*d*x^4 + 270*b*c*x^3 - 108*a*e*x^2 + 324*a*d*x + 2*(a^2*b*x^5
+ a^3*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*
e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*
b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^
(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^
7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3
- 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*log(1/81*((-I*sqrt(3) +
 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d
 + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c
^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1
)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 6
4*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a
*b)/a^8)^(1/3) + 54*e/a^2)^2*a^6*d + 160*a*b*c*d^2 - 75*a*b*c^2*e + 36*a^2*
d*e^2 + 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20
*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 +
1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*
(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1
/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1
458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*
e/a^2) + (125*b^2*c^3 + 64*a*b*d^3)*x) + 162*a*c + (162*b*e*x^5 + 162*a*e*x
^2 - (a^2*b*x^5 + a^3*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e
^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*
c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c
*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d
 + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c
^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) - 3*sqr
t(1/3)*(a^2*b*x^5 + a^3*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*
d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/145
8*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d
^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*
(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(
125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2
)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/
27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^
3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^
8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e
/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e
^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c*d
+ 2916*a*e^2)/a^5))*log(-1/81*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*
a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125
*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 4
5*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*
c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^
2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^
6*d - 160*a*b*c*d^2 + 75*a*b*c^2*e - 36*a^2*d*e^2 - 1/18*(25*a^3*b*c^2 - 24
*a^4*d*e)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e
^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b
/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(
1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7
 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 -
 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) + 2*(125*b^2*c^3 + 64*a*
b*d^3)*x + 1/54*sqrt(1/3)*(2*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a
*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*
```

```
b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45
*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c
*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2
*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^6*d
 - 225*a^3*b*c^2 - 108*a^4*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b
*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/
1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(1
6*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/1
62*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/145
8*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/
a^2)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(
-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a
*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)
/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2
)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^
2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c
*d + 2916*a*e^2)/a^5)) + (162*b*e*x^5 + 162*a*e*x^2 - (a^2*b*x^5 + a^3*x^2)
*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 +
1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/
1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81
*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458
*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^
3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) + 3*sqrt(1/3)*(a^2*b*x^5 + a^3*x^
2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e
^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b
/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(
1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7
 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 -
 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^5 - 108*((-I*sqrt(3)
 + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c
*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2
*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) +
 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 +
 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)
*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c*d + 2916*a*e^2)/a^5))*log(-1
/81*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6
 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 -
 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) +
 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1
458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16
*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^6*d - 160*a*b*c*d^2 + 75*a
*b*c^2*e - 36*a^2*d*e^2 - 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*sqrt(3) + 1
)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d +
 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3
 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*
(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*
a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b
)/a^8)^(1/3) + 54*e/a^2) + 2*(125*b^2*c^3 + 64*a*b*d^3)*x - 1/54*sqrt(1/3)*
(2*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6
+ 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 -
1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) +
81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/14
58*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*
d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^6*d - 225*a^3*b*c^2 - 108*a^4
*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/2
7*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3
)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8
)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/
a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^
```

```
3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^5 - 108*((-I*sqrt
(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*
b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*
b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3
) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^
3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d
*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c*d + 2916*a*e^2)/a^5)) - 3
24*(b*e*x^5 + a*e*x^2)*log(x))/(a^2*b*x^5 + a^3*x^2)
```

giac [A] time = 0.20, size = 248, normalized size = 1.02

$$-\frac{e\log\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{e\log\left(\left|x\right|\right)}{a^2}-\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bc-4\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}-\frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}bc+4\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b}+\frac{\left(4\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}-\frac{8\,bdx^4+5\,bcx^3-2\,ax^2e+6\,adx+3\,ac}{6\left(bx^3+a\right)a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 - 1/9*sqrt(3)*(5*(-a*b^2
)^(1/3)*b*c - 4*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a^3*b) - 1/18*(5*(-a*b^2)^(1/3)*b*c + 4*(-a*b^2)^(2/3)*d)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 1/9*(4*a^2*b^2*d*(-a/b)^(1/
3) + 5*a^2*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) - 1/6*(8*
b*d*x^4 + 5*b*c*x^3 - 2*a*x^2*e + 6*a*d*x + 3*a*c)/((b*x^3 + a)*a^2*x^2)
```

maple [A] time = 0.06, size = 276, normalized size = 1.14

$$-\frac{bd\,x^2}{3\left(b\,x^3+a\right)a^2}-\frac{bcx}{3\left(b\,x^3+a\right)a^2}+\frac{e}{3\left(b\,x^3+a\right)a}-\frac{5\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{5c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{5c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{4\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{4d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{2d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{e\ln(x)}{a^2}-\frac{e\ln\left(b\,x^3+a\right)}{3a^2}-\frac{d}{a^2x}-\frac{c}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] -1/3/(b*x^3+a)/a^2*b*d*x^2-1/3/a^2*b*x/(b*x^3+a)*c+1/3/a/(b*x^3+a)*e-5/9/a^
2*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/18/a^2*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))-5/9/a^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))+4/9/(a/b)^(1/3)/a^2*d*ln(x+(a/b)^(1/3))-2/9/(a/b)^(1/3)/a^2*d*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9*3^(1/2)/(a/b)^(1/3)/a^2*d*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/a^2*e*ln(b*x^3+a)-1/a^2*d/x+1/a^2*e*ln(x)-1/
2/a^2*c/x^2
```

maxima [A] time = 2.86, size = 220, normalized size = 0.91

$$-\frac{8\,bdx^4+5\,bcx^3-2\,aex^2+6\,adx+3\,ac}{6\left(a^2bx^5+a^3x^2\right)}+\frac{e\log(x)}{a^2}-\frac{\sqrt{3}\left(4\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3}-\frac{\left(6\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(8*b*d*x^4 + 5*b*c*x^3 - 2*a*e*x^2 + 6*a*d*x + 3*a*c)/(a^2*b*x^5 + a^3
*x^2) + e*log(x)/a^2 - 1/9*sqrt(3)*(4*b*d*(a/b)^(2/3) + 5*b*c*(a/b)^(1/3))*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/18*(6*e*(a/b)^(
2/3) + 4*d*(a/b)^(1/3) - 5*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(
a/b)^(2/3)) - 1/9*(3*e*(a/b)^(2/3) - 4*d*(a/b)^(1/3) + 5*c)*log(x + (a/b)^(
1/3))/(a^2*(a/b)^(2/3))
```

mupad [B] time = 5.39, size = 733, normalized size = 3.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] symsum(log(-(b^3*(108*root(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z +
243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k
)^2*a^6*d - 36*a^2*d*e^2 + 972*root(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b
*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*
c^3, z, k)^3*a^8*x + 125*b^2*c^3*x - 72*root(729*a^8*z^3 + 729*a^6*e*z^2 +
540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 +
 125*b^2*c^3, z, k)*a^4*d*e - 75*a*b*c^2*e - 64*a*b*d^3*x + 75*root(729*a^8
*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64
*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k)*a^3*b*c^2 + 108*root(729*a^8*z^3
 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b
*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k)*a^4*e^2*x + 648*root(729*a^8*z^3 + 7
29*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3
 + 27*a^2*e^3 + 125*b^2*c^3, z, k)^2*a^6*e*x + 600*root(729*a^8*z^3 + 729*a
^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 2
7*a^2*e^3 + 125*b^2*c^3, z, k)*a^3*b*c*d*x + 120*a*b*c*d*e*x))/(27*a^6))*ro
ot(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*
c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k), k, 1, 3) - (c/(2*a) -
 (e*x^2)/(3*a) + (d*x)/a + (5*b*c*x^3)/(6*a^2) + (4*b*d*x^4)/(3*a^2))/(a*x^
2 + b*x^5) + (e*log(x))/a^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.297 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2} \, dx$

Optimal. Leaf size=262

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(4\sqrt[3]{a}\,e+5\sqrt[3]{b}\right.}{3\sqrt{}}$$

Rubi [A] time = 0.40, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{8/3}}+\frac{2bc\log\left(a+bx^3\right)}{3a^3}-\frac{2bc\log(x)}{a^3}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(4\sqrt[3]{a}\,e+5\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/
a))/(3*a^2*(a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (2*b*c*Log[x])
/a^3 - (b^(1/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^
(8/3)) + (b^(1/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(18*a^(8/3)) + (2*b*c*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-3bdx-3bex^2+\frac{3b^2cx^3}{a}+\frac{2b^2dx^4}{a}+\frac{b^2ex^5}{a}}{x^4\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax^4}-\frac{3bd}{ax^3}-\frac{3be}{ax^2}+\frac{6b^2c}{a^2x}+\frac{b^2\left(5ad+4aex-6bcx^2\right)}{a^2\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}-\frac{2bc\log(x)}{a^3}-\frac{b\int \frac{5ad+4aex-6bcx^2}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}-\frac{2bc\log(x)}{a^3}-\frac{b\int \frac{5ad+4aex}{a+bx^3}\,dx}{3a^3}+\frac{\left(2b^2c\right)\cdots}{\cdots}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}-\frac{2bc\log(x)}{a^3}+\frac{2bc\log\left(a+bx^3\right)}{3a^3}-\frac{b^{2/3}\cdots}{\cdots}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}-\frac{2bc\log(x)}{a^3}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\cdots\right.}{9a^{8/3}}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}-\frac{2bc\log(x)}{a^3}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\cdots\right.}{9a^{8/3}}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\,d+4\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}\cdots$$

Mathematica [A] time = 0.30, size = 225, normalized size = 0.86

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}\,d-4a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(4a^{2/3}e-5\sqrt[3]{a}\sqrt[3]{b}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\frac{6ab(c+x(d+ex))}{a+bx^3}+12bc\log\left(a+bx^3\right)+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(4\sqrt[3]{a}\,e+5\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{6ac}{x^3}-\frac{9ad}{x^2}-\frac{18ae}{x}-36bc\log(x)}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x - (6*a*b*(c + x*(d + e*x)))/(a + b
*x^3) + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 36*b*c*Log[x] + 2*b^(1/3)*(-5*a^(1/3)*b^(1
/3)*d + 4*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1/3)*
d - 4*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 12*b*c*Lo
g[a + b*x^3])/(18*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2), x]
```

fricas [C] time = 1.56, size = 5373, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(48*a*b*e*x^5 + 30*a*b*d*x^4 + 24*a*b*c*x^3 + 36*a^2*e*x^2 + 18*a^2*d
*x + 12*a^2*c + 2*(a^3*b*x^6 + a^4*x^3)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*
b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 +
64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^
2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*
b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)
/a^9)^(1/3) - 12*b*c/a^3)*log((8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^
6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*
b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 -
5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*
b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c
/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3
) - 12*b*c/a^3)^2*a^6*e + 150*b^2*c*d^2 + 144*b^2*c^2*e + 160*a*b*d*e^2 + 1
/2*(25*a^3*b*d^2 + 48*a^3*b*c*e)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2
/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^
3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3
 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(4
32*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*
b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(
1/3) - 12*b*c/a^3) + (125*b^2*d^3 + 64*a*b*e^3)*x) - (36*b^2*c*x^6 + 36*a*b
*c*x^3 + (a^3*b*x^6 + a^4*x^3)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a
^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)
*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 -
 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432
*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*
c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/
3) - 12*b*c/a^3) + 3*sqrt(1/3)*(a^3*b*x^6 + a^4*x^3)*sqrt(-((8*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3
/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 +
(216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 -
72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^
3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*a^6 + 24*(8*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^
9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (21
6*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^
(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*
(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^3*b*c + 144*b^2*c^2 + 320*a*b*
d*e)/a^6))*log(-(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2
 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*
b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72
*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 +
(125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^
3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3
)^2*a^6*e - 150*b^2*c*d^2 - 144*b^2*c^2*e - 160*a*b*d*e^2 - 1/2*(25*a^3*b*d
^2 + 48*a^3*b*c*e)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*
c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*
(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9
 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216
*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/
```

a^3) + 2*(125*b^2*d^3 + 64*a*b*e^3)*x + 3/2*sqrt(1/3)*(2*(8*(1/2)^(2/3)*(-I
sqrt(3) + 1)(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^
9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (21
6*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^
(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*
(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^6*e - 25*a^3*b*d^2 + 24*a^3*b*
c*e)*sqrt(-((8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5
*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*
c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d
*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125
*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^
3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*
a^6 + 24*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*
b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2
 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)
*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*
d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 +
 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^3*b
*c + 144*b^2*c^2 + 320*a*b*d*e)/a^6)) - (36*b^2*c*x^6 + 36*a*b*c*x^3 + (a^3
*b*x^6 + a^4*x^3)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c
^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(
9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9
+ (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*
b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a
^3) - 3*sqrt(1/3)*(a^3*b*x^6 + a^4*x^3)*sqrt(-((8*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b
*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3
+ 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2
 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)
*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*a^6 + 24*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^
3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 6
4*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 +
5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*
b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^3*b*c + 144*b^2*c^2 + 320*a*b*d*e)/a^6))*lo
g(-(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)
/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a
*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2
)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 +
64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^
2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*a^6*e - 1
50*b^2*c*d^2 - 144*b^2*c^2*e - 160*a*b*d*e^2 - 1/2*(25*a^3*b*d^2 + 48*a^3*b
*c*e)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d
*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 +
5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*
b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3
 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64
*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3) + 2*(125
*b^2*d^3 + 64*a*b*e^3)*x - 3/2*sqrt(1/3)*(2*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^
3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 6
4*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 +
5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*
b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^6*e - 25*a^3*b*d^2 + 24*a^3*b*c*e)*sqrt(-((

```
8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6
)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d
*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^
9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a
*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*
e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*a^6 + 24*(8*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(
432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)
*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^
(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^
3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3
 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^3*b*c + 144*b^2*
c^2 + 320*a*b*d*e)/a^6)) + 72*(b^2*c*x^6 + a*b*c*x^3)*log(x))/(a^3*b*x^6 +
a^4*x^3)
```

giac [A] time = 0.18, size = 269, normalized size = 1.03

$$\frac{2\,bc\log\left(\left|bx^3+a\right|\right)}{3\,a^3}-\frac{2\,bc\log\left(\left|x\right|\right)}{a^3}-\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bd-4\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}-\frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}bd+4\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b}+\frac{\left(4\,a^4b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5\,a^4b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7b}-\frac{8\,abx^5e+5\,abdx^4+4\,abcx^3+6\,a^2x^2e+3\,a^2dx+2\,a^2c}{6\left(bx^3+a\right)a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*b*c*log(abs(b*x^3 + a))/a^3 - 2*b*c*log(abs(x))/a^3 - 1/9*sqrt(3)*(5*(-
a*b^2)^(1/3)*b*d - 4*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^3*b) - 1/18*(5*(-a*b^2)^(1/3)*b*d + 4*(-a*b^2)^(2/3)*e
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 1/9*(4*a^4*b^2*(-a/b)^
(1/3)*e + 5*a^4*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b) - 1/
6*(8*a*b*x^5*e + 5*a*b*d*x^4 + 4*a*b*c*x^3 + 6*a^2*x^2*e + 3*a^2*d*x + 2*a^
2*c)/((b*x^3 + a)*a^3*x^3)
```

maple [A] time = 0.06, size = 289, normalized size = 1.10

$$-\frac{be\,x^2}{3\left(b\,x^3+a\right)a^2}-\frac{bdx}{3\left(b\,x^3+a\right)a^2}-\frac{bc}{3\left(b\,x^3+a\right)a^2}-\frac{5\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{5d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{5d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{4\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{4e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{2e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{2bc\ln(x)}{a^3}+\frac{2bc\ln\left(b\,x^3+a\right)}{3a^3}-\frac{e}{a^2x}-\frac{d}{2a^2x^2}-\frac{c}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x)
```

```
[Out] -1/3/(b*x^3+a)/a^2*b*e*x^2-1/3/(b*x^3+a)/a^2*b*d*x-1/3*b/a^2/(b*x^3+a)*c-5/
9/(a/b)^(2/3)/a^2*d*ln(x+(a/b)^(1/3))+5/18/(a/b)^(2/3)/a^2*d*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))-5/9/(a/b)^(2/3)*3^(1/2)/a^2*d*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))+4/9/(a/b)^(1/3)/a^2*e*ln(x+(a/b)^(1/3))-2/9/a^2*e/(a/b)^(1/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/a^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/3/a^3*b*c*ln(b*x^3+a)-1/a^2*e/x-1/3/a^2*c/
x^3-1/2/a^2*d/x^2-2/a^3*b*c*ln(x)
```

maxima [A] time = 3.07, size = 236, normalized size = 0.90

$$-\frac{8\,bex^5+5\,bdx^4+4\,bcx^3+6\,aex^2+3\,adx+2\,ac}{6\left(a^2bx^6+a^3x^3\right)}-\frac{2\,bc\log(x)}{a^3}-\frac{\sqrt{3}\left(4\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4}+\frac{\left(12\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(6\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(8*b*e*x^5 + 5*b*d*x^4 + 4*b*c*x^3 + 6*a*e*x^2 + 3*a*d*x + 2*a*c)/(a^2
*b*x^6 + a^3*x^3) - 2*b*c*log(x)/a^3 - 1/9*sqrt(3)*(4*a*e*(a/b)^(2/3) + 5*a
*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 +
```

```
1/18*(12*b*c*(a/b)^(2/3) - 4*a*e*(a/b)^(1/3) + 5*a*d)*log(x^2 - x*(a/b)^(1
/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/3)) + 1/9*(6*b*c*(a/b)^(2/3) + 4*a*e*(a/b)
^(1/3) - 5*a*d)*log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

mupad [B] time = 5.48, size = 537, normalized size = 2.05

$$\left(\sum_{k=1}^{3}\ln\left(-\frac{50\,b^5\,c\,d^2-48\,b^5\,c^2\,e}{9\,a^6}-\mathrm{root}\left(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3\,c^3,z,k\right)\left(\frac{25\,a^3\,b^4\,d^2+48\,c\,e\,a^3\,b^4}{9\,a^6}+\mathrm{root}\left(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3\,c^3,z,k\right)\left(4\,b^3\,e+\mathrm{root}\left(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3\,c^3,z,k\right)\,a^2\,b^3\,x\,36-\frac{48\,b^4\,c\,x}{a}\right)+\frac{x\,\left(600\,d\,e\,a^3\,b^4+432\,a^2\,b^5\,c^2\right)}{27\,a^6}\right)+\frac{x\,\left(-125\,b^5\,d^3+240\,c\,b^5\,d\,e+64\,a\,b^4\,e^3\right)}{27\,a^6}\right)\,\mathrm{root}\left(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3\,c^3,z,k\right)\right)-\frac{\frac{c}{3\,a}+\frac{e\,x^2}{a}+\frac{d\,x}{2\,a}+\frac{2\,b\,c\,x^3}{3\,a^2}+\frac{5\,b\,d\,x^4}{6\,a^2}+\frac{4\,b\,e\,x^5}{3\,a^2}}{b\,x^6+a\,x^3}-\frac{2\,b\,c\,\ln\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((x*(64*a*b^4*e^3 - 125*b^5*d^3 + 240*b^5*c*d*e))/(27*a^6) - root
(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360
*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*b^2*d^3 - 216*b^3*c^3, z, k)*((25*a^3*b
^4*d^2 + 48*a^3*b^4*c*e)/(9*a^6) + root(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 54
0*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*
b^2*d^3 - 216*b^3*c^3, z, k)*(4*b^3*e + 36*root(729*a^9*z^3 - 1458*a^6*b*c*
z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3
+ 125*a*b^2*d^3 - 216*b^3*c^3, z, k)*a^2*b^3*x - (48*b^4*c*x)/a) + (x*(432*
a^2*b^5*c^2 + 600*a^3*b^4*d*e))/(27*a^6)) - (50*b^5*c*d^2 - 48*b^5*c^2*e)/(
9*a^6))*root(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2
*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*b^2*d^3 - 216*b^3*c^3, z, k
), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x)/(2*a) + (2*b*c*x^3)/(3*a^2) + (5
*b*d*x^4)/(6*a^2) + (4*b*e*x^5)/(3*a^2))/(a*x^3 + b*x^6) - (2*b*c*log(x))/a
^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.298 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=215

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{5/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}}-\frac{c}{6b}$$

Rubi [A] time = 0.20, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1823, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{5/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}}-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] -(c + d*x + e*x^2)/(6*b*(a + b*x^3)^2) + (x*(d + 2*e*x))/(18*a*b*(a + b*x^3
)) - ((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(9*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) +
b^(1/3)*x])/(27*a^(5/3)*b^(5/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^3}\,dx &= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{\int \frac{d+2ex}{\left(a+bx^3\right)^2}\,dx}{6b} \\
&= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}-\frac{\int \frac{-2d-2ex}{a+bx^3}\,dx}{18ab} \\
&= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}-\frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{54a^{5/3}b^{4/3}}+\frac{\left(d-\cdots\right.}{\cdots} \\
&= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}+\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\int\cdots}{18a\cdots} \\
&= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}+\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}}-\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\log\left(a^{\cdots}\right.}{54\cdots} \\
&= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}-\frac{\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{5/3}}+\frac{\left(d-\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)\text{lo}\cdots}{27a\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 198, normalized size = 0.92

$$
\frac{\frac{\left(\sqrt[3]{a}\,e-\sqrt[3]{b}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{2\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}-\frac{2\sqrt{3}\left(\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}-\frac{9b^{2/3}(c+x(d+ex))}{\left(a+bx^3\right)^2}+\frac{3b^{2/3}x(d+2ex)}{a\left(a+bx^3\right)}}{54b^{5/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] ((3*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)) - (9*b^(2/3)*(c + x*(d + e*x)))/
(a + b*x^3)^2 - (2*Sqrt[3]*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(
1/3)*x])/a^(5/3) + ((-(b^(1/3)*d) + a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3, x]
```

fricas [C] time = 1.28, size = 2163, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(12*b*e*x^5 + 6*b*d*x^4 - 6*a*e*x^2 - 12*a*d*x - 2*(a*b^3*x^6 + 2*a^2
*b^2*x^3 + a^3*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) +
 (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3
*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*log(1/
4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)
/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a
*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^4*b^3*e - 1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*
b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/
(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^2*b^2*d^2 + 2*a*d*e^2 + (b
*d^3 + a*e^3)*x) - 18*a*c + ((a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^
(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b
^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3))) + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b
^2*x^3 + a^3*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b
^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)
/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2
*a^3*b^3 + 16*d*e)/(a^3*b^3)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^
3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e
*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3)))^2*a^4*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*
e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*s
qrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5)
)^(1/3)))*a^2*b^2*d^2 - 2*a*d*e^2 + 2*(b*d^3 + a*e^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3
)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^4*b^3*e + 2*a^2*b^2*d^2)
*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 -
a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d
^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 + 16*d
*e)/(a^3*b^3))) + ((a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*((1/2)^(1/3)*(I*sqrt
(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*
(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d
^3 - a*e^3)/(a^5*b^5))^(1/3))) - 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a
^3*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d
^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*
((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 +
```

```
16*d*e)/(a^3*b^3)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)
/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(
3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1
/3)))^2*a^4*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*
b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1
)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*
a^2*b^2*d^2 - 2*a*d*e^2 + 2*(b*d^3 + a*e^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)
*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1
/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5
) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^4*b^3*e + 2*a^2*b^2*d^2)*sqrt(-(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3
)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 + 16*d*e)/(a^3*b
^3))))/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)
```

giac [A] time = 0.21, size = 208, normalized size = 0.97

$$-\frac{\sqrt{3}\left(bd-\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{2}{3}}ab}-\frac{\left(bd+\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^{2}\right)^{\frac{2}{3}}ab}-\frac{\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{2}b}+\frac{2\,bx^{5}e+bdx^{4}-ax^{2}e-2\,adx-3\,ac}{18\left(bx^{3}+a\right)^{2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/54*(b*d + (-a*b^2)^(1/3)*e)*log(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) - 1/27*((-a/b)^(1
/3)*e + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/18*(2*b*x^5*
e + b*d*x^4 - a*x^2*e - 2*a*d*x - 3*a*c)/((b*x^3 + a)^2*a*b)
```

maple [A] time = 0.06, size = 255, normalized size = 1.19

$$\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}-\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}-\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{\frac{e\,x^{5}}{9a}+\frac{d\,x^{4}}{18a}-\frac{e\,x^{2}}{18b}-\frac{dx}{9b}-\frac{c}{6b}}{\left(b\,x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (1/9/a*e*x^5+1/18*d/a*x^4-1/18/b*e*x^2-1/9/b*d*x-1/6/b*c)/(b*x^3+a)^2+1/27/
(a/b)^(2/3)/a/b^2*d*ln(x+(a/b)^(1/3))-1/54/(a/b)^(2/3)/a/b^2*d*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))+1/27/(a/b)^(2/3)*3^(1/2)/a/b^2*d*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))-1/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*e+1/54/a/b^2/(
a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/27/a/b^2*3^(1/2)/(a/b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e
```

maxima [A] time = 3.02, size = 203, normalized size = 0.94

$$\frac{2\,bex^{5}+bdx^{4}-aex^{2}-2\,adx-3\,ac}{18\left(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b\right)}+\frac{\sqrt{3}\left(e\left(\frac{a}{b}\right)^{\frac{1}{3}}+d\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*b*e*x^5 + b*d*x^4 - a*e*x^2 - 2*a*d*x - 3*a*c)/(a*b^3*x^6 + 2*a^2*b
^2*x^3 + a^3*b) + 1/27*sqrt(3)*(e*(a/b)^(1/3) + d)*arctan(1/3*sqrt(3)*(2*x
```

```
- (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) + 1/54*(e*(a/b)^(1/3) - d)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/27*(e*(a/b)^
(1/3) - d)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [B] time = 0.23, size = 216, normalized size = 1.00

$$\left(\sum_{k=1}^{3}\ln\left(\frac{d\,e+e^2\,x+\mathrm{root}\left(19683\,a^5\,b^5\,z^3+81\,a^2\,b^2\,d\,e\,z+a\,e^3-b\,d^3,z,k\right)^2\,a^3\,b^3\,729+\mathrm{root}\left(19683\,a^5\,b^5\,z^3+81\,a^2\,b^2\,d\,e\,z+a\,e^3-b\,d^3,z,k\right)\,a\,b^2\,d\,x\,27}{a^2\,b\,81}\right)\,\mathrm{root}\left(19683\,a^5\,b^5\,z^3+81\,a^2\,b^2\,d\,e\,z+a\,e^3-b\,d^3,z,k\right)\right)-\frac{\frac{c}{6\,b}-\frac{d\,x^4}{18\,a}-\frac{e\,x^5}{9\,a}+\frac{e\,x^2}{18\,b}+\frac{d\,x}{9\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log((d*e + e^2*x + 729*root(19683*a^5*b^5*z^3 + 81*a^2*b^2*d*e*z + a
*e^3 - b*d^3, z, k)^2*a^3*b^3 + 27*root(19683*a^5*b^5*z^3 + 81*a^2*b^2*d*e*
z + a*e^3 - b*d^3, z, k)*a*b^2*d*x)/(81*a^2*b))*root(19683*a^5*b^5*z^3 + 81
*a^2*b^2*d*e*z + a*e^3 - b*d^3, z, k), k, 1, 3) - (c/(6*b) - (d*x^4)/(18*a)
 - (e*x^5)/(9*a) + (e*x^2)/(18*b) + (d*x)/(9*b))/(a^2 + b^2*x^6 + 2*a*b*x^3
)
```

sympy [A] time = 6.26, size = 148, normalized size = 0.69

$$\mathrm{RootSum}\left(19683t^3a^5b^5+81ta^2b^2de+ae^3-bd^3,\left(t\mapsto t\log\left(x+\frac{729t^2a^4b^3e+27ta^2b^2d^2+2ade^2}{ae^3+bd^3}\right)\right)\right)+\frac{-3ac-2adx-aex^2+bdx^4+2bex^5}{18a^3b+36a^2b^2x^3+18ab^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**5*b**5 + 81*_t*a**2*b**2*d*e + a*e**3 - b*d**3, Lamb
da(_t, _t*log(x + (729*_t**2*a**4*b**3*e + 27*_t*a**2*b**2*d**2 + 2*a*d*e**
2)/(a*e**3 + b*d**3)))) + (-3*a*c - 2*a*d*x - a*e*x**2 + b*d*x**4 + 2*b*e*x
**5)/(18*a**3*b + 36*a**2*b**2*x**3 + 18*a*b**3*x**6)
```

3.299 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=239

$$\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}}-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\ldots}{\sqrt{3}\sqrt[3]{\ldots}}\right.}{9\sqrt{3}\,a^{7/3}b^{4/3}}$$

Rubi [A] time = 0.20, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1828, 1854, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}}-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{4/3}}-\frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)}-\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(6*a*b*(a + b*x^3)^2) - (3*a*d - x*(a*e + 4*b*
c*x))/(18*a^2*b*(a + b*x^3)) - ((2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(4/3)) - ((2*b^(2/3)
*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(4/3)) + ((2*b^(2/3
)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3
)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^3}\,dx = -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}-\frac{\int \frac{-ae-4bcx-3bdx^2}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}-\frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)}+\frac{\int \frac{2ae+4bcx}{a+bx^3}\,dx}{18a^2b}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}-\frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)}+\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{a}\,bc+4a\sqrt[3]{b}\,e\right)+\sqrt[3]{b}\left(4\sqrt[3]{a}\,bc-2a\sqrt[3]{b}\,e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{4/3}} -$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}-\frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{4/3}}+\frac{\left(2b^{2/3}\right.}{}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}-\frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{4/3}}+\frac{\left(2b^{2/3}\right.}{}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}-\frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)}-\frac{\left(2b^{2/3}c+a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{4/3}}-\frac{\left(2b^{2/3}\right.}{}$$

Mathematica [A] time = 0.40, size = 214, normalized size = 0.90

$$\frac{\left(2a^{2/3}bc-a^{4/3}\sqrt[3]{b}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\sqrt{3}\,a^{2/3}\sqrt[3]{b}\left(a^{2/3}e+2b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(a^{4/3}\sqrt[3]{b}\,e-2a^{2/3}bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\frac{3ab^{2/3}\left(-a^2(3d+2ex)+abx^2\left(7c+ex^2\right)+4b^2cx^5\right)}{\left(a+bx^3\right)^2}}{54a^3b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] ((3*a*b^(2/3)*(4*b^2*c*x^5 - a^2*(3*d + 2*e*x) + a*b*x^2*(7*c + e*x^2)))/(a
 + b*x^3)^2 - 2*Sqrt[3]*a^(2/3)*b^(1/3)*(2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*a^(2/3)*b*c + a^(4/3)*b^(1/3)*e)
*Log[a^(1/3) + b^(1/3)*x] + (2*a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^3*b^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x + e*x^2))/(a + b*x^3)^3, x]
```

fricas [C] time = 1.32, size = 2519, normalized size = 10.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(24*b^2*c*x^5 + 6*a*b*e*x^4 + 42*a*b*c*x^2 - 12*a^2*e*x - 18*a^2*d -
2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^
2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/
2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8
*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)
+ 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^
4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^5*b^3*c - 1/2*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/
(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
 a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 +
8*a*b*c^2*e + (8*b^2*c^3 + a^2*e^3)*x) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^
4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2
*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b
^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3
))) + 3*sqrt(1/3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)
*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(
a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 3
2*c*e)/(a^4*b^2)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*
e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e
*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a
^2*e^3)/(a^7*b^4))^(1/3)))^2*a^5*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) +
4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4)
 - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 - 8*a*b*c^2*e + 2*(8*
b^2*c^3 + a^2*e^3)*x + 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*
c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)
^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b
^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^5*b^3*c + a^4*b*e^2)*sqrt(-(((1/2)^(
1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^
3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^
3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2
```

```
 + 32*c*e)/(a^4*b^2))) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/
(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
 a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3))) - 3*sqrt(1/3
)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)
*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)
+ 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^
4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 32*c*e)/(a^4*b^2)
))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) -
 (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)
/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4
))^(1/3)))^2*a^5*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2
*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*
e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 -
a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 - 8*a*b*c^2*e + 2*(8*b^2*c^3 + a^2*e^
3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(
a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sq
rt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3
)/(a^7*b^4))^(1/3)))*a^5*b^3*c + a^4*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1
/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^
7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 32*c*e)/(a^4*
b^2))))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

giac [A] time = 0.20, size = 215, normalized size = 0.90

$$-\frac{\sqrt{3}\left(ae-2\left(-ab^{2}\right)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{\left(ae+2\left(-ab^{2}\right)^{\frac{1}{3}}c\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{\left(2bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^{3}b}+\frac{4b^{2}cx^{5}+abx^{4}e+7abcx^{2}-2a^{2}xe-3a^{2}d}{18\left(bx^{3}+a\right)^{2}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(a*e - 2*(-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(a*e + 2*(-a*b^2)^(1/3)*c)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(2*b*c
*(-a/b)^(1/3) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18
*(4*b^2*c*x^5 + a*b*x^4*e + 7*a*b*c*x^2 - 2*a^2*x*e - 3*a^2*d)/((b*x^3 + a)
^2*a^2*b)
```

maple [A] time = 0.05, size = 256, normalized size = 1.07

$$\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}-\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{\frac{2bc\,x^{5}}{9a^{2}}+\frac{e\,x^{4}}{18a}+\frac{7c\,x^{2}}{18a}-\frac{ex}{9b}-\frac{d}{6b}}{\left(b\,x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (2/9/a^2*c*b*x^5+1/18/a*e*x^4+7/18/a*c*x^2-1/9/b*e*x-1/6/b*d)/(b*x^3+a)^2+1
/27/(a/b)^(2/3)/a/b^2*e*ln(x+(a/b)^(1/3))-1/54/(a/b)^(2/3)/a/b^2*e*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))+1/27/(a/b)^(2/3)*3^(1/2)/a/b^2*e*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))-2/27/(a/b)^(1/3)/a^2/b*c*ln(x+(a/b)^(1/3))+1/27/(a/
b)^(1/3)/a^2/b*c*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/27*3^(1/2)/(a/b)^(1/3)
/a^2/b*c*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 3.06, size = 223, normalized size = 0.93

$$\frac{4\,b^2cx^5 + abex^4 + 7\,abcx^2 - 2\,a^2ex - 3\,a^2d}{18\left(a^2b^3x^6 + 2\,a^3b^2x^3 + a^4b\right)} + \frac{\sqrt{3}\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}} + ae\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - ae\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - ae\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b^2*c*x^5 + a*b*e*x^4 + 7*a*b*c*x^2 - 2*a^2*e*x - 3*a^2*d)/(a^2*b^3
*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(2*b*c*(a/b)^(1/3) + a*e)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/5
4*(2*b*c*(a/b)^(1/3) - a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2
*(a/b)^(2/3)) - 1/27*(2*b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(a^2*b^
2*(a/b)^(2/3))
```

mupad [B] time = 0.23, size = 232, normalized size = 0.97

$$\frac{\frac{7\,c\,x^2}{18\,a} - \frac{d}{6\,b} + \frac{e\,x^4}{18\,a} - \frac{e\,x}{9\,b} + \frac{2\,b\,c\,x^5}{9\,a^2}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6} + \left(\sum_{k=1}^{3}\ln\left(\frac{2\,a\,c\,e + \mathrm{root}\left(19683\,a^7\,b^4\,z^3 + 162\,a^3\,b^2\,c\,e\,z + 8\,b^2\,c^3 - a^2\,e^3, z, k\right)^2\,a^5\,b^2\,729 + 4\,b\,c^2\,x + \mathrm{root}\left(19683\,a^7\,b^4\,z^3 + 162\,a^3\,b^2\,c\,e\,z + 8\,b^2\,c^3 - a^2\,e^3, z, k\right)\,a^3\,b\,e\,x\,27}{a^4\,81}\right)\,\mathrm{root}\left(19683\,a^7\,b^4\,z^3 + 162\,a^3\,b^2\,c\,e\,z + 8\,b^2\,c^3 - a^2\,e^3, z, k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^3,x)
```

```
[Out] ((7*c*x^2)/(18*a) - d/(6*b) + (e*x^4)/(18*a) - (e*x)/(9*b) + (2*b*c*x^5)/(9
*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((2*a*c*e + 729*root(19683*a
^7*b^4*z^3 + 162*a^3*b^2*c*e*z + 8*b^2*c^3 - a^2*e^3, z, k)^2*a^5*b^2 + 4*b
*c^2*x + 27*root(19683*a^7*b^4*z^3 + 162*a^3*b^2*c*e*z + 8*b^2*c^3 - a^2*e^
3, z, k)*a^3*b*e*x)/(81*a^4))*root(19683*a^7*b^4*z^3 + 162*a^3*b^2*c*e*z +
8*b^2*c^3 - a^2*e^3, z, k), k, 1, 3)
```

sympy [A] time = 3.99, size = 170, normalized size = 0.71

$$\operatorname{RootSum}\left(19683t^3a^7b^4 + 162ta^3b^2ce - a^2e^3 + 8b^2c^3, \left(t \mapsto t\log\left(x + \frac{1458t^2a^5b^3c + 27ta^4be^2 + 8abc^2e}{a^2e^3 + 8b^2c^3}\right)\right)\right) + \frac{-3a^2d - 2a^2ex + 7abcx^2 + abex^4 + 4b^2cx^5}{18a^4b + 36a^3b^2x^3 + 18a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**7*b**4 + 162*_t*a**3*b**2*c*e - a**2*e**3 + 8*b**2*c
**3, Lambda(_t, _t*log(x + (1458*_t**2*a**5*b**3*c + 27*_t*a**4*b*e**2 + 8*
a*b*c**2*e)/(a**2*e**3 + 8*b**2*c**3)))) + (-3*a**2*d - 2*a**2*e*x + 7*a*b*
c*x**2 + a*b*e*x**4 + 4*b**2*c*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a*
*2*b**3*x**6)
```

3.300 $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}\,d+5\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}}{\sqrt{}}\right.}{9\sqrt{3}\,a^{8/3}b^{2/3}}$$

Rubi [A] time = 0.19, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1854, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}\,c-2\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}\,d+5\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}}+\frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(6*a*b*(a +
b*x^3)^2) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)
*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)
*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^3}\,dx = -\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2} - \frac{\int \frac{-5c-4dx}{\left(a+bx^3\right)^2}\,dx}{6a}$$

$$= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} - \frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2} + \frac{\int \frac{10c+4dx}{a+bx^3}\,dx}{18a^2}$$

$$= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} - \frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2} + \frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}c+4\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-10\sqrt[3]{b}c+4\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}\sqrt[3]{b}} + \frac{\left(5c-\frac{2\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}\right.\cdots}{\cdots}$$

$$= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} - \frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}\cdots\right.}{5\cdots}$$

$$= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} - \frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}\cdots\right.}{\cdots}$$

$$= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} - \frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2} - \frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}\cdots\right.}{27\cdots}$$

Mathematica [A] time = 0.37, size = 213, normalized size = 0.95

$$\frac{\sqrt[3]{a}\sqrt[3]{b}\left(2\sqrt[3]{a}d-5\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{3a\left(-3a^2e+abx(8c+7dx)+b^2x^4(5c+4dx)\right)}{\left(a+bx^3\right)^2}-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] ((3*a*(-3*a^2*e + b^2*x^4*(5*c + 4*d*x) + a*b*x*(8*c + 7*d*x)))/(a + b*x^3)
^2 - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)
*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*b^(1/3)*(-5*b^(1/3)*c + 2*a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^3*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2}{\left(a + bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^3)^3, x]
```

fricas [C] time = 1.19, size = 2251, normalized size = 10.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(24*b^2*d*x^5 + 30*b^2*c*x^4 + 42*a*b*d*x^2 + 48*a*b*c*x - 18*a^2*e -
 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125
*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(
1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (
125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)
 - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b
^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/
(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 +
 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 +
40*a*c*d^2 + (125*b*c^3 + 8*a*d^3)*x) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4
*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*
c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*
b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)
)) + 3*sqrt(1/3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*
(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a
^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*
c*d)/(a^5*b)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)
/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-
I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d
^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*
b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^
3 + 8*a*d^3)*x + 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 +
8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3
)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*
(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a
^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*
c*d)/(a^5*b))) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2
))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3
)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) - 3*sqrt(1/3)*(a^2*b
```

```
^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/
2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (12
5*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2
*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^
2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b
^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(
3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^
8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x - 3/2*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/
(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/
2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (12
5*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b))))/(a^2*b^
3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

giac [A] time = 0.21, size = 210, normalized size = 0.93

$$-\frac{\sqrt{3}\left(5\,bc-2\,\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{\left(5\,bc+2\,\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{3}}+\frac{4\,b^{2}dx^{5}+5\,b^{2}cx^{4}+7\,abdx^{2}+8\,abcx-3\,a^{2}e}{18\,\left(bx^{3}+a\right)^{2}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b*c - 2*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*c + 2*(-a*b^2)^(1/3)
*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(2
*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/18*(
4*b^2*d*x^5 + 5*b^2*c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3*a^2*e)/((b*x^3 + a)
^2*a^2*b)
```

maple [A] time = 0.05, size = 308, normalized size = 1.37

$$\frac{e\,x^{3}}{6\left(b\,x^{3}+a\right)^{2}a}+\frac{d\,x^{2}}{6\left(b\,x^{3}+a\right)^{2}a}+\frac{cx}{6\left(b\,x^{3}+a\right)^{2}a}+\frac{2d\,x^{2}}{9\left(b\,x^{3}+a\right)a^{2}}+\frac{5cx}{18\left(b\,x^{3}+a\right)a^{2}}-\frac{e}{6\left(b\,x^{3}+a\right)ab}+\frac{5\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{5c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{5c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{2\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6/(b*x^3+a)^2/a*c*x+5/18*c/a^2*x/(b*x^3+a)+5/27/(a/b)^(2/3)/a^2/b*c*ln(x+
(a/b)^(1/3))-5/54/(a/b)^(2/3)/a^2/b*c*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+5/2
7/(a/b)^(2/3)*3^(1/2)/a^2/b*c*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/6/a
/(b*x^3+a)^2*x^2*d+2/9*d/a^2*x^2/(b*x^3+a)-2/27/a^2/b/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))*d+1/27/a^2/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+2/27/
a^2/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/6*e/a
*x^3/(b*x^3+a)^2-1/6*e/a/b/(b*x^3+a)
```

maxima [A] time = 2.99, size = 219, normalized size = 0.97

$$\frac{4\,b^{2}dx^{5}+5\,b^{2}cx^{4}+7\,abdx^{2}+8\,abcx-3\,a^{2}e}{18\left(a^{2}b^{3}x^{6}+2\,a^{3}b^{2}x^{3}+a^{4}b\right)}+\frac{\sqrt{3}\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b^2*d*x^5 + 5*b^2*c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3*a^2*e)/(a^2*b
^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(2*d*(a/b)^(1/3) + 5*c)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) + 1/54*
(2*d*(a/b)^(1/3) - 5*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)
^(2/3)) - 1/27*(2*d*(a/b)^(1/3) - 5*c)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2
/3))
```

mupad [B] time = 0.26, size = 212, normalized size = 0.94

$$\frac{\frac{7\,d\,x^2}{18\,a}-\frac{e}{6\,b}+\frac{4\,c\,x}{9\,a}+\frac{5\,b\,c\,x^4}{18\,a^2}+\frac{2\,b\,d\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\left(\sum_{k=1}^{3}\ln\left(\frac{b\,\left(10\,c\,d+4\,d^2\,x+\mathrm{root}\left(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k\right)^2\,a^5\,b\,729+\mathrm{root}\left(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k\right)\,a^2\,b\,c\,x\,135\right)}{a^4\,81}\right)\,\mathrm{root}\left(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^3,x)
```

```
[Out] ((7*d*x^2)/(18*a) - e/(6*b) + (4*c*x)/(9*a) + (5*b*c*x^4)/(18*a^2) + (2*b*d
*x^5)/(9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((b*(10*c*d + 4*d^2*
x + 729*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z,
k)^2*a^5*b + 135*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a
*d^3, z, k)*a^2*b*c*x))/(81*a^4))*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z
- 125*b*c^3 + 8*a*d^3, z, k), k, 1, 3)
```

sympy [A] time = 2.28, size = 163, normalized size = 0.72

$$\operatorname{RootSum}\left(19683t^3a^8b^2+810ta^3bcd+8ad^3-125bc^3,\left(t\mapsto t\log\left(x+\frac{1458t^2a^6bd+675ta^3bc^2+40acd^2}{8ad^3+125bc^3}\right)\right)\right)+\frac{-3a^2e+8abcx+7abdx^2+5b^2cx^4+4b^2dx^5}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*d*
*2)/(8*a*d**3 + 125*b*c**3)))) + (-3*a**2*e + 8*a*b*c*x + 7*a*b*d*x**2 + 5*
b**2*c*x**4 + 4*b**2*d*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a**2*b**3*
x**6)
```

3.301 $\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=257

$$-\frac{\left(5\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}\,+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}\,e+5\sqrt[3]{b}\,d\right)\tan^{-1}\left(\right.}{9\sqrt{3}\,a^{8/3}b^{2/3}}$$

Rubi [A] time = 0.41, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(5\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}\,+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}\,e+5\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}}+\frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)}+\frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{c\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*d + 4*a*e*x - 9
*b*c*x^2))/(18*a^3*(a + b*x^3)) - ((5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + (c*Log
[x])/a^3 + ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/
3)*b^(2/3)) - ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx = \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6bc-5bdx-4bex^2+\frac{3b^2cx^3}{a}}{x\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^2c+10b^2dx+4b^2ex^2}{x\left(a+bx^3\right)}\,dx}{18a^2b^2}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^2c}{ax} + \frac{2b^2\left(5ad+2aex-9bcx^2\right)}{a\left(a+bx^3\right)}\right)dx}{18a^2b^2}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\int \frac{5ad+2aex-9bcx^2}{a+bx^3}\,dx}{9a^3}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\int \frac{5ad+2aex}{a+bx^3}\,dx}{9a^3} - \frac{(bc)\int \frac{x^2}{a+bx^3}}{a^3}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} - \frac{c\log\left(a+bx^3\right)}{3a^3} + \frac{\int \frac{\sqrt[3]{a}\left(10a\sqrt[3]{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\left(5\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\cdots\right.}{27a^{8/3}b^{2/3}}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\left(5\sqrt[3]{b}\,d-2\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\cdots\right.}{27a^{8/3}b^{2/3}}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} - \frac{\left(5\sqrt[3]{b}\,d+2\sqrt[3]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{2/3}} + \frac{c\cdots}{\cdots}$$

Mathematica [A] time = 0.21, size = 229, normalized size = 0.89

$$\frac{\frac{\left(2a^{2/3}e-5\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}d-2a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{9a^2(c+x(d+ex))}{\left(a+bx^3\right)^2} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{3a(6c+x(5d+4ex))}{a+bx^3} - 18c\log\left(a+bx^3\right) + 54c\log(x)}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((9*a^2*(c + x*(d + e*x)))/(a + b*x^3)^2 + (3*a*(6*c + x*(5*d + 4*e*x)))/(a
 + b*x^3) - (2*Sqrt[3]*a^(1/3)*(5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*c*Log[x] + (2*(5*a^(1/3)*b^(1/3)*
d - 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*d
 + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 1
8*c*Log[a + b*x^3])/(54*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x*(a + b*x^3)^3), x]
```

fricas [C] time = 1.44, size = 5229, normalized size = 20.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2916*(648*a*b*e*x^5 + 810*a*b*d*x^4 + 972*a*b*c*x^3 + 1134*a^2*e*x^2 + 12
96*a^2*d*x + 1458*a^2*c - 2*(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*((-I*sqrt(3)
+ 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(
81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) -
1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1
/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(
a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8
*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*log(1/1
458*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c
^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*
e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a
*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^
2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366
*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 4
86*c/a^3)^2*a^6*b*e + 225*b*c*d^2 + 162*b*c^2*e + 40*a*d*e^2 - 1/54*(25*a^3
*b*d^2 + 36*a^3*b*c*e)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e
)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/3936
6*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(2
5*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^
9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/
(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(
a^9*b^2))^(1/3) + 486*c/a^3) + (125*b*d^3 + 8*a*e^3)*x) - (1458*b^2*c*x^6 +
 2916*a*b*c*x^3 + 1458*a^2*c - (a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*((-I*sqrt(
3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/145
8*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2)
 - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))
^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*
c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3
+ 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3) - 3*
sqrt(1/3)*(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^
2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 +
10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(72
9*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(
I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/
39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 -
5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b - 972*((-I
*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 +
 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^
8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9
*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a
*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^
2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3
)*a^3*b*c + 236196*b*c^2 + 116640*a*d*e)/(a^6*b)))*log(-1/1458*((-I*sqrt(3)
 + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*
(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) -
 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(
1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/
(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 +
```

```
8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*
b*e - 225*b*c*d^2 - 162*b*c^2*e - 40*a*d*e^2 + 1/54*(25*a^3*b*d^2 + 36*a^3*
b*c*e)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/2
7*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8
*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e
)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b
*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39
366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3)
 + 486*c/a^3) + 2*(125*b*d^3 + 8*a*e^3)*x + 1/486*sqrt(1/3)*(((-I*sqrt(3) +
1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81
*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/
39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3
) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^
9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a
^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^6*b*e +
 675*a^3*b*d^2 - 486*a^3*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*
b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/
(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 +
8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) +
1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b
*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 -
54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b - 972*((-I*sqrt(3) + 1
)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*
b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/3
9366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3)
 + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9
*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^
2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^3*b*c +
236196*b*c^2 + 116640*a*d*e)/(a^6*b))) - (1458*b^2*c*x^6 + 2916*a*b*c*x^3 +
 1458*a^2*c - (a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*((-I*sqrt(3) + 1)*(81*c^2/a
^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*
a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b
^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*s
qrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/393
66*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(
25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3) + 3*sqrt(1/3)*(a^3*b^
2*x^6 + 2*a^4*b*x^3 + a^5)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2
 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*
b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2
*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-
1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3
+ 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*
d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b - 972*((-I*sqrt(3) + 1)*(81
*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2
 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*
(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 72
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) +
 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3
 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^3*b*c + 23619
6*b*c^2 + 116640*a*d*e)/(a^6*b)))*log(-1/1458*((-I*sqrt(3) + 1)*(81*c^2/a^6
 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*
d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2
*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqr
t(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366
*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25
*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b*e - 225*b*c*d^2
 - 162*b*c^2*e - 40*a*d*e^2 + 1/54*(25*a^3*b*d^2 + 36*a^3*b*c*e)*((-I*sqrt(
3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/145
8*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2)
```

```
 - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))
^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*
c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3
+ 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3) + 2*
(125*b*d^3 + 8*a*e^3)*x - 1/486*sqrt(1/3)*(((-I*sqrt(3) + 1)*(81*c^2/a^6 -
(81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e
)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^
3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3
) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(1
25*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^
3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^6*b*e + 675*a^3*b*d^2 -
486*a^3*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)
/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366
*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25
*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9
 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(
a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a
^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b - 972*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (
81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)
*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3
 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3)
 + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(12
5*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3
 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^3*b*c + 236196*b*c^2 + 11
6640*a*d*e)/(a^6*b))) + 2916*(b^2*c*x^6 + 2*a*b*c*x^3 + a^2*c)*log(x))/(a^3
*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

giac [A] time = 0.24, size = 253, normalized size = 0.98

$$-\frac{\sqrt{3}\left(5\,bd-2\,\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{\left(5\,bd+2\,\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{c\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{3}}+\frac{c\log\left(\left|x\right|\right)}{a^{3}}+\frac{4\,abx^{5}e+5\,abdx^{4}+6\,abcx^{3}+7\,a^{2}x^{2}e+8\,a^{2}dx+9\,a^{2}c}{18\,\left(bx^{3}+a\right)^{2}a^{3}}-\frac{\left(2\,a^{4}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5\,a^{4}bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{7}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b*d - 2*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*d + 2*(-a*b^2)^(1/3)
*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/3*c*l
og(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 + 1/18*(4*a*b*x^5*e + 5*a*b*d*x^
4 + 6*a*b*c*x^3 + 7*a^2*x^2*e + 8*a^2*d*x + 9*a^2*c)/((b*x^3 + a)^2*a^3) -
1/27*(2*a^4*b*(-a/b)^(1/3)*e + 5*a^4*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(
1/3)))/(a^7*b)
```

maple [A] time = 0.07, size = 331, normalized size = 1.29

$$\frac{2be\,x^{5}}{9\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{5bd\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{bc\,x^{3}}{3\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{7e\,x^{2}}{18\left(b\,x^{3}+a\right)^{2}a}+\frac{4dx}{9\left(b\,x^{3}+a\right)^{2}a}+\frac{c}{2\left(b\,x^{3}+a\right)^{2}a}+\frac{5\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{5d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{5d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{2\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{c\ln(x)}{a^{3}}-\frac{c\ln\left(b\,x^{3}+a\right)}{3a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^3,x)
```

```
[Out] 2/9/a^2/(b*x^3+a)^2*b*e*x^5+5/18/(b*x^3+a)^2/a^2*b*d*x^4+1/3/a^2/(b*x^3+a)^
2*x^3*c*b+7/18/a/(b*x^3+a)^2*e*x^2+4/9/(b*x^3+a)^2/a*d*x+1/2/(b*x^3+a)^2/a*
c+5/27/(a/b)^(2/3)/a^2/b*d*ln(x+(a/b)^(1/3))-5/54/(a/b)^(2/3)/a^2/b*d*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+5/27/(a/b)^(2/3)*3^(1/2)/a^2/b*d*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))-2/27/a^2*e/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/27/
a^2*e/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/27/a^2*e*3^(1/2)/b/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/a^3*c*ln(b*x^3+a)+1
/a^3*c*ln(x)
```

maxima [A] time = 3.00, size = 246, normalized size = 0.96

$$\frac{4\,bex^5+5\,bdx^4+6\,bcx^3+7\,aex^2+8\,adx+9\,ac}{18\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}+\frac{c\log(x)}{a^3}+\frac{\sqrt{3}\left(2\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}-\frac{\left(18\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*e*x^5 + 5*b*d*x^4 + 6*b*c*x^3 + 7*a*e*x^2 + 8*a*d*x + 9*a*c)/(a^2
*b^2*x^6 + 2*a^3*b*x^3 + a^4) + c*log(x)/a^3 + 1/27*sqrt(3)*(2*a*e*(a/b)^(2
/3) + 5*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/a^4 - 1/54*(18*b*c*(a/b)^(2/3) - 2*a*e*(a/b)^(1/3) + 5*a*d)*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 1/27*(9*b*c*(a/b)^(2/3) + 2
*a*e*(a/b)^(1/3) - 5*a*d)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

mupad [B] time = 5.44, size = 540, normalized size = 2.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^3),x)
```

```
[Out] (c/(2*a) + (7*e*x^2)/(18*a) + (4*d*x)/(9*a) + (b*c*x^3)/(3*a^2) + (5*b*d*x^
4)/(18*a^2) + (2*b*e*x^5)/(9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log
((25*b^2*c*d^2 - 18*b^2*c^2*e)/(81*a^6) - root(19683*a^9*b^2*z^3 + 19683*a^
6*b^2*c*z^2 + 810*a^4*b*d*e*z + 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*
b*d^3 + 8*a^2*e^3 + 729*b^2*c^3, z, k)*((25*a^3*b^2*d^2 + 36*a^3*b^2*c*e)/(
81*a^6) + root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b*d*e*z +
6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 729*b^2*c^3,
 z, k)*(36*root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b*d*e*z +
 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 729*b^2*c^3
, z, k)*a^2*b^3*x - (2*b^2*e)/3 + (24*b^3*c*x)/a) + (x*(2916*a^2*b^3*c^2 +
900*a^3*b^2*d*e))/(729*a^6)) - (x*(8*a*b*e^3 - 125*b^2*d^3 + 180*b^2*c*d*e)
)/(729*a^6))*root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b*d*e*z
 + 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 729*b^2*c
^3, z, k), k, 1, 3) + (c*log(x))/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.302 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx$

Optimal. Leaf size=267

$$-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}}+\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{10/3}\sqrt[3]{b}}+\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\tan^{-1}\left(\right.}{9\sqrt{3}\,a^{10/3}\sqrt[3]{b}}$$

Rubi [A] time = 0.46, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}}+\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{10/3}\sqrt[3]{b}}+\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{10/3}\sqrt[3]{b}}+\frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)}+\frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2}-\frac{d\log\left(a+bx^3\right)}{3a^3}-\frac{c}{a^3x}+\frac{d\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) + (x*(a*e - b*c*x - b*d*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*
e - 10*b*c*x - 9*b*d*x^2))/(18*a^3*(a + b*x^3)) + ((14*b^(2/3)*c - 5*a^(2/3
)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)
*b^(1/3)) + (d*Log[x])/a^3 + ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(1/3) + b^
(1/3)*x])/(27*a^(10/3)*b^(1/3)) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(1/3)) - (d*Log[a + b*x
^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^3}\,dx = \frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6bc-6bdx-5bex^2+\frac{4b^2cx^3}{a}+\frac{3b^2dx^4}{a}}{x^2\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^3c+18b^3dx+10b^3ex^2-\frac{10b^4cx^3}{a}}{x^2\left(a+bx^3\right)}\,dx}{18a^2b^3}$$

$$= \frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^3c}{ax^2}+\frac{18b^3d}{ax}+\frac{2b^3\left(5ae-14bcx-9bdx^2\right)}{a\left(a+bx^3\right)}\right) dx}{18a^2b^3}$$

$$= -\frac{c}{a^3x} + \frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{d\log(x)}{a^3} + \frac{\int \frac{5ae-14bcx-9bdx^2}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{c}{a^3x} + \frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{d\log(x)}{a^3} + \frac{\int \frac{5ae-14bcx}{a+bx^3}\,dx}{9a^3} - \frac{(bd)}{}$$

$$= -\frac{c}{a^3x} + \frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{d\log(x)}{a^3} - \frac{d\log\left(a+bx^3\right)}{3a^3} + \frac{\int}{}$$

$$= -\frac{c}{a^3x} + \frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{d\log(x)}{a^3} + \frac{\left(14b^{2/3}c+5a^{2/3}e\right)\log}{27a^{10/3}\sqrt[3]{}}$$

$$= -\frac{c}{a^3x} + \frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{d\log(x)}{a^3} + \frac{\left(14b^{2/3}c+5a^{2/3}e\right)\log}{27a^{10/3}\sqrt[3]{}}$$

$$= -\frac{c}{a^3x} + \frac{x\left(ae-bcx-bdx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ae-10bcx-9bdx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{10/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.33, size = 248, normalized size = 0.93

$$\frac{-\frac{\left(14a^{2/3}b^{2/3}c+5a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+\frac{2\left(14a^{2/3}b^{2/3}c+5a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\sqrt{3}a^{2/3}\left(5a^{2/3}e-14b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{9a^2\left(a(d+ex)-bcx^2\right)}{\left(a+bx^3\right)^2}+\frac{3a\left(6ad+5aex-10bcx^2\right)}{a+bx^3}-18ad\log\left(a+bx^3\right)-\frac{54ac}{x}+54ad\log(x)}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a*c)/x + (3*a*(6*a*d + 5*a*e*x - 10*b*c*x^2))/(a + b*x^3) + (9*a^2*(-
(b*c*x^2) + a*(d + e*x)))/(a + b*x^3)^2 - (2*Sqrt[3]*a^(2/3)*(-14*b^(2/3)*c
 + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + 54*a
*d*Log[x] + (2*(14*a^(2/3)*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x
])/b^(1/3) - ((14*a^(2/3)*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/b^(1/3) - 18*a*d*Log[a + b*x^3])/(54*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3), x]
```

fricas [C] time = 1.53, size = 5112, normalized size = 19.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2916*(4536*b^2*c*x^6 - 810*a*b*e*x^5 - 972*a*b*d*x^4 + 7938*a*b*c*x^3 -
1296*a^2*e*x^2 - 1458*a^2*d*x + 2916*a^2*c + 2*(a^3*b^2*x^7 + 2*a^4*b*x^4 +
 a^5*x)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a
^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 -
 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3
)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 7
0*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)
*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486
*d/a^3)*log(-7/1458*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/
(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 1
25*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 -
 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458
*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3
 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))
^(1/3) + 486*d/a^3)^2*a^7*b*c - 1134*a*b*c*d^2 + 1960*a*b*c^2*e + 225*a^2*d
*e^2 + 1/54*(252*a^4*b*c*d - 25*a^5*e^2)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (8
1*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39
366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/
39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-
1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125
*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 1
25*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3) - (2744*b^2*c^3 - 125*a^2*e^3)*x)
 + (1458*b^2*d*x^7 + 2916*a*b*d*x^4 + 1458*a^2*d*x - (a^3*b^2*x^7 + 2*a^4*b*
x^4 + a^5*x)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*
d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*
e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^
2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^
2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c
*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3)
 + 486*d/a^3) + 3*sqrt(1/3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*sqrt(-(((-I*
sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(
81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 -
 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(
1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9
 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b
) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^6
 - 972*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^
9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 -
27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)
/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70
*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*
a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*
d/a^3)*a^3*d + 236196*d^2 - 816480*c*e)/a^6))*log(7/1458*((-I*sqrt(3) + 1)*
(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c
*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*
b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I
*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(27
44*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(
```

2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^7*b*c + 1134*a
*b*c*d^2 - 1960*a*b*c^2*e - 225*a^2*d*e^2 - 1/54*(252*a^4*b*c*d - 25*a^5*e^
2)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 +
1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(
27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e
)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)
/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^
3) - 2*(2744*b^2*c^3 - 125*a^2*e^3)*x + 1/486*sqrt(1/3)*(7*((-I*sqrt(3) + 1
)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70
*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*
a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*
(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(
2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366
*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)*a^7*b*c - 3402*a
^4*b*c*d - 675*a^5*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70
*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*
b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(274
4*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a
^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 -
27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3
)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^6 - 972*((-I*sqrt(3) + 1)*(81*d^2/a^6 -
(81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/
39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) -
1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*
(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 1
25*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 -
125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)*a^3*d + 236196*d^2 - 816480*c*e)
/a^6)) + (1458*b^2*d*x^7 + 2916*a*b*d*x^4 + 1458*a^2*d*x - (a^3*b^2*x^7 + 2
*a^4*b*x^4 + a^5*x)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/
(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 1
25*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 -
125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458
*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3
- 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))
^(1/3) + 486*d/a^3) - 3*sqrt(1/3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*sqrt(
-(((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1
/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(2
7*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^1
0*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)
d/a^9 + 1/39366(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/
(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3
)^2*a^6 - 972*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27
d^3/a^9 + 1/1458(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2
e^3 - 27(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a
^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d
^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*
c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3)
+ 486*d/a^3)*a^3*d + 236196*d^2 - 816480*c*e)/a^6))*log(7/1458*((-I*sqrt(3
) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2
- 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*
d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) +
729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39
366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/
39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^7*b*c +
1134*a*b*c*d^2 - 1960*a*b*c^2*e - 225*a^2*d*e^2 - 1/54*(252*a^4*b*c*d - 25
*a^5*e^2)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3
/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3
- 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e

```
^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 -
 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*
e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 4
86*d/a^3) - 2*(2744*b^2*c^3 - 125*a^2*e^3)*x - 1/486*sqrt(1/3)*(7*((-I*sqrt
(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d
^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*
c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3)
 + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/
39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) -
1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)*a^7*b*c -
 3402*a^4*b*c*d - 675*a^5*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d
^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366
*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/393
66*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/2
7*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^
2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*
a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^6 - 972*((-I*sqrt(3) + 1)*(81*d^2
/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a
^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^1
0*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3
) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*
c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^
2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)*a^3*d + 236196*d^2 - 8164
80*c*e)/a^6)) - 2916*(b^2*d*x^7 + 2*a*b*d*x^4 + a^2*d*x)*log(x))/(a^3*b^2*x
^7 + 2*a^4*b*x^4 + a^5*x)
```

giac [A] time = 0.21, size = 273, normalized size = 1.02

$$-\frac{d\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{d\log\left(\left|x\right|\right)}{a^3}+\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}ae+14\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}+\frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}ae-14\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b}-\frac{28\,b^2cx^6-5\,abx^5e-6\,abdx^4+49\,abcx^3-8\,a^2x^2e-9\,a^2dx+18\,a^2c}{18\left(bx^3+a\right)^2a^3x}+\frac{\left(14\,a^3b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^4be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^3 + d*log(abs(x))/a^3 + 1/27*sqrt(3)*(5*(-a*b^
2)^(1/3)*a*e + 14*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^4*b) + 1/54*(5*(-a*b^2)^(1/3)*a*e - 14*(-a*b^2)^(2/3)*c)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/18*(28*b^2*c*x^6 - 5*a
*b*x^5*e - 6*a*b*d*x^4 + 49*a*b*c*x^3 - 8*a^2*x^2*e - 9*a^2*d*x + 18*a^2*c)
/((b*x^3 + a)^2*a^3*x) + 1/27*(14*a^3*b^2*c*(-a/b)^(1/3) - 5*a^4*b*e)*(-a/b
)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

maple [A] time = 0.06, size = 334, normalized size = 1.25

$$-\frac{5b^2c\,x^5}{9\left(b\,x^3+a\right)^2a^3}+\frac{5be\,x^4}{18\left(b\,x^3+a\right)^2a^2}+\frac{bd\,x^3}{3\left(b\,x^3+a\right)^2a^2}-\frac{13bc\,x^2}{18\left(b\,x^3+a\right)^2a^2}+\frac{4ex}{9\left(b\,x^3+a\right)^2a}+\frac{d}{2\left(b\,x^3+a\right)^2a}+\frac{5\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{5e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}-\frac{5e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}-\frac{14\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{14c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{7c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{d\ln(x)}{a^3}-\frac{d\ln\left(b\,x^3+a\right)}{3a^3}-\frac{c}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x)
```

```
[Out] -5/9/(b*x^3+a)^2/a^3*b^2*c*x^5+5/18/(b*x^3+a)^2/a^2*b*e*x^4+1/3/a^2/(b*x^3+
a)^2*b*d*x^3-13/18/(b*x^3+a)^2/a^2*b*c*x^2+4/9/(b*x^3+a)^2/a*e*x+1/2/(b*x^3
+a)^2/a*d+5/27/(a/b)^(2/3)/a^2/b*e*ln(x+(a/b)^(1/3))-5/54/(a/b)^(2/3)/a^2/b
*e*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+5/27/(a/b)^(2/3)*3^(1/2)/a^2/b*e*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+14/27/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*
c-7/27/a^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c-14/27/a^3*3^(1/2
)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/3/a^3*d*ln(b*x^3+
a)-1/a^3*c/x+1/a^3*d*ln(x)
```

maxima [A] time = 3.09, size = 266, normalized size = 1.00

$$-\frac{28\,b^2cx^6 - 5\,abex^5 - 6\,abdx^4 + 49\,abcx^3 - 8\,a^2ex^2 - 9\,a^2dx + 18\,a^2c}{18\left(a^3b^2x^7 + 2\,a^4bx^4 + a^5x\right)} + \frac{d\log(x)}{a^3} - \frac{\sqrt{3}\left(14\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}} - 5\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4} - \frac{\left(18\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}} + 14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}} + 5\,ae\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(9\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}} - 14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - 5\,ae\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*c*x^6 - 5*a*b*e*x^5 - 6*a*b*d*x^4 + 49*a*b*c*x^3 - 8*a^2*e*x^
2 - 9*a^2*d*x + 18*a^2*c)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x) + d*log(x)/a^
3 - 1/27*sqrt(3)*(14*b*c*(a/b)^(2/3) - 5*a*e*(a/b)^(1/3))*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 - 1/54*(18*b*d*(a/b)^(2/3) + 14*b*c*
(a/b)^(1/3) + 5*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2
/3)) - 1/27*(9*b*d*(a/b)^(2/3) - 14*b*c*(a/b)^(1/3) - 5*a*e)*log(x + (a/b)^
(1/3))/(a^3*b*(a/b)^(2/3))
```

mupad [B] time = 5.46, size = 793, normalized size = 2.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] ((4*e*x^2)/(9*a) - c/a + (d*x)/(2*a) - (14*b^2*c*x^6)/(9*a^3) - (49*b*c*x^3
)/(18*a^2) + (b*d*x^4)/(3*a^2) + (5*b*e*x^5)/(18*a^2))/(a^2*x + b^2*x^7 + 2
*a*b*x^4) + symsum(log((b^2*(225*a^2*d*e^2 - 225*root(19683*a^10*b*z^3 + 19
683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 72
9*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^5*e^2 + 2744*b^2*c^3*x + 12
5*a^2*e^3*x + 1134*a*b*c*d^2 - 3402*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z
^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 1
25*a^2*e^3 - 2744*b^2*c^3, z, k)^2*a^7*b*c - 26244*root(19683*a^10*b*z^3 +
19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e +
729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)^3*a^10*b*x - 2916*root(1968
3*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 18
90*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^4*b*d^2*x
- 17496*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561
*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z
, k)^2*a^7*b*d*x + 2268*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^
4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 -
 2744*b^2*c^3, z, k)*a^4*b*c*d + 6300*root(19683*a^10*b*z^3 + 19683*a^7*b*d
*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 -
 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^4*b*c*e*x + 1260*a*b*c*d*e*x))/(729*a^
8))*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4
*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)
, k, 1, 3) + (d*log(x))/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.303 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3} dx$

Optimal. Leaf size=276

$$\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}}+\frac{2\sqrt[3]{b}\left(7\sqrt[3]{a}\,d+\right.}{}$$

Rubi [A] time = 0.50, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)}-\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}}+\frac{2\sqrt[3]{b}\left(7\sqrt[3]{a}\,d+10\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}}-\frac{e\log\left(a+bx^3\right)}{3a^3}-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c + b*d*x + b*e*x^2))/(6*a^2*(a + b*x^3)
^2) - (x*(11*b*c + 10*b*d*x + 9*b*e*x^2))/(18*a^3*(a + b*x^3)) + (2*b^(1/3)
*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(9*Sqrt[3]*a^(11/3)) + (e*Log[x])/a^3 - (2*b^(1/3)*(10*b^(1/3)*c - 7*
a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*c
 - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(11/3
)) - (e*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx &= -\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6bc-6bdx-6bex^2+\frac{5b^2cx^3}{a}+\frac{4b^2dx^4}{a}+\frac{3b^2ex^5}{a}}{x^3\left(a+bx^3\right)^2}\,dx}{6ab} \\
&= -\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^3c+18b^3dx+18b^3ex^2-\frac{22b^4cx^3}{a}-\frac{10b^4dx^4}{a}}{x^3\left(a+bx^3\right)}}{18a^2b^3} \\
&= -\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^3c}{ax^3}+\frac{18b^3d}{ax^2}+\frac{18b^3e}{ax}-\frac{2b^4\left(20c+14\right.}{a\left(a+b\right.}\right.}{18a^2b^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{b\int \frac{20c+1}{a}}{9} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{b\int \frac{20c+1}{a+b}}{9a} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{e\log\left(a\right.}{3a} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - 2\sqrt[3]{b}\left(10\right. \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - 2\sqrt[3]{b}\left(10\right. \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c+7\sqrt[3]{\right.}{9\sqrt{3}}
\end{aligned}
$$

Mathematica [A] time = 0.33, size = 253, normalized size = 0.92

$$
\frac{2\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+4\sqrt[3]{b}\left(7a^{2/3}d-10\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{9a^2(ae-bx(c+dx))}{\left(a+bx^3\right)^2}+\frac{3a(6ae-bx(11c+10dx))}{a+bx^3}+4\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(7\sqrt[3]{a}d+10\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-18ae\log\left(a+bx^3\right)-\frac{27ac}{x^2}-\frac{54ad}{x}+54ae\log(x)}{54a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a*c)/x^2 - (54*a*d)/x + (9*a^2*(a*e - b*x*(c + d*x)))/(a + b*x^3)^2 +
 (3*a*(6*a*e - b*x*(11*c + 10*d*x)))/(a + b*x^3) + 4*Sqrt[3]*a^(1/3)*b^(1/3
)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
+ 54*a*e*Log[x] + 4*b^(1/3)*(-10*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/
3) + b^(1/3)*x] + 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 18*a*e*Log[a + b*x^3])/(54*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3), x]
```

fricas [C] time = 1.43, size = 4911, normalized size = 17.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2916*(4536*b^2*d*x^7 + 3240*b^2*c*x^6 - 972*a*b*e*x^5 + 7938*a*b*d*x^4 +
 5184*a*b*c*x^3 - 1458*a^2*e*x^2 + 2916*a^2*d*x + 1458*a^2*c + 2*(a^3*b^2*x
^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81
*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/1968
3*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 5
6*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^
9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)
*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b
)/a^11)^(1/3) + 486*e/a^3)*log(7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*
b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^1
0 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a
^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1
/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 +
343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*
c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^8*d + 3920*a*b*c*d^2 - 1800*a*b*c^
2*e + 567*a^2*d*e^2 + 1/27*(100*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(
81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d
 + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(80
00*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(
I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19
683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 -
 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3) + 4*(1000*b^2*c^3 +
343*a*b*d^3)*x) + (1458*b^2*e*x^8 + 2916*a*b*e*x^5 + 1458*a^2*e*x^2 - (a^3*
b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d
 + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4
/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^
3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e
^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a
*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e
)*a*b)/a^11)^(1/3) + 486*e/a^3) - 3*sqrt(1/3)*(a^3*b^2*x^8 + 2*a^4*b*x^5 +
a^5*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)
/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^
3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 -
135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(
280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/
39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3
) + 486*e/a^3)^2*a^7 - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*
a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683
*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56
*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9
 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*
b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)
/a^11)^(1/3) + 486*e/a^3)*a^4*e + 3265920*b*c*d + 236196*a*e^2)/a^7))*log(-
7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e
^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a
*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e
)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d
 + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(80
```

```
00*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e
/a^3)^2*a^8*d - 3920*a*b*c*d^2 + 1800*a*b*c^2*e - 567*a^2*d*e^2 - 1/27*(100
*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*
e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(
1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(
49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 +
 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/
a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a
^11)^(1/3) + 486*e/a^3) + 8*(1000*b^2*c^3 + 343*a*b*d^3)*x + 1/972*sqrt(1/3
)*(7*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3
/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d
^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*
a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d +
 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000
*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a
^3)*a^8*d - 10800*a^4*b*c^2 - 3402*a^5*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^
2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81
*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^
2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqr
t(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(
1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(
49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^7 - 972*((-I*sqrt(3)
+ 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280
*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/393
66*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) +
 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10
+ 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2
*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)*a^4*e + 326592
0*b*c*d + 236196*a*e^2)/a^7)) + (1458*b^2*e*x^8 + 2916*a*b*e*x^5 + 1458*a^2
*e*x^2 - (a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^
6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e
^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^
3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3)
 + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000
*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d
^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3) + 3*sqrt(1/3)*(a^3*b^2*x^8 +
2*a^4*b*x^5 + a^5*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d +
81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19
683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 -
 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/
a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^
3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a
*b)/a^11)^(1/3) + 486*e/a^3)^2*a^7 - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (2
80*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/
a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 72
9*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*
(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3
 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 1
35*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)*a^4*e + 3265920*b*c*d + 236196*a*e^
2)/a^7))*log(-7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)
/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000
*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d
^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1
458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11
 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)
^(1/3) + 486*e/a^3)^2*a^8*d - 3920*a*b*c*d^2 + 1800*a*b*c^2*e - 567*a^2*d*e
^2 - 1/27*(100*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280
*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^
10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*
```

```
a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-
1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 +
 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135
*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3) + 8*(1000*b^2*c^3 + 343*a*b*d^3)*x -
1/972*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a
^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b
*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3
 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/145
8*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 -
 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(
1/3) + 486*e/a^3)*a^8*d - 10800*a^4*b*c^2 - 3402*a^5*d*e)*sqrt(-(((-I*sqrt(
3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(
280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/
39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3
) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^
10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*
a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^7 - 972
*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9
 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*
b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)
/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*
a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2
*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)*
a^4*e + 3265920*b*c*d + 236196*a*e^2)/a^7)) - 2916*(b^2*e*x^8 + 2*a*b*e*x^5
 + a^2*e*x^2)*log(x))/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)
```

giac [A] time = 0.19, size = 282, normalized size = 1.02

$$-\frac{e\log\left(\left|bx^3+a\right|\right)}{3a^3}+\frac{e\log\left(|x|\right)}{a^3}-\frac{2\sqrt{3}\left(10\left(-ab^2\right)^{\frac{1}{3}}bc-7\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^4b}-\frac{\left(10\left(-ab^2\right)^{\frac{1}{3}}bc+7\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^4b}-\frac{28b^2dx^7+20b^2cx^6-6abx^5e+49abdx^4+32abcx^3-9a^2x^2e+18a^2dx+9a^2c}{18\left(bx^4+ax\right)^2a^3}+\frac{2\left(7a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10a^3b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 - 2/27*sqrt(3)*(10*(-a*b
^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^4*b) - 1/27*(10*(-a*b^2)^(1/3)*b*c + 7*(-a*b^2)^(2/3)*d)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/18*(28*b^2*d*x^7 + 20*
b^2*c*x^6 - 6*a*b*x^5*e + 49*a*b*d*x^4 + 32*a*b*c*x^3 - 9*a^2*x^2*e + 18*a^
2*d*x + 9*a^2*c)/((b*x^4 + a*x)^2*a^3) + 2/27*(7*a^3*b^2*d*(-a/b)^(1/3) + 1
0*a^3*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

maple [A] time = 0.06, size = 337, normalized size = 1.22

$$-\frac{5b^2dx^5}{9\left(bx^3+a\right)^2a^3}-\frac{11b^2cx^4}{18\left(bx^3+a\right)^2a^3}+\frac{bex^3}{3\left(bx^3+a\right)^2a^2}-\frac{13bdx^2}{18\left(bx^3+a\right)^2a^2}-\frac{7bcx}{9\left(bx^3+a\right)^2a^2}+\frac{e}{2\left(bx^3+a\right)^2a}-\frac{20\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{20c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}+\frac{10c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{14\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{14d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{7d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{e\ln(x)}{a^3}-\frac{e\ln\left(bx^3+a\right)}{3a^3}-\frac{d}{a^3x}-\frac{c}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x)
```

```
[Out] -5/9/(b*x^3+a)^2/a^3*b^2*d*x^5-11/18/(b*x^3+a)^2/a^3*b^2*c*x^4+1/3*b/a^2/(b
*x^3+a)^2*e*x^3-13/18/(b*x^3+a)^2/a^2*b*d*x^2-7/9/(b*x^3+a)^2/a^2*b*c*x+1/2
/a/(b*x^3+a)^2*e-20/27/(a/b)^(2/3)/a^3*c*ln(x+(a/b)^(1/3))+10/27/(a/b)^(2/3
)/a^3*c*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-20/27/(a/b)^(2/3)*3^(1/2)/a^3*c*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+14/27/a^3*d/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))-7/27/a^3*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-14/27/a^3*d*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/a^3*e*ln(b*
x^3+a)-1/a^3*d/x+1/a^3*e*ln(x)-1/2/a^3*c/x^2
```

maxima [A] time = 3.11, size = 265, normalized size = 0.96

$$-\frac{28\,b^2dx^7+20\,b^2cx^6-6\,abex^5+49\,abdx^4+32\,abcx^3-9\,a^2ex^2+18\,a^2dx+9\,a^2c}{18\left(a^3b^2x^8+2\,a^4bx^5+a^5x^2\right)}+\frac{e\log(x)}{a^3}-\frac{2\sqrt{3}\left(7\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+10\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}-\frac{\left(9\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-10\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*d*x^7 + 20*b^2*c*x^6 - 6*a*b*e*x^5 + 49*a*b*d*x^4 + 32*a*b*c*
x^3 - 9*a^2*e*x^2 + 18*a^2*d*x + 9*a^2*c)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*
x^2) + e*log(x)/a^3 - 2/27*sqrt(3)*(7*b*d*(a/b)^(2/3) + 10*b*c*(a/b)^(1/3))
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 - 1/27*(9*e*(a/b)^
(2/3) + 7*d*(a/b)^(1/3) - 10*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3
*(a/b)^(2/3)) - 1/27*(9*e*(a/b)^(2/3) - 14*d*(a/b)^(1/3) + 20*c)*log(x + (a
/b)^(1/3))/(a^3*(a/b)^(2/3))
```

mupad [B] time = 5.36, size = 778, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] symsum(log(-(2*b^3*(1701*root(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*
b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 80
00*b^2*c^3, z, k)^2*a^8*d - 567*a^2*d*e^2 + 13122*root(19683*a^11*z^3 + 196
83*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a
*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)^3*a^11*x + 4000*b^2*c^3*x - 1134
*root(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z
 + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^5*d*
e - 1800*a*b*c^2*e - 1372*a*b*d^3*x + 1800*root(19683*a^11*z^3 + 19683*a^8*
e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3
+ 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^4*b*c^2 + 1458*root(19683*a^11*z^3 +
19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 274
4*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^5*e^2*x + 8748*root(19683*a
^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c
*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)^2*a^8*e*x + 12600*r
oot(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z +
 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^4*b*c*
d*x + 2520*a*b*c*d*e*x))/(729*a^9))*root(19683*a^11*z^3 + 19683*a^8*e*z^2 +
 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a
^2*e^3 + 8000*b^2*c^3, z, k), k, 1, 3) - (c/(2*a) - (e*x^2)/(2*a) + (d*x)/a
 + (10*b^2*c*x^6)/(9*a^3) + (14*b^2*d*x^7)/(9*a^3) + (16*b*c*x^3)/(9*a^2) +
 (49*b*d*x^4)/(18*a^2) - (b*e*x^5)/(3*a^2))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5)
 + (e*log(x))/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.304 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx$

Optimal. Leaf size=298

$$\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}}+\frac{2\sqrt[3]{b}\left(7\sqrt[3]{a}\,e+10\right.}{9\cdots}$$

Rubi [A] time = 0.59, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{x\left(-\frac{15b^2cx^2}{a}+11bd+10bex\right)}{18a^3\left(a+bx^3\right)}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2\left(a+bx^3\right)^2}+\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{11/3}}+\frac{bc\log\left(a+bx^3\right)}{a^4}-\frac{3bc\log(x)}{a^4}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}}+\frac{2\sqrt[3]{b}\left(7\sqrt[3]{a}\,e+10\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}}-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/
a))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d + 10*b*e*x - (15*b^2*c*x^2)/a))/(18*
a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)) - (3*b*c*Log[x])/a^
4 - (2*b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(11/3)) + (b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(27*a^(11/3)) + (b*c*Log[a + b*x^3])/a^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6bc-6bdx-6bex^2+\frac{6b^2cx^3}{a}+\frac{5b^2dx^4}{a}+\frac{4b^2ex^5}{a}-\frac{3b^3cx^6}{a^2}}{x^4\left(a+bx^3\right)^2}dx}{6ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^3c+18b^3dx+18b^3ex^2-\frac{36b^4cx^3}{a}-\frac{22b^4dx^4}{a}-10\cdots}{x^4\left(a+bx^3\right)}}{18a^2b^3}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^3c}{ax^4}+\frac{18b^3d}{ax^3}+\frac{18b^3e}{ax^2}-\frac{54b^4c}{a^2x}-\frac{2b^4(\cdots}{\cdots}\right.}{18a^2b^3}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} - \frac{3bc\log(x)}{a^4} - \cdots$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} - \frac{3bc\log(x)}{a^4} - \cdots$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} - \frac{3bc\log(x)}{a^4} + \cdots$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} - \frac{3bc\log(x)}{a^4} - \cdots$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} - \frac{3bc\log(x)}{a^4} - \cdots$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} + \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.46, size = 255, normalized size = 0.86

$$-\frac{-2\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+4\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{9a^2b(c+x(d+ex))}{\left(a+bx^3\right)^2}+\frac{3ab(12c+x(11d+10ex))}{a+bx^3}-54bc\log\left(a+bx^3\right)-4\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(7\sqrt[3]{a}e+10\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\frac{18ac}{x^3}+\frac{27ad}{x^2}+\frac{54ae}{x}+162bc\log(x)}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/54*((18*a*c)/x^3 + (27*a*d)/x^2 + (54*a*e)/x + (9*a^2*b*(c + x*(d + e*x)
))/(a + b*x^3)^2 + (3*a*b*(12*c + x*(11*d + 10*e*x)))/(a + b*x^3) - 4*Sqrt[
3]*a^(1/3)*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 162*b*c*Log[x] + 4*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7*a^(
2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7*a^(2
/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 54*b*c*Log[a + b*x^
3])/a^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3), x]
```

fricas [C] time = 1.69, size = 5550, normalized size = 18.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(168*a*b^2*e*x^8 + 120*a*b^2*d*x^7 + 108*a*b^2*c*x^6 + 294*a^2*b*e*x
^5 + 192*a^2*b*d*x^4 + 162*a^2*b*c*x^3 + 108*a^3*e*x^2 + 54*a^3*d*x + 36*a^
3*c + 2*(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12
+ 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^
12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12
)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 +
 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c
^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/
a^4)*log(7/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^
2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^
11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*
e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*
c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3
 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8*e + 5400*b^2*c*d^2 + 5
103*b^2*c^2*e + 3920*a*b*d*e^2 + (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^(
2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(
39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 +
280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567
*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^1
2 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/
a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^
12)^(1/3) - 54*b*c/a^4) + 4*(1000*b^2*d^3 + 343*a*b*e^3)*x) - (162*b^3*c*x^
9 + 324*a*b^2*c*x^6 + 162*a^2*b*c*x^3 + (a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^
3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*
b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(7
29*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(
200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(3936
6*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*
a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d
*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4) + 3*sqrt(1/3)*(a^4*b^2*x^9 + 2*a^5*b*x
^6 + a^6*x^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (72
9*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e
^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 274
4*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*
sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(
729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*
(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)
^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)
/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2
+ 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 5
67*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a
^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*
c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/
a^12)^(1/3) - 54*b*c/a^4)*a^4*b*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8))*log(
-7/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*
a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*
```

(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8*e - 5400*b^2*c*d^2 - 5103*b^2*c^2*e - 3920*a*b*d*e^2 - (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4) + 8*(1000*b^2*d^3 + 343*a*b*e^3)*x + 3/4*sqrt(1/3)*(7*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)*a^8*e - 400*a^4*b*d^2 + 378*a^4*b*c*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)*a^4*b*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8)) - (162*b^3*c*x^9 + 324*a*b^2*c*x^6 + 162*a^2*b*c*x^3 + (a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4) - 3*sqrt(1/3)*(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)*a^4*b*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8))*log(-7/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3

```
- 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^
3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)
*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^
2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8*e - 5400*b^2*c*d^2 - 5103*b^2*c^2*e - 39
20*a*b*d*e^2 - (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12
 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a
^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^1
2)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3
+ 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*
c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c
/a^4) + 8*(1000*b^2*d^3 + 343*a*b*e^3)*x - 3/4*sqrt(1/3)*(7*(2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366
*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a
*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*
e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8
*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12
+ (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(
1/3) - 54*b*c/a^4)*a^8*e - 400*a^4*b*d^2 + 378*a^4*b*c*e)*sqrt(-((2*(1/2)^(
2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(
39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 +
280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567
*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^1
2 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/
a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^
12)^(1/3) - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^
2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*
b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (1968
3*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3
)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*
a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)*a^4*b
*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8)) + 324*(b^3*c*x^9 + 2*a*b^2*c*x^6 +
a^2*b*c*x^3)*log(x))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

giac [A] time = 0.26, size = 305, normalized size = 1.02

$$\frac{bc\log\left(\left|bx^3+a\right|\right)}{a^4}-\frac{3\,bc\log\left(\left|x\right|\right)}{a^4}-\frac{2\,\sqrt{3}\left(10\,\left(-ab^2\right)^{\frac{1}{3}}bd-7\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}-\frac{\left(10\,\left(-ab^2\right)^{\frac{1}{3}}bd+7\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b}+\frac{2\left(7\,a^5b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+10\,a^5b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^9b}-\frac{28\,ab^2x^8e+20\,ab^2dx^7+18\,ab^2cx^6+49\,a^2bx^5e+32\,a^2bdx^4+27\,a^2bcx^3+18\,a^3x^2e+9\,a^3dx+6\,a^3c}{18\left(bx^3+a\right)^2a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] b*c*log(abs(b*x^3 + a))/a^4 - 3*b*c*log(abs(x))/a^4 - 2/27*sqrt(3)*(10*(-a*
b^2)^(1/3)*b*d - 7*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(a^4*b) - 1/27*(10*(-a*b^2)^(1/3)*b*d + 7*(-a*b^2)^(2/3)*e)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 2/27*(7*a^5*b^2*(-a/b)^
(1/3)*e + 10*a^5*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b) - 1
/18*(28*a*b^2*x^8*e + 20*a*b^2*d*x^7 + 18*a*b^2*c*x^6 + 49*a^2*b*x^5*e + 32
*a^2*b*d*x^4 + 27*a^2*b*c*x^3 + 18*a^3*x^2*e + 9*a^3*d*x + 6*a^3*c)/((b*x^3
 + a)^2*a^4*x^3)
```

maple [A] time = 0.06, size = 351, normalized size = 1.18

$$-\frac{5b^2e\,x^5}{9\left(b\,x^3+a\right)^2a^3}-\frac{11b^2d\,x^4}{18\left(b\,x^3+a\right)^2a^3}-\frac{2b^2c\,x^3}{3\left(b\,x^3+a\right)^2a^3}-\frac{13be\,x^2}{18\left(b\,x^3+a\right)^2a^2}-\frac{7bdx}{9\left(b\,x^3+a\right)^2a^2}-\frac{5bc}{6\left(b\,x^3+a\right)^2a^2}-\frac{20\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{20d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}+\frac{10d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{14\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{14e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{7e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{3bc\ln(x)}{a^4}+\frac{bc\ln\left(b\,x^3+a\right)}{a^4}-\frac{e}{a^3x}-\frac{d}{2a^3x^2}-\frac{c}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x)
```

[Out] -5/9/(b*x^3+a)^2/a^3*b^2*e*x^5-11/18/(b*x^3+a)^2/a^3*b^2*d*x^4-2/3/a^3*b^2/(b*x^3+a)^2*x^3*c-13/18/a^2/(b*x^3+a)^2*x^2*b*e-7/9/(b*x^3+a)^2/a^2*b*d*x-5/6/(b*x^3+a)^2/a^2*b*c-20/27/(a/b)^(2/3)/a^3*d*ln(x+(a/b)^(1/3))+10/27/(a/b)^(2/3)/a^3*d*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-20/27/(a/b)^(2/3)*3^(1/2)/a^3*d*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+14/27/a^3*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-7/27/(a/b)^(1/3)/a^3*e*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-14/27*3^(1/2)/(a/b)^(1/3)/a^3*e*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/a^4*b*c*ln(b*x^3+a)-1/3/a^3*c/x^3-1/2/a^3*d/x^2-1/a^3*e/x-3/a^4*b*c*ln(x)

maxima [A] time = 3.03, size = 283, normalized size = 0.95

$$-\frac{28\,b^{2}ex^{8}+20\,b^{2}dx^{7}+18\,b^{2}cx^{6}+49\,abex^{5}+32\,abdx^{4}+27\,abcx^{3}+18\,a^{2}ex^{2}+9\,a^{2}dx+6\,a^{2}c}{18\left(a^{3}b^{2}x^{9}+2\,a^{4}bx^{6}+a^{5}x^{3}\right)}-\frac{3\,bc\log(x)}{a^{4}}-\frac{2\sqrt{3}\left(7\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+10\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{5}}+\frac{\left(27\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-7\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+10\,ad\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(27\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")

[Out] -1/18*(28*b^2*e*x^8 + 20*b^2*d*x^7 + 18*b^2*c*x^6 + 49*a*b*e*x^5 + 32*a*b*d*x^4 + 27*a*b*c*x^3 + 18*a^2*e*x^2 + 9*a^2*d*x + 6*a^2*c)/(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3) - 3*b*c*log(x)/a^4 - 2/27*sqrt(3)*(7*a*e*(a/b)^(2/3) + 10*a*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 + 1/27*(27*b*c*(a/b)^(2/3) - 7*a*e*(a/b)^(1/3) + 10*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) + 1/27*(27*b*c*(a/b)^(2/3) + 14*a*e*(a/b)^(1/3) - 20*a*d)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3))

mupad [B] time = 0.46, size = 870, normalized size = 2.92

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x)

[Out] symsum(log(-(2*b^3*(1701*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)^2*a^8*e + 5400*b^2*c*d^2 - 5103*b^2*c^2*e + 13122*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)^3*a^11*x + 4000*b^2*d^3*x - 1372*a*b*e^3*x + 1800*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)*a^4*b*d^2 - 26244*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)^2*a^7*b*c*x + 13122*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)*a^3*b^2*c^2*x + 3402*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)*a^4*b*c*e - 7560*b^2*c*d*e*x + 12600*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)*a^4*b*d*e*x))/(729*a^9))*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x)/(2*a) + (b^2*c*x^6)/a^3 + (10*b^2*d*x^7)/(9*a^3) + (14*b^2*e*x^8)/(9*a^3) + (3*b*c*x^3)/(2*a^2) + (16*b*d*x^4)/(9*a^2) + (49*b*e*x^5)/(18*a^2))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (3*b*c*log(x))/a^4

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.305 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4} dx$

Optimal. Leaf size=248

$$-\frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}}+\frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{8/3}b^{5/3}}-\frac{\left(4\sqrt[3]{a}\,e+5\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}}{\sqrt{}}\right.}{81\sqrt{3}\,a^{8/3}b^{5/3}}$$

Rubi [A] time = 0.24, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1823, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}}+\frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{8/3}b^{5/3}}-\frac{\left(4\sqrt[3]{a}\,e+5\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{8/3}b^{5/3}}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}-\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] -(c + d*x + e*x^2)/(9*b*(a + b*x^3)^3) + (x*(d + 2*e*x))/(54*a*b*(a + b*x^3
)^2) + (x*(5*d + 8*e*x))/(162*a^2*b*(a + b*x^3)) - ((5*b^(1/3)*d + 4*a^(1/3
)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(8/3)
*b^(5/3)) + ((5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(
8/3)*b^(5/3)) - ((5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(486*a^(8/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^4}\,dx = -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{\int \frac{d+2ex}{\left(a+bx^3\right)^3}\,dx}{9b}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}-\frac{\int \frac{-5d-8ex}{\left(a+bx^3\right)^2}\,dx}{54ab}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}+\frac{\int \frac{10d+8ex}{a+bx^3}\,dx}{162a^2b}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}+\frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}\,d+8\sqrt[3]{a}\,e\right)+\sqrt[3]{b}\left(-10 \cdots\right.}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3} \cdots}}{486a^{8/3}b^{4/3}}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}+\frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a} \cdots\right.}{243a^{8/3}b^{5/3}}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}+\frac{\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a} \cdots\right.}{243a^{8/3}b^{5/3}}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}-\frac{\left(5\sqrt[3]{b}\,d+4\sqrt[3]{a}\,e\right)\tan^{-1}\left(\cdots\right.}{81\sqrt{3}\,a^{8/3}b^{5/3}}$$

Mathematica [A] time = 0.29, size = 230, normalized size = 0.93

$$\frac{\frac{\left(4\sqrt[3]{a}\,e-5\sqrt[3]{b}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{8/3}}+\frac{2\left(5\sqrt[3]{b}\,d-4\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{8/3}}-\frac{2\sqrt{3}\left(4\sqrt[3]{a}\,e+5\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}+\frac{3b^{2/3}x(5d+8ex)}{a^2\left(a+bx^3\right)}-\frac{54b^{2/3}(c+x(d+ex))}{\left(a+bx^3\right)^3}+\frac{9b^{2/3}x(d+2ex)}{a\left(a+bx^3\right)^2}}{486b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] ((9*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)^2) + (3*b^(2/3)*x*(5*d + 8*e*x))/
(a^2*(a + b*x^3)) - (54*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3)^3 - (2*Sqrt[
3]*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/a^(8/3) + (2*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3)
 + ((-5*b^(1/3)*d + 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/a^(8/3))/(486*b^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(c + dx + ex^2\right)}{\left(a + bx^3\right)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4, x]
```

fricas [C] time = 1.52, size = 2364, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(48*b^2*e*x^8 + 30*b^2*d*x^7 + 132*a*b*e*x^5 + 78*a*b*d*x^4 - 24*a^2*
e*x^2 - 60*a^2*d*x - 108*a^2*c - 2*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2
*x^3 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5
) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3
) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/
(a^8*b^5))^(1/3)))*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)
/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(
-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 6
4*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3*e - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a
^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))*a^3*b^2*d^2 + 160*a*d*e
^2 + (125*b*d^3 + 64*a*e^3)*x) + ((a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*
x^3 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5)
 + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3)
 + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(
a^8*b^5))^(1/3))) + 3*sqrt(1/3)*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^
3 + a^5*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8
*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sq
rt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e
^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)))*log(-((1/2)^(1/3)*(
I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(
a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3
+ 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3
*e + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) +
(125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) +
1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8
*b^5))^(1/3)))*a^3*b^2*d^2 - 160*a*d*e^2 + 2*(125*b*d^3 + 64*a*e^3)*x + 3/2
*sqrt(1/3)*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5
) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3
) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/
(a^8*b^5))^(1/3)))*a^6*b^3*e + 25*a^3*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(
```

```
3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5
))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*
e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*
d*e)/(a^5*b^3))) + ((a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)*(
(1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3
- 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
))) - 3*sqrt(1/3)*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b
*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^
5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))
^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)))*log(-((1/2)^(1/3)*(I*sqrt(3) + 1)
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a
^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3*e + 25/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 6
4*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((
125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))
*a^3*b^2*d^2 - 160*a*d*e^2 + 2*(125*b*d^3 + 64*a*e^3)*x - 3/2*sqrt(1/3)*(2*
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3
 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^
3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/
3)))*a^6*b^3*e + 25*a^3*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*
b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*
(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5)
 + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)
)))/(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)
```

giac [A] time = 0.21, size = 242, normalized size = 0.98

$$-\frac{\sqrt{3}\left(5\,bd-4\,\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b}-\frac{\left(5\,bd+4\,\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b}-\frac{\left(4\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5\,d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{3}b}+\frac{8\,b^{2}x^{8}e+5\,b^{2}dx^{7}+22\,abx^{5}e+13\,abdx^{4}-4\,a^{2}x^{2}e-10\,a^{2}dx-18\,a^{2}c}{162\,\left(bx^{3}+a\right)^{3}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/243*sqrt(3)*(5*b*d - 4*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) - 1/486*(5*b*d + 4*(-a*b^2)^(
1/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1
/243*(4*(-a/b)^(1/3)*e + 5*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*
b) + 1/162*(8*b^2*x^8*e + 5*b^2*d*x^7 + 22*a*b*x^5*e + 13*a*b*d*x^4 - 4*a^2
*x^2*e - 10*a^2*d*x - 18*a^2*c)/((b*x^3 + a)^3*a^2*b)
```

maple [A] time = 0.06, size = 275, normalized size = 1.11

$$\frac{5\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b^{2}}+\frac{5d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b^{2}}-\frac{5d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b^{2}}+\frac{4\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b^{2}}-\frac{4e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b^{2}}+\frac{2e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b^{2}}+\frac{\frac{4be\,x^{8}}{81a^{2}}+\frac{5bd\,x^{7}}{162a^{2}}+\frac{11e\,x^{5}}{81a}+\frac{13d\,x^{4}}{162a}-\frac{2e\,x^{2}}{81b}-\frac{5dx}{81b}-\frac{c}{9b}}{\left(b\,x^{3}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] (4/81/a^2*b*e*x^8+5/162/a^2*d*b*x^7+11/81/a*e*x^5+13/162/a*d*x^4-2/81/b*e*x
^2-5/81/b*d*x-1/9/b*c)/(b*x^3+a)^3+5/243/a^2/b^2*d/(a/b)^(2/3)*ln(x+(a/b)^(
1/3))-5/486/a^2/b^2*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+5/243/a
^2/b^2*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-4/243/
a^2/b^2*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+2/243/a^2/b^2*e/(a/b)^(1/3)*ln(x^2-
```

```
(a/b)^(1/3)*x+(a/b)^(2/3))+4/243/a^2/b^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 3.01, size = 248, normalized size = 1.00

$$\frac{8\,b^{2}ex^{8}+5\,b^{2}dx^{7}+22\,abex^{5}+13\,abdx^{4}-4\,a^{2}ex^{2}-10\,a^{2}dx-18\,a^{2}c}{162\,\left(a^{2}b^{4}x^{9}+3\,a^{3}b^{3}x^{6}+3\,a^{4}b^{2}x^{3}+a^{5}b\right)}+\frac{\sqrt{3}\left(4\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(4\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,d\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(4\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(8*b^2*e*x^8 + 5*b^2*d*x^7 + 22*a*b*e*x^5 + 13*a*b*d*x^4 - 4*a^2*e*x^
2 - 10*a^2*d*x - 18*a^2*c)/(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a
^5*b) + 1/243*sqrt(3)*(4*e*(a/b)^(1/3) + 5*d)*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/486*(4*e*(a/b)^(1/3) - 5*d
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) - 1/243*(4*e
*(a/b)^(1/3) - 5*d)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

mupad [B] time = 0.27, size = 253, normalized size = 1.02

$$\left(\sum_{k=1}^{3}\ln\left(\frac{20\,d\,e+16\,e^{2}\,x+\mathrm{root}\left(14348907\,a^{8}\,b^{5}\,z^{3}+14580\,a^{3}\,b^{2}\,d\,e\,z-125\,b\,d^{3}+64\,a\,e^{3},z,k\right)^{2}\,a^{5}\,b^{3}\,59049+\mathrm{root}\left(14348907\,a^{8}\,b^{5}\,z^{3}+14580\,a^{3}\,b^{2}\,d\,e\,z-125\,b\,d^{3}+64\,a\,e^{3},z,k\right)\,a^{2}\,b^{2}\,d\,x\,1215}{a^{4}\,b\,6561}\right)\,\mathrm{root}\left(14348907\,a^{8}\,b^{5}\,z^{3}+14580\,a^{3}\,b^{2}\,d\,e\,z-125\,b\,d^{3}+64\,a\,e^{3},z,k\right)\right)+\frac{\frac{13\,d\,x^{4}}{162\,a}-\frac{c}{9\,b}+\frac{11\,e\,x^{5}}{81\,a}-\frac{2\,e\,x^{2}}{81\,b}-\frac{5\,d\,x}{81\,b}+\frac{5\,b\,d\,x^{7}}{162\,a^{2}}+\frac{4\,b\,e\,x^{8}}{81\,a^{2}}}{a^{3}+3\,a^{2}\,b\,x^{3}+3\,a\,b^{2}\,x^{6}+b^{3}\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x)
```

```
[Out] symsum(log((20*d*e + 16*e^2*x + 59049*root(14348907*a^8*b^5*z^3 + 14580*a^3
*b^2*d*e*z - 125*b*d^3 + 64*a*e^3, z, k)^2*a^5*b^3 + 1215*root(14348907*a^8
*b^5*z^3 + 14580*a^3*b^2*d*e*z - 125*b*d^3 + 64*a*e^3, z, k)*a^2*b^2*d*x)/(
6561*a^4*b))*root(14348907*a^8*b^5*z^3 + 14580*a^3*b^2*d*e*z - 125*b*d^3 +
64*a*e^3, z, k), k, 1, 3) + ((13*d*x^4)/(162*a) - c/(9*b) + (11*e*x^5)/(81*
a) - (2*e*x^2)/(81*b) - (5*d*x)/(81*b) + (5*b*d*x^7)/(162*a^2) + (4*b*e*x^8
)/(81*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6)
```

sympy [A] time = 17.94, size = 201, normalized size = 0.81

$$\operatorname{RootSum}\left(14348907t^{3}a^{8}b^{5}+14580ta^{3}b^{2}de+64ae^{3}-125bd^{3},\left(t\mapsto t\log\left(x+\frac{236196t^{2}a^{6}b^{3}e+6075ta^{3}b^{2}d^{2}+160ade^{2}}{64ae^{3}+125bd^{3}}\right)\right)\right)+\frac{-18a^{2}c-10a^{2}dx-4a^{2}ex^{2}+13abdx^{4}+22abex^{5}+5b^{2}dx^{7}+8b^{2}ex^{8}}{162a^{5}b+486a^{4}b^{2}x^{3}+486a^{3}b^{3}x^{6}+162a^{2}b^{4}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**8*b**5 + 14580*_t*a**3*b**2*d*e + 64*a*e**3 - 125
*b*d**3, Lambda(_t, _t*log(x + (236196*_t**2*a**6*b**3*e + 6075*_t*a**3*b**
2*d**2 + 160*a*d*e**2)/(64*a*e**3 + 125*b*d**3)))) + (-18*a**2*c - 10*a**2*
d*x - 4*a**2*e*x**2 + 13*a*b*d*x**4 + 22*a*b*e*x**5 + 5*b**2*d*x**7 + 8*b**
2*e*x**8)/(162*a**5*b + 486*a**4*b**2*x**3 + 486*a**3*b**3*x**6 + 162*a**2*
b**4*x**9)
```

3.306 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$

Optimal. Leaf size=270

$$\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{10/3}b^{4/3}}-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\tan^{-1}}{81\sqrt{3}\,a^{10/3}b^{4/}}$$

Rubi [A] time = 0.25, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1828, 1854, 1855, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{10/3}b^{4/3}}-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{10/3}b^{4/3}}-\frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2}+\frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)}-\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(9*a*b*(a + b*x^3)^3) + (x*(5*a*e + 28*b*c*x))
/(162*a^3*b*(a + b*x^3)) - (6*a*d - x*(a*e + 7*b*c*x))/(54*a^2*b*(a + b*x^3
)^2) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(81*Sqrt[3]*a^(10/3)*b^(4/3)) - ((14*b^(2/3)*c - 5*a^(2/3)*e)*
Log[a^(1/3) + b^(1/3)*x])/(243*a^(10/3)*b^(4/3)) + ((14*b^(2/3)*c - 5*a^(2/
3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(486*a^(10/3)*b^(4/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^4}\,dx = -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} - \frac{\int \frac{-ae-7bcx-6bdx^2}{\left(a+bx^3\right)^3}\,dx}{9ab}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} + \frac{\int \frac{5ae+28bcx}{\left(a+bx^3\right)^2}\,dx}{54a^2b}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} + \frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{-10ae-28bcx}{a+bx^3}\,dx}{162a^3b}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} + \frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{\sqrt[3]{a}\left(-28\sqrt[3]{a}\,bc-20a\sqrt[3]{\ldots}\right.}{a^{2/3}-\ldots}}{48\ldots}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} + \frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} - \frac{\left(14b^{2/3}c-5a^{2/3}e\right)1\ldots}{243a^{10/\ldots}}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} + \frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} - \frac{\left(14b^{2/3}c-5a^{2/3}e\right)1\ldots}{243a^{10/\ldots}}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} + \frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} - \frac{\left(14b^{2/3}c+5a^{2/3}e\right)t\ldots}{81\sqrt{3}\,a^{1\ldots}}$$

Mathematica [A] time = 0.43, size = 241, normalized size = 0.89

$$\frac{a^{2/3}\sqrt[3]{b}\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt{3}a^{2/3}\sqrt[3]{b}\left(5a^{2/3}e+14b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(5a^{4/3}\sqrt[3]{b}e-14a^{2/3}bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{3ab^{2/3}\left(-2a^3(9d+5ex)+a^2bx^2\left(67c+13ex^2\right)+ab^2x^5\left(77c+5ex^2\right)+28b^3cx^8\right)}{\left(a+bx^3\right)^3}}{486a^4b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] ((3*a*b^(2/3)*(28*b^3*c*x^8 - 2*a^3*(9*d + 5*e*x) + a*b^2*x^5*(77*c + 5*e*x
^2) + a^2*b*x^2*(67*c + 13*e*x^2)))/(a + b*x^3)^3 - 2*Sqrt[3]*a^(2/3)*b^(1/
3)*(14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
 + 2*(-14*a^(2/3)*b*c + 5*a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] + a^(
2/3)*b^(1/3)*(14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(486*a^4*b^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x + e*x^2))/(a + b*x^3)^4, x]
```

fricas [C] time = 1.45, size = 2646, normalized size = 9.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^3*c*x^8 + 30*a*b^2*e*x^7 + 462*a*b^2*c*x^5 + 78*a^2*b*e*x^4 +
402*a^2*b*c*x^2 - 60*a^3*e*x - 108*a^3*d - 2*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 +
 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a
^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(
1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^1
0*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*log(7/2*((1/2)^(1
/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^
3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(
a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*
e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((
2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10
*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3
 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)
))*a^5*b*e^2 + 1960*a*b*c^2*e + (2744*b^2*c^3 + 125*a^2*e^3)*x) + ((a^3*b^4
*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*
((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c
^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/
3))) + 3*sqrt(1/3)*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sq
rt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
 - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*
sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*
c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 1120*c*e)/(a^6*b^2)))*lo
g(-7/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4
) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-
I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^
2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c + 25/2*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 12
5*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^
2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(
a^10*b^4))^(1/3)))*a^5*b*e^2 - 1960*a*b*c^2*e + 2*(2744*b^2*c^3 + 125*a^2*e
^3)*x + 3/2*sqrt(1/3)*(7*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*
a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*
(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^
10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*a^7*b^3*c + 25*a
^5*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)
/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2
/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
 - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 1120*c*e)/(a
^6*b^2))) + ((a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(
1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c
^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/
(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2
*e^3)/(a^10*b^4))^(1/3))) - 3*sqrt(1/3)*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^
5*b^2*x^3 + a^6*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125
*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140
*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a
^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 112
0*c*e)/(a^6*b^2)))*log(-7/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 1
25*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 1
40*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/
(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c +
 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
 - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I
*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2
*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*a^5*b*e^2 - 1960*a*b*c^2*e + 2*(274
```

```
4*b^2*c^3 + 125*a^2*e^3)*x - 3/2*sqrt(1/3)*(7*((1/2)^(1/3)*(I*sqrt(3) + 1)*
((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c
^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/
3)))*a^7*b^3*c + 25*a^5*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b
^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))
^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125
*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a
^6*b^2 + 1120*c*e)/(a^6*b^2))))/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^
3 + a^6*b)
```

giac [A] time = 0.24, size = 244, normalized size = 0.90

$$-\frac{\sqrt{3}\left(5\,ae-14\,\left(-ab^2\right)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,\left(-ab^2\right)^{\frac{2}{3}}a^3}-\frac{\left(5\,ae+14\,\left(-ab^2\right)^{\frac{1}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,\left(-ab^2\right)^{\frac{2}{3}}a^3}-\frac{\left(14\,bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4b}+\frac{28\,b^3cx^8+5\,ab^2x^7e+77\,ab^2cx^5+13\,a^2bx^4e+67\,a^2bcx^2-10\,a^3xe-18\,a^3d}{162\,\left(bx^3+a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/243*sqrt(3)*(5*a*e - 14*(-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/486*(5*a*e + 14*(-a*b^2)^(
1/3)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 1/2
43*(14*b*c*(-a/b)^(1/3) + 5*a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a
^4*b) + 1/162*(28*b^3*c*x^8 + 5*a*b^2*x^7*e + 77*a*b^2*c*x^5 + 13*a^2*b*x^4
*e + 67*a^2*b*c*x^2 - 10*a^3*x*e - 18*a^3*d)/((b*x^3 + a)^3*a^3*b)
```

maple [A] time = 0.06, size = 278, normalized size = 1.03

$$\frac{5\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b^2}+\frac{5e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b^2}-\frac{5e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b^2}+\frac{14\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3b}-\frac{14c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3b}+\frac{7c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3b}+\frac{\frac{14b^2c\,x^8}{81a^3}+\frac{5be\,x^7}{162a^2}+\frac{77bc\,x^5}{162a^2}+\frac{13e\,x^4}{162a}+\frac{67c\,x^2}{162a}-\frac{5ex}{81b}-\frac{d}{9b}}{\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] (14/81*c/a^3*b^2*x^8+5/162/a^2*b*e*x^7+77/162/a^2*b*c*x^5+13/162/a*e*x^4+67
/162/a*c*x^2-5/81/b*e*x-1/9/b*d)/(b*x^3+a)^3+5/243/a^2/b^2*e/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))-5/486/a^2/b^2*e/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+5/243/a^2/b^2*e/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))-14/243/a^3/b*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7/243/a^3/b*c/(a/b)^(1/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+14/243/a^3/b*c*3^(1/2)/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 3.03, size = 260, normalized size = 0.96

$$\frac{28\,b^3cx^8+5\,ab^2ex^7+77\,ab^2cx^5+13\,a^2bex^4+67\,a^2bcx^2-10\,a^3ex-18\,a^3d}{162\left(a^3b^4x^9+3\,a^4b^3x^6+3\,a^5b^2x^3+a^6b\right)}+\frac{\sqrt{3}\left(14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ae\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(28*b^3*c*x^8 + 5*a*b^2*e*x^7 + 77*a*b^2*c*x^5 + 13*a^2*b*e*x^4 + 67*
a^2*b*c*x^2 - 10*a^3*e*x - 18*a^3*d)/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b
^2*x^3 + a^6*b) + 1/243*sqrt(3)*(14*b*c*(a/b)^(1/3) + 5*a*e)*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) + 1/486*(14*b*c
*(a/b)^(1/3) - 5*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)
^(2/3)) - 1/243*(14*b*c*(a/b)^(1/3) - 5*a*e)*log(x + (a/b)^(1/3))/(a^3*b^2*
(a/b)^(2/3))
```

mupad [B] time = 0.24, size = 265, normalized size = 0.98

$$\frac{\frac{67\,c\,x^2}{162\,a}-\frac{d}{9\,b}+\frac{13\,e\,x^4}{162\,a}-\frac{5\,e\,x}{81\,b}+\frac{14\,b^2\,c\,x^8}{81\,a^3}+\frac{77\,b\,c\,x^5}{162\,a^2}+\frac{5\,b\,e\,x^7}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}+\left(\sum_{k=1}^{3}\ln\left(\frac{70\,a\,c\,e+\mathrm{root}\left(14348907\,a^{10}\,b^4\,z^3+51030\,a^4\,b^2\,c\,e\,z-125\,a^2\,e^3+2744\,b^2\,c^3,z,k\right)^2\,a^7\,b^2\,59049+196\,b\,c^2\,x+\mathrm{root}\left(14348907\,a^{10}\,b^4\,z^3+51030\,a^4\,b^2\,c\,e\,z-125\,a^2\,e^3+2744\,b^2\,c^3,z,k\right)\,a^4\,b\,e\,x\,1215}{a^6\,6561}\right)\,\mathrm{root}\left(14348907\,a^{10}\,b^4\,z^3+51030\,a^4\,b^2\,c\,e\,z-125\,a^2\,e^3+2744\,b^2\,c^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2))/(a + b*x^3)^4,x)
```

```
[Out] ((67*c*x^2)/(162*a) - d/(9*b) + (13*e*x^4)/(162*a) - (5*e*x)/(81*b) + (14*b
^2*c*x^8)/(81*a^3) + (77*b*c*x^5)/(162*a^2) + (5*b*e*x^7)/(162*a^2))/(a^3 +
 b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6) + symsum(log((70*a*c*e + 59049*root(1
4348907*a^10*b^4*z^3 + 51030*a^4*b^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3, z,
 k)^2*a^7*b^2 + 196*b*c^2*x + 1215*root(14348907*a^10*b^4*z^3 + 51030*a^4*b
^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3, z, k)*a^4*b*e*x)/(6561*a^6))*root(14
348907*a^10*b^4*z^3 + 51030*a^4*b^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3, z,
k), k, 1, 3)
```

sympy [A] time = 8.79, size = 214, normalized size = 0.79

$$\operatorname{RootSum}\left(14348907t^3a^{10}b^4+51030ta^4b^2ce-125a^2e^3+2744b^2c^3,\left(t\mapsto t\log\left(x+\frac{826686t^2a^7b^3c+6075ta^5be^2+1960abc^2e}{125a^2e^3+2744b^2c^3}\right)\right)\right)+\frac{-18a^3d-10a^3ex+67a^2bcx^2+13a^2bex^4+77ab^2cx^5+5ab^2ex^7+28b^3cx^8}{162a^6b+486a^5b^2x^3+486a^4b^3x^6+162a^3b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**10*b**4 + 51030*_t*a**4*b**2*c*e - 125*a**2*e**3
+ 2744*b**2*c**3, Lambda(_t, _t*log(x + (826686*_t**2*a**7*b**3*c + 6075*_t
*a**5*b*e**2 + 1960*a*b*c**2*e)/(125*a**2*e**3 + 2744*b**2*c**3)))) + (-18*
a**3*d - 10*a**3*e*x + 67*a**2*b*c*x**2 + 13*a**2*b*e*x**4 + 77*a*b**2*c*x*
*5 + 5*a*b**2*e*x**7 + 28*b**3*c*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 4
86*a**4*b**3*x**6 + 162*a**3*b**4*x**9)
```

3.307 $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^4} \, dx$

Optimal. Leaf size=250

$$-\frac{\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}\,d+20\sqrt[3]{b}\,c\right)}{81\sqrt{3}\,a^{11}}$$

Rubi [A] time = 0.22, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1854, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}\,d+20\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{11/3}b^{2/3}}+\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a
+ b*x^3)) - (a*e - b*x*(c + d*x))/(9*a*b*(a + b*x^3)^3) - (2*(20*b^(1/3)*c
+ 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[
3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3
)*x])/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^4}\,dx = -\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}-\frac{\int \frac{-8c-7dx}{\left(a+bx^3\right)^3}\,dx}{9a}$$

$$=\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}+\frac{\int \frac{40c+28dx}{\left(a+bx^3\right)^2}\,dx}{54a^2}$$

$$=\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}-\frac{\int \frac{-80c-28dx}{a+bx^3}\,dx}{162a^3}$$

$$=\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}-\frac{\int \frac{\sqrt[3]{a}\left(-160\sqrt[3]{b}c-28\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(80\sqrt[3]{b}c-28\sqrt[3]{a}\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{486a^{11/3}\sqrt[3]{b}}$$

$$=\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}$$

$$=\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}$$

$$=\frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2}+\frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}-\frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{11/3}b^{2/3}}$$

Mathematica [A] time = 0.28, size = 239, normalized size = 0.96

$$\frac{\frac{2\left(7a^{2/3}d-20\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{54a^3(ae-bx(c+dx))}{b\left(a+bx^3\right)^3}+\frac{9a^2x(8c+7dx)}{\left(a+bx^3\right)^2}-\frac{4\sqrt{3}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{12ax(10c+7dx)}{a+bx^3}}{486a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] ((9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3
) - (54*a^3*(a*e - b*x*(c + d*x)))/(b*(a + b*x^3)^3) - (4*Sqrt[3]*a^(1/3)*(
20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(2/3) + (4*(20*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b
^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/b^(2/3))/(486*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2}{\left(a + bx^3\right)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(a + b*x^3)^4, x]
```

fricas [C] time = 1.22, size = 2344, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^3*d*x^8 + 240*b^3*c*x^7 + 462*a*b^2*d*x^5 + 624*a*b^2*c*x^4 +
402*a^2*b*d*x^2 + 492*a^2*b*c*x - 108*a^3*e - 2*(a^3*b^4*x^9 + 3*a^4*b^3*x^
6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^
3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*
d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
 - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400*(4^(1/3)
*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a
*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*
c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*
a^4*b*c^2 + 7840*a*c*d^2 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a^3*b^4*x^9 +
3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^
3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 14
0*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2)
+ (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3))) + 3*sqrt(1/3)*(a^3*b^4*x^9 +
 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8
000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1
/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^
11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/
(a^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
 - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2
 + 8*(8000*b*c^3 + 343*a*d^3)*x + 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)
*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2)
)^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)
/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a
```

```
^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*
d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b))) + ((a^3*b^4*x^9 + 3*
a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3
+ 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*
4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3))) - 3*sqrt(1/3)*(a^3*b^4*x^9 + 3
*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((800
0*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3
) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a
^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*
d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*
c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) -
140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 +
 8*(8000*b*c^3 + 343*a*d^3)*x - 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*(
(8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^
(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(
a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a^4
*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3
) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^
3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b))))/(a^3*b^4*x^9 + 3*a^4*
b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)
```

giac [A] time = 0.21, size = 234, normalized size = 0.94

$$-\frac{2\sqrt{3}\left(20\,bc-7\,\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}}-\frac{\left(20\,bc+7\,\left(-ab^{2}\right)^{\frac{1}{3}}d\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}}-\frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{4}}+\frac{28\,b^{3}dx^{8}+40\,b^{3}cx^{7}+77\,ab^{2}dx^{5}+104\,ab^{2}cx^{4}+67\,a^{2}bdx^{2}+82\,a^{2}bcx-18\,a^{3}e}{162\,\left(bx^{3}+a\right)^{3}a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*sqrt(3)*(20*b*c - 7*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*c + 7*(-a*b^2)^(
1/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 2/2
43*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 +
1/162*(28*b^3*d*x^8 + 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104*a*b^2*c*x^4 + 67*
a^2*b*d*x^2 + 82*a^2*b*c*x - 18*a^3*e)/((b*x^3 + a)^3*a^3*b)
```

maple [A] time = 0.06, size = 360, normalized size = 1.44

$$\frac{e\,x^{3}}{9\left(b\,x^{3}+a\right)^{3}a}+\frac{d\,x^{2}}{9\left(b\,x^{3}+a\right)^{3}a}+\frac{e\,x^{3}}{9\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{cx}{9\left(b\,x^{3}+a\right)^{3}a}+\frac{7d\,x^{2}}{54\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{4cx}{27\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{14d\,x^{2}}{81\left(b\,x^{3}+a\right)a^{3}}+\frac{20cx}{81\left(b\,x^{3}+a\right)a^{3}}-\frac{e}{9\left(b\,x^{3}+a\right)a^{2}b}+\frac{40\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}b}+\frac{40c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}b}-\frac{20c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}b}+\frac{14\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}b}-\frac{14d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}b}+\frac{7d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] 1/9*c/a*x/(b*x^3+a)^3+4/27*c/a^2*x/(b*x^3+a)^2+20/81*c/a^3*x/(b*x^3+a)+40/2
43*c/a^3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-20/243*c/a^3/b/(a/b)^(2/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+40/243*c/a^3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))+1/9*d/a*x^2/(b*x^3+a)^3+7/54*d/a^2*x^2/(b*x^3+a)
^2+14/81*d/a^3*x^2/(b*x^3+a)-14/243*d/a^3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7
/243*d/a^3/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+14/243*d/a^3*3^(
```

```
1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9*e/a*x^3/(b*x
^3+a)^3+1/9*e/a^2*x^3/(b*x^3+a)^2-1/9*e/a^2/b/(b*x^3+a)
```

maxima [A] time = 2.99, size = 254, normalized size = 1.02

$$\frac{28\,b^3dx^8+40\,b^3cx^7+77\,ab^2dx^5+104\,ab^2cx^4+67\,a^2bdx^2+82\,a^2bcx-18\,a^3e}{162\left(a^3b^4x^9+3\,a^4b^3x^6+3\,a^5b^2x^3+a^6b\right)}+\frac{2\sqrt{3}\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(28*b^3*d*x^8 + 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104*a*b^2*c*x^4 + 67*
a^2*b*d*x^2 + 82*a^2*b*c*x - 18*a^3*e)/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5
*b^2*x^3 + a^6*b) + 2/243*sqrt(3)*(7*d*(a/b)^(1/3) + 20*c)*arctan(1/3*sqrt(
3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/243*(7*d*(a/b)^
(1/3) - 20*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) -
2/243*(7*d*(a/b)^(1/3) - 20*c)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

mupad [B] time = 0.28, size = 247, normalized size = 0.99

$$\frac{\frac{67\,d\,x^2}{162\,a}-\frac{e}{9\,b}+\frac{41\,c\,x}{81\,a}+\frac{20\,b^2\,c\,x^7}{81\,a^3}+\frac{14\,b^2\,d\,x^8}{81\,a^3}+\frac{52\,b\,c\,x^4}{81\,a^2}+\frac{77\,b\,d\,x^5}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}+\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(560\,c\,d+196\,d^2\,x+\mathrm{root}\left(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right)^2\,a^7\,b\,59049+\mathrm{root}\left(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right)\,a^3\,b\,c\,x\,9720\right)}{a^6\,6561}\right)\mathrm{root}\left(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(a + b*x^3)^4,x)
```

```
[Out] ((67*d*x^2)/(162*a) - e/(9*b) + (41*c*x)/(81*a) + (20*b^2*c*x^7)/(81*a^3) +
 (14*b^2*d*x^8)/(81*a^3) + (52*b*c*x^4)/(81*a^2) + (77*b*d*x^5)/(162*a^2))/
(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6) + symsum(log((b*(560*c*d + 196*
d^2*x + 59049*root(14348907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3
 + 2744*a*d^3, z, k)^2*a^7*b + 9720*root(14348907*a^11*b^2*z^3 + 408240*a^4
*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)*a^3*b*c*x))/(6561*a^6))*root(143
48907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k),
k, 1, 3)
```

sympy [A] time = 4.47, size = 202, normalized size = 0.81

$$\mathrm{RootSum}\left(14348907t^3a^{11}b^2+408240ta^4bcd+2744ad^3-64000bc^3,\left(t\mapsto t\log\left(x+\frac{413343t^2a^8bd+194400ta^4bc^2+7840acd^2}{1372ad^3+32000bc^3}\right)\right)\right)+\frac{-18a^3e+82a^2bcx+67a^2bdx^2+104ab^2cx^4+77ab^2dx^5+40b^3cx^7+28b^3dx^8}{162a^6b+486a^5b^2x^3+486a^4b^3x^6+162a^3b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 64
000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4*b
*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (-18*a**3*e + 82*a
**2*b*c*x + 67*a**2*b*d*x**2 + 104*a*b**2*c*x**4 + 77*a*b**2*d*x**5 + 40*b*
*3*c*x**7 + 28*b**3*d*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 486*a**4*b**
3*x**6 + 162*a**3*b**4*x**9)
```

3.308 $\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4} \, dx$

Optimal. Leaf size=291

$$-\frac{\left(20\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}\,e+20\sqrt[3]{b}\,d\right)\tan}{81\sqrt{3}\,a^{11/3}b}$$

Rubi [A] time = 0.52, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(20\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}\,e+20\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{11/3}b^{2/3}}+\frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)}+\frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2}+\frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3}-\frac{c\log\left(a+bx^3\right)}{3a^4}+\frac{c\log(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*d + 7*a*e*x - 1
5*b*c*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*d + 28*a*e*x - 99*b*c*x^2))/(
162*a^4*(a + b*x^3)) - (2*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (c*Log[x])/a
^4 + (2*(20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3
)*b^(2/3)) - ((20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx = \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-8bdx-7bex^2+\frac{6b^2cx^3}{a}}{x\left(a+bx^3\right)^3}\,dx}{9ab}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^2c+40b^2dx+28b^2ex^2-\frac{45b^3cx^3}{a}}{x\left(a+bx^3\right)^2}\,dx}{54a^2b^2}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int \frac{-162b^3c-8\cdots}{x(a\cdots}}{16\cdots}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int \left(-\frac{162b^3c}{ax}\cdots\right.}{\cdots}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{c\log(x)}{a^4} + \cdots$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{c\log(x)}{a^4} + \cdots$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{c\log(x)}{a^4} - \cdots$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{c\log(x)}{a^4} + \cdots$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{c\log(x)}{a^4} + \cdots$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} - \frac{2\left(20\sqrt[3]{b}\,d\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.31, size = 259, normalized size = 0.89

$$\frac{\frac{2\left(7a^{2/3}e-20\sqrt[3]{a}\sqrt[3]{b}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}\,d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{54a^3(c+x(d+ex))}{\left(a+bx^3\right)^3}+\frac{9a^2(9c+x(8d+7ex))}{\left(a+bx^3\right)^2}-\frac{4\sqrt{3}\sqrt[3]{a}\left(7\sqrt[3]{a}\,e+20\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6a(27c+2x(10d+7ex))}{a+bx^3}-162c\log\left(a+bx^3\right)+486c\log(x)}{486a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

```
[Out] ((54*a^3*(c + x*(d + e*x)))/(a + b*x^3)^3 + (9*a^2*(9*c + x*(8*d + 7*e*x)))
/(a + b*x^3)^2 + (6*a*(27*c + 2*x*(10*d + 7*e*x)))/(a + b*x^3) - (4*Sqrt[3]
*a^(1/3)*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(2/3) + 486*c*Log[x] + (4*(20*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Lo
g[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*d + 7*a^(2/3)*e)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 162*c*Log[a + b*x
^3])/(486*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x*(a + b*x^3)^4), x]
```

fricas [C] time = 1.55, size = 5370, normalized size = 18.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/236196*(40824*a*b^2*e*x^8 + 58320*a*b^2*d*x^7 + 78732*a*b^2*c*x^6 + 11226
6*a^2*b*e*x^5 + 151632*a^2*b*d*x^4 + 196830*a^2*b*c*x^3 + 97686*a^3*e*x^2 +
 119556*a^3*d*x + 144342*a^3*c - 2*(a^4*b^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x
^3 + a^7)*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b
))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348
907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744
*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt
(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4
/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3
+ 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c
/a^4)*log(7/236196*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*
e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b)
+ 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c
^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 5904
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^
12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441
*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3)
+ 39366*c/a^4)^2*a^8*b*e + 64800*b*c*d^2 + 45927*b*c^2*e + 7840*a*d*e^2 - 1
/243*(400*a^4*b*d^2 + 567*a^4*b*c*e)*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (656
1*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*
a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/2869
7814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*
b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2
+ 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) -
1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/
(a^12*b^2))^(1/3) + 39366*c/a^4) + 4*(8000*b*d^3 + 343*a*e^3)*x) - (118098*
b^3*c*x^9 + 354294*a*b^2*c*x^6 + 354294*a^2*b*c*x^3 + 118098*a^3*c - (a^4*b
^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)*((-I*sqrt(3) + 1)*(6561*c^2/a^8
 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2
 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) -
 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)
/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561
*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*
b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e
)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) - 3*sqrt(1/3)*(a^4*b^3*x^9 + 3*a^5*
b^2*x^6 + 3*a^6*b*x^3 + a^7)*sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561
*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a
*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697
814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b
^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 +
 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1
/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(
```

a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(6561*c^2
/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b
*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^
2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*
a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(
6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a
^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c
*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 + 52
9079040*a*d*e)/(a^8*b)))*log(-7/236196*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6
561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 56
0*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28
697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^1
2*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^
2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2)
- 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b
)/(a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b*e - 64800*b*c*d^2 - 45927*b*c^2*
e - 7840*a*d*e^2 + 1/243*(400*a^4*b*d^2 + 567*a^4*b*c*e)*((-I*sqrt(3) + 1)*
(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/11809
8*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)
/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 170
1*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1
/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*
a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3
 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) + 8*(8000*b*d^3 + 343*
a*e^3)*x + 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c
^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e
)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*
(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))
^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560
*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/286
97814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12
*b^2))^(1/3) + 39366*c/a^4)*a^8*b*e + 388800*a^4*b*d^2 - 275562*a^4*b*c*e)*
sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/
(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907
*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^
2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3)
 + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14
348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2
744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^
4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*
e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b)
+ 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c
^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 5904
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^
12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441
*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3)
+ 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 + 529079040*a*d*e)/(a^8*b))) - (1
18098*b^3*c*x^9 + 354294*a*b^2*c*x^6 + 354294*a^2*b*c*x^3 + 118098*a^3*c -
(a^4*b^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)*((-I*sqrt(3) + 1)*(6561*c
^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561
*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*
b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e
)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098
*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/
(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701
*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) + 3*sqrt(1/3)*(a^4*b^3*x^9 +
3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)*sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 -
 (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 +
 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1

```
/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(
a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b
*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^
2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*
a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(65
61*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(
6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a
^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c
*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/11
8098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e
^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 -
1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4)*a^4*b*c + 1549681956*b*c^
2 + 529079040*a*d*e)/(a^8*b)))*log(-7/236196*((-I*sqrt(3) + 1)*(6561*c^2/a^
8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^
2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2)
- 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b
)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(656
1*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11
*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*
e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b*e - 64800*b*c*d^2 - 45927*
b*c^2*e - 7840*a*d*e^2 + 1/243*(400*a^4*b*d^2 + 567*a^4*b*c*e)*((-I*sqrt(3)
 + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1
/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*
a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3
 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^
12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3
+ 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(8
00*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) + 8*(8000*b*d^3
+ 343*a*e^3)*x - 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (65
61*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560
*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/286
97814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12
*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2
 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) -
 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)
/(a^12*b^2))^(1/3) + 39366*c/a^4)*a^8*b*e + 388800*a^4*b*d^2 - 275562*a^4*b
*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^
8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14
348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2
744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*s
qrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b)
+ 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c
^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 3936
6*c/a^4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 56
0*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^
12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441
*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3)
+ 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)
*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(
531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^
(1/3) + 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 + 529079040*a*d*e)/(a^8*b))
) + 236196*(b^3*c*x^9 + 3*a*b^2*c*x^6 + 3*a^2*b*c*x^3 + a^3*c)*log(x))/(a^4
*b^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)
```

giac [A] time = 0.24, size = 290, normalized size = 1.00

$$-\frac{2\sqrt{3}\left(20\,bd - 7\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}} - \frac{\left(20\,bd + 7\left(-ab^{2}\right)^{\frac{1}{3}}e\right)\log\left(x^{2} + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}} - \frac{c\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{4}} + \frac{c\log\left(\left|x\right|\right)}{a^{4}} + \frac{28\,ab^{2}x^{8}e + 40\,ab^{2}dx^{7} + 54\,ab^{2}cx^{6} + 77\,a^{2}bx^{5}e + 104\,a^{2}bdx^{4} + 135\,a^{2}bcx^{3} + 67\,a^{3}x^{2}e + 82\,a^{3}dx + 99\,a^{3}c}{162\left(bx^{3}+a\right)^{3}a^{4}} - \frac{2\left(7\,a^{5}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e + 20\,a^{5}bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{9}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*sqrt(3)*(20*b*d - 7*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*d + 7*(-a*b^2)^(
1/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 1/3
*c*log(abs(b*x^3 + a))/a^4 + c*log(abs(x))/a^4 + 1/162*(28*a*b^2*x^8*e + 40
*a*b^2*d*x^7 + 54*a*b^2*c*x^6 + 77*a^2*b*x^5*e + 104*a^2*b*d*x^4 + 135*a^2*
b*c*x^3 + 67*a^3*x^2*e + 82*a^3*d*x + 99*a^3*c)/((b*x^3 + a)^3*a^4) - 2/243
*(7*a^5*b*(-a/b)^(1/3)*e + 20*a^5*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a^9*b)
```

maple [A] time = 0.06, size = 394, normalized size = 1.35

$$\frac{14b^2e x^8}{81\left(b x^3+a\right)^3 a^3}+\frac{20b^2d x^7}{81\left(b x^3+a\right)^3 a^3}+\frac{b^2c x^6}{3\left(b x^3+a\right)^3 a^3}+\frac{77be x^5}{162\left(b x^3+a\right)^3 a^2}+\frac{52bd x^4}{81\left(b x^3+a\right)^3 a^2}+\frac{5bc x^3}{6\left(b x^3+a\right)^3 a^2}+\frac{67e x^2}{162\left(b x^3+a\right)^3 a}+\frac{41dx}{81\left(b x^3+a\right)^3 a}+\frac{11c}{18\left(b x^3+a\right)^3 a}+\frac{40\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}} a^3b}+\frac{40d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}} a^3b}-\frac{20d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}} a^3b}+\frac{14\sqrt{3}\, e \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3b}-\frac{14e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3b}+\frac{7e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}} a^3b}+\frac{c\ln(x)}{a^4}-\frac{c\ln\left(b x^3+a\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^4,x)
```

```
[Out] 14/81/a^3/(b*x^3+a)^3*b^2*e*x^8+20/81/a^3/(b*x^3+a)^3*b^2*d*x^7+1/3/a^3/(b*
x^3+a)^3*b^2*c*x^6+77/162/a^2/(b*x^3+a)^3*b*e*x^5+52/81/a^2/(b*x^3+a)^3*b*d
*x^4+5/6/a^2/(b*x^3+a)^3*b*c*x^3+67/162/a/(b*x^3+a)^3*e*x^2+41/81/a/(b*x^3+
a)^3*d*x+11/18/a/(b*x^3+a)^3*c+40/243/a^3*d/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))
-20/243/a^3*d/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+40/243/a^3*d/
b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-14/243/a^3*e/
b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+7/243/a^3*e/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+14/243/a^3*e*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))-1/3*c*ln(b*x^3+a)/a^4+c*ln(x)/a^4
```

maxima [A] time = 3.04, size = 293, normalized size = 1.01

$$\frac{28\,b^2ex^8+40\,b^2dx^7+54\,b^2cx^6+77\,abex^5+104\,abdx^4+135\,abcx^3+67\,a^2ex^2+82\,a^2dx+99\,a^2c}{162\left(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6\right)}+\frac{c\log(x)}{a^4}+\frac{2\sqrt{3}\left(7\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+20\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5}-\frac{\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-7\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-40\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/162*(28*b^2*e*x^8 + 40*b^2*d*x^7 + 54*b^2*c*x^6 + 77*a*b*e*x^5 + 104*a*b*
d*x^4 + 135*a*b*c*x^3 + 67*a^2*e*x^2 + 82*a^2*d*x + 99*a^2*c)/(a^3*b^3*x^9
+ 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6) + c*log(x)/a^4 + 2/243*sqrt(3)*(7*a*e*
(a/b)^(2/3) + 20*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a
/b)^(1/3))/a^5 - 1/243*(81*b*c*(a/b)^(2/3) - 7*a*e*(a/b)^(1/3) + 20*a*d)*lo
g(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) - 1/243*(81*b*c*(a
/b)^(2/3) + 14*a*e*(a/b)^(1/3) - 40*a*d)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^
(2/3))
```

mupad [B] time = 5.40, size = 871, normalized size = 2.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x*(a + b*x^3)^4),x)
```

```
[Out] ((11*c)/(18*a) + (67*e*x^2)/(162*a) + (41*d*x)/(81*a) + (b^2*c*x^6)/(3*a^3)
 + (20*b^2*d*x^7)/(81*a^3) + (14*b^2*e*x^8)/(81*a^3) + (5*b*c*x^3)/(6*a^2)
+ (52*b*d*x^4)/(81*a^2) + (77*b*e*x^5)/(162*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*
x^3 + 3*a*b^2*x^6) + symsum(log(-(2*b*(45927*b*c^2*e - 64800*b*c*d^2 + 1372
*a*e^3*x - 32000*b*d^3*x + 9565938*root(14348907*a^12*b^2*z^3 + 14348907*a^
8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e
```

```
 - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)^3*a^11*b^2*x + 6480
0*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z
+ 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 +
 531441*b^2*c^3, z, k)*a^4*b*d^2 - 137781*root(14348907*a^12*b^2*z^3 + 1434
8907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*
b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)^2*a^8*b*e +
45360*b*c*d*e*x + 1062882*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z
^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*
a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)*a^3*b^2*c^2*x + 6377292*root
(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782
969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 53144
1*b^2*c^3, z, k)^2*a^7*b^2*c*x + 91854*root(14348907*a^12*b^2*z^3 + 1434890
7*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c
*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)*a^4*b*c*e + 226
800*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*
z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3
 + 531441*b^2*c^3, z, k)*a^4*b*d*e*x))/(531441*a^9))*root(14348907*a^12*b^2
*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z
+ 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k),
k, 1, 3) + (c*log(x))/a^4
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

3.309 $\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx$

Optimal. Leaf size=301

$$-\frac{10\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(7b^{2/3}c-2a^{2/3}e\right)\text{ta}}{81\sqrt{3}\,a^{13/3}}$$

Rubi [A] time = 0.60, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{10\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(7b^{2/3}c-2a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{13/3}\sqrt[3]{b}}+\frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)}+\frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2}+\frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3}-\frac{d\log\left(a+bx^3\right)}{3a^4}-\frac{c}{a^4x}+\frac{d\log(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

```
[Out] -(c/(a^4*x)) + (x*(a*e - b*c*x - b*d*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*
e - 16*b*c*x - 15*b*d*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*e - 118*b*c*x
 - 99*b*d*x^2))/(162*a^4*(a + b*x^3)) + (20*(7*b^(2/3)*c - 2*a^(2/3)*e)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(13/3)*b^(1/3
)) + (d*Log[x])/a^4 + (20*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)
*x])/(243*a^(13/3)*b^(1/3)) - (10*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(13/3)*b^(1/3)) - (d*Log[a + b*x^
3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx = \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-9bdx-8bex^2+\frac{7b^2cx^3}{a}+\frac{6b^2dx^4}{a}}{x^2\left(a+bx^3\right)^3}dx}{9ab}$$

$$= \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^3c+54b^3dx+40b^3ex^2-\frac{64b^4cx^3}{a}-\frac{45b^4dx^4}{a}}{x^2\left(a+bx^3\right)^2}dx}{54a^2b^3}$$

$$= \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int \frac{-162b^5c}{}}{}$$

$$= \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int\left(-\frac{162b}{ax^2}\right.}{}$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} +$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} +$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} +$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} +$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} +$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} +$$

Mathematica [A] time = 0.31, size = 279, normalized size = 0.93

$$\frac{-\frac{20\left(7a^{2/3}b^{2/3}c+2a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+\frac{40\left(7a^{2/3}b^{2/3}c+2a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{40\sqrt{3}a^{2/3}\left(2a^{2/3}e-7b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{54a^3\left(a(d+ex)-bcx^2\right)}{\left(a+bx^3\right)^3}+\frac{9a^2\left(9ad+8aex-16bcx^2\right)}{\left(a+bx^3\right)^2}+\frac{6a\left(27ad+20aex-59bcx^2\right)}{a+bx^3}-162ad\log\left(a+bx^3\right)-\frac{486ac}{x}+486ad\log(x)}{486a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

```
[Out] ((-486*a*c)/x + (9*a^2*(9*a*d + 8*a*e*x - 16*b*c*x^2))/(a + b*x^3)^2 + (6*a
*(27*a*d + 20*a*e*x - 59*b*c*x^2))/(a + b*x^3) + (54*a^3*(-(b*c*x^2) + a*(d
 + e*x)))/(a + b*x^3)^3 - (40*Sqrt[3]*a^(2/3)*(-7*b^(2/3)*c + 2*a^(2/3)*e)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + 486*a*d*Log[x] + (40
*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (2
0*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/b^(1/3) - 162*a*d*Log[a + b*x^3])/(486*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2}{x^2 \left(a + bx^3\right)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4), x]
```

fricas [C] time = 1.56, size = 5250, normalized size = 17.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] -1/236196*(408240*b^3*c*x^9 - 58320*a*b^2*e*x^8 - 78732*a*b^2*d*x^7 + 11226
60*a*b^2*c*x^6 - 151632*a^2*b*e*x^5 - 196830*a^2*b*d*x^4 + 976860*a^2*b*c*x
^3 - 119556*a^3*e*x^2 - 144342*a^3*d*x + 236196*a^3*c + 2*(a^4*b^3*x^10 + 3
*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561
*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^
12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d
*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3)
 + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d
/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*
c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1
/3) + 39366*d/a^4)*log(-7/236196*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^
2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12
+ 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)
*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) +
59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^
12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d
*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3)
 + 39366*d/a^4)^2*a^9*b*c - 45927*a*b*c*d^2 + 78400*a*b*c^2*e + 6480*a^2*d*
e^2 + 1/243*(567*a^5*b*c*d - 40*a^6*e^2)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 -
(6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)
*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 560
0*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^
(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c
*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 -
5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b
))^(1/3) + 39366*d/a^4) - 400*(343*b^2*c^3 - 8*a^2*e^3)*x) + (118098*b^3*d*
x^10 + 354294*a*b^2*d*x^7 + 354294*a^2*b*d*x^4 + 118098*a^3*d*x - (a^4*b^3*
x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*((-I*sqrt(3) + 1)*(6561*d^2/a^8
 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c
*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 -
5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b
))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 560
0*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3
 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^1
3*b))^(1/3) + 39366*d/a^4) + 3*sqrt(1/3)*(a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*
a^6*b*x^4 + a^7*x)*sqrt(-(((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 560
0*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/286
97814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(
a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(
I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/
28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b
)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 3936
```

```
6*d/a^4)^2*a^8 - 78732*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c
*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/286978
14*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^1
3*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*s
qrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/286
97814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(
a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d
/a^4)*a^4*d + 1549681956*d^2 - 5290790400*c*e)/a^8))*log(7/236196*((-I*sqrt
(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118
098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*
e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^
3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/
118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a
^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2
*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)^2*a^9*b*c + 45927*a*b*c*d^
2 - 78400*a*b*c^2*e - 6480*a^2*d*e^2 - 1/243*(567*a^5*b*c*d - 40*a^6*e^2)*(
(-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12
 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64
000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(34
3*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a
^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 +
 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*
(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4) - 800*(343*b^2*c^3
 - 8*a^2*e^3)*x + 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6
561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d
/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*
c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1
/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e
)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 56
00*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))
^(1/3) + 39366*d/a^4)*a^9*b*c - 275562*a^5*b*c*d - 38880*a^6*e^2)*sqrt(-(((
-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12
+ 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 640
00*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343
*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^
12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 +
64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(
343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)^2*a^8 - 78732*((-I*
sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1
/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*
a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^
2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12
+ 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 640
00*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343
*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)*a^4*d + 1549681956*d^2
 - 5290790400*c*e)/a^8)) + (118098*b^3*d*x^10 + 354294*a*b^2*d*x^7 + 354294
*a^2*b*d*x^4 + 118098*a^3*d*x - (a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4
 + a^7*x)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/
27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b
^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/1
4348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(
-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(274400
0*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 400
0/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4) - 3*sqr
t(1/3)*(a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*sqrt(-(((-I*sqr
t(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/11
8098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2
*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c
^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1
```

```
/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*
a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^
2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)^2*a^8 - 78732*((-I*sqrt(3
) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/11809
8*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^
3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3
- 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/11
8098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2
*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c
^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)*a^4*d + 1549681956*d^2 - 529
0790400*c*e)/a^8))*log(7/236196*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2
 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 +
 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*
a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 5
9049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^1
2 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*
e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3)
+ 39366*d/a^4)^2*a^9*b*c + 45927*a*b*c*d^2 - 78400*a*b*c^2*e - 6480*a^2*d*e
^2 - 1/243*(567*a^5*b*c*d - 40*a^6*e^2)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (
6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*
d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600
*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(
1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*
e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5
600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b)
)^(1/3) + 39366*d/a^4) - 800*(343*b^2*c^3 - 8*a^2*e^3)*x - 1/78732*sqrt(1/3
)*(7*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^
3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^
3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/143489
07*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27
*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2
*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/143
48907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)*a^9*b*c - 27
5562*a^5*b*c*d - 38880*a^6*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6
561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d
/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*
c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1
/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e
)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 56
00*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))
^(1/3) + 39366*d/a^4)^2*a^8 - 78732*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561
*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^
12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d
*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3)
 + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d
/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*
c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1
/3) + 39366*d/a^4)*a^4*d + 1549681956*d^2 - 5290790400*c*e)/a^8)) - 236196*
(b^3*d*x^10 + 3*a*b^2*d*x^7 + 3*a^2*b*d*x^4 + a^3*d*x)*log(x))/(a^4*b^3*x^1
0 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)
```

<u>**giac**</u> **[A]** time = 0.18, size = 310, normalized size = 1.03

$$-\frac{d\log\left(\left|bx^3+a\right|\right)}{3\,a^4}+\frac{d\log\left(\left|x\right|\right)}{a^4}+\frac{20\,\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ae+7\,\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b}+\frac{10\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ae-7\,\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}-\frac{280\,b^3cx^9-40\,ab^2x^8e-54\,ab^2dx^7+770\,ab^2cx^6-104\,a^2bx^5e-135\,a^2bdx^4+670\,a^2bcx^3-82\,a^3x^2e-99\,a^3dx+162\,a^3c}{162\left(bx^3+a\right)^3a^4x}+\frac{20\left(7\,a^4b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^5be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^4 + d*log(abs(x))/a^4 + 20/243*sqrt(3)*(2*(-a*
b^2)^(1/3)*a*e + 7*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
```

```
)/(-a/b)^(1/3))/(a^5*b) + 10/243*(2*(-a*b^2)^(1/3)*a*e - 7*(-a*b^2)^(2/3)*c
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/162*(280*b^3*c*x^9 -
 40*a*b^2*x^8*e - 54*a*b^2*d*x^7 + 770*a*b^2*c*x^6 - 104*a^2*b*x^5*e - 135*
a^2*b*d*x^4 + 670*a^2*b*c*x^3 - 82*a^3*x^2*e - 99*a^3*d*x + 162*a^3*c)/((b*
x^3 + a)^3*a^4*x) + 20/243*(7*a^4*b^2*c*(-a/b)^(1/3) - 2*a^5*b*e)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b)
```

maple [A] time = 0.07, size = 397, normalized size = 1.32

$$-\frac{59b^3c\,x^8}{81\left(b\,x^3+a\right)^3a^4}+\frac{20b^2e\,x^7}{81\left(b\,x^3+a\right)^3a^3}+\frac{b^2d\,x^6}{3\left(b\,x^3+a\right)^3a^3}-\frac{142b^2c\,x^5}{81\left(b\,x^3+a\right)^3a^3}+\frac{52be\,x^4}{81\left(b\,x^3+a\right)^3a^2}+\frac{5bd\,x^3}{6\left(b\,x^3+a\right)^3a^2}-\frac{92bc\,x^2}{81\left(b\,x^3+a\right)^3a^2}+\frac{41ex}{81\left(b\,x^3+a\right)^3a}+\frac{11d}{18\left(b\,x^3+a\right)^3a}+\frac{40\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3b}+\frac{40e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3b}-\frac{20e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3b}-\frac{140\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}+\frac{140c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}-\frac{70c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}+\frac{d\ln(x)}{a^4}-\frac{d\ln\left(b\,x^3+a\right)}{3a^4}-\frac{c}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x)
```

```
[Out] -59/81/a^4/(b*x^3+a)^3*b^3*c*x^8+20/81/a^3/(b*x^3+a)^3*b^2*e*x^7+1/3/a^3/(b
*x^3+a)^3*b^2*d*x^6-142/81/a^3/(b*x^3+a)^3*b^2*c*x^5+52/81/a^2/(b*x^3+a)^3*
b*e*x^4+5/6/a^2/(b*x^3+a)^3*b*d*x^3-92/81/a^2/(b*x^3+a)^3*b*c*x^2+41/81/a/(
b*x^3+a)^3*e*x+11/18/a/(b*x^3+a)^3*d+40/243/a^3*e/b/(a/b)^(2/3)*ln(x+(a/b)^
(1/3))-20/243/a^3*e/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+40/243/
a^3*e/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+140/243
/a^4*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-70/243/a^4*c/(a/b)^(1/3)*ln(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3))-140/243/a^4*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))-1/3*d*ln(b*x^3+a)/a^4-c/a^4/x+d*ln(x)/a^4
```

maxima [A] time = 3.00, size = 313, normalized size = 1.04

$$-\frac{280\,b^3cx^9-40\,ab^2ex^8-54\,ab^2dx^7+770\,ab^2cx^6-104\,a^2bex^5-135\,a^2bdx^4+670\,a^2bcx^3-82\,a^3ex^2-99\,a^3dx+162\,a^3c}{162\left(a^4b^3x^{10}+3\,a^5b^2x^7+3\,a^6bx^4+a^7x\right)}+\frac{d\log(x)}{a^4}-\frac{20\sqrt{3}\left(7\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5}-\frac{\left(81\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+70\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(81\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-140\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-40\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] -1/162*(280*b^3*c*x^9 - 40*a*b^2*e*x^8 - 54*a*b^2*d*x^7 + 770*a*b^2*c*x^6 -
 104*a^2*b*e*x^5 - 135*a^2*b*d*x^4 + 670*a^2*b*c*x^3 - 82*a^3*e*x^2 - 99*a^
3*d*x + 162*a^3*c)/(a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x) + d
*log(x)/a^4 - 20/243*sqrt(3)*(7*b*c*(a/b)^(2/3) - 2*a*e*(a/b)^(1/3))*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 - 1/243*(81*b*d*(a/b)^(2/
3) + 70*b*c*(a/b)^(1/3) + 20*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^4*b*(a/b)^(2/3)) - 1/243*(81*b*d*(a/b)^(2/3) - 140*b*c*(a/b)^(1/3) - 40*a*
e)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

mupad [B] time = 5.43, size = 840, normalized size = 2.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x)
```

```
[Out] ((41*e*x^2)/(81*a) - c/a + (11*d*x)/(18*a) - (385*b^2*c*x^6)/(81*a^3) - (14
0*b^3*c*x^9)/(81*a^4) + (b^2*d*x^7)/(3*a^3) + (20*b^2*e*x^8)/(81*a^3) - (33
5*b*c*x^3)/(81*a^2) + (5*b*d*x^4)/(6*a^2) + (52*b*e*x^5)/(81*a^2))/(a^3*x +
 b^3*x^10 + 3*a^2*b*x^4 + 3*a*b^2*x^7) + symsum(log((4*b^2*(32400*a^2*d*e^2
 - 32400*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*
e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*
e^3 - 2744000*b^2*c^3, z, k)*a^6*e^2 + 686000*b^2*c^3*x + 16000*a^2*e^3*x +
 229635*a*b*c*d^2 - 688905*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2
- 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*
b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)^2*a^9*b*c - 4782969*root(143
```

```
48907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5
*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2
*c^3, z, k)^3*a^13*b*x - 531441*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d
*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 5314
41*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)*a^5*b*d^2*x - 3188646*r
oot(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782
969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744
000*b^2*c^3, z, k)^2*a^9*b*d*x + 459270*root(14348907*a^13*b*z^3 + 14348907
*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*
e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)*a^5*b*c*d + 113
4000*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z
+ 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3
- 2744000*b^2*c^3, z, k)*a^5*b*c*e*x + 226800*a*b*c*d*e*x))/(531441*a^11))*
root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 478
2969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 274
4000*b^2*c^3, z, k), k, 1, 3) + (d*log(x))/a^4
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**4,x)
[Out] Timed out
```

3.310 $\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx$

Optimal. Leaf size=310

$$\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{14/3}}+\frac{20\sqrt[3]{b}\left(7\sqrt[3]{a}\,d\right.}{}$$

Rubi [A] time = 0.66, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{x\left(139bc+118bdx+99bex^2\right)}{162a^4\left(a+bx^3\right)}-\frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}\,c-7\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{14/3}}+\frac{20\sqrt[3]{b}\left(7\sqrt[3]{a}\,d+11\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{14/3}}-\frac{e\log\left(a+bx^3\right)}{3a^4}-\frac{c}{2a^4x^2}-\frac{d}{a^4x}+\frac{e\log(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

```
[Out] -c/(2*a^4*x^2) - d/(a^4*x) - (x*(b*c + b*d*x + b*e*x^2))/(9*a^2*(a + b*x^3)
^3) - (x*(17*b*c + 16*b*d*x + 15*b*e*x^2))/(54*a^3*(a + b*x^3)^2) - (x*(139
*b*c + 118*b*d*x + 99*b*e*x^2))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(
1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(8
1*Sqrt[3]*a^(14/3)) + (e*Log[x])/a^4 - (20*b^(1/3)*(11*b^(1/3)*c - 7*a^(1/3
)*d)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*c -
 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/3)
) - (e*Log[a + b*x^3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx = -\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-9bdx-9bex^2+\frac{8b^2cx^3}{a}+\frac{7b^2dx^4}{a}+\frac{6b^2ex^5}{a}}{x^3\left(a+bx^3\right)^3}\,dx}{9ab}$$

$$= -\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^3c+54b^3dx+54b^3ex^2-\frac{85b^4cx^3}{a}-\frac{64b^4dx^4}{a}-4\cdots}{x^3\left(a+bx^3\right)^2}}{54a^2b^3}$$

$$= -\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+99bex^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int \frac{-1\cdots}{\cdots}}{\cdots}$$

$$= -\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+99bex^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int \left(-\cdots\right.}{\cdots}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+\cdots\right.}{162a^4\left(a+bx^3\cdots\right.}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+\cdots\right.}{162a^4\left(a+bx^3\cdots\right.}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+\cdots\right.}{162a^4\left(a+bx^3\cdots\right.}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+\cdots\right.}{162a^4\left(a+bx^3\cdots\right.}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+\cdots\right.}{162a^4\left(a+bx^3\cdots\right.}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+\cdots\right.}{162a^4\left(a+bx^3\cdots\right.}$$

Mathematica [A] time = 0.31, size = 284, normalized size = 0.92

$$\frac{20\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+40\sqrt[3]{b}\left(7a^{2/3}d-11\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{54a^3(ae-bx(c+dx))}{\left(a+bx^3\right)^3}+\frac{9a^2(9ae-bx(17c+16dx))}{\left(a+bx^3\right)^2}+\frac{3a(54ae-bx(139c+118dx))}{a+bx^3}+40\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(7\sqrt[3]{a}d+11\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-162ae\log\left(a+bx^3\right)-\frac{243ac}{x^2}-\frac{486ad}{x}+486ae\log(x)}{486a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

```
[Out] ((-243*a*c)/x^2 - (486*a*d)/x + (54*a^3*(a*e - b*x*(c + d*x)))/(a + b*x^3)^
3 + (9*a^2*(9*a*e - b*x*(17*c + 16*d*x)))/(a + b*x^3)^2 + (3*a*(54*a*e - b*
x*(139*c + 118*d*x)))/(a + b*x^3) + 40*Sqrt[3]*a^(1/3)*b^(1/3)*(11*b^(1/3)*
c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 486*a*e*Log[
x] + 40*b^(1/3)*(-11*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)
*x] + 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2] - 162*a*e*Log[a + b*x^3])/(486*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2}{x^3 \left(a + bx^3\right)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4), x]
```

fricas [C] time = 1.45, size = 5049, normalized size = 16.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] -1/236196*(408240*b^3*d*x^10 + 320760*b^3*c*x^9 - 78732*a*b^2*e*x^8 + 11226
60*a*b^2*d*x^7 + 833976*a*b^2*c*x^6 - 196830*a^2*b*e*x^5 + 976860*a^2*b*d*x
^4 + 657558*a^2*b*c*x^3 - 144342*a^3*e*x^2 + 236196*a^3*d*x + 118098*a^3*c
+ 2*(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*((-I*sqrt(3) + 1
)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/11809
8*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3
)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 -
2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/11
8098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*
d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3
 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*log(7/236196*((-I*sqrt(3) +
1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/1180
98*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^
3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 -
 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/1
18098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a
*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^
3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^10*d + 431200*a*b*c*d^2
 - 196020*a*b*c^2*e + 45927*a^2*d*e^2 + 1/243*(1210*a^5*b*c^2 - 567*a^6*d*e
)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*
e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331
*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3
- 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/
27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1
331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e
^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4) + 400*(133
1*b^2*c^3 + 343*a*b*d^3)*x) + (118098*b^3*e*x^11 + 354294*a*b^2*e*x^8 + 354
294*a^2*b*e*x^5 + 118098*a^3*e*x^2 - (a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*
b*x^5 + a^7*x^2)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^
2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000
/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 +
531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sq
rt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4
000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3
 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/
a^4) - 3*sqrt(1/3)*(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*s
qrt(-(((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1
/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(
1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*
e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*
(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/1434890
7*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a
```

^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^9 - 78732*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907 *(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^ 2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348 907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441 *a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*a^5* e + 29099347200*b*c*d + 1549681956*a*e^2)/a^9))*log(-7/236196*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/11 8098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a* d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1 /118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343 *a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980* d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^10*d - 431200*a*b*c*d ^2 + 196020*a*b*c^2*e - 45927*a^2*d*e^2 - 1/243*(1210*a^5*b*c^2 - 567*a^6*d *e)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/2 7*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(13 31*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^ 3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(- 1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907* (1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2 *e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4) + 800*(1 331*b^2*c^3 + 343*a*b*d^3)*x + 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(6561 *e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(3080 0*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^1 4 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c* d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(3 0800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/ a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673 *c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*a^10*d - 1176120*a^5*b*c^2 - 275562 *a^6*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^ 2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000 /14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sq rt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4 000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/ a^4)^2*a^9 - 78732*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a* e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 40 00/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I* sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c ^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366* e/a^4)*a^5*e + 29099347200*b*c*d + 1549681956*a*e^2)/a^9)) + (118098*b^3*e* x^11 + 354294*a*b^2*e*x^8 + 354294*a^2*b*e*x^5 + 118098*a^3*e*x^2 - (a^4*b^ 3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*((-I*sqrt(3) + 1)*(6561*e^2 /a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b* c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e) *a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800 *b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d *e)*a*b)/a^14)^(1/3) + 39366*e/a^4) + 3*sqrt(1/3)*(a^4*b^3*x^11 + 3*a^5*b^2 *x^8 + 3*a^6*b*x^5 + a^7*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (308 00*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561* a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*

```
(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)
^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 65
61*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/286978
14*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^
14)^(1/3) + 39366*e/a^4)^2*a^9 - 78732*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (3
0800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 656
1*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/2869781
4*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^1
4)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d +
6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/2869
7814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/
a^14)^(1/3) + 39366*e/a^4)*a^5*e + 29099347200*b*c*d + 1549681956*a*e^2)/a^
9))*log(-7/236196*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e
^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 400
0/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 +
 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*s
qrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 +
4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^
3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e
/a^4)^2*a^10*d - 431200*a*b*c*d^2 + 196020*a*b*c^2*e - 45927*a^2*d*e^2 - 1/
243*(1210*a^5*b*c^2 - 567*a^6*d*e)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800
*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*
e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(1
0648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(
1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561
*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814
*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14
)^(1/3) + 39366*e/a^4) + 800*(1331*b^2*c^3 + 343*a*b*d^3)*x - 1/78732*sqrt(
1/3)*(7*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(
-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907
*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^
2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1
)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348
907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441
*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*a^10
*d - 1176120*a^5*b*c^2 - 275562*a^6*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(6561*e^2
/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*
c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 -
1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)
*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800
*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14
 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d
*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^9 - 78732*((-I*sqrt(3) + 1)*(6561*e
^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*
b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14
- 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*
e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(308
00*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^
14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c
*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*a^5*e + 29099347200*b*c*d + 154968195
6*a*e^2)/a^9)) - 236196*(b^3*e*x^11 + 3*a*b^2*e*x^8 + 3*a^2*b*e*x^5 + a^3*e
*x^2)*log(x))/(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)
```

giac [A] time = 0.18, size = 320, normalized size = 1.03

$$-\frac{e\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{4}}+\frac{e\log\left(\left|x\right|\right)}{a^{4}}-\frac{20\,\sqrt{3}\left(11\,\left(-ab^{2}\right)^{\frac{1}{3}}bc-7\,\left(-ab^{2}\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^{5}b}-\frac{10\left(11\,\left(-ab^{2}\right)^{\frac{1}{3}}bc+7\,\left(-ab^{2}\right)^{\frac{2}{3}}d\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^{5}b}+\frac{20\left(7\,a^{4}b^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+11\,a^{4}b^{2}c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{9}b}-\frac{280\,b^{3}dx^{10}+220\,b^{3}cx^{9}-54\,ab^{2}x^{8}e+770\,ab^{2}dx^{7}+572\,ab^{2}cx^{6}-135\,a^{2}bx^{5}e+670\,a^{2}bdx^{4}+451\,a^{2}bcx^{3}-99\,a^{3}x^{2}e+162\,a^{3}dx+81\,a^{3}c}{162\,\left(bx^{3}+a\right)^{3}a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^4 + e*log(abs(x))/a^4 - 20/243*sqrt(3)*(11*(-a
*b^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(a^5*b) - 10/243*(11*(-a*b^2)^(1/3)*b*c + 7*(-a*b^2)^(2/3)
*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) + 20/243*(7*a^4*b^2*d*
(-a/b)^(1/3) + 11*a^4*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b
) - 1/162*(280*b^3*d*x^10 + 220*b^3*c*x^9 - 54*a*b^2*x^8*e + 770*a*b^2*d*x^
7 + 572*a*b^2*c*x^6 - 135*a^2*b*x^5*e + 670*a^2*b*d*x^4 + 451*a^2*b*c*x^3 -
 99*a^3*x^2*e + 162*a^3*d*x + 81*a^3*c)/((b*x^3 + a)^3*a^4*x^2)
```

maple [A] time = 0.07, size = 400, normalized size = 1.29

$$-\frac{59b^3dx^8}{81\left(bx^3+a\right)^3a^4}-\frac{139b^3cx^7}{162\left(bx^3+a\right)^3a^4}+\frac{b^2ex^6}{3\left(bx^3+a\right)^3a^3}-\frac{142b^2dx^5}{81\left(bx^3+a\right)^3a^3}-\frac{329b^2cx^4}{162\left(bx^3+a\right)^3a^3}+\frac{5bex^3}{6\left(bx^3+a\right)^3a^2}-\frac{92bdx^2}{81\left(bx^3+a\right)^3a^2}-\frac{104bcx}{81\left(bx^3+a\right)^3a^2}+\frac{11e}{18\left(bx^3+a\right)^3a}-\frac{220\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4}-\frac{220c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4}+\frac{110c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4}-\frac{140\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}+\frac{140d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}-\frac{70d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}+\frac{e\ln(x)}{a^4}-\frac{e\ln\left(bx^3+a\right)}{3a^4}-\frac{d}{a^4x}-\frac{c}{2a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x)
```

```
[Out] -59/81/a^4*b^3/(b*x^3+a)^3*d*x^8-139/162/a^4*b^3/(b*x^3+a)^3*c*x^7+1/3/a^3*
b^2/(b*x^3+a)^3*e*x^6-142/81/a^3*b^2/(b*x^3+a)^3*d*x^5-329/162/a^3*b^2/(b*x
^3+a)^3*c*x^4+5/6/a^2*b/(b*x^3+a)^3*e*x^3-92/81/a^2*b/(b*x^3+a)^3*d*x^2-104
/81/a^2*b/(b*x^3+a)^3*c*x+11/18/a/(b*x^3+a)^3*e-220/243/a^4*c/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))+110/243/a^4*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)-220/243/a^4*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
+140/243/a^4*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-70/243/a^4*d/(a/b)^(1/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))-140/243/a^4*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*e*ln(b*x^3+a)/a^4-1/2*c/a^4/x^2-d/a^4/x+e*l
n(x)/a^4
```

maxima [A] time = 3.10, size = 312, normalized size = 1.01

$$-\frac{280\,b^3dx^{10}+220\,b^3cx^9-54\,ab^2ex^8+770\,ab^2dx^7+572\,ab^2cx^6-135\,a^2bex^5+670\,a^2bdx^4+451\,a^2bcx^3-99\,a^3ex^2+162\,a^3dx+81\,a^3c}{162\left(a^4b^3x^{11}+3\,a^5b^2x^8+3\,a^6bx^5+a^7x^2\right)}+\frac{e\log(x)}{a^4}-\frac{20\sqrt{3}\left(7\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+11\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5}-\frac{\left(81\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+70\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-110\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(81\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}-140\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+220\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] -1/162*(280*b^3*d*x^10 + 220*b^3*c*x^9 - 54*a*b^2*e*x^8 + 770*a*b^2*d*x^7 +
 572*a*b^2*c*x^6 - 135*a^2*b*e*x^5 + 670*a^2*b*d*x^4 + 451*a^2*b*c*x^3 - 99
*a^3*e*x^2 + 162*a^3*d*x + 81*a^3*c)/(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*
b*x^5 + a^7*x^2) + e*log(x)/a^4 - 20/243*sqrt(3)*(7*b*d*(a/b)^(2/3) + 11*b*
c*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 - 1/
243*(81*e*(a/b)^(2/3) + 70*d*(a/b)^(1/3) - 110*c)*log(x^2 - x*(a/b)^(1/3) +
 (a/b)^(2/3))/(a^4*(a/b)^(2/3)) - 1/243*(81*e*(a/b)^(2/3) - 140*d*(a/b)^(1/
3) + 220*c)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3))
```

mupad [B] time = 5.38, size = 825, normalized size = 2.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x)
```

```
[Out] symsum(log(-(4*b^3*(688905*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 2
2453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d
^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)^2*a^10*d - 229635*a^2*d*e^2 +
 4782969*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*
z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^
3 + 10648000*b^2*c^3, z, k)^3*a^14*x + 2662000*b^2*c^3*x - 459270*root(1434
8907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^
```

```
2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c
^3, z, k)*a^6*d*e - 980100*a*b*c^2*e - 686000*a*b*d^3*x + 980100*root(14348
907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^2
*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c^
3, z, k)*a^5*b*c^2 + 531441*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 +
22453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*
d^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)*a^6*e^2*x + 3188646*root(143
48907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e
^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*
c^3, z, k)^2*a^10*e*x + 6237000*root(14348907*a^14*z^3 + 14348907*a^10*e*z^
2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*
a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)*a^5*b*c*d*x + 1247400*a*
b*c*d*e*x))/(531441*a^12))*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 2
2453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d
^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k), k, 1, 3) - (c/(2*a) - (11*e*
x^2)/(18*a) + (d*x)/a + (286*b^2*c*x^6)/(81*a^3) + (110*b^3*c*x^9)/(81*a^4)
 + (385*b^2*d*x^7)/(81*a^3) + (140*b^3*d*x^10)/(81*a^4) - (b^2*e*x^8)/(3*a^
3) + (451*b*c*x^3)/(162*a^2) + (335*b*d*x^4)/(81*a^2) - (5*b*e*x^5)/(6*a^2)
)/(a^3*x^2 + b^3*x^11 + 3*a^2*b*x^5 + 3*a*b^2*x^8) + (e*log(x))/a^4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

3.311 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx$

Optimal. Leaf size=340

$$\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{14/3}}+\frac{20\sqrt[3]{b}\left(7\sqrt[3]{a}\,e\right.}{}$$

Rubi [A] time = 0.77, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{x\left(-\frac{234b^2cx^2}{a}+139bd+118bex\right)}{162a^4\left(a+bx^3\right)}-\frac{x\left(-\frac{24b^2cx^2}{a}+17bd+16bex\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2\left(a+bx^3\right)^3}+\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{243a^{14/3}}+\frac{4bc\log\left(a+bx^3\right)}{3a^5}-\frac{4bc\log(x)}{a^5}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}\,d-7\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{243a^{14/3}}+\frac{20\sqrt[3]{b}\left(7\sqrt[3]{a}\,e+11\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{81\sqrt{3}\,a^{14/3}}-\frac{c}{3a^4x^3}-\frac{d}{2a^4x^2}-\frac{e}{a^4x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

```
[Out] -c/(3*a^4*x^3) - d/(2*a^4*x^2) - e/(a^4*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/
a))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*d + 16*b*e*x - (24*b^2*c*x^2)/a))/(54*
a^3*(a + b*x^3)^2) - (x*(139*b*d + 118*b*e*x - (234*b^2*c*x^2)/a))/(162*a^4
*(a + b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(14/3)) - (4*b*c*Log[x])/a^5
 - (20*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*
a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/3)) + (4*b*c*Log[a + b*x^3])/(3*a^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-9bdx-9bex^2+\frac{9b^2cx^3}{a}+\frac{8b^2dx^4}{a}+\frac{7b^2ex^5}{a}-\frac{6b^3cx^6}{a^2}}{x^4\left(a+bx^3\right)^3}\,dx}{9ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^3c+54b^3dx+54b^3ex^2-\frac{108b^4cx^3}{a}-\frac{85b^4dx^4}{a}-\cdots}{x^4\left(a+bx^3\right)^2}}{54a^2b^3}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)} - \int \frac{-1\cdots}{\cdots}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)} - \frac{\int \left(-\cdots\right.}{\cdots}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+1\cdots\right.}{162a^4\cdots}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+1\cdots\right.}{162a^4\cdots}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+1\cdots\right.}{162a^4\cdots}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+1\cdots\right.}{162a^4\cdots}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+1\cdots\right.}{162a^4\cdots}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+1\cdots\right.}{162a^4\cdots}$$

<u>**Mathematica**</u> **[A]** time = 0.60, size = 284, normalized size = 0.84

$$-\frac{-20\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+40\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{54a^3b(c+x(d+ex))}{\left(a+bx^3\right)^3}+\frac{9a^2b(18c+x(17d+16ex))}{\left(a+bx^3\right)^2}+\frac{3ab(162c+x(139d+118ex))}{a+bx^3}-648bc\log\left(a+bx^3\right)-40\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(7\sqrt[3]{a}e+11\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\frac{162ac}{x^3}+\frac{243ad}{x^2}+\frac{486ae}{x}+1944bc\log(x)}{486a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

```
[Out] -1/486*((162*a*c)/x^3 + (243*a*d)/x^2 + (486*a*e)/x + (54*a^3*b*(c + x*(d +
 e*x)))/(a + b*x^3)^3 + (9*a^2*b*(18*c + x*(17*d + 16*e*x)))/(a + b*x^3)^2
+ (3*a*b*(162*c + x*(139*d + 118*e*x)))/(a + b*x^3) - 40*Sqrt[3]*a^(1/3)*b^
(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 1944*b*c*Log[x] + 40*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log
[a^(1/3) + b^(1/3)*x] - 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 648*b*c*Log[a + b*x^3])/a^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4), x]
```

fricas [C] time = 2.03, size = 5670, normalized size = 16.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] -1/486*(840*a*b^3*e*x^11 + 660*a*b^3*d*x^10 + 648*a*b^3*c*x^9 + 2310*a^2*b^
2*e*x^8 + 1716*a^2*b^2*d*x^7 + 1620*a^2*b^2*c*x^6 + 2010*a^3*b*e*x^5 + 1353
*a^3*b*d*x^4 + 1188*a^3*b*c*x^3 + 486*a^4*e*x^2 + 243*a^4*d*x + 162*a^4*c +
 2*(a^5*b^3*x^12 + 3*a^6*b^2*x^9 + 3*a^7*b*x^6 + a^8*x^3)*(4^(2/3)*(-I*sqrt
(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*
b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 19
25*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1
701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^
15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e
)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)
*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*log(7*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^
2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 12
5*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a
^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)
/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*
d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531
441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/
3) - 324*b*c/a^5)^2*a^10*e + 784080*b^2*c*d^2 + 734832*b^2*c^2*e + 431200*a
*b*d*e^2 + 4*(605*a^5*b*d^2 + 1134*a^5*b*c*e)*(4^(2/3)*(-I*sqrt(3) + 1)*(65
61*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15
 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*
b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a
*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(13
31*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 +
 (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15
)^(1/3) - 324*b*c/a^5) + 400*(1331*b^2*d^3 + 343*a*b*e^3)*x) - (972*b^4*c*x
^12 + 2916*a*b^3*c*x^9 + 2916*a^2*b^2*c*x^6 + 972*a^3*b*c*x^3 + (a^5*b^3*x^
12 + 3*a^6*b^2*x^9 + 3*a^7*b*x^6 + a^8*x^3)*(4^(2/3)*(-I*sqrt(3) + 1)*(6561
*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 +
 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*
c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b
^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331
*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (
531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^
(1/3) - 324*b*c/a^5) + 3*sqrt(1/3)*(a^5*b^3*x^12 + 3*a^6*b^2*x^9 + 3*a^7*b*
x^6 + a^8*x^3)*sqrt(-((4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*
b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343
*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c
^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1
/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/
a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875
*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)^2
```

```
*a^10 + 648*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 +
1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/
a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875
*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqr
t(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243
*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3
 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^5*b*c + 1
04976*b^2*c^2 + 123200*a*b*d*e)/a^10))*log(-7*(4^(2/3)*(-I*sqrt(3) + 1)*(65
61*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15
 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*
b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a
*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(13
31*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 +
 (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15
)^(1/3) - 324*b*c/a^5)^2*a^10*e - 784080*b^2*c*d^2 - 734832*b^2*c^2*e - 431
200*a*b*d*e^2 - 4*(605*a^5*b*d^2 + 1134*a^5*b*c*e)*(4^(2/3)*(-I*sqrt(3) + 1
)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3
/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*
d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d
*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 12
5*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a
^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)
/a^15)^(1/3) - 324*b*c/a^5) + 800*(1331*b^2*d^3 + 343*a*b*e^3)*x + 3*sqrt(1
/3)*(7*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*
a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14
- 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*
b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3)
+ 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(656
1*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 27
5*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^10*e - 2420*a^
5*b*d^2 + 2268*a^5*b*c*e)*sqrt(-((4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^
10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*
b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (5
31441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(
1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 34
3*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*
c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324
*b*c/a^5)^2*a^10 + 648*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561
*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 34
3*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*
c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(
1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b
/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 4287
5*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*
a^5*b*c + 104976*b^2*c^2 + 123200*a*b*d*e)/a^10)) - (972*b^4*c*x^12 + 2916*
a*b^3*c*x^9 + 2916*a^2*b^2*c*x^6 + 972*a^3*b*c*x^3 + (a^5*b^3*x^12 + 3*a^6*
b^2*x^9 + 3*a^7*b*x^6 + a^8*x^3)*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^
10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*
b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (5
31441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(
1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 34
3*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*
c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324
*b*c/a^5) - 3*sqrt(1/3)*(a^5*b^3*x^12 + 3*a^6*b^2*x^9 + 3*a^7*b*x^6 + a^8*x
^3)*sqrt(-((4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1
925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a
^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*
a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt
(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*
```

```
(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3
- 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)^2*a^10 + 648
*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*
e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*
(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3
- 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(
1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*
c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605
*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^5*b*c + 104976*b^2*c
^2 + 123200*a*b*d*e)/a^10))*log(-7*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/
a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(133
1*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 +
(531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)
^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 +
343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^
3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 3
24*b*c/a^5)^2*a^10*e - 784080*b^2*c*d^2 - 734832*b^2*c^2*e - 431200*a*b*d*e
^2 - 4*(605*a^5*b*d^2 + 1134*a^5*b*c*e)*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2
*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125
*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^
15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/
a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d
^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (5314
41*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3
) - 324*b*c/a^5) + 800*(1331*b^2*d^3 + 343*a*b*e^3)*x - 3*sqrt(1/3)*(7*(4^(
2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^
10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561
*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275
*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(10628
82*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 +
 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3
- 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^10*e - 2420*a^5*b*d^2 + 2
268*a^5*b*c*e)*sqrt(-((4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*
b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343
*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c
^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1
/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/
a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875
*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)^2
*a^10 + 648*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 +
1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/
a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875
*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqr
t(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243
*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3
 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^5*b*c + 1
04976*b^2*c^2 + 123200*a*b*d*e)/a^10)) + 1944*(b^4*c*x^12 + 3*a*b^3*c*x^9 +
 3*a^2*b^2*c*x^6 + a^3*b*c*x^3)*log(x))/(a^5*b^3*x^12 + 3*a^6*b^2*x^9 + 3*a
^7*b*x^6 + a^8*x^3)
```

giac [**A**] time = 0.20, size = 333, normalized size = 0.98

$$\frac{4\,bc\log\left(\left|bx^3+a\right|\right)}{3\,a^5}-\frac{4\,bc\log\left(\left|x\right|\right)}{a^5}-\frac{20\,\sqrt{3}\left(11\,\left(-ab^2\right)^{\frac{1}{3}}bd-7\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b}-\frac{10\left(11\,\left(-ab^2\right)^{\frac{1}{3}}bd+7\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}-\frac{280\,b^3x^{11}e+220\,b^3dx^{10}+216\,b^3cx^9+770\,ab^2x^8e+572\,ab^2dx^7+540\,ab^2cx^6+670\,a^2bx^5e+451\,a^2bdx^4+396\,a^2bcx^3+162\,a^3x^2e+81\,a^3dx+54\,a^3c}{162\left(bx^4+ax\right)^3a^4}+\frac{20\left(7\,a^6b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+11\,a^6b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{11}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 4/3*b*c*log(abs(b*x^3 + a))/a^5 - 4*b*c*log(abs(x))/a^5 - 20/243*sqrt(3)*(1
1*(-a*b^2)^(1/3)*b*d - 7*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
```

```
^(1/3))/(-a/b)^(1/3))/(a^5*b) - 10/243*(11*(-a*b^2)^(1/3)*b*d + 7*(-a*b^2)^
(2/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/162*(280*b^3*
x^11*e + 220*b^3*d*x^10 + 216*b^3*c*x^9 + 770*a*b^2*x^8*e + 572*a*b^2*d*x^7
 + 540*a*b^2*c*x^6 + 670*a^2*b*x^5*e + 451*a^2*b*d*x^4 + 396*a^2*b*c*x^3 +
162*a^3*x^2*e + 81*a^3*d*x + 54*a^3*c)/((b*x^4 + a*x)^3*a^4) + 20/243*(7*a^
6*b^2*(-a/b)^(1/3)*e + 11*a^6*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3))
)/(a^11*b)
```

maple [A] time = 0.07, size = 415, normalized size = 1.22

$$-\frac{59b^3ex^8}{81(bx^3+a)^3a^4}-\frac{139b^3dx^7}{162(bx^3+a)^3a^4}-\frac{b^3cx^6}{(bx^3+a)^3a^4}-\frac{142b^2ex^5}{81(bx^3+a)^3a^3}-\frac{329b^2dx^4}{162(bx^3+a)^3a^3}-\frac{7b^2cx^3}{3(bx^3+a)^3a^3}-\frac{92bex^2}{81(bx^3+a)^3a^2}-\frac{104bdx}{81(bx^3+a)^3a^2}-\frac{13bc}{9(bx^3+a)^3a^2}-\frac{220\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4}-\frac{220d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4}+\frac{110d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4}-\frac{140\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}+\frac{140e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}-\frac{70e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}-\frac{4bc\ln(x)}{a^5}+\frac{4bc\ln\left(bx^3+a\right)}{3a^5}-\frac{e}{a^4x}-\frac{d}{2a^4x^2}-\frac{c}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x)
```

```
[Out] -59/81/a^4*b^3/(b*x^3+a)^3*e*x^8-139/162/a^4*b^3/(b*x^3+a)^3*d*x^7-1/a^4*b^
3/(b*x^3+a)^3*c*x^6-142/81/a^3*b^2/(b*x^3+a)^3*e*x^5-329/162/a^3*b^2/(b*x^3
+a)^3*d*x^4-7/3/a^3*b^2/(b*x^3+a)^3*c*x^3-92/81/a^2*b/(b*x^3+a)^3*e*x^2-104
/81/a^2*b/(b*x^3+a)^3*d*x-13/9/a^2*b/(b*x^3+a)^3*c-220/243/a^4*d/(a/b)^(2/3
)*ln(x+(a/b)^(1/3))+110/243/a^4*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))-220/243/a^4*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))+140/243/a^4*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-70/243/a^4*e/(a/b)^(1/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-140/243/a^4*e*3^(1/2)/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+4/3*b*c*ln(b*x^3+a)/a^5-1/3*c/a^4/x^3-1/2*d/
a^4/x^2-e/a^4/x-4*b*c*ln(x)/a^5
```

maxima [A] time = 3.08, size = 330, normalized size = 0.97

$$-\frac{280\,b^3ex^{11}+220\,b^3dx^{10}+216\,b^3cx^9+770\,ab^2ex^8+572\,ab^2dx^7+540\,ab^2cx^6+670\,a^2bex^5+451\,a^2bdx^4+396\,a^2bcx^3+162\,a^3ex^2+81\,a^3dx+54\,a^3c}{162\left(a^4b^3x^{12}+3\,a^5b^2x^9+3\,a^6bx^6+a^7x^3\right)}-\frac{4\,bc\log(x)}{a^5}-\frac{20\,\sqrt{3}\left(7\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+11\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^6}+\frac{2\left(162\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-35\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+55\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+35\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-55\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] -1/162*(280*b^3*e*x^11 + 220*b^3*d*x^10 + 216*b^3*c*x^9 + 770*a*b^2*e*x^8 +
 572*a*b^2*d*x^7 + 540*a*b^2*c*x^6 + 670*a^2*b*e*x^5 + 451*a^2*b*d*x^4 + 39
6*a^2*b*c*x^3 + 162*a^3*e*x^2 + 81*a^3*d*x + 54*a^3*c)/(a^4*b^3*x^12 + 3*a^
5*b^2*x^9 + 3*a^6*b*x^6 + a^7*x^3) - 4*b*c*log(x)/a^5 - 20/243*sqrt(3)*(7*a
*e*(a/b)^(2/3) + 11*a*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3
))/(a/b)^(1/3))/a^6 + 2/243*(162*b*c*(a/b)^(2/3) - 35*a*e*(a/b)^(1/3) + 55*
a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(2/3)) + 4/243*(81*b
*c*(a/b)^(2/3) + 35*a*e*(a/b)^(1/3) - 55*a*d)*log(x + (a/b)^(1/3))/(a^5*(a/
b)^(2/3))
```

mupad [B] time = 0.52, size = 918, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x)
```

```
[Out] symsum(log(-(4*b^3*(688905*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 +
 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 274
4000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)^2*a^10*e + 39
20400*b^2*c*d^2 - 3674160*b^2*c^2*e + 4782969*root(14348907*a^15*z^3 - 5739
5628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 2993760
0*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3,
z, k)^3*a^14*x + 2662000*b^2*d^3*x - 686000*a*b*e^3*x + 980100*root(1434890
```

```
7*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^
2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 3
4012224*b^3*c^3, z, k)*a^5*b*d^2 - 12754584*root(14348907*a^15*z^3 - 573956
28*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*
a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z,
 k)^2*a^9*b*c*x + 8503056*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 +
22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744
000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^4*b^2*c^2*x
+ 1837080*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d
*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 +
10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^5*b*c*e - 4989600*b^2*c*d*e*
x + 6237000*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b
*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3
+ 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^5*b*d*e*x))/(531441*a^12))
*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76
527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*
a*b^2*d^3 - 34012224*b^3*c^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x
)/(2*a) + (10*b^2*c*x^6)/(3*a^3) + (4*b^3*c*x^9)/(3*a^4) + (286*b^2*d*x^7)/
(81*a^3) + (110*b^3*d*x^10)/(81*a^4) + (385*b^2*e*x^8)/(81*a^3) + (140*b^3*
e*x^11)/(81*a^4) + (22*b*c*x^3)/(9*a^2) + (451*b*d*x^4)/(162*a^2) + (335*b*
e*x^5)/(81*a^2))/(a^3*x^3 + b^3*x^12 + 3*a^2*b*x^6 + 3*a*b^2*x^9) - (4*b*c*
log(x))/a^5
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

3.312 $\int \frac{2ax-x^2}{a^3+x^3} dx$

Optimal. Leaf size=29

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1593, 1868, 31, 617, 204}

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a*x - x^2)/(a^3 + x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{2ax-x^2}{a^3+x^3}\,dx &= \int \frac{(2a-x)x}{a^3+x^3}\,dx\\ &= a\int \frac{1}{a^2-ax+x^2}\,dx - \int \frac{1}{a+x}\,dx\\ &= -\log(a+x) + 2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.97

$$\frac{1}{3}\left(-\log\left(a^3+x^3\right)+\log\left(a^2-ax+x^2\right)-2\log(a+x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a*x - x^2)/(a^3 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
x^2] - Log[a^3 + x^3])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax-x^2}{a^3+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*a*x - x^2)/(a^3 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(2*a*x - x^2)/(a^3 + x^3), x]
```

fricas [A] time = 0.40, size = 26, normalized size = 0.90

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

giac [A] time = 0.15, size = 27, normalized size = 0.93

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(|a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))
```

maple [A] time = 0.05, size = 29, normalized size = 1.00

$$\frac{2\sqrt{3}\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x-x^2)/(a^3+x^3),x)
```

```
[Out] -ln(a+x)+2/3*3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a)
```

maxima [A] time = 2.89, size = 26, normalized size = 0.90

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

mupad [B] time = 4.97, size = 26, normalized size = 0.90

$$-\ln(a+x)-\frac{2\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}\,a}{a-2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x - x^2)/(a^3 + x^3),x)
```

```
[Out] - log(a + x) - (2*3^(1/2)*atan(-(3^(1/2)*a)/(a - 2*x)))/3
```

sympy [C] time = 0.18, size = 54, normalized size = 1.86

$$-\log(a+x)-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2}-\frac{\sqrt{3}\,ia}{2}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2}+\frac{\sqrt{3}\,ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x**2)/(a**3+x**3),x)
```

```
[Out] -log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)*I*log(-a/
2 + sqrt(3)*I*a/2 + x)/3
```

3.313 $\int \frac{(2a-x)x}{a^3+x^3}\,dx$

Optimal. Leaf size=29

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = $\{1868, 31, 617, 204\}$

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2*a - x)*x)/(a^3 + x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{(2a-x)x}{a^3+x^3}\,dx &= a\int \frac{1}{a^2-ax+x^2}\,dx-\int \frac{1}{a+x}\,dx\\ &= -\log(a+x)+2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.97

$$\frac{1}{3}\left(-\log\left(a^3+x^3\right)+\log\left(a^2-ax+x^2\right)-2\log(a+x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2*a - x)*x)/(a^3 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x + x^2] - Log[a^3 + x^3])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2a-x)x}{a^3+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((2*a - x)*x)/(a^3 + x^3),x]
```

```
[Out] IntegrateAlgebraic[((2*a - x)*x)/(a^3 + x^3), x]
```

fricas [A] time = 0.41, size = 26, normalized size = 0.90

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

giac [A] time = 0.15, size = 27, normalized size = 0.93

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(|a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))
```

maple [A] time = 0.05, size = 29, normalized size = 1.00

$$\frac{2\sqrt{3}\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a-x)*x/(a^3+x^3),x)
```

```
[Out] 2/3*3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a)-ln(a+x)
```

maxima [A] time = 2.97, size = 26, normalized size = 0.90

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

mupad [B] time = 0.03, size = 26, normalized size = 0.90

$$-\ln\left(a+x\right)-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}\,a}{a-2\,x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*a - x))/(a^3 + x^3),x)
```

```
[Out] - log(a + x) - (2*3^(1/2)*atan(-(3^(1/2)*a)/(a - 2*x)))/3
```

sympy [C] time = 0.17, size = 54, normalized size = 1.86

$$-\log\left(a+x\right)-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2}-\frac{\sqrt{3}\,ia}{2}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2}+\frac{\sqrt{3}\,ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a**3+x**3),x)
```

```
[Out] -log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)*I*log(-a/
2 + sqrt(3)*I*a/2 + x)/3
```

3.314 $\int \frac{2ax+x^2}{a^3-x^3}\,dx$

Optimal. Leaf size=31

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1593, 1868, 31, 617, 204}

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a*x + x^2)/(a^3 - x^3),x]
```

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{2ax+x^2}{a^3-x^3}\,dx &= \int \frac{x(2a+x)}{a^3-x^3}\,dx\\ &= -\left(a\int \frac{1}{a^2+ax+x^2}\,dx\right) - \int \frac{1}{-a+x}\,dx\\ &= -\log(a-x) + 2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 58, normalized size = 1.87

$$\frac{1}{3}\left(-\log\left(x^3-a^3\right)+\log\left(a^2+ax+x^2\right)-2\log(x-a)-2\sqrt{3}\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a*x + x^2)/(a^3 - x^3),x]
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x +
 x^2] - Log[-a^3 + x^3])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*a*x + x^2)/(a^3 - x^3),x]
[Out] IntegrateAlgebraic[(2*a*x + x^2)/(a^3 - x^3), x]
```

fricas [A] time = 0.43, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

giac [A] time = 0.15, size = 29, normalized size = 0.94

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(|-a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="giac")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))
```

maple [A] time = 0.05, size = 29, normalized size = 0.94

$$-\frac{2\sqrt{3}\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)}{3}-\ln(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x+x^2)/(a^3-x^3),x)
```

```
[Out] -2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)-ln(x-a)
```

maxima [A] time = 2.92, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

mupad [B] time = 4.95, size = 27, normalized size = 0.87

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,a}{a+2x}\right)}{3}-\ln(x-a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x + x^2)/(a^3 - x^3),x)
```

```
[Out] (2*3^(1/2)*atan((3^(1/2)*a)/(a + 2*x)))/3 - log(x - a)
```

sympy [C] time = 0.17, size = 54, normalized size = 1.74

$$-\log(-a+x)+\frac{\sqrt{3}\,i\log\left(\frac{a}{2}-\frac{\sqrt{3}\,ia}{2}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(\frac{a}{2}+\frac{\sqrt{3}\,ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x**2)/(a**3-x**3),x)
```

```
[Out] -log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)*I*log(a/2
 + sqrt(3)*I*a/2 + x)/3
```

3.315 $\int \frac{x(2a+x)}{a^3-x^3}\,dx$

Optimal. Leaf size=31

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1868, 31, 617, 204}

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*a + x))/(a^3 - x^3),x]
```

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{x(2a+x)}{a^3-x^3}\,dx &= -\left(a\int \frac{1}{a^2+ax+x^2}\,dx\right)-\int \frac{1}{-a+x}\,dx\\ &= -\log(a-x)+2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}\,a}\right)}{\sqrt{3}}-\log(a-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 58, normalized size = 1.87

$$\frac{1}{3}\left(-\log\left(x^3-a^3\right)+\log\left(a^2+ax+x^2\right)-2\log(x-a)-2\sqrt{3}\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*a + x))/(a^3 - x^3),x]
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x + x^2] - Log[-a^3 + x^3])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(2*a + x))/(a^3 - x^3),x]
[Out] IntegrateAlgebraic[(x*(2*a + x))/(a^3 - x^3), x]
```

fricas [A] time = 0.40, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

giac [A] time = 0.15, size = 29, normalized size = 0.94

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(|-a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="giac")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))
```

maple [A] time = 0.06, size = 29, normalized size = 0.94

$$-\frac{2\sqrt{3}\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)}{3}-\ln(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*a+x)/(a^3-x^3),x)
[Out] -2/3*3^(1/2)*arctan(1/3*(a+2*x)*3^(1/2)/a)-ln(-a+x)
```

maxima [A] time = 2.84, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

mupad [B] time = 0.03, size = 27, normalized size = 0.87

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,a}{a+2\,x}\right)}{3}-\ln\left(x-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*a + x))/(a^3 - x^3),x)
```

```
[Out] (2*3^(1/2)*atan((3^(1/2)*a)/(a + 2*x)))/3 - log(x - a)
```

sympy [C] time = 0.17, size = 54, normalized size = 1.74

$$-\log\left(-a+x\right)+\frac{\sqrt{3}\,i\log\left(\frac{a}{2}-\frac{\sqrt{3}\,ia}{2}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(\frac{a}{2}+\frac{\sqrt{3}\,ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a**3-x**3),x)
```

```
[Out] -log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)*I*log(a/2
 + sqrt(3)*I*a/2 + x)/3
```

3.316 $\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

Optimal. Leaf size=50

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1867, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(1
/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b} - \frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$
$$= \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b} - \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$
$$= \frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b} + \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] time = 0.04, size = 146, normalized size = 2.92

$$\frac{C\left(-\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a+bx^3\right)+2\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3\sqrt[3]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(1/3)*b^(1/3)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b*x^3]))/(3
*a^(1/3)*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3), x]
```

fricas [A] time = 0.43, size = 52, normalized size = 1.04

$$-\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-3C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) - 3
*C*log(x + (a/b)^(1/3)))/b
```

giac [B] time = 0.48, size = 174, normalized size = 3.48

$$-\frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-2\left(ab^2\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}-\frac{\sqrt{3}\left(ab^2-\sqrt{3}\sqrt{a^2b^4}\,i\right)C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3}+\frac{\left(3ab^2-\sqrt{3}\sqrt{a^2b^4}\,i\right)C\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(C*b*(-a/b)^(2/3) - 2*(a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a*b) - 1/3*sqrt(3)*(a*b^2 - sqrt(3)*sqrt(a^2*b^4)*i)
*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/6*(3*a
*b^2 - sqrt(3)*sqrt(a^2*b^4)*i)*C*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
(a*b^3)
```

maple [A] time = 0.05, size = 87, normalized size = 1.74

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b}+\frac{2C\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}-\frac{C\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b}+\frac{C\ln\left(b\,x^{3}+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x)
```

```
[Out] 2/3*C/b*ln(x+(a/b)^(1/3))-1/3*C/b*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-2/3*3^(
1/2)*C/b*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)
```

maxima [A] time = 3.07, size = 51, normalized size = 1.02

$$-\frac{2\,\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}+\frac{C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/b + C*lo
g(x + (a/b)^(1/3))/b
```

mupad [B] time = 5.22, size = 154, normalized size = 3.08

$$\sum_{k=1}^{3}\ln\left(\frac{C^2\,a+\mathrm{root}\left(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k\right)^2\,a\,b^2\,9-C\,\mathrm{root}\left(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k\right)\,a\,b\,6+4\,C^2\,b\,x\,{\left(\frac{a}{b}\right)}^{2/3}}{b^3}\right)\,\mathrm{root}\left(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(C*x - 2*C*(a/b)^(1/3)))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a + 9*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*
C^3*a, z, k)^2*a*b^2 - 6*C*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z
 - 9*C^3*a, z, k)*a*b + 4*C^2*b*x*(a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 - 27*
C*a*b^2*z^2 + 9*C^2*a*b*z - 9*C^3*a, z, k), k, 1, 3)
```

sympy [C] time = 0.32, size = 100, normalized size = 2.00

$$\frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) - sqrt
(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

3.317 $\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Rubi [A] time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1867, 31, 617, 204}

$$-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a
/b))^(1/3) + x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rubi steps

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b} + \frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}\,x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b} + \frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= -\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b} - \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] time = 0.08, size = 149, normalized size = 2.81

$$-\frac{C\left(\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a-bx^3\right)-2\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)-2\sqrt{3}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{3\sqrt[3]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] -1/3*(C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^
(1/3)*b^(1/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[
a - b*x^3]))/(a^(1/3)*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3), x]
```

fricas [A] time = 0.44, size = 53, normalized size = 1.00

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}\,a}{3a}\right)+3C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a) +
3*C*log(x + (-a/b)^(1/3)))/b
```

giac [B] time = 0.21, size = 165, normalized size = 3.11

$$-\frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\left(-ab^2\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}+\frac{\sqrt{3}\left(ab^2+\sqrt{3}\sqrt{a^2b^4}\,i\right)C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3}-\frac{\left(3ab^2+\sqrt{3}\sqrt{a^2b^4}\,i\right)C\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(C*b*(a/b)^(2/3) - 2*(-a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/3)*log(abs
(x - (a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(a*b^2 + sqrt(3)*sqrt(a^2*b^4)*i)*C*
arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/6*(3*a*b^2
+ sqrt(3)*sqrt(a^2*b^4)*i)*C*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3)
```

maple [B] time = 0.05, size = 135, normalized size = 2.55

$$\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sqrt{3}\,C\arctan\left(\frac{\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)\sqrt{3}}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}C\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}C\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{C\ln\left(b\,x^{3}-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x)
```

```
[Out] 2/3*C*(-a/b)^(1/3)/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))-1/3*C*(-a/b)^(1/3)/b/(a/
b)^(1/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+2/3*C*(-a/b)^(1/3)*3^(1/2)/b/(a/
b)^(1/3)*arctan(1/3*(2/(a/b)^(1/3)*x+1)*3^(1/2))-1/3*C/b*ln(b*x^3-a)
```

maxima [B] time = 3.02, size = 166, normalized size = 3.13

$$-\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}+C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^{2}+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\sqrt{3}\left(Ca-\left(3\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*(C*(a/b)^(1/3) + C*(-a/b)^(1/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3)
)/(b*(a/b)^(1/3)) - 1/3*(C*(a/b)^(1/3) - 2*C*(-a/b)^(1/3))*log(x - (a/b)^(1
/3))/(b*(a/b)^(1/3)) - 2/9*sqrt(3)*(C*a - (3*C*(a/b)^(2/3)*(-a/b)^(1/3) + C
*a/b)*b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

mupad [B] time = 5.25, size = 156, normalized size = 2.94

$$\sum_{k=1}^{3}\ln\left(-\frac{C^{2}\,a+\mathrm{root}\left(27\,a\,b^{3}\,z^{3}+27\,C\,a\,b^{2}\,z^{2}+9\,C^{2}\,a\,b\,z+9\,C^{3}\,a,z,k\right)^{2}\,a\,b^{2}\,9+C\,\mathrm{root}\left(27\,a\,b^{3}\,z^{3}+27\,C\,a\,b^{2}\,z^{2}+9\,C^{2}\,a\,b\,z+9\,C^{3}\,a,z,k\right)\,a\,b\,6-4\,C^{2}\,b\,x\left(-\frac{a}{b}\right)^{2/3}}{b^{3}}\right)\mathrm{root}\left(27\,a\,b^{3}\,z^{3}+27\,C\,a\,b^{2}\,z^{2}+9\,C^{2}\,a\,b\,z+9\,C^{3}\,a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(C*x - 2*C*(-a/b)^(1/3)))/(a - b*x^3),x)
```

```
[Out] symsum(log(-(C^2*a + 9*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*z + 9
*C^3*a, z, k)^2*a*b^2 + 6*C*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*
z + 9*C^3*a, z, k)*a*b - 4*C^2*b*x*(-a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 + 2
7*C*a*b^2*z^2 + 9*C^2*a*b*z + 9*C^3*a, z, k), k, 1, 3)
```

sympy [C] time = 0.35, size = 110, normalized size = 2.08

$$-\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3)) - s
qrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

3.318 $\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

Optimal. Leaf size=54

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Rubi [A] time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1869, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(a/
b))^(1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=-\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b}+\frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$
$$=\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$
$$=\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] time = 0.05, size = 148, normalized size = 2.74

$$\frac{C\left(\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a+bx^3\right)-2\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] + (-(a/b))^(1/3)
*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b
*x^3]))/(3*a^(1/3)*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3), x]
```

fricas [A] time = 0.44, size = 56, normalized size = 1.04

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-3\,C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) -
3*C*log(x - (-a/b)^(1/3)))/b
```

giac [B] time = 0.19, size = 97, normalized size = 1.80

$$\frac{2\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}-\frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/3
*(C*b*(-a/b)^(2/3) + 2*(-a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a*b)
```

maple [B] time = 0.06, size = 132, normalized size = 2.44

$$\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}C\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}C\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{C\ln\left(bx^{3}+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x)
```

```
[Out] -2/3*C*(-a/b)^(1/3)/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/3*C*(-a/b)^(1/3)/b/(a
/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/3*C*(-a/b)^(1/3)*3^(1/2)/b/(a
/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)
```

maxima [B] time = 2.99, size = 167, normalized size = 3.09

$$\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}+C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\sqrt{3}\left(Ca-\left(3\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(C*(a/b)^(1/3) + C*(-a/b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(b*(a/b)^(1/3)) + 1/3*(C*(a/b)^(1/3) - 2*C*(-a/b)^(1/3))*log(x + (a/b)^(1/
3))/(b*(a/b)^(1/3)) - 2/9*sqrt(3)*(C*a - (3*C*(a/b)^(2/3)*(-a/b)^(1/3) + C*
a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

mupad [B] time = 5.22, size = 155, normalized size = 2.87

$$\sum_{k=1}^{3}\ln\left(\frac{C^{2}\,a+\mathrm{root}\left(27\,a\,b^{3}\,z^{3}-27\,C\,a\,b^{2}\,z^{2}+9\,C^{2}\,a\,b\,z-9\,C^{3}\,a,z,k\right)^{2}\,a\,b^{2}\,9-C\,\mathrm{root}\left(27\,a\,b^{3}\,z^{3}-27\,C\,a\,b^{2}\,z^{2}+9\,C^{2}\,a\,b\,z-9\,C^{3}\,a,z,k\right)\,a\,b\,6+4\,C^{2}\,b\,x\left(-\frac{a}{b}\right)^{2/3}}{b^{3}}\right)\mathrm{root}\left(27\,a\,b^{3}\,z^{3}-27\,C\,a\,b^{2}\,z^{2}+9\,C^{2}\,a\,b\,z-9\,C^{3}\,a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(C*x + 2*C*(-a/b)^(1/3)))/(a + b*x^3),x)
```

```
[Out] symsum(log((C^2*a + 9*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*
C^3*a, z, k)^2*a*b^2 - 6*C*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z
 - 9*C^3*a, z, k)*a*b + 4*C^2*b*x*(-a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 - 27
*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*C^3*a, z, k), k, 1, 3)
```

sympy [C] time = 0.32, size = 109, normalized size = 2.02

$$\frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}\,i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3)) - sq
rt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

3.319 $\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1869, 31, 617, 204}

$$-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(
1/3) - x])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b} - \frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$
$$= -\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b} + \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$
$$= -\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b} - \frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] time = 0.06, size = 147, normalized size = 2.77

$$-\frac{C\left(-\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a-bx^3\right)+2\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{3\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] -1/3*(C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] - (a/b)^(1/3)*b^(1
/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a - b*x^3]
))/(a^(1/3)*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3), x]
```

fricas [A] time = 0.43, size = 53, normalized size = 1.00

$$-\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+3C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a) + 3
*C*log(x - (a/b)^(1/3)))/b
```

giac [A] time = 0.20, size = 90, normalized size = 1.70

$$-\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b} - \frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - 1/3*
(C*b*(a/b)^(2/3) + 2*(a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/3)*log(abs(x - (
a/b)^(1/3)))/(a*b)
```

maple [A] time = 0.05, size = 90, normalized size = 1.70

$$-\frac{2\sqrt{3}\,C\arctan\left(\frac{\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)\sqrt{3}}{3}\right)}{3b}-\frac{2C\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}+\frac{C\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b}-\frac{C\ln\left(b\,x^{3}-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x)
```

```
[Out] -2/3*C/b*ln(x-(a/b)^(1/3))+1/3*C/b*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-2/3*3^
(1/2)*C/b*arctan(1/3*(2/(a/b)^(1/3)*x+1)*3^(1/2))-1/3*C/b*ln(b*x^3-a)
```

maxima [A] time = 3.02, size = 52, normalized size = 0.98

$$-\frac{2\,\sqrt{3}\,C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b}-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - C*lo
g(x - (a/b)^(1/3))/b
```

mupad [B] time = 5.23, size = 155, normalized size = 2.92

$$\sum_{k=1}^{3}\ln\left(-\frac{C^{2}\,a+\mathrm{root}\left(27\,a\,b^{3}\,z^{3}+27\,C\,a\,b^{2}\,z^{2}+9\,C^{2}\,a\,b\,z+9\,C^{3}\,a,z,k\right)^{2}a\,b^{2}\,9+C\,\mathrm{root}\left(27\,a\,b^{3}\,z^{3}+27\,C\,a\,b^{2}\,z^{2}+9\,C^{2}\,a\,b\,z+9\,C^{3}\,a,z,k\right)a\,b\,6-4\,C^{2}\,b\,x\left(\frac{a}{b}\right)^{2/3}}{b^{3}}\right)\mathrm{root}\left(27\,a\,b^{3}\,z^{3}+27\,C\,a\,b^{2}\,z^{2}+9\,C^{2}\,a\,b\,z+9\,C^{3}\,a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(C*x + 2*C*(a/b)^(1/3)))/(a - b*x^3),x)
```

```
[Out] symsum(log(-(C^2*a + 9*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*z + 9
*C^3*a, z, k)^2*a*b^2 + 6*C*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*
z + 9*C^3*a, z, k)*a*b - 4*C^2*b*x*(a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 + 27
*C*a*b^2*z^2 + 9*C^2*a*b*z + 9*C^3*a, z, k), k, 1, 3)
```

sympy [C] time = 0.37, size = 102, normalized size = 1.92

$$-\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}\,i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) - sqr
t(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

3.320 $\int x^4 \left(a + bx^3\right) \left(c + dx + ex^2 + fx^3 + gx^4 + hx^5\right) dx$

Optimal. Leaf size=97

$$\frac{1}{8}x^8(af+bc)+\frac{1}{9}x^9(ag+bd)+\frac{1}{10}x^{10}(ah+be)+\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\frac{1}{8}x^8(af + bc) + \frac{1}{9}x^9(ag + bd) + \frac{1}{10}x^{10}(ah + be) + \frac{1}{5}acx^5 + \frac{1}{6}adx^6 + \frac{1}{7}aex^7 + \frac{1}{11}bfx^{11} + \frac{1}{12}bgx^{12} + \frac{1}{13}bhx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g
)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^1
3)/13
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^4 \left(a + bx^3\right) \left(c + dx + ex^2 + fx^3 + gx^4 + hx^5\right) dx = \int \left(acx^4 + adx^5 + aex^6 + (bc + af)x^7 + (bd + ag)x^8 + (be\right.$$
$$= \frac{1}{5}acx^5 + \frac{1}{6}adx^6 + \frac{1}{7}aex^7 + \frac{1}{8}(bc + af)x^8 + \frac{1}{9}(bd + ag)x^9 -$$

Mathematica [A] time = 0.04, size = 97, normalized size = 1.00

$$\frac{1}{8}x^8(af + bc) + \frac{1}{9}x^9(ag + bd) + \frac{1}{10}x^{10}(ah + be) + \frac{1}{5}acx^5 + \frac{1}{6}adx^6 + \frac{1}{7}aex^7 + \frac{1}{11}bfx^{11} + \frac{1}{12}bgx^{12} + \frac{1}{13}bhx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g
)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^1
3)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + bx^3\right) \left(c + dx + ex^2 + fx^3 + gx^4 + hx^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5
),x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5
), x]
```

fricas [A] time = 0.36, size = 85, normalized size = 0.88

$$\frac{1}{13}x^{13}hb+\frac{1}{12}x^{12}gb+\frac{1}{11}x^{11}fb+\frac{1}{10}x^{10}eb+\frac{1}{10}x^{10}ha+\frac{1}{9}x^{9}db+\frac{1}{9}x^{9}ga+\frac{1}{8}x^{8}cb+\frac{1}{8}x^{8}fa+\frac{1}{7}x^{7}ea+\frac{1}{6}x^{6}da+\frac{1}{5}x^{5}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/13*x^13*h*b + 1/12*x^12*g*b + 1/11*x^11*f*b + 1/10*x^10*e*b + 1/10*x^10*h
*a + 1/9*x^9*d*b + 1/9*x^9*g*a + 1/8*x^8*c*b + 1/8*x^8*f*a + 1/7*x^7*e*a +
1/6*x^6*d*a + 1/5*x^5*c*a
```

giac [A] time = 0.15, size = 87, normalized size = 0.90

$$\frac{1}{13}bhx^{13}+\frac{1}{12}bgx^{12}+\frac{1}{11}bfx^{11}+\frac{1}{10}ahx^{10}+\frac{1}{10}bx^{10}e+\frac{1}{9}bdx^{9}+\frac{1}{9}agx^{9}+\frac{1}{8}bcx^{8}+\frac{1}{8}afx^{8}+\frac{1}{7}ax^{7}e+\frac{1}{6}adx^{6}+\frac{1}{5}acx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*a*h*x^10 + 1/10*b*x^10
*e + 1/9*b*d*x^9 + 1/9*a*g*x^9 + 1/8*b*c*x^8 + 1/8*a*f*x^8 + 1/7*a*x^7*e +
1/6*a*d*x^6 + 1/5*a*c*x^5
```

maple [A] time = 0.05, size = 80, normalized size = 0.82

$$\frac{bh\,x^{13}}{13}+\frac{bg\,x^{12}}{12}+\frac{bf\,x^{11}}{11}+\frac{(ah+be)\,x^{10}}{10}+\frac{ae\,x^{7}}{7}+\frac{\left(ag+bd\right)x^{9}}{9}+\frac{ad\,x^{6}}{6}+\frac{\left(af+bc\right)x^{8}}{8}+\frac{ac\,x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/5*a*c*x^5+1/6*a*d*x^6+1/7*a*e*x^7+1/8*(a*f+b*c)*x^8+1/9*(a*g+b*d)*x^9+1/1
0*(a*h+b*e)*x^10+1/11*b*f*x^11+1/12*b*g*x^12+1/13*b*h*x^13
```

maxima [A] time = 1.35, size = 79, normalized size = 0.81

$$\frac{1}{13}bhx^{13}+\frac{1}{12}bgx^{12}+\frac{1}{11}bfx^{11}+\frac{1}{10}(be+ah)x^{10}+\frac{1}{9}\left(bd+ag\right)x^{9}+\frac{1}{7}aex^{7}+\frac{1}{8}\left(bc+af\right)x^{8}+\frac{1}{6}adx^{6}+\frac{1}{5}acx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*(b*e + a*h)*x^10 + 1/9
*(b*d + a*g)*x^9 + 1/7*a*e*x^7 + 1/8*(b*c + a*f)*x^8 + 1/6*a*d*x^6 + 1/5*a*
c*x^5
```

mupad [B] time = 0.05, size = 82, normalized size = 0.85

$$\frac{b\,h\,x^{13}}{13}+\frac{b\,g\,x^{12}}{12}+\frac{b\,f\,x^{11}}{11}+\left(\frac{b\,e}{10}+\frac{a\,h}{10}\right)x^{10}+\left(\frac{b\,d}{9}+\frac{a\,g}{9}\right)x^{9}+\left(\frac{b\,c}{8}+\frac{a\,f}{8}\right)x^{8}+\frac{a\,e\,x^{7}}{7}+\frac{a\,d\,x^{6}}{6}+\frac{a\,c\,x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^8*((b*c)/8 + (a*f)/8) + x^9*((b*d)/9 + (a*g)/9) + x^10*((b*e)/10 + (a*h)/
10) + (b*h*x^13)/13 + (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + (b*f*x^11)/
11 + (b*g*x^12)/12
```

sympy [A] time = 0.09, size = 90, normalized size = 0.93

$$\frac{acx^5}{5} + \frac{adx^6}{6} + \frac{aex^7}{7} + \frac{bfx^{11}}{11} + \frac{bgx^{12}}{12} + \frac{bhx^{13}}{13} + x^{10}\left(\frac{ah}{10} + \frac{be}{10}\right) + x^9\left(\frac{ag}{9} + \frac{bd}{9}\right) + x^8\left(\frac{af}{8} + \frac{bc}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**5/5 + a*d*x**6/6 + a*e*x**7/7 + b*f*x**11/11 + b*g*x**12/12 + b*h*x*
*13/13 + x**10*(a*h/10 + b*e/10) + x**9*(a*g/9 + b*d/9) + x**8*(a*f/8 + b*c
/8)
```

3.321 $\int x^3 \left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=97

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g
)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)
/12
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^3\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx=\int\left(acx^3+adx^4+aex^5+(bc+af)x^6+(bd+ag)x^7+\right.$$
$$=\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}(bc+af)x^7+\frac{1}{8}(bd+ag)$$

Mathematica [A] time = 0.04, size = 97, normalized size = 1.00

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g
)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)
/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5
),x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5
), x]
```

fricas [A] time = 0.36, size = 85, normalized size = 0.88

$$\frac{1}{12}x^{12}hb+\frac{1}{11}x^{11}gb+\frac{1}{10}x^{10}fb+\frac{1}{9}x^{9}eb+\frac{1}{9}x^{9}ha+\frac{1}{8}x^{8}db+\frac{1}{8}x^{8}ga+\frac{1}{7}x^{7}cb+\frac{1}{7}x^{7}fa+\frac{1}{6}x^{6}ea+\frac{1}{5}x^{5}da+\frac{1}{4}x^{4}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/12*x^12*h*b + 1/11*x^11*g*b + 1/10*x^10*f*b + 1/9*x^9*e*b + 1/9*x^9*h*a +
 1/8*x^8*d*b + 1/8*x^8*g*a + 1/7*x^7*c*b + 1/7*x^7*f*a + 1/6*x^6*e*a + 1/5*
x^5*d*a + 1/4*x^4*c*a
```

giac [A] time = 0.16, size = 87, normalized size = 0.90

$$\frac{1}{12}bhx^{12}+\frac{1}{11}bgx^{11}+\frac{1}{10}bfx^{10}+\frac{1}{9}ahx^{9}+\frac{1}{9}bx^{9}e+\frac{1}{8}bdx^{8}+\frac{1}{8}agx^{8}+\frac{1}{7}bcx^{7}+\frac{1}{7}afx^{7}+\frac{1}{6}ax^{6}e+\frac{1}{5}adx^{5}+\frac{1}{4}acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*a*h*x^9 + 1/9*b*x^9*e +
 1/8*b*d*x^8 + 1/8*a*g*x^8 + 1/7*b*c*x^7 + 1/7*a*f*x^7 + 1/6*a*x^6*e + 1/5*
a*d*x^5 + 1/4*a*c*x^4
```

maple [A] time = 0.05, size = 80, normalized size = 0.82

$$\frac{bh\,x^{12}}{12}+\frac{bg\,x^{11}}{11}+\frac{bf\,x^{10}}{10}+\frac{(ah+be)\,x^{9}}{9}+\frac{ae\,x^{6}}{6}+\frac{\left(ag+bd\right)x^{8}}{8}+\frac{ad\,x^{5}}{5}+\frac{\left(af+bc\right)x^{7}}{7}+\frac{ac\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*(a*f+b*c)*x^7+1/8*(a*g+b*d)*x^8+1/9
*(a*h+b*e)*x^9+1/10*b*f*x^10+1/11*b*g*x^11+1/12*b*h*x^12
```

maxima [A] time = 1.35, size = 79, normalized size = 0.81

$$\frac{1}{12}bhx^{12}+\frac{1}{11}bgx^{11}+\frac{1}{10}bfx^{10}+\frac{1}{9}(be+ah)x^{9}+\frac{1}{8}\left(bd+ag\right)x^{8}+\frac{1}{6}aex^{6}+\frac{1}{7}\left(bc+af\right)x^{7}+\frac{1}{5}adx^{5}+\frac{1}{4}acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*(b*e + a*h)*x^9 + 1/8*(
b*d + a*g)*x^8 + 1/6*a*e*x^6 + 1/7*(b*c + a*f)*x^7 + 1/5*a*d*x^5 + 1/4*a*c*
x^4
```

mupad [B] time = 0.04, size = 82, normalized size = 0.85

$$\frac{b\,h\,x^{12}}{12}+\frac{b\,g\,x^{11}}{11}+\frac{b\,f\,x^{10}}{10}+\left(\frac{b\,e}{9}+\frac{a\,h}{9}\right)\,x^9+\left(\frac{b\,d}{8}+\frac{a\,g}{8}\right)\,x^8+\left(\frac{b\,c}{7}+\frac{a\,f}{7}\right)\,x^7+\frac{a\,e\,x^6}{6}+\frac{a\,d\,x^5}{5}+\frac{a\,c\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^7*((b*c)/7 + (a*f)/7) + x^8*((b*d)/8 + (a*g)/8) + x^9*((b*e)/9 + (a*h)/9)
 + (b*h*x^12)/12 + (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (b*f*x^10)/10
+ (b*g*x^11)/11
```

sympy [**A**] time = 0.09, size = 90, normalized size = 0.93

$$\frac{acx^4}{4}+\frac{adx^5}{5}+\frac{aex^6}{6}+\frac{bfx^{10}}{10}+\frac{bgx^{11}}{11}+\frac{bhx^{12}}{12}+x^9\left(\frac{ah}{9}+\frac{be}{9}\right)+x^8\left(\frac{ag}{8}+\frac{bd}{8}\right)+x^7\left(\frac{af}{7}+\frac{bc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + b*f*x**10/10 + b*g*x**11/11 + b*h*x*
*12/12 + x**9*(a*h/9 + b*e/9) + x**8*(a*g/8 + b*d/8) + x**7*(a*f/7 + b*c/7)
```

3.322 $\int x^2 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=97

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g
)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/1
1
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^2 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \int \left(acx^2+adx^3+aex^4+(bc+af)x^5+(bd+ag)x^6+(be$$
$$=\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}(bc+af)x^6+\frac{1}{7}(bd+ag)x^7$$

Mathematica [A] time = 0.03, size = 97, normalized size = 1.00

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g
)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/1
1
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5
),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5
), x]
```

fricas [A] time = 0.37, size = 85, normalized size = 0.88

$$\frac{1}{11}x^{11}hb+\frac{1}{10}x^{10}gb+\frac{1}{9}x^{9}fb+\frac{1}{8}x^{8}eb+\frac{1}{8}x^{8}ha+\frac{1}{7}x^{7}db+\frac{1}{7}x^{7}ga+\frac{1}{6}x^{6}cb+\frac{1}{6}x^{6}fa+\frac{1}{5}x^{5}ea+\frac{1}{4}x^{4}da+\frac{1}{3}x^{3}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/11*x^11*h*b + 1/10*x^10*g*b + 1/9*x^9*f*b + 1/8*x^8*e*b + 1/8*x^8*h*a + 1
/7*x^7*d*b + 1/7*x^7*g*a + 1/6*x^6*c*b + 1/6*x^6*f*a + 1/5*x^5*e*a + 1/4*x^
4*d*a + 1/3*x^3*c*a
```

giac [A] time = 0.16, size = 87, normalized size = 0.90

$$\frac{1}{11}bhx^{11}+\frac{1}{10}bgx^{10}+\frac{1}{9}bfx^{9}+\frac{1}{8}ahx^{8}+\frac{1}{8}bx^{8}e+\frac{1}{7}bdx^{7}+\frac{1}{7}agx^{7}+\frac{1}{6}bcx^{6}+\frac{1}{6}afx^{6}+\frac{1}{5}ax^{5}e+\frac{1}{4}adx^{4}+\frac{1}{3}acx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*a*h*x^8 + 1/8*b*x^8*e + 1
/7*b*d*x^7 + 1/7*a*g*x^7 + 1/6*b*c*x^6 + 1/6*a*f*x^6 + 1/5*a*x^5*e + 1/4*a*
d*x^4 + 1/3*a*c*x^3
```

maple [A] time = 0.05, size = 80, normalized size = 0.82

$$\frac{bh\,x^{11}}{11}+\frac{bg\,x^{10}}{10}+\frac{bf\,x^{9}}{9}+\frac{(ah+be)\,x^{8}}{8}+\frac{ae\,x^{5}}{5}+\frac{\left(ag+bd\right)x^{7}}{7}+\frac{ad\,x^{4}}{4}+\frac{\left(af+bc\right)x^{6}}{6}+\frac{ac\,x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*(a*f+b*c)*x^6+1/7*(a*g+b*d)*x^7+1/8
*(a*h+b*e)*x^8+1/9*b*f*x^9+1/10*b*g*x^10+1/11*b*h*x^11
```

maxima [A] time = 1.34, size = 79, normalized size = 0.81

$$\frac{1}{11}bhx^{11}+\frac{1}{10}bgx^{10}+\frac{1}{9}bfx^{9}+\frac{1}{8}(be+ah)x^{8}+\frac{1}{7}\left(bd+ag\right)x^{7}+\frac{1}{5}aex^{5}+\frac{1}{6}\left(bc+af\right)x^{6}+\frac{1}{4}adx^{4}+\frac{1}{3}acx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*(b*e + a*h)*x^8 + 1/7*(b*
d + a*g)*x^7 + 1/5*a*e*x^5 + 1/6*(b*c + a*f)*x^6 + 1/4*a*d*x^4 + 1/3*a*c*x^
3
```

mupad [B] time = 0.04, size = 82, normalized size = 0.85

$$\frac{b\,h\,x^{11}}{11}+\frac{b\,g\,x^{10}}{10}+\frac{b\,f\,x^{9}}{9}+\left(\frac{b\,e}{8}+\frac{a\,h}{8}\right)x^{8}+\left(\frac{b\,d}{7}+\frac{a\,g}{7}\right)x^{7}+\left(\frac{b\,c}{6}+\frac{a\,f}{6}\right)x^{6}+\frac{a\,e\,x^{5}}{5}+\frac{a\,d\,x^{4}}{4}+\frac{a\,c\,x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^6*((b*c)/6 + (a*f)/6) + x^7*((b*d)/7 + (a*g)/7) + x^8*((b*e)/8 + (a*h)/8)
 + (b*h*x^11)/11 + (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*f*x^9)/9 +
(b*g*x^10)/10
```

sympy [A] time = 0.08, size = 90, normalized size = 0.93

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{bfx^9}{9} + \frac{bgx^{10}}{10} + \frac{bhx^{11}}{11} + x^8\left(\frac{ah}{8} + \frac{be}{8}\right) + x^7\left(\frac{ag}{7} + \frac{bd}{7}\right) + x^6\left(\frac{af}{6} + \frac{bc}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*f*x**9/9 + b*g*x**10/10 + b*h*x**1
1/11 + x**8*(a*h/8 + b*e/8) + x**7*(a*g/7 + b*d/7) + x**6*(a*f/6 + b*c/6)
```

3.323 $\int x \left(a+b x^3\right) \left(c+d x+e x^2+f x^3+g x^4+h x^5\right) \, dx$

Optimal. Leaf size=97

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1820}

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g
)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int x \left(a+b x^3\right) \left(c+d x+e x^2+f x^3+g x^4+h x^5\right) \, dx &= \int \left(acx+adx^2+aex^3+(bc+af)x^4+(bd+ag)x^5+(b\right.\\ &= \frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}(bc+af)x^5+\frac{1}{6}(bd+ag)x^6 \end{aligned}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 1.00

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g
)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a+b x^3\right) \left(c+d x+e x^2+f x^3+g x^4+h x^5\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),
x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),
 x]
```

fricas [A] time = 0.37, size = 85, normalized size = 0.88

$$\frac{1}{10}x^{10}hb+\frac{1}{9}x^{9}gb+\frac{1}{8}x^{8}fb+\frac{1}{7}x^{7}eb+\frac{1}{7}x^{7}ha+\frac{1}{6}x^{6}db+\frac{1}{6}x^{6}ga+\frac{1}{5}x^{5}cb+\frac{1}{5}x^{5}fa+\frac{1}{4}x^{4}ea+\frac{1}{3}x^{3}da+\frac{1}{2}x^{2}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/10*x^10*h*b + 1/9*x^9*g*b + 1/8*x^8*f*b + 1/7*x^7*e*b + 1/7*x^7*h*a + 1/6
*x^6*d*b + 1/6*x^6*g*a + 1/5*x^5*c*b + 1/5*x^5*f*a + 1/4*x^4*e*a + 1/3*x^3*
d*a + 1/2*x^2*c*a
```

giac [A] time = 0.15, size = 87, normalized size = 0.90

$$\frac{1}{10}bhx^{10}+\frac{1}{9}bgx^{9}+\frac{1}{8}bfx^{8}+\frac{1}{7}ahx^{7}+\frac{1}{7}bx^{7}e+\frac{1}{6}bdx^{6}+\frac{1}{6}agx^{6}+\frac{1}{5}bcx^{5}+\frac{1}{5}afx^{5}+\frac{1}{4}ax^{4}e+\frac{1}{3}adx^{3}+\frac{1}{2}acx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*a*h*x^7 + 1/7*b*x^7*e + 1/6
*b*d*x^6 + 1/6*a*g*x^6 + 1/5*b*c*x^5 + 1/5*a*f*x^5 + 1/4*a*x^4*e + 1/3*a*d*
x^3 + 1/2*a*c*x^2
```

maple [A] time = 0.05, size = 80, normalized size = 0.82

$$\frac{bh\,x^{10}}{10}+\frac{bg\,x^{9}}{9}+\frac{bf\,x^{8}}{8}+\frac{(ah+be)\,x^{7}}{7}+\frac{ae\,x^{4}}{4}+\frac{\left(ag+bd\right)x^{6}}{6}+\frac{ad\,x^{3}}{3}+\frac{\left(af+bc\right)x^{5}}{5}+\frac{ac\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*(a*f+b*c)*x^5+1/6*(a*g+b*d)*x^6+1/7
*(a*h+b*e)*x^7+1/8*b*f*x^8+1/9*b*g*x^9+1/10*b*h*x^10
```

maxima [A] time = 1.37, size = 79, normalized size = 0.81

$$\frac{1}{10}bhx^{10}+\frac{1}{9}bgx^{9}+\frac{1}{8}bfx^{8}+\frac{1}{7}(be+ah)x^{7}+\frac{1}{6}\left(bd+ag\right)x^{6}+\frac{1}{4}aex^{4}+\frac{1}{5}\left(bc+af\right)x^{5}+\frac{1}{3}adx^{3}+\frac{1}{2}acx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*(b*e + a*h)*x^7 + 1/6*(b*d
+ a*g)*x^6 + 1/4*a*e*x^4 + 1/5*(b*c + a*f)*x^5 + 1/3*a*d*x^3 + 1/2*a*c*x^2
```

mupad [B] time = 0.04, size = 82, normalized size = 0.85

$$\frac{b\,h\,x^{10}}{10}+\frac{b\,g\,x^{9}}{9}+\frac{b\,f\,x^{8}}{8}+\left(\frac{b\,e}{7}+\frac{a\,h}{7}\right)\,x^{7}+\left(\frac{b\,d}{6}+\frac{a\,g}{6}\right)\,x^{6}+\left(\frac{b\,c}{5}+\frac{a\,f}{5}\right)\,x^{5}+\frac{a\,e\,x^{4}}{4}+\frac{a\,d\,x^{3}}{3}+\frac{a\,c\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^5*((b*c)/5 + (a*f)/5) + x^6*((b*d)/6 + (a*g)/6) + x^7*((b*e)/7 + (a*h)/7)
 + (b*h*x^10)/10 + (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*f*x^8)/8 +
(b*g*x^9)/9
```

sympy [A] time = 0.08, size = 90, normalized size = 0.93

$$\frac{acx^2}{2} + \frac{adx^3}{3} + \frac{aex^4}{4} + \frac{bfx^8}{8} + \frac{bgx^9}{9} + \frac{bhx^{10}}{10} + x^7\left(\frac{ah}{7} + \frac{be}{7}\right) + x^6\left(\frac{ag}{6} + \frac{bd}{6}\right) + x^5\left(\frac{af}{5} + \frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*f*x**8/8 + b*g*x**9/9 + b*h*x**10/
10 + x**7*(a*h/7 + b*e/7) + x**6*(a*g/6 + b*d/6) + x**5*(a*f/5 + b*c/5)
```

3.324 $\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal. Leaf size=92

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {1850}

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)
/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \int (ac+adx+aex^2+(bc+af)x^3+(bd+ag)x^4+(be+ah)$$
$$= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{5}(bd+ag)x^5+\frac{1}{6}(b$$

Mathematica [A] time = 0.02, size = 92, normalized size = 1.00

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)
/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x
]
```

fricas [A] time = 0.37, size = 82, normalized size = 0.89

$$\frac{1}{9}x^9hb+\frac{1}{8}x^8gb+\frac{1}{7}x^7fb+\frac{1}{6}x^6eb+\frac{1}{6}x^6ha+\frac{1}{5}x^5db+\frac{1}{5}x^5ga+\frac{1}{4}x^4cb+\frac{1}{4}x^4fa+\frac{1}{3}x^3ea+\frac{1}{2}x^2da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*h*b + 1/8*x^8*g*b + 1/7*x^7*f*b + 1/6*x^6*e*b + 1/6*x^6*h*a + 1/5*x
^5*d*b + 1/5*x^5*g*a + 1/4*x^4*c*b + 1/4*x^4*f*a + 1/3*x^3*e*a + 1/2*x^2*d*
a + x*c*a
```

giac [A] time = 0.15, size = 84, normalized size = 0.91

$$\frac{1}{9}bhx^9 + \frac{1}{8}bgx^8 + \frac{1}{7}bfx^7 + \frac{1}{6}ahx^6 + \frac{1}{6}bx^6e + \frac{1}{5}bdx^5 + \frac{1}{5}agx^5 + \frac{1}{4}bcx^4 + \frac{1}{4}afx^4 + \frac{1}{3}ax^3e + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*a*h*x^6 + 1/6*b*x^6*e + 1/5*b
*d*x^5 + 1/5*a*g*x^5 + 1/4*b*c*x^4 + 1/4*a*f*x^4 + 1/3*a*x^3*e + 1/2*a*d*x^
2 + a*c*x
```

maple [A] time = 0.04, size = 77, normalized size = 0.84

$$\frac{bh\,x^9}{9} + \frac{bg\,x^8}{8} + \frac{bf\,x^7}{7} + \frac{(ah+be)\,x^6}{6} + \frac{ae\,x^3}{3} + \frac{\left(ag+bd\right)x^5}{5} + \frac{ad\,x^2}{2} + \frac{\left(af+bc\right)x^4}{4} + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*(a*f+b*c)*x^4+1/5*(a*g+b*d)*x^5+1/6*(a*h+
b*e)*x^6+1/7*b*f*x^7+1/8*b*g*x^8+1/9*b*h*x^9
```

maxima [A] time = 1.40, size = 76, normalized size = 0.83

$$\frac{1}{9}bhx^9 + \frac{1}{8}bgx^8 + \frac{1}{7}bfx^7 + \frac{1}{6}(be+ah)x^6 + \frac{1}{5}\left(bd+ag\right)x^5 + \frac{1}{3}aex^3 + \frac{1}{4}\left(bc+af\right)x^4 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*(b*e + a*h)*x^6 + 1/5*(b*d +
a*g)*x^5 + 1/3*a*e*x^3 + 1/4*(b*c + a*f)*x^4 + 1/2*a*d*x^2 + a*c*x
```

mupad [B] time = 0.04, size = 79, normalized size = 0.86

$$\frac{b\,h\,x^9}{9} + \frac{b\,g\,x^8}{8} + \frac{b\,f\,x^7}{7} + \left(\frac{b\,e}{6} + \frac{a\,h}{6}\right)x^6 + \left(\frac{b\,d}{5} + \frac{a\,g}{5}\right)x^5 + \left(\frac{b\,c}{4} + \frac{a\,f}{4}\right)x^4 + \frac{a\,e\,x^3}{3} + \frac{a\,d\,x^2}{2} + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^4*((b*c)/4 + (a*f)/4) + x^5*((b*d)/5 + (a*g)/5) + x^6*((b*e)/6 + (a*h)/6)
 + (b*h*x^9)/9 + a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*f*x^7)/7 + (b*g*x^8
)/8
```

sympy [A] time = 0.08, size = 87, normalized size = 0.95

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{bfx^7}{7} + \frac{bgx^8}{8} + \frac{bhx^9}{9} + x^6\left(\frac{ah}{6} + \frac{be}{6}\right) + x^5\left(\frac{ag}{5} + \frac{bd}{5}\right) + x^4\left(\frac{af}{4} + \frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*f*x**7/7 + b*g*x**8/8 + b*h*x**9/9 + x*
*6*(a*h/6 + b*e/6) + x**5*(a*g/5 + b*d/5) + x**4*(a*f/4 + b*c/4)
```

3.325 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx$

Optimal. Leaf size=88

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a
*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx = \int \left(ad+\frac{ac}{x}+aex+(bc+af)x^2+(bd+ag)x^3+(be+ah)x^4\right.$$
$$= adx+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+\frac{1}{4}(bd+ag)x^4+\frac{1}{5}(be+ah)x^5$$

Mathematica [A] time = 0.07, size = 88, normalized size = 1.00

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a
*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/
x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/
x, x]
```

fricas [A] time = 0.41, size = 74, normalized size = 0.84

$$\frac{1}{8}bhx^8 + \frac{1}{7}bgx^7 + \frac{1}{6}bfx^6 + \frac{1}{5}(be + ah)x^5 + \frac{1}{4}\left(bd + ag\right)x^4 + \frac{1}{2}aex^2 + \frac{1}{3}\left(bc + af\right)x^3 + adx + ac\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas"
)
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1/4*(b*d +
a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x + a*c*log(x)
```

giac [A] time = 0.15, size = 83, normalized size = 0.94

$$\frac{1}{8}bhx^8 + \frac{1}{7}bgx^7 + \frac{1}{6}bfx^6 + \frac{1}{5}ahx^5 + \frac{1}{5}bx^5e + \frac{1}{4}bdx^4 + \frac{1}{4}agx^4 + \frac{1}{3}bcx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx + ac\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*a*h*x^5 + 1/5*b*x^5*e + 1/4*b
*d*x^4 + 1/4*a*g*x^4 + 1/3*b*c*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*e + a*d*x + a*
c*log(abs(x))
```

maple [A] time = 0.05, size = 81, normalized size = 0.92

$$\frac{bh\,x^8}{8} + \frac{bg\,x^7}{7} + \frac{bf\,x^6}{6} + \frac{ah\,x^5}{5} + \frac{be\,x^5}{5} + \frac{ag\,x^4}{4} + \frac{bd\,x^4}{4} + \frac{af\,x^3}{3} + \frac{bc\,x^3}{3} + \frac{ae\,x^2}{2} + ac\ln(x) + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] 1/8*b*h*x^8+1/7*b*g*x^7+1/6*b*f*x^6+1/5*x^5*a*h+1/5*b*e*x^5+1/4*x^4*a*g+1/4
*b*d*x^4+1/3*x^3*a*f+1/3*b*c*x^3+1/2*a*e*x^2+a*d*x+a*c*ln(x)
```

maxima [A] time = 1.34, size = 74, normalized size = 0.84

$$\frac{1}{8}bhx^8 + \frac{1}{7}bgx^7 + \frac{1}{6}bfx^6 + \frac{1}{5}(be + ah)x^5 + \frac{1}{4}\left(bd + ag\right)x^4 + \frac{1}{2}aex^2 + \frac{1}{3}\left(bc + af\right)x^3 + adx + ac\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima"
)
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1/4*(b*d +
a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x + a*c*log(x)
```

mupad [B] time = 0.05, size = 77, normalized size = 0.88

$$x^3\left(\frac{b\,c}{3} + \frac{a\,f}{3}\right) + x^4\left(\frac{b\,d}{4} + \frac{a\,g}{4}\right) + x^5\left(\frac{b\,e}{5} + \frac{a\,h}{5}\right) + \frac{b\,h\,x^8}{8} + a\,c\,\ln(x) + a\,d\,x + \frac{a\,e\,x^2}{2} + \frac{b\,f\,x^6}{6} + \frac{b\,g\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

```
[Out] x^3*((b*c)/3 + (a*f)/3) + x^4*((b*d)/4 + (a*g)/4) + x^5*((b*e)/5 + (a*h)/5)
 + (b*h*x^8)/8 + a*c*log(x) + a*d*x + (a*e*x^2)/2 + (b*f*x^6)/6 + (b*g*x^7)
/7
```

sympy [A] time = 0.22, size = 85, normalized size = 0.97

$$ac\log(x) + adx + \frac{aex^2}{2} + \frac{bfx^6}{6} + \frac{bgx^7}{7} + \frac{bhx^8}{8} + x^5\left(\frac{ah}{5} + \frac{be}{5}\right) + x^4\left(\frac{ag}{4} + \frac{bd}{4}\right) + x^3\left(\frac{af}{3} + \frac{bc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a*c*log(x) + a*d*x + a*e*x**2/2 + b*f*x**6/6 + b*g*x**7/7 + b*h*x**8/8 + x*
*5*(a*h/5 + b*e/5) + x**4*(a*g/4 + b*d/4) + x**3*(a*f/3 + b*c/3)
```

3.326 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} dx$

Optimal. Leaf size=86

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a*
h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^2}\,dx = \int \left(ae+\frac{ac}{x^2}+\frac{ad}{x}+(bc+af)x+(bd+ag)x^2+(be+ah)x$$
$$= -\frac{ac}{x}+aex+\frac{1}{2}(bc+af)x^2+\frac{1}{3}(bd+ag)x^3+\frac{1}{4}(be+ah)x$$

Mathematica [A] time = 0.07, size = 86, normalized size = 1.00

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a*
h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/
x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/
x^2, x]
```

fricas [A] time = 0.40, size = 81, normalized size = 0.94

$$\frac{60\,bhx^{8}+70\,bgx^{7}+84\,bfx^{6}+105\,(be+ah)x^{5}+140\,\left(bd+ag\right)x^{4}+420\,aex^{2}+210\,\left(bc+af\right)x^{3}+420\,adx\log(x)-420\,ac}{420\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="frica
s")
```

```
[Out] 1/420*(60*b*h*x^8 + 70*b*g*x^7 + 84*b*f*x^6 + 105*(b*e + a*h)*x^5 + 140*(b*
d + a*g)*x^4 + 420*a*e*x^2 + 210*(b*c + a*f)*x^3 + 420*a*d*x*log(x) - 420*a
*c)/x
```

giac [A] time = 0.16, size = 83, normalized size = 0.97

$$\frac{1}{7}\,bhx^{7}+\frac{1}{6}\,bgx^{6}+\frac{1}{5}\,bfx^{5}+\frac{1}{4}\,ahx^{4}+\frac{1}{4}\,bx^{4}e+\frac{1}{3}\,bdx^{3}+\frac{1}{3}\,agx^{3}+\frac{1}{2}\,bcx^{2}+\frac{1}{2}\,afx^{2}+axe+ad\log\left(|x|\right)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac"
)
```

```
[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*a*h*x^4 + 1/4*b*x^4*e + 1/3*b
*d*x^3 + 1/3*a*g*x^3 + 1/2*b*c*x^2 + 1/2*a*f*x^2 + a*x*e + a*d*log(abs(x))
- a*c/x
```

maple [A] time = 0.05, size = 81, normalized size = 0.94

$$\frac{bh\,x^{7}}{7}+\frac{bg\,x^{6}}{6}+\frac{bf\,x^{5}}{5}+\frac{ah\,x^{4}}{4}+\frac{be\,x^{4}}{4}+\frac{ag\,x^{3}}{3}+\frac{bd\,x^{3}}{3}+\frac{af\,x^{2}}{2}+\frac{bc\,x^{2}}{2}+ad\ln(x)+aex-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] 1/7*b*h*x^7+1/6*b*g*x^6+1/5*b*f*x^5+1/4*x^4*a*h+1/4*b*e*x^4+1/3*x^3*a*g+1/3
*b*d*x^3+1/2*x^2*a*f+1/2*b*c*x^2+a*e*x-a*c/x+a*d*ln(x)
```

maxima [A] time = 1.35, size = 74, normalized size = 0.86

$$\frac{1}{7}\,bhx^{7}+\frac{1}{6}\,bgx^{6}+\frac{1}{5}\,bfx^{5}+\frac{1}{4}\,(be+ah)x^{4}+\frac{1}{3}\,\left(bd+ag\right)x^{3}+aex+\frac{1}{2}\,\left(bc+af\right)x^{2}+ad\log(x)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxim
a")
```

```
[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*(b*e + a*h)*x^4 + 1/3*(b*d +
a*g)*x^3 + a*e*x + 1/2*(b*c + a*f)*x^2 + a*d*log(x) - a*c/x
```

mupad [B] time = 0.05, size = 77, normalized size = 0.90

$$x^{2}\left(\frac{b\,c}{2}+\frac{a\,f}{2}\right)+x^{3}\left(\frac{b\,d}{3}+\frac{a\,g}{3}\right)+x^{4}\left(\frac{b\,e}{4}+\frac{a\,h}{4}\right)+\frac{b\,h\,x^{7}}{7}+a\,d\,\ln(x)+a\,e\,x-\frac{a\,c}{x}+\frac{b\,f\,x^{5}}{5}+\frac{b\,g\,x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

```
[Out] x^2*((b*c)/2 + (a*f)/2) + x^3*((b*d)/3 + (a*g)/3) + x^4*((b*e)/4 + (a*h)/4)
 + (b*h*x^7)/7 + a*d*log(x) + a*e*x - (a*c)/x + (b*f*x^5)/5 + (b*g*x^6)/6
```

sympy [A] time = 0.23, size = 82, normalized size = 0.95

$$-\frac{ac}{x}+ad\log{(x)}+aex+\frac{bfx^{5}}{5}+\frac{bgx^{6}}{6}+\frac{bhx^{7}}{7}+x^{4}\left(\frac{ah}{4}+\frac{be}{4}\right)+x^{3}\left(\frac{ag}{3}+\frac{bd}{3}\right)+x^{2}\left(\frac{af}{2}+\frac{bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a*c/x + a*d*log(x) + a*e*x + b*f*x**5/5 + b*g*x**6/6 + b*h*x**7/7 + x**4*(
a*h/4 + b*e/4) + x**3*(a*g/3 + b*d/3) + x**2*(a*f/2 + b*c/2)
```

3.327 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx$

Optimal. Leaf size=86

$$x(af+bc)+\frac{1}{2}x^2(ag+bd)+\frac{1}{3}x^3(ah+be)-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6$$

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1820}

$$x(af+bc)+\frac{1}{2}x^2(ag+bd)+\frac{1}{3}x^3(ah+be)-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -(a*c)/(2*x^2) - (a*d)/x + (b*c + a*f)*x + ((b*d + a*g)*x^2)/2 + ((b*e + a*
h)*x^3)/3 + (b*f*x^4)/4 + (b*g*x^5)/5 + (b*h*x^6)/6 + a*e*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx = \int \left(bc\left(1+\frac{af}{bc}\right)+\frac{ac}{x^3}+\frac{ad}{x^2}+\frac{ae}{x}+(bd+ag)x+(be+ah)x^2\right.$$
$$= -\frac{ac}{2x^2}-\frac{ad}{x}+(bc+af)x+\frac{1}{2}(bd+ag)x^2+\frac{1}{3}(be+ah)x^3+\frac{1}{4}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 0.91

$$\frac{a\left(-3c-6dx+6fx^3+3gx^4+2hx^5\right)}{6x^2}+ae\log(x)+bcx+\frac{1}{60}bx^2\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] b*c*x + (a*(-3*c - 6*d*x + 6*f*x^3 + 3*g*x^4 + 2*h*x^5))/(6*x^2) + (b*x^2*(
30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3)))/60 + a*e*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/
x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/
x^3, x]
```

fricas [A] time = 0.41, size = 81, normalized size = 0.94

$$\frac{10\,bhx^8+12\,bgx^7+15\,bfx^6+20\,(be+ah)x^5+30\,\left(bd+ag\right)x^4+60\,aex^2\log(x)+60\,\left(bc+af\right)x^3-60\,adx-30\,ac}{60\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="frica
s")
```

```
[Out] 1/60*(10*b*h*x^8 + 12*b*g*x^7 + 15*b*f*x^6 + 20*(b*e + a*h)*x^5 + 30*(b*d +
 a*g)*x^4 + 60*a*e*x^2*log(x) + 60*(b*c + a*f)*x^3 - 60*a*d*x - 30*a*c)/x^2
```

giac [A] time = 0.16, size = 80, normalized size = 0.93

$$\frac{1}{6}\,bhx^6+\frac{1}{5}\,bgx^5+\frac{1}{4}\,bfx^4+\frac{1}{3}\,ahx^3+\frac{1}{3}\,bx^3e+\frac{1}{2}\,bdx^2+\frac{1}{2}\,agx^2+bcx+afx+ae\log\left(|x|\right)-\frac{2\,adx+ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac"
)
```

```
[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*a*h*x^3 + 1/3*b*x^3*e + 1/2*b
*d*x^2 + 1/2*a*g*x^2 + b*c*x + a*f*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a*c
)/x^2
```

maple [A] time = 0.05, size = 78, normalized size = 0.91

$$\frac{bh\,x^6}{6}+\frac{bg\,x^5}{5}+\frac{bf\,x^4}{4}+\frac{ah\,x^3}{3}+\frac{be\,x^3}{3}+\frac{ag\,x^2}{2}+\frac{bd\,x^2}{2}+ae\ln(x)+afx+bcx-\frac{ad}{x}-\frac{ac}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] 1/6*b*h*x^6+1/5*b*g*x^5+1/4*b*f*x^4+1/3*x^3*a*h+1/3*b*e*x^3+1/2*x^2*a*g+1/2
*b*d*x^2+a*f*x+b*c*x-1/2*a*c/x^2-a*d/x+a*e*ln(x)
```

maxima [A] time = 1.34, size = 74, normalized size = 0.86

$$\frac{1}{6}\,bhx^6+\frac{1}{5}\,bgx^5+\frac{1}{4}\,bfx^4+\frac{1}{3}\,(be+ah)x^3+\frac{1}{2}\,\left(bd+ag\right)x^2+ae\log(x)+\left(bc+af\right)x-\frac{2\,adx+ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxim
a")
```

```
[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*(b*e + a*h)*x^3 + 1/2*(b*d +
a*g)*x^2 + a*e*log(x) + (b*c + a*f)*x - 1/2*(2*a*d*x + a*c)/x^2
```

mupad [B] time = 0.04, size = 76, normalized size = 0.88

$$x\,\left(b\,c+a\,f\right)-\frac{\frac{a\,c}{2}+a\,d\,x}{x^2}+x^2\,\left(\frac{b\,d}{2}+\frac{a\,g}{2}\right)+x^3\,\left(\frac{b\,e}{3}+\frac{a\,h}{3}\right)+\frac{b\,h\,x^6}{6}+a\,e\,\ln(x)+\frac{b\,f\,x^4}{4}+\frac{b\,g\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

```
[Out] x*(b*c + a*f) - ((a*c)/2 + a*d*x)/x^2 + x^2*((b*d)/2 + (a*g)/2) + x^3*((b*e
)/3 + (a*h)/3) + (b*h*x^6)/6 + a*e*log(x) + (b*f*x^4)/4 + (b*g*x^5)/5
```

<u>**sympy**</u> **[A]** time = 0.31, size = 83, normalized size = 0.97

$$ae\log(x) + \frac{bfx^4}{4} + \frac{bgx^5}{5} + \frac{bhx^6}{6} + x^3\left(\frac{ah}{3} + \frac{be}{3}\right) + x^2\left(\frac{ag}{2} + \frac{bd}{2}\right) + x\left(af + bc\right) + \frac{-ac - 2adx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a*e*log(x) + b*f*x**4/4 + b*g*x**5/5 + b*h*x**6/6 + x**3*(a*h/3 + b*e/3) +
x**2*(a*g/2 + b*d/2) + x*(a*f + b*c) + (-a*c - 2*a*d*x)/(2*x**2)
```

3.328 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$

Optimal. Leaf size=86

$$\log(x)(af+bc)+x(ag+bd)+\frac{1}{2}x^2(ah+be)-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5$$

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\log(x)(af+bc)+x(ag+bd)+\frac{1}{2}x^2(ah+be)-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a*c)/(3*x^3) - (a*d)/(2*x^2) - (a*e)/x + (b*d + a*g)*x + ((b*e + a*h)*x^2
)/2 + (b*f*x^3)/3 + (b*g*x^4)/4 + (b*h*x^5)/5 + (b*c + a*f)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = \int \left(bd\left(1+\frac{ag}{bd}\right)+\frac{ac}{x^4}+\frac{ad}{x^3}+\frac{ae}{x^2}+\frac{bc+af}{x}+(be+ah)x\right.$$

$$= -\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+(bd+ag)x+\frac{1}{2}(be+ah)x^2+\frac{1}{3}bfx^3+$$

Mathematica [A] time = 0.07, size = 76, normalized size = 0.88

$$\log(x)(af+bc)-\frac{a\left(2c+3x\left(d+2ex-\left(x^3(2g+hx)\right)\right)\right)}{6x^3}+\frac{1}{60}bx\left(60d+x\left(30e+x\left(20f+15gx+12hx^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/6*(a*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (b*x*(60*d + x*(30
*e + x*(20*f + 15*g*x + 12*h*x^2))))/60 + (b*c + a*f)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/
x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/
x^4, x]
```

fricas [A] time = 0.40, size = 81, normalized size = 0.94

$$\frac{12\,bhx^{8}+15\,bgx^{7}+20\,bfx^{6}+30\,(be+ah)x^{5}+60\,\left(bd+ag\right)x^{4}+60\,\left(bc+af\right)x^{3}\log(x)-60\,aex^{2}-30\,adx-20\,ac}{60\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="frica
s")
```

```
[Out] 1/60*(12*b*h*x^8 + 15*b*g*x^7 + 20*b*f*x^6 + 30*(b*e + a*h)*x^5 + 60*(b*d +
 a*g)*x^4 + 60*(b*c + a*f)*x^3*log(x) - 60*a*e*x^2 - 30*a*d*x - 20*a*c)/x^3
```

giac [A] time = 0.17, size = 79, normalized size = 0.92

$$\frac{1}{5}\,bhx^{5}+\frac{1}{4}\,bgx^{4}+\frac{1}{3}\,bfx^{3}+\frac{1}{2}\,ahx^{2}+\frac{1}{2}\,bx^{2}e+bdx+agx+\left(bc+af\right)\log\left(|x|\right)-\frac{6\,ax^{2}e+3\,adx+2\,ac}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac"
)
```

```
[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*a*h*x^2 + 1/2*b*x^2*e + b*d*x
 + a*g*x + (b*c + a*f)*log(abs(x)) - 1/6*(6*a*x^2*e + 3*a*d*x + 2*a*c)/x^3
```

maple [A] time = 0.05, size = 76, normalized size = 0.88

$$\frac{bh\,x^{5}}{5}+\frac{bg\,x^{4}}{4}+\frac{bf\,x^{3}}{3}+\frac{ah\,x^{2}}{2}+\frac{be\,x^{2}}{2}+af\ln(x)+agx+bc\ln(x)+bdx-\frac{ae}{x}-\frac{ad}{2x^{2}}-\frac{ac}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] 1/5*b*h*x^5+1/4*b*g*x^4+1/3*b*f*x^3+1/2*x^2*a*h+1/2*b*e*x^2+a*g*x+x*b*d-1/3
*a*c/x^3-1/2*a*d/x^2-a*e/x+ln(x)*a*f+ln(x)*b*c
```

maxima [A] time = 1.36, size = 75, normalized size = 0.87

$$\frac{1}{5}\,bhx^{5}+\frac{1}{4}\,bgx^{4}+\frac{1}{3}\,bfx^{3}+\frac{1}{2}\,(be+ah)x^{2}+\left(bd+ag\right)x+\left(bc+af\right)\log(x)-\frac{6\,aex^{2}+3\,adx+2\,ac}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*(b*e + a*h)*x^2 + (b*d + a*g)
*x + (b*c + a*f)*log(x) - 1/6*(6*a*e*x^2 + 3*a*d*x + 2*a*c)/x^3
```

mupad [B] time = 0.04, size = 75, normalized size = 0.87

$$x\,\left(b\,d+a\,g\right)-\frac{a\,e\,x^{2}+\frac{a\,d\,x}{2}+\frac{a\,c}{3}}{x^{3}}+x^{2}\,\left(\frac{b\,e}{2}+\frac{a\,h}{2}\right)+\ln(x)\,\left(b\,c+a\,f\right)+\frac{b\,h\,x^{5}}{5}+\frac{b\,f\,x^{3}}{3}+\frac{b\,g\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

```
[Out] x*(b*d + a*g) - ((a*c)/3 + (a*d*x)/2 + a*e*x^2)/x^3 + x^2*((b*e)/2 + (a*h)/
2) + log(x)*(b*c + a*f) + (b*h*x^5)/5 + (b*f*x^3)/3 + (b*g*x^4)/4
```

sympy [A] time = 0.67, size = 83, normalized size = 0.97

$$\frac{bfx^3}{3}+\frac{bgx^4}{4}+\frac{bhx^5}{5}+x^2\left(\frac{ah}{2}+\frac{be}{2}\right)+x\left(ag+bd\right)+\left(af+bc\right)\log\left(x\right)+\frac{-2ac-3adx-6aex^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] b*f*x**3/3 + b*g*x**4/4 + b*h*x**5/5 + x**2*(a*h/2 + b*e/2) + x*(a*g + b*d)
 + (a*f + b*c)*log(x) + (-2*a*c - 3*a*d*x - 6*a*e*x**2)/(6*x**3)
```

3.329 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$

Optimal. Leaf size=86

$$-\frac{af+bc}{x}+\log(x)(ag+bd)+x(ah+be)-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4$$

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$-\frac{af+bc}{x}+\log(x)(ag+bd)+x(ah+be)-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -(a*c)/(4*x^4) - (a*d)/(3*x^3) - (a*e)/(2*x^2) - (b*c + a*f)/x + (b*e + a*h
)*x + (b*f*x^2)/2 + (b*g*x^3)/3 + (b*h*x^4)/4 + (b*d + a*g)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx = \int \left(be\left(1+\frac{ah}{be}\right)+\frac{ac}{x^5}+\frac{ad}{x^4}+\frac{ae}{x^3}+\frac{bc+af}{x^2}+\frac{bd+ag}{x}+bfx\right.$$
$$= -\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}-\frac{bc+af}{x}+(be+ah)x+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.90

$$\log(x)(ag+bd)-\frac{a\left(3c+4dx+6x^2\left(e+2fx-2hx^3\right)\right)}{12x^4}+b\left(-\frac{c}{x}+ex+\frac{1}{12}x^2\left(6f+4gx+3hx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] b*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) - (a*(3*c + 4*d*x + 6*x
^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) + (b*d + a*g)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/
x^5,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/
x^5, x]
```

fricas [A] time = 0.40, size = 81, normalized size = 0.94

$$\frac{3\,bhx^8 + 4\,bgx^7 + 6\,bfx^6 + 12\,(be + ah)x^5 + 12\,(bd + ag)x^4\log(x) - 6\,aex^2 - 12\,(bc + af)x^3 - 4\,adx - 3\,ac}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="frica
s")
```

```
[Out] 1/12*(3*b*h*x^8 + 4*b*g*x^7 + 6*b*f*x^6 + 12*(b*e + a*h)*x^5 + 12*(b*d + a*
g)*x^4*log(x) - 6*a*e*x^2 - 12*(b*c + a*f)*x^3 - 4*a*d*x - 3*a*c)/x^4
```

giac [A] time = 0.15, size = 77, normalized size = 0.90

$$\frac{1}{4}\,bhx^4 + \frac{1}{3}\,bgx^3 + \frac{1}{2}\,bfx^2 + ahx + bxe + (bd + ag)\log(|x|) - \frac{12\,(bc + af)x^3 + 6\,ax^2e + 4\,adx + 3\,ac}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac"
)
```

```
[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + a*h*x + b*x*e + (b*d + a*g)*log(a
bs(x)) - 1/12*(12*(b*c + a*f)*x^3 + 6*a*x^2*e + 4*a*d*x + 3*a*c)/x^4
```

maple [A] time = 0.05, size = 76, normalized size = 0.88

$$\frac{bh\,x^4}{4} + \frac{bg\,x^3}{3} + \frac{bf\,x^2}{2} + ag\ln(x) + ahx + bd\ln(x) + bex - \frac{af}{x} - \frac{bc}{x} - \frac{ae}{2x^2} - \frac{ad}{3x^3} - \frac{ac}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] 1/4*b*h*x^4+1/3*b*g*x^3+1/2*b*f*x^2+a*h*x+b*e*x-1/4*a*c/x^4-1/3*a*d/x^3-1/2
*a*e/x^2-1/x*a*f-1/x*b*c+ln(x)*a*g+ln(x)*b*d
```

maxima [A] time = 1.34, size = 75, normalized size = 0.87

$$\frac{1}{4}\,bhx^4 + \frac{1}{3}\,bgx^3 + \frac{1}{2}\,bfx^2 + (be + ah)x + (bd + ag)\log(x) - \frac{6\,aex^2 + 12\,(bc + af)x^3 + 4\,adx + 3\,ac}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxim
a")
```

```
[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + (b*e + a*h)*x + (b*d + a*g)*log(x
) - 1/12*(6*a*e*x^2 + 12*(b*c + a*f)*x^3 + 4*a*d*x + 3*a*c)/x^4
```

mupad [B] time = 4.98, size = 74, normalized size = 0.86

$$x\,(b\,e + a\,h) - \frac{(b\,c + a\,f)\,x^3 + \frac{a\,e\,x^2}{2} + \frac{a\,d\,x}{3} + \frac{a\,c}{4}}{x^4} + \ln(x)\,(b\,d + a\,g) + \frac{b\,h\,x^4}{4} + \frac{b\,f\,x^2}{2} + \frac{b\,g\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

```
[Out] x*(b*e + a*h) - ((a*c)/4 + x^3*(b*c + a*f) + (a*d*x)/3 + (a*e*x^2)/2)/x^4 +
 log(x)*(b*d + a*g) + (b*h*x^4)/4 + (b*f*x^2)/2 + (b*g*x^3)/3
```

sympy [A] time = 2.57, size = 83, normalized size = 0.97

$$\frac{bfx^2}{2} + \frac{bgx^3}{3} + \frac{bhx^4}{4} + x(ah + be) + \left(ag + bd\right)\log(x) + \frac{-3ac - 4adx - 6aex^2 + x^3\left(-12af - 12bc\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] b*f*x**2/2 + b*g*x**3/3 + b*h*x**4/4 + x*(a*h + b*e) + (a*g + b*d)*log(x) +
 (-3*a*c - 4*a*d*x - 6*a*e*x**2 + x**3*(-12*a*f - 12*b*c))/(12*x**4)
```

3.330 $\int x^4 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=163

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2ah+b$$

Rubi [A] time = 0.21, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1820}

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2ah+be)+\frac{1}{10}ax^{10}(ah+2be)+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (
a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)
/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/
14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx &= \int \left(a^2cx^4+a^2dx^5+a^2ex^6+a(2bc+af)x^7+a(2bd+\right.\\ &= \frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{8}a(2bc+af)x^8+\frac{1}{9}a(2\end{aligned}$$

Mathematica [A] time = 0.05, size = 163, normalized size = 1.00

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2ah+be)+\frac{1}{10}ax^{10}(ah+2be)+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (
a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)
/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/
14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5),x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5), x]
```

fricas [A] time = 0.37, size = 157, normalized size = 0.96

$$\frac{1}{16}x^{16}hb^2+\frac{1}{15}x^{15}gb^2+\frac{1}{14}x^{14}fb^2+\frac{1}{13}x^{13}eb^2+\frac{2}{13}x^{13}hba+\frac{1}{12}x^{12}db^2+\frac{1}{6}x^{12}gba+\frac{1}{11}x^{11}cb^2+\frac{2}{11}x^{11}fba+\frac{1}{5}x^{10}eba+\frac{1}{10}x^{10}ha^2+\frac{2}{9}x^9dba+\frac{1}{9}x^9ga^2+\frac{1}{4}x^8cba+\frac{1}{8}x^8fa^2+\frac{1}{7}x^7ea^2+\frac{1}{6}x^6da^2+\frac{1}{5}x^5ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/16*x^16*h*b^2 + 1/15*x^15*g*b^2 + 1/14*x^14*f*b^2 + 1/13*x^13*e*b^2 + 2/1
3*x^13*h*b*a + 1/12*x^12*d*b^2 + 1/6*x^12*g*b*a + 1/11*x^11*c*b^2 + 2/11*x^
11*f*b*a + 1/5*x^10*e*b*a + 1/10*x^10*h*a^2 + 2/9*x^9*d*b*a + 1/9*x^9*g*a^2
 + 1/4*x^8*c*b*a + 1/8*x^8*f*a^2 + 1/7*x^7*e*a^2 + 1/6*x^6*d*a^2 + 1/5*x^5*
c*a^2
```

giac [A] time = 0.15, size = 160, normalized size = 0.98

$$\frac{1}{16}b^2hx^{16}+\frac{1}{15}b^2gx^{15}+\frac{1}{14}b^2fx^{14}+\frac{2}{13}abhx^{13}+\frac{1}{13}b^2x^{13}e+\frac{1}{12}b^2dx^{12}+\frac{1}{6}abgx^{12}+\frac{1}{11}b^2cx^{11}+\frac{2}{11}abfx^{11}+\frac{1}{10}a^2hx^{10}+\frac{1}{5}abx^{10}e+\frac{2}{9}abdx^9+\frac{1}{9}a^2gx^9+\frac{1}{4}abcx^8+\frac{1}{8}a^2fx^8+\frac{1}{7}a^2x^7e+\frac{1}{6}a^2dx^6+\frac{1}{5}a^2cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 2/13*a*b*h*x^13 + 1/1
3*b^2*x^13*e + 1/12*b^2*d*x^12 + 1/6*a*b*g*x^12 + 1/11*b^2*c*x^11 + 2/11*a*
b*f*x^11 + 1/10*a^2*h*x^10 + 1/5*a*b*x^10*e + 2/9*a*b*d*x^9 + 1/9*a^2*g*x^9
 + 1/4*a*b*c*x^8 + 1/8*a^2*f*x^8 + 1/7*a^2*x^7*e + 1/6*a^2*d*x^6 + 1/5*a^2*
c*x^5
```

maple [A] time = 0.04, size = 152, normalized size = 0.93

$$\frac{b^2hx^{16}}{16}+\frac{b^2gx^{15}}{15}+\frac{b^2fx^{14}}{14}+\frac{\left(2abh+b^2e\right)x^{13}}{13}+\frac{\left(2abg+b^2d\right)x^{12}}{12}+\frac{\left(2abf+cb^2\right)x^{11}}{11}+\frac{a^2ex^7}{7}+\frac{\left(a^2h+2bea\right)x^{10}}{10}+\frac{a^2dx^6}{6}+\frac{\left(a^2g+2bda\right)x^9}{9}+\frac{a^2cx^5}{5}+\frac{\left(a^2f+2abc\right)x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/16*b^2*h*x^16+1/15*b^2*g*x^15+1/14*b^2*f*x^14+1/13*(2*a*b*h+b^2*e)*x^13+1
/12*(2*a*b*g+b^2*d)*x^12+1/11*(2*a*b*f+b^2*c)*x^11+1/10*(a^2*h+2*a*b*e)*x^1
0+1/9*(a^2*g+2*a*b*d)*x^9+1/8*(a^2*f+2*a*b*c)*x^8+1/7*a^2*e*x^7+1/6*a^2*d*x
^6+1/5*a^2*c*x^5
```

maxima [A] time = 1.37, size = 151, normalized size = 0.93

$$\frac{1}{16}b^2hx^{16}+\frac{1}{15}b^2gx^{15}+\frac{1}{14}b^2fx^{14}+\frac{1}{13}\left(b^2e+2\,abh\right)x^{13}+\frac{1}{12}\left(b^2d+2\,abg\right)x^{12}+\frac{1}{11}\left(b^2c+2\,abf\right)x^{11}+\frac{1}{10}\left(2\,abe+a^2h\right)x^{10}+\frac{1}{7}a^2ex^7+\frac{1}{9}\left(2\,abd+a^2g\right)x^9+\frac{1}{6}a^2dx^6+\frac{1}{8}\left(2\,abc+a^2f\right)x^8+\frac{1}{5}a^2cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*(b^2*e + 2*a*b*h
)*x^13 + 1/12*(b^2*d + 2*a*b*g)*x^12 + 1/11*(b^2*c + 2*a*b*f)*x^11 + 1/10*(
2*a*b*e + a^2*h)*x^10 + 1/7*a^2*e*x^7 + 1/9*(2*a*b*d + a^2*g)*x^9 + 1/6*a^2
*d*x^6 + 1/8*(2*a*b*c + a^2*f)*x^8 + 1/5*a^2*c*x^5
```

mupad [B] time = 0.10, size = 151, normalized size = 0.93

$$x^8\left(\frac{f\,a^2}{8}+\frac{b\,c\,a}{4}\right)+x^{11}\left(\frac{c\,b^2}{11}+\frac{2\,a\,f\,b}{11}\right)+x^9\left(\frac{g\,a^2}{9}+\frac{2\,b\,d\,a}{9}\right)+x^{12}\left(\frac{d\,b^2}{12}+\frac{a\,g\,b}{6}\right)+x^{10}\left(\frac{h\,a^2}{10}+\frac{b\,e\,a}{5}\right)+x^{13}\left(\frac{e\,b^2}{13}+\frac{2\,a\,h\,b}{13}\right)+\frac{a^2\,c\,x^5}{5}+\frac{a^2\,d\,x^6}{6}+\frac{a^2\,e\,x^7}{7}+\frac{b^2\,f\,x^{14}}{14}+\frac{b^2\,g\,x^{15}}{15}+\frac{b^2\,h\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^8*((a^2*f)/8 + (a*b*c)/4) + x^11*((b^2*c)/11 + (2*a*b*f)/11) + x^9*((a^2*
g)/9 + (2*a*b*d)/9) + x^12*((b^2*d)/12 + (a*b*g)/6) + x^10*((a^2*h)/10 + (a
*b*e)/5) + x^13*((b^2*e)/13 + (2*a*b*h)/13) + (a^2*c*x^5)/5 + (a^2*d*x^6)/6
 + (a^2*e*x^7)/7 + (b^2*f*x^14)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

sympy [A] time = 0.10, size = 167, normalized size = 1.02

$$\frac{a^2cx^5}{5} + \frac{a^2dx^6}{6} + \frac{a^2ex^7}{7} + \frac{b^2fx^{14}}{14} + \frac{b^2gx^{15}}{15} + \frac{b^2hx^{16}}{16} + x^{13}\left(\frac{2abh}{13} + \frac{b^2e}{13}\right) + x^{12}\left(\frac{abg}{6} + \frac{b^2d}{12}\right) + x^{11}\left(\frac{2abf}{11} + \frac{b^2c}{11}\right) + x^{10}\left(\frac{a^2h}{10} + \frac{abe}{5}\right) + x^9\left(\frac{a^2g}{9} + \frac{2abd}{9}\right) + x^8\left(\frac{a^2f}{8} + \frac{abc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**5/5 + a**2*d*x**6/6 + a**2*e*x**7/7 + b**2*f*x**14/14 + b**2*g*x*
*15/15 + b**2*h*x**16/16 + x**13*(2*a*b*h/13 + b**2*e/13) + x**12*(a*b*g/6
+ b**2*d/12) + x**11*(2*a*b*f/11 + b**2*c/11) + x**10*(a**2*h/10 + a*b*e/5)
 + x**9*(a**2*g/9 + 2*a*b*d/9) + x**8*(a**2*f/8 + a*b*c/4)
```

3.331 $\int x^3 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=163

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah+be)+$$

Rubi [A] time = 0.16, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah+be)+\frac{1}{9}ax^9(ah+2be)+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (
a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/1
0 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13
 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^3 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \int \left(a^2cx^3+a^2dx^4+a^2ex^5+a(2bc+af)x^6+a(2bd+ag\right.$$
$$=\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a(2bc+af)x^7+\frac{1}{8}a(2bd$$

Mathematica [A] time = 0.03, size = 163, normalized size = 1.00

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah+be)+\frac{1}{9}ax^9(ah+2be)+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (
a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/1
0 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13
 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5),x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5), x]
```

fricas [A] time = 0.37, size = 157, normalized size = 0.96

$$\frac{1}{15}x^{15}hb^2+\frac{1}{14}x^{14}gb^2+\frac{1}{13}x^{13}fb^2+\frac{1}{12}x^{12}eb^2+\frac{1}{6}x^{12}hba+\frac{1}{11}x^{11}db^2+\frac{2}{11}x^{11}gba+\frac{1}{10}x^{10}cb^2+\frac{1}{5}x^{10}fba+\frac{2}{9}x^9eba+\frac{1}{9}x^9ha^2+\frac{1}{4}x^8dba+\frac{1}{8}x^8ga^2+\frac{2}{7}x^7cba+\frac{1}{7}x^7fa^2+\frac{1}{6}x^6ea^2+\frac{1}{5}x^5da^2+\frac{1}{4}x^4ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/15*x^15*h*b^2 + 1/14*x^14*g*b^2 + 1/13*x^13*f*b^2 + 1/12*x^12*e*b^2 + 1/6
*x^12*h*b*a + 1/11*x^11*d*b^2 + 2/11*x^11*g*b*a + 1/10*x^10*c*b^2 + 1/5*x^1
0*f*b*a + 2/9*x^9*e*b*a + 1/9*x^9*h*a^2 + 1/4*x^8*d*b*a + 1/8*x^8*g*a^2 + 2
/7*x^7*c*b*a + 1/7*x^7*f*a^2 + 1/6*x^6*e*a^2 + 1/5*x^5*d*a^2 + 1/4*x^4*c*a^
2
```

giac [A] time = 0.15, size = 160, normalized size = 0.98

$$\frac{1}{15}b^2hx^{15}+\frac{1}{14}b^2gx^{14}+\frac{1}{13}b^2fx^{13}+\frac{1}{6}abhx^{12}+\frac{1}{12}b^2x^{12}e+\frac{1}{11}b^2dx^{11}+\frac{2}{11}abgx^{11}+\frac{1}{10}b^2cx^{10}+\frac{1}{5}abfx^{10}+\frac{1}{9}a^2hx^9+\frac{2}{9}abx^9e+\frac{1}{4}abdx^8+\frac{1}{8}a^2gx^8+\frac{2}{7}abcx^7+\frac{1}{7}a^2fx^7+\frac{1}{6}a^2x^6e+\frac{1}{5}a^2dx^5+\frac{1}{4}a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/6*a*b*h*x^12 + 1/12
*b^2*x^12*e + 1/11*b^2*d*x^11 + 2/11*a*b*g*x^11 + 1/10*b^2*c*x^10 + 1/5*a*b
*f*x^10 + 1/9*a^2*h*x^9 + 2/9*a*b*x^9*e + 1/4*a*b*d*x^8 + 1/8*a^2*g*x^8 + 2
/7*a*b*c*x^7 + 1/7*a^2*f*x^7 + 1/6*a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^
4
```

maple [A] time = 0.04, size = 152, normalized size = 0.93

$$\frac{b^2hx^{15}}{15}+\frac{b^2gx^{14}}{14}+\frac{b^2fx^{13}}{13}+\frac{\left(2abh+b^2e\right)x^{12}}{12}+\frac{\left(2abg+b^2d\right)x^{11}}{11}+\frac{\left(2abf+cb^2\right)x^{10}}{10}+\frac{a^2ex^6}{6}+\frac{\left(a^2h+2bea\right)x^9}{9}+\frac{a^2dx^5}{5}+\frac{\left(a^2g+2bda\right)x^8}{8}+\frac{a^2cx^4}{4}+\frac{\left(a^2f+2abc\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/15*b^2*h*x^15+1/14*b^2*g*x^14+1/13*b^2*f*x^13+1/12*(2*a*b*h+b^2*e)*x^12+1
/11*(2*a*b*g+b^2*d)*x^11+1/10*(2*a*b*f+b^2*c)*x^10+1/9*(a^2*h+2*a*b*e)*x^9+
1/8*(a^2*g+2*a*b*d)*x^8+1/7*(a^2*f+2*a*b*c)*x^7+1/6*a^2*e*x^6+1/5*a^2*d*x^5
+1/4*a^2*c*x^4
```

maxima [A] time = 1.35, size = 151, normalized size = 0.93

$$\frac{1}{15}b^2hx^{15}+\frac{1}{14}b^2gx^{14}+\frac{1}{13}b^2fx^{13}+\frac{1}{12}\left(b^2e+2abh\right)x^{12}+\frac{1}{11}\left(b^2d+2abg\right)x^{11}+\frac{1}{10}\left(b^2c+2abf\right)x^{10}+\frac{1}{9}\left(2abe+a^2h\right)x^9+\frac{1}{6}a^2ex^6+\frac{1}{8}\left(2abd+a^2g\right)x^8+\frac{1}{5}a^2dx^5+\frac{1}{7}\left(2abc+a^2f\right)x^7+\frac{1}{4}a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*(b^2*e + 2*a*b*h
)*x^12 + 1/11*(b^2*d + 2*a*b*g)*x^11 + 1/10*(b^2*c + 2*a*b*f)*x^10 + 1/9*(2
*a*b*e + a^2*h)*x^9 + 1/6*a^2*e*x^6 + 1/8*(2*a*b*d + a^2*g)*x^8 + 1/5*a^2*d
*x^5 + 1/7*(2*a*b*c + a^2*f)*x^7 + 1/4*a^2*c*x^4
```

mupad [B] time = 0.09, size = 151, normalized size = 0.93

$$x^7\left(\frac{fa^2}{7}+\frac{2bca}{7}\right)+x^{10}\left(\frac{cb^2}{10}+\frac{afb}{5}\right)+x^8\left(\frac{ga^2}{8}+\frac{bda}{4}\right)+x^{11}\left(\frac{db^2}{11}+\frac{2agb}{11}\right)+x^9\left(\frac{ha^2}{9}+\frac{2bea}{9}\right)+x^{12}\left(\frac{eb^2}{12}+\frac{ahb}{6}\right)+\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{b^2fx^{13}}{13}+\frac{b^2gx^{14}}{14}+\frac{b^2hx^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^7*((a^2*f)/7 + (2*a*b*c)/7) + x^10*((b^2*c)/10 + (a*b*f)/5) + x^8*((a^2*g
)/8 + (a*b*d)/4) + x^11*((b^2*d)/11 + (2*a*b*g)/11) + x^9*((a^2*h)/9 + (2*a
*b*e)/9) + x^12*((b^2*e)/12 + (a*b*h)/6) + (a^2*c*x^4)/4 + (a^2*d*x^5)/5 +
(a^2*e*x^6)/6 + (b^2*f*x^13)/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

sympy [A] time = 0.11, size = 167, normalized size = 1.02

$$\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{b^2fx^{13}}{13}+\frac{b^2gx^{14}}{14}+\frac{b^2hx^{15}}{15}+x^{12}\left(\frac{abh}{6}+\frac{b^2e}{12}\right)+x^{11}\left(\frac{2abg}{11}+\frac{b^2d}{11}\right)+x^{10}\left(\frac{abf}{5}+\frac{b^2c}{10}\right)+x^9\left(\frac{a^2h}{9}+\frac{2abe}{9}\right)+x^8\left(\frac{a^2g}{8}+\frac{abd}{4}\right)+x^7\left(\frac{a^2f}{7}+\frac{2abc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + b**2*f*x**13/13 + b**2*g*x*
*14/14 + b**2*h*x**15/15 + x**12*(a*b*h/6 + b**2*e/12) + x**11*(2*a*b*g/11
+ b**2*d/11) + x**10*(a*b*f/5 + b**2*c/10) + x**9*(a**2*h/9 + 2*a*b*e/9) +
x**8*(a**2*g/8 + a*b*d/4) + x**7*(a**2*f/7 + 2*a*b*c/7)
```

3.332 $\int x^2 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=158

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{c\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ag+bd)+\frac{1}{7}ax^7(ag+2bd)+\frac{1}{11}bx^{11}(2ah+be)+\frac{1}{8}ax^8(ah+2be)+$$

Rubi [A] time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1582, 1850}

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{c\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ag+bd)+\frac{1}{7}ax^7(ag+2bd)+\frac{1}{11}bx^{11}(2ah+be)+\frac{1}{8}ax^8(ah+2be)+\frac{2}{9}abfx^9+\frac{1}{12}b^2fx^{12}+\frac{1}{13}b^2gx^{13}+\frac{1}{14}b^2hx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a^2*f*x^6)/6 + (a*(2*b*d + a*g)*x^7)/7 + (
a*(2*b*e + a*h)*x^8)/8 + (2*a*b*f*x^9)/9 + (b*(b*d + 2*a*g)*x^10)/10 + (b*(
b*e + 2*a*h)*x^11)/11 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14
 + (c*(a + b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx &= \frac{c\left(a+bx^3\right)^3}{9b}+\int \left(a+bx^3\right)^2\left(-cx^2+x^2\left(c+dx+ex^2\right.\right.\\ &= \frac{c\left(a+bx^3\right)^3}{9b}+\int \left(a^2dx^3+a^2ex^4+a^2fx^5+a(2bd+a\right.\\ &= \frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{1}{7}a(2bd+ag)x^7+\frac{1}{8}a(2\end{aligned}$$

Mathematica [A] time = 0.08, size = 150, normalized size = 0.95

$$a^2\left(\frac{cx^3}{3}+\frac{dx^4}{4}+\frac{ex^5}{5}+\frac{fx^6}{6}+\frac{gx^7}{7}+\frac{hx^8}{8}\right)+ab\left(\frac{cx^6}{3}+\frac{2dx^7}{7}+\frac{ex^8}{4}+\frac{2fx^9}{9}+\frac{gx^{10}}{5}+\frac{2hx^{11}}{11}\right)+\frac{b^2x^9\left(20020c+3x\left(6006d+5460ex+55x^2\left(91f+84gx+78hx^2\right)\right)\right)}{180180}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^2*((c*x^3)/3 + (d*x^4)/4 + (e*x^5)/5 + (f*x^6)/6 + (g*x^7)/7 + (h*x^8)/8)
 + a*b*((c*x^6)/3 + (2*d*x^7)/7 + (e*x^8)/4 + (2*f*x^9)/9 + (g*x^10)/5 + (2
*h*x^11)/11) + (b^2*x^9*(20020*c + 3*x*(6006*d + 5460*e*x + 55*x^2*(91*f +
84*g*x + 78*h*x^2))))/180180
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^3\right)^2 \left(c + dx + ex^2 + fx^3 + gx^4 + hx^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5), x]
```

fricas [A] time = 0.34, size = 157, normalized size = 0.99

$$\frac{1}{14}x^{14}hb^2 + \frac{1}{13}x^{13}gb^2 + \frac{1}{12}x^{12}fb^2 + \frac{1}{11}x^{11}eb^2 + \frac{2}{11}x^{11}hba + \frac{1}{10}x^{10}db^2 + \frac{1}{5}x^{10}gba + \frac{1}{9}x^9cb^2 + \frac{2}{9}x^9fba + \frac{1}{4}x^8eba + \frac{1}{8}x^8ha^2 + \frac{2}{7}x^7dba + \frac{1}{7}x^7ga^2 + \frac{1}{3}x^6cba + \frac{1}{6}x^6fa^2 + \frac{1}{5}x^5ea^2 + \frac{1}{4}x^4da^2 + \frac{1}{3}x^3ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/14*x^14*h*b^2 + 1/13*x^13*g*b^2 + 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 2/1
1*x^11*h*b*a + 1/10*x^10*d*b^2 + 1/5*x^10*g*b*a + 1/9*x^9*c*b^2 + 2/9*x^9*f
*b*a + 1/4*x^8*e*b*a + 1/8*x^8*h*a^2 + 2/7*x^7*d*b*a + 1/7*x^7*g*a^2 + 1/3*
x^6*c*b*a + 1/6*x^6*f*a^2 + 1/5*x^5*e*a^2 + 1/4*x^4*d*a^2 + 1/3*x^3*c*a^2
```

giac [A] time = 0.18, size = 160, normalized size = 1.01

$$\frac{1}{14}b^2hx^{14} + \frac{1}{13}b^2gx^{13} + \frac{1}{12}b^2fx^{12} + \frac{2}{11}abhx^{11} + \frac{1}{11}b^2x^{11}e + \frac{1}{10}b^2dx^{10} + \frac{1}{5}abgx^{10} + \frac{1}{9}b^2cx^9 + \frac{2}{9}abfx^9 + \frac{1}{8}a^2hx^8 + \frac{1}{4}abx^8e + \frac{2}{7}abdx^7 + \frac{1}{7}a^2gx^7 + \frac{1}{3}abcx^6 + \frac{1}{6}a^2fx^6 + \frac{1}{5}a^2x^5e + \frac{1}{4}a^2dx^4 + \frac{1}{3}a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 2/11*a*b*h*x^11 + 1/1
1*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/5*a*b*g*x^10 + 1/9*b^2*c*x^9 + 2/9*a*b*f
*x^9 + 1/8*a^2*h*x^8 + 1/4*a*b*x^8*e + 2/7*a*b*d*x^7 + 1/7*a^2*g*x^7 + 1/3*
a*b*c*x^6 + 1/6*a^2*f*x^6 + 1/5*a^2*x^5*e + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

maple [A] time = 0.04, size = 152, normalized size = 0.96

$$\frac{b^2 h x^{14}}{14} + \frac{b^2 g x^{13}}{13} + \frac{b^2 f x^{12}}{12} + \frac{\left(2abh + b^2 e\right) x^{11}}{11} + \frac{\left(2abg + b^2 d\right) x^{10}}{10} + \frac{\left(2abf + c b^2\right) x^9}{9} + \frac{a^2 e x^5}{5} + \frac{\left(a^2 h + 2bea\right) x^8}{8} + \frac{a^2 d x^4}{4} + \frac{\left(a^2 g + 2bda\right) x^7}{7} + \frac{a^2 c x^3}{3} + \frac{\left(a^2 f + 2abc\right) x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/14*b^2*h*x^14+1/13*b^2*g*x^13+1/12*b^2*f*x^12+1/11*(2*a*b*h+b^2*e)*x^11+1
/10*(2*a*b*g+b^2*d)*x^10+1/9*(2*a*b*f+b^2*c)*x^9+1/8*(a^2*h+2*a*b*e)*x^8+1/
7*(a^2*g+2*a*b*d)*x^7+1/6*(a^2*f+2*a*b*c)*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1
/3*a^2*c*x^3
```

maxima [A] time = 1.38, size = 151, normalized size = 0.96

$$\frac{1}{14}b^2hx^{14} + \frac{1}{13}b^2gx^{13} + \frac{1}{12}b^2fx^{12} + \frac{1}{11}\left(b^2e + 2\,abh\right)x^{11} + \frac{1}{10}\left(b^2d + 2\,abg\right)x^{10} + \frac{1}{9}\left(b^2c + 2\,abf\right)x^9 + \frac{1}{8}\left(2\,abe + a^2h\right)x^8 + \frac{1}{5}a^2ex^5 + \frac{1}{7}\left(2\,abd + a^2g\right)x^7 + \frac{1}{4}a^2dx^4 + \frac{1}{6}\left(2\,abc + a^2f\right)x^6 + \frac{1}{3}a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*(b^2*e + 2*a*b*h
)*x^11 + 1/10*(b^2*d + 2*a*b*g)*x^10 + 1/9*(b^2*c + 2*a*b*f)*x^9 + 1/8*(2*a
*b*e + a^2*h)*x^8 + 1/5*a^2*e*x^5 + 1/7*(2*a*b*d + a^2*g)*x^7 + 1/4*a^2*d*x
^4 + 1/6*(2*a*b*c + a^2*f)*x^6 + 1/3*a^2*c*x^3
```

mupad [B] time = 0.09, size = 151, normalized size = 0.96

$$x^6\left(\frac{f\,a^2}{6}+\frac{b\,c\,a}{3}\right)+x^9\left(\frac{c\,b^2}{9}+\frac{2\,a\,f\,b}{9}\right)+x^7\left(\frac{g\,a^2}{7}+\frac{2\,b\,d\,a}{7}\right)+x^{10}\left(\frac{d\,b^2}{10}+\frac{a\,g\,b}{5}\right)+x^8\left(\frac{h\,a^2}{8}+\frac{b\,e\,a}{4}\right)+x^{11}\left(\frac{e\,b^2}{11}+\frac{2\,a\,h\,b}{11}\right)+\frac{a^2\,c\,x^3}{3}+\frac{a^2\,d\,x^4}{4}+\frac{a^2\,e\,x^5}{5}+\frac{b^2\,f\,x^{12}}{12}+\frac{b^2\,g\,x^{13}}{13}+\frac{b^2\,h\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^6*((a^2*f)/6 + (a*b*c)/3) + x^9*((b^2*c)/9 + (2*a*b*f)/9) + x^7*((a^2*g)/
7 + (2*a*b*d)/7) + x^10*((b^2*d)/10 + (a*b*g)/5) + x^8*((a^2*h)/8 + (a*b*e)
/4) + x^11*((b^2*e)/11 + (2*a*b*h)/11) + (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a
^2*e*x^5)/5 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14
```

sympy [A] time = 0.10, size = 167, normalized size = 1.06

$$\frac{a^2cx^3}{3}+\frac{a^2dx^4}{4}+\frac{a^2ex^5}{5}+\frac{b^2fx^{12}}{12}+\frac{b^2gx^{13}}{13}+\frac{b^2hx^{14}}{14}+x^{11}\left(\frac{2abh}{11}+\frac{b^2e}{11}\right)+x^{10}\left(\frac{abg}{5}+\frac{b^2d}{10}\right)+x^9\left(\frac{2abf}{9}+\frac{b^2c}{9}\right)+x^8\left(\frac{a^2h}{8}+\frac{abe}{4}\right)+x^7\left(\frac{a^2g}{7}+\frac{2abd}{7}\right)+x^6\left(\frac{a^2f}{6}+\frac{abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + b**2*f*x**12/12 + b**2*g*x*
*13/13 + b**2*h*x**14/14 + x**11*(2*a*b*h/11 + b**2*e/11) + x**10*(a*b*g/5
+ b**2*d/10) + x**9*(2*a*b*f/9 + b**2*c/9) + x**8*(a**2*h/8 + a*b*e/4) + x*
*7*(a**2*g/7 + 2*a*b*d/7) + x**6*(a**2*f/6 + a*b*c/3)
```

3.333 $\int x\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=158

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{6}a^2gx^6+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+\frac{2}{9}abg$$

Rubi [A] time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1582, 1850}

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{6}a^2gx^6+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+\frac{2}{9}abgx^9+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (a^2*g*x^6)/6 + (
a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8 + (2*a*b*g*x^9)/9 + (b*(b*
e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13 +
 (d*(a + b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx &= \frac{d\left(a+bx^3\right)^3}{9b}+\int\left(a+bx^3\right)^2\left(-dx^2+x\left(c+dx+ex^2+fx\right.\right.\\ &= \frac{d\left(a+bx^3\right)^3}{9b}+\int\left(a^2cx+a^2ex^3+a(2bc+af)x^4+a^2gx^5\right.\\ &= \frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{5}a(2bc+af)x^5+\frac{1}{6}a^2gx^6+\frac{1}{7}a(2be+\end{aligned}$$

Mathematica [A] time = 0.03, size = 163, normalized size = 1.03

$$\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{1}{9}bx^9(2ag+bd)+\frac{1}{6}ax^6(ag+2bd)+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (
a*(2*b*d + a*g)*x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8
+ (b*(b*d + 2*a*g)*x^9)/9 + (b*(b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (
b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5
),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5
), x]
```

fricas [A] time = 0.37, size = 157, normalized size = 0.99

$$\frac{1}{13}x^{13}hb^2+\frac{1}{12}x^{12}gb^2+\frac{1}{11}x^{11}fb^2+\frac{1}{10}x^{10}eb^2+\frac{1}{5}x^{10}hba+\frac{1}{9}x^9db^2+\frac{2}{9}x^9gba+\frac{1}{8}x^8cb^2+\frac{1}{4}x^8fba+\frac{2}{7}x^7eba+\frac{1}{7}x^7ha^2+\frac{1}{3}x^6dba+\frac{1}{6}x^6ga^2+\frac{2}{5}x^5cba+\frac{1}{5}x^5fa^2+\frac{1}{4}x^4ea^2+\frac{1}{3}x^3da^2+\frac{1}{2}x^2ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/13*x^13*h*b^2 + 1/12*x^12*g*b^2 + 1/11*x^11*f*b^2 + 1/10*x^10*e*b^2 + 1/5
*x^10*h*b*a + 1/9*x^9*d*b^2 + 2/9*x^9*g*b*a + 1/8*x^8*c*b^2 + 1/4*x^8*f*b*a
 + 2/7*x^7*e*b*a + 1/7*x^7*h*a^2 + 1/3*x^6*d*b*a + 1/6*x^6*g*a^2 + 2/5*x^5*
c*b*a + 1/5*x^5*f*a^2 + 1/4*x^4*e*a^2 + 1/3*x^3*d*a^2 + 1/2*x^2*c*a^2
```

giac [A] time = 0.19, size = 160, normalized size = 1.01

$$\frac{1}{13}b^2hx^{13}+\frac{1}{12}b^2gx^{12}+\frac{1}{11}b^2fx^{11}+\frac{1}{5}abhx^{10}+\frac{1}{10}b^2x^{10}e+\frac{1}{9}b^2dx^9+\frac{2}{9}abgx^9+\frac{1}{8}b^2cx^8+\frac{1}{4}abfx^8+\frac{1}{7}a^2hx^7+\frac{2}{7}abx^7e+\frac{1}{3}abdx^6+\frac{1}{6}a^2gx^6+\frac{2}{5}abcx^5+\frac{1}{5}a^2fx^5+\frac{1}{4}a^2x^4e+\frac{1}{3}a^2dx^3+\frac{1}{2}a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/5*a*b*h*x^10 + 1/10
*b^2*x^10*e + 1/9*b^2*d*x^9 + 2/9*a*b*g*x^9 + 1/8*b^2*c*x^8 + 1/4*a*b*f*x^8
 + 1/7*a^2*h*x^7 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 1/6*a^2*g*x^6 + 2/5*a*b*
c*x^5 + 1/5*a^2*f*x^5 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

maple [A] time = 0.04, size = 152, normalized size = 0.96

$$\frac{b^2hx^{13}}{13}+\frac{b^2gx^{12}}{12}+\frac{b^2fx^{11}}{11}+\frac{\left(2abh+b^2e\right)x^{10}}{10}+\frac{\left(2abg+b^2d\right)x^9}{9}+\frac{\left(2abf+cb^2\right)x^8}{8}+\frac{a^2ex^4}{4}+\frac{\left(a^2h+2bea\right)x^7}{7}+\frac{a^2dx^3}{3}+\frac{\left(a^2g+2bda\right)x^6}{6}+\frac{a^2cx^2}{2}+\frac{\left(a^2f+2abc\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/13*b^2*h*x^13+1/12*b^2*g*x^12+1/11*b^2*f*x^11+1/10*(2*a*b*h+b^2*e)*x^10+1
/9*(2*a*b*g+b^2*d)*x^9+1/8*(2*a*b*f+b^2*c)*x^8+1/7*(a^2*h+2*a*b*e)*x^7+1/6*
(a^2*g+2*a*b*d)*x^6+1/5*(a^2*f+2*a*b*c)*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2
*a^2*c*x^2
```

maxima [A] time = 1.28, size = 151, normalized size = 0.96

$$\frac{1}{13}b^2hx^{13}+\frac{1}{12}b^2gx^{12}+\frac{1}{11}b^2fx^{11}+\frac{1}{10}\left(b^2e+2\,abh\right)x^{10}+\frac{1}{9}\left(b^2d+2\,abg\right)x^9+\frac{1}{8}\left(b^2c+2\,abf\right)x^8+\frac{1}{7}\left(2\,abe+a^2h\right)x^7+\frac{1}{4}a^2ex^4+\frac{1}{6}\left(2\,abd+a^2g\right)x^6+\frac{1}{3}a^2dx^3+\frac{1}{5}\left(2\,abc+a^2f\right)x^5+\frac{1}{2}a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*(b^2*e + 2*a*b*h
)*x^10 + 1/9*(b^2*d + 2*a*b*g)*x^9 + 1/8*(b^2*c + 2*a*b*f)*x^8 + 1/7*(2*a*b
*e + a^2*h)*x^7 + 1/4*a^2*e*x^4 + 1/6*(2*a*b*d + a^2*g)*x^6 + 1/3*a^2*d*x^3
 + 1/5*(2*a*b*c + a^2*f)*x^5 + 1/2*a^2*c*x^2
```

mupad [B] time = 0.09, size = 151, normalized size = 0.96

$$x^5\left(\frac{f\,a^2}{5}+\frac{2\,b\,c\,a}{5}\right)+x^8\left(\frac{c\,b^2}{8}+\frac{a\,f\,b}{4}\right)+x^6\left(\frac{g\,a^2}{6}+\frac{b\,d\,a}{3}\right)+x^9\left(\frac{d\,b^2}{9}+\frac{2\,a\,g\,b}{9}\right)+x^7\left(\frac{h\,a^2}{7}+\frac{2\,b\,e\,a}{7}\right)+x^{10}\left(\frac{e\,b^2}{10}+\frac{a\,h\,b}{5}\right)+\frac{a^2\,c\,x^2}{2}+\frac{a^2\,d\,x^3}{3}+\frac{a^2\,e\,x^4}{4}+\frac{b^2\,f\,x^{11}}{11}+\frac{b^2\,g\,x^{12}}{12}+\frac{b^2\,h\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^5*((a^2*f)/5 + (2*a*b*c)/5) + x^8*((b^2*c)/8 + (a*b*f)/4) + x^6*((a^2*g)/
6 + (a*b*d)/3) + x^9*((b^2*d)/9 + (2*a*b*g)/9) + x^7*((a^2*h)/7 + (2*a*b*e)
/7) + x^10*((b^2*e)/10 + (a*b*h)/5) + (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*
e*x^4)/4 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

sympy [A] time = 0.10, size = 167, normalized size = 1.06

$$\frac{a^{2}cx^{2}}{2}+\frac{a^{2}dx^{3}}{3}+\frac{a^{2}ex^{4}}{4}+\frac{b^{2}fx^{11}}{11}+\frac{b^{2}gx^{12}}{12}+\frac{b^{2}hx^{13}}{13}+x^{10}\left(\frac{abh}{5}+\frac{b^{2}e}{10}\right)+x^{9}\left(\frac{2abg}{9}+\frac{b^{2}d}{9}\right)+x^{8}\left(\frac{abf}{4}+\frac{b^{2}c}{8}\right)+x^{7}\left(\frac{a^{2}h}{7}+\frac{2abe}{7}\right)+x^{6}\left(\frac{a^{2}g}{6}+\frac{abd}{3}\right)+x^{5}\left(\frac{a^{2}f}{5}+\frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + b**2*f*x**11/11 + b**2*g*x*
*12/12 + b**2*h*x**13/13 + x**10*(a*b*h/5 + b**2*e/10) + x**9*(2*a*b*g/9 +
b**2*d/9) + x**8*(a*b*f/4 + b**2*c/8) + x**7*(a**2*h/7 + 2*a*b*e/7) + x**6*
(a**2*g/6 + a*b*d/3) + x**5*(a**2*f/5 + 2*a*b*c/5)
```

3.334 $\int \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=153

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{6}a^2hx^6+\frac{1}{7}bx^7(2af+bc)+\frac{1}{4}ax^4(af+2bc)+\frac{1}{8}bx^8(2ag+bd)+\frac{1}{5}ax^5(ag+2bd)+\frac{e\left(a+bx^3\right)^3}{9b}+\frac{2}{9}abhx$$

Rubi [A] time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1582, 1850}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{6}a^2hx^6+\frac{1}{7}bx^7(2af+bc)+\frac{1}{4}ax^4(af+2bc)+\frac{1}{8}bx^8(2ag+bd)+\frac{1}{5}ax^5(ag+2bd)+\frac{e\left(a+bx^3\right)^3}{9b}+\frac{2}{9}abhx^9+\frac{1}{10}b^2fx^{10}+\frac{1}{11}b^2gx^{11}+\frac{1}{12}b^2hx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*(2*b*c + a*f)*x^4)/4 + (a*(2*b*d + a*g)*x^5)/5
 + (a^2*h*x^6)/6 + (b*(b*c + 2*a*f)*x^7)/7 + (b*(b*d + 2*a*g)*x^8)/8 + (2*a
*b*h*x^9)/9 + (b^2*f*x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + (e*(a +
 b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx &= \frac{e\left(a+bx^3\right)^3}{9b}+\int \left(a+bx^3\right)^2 \left(c+dx+fx^3+gx^4+hx^5\right)\\ &= \frac{e\left(a+bx^3\right)^3}{9b}+\int \left(a^2c+a^2dx+a(2bc+af)x^3+a(2bd+\right.\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{4}a(2bc+af)x^4+\frac{1}{5}a(2bd+ag)x^5+\frac{1}{6}\end{aligned}$$

Mathematica [A] time = 0.09, size = 125, normalized size = 0.82

$$\frac{462a^2x\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+22abx^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))+b^2x^7\left(3960c+7x\left(495d+440ex+6x^2\left(66f+60gx+55hx^2\right)\right)\right)}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (b^2*x^7*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2))
) + 462*a^2*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))) +
 22*a*b*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x)))))
)/27720
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),
x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),
 x]
```

fricas [A] time = 0.37, size = 154, normalized size = 1.01

$$\frac{1}{12}x^{12}hb^2 + \frac{1}{11}x^{11}gb^2 + \frac{1}{10}x^{10}fb^2 + \frac{1}{9}x^9eb^2 + \frac{2}{9}x^9hba + \frac{1}{8}x^8db^2 + \frac{1}{4}x^8gba + \frac{1}{7}x^7cb^2 + \frac{2}{7}x^7fba + \frac{1}{3}x^6eba + \frac{1}{6}x^6ha^2 + \frac{2}{5}x^5dba + \frac{1}{5}x^5ga^2 + \frac{1}{2}x^4cba + \frac{1}{4}x^4fa^2 + \frac{1}{3}x^3ea^2 + \frac{1}{2}x^2da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/12*x^12*h*b^2 + 1/11*x^11*g*b^2 + 1/10*x^10*f*b^2 + 1/9*x^9*e*b^2 + 2/9*x
^9*h*b*a + 1/8*x^8*d*b^2 + 1/4*x^8*g*b*a + 1/7*x^7*c*b^2 + 2/7*x^7*f*b*a +
1/3*x^6*e*b*a + 1/6*x^6*h*a^2 + 2/5*x^5*d*b*a + 1/5*x^5*g*a^2 + 1/2*x^4*c*b
*a + 1/4*x^4*f*a^2 + 1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2
```

giac [A] time = 0.16, size = 157, normalized size = 1.03

$$\frac{1}{12}b^2hx^{12} + \frac{1}{11}b^2gx^{11} + \frac{1}{10}b^2fx^{10} + \frac{2}{9}abhx^9 + \frac{1}{9}b^2x^9e + \frac{1}{8}b^2dx^8 + \frac{1}{4}abgx^8 + \frac{1}{7}b^2cx^7 + \frac{2}{7}abfx^7 + \frac{1}{6}a^2hx^6 + \frac{1}{3}abx^6e + \frac{2}{5}abdx^5 + \frac{1}{5}a^2gx^5 + \frac{1}{2}abcx^4 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2x^3e + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 2/9*a*b*h*x^9 + 1/9*b
^2*x^9*e + 1/8*b^2*d*x^8 + 1/4*a*b*g*x^8 + 1/7*b^2*c*x^7 + 2/7*a*b*f*x^7 +
1/6*a^2*h*x^6 + 1/3*a*b*x^6*e + 2/5*a*b*d*x^5 + 1/5*a^2*g*x^5 + 1/2*a*b*c*x
^4 + 1/4*a^2*f*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

maple [A] time = 0.04, size = 149, normalized size = 0.97

$$\frac{b^2h\,x^{12}}{12} + \frac{b^2g\,x^{11}}{11} + \frac{b^2f\,x^{10}}{10} + \frac{\left(2abh+b^2e\right)x^9}{9} + \frac{\left(2abg+b^2d\right)x^8}{8} + \frac{\left(2abf+c\,b^2\right)x^7}{7} + \frac{a^2e\,x^3}{3} + \frac{\left(a^2h+2bea\right)x^6}{6} + \frac{a^2d\,x^2}{2} + \frac{\left(a^2g+2bda\right)x^5}{5} + a^2cx + \frac{\left(a^2f+2abc\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/12*b^2*h*x^12+1/11*b^2*g*x^11+1/10*b^2*f*x^10+1/9*(2*a*b*h+b^2*e)*x^9+1/8
*(2*a*b*g+b^2*d)*x^8+1/7*(2*a*b*f+b^2*c)*x^7+1/6*(a^2*h+2*a*b*e)*x^6+1/5*(a
^2*g+2*a*b*d)*x^5+1/4*(a^2*f+2*a*b*c)*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c
*x
```

maxima [A] time = 1.32, size = 148, normalized size = 0.97

$$\frac{1}{12}b^2hx^{12} + \frac{1}{11}b^2gx^{11} + \frac{1}{10}b^2fx^{10} + \frac{1}{9}\left(b^2e+2\,abh\right)x^9 + \frac{1}{8}\left(b^2d+2\,abg\right)x^8 + \frac{1}{7}\left(b^2c+2\,abf\right)x^7 + \frac{1}{6}\left(2\,abe+a^2h\right)x^6 + \frac{1}{3}a^2ex^3 + \frac{1}{5}\left(2\,abd+a^2g\right)x^5 + \frac{1}{2}a^2dx^2 + \frac{1}{4}\left(2\,abc+a^2f\right)x^4 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*(b^2*e + 2*a*b*h)
*x^9 + 1/8*(b^2*d + 2*a*b*g)*x^8 + 1/7*(b^2*c + 2*a*b*f)*x^7 + 1/6*(2*a*b*e
 + a^2*h)*x^6 + 1/3*a^2*e*x^3 + 1/5*(2*a*b*d + a^2*g)*x^5 + 1/2*a^2*d*x^2 +
 1/4*(2*a*b*c + a^2*f)*x^4 + a^2*c*x
```

mupad [B] time = 0.09, size = 148, normalized size = 0.97

$$x^4\left(\frac{f\,a^2}{4}+\frac{b\,c\,a}{2}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,f\,b}{7}\right)+x^5\left(\frac{g\,a^2}{5}+\frac{2\,b\,d\,a}{5}\right)+x^8\left(\frac{d\,b^2}{8}+\frac{a\,g\,b}{4}\right)+x^6\left(\frac{h\,a^2}{6}+\frac{b\,e\,a}{3}\right)+x^9\left(\frac{e\,b^2}{9}+\frac{2\,a\,h\,b}{9}\right)+\frac{a^2\,d\,x^2}{2}+\frac{a^2\,e\,x^3}{3}+\frac{b^2\,f\,x^{10}}{10}+\frac{b^2\,g\,x^{11}}{11}+\frac{b^2\,h\,x^{12}}{12}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^4*((a^2*f)/4 + (a*b*c)/2) + x^7*((b^2*c)/7 + (2*a*b*f)/7) + x^5*((a^2*g)/
5 + (2*a*b*d)/5) + x^8*((b^2*d)/8 + (a*b*g)/4) + x^6*((a^2*h)/6 + (a*b*e)/3
) + x^9*((b^2*e)/9 + (2*a*b*h)/9) + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (b^2*f*
x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + a^2*c*x
```

sympy [A] time = 0.10, size = 163, normalized size = 1.07

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{b^2fx^{10}}{10}+\frac{b^2gx^{11}}{11}+\frac{b^2hx^{12}}{12}+x^9\left(\frac{2abh}{9}+\frac{b^2e}{9}\right)+x^8\left(\frac{abg}{4}+\frac{b^2d}{8}\right)+x^7\left(\frac{2abf}{7}+\frac{b^2c}{7}\right)+x^6\left(\frac{a^2h}{6}+\frac{abe}{3}\right)+x^5\left(\frac{a^2g}{5}+\frac{2abd}{5}\right)+x^4\left(\frac{a^2f}{4}+\frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + b**2*f*x**10/10 + b**2*g*x**11/1
1 + b**2*h*x**12/12 + x**9*(2*a*b*h/9 + b**2*e/9) + x**8*(a*b*g/4 + b**2*d/
8) + x**7*(2*a*b*f/7 + b**2*c/7) + x**6*(a**2*h/6 + a*b*e/3) + x**5*(a**2*g
/5 + 2*a*b*d/5) + x**4*(a**2*f/4 + a*b*c/2)
```

3.335 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal. Leaf size=149

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{f(a+bx^3)^3}{9b}$$

Rubi [A] time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1583, 1820}

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{f(a+bx^3)^3}{9b}+\frac{1}{6}b^2cx^6+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4 + (a*(2
*b*e + a*h)*x^5)/5 + (b^2*c*x^6)/6 + (b*(b*d + 2*a*g)*x^7)/7 + (b*(b*e + 2*
a*h)*x^8)/8 + (b^2*g*x^10)/10 + (b^2*h*x^11)/11 + (f*(a + b*x^3)^3)/(9*b) +
 a^2*c*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx=\frac{f(a+bx^3)^3}{9b}+\int \frac{(a+bx^3)^2(c+dx+ex^2+gx^4+hx^5)}{x}\,dx$$
$$=\frac{f(a+bx^3)^3}{9b}+\int \left(a^2d+\frac{a^2c}{x}+a^2ex+2abcx^2+a(2bd+ag\right.$$
$$=a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{4}a(2bd+ag)x^4+\frac{1}{5}a(2be+ah)$$

Mathematica [A] time = 0.05, size = 154, normalized size = 1.03

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{6}bx^6(2af+bc)+\frac{1}{3}ax^3(af+2bc)+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{1}{9}b^2fx^9+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*c + a*f)*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4
 + (a*(2*b*e + a*h)*x^5)/5 + (b*(b*c + 2*a*f)*x^6)/6 + (b*(b*d + 2*a*g)*x^7
```

```
)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*f*x^9)/9 + (b^2*g*x^10)/10 + (b^2*h*x^
11)/11 + a^2*c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x, x]
```

fricas [A] time = 0.40, size = 146, normalized size = 0.98

$$\frac{1}{11}b^2hx^{11}+\frac{1}{10}b^2gx^{10}+\frac{1}{9}b^2fx^9+\frac{1}{8}\left(b^2e+2\,abh\right)x^8+\frac{1}{7}\left(b^2d+2\,abg\right)x^7+\frac{1}{6}\left(b^2c+2\,abf\right)x^6+\frac{1}{5}\left(2\,abe+a^2h\right)x^5+\frac{1}{2}a^2ex^2+\frac{1}{4}\left(2\,abd+a^2g\right)x^4+a^2dx+\frac{1}{3}\left(2\,abc+a^2f\right)x^3+a^2c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="frica
s")
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*x
^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e +
 a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*(2*
a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

giac [A] time = 0.15, size = 156, normalized size = 1.05

$$\frac{1}{11}b^2hx^{11}+\frac{1}{10}b^2gx^{10}+\frac{1}{9}b^2fx^9+\frac{1}{4}abhx^8+\frac{1}{8}b^2x^8e+\frac{1}{7}b^2dx^7+\frac{2}{7}abgx^7+\frac{1}{6}b^2cx^6+\frac{1}{3}abfx^6+\frac{1}{5}a^2hx^5+\frac{2}{5}abx^5e+\frac{1}{2}abdx^4+\frac{1}{4}a^2gx^4+\frac{2}{3}abcx^3+\frac{1}{3}a^2fx^3+\frac{1}{2}a^2x^2e+a^2dx+a^2c\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac"
)
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/4*a*b*h*x^8 + 1/8*b^2
*x^8*e + 1/7*b^2*d*x^7 + 2/7*a*b*g*x^7 + 1/6*b^2*c*x^6 + 1/3*a*b*f*x^6 + 1/
5*a^2*h*x^5 + 2/5*a*b*x^5*e + 1/2*a*b*d*x^4 + 1/4*a^2*g*x^4 + 2/3*a*b*c*x^3
 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x + a^2*c*log(abs(x))
```

maple [A] time = 0.04, size = 153, normalized size = 1.03

$$\frac{b^2hx^{11}}{11}+\frac{b^2gx^{10}}{10}+\frac{b^2fx^9}{9}+\frac{abhx^8}{4}+\frac{b^2ex^8}{8}+\frac{2abgx^7}{7}+\frac{b^2dx^7}{7}+\frac{abfx^6}{3}+\frac{b^2cx^6}{6}+\frac{a^2hx^5}{5}+\frac{2abex^5}{5}+\frac{a^2gx^4}{4}+\frac{abdx^4}{2}+\frac{a^2fx^3}{3}+\frac{2abcx^3}{3}+\frac{a^2ex^2}{2}+a^2c\ln(x)+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] 1/11*b^2*h*x^11+1/10*b^2*g*x^10+1/9*x^9*f*b^2+1/4*x^8*a*b*h+1/8*b^2*e*x^8+2
/7*x^7*a*b*g+1/7*b^2*d*x^7+1/3*x^6*a*b*f+1/6*b^2*c*x^6+1/5*x^5*a^2*h+2/5*a*
b*e*x^5+1/4*x^4*a^2*g+1/2*a*b*d*x^4+1/3*x^3*a^2*f+2/3*a*b*c*x^3+1/2*a^2*e*x
^2+a^2*d*x+a^2*c*ln(x)
```

maxima [A] time = 1.31, size = 146, normalized size = 0.98

$$\frac{1}{11}b^2hx^{11}+\frac{1}{10}b^2gx^{10}+\frac{1}{9}b^2fx^9+\frac{1}{8}\left(b^2e+2\,abh\right)x^8+\frac{1}{7}\left(b^2d+2\,abg\right)x^7+\frac{1}{6}\left(b^2c+2\,abf\right)x^6+\frac{1}{5}\left(2\,abe+a^2h\right)x^5+\frac{1}{2}a^2ex^2+\frac{1}{4}\left(2\,abd+a^2g\right)x^4+a^2dx+\frac{1}{3}\left(2\,abc+a^2f\right)x^3+a^2c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxim
a")
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*x
^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e +
 a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*(2*
a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

mupad [B] time = 0.10, size = 146, normalized size = 0.98

$$x^3\left(\frac{f\,a^2}{3}+\frac{2\,b\,c\,a}{3}\right)+x^6\left(\frac{c\,b^2}{6}+\frac{a\,f\,b}{3}\right)+x^4\left(\frac{g\,a^2}{4}+\frac{b\,d\,a}{2}\right)+x^7\left(\frac{d\,b^2}{7}+\frac{2\,a\,g\,b}{7}\right)+x^5\left(\frac{h\,a^2}{5}+\frac{2\,b\,e\,a}{5}\right)+x^8\left(\frac{e\,b^2}{8}+\frac{a\,h\,b}{4}\right)+\frac{a^2\,e\,x^2}{2}+\frac{b^2\,f\,x^9}{9}+\frac{b^2\,g\,x^{10}}{10}+\frac{b^2\,h\,x^{11}}{11}+a^2\,c\,\ln(x)+a^2\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

```
[Out] x^3*((a^2*f)/3 + (2*a*b*c)/3) + x^6*((b^2*c)/6 + (a*b*f)/3) + x^4*((a^2*g)/
4 + (a*b*d)/2) + x^7*((b^2*d)/7 + (2*a*b*g)/7) + x^5*((a^2*h)/5 + (2*a*b*e)
/5) + x^8*((b^2*e)/8 + (a*b*h)/4) + (a^2*e*x^2)/2 + (b^2*f*x^9)/9 + (b^2*g*
x^10)/10 + (b^2*h*x^11)/11 + a^2*c*log(x) + a^2*d*x
```

sympy [A] time = 0.34, size = 162, normalized size = 1.09

$$a^{2}c\log{\left(x \right)}+a^{2}dx+\frac{a^{2}ex^{2}}{2}+\frac{b^{2}fx^{9}}{9}+\frac{b^{2}gx^{10}}{10}+\frac{b^{2}hx^{11}}{11}+x^{8}\left(\frac{abh}{4}+\frac{b^{2}e}{8}\right)+x^{7}\left(\frac{2abg}{7}+\frac{b^{2}d}{7}\right)+x^{6}\left(\frac{abf}{3}+\frac{b^{2}c}{6}\right)+x^{5}\left(\frac{a^{2}h}{5}+\frac{2abe}{5}\right)+x^{4}\left(\frac{a^{2}g}{4}+\frac{abd}{2}\right)+x^{3}\left(\frac{a^{2}f}{3}+\frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + b**2*f*x**9/9 + b**2*g*x**10/10
+ b**2*h*x**11/11 + x**8*(a*b*h/4 + b**2*e/8) + x**7*(2*a*b*g/7 + b**2*d/7)
 + x**6*(a*b*f/3 + b**2*c/6) + x**5*(a**2*h/5 + 2*a*b*e/5) + x**4*(a**2*g/4
 + a*b*d/2) + x**3*(a**2*f/3 + 2*a*b*c/3)
```

3.336 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{2}{3}abdx^3+\frac{1}{7}bx^7(2ah+be)+\frac{1}{4}ax^4(ah+2be)+\frac{g(a+bx^3)}{9b}$$

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1583, 1820}

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{2}{3}abdx^3+\frac{1}{7}bx^7(2ah+be)+\frac{1}{4}ax^4(ah+2be)+\frac{g(a+bx^3)^3}{9b}+\frac{1}{6}b^2dx^6+\frac{1}{8}b^2fx^8+\frac{1}{10}b^2hx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (2*a*b*d*x^3)/3 + (a*(2*
b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b^2*d*x^6)/6 + (b*(b*e + 2*a
*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*h*x^10)/10 + (g*(a + b*x^3)^3)/(9*b) + a^
2*d*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = \frac{g(a+bx^3)^3}{9b}+\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+hx^5)}{x^2}$$
$$= \frac{g(a+bx^3)^3}{9b}+\int \left(a^2e+\frac{a^2c}{x^2}+\frac{a^2d}{x}+a(2bc+af)x+2a\right.$$
$$= -\frac{a^2c}{x}+a^2ex+\frac{1}{2}a(2bc+af)x^2+\frac{2}{3}abdx^3+\frac{1}{4}a(2be+ah$$

Mathematica [A] time = 0.07, size = 152, normalized size = 1.03

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{1}{6}bx^6(2ag+bd)+\frac{1}{3}ax^3(ag+2bd)+\frac{1}{7}bx^7(2ah+be)+\frac{1}{4}ax^4(ah+2be)+\frac{1}{8}b^2fx^8+\frac{1}{9}b^2gx^9+\frac{1}{10}b^2hx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (a*(2*b*d + a*g)*x^3)/3
+ (a*(2*b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b*(b*d + 2*a*g)*x^6)
```

```
/6 + (b*(b*e + 2*a*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*g*x^9)/9 + (b^2*h*x^10)
/10 + a^2*d*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^2, x]
```

fricas [A] time = 0.41, size = 153, normalized size = 1.04

$$\frac{252\,b^2hx^{11}+280\,b^2gx^{10}+315\,b^2fx^9+360\left(b^2e+2\,abh\right)x^8+420\left(b^2d+2\,abg\right)x^7+504\left(b^2c+2\,abf\right)x^6+630\left(2\,abe+a^2h\right)x^5+2520\,a^2ex^2+840\left(2\,abd+a^2g\right)x^4+2520\,a^2dx\log(x)+1260\left(2\,abc+a^2f\right)x^3-2520\,a^2c}{2520\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(252*b^2*h*x^11 + 280*b^2*g*x^10 + 315*b^2*f*x^9 + 360*(b^2*e + 2*a*
b*h)*x^8 + 420*(b^2*d + 2*a*b*g)*x^7 + 504*(b^2*c + 2*a*b*f)*x^6 + 630*(2*a
*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 2520*a^2*d
*x*log(x) + 1260*(2*a*b*c + a^2*f)*x^3 - 2520*a^2*c)/x
```

giac [A] time = 0.15, size = 155, normalized size = 1.05

$$\frac{1}{10}\,b^2hx^{10}+\frac{1}{9}\,b^2gx^9+\frac{1}{8}\,b^2fx^8+\frac{2}{7}\,abhx^7+\frac{1}{7}\,b^2x^7e+\frac{1}{6}\,b^2dx^6+\frac{1}{3}\,abgx^6+\frac{1}{5}\,b^2cx^5+\frac{2}{5}\,abfx^5+\frac{1}{4}\,a^2hx^4+\frac{1}{2}\,abx^4e+\frac{2}{3}\,abdx^3+\frac{1}{3}\,a^2gx^3+abcx^2+\frac{1}{2}\,a^2fx^2+a^2xe+a^2d\log\left(|x|\right)-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="gia
c")
```

```
[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 2/7*a*b*h*x^7 + 1/7*b^2*x
^7*e + 1/6*b^2*d*x^6 + 1/3*a*b*g*x^6 + 1/5*b^2*c*x^5 + 2/5*a*b*f*x^5 + 1/4*
a^2*h*x^4 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2*g*x^3 + a*b*c*x^2 + 1/2
*a^2*f*x^2 + a^2*x*e + a^2*d*log(abs(x)) - a^2*c/x
```

maple [A] time = 0.05, size = 152, normalized size = 1.03

$$\frac{b^2h\,x^{10}}{10}+\frac{b^2g\,x^9}{9}+\frac{b^2f\,x^8}{8}+\frac{2abh\,x^7}{7}+\frac{b^2e\,x^7}{7}+\frac{abg\,x^6}{3}+\frac{b^2d\,x^6}{6}+\frac{2abf\,x^5}{5}+\frac{b^2c\,x^5}{5}+\frac{a^2h\,x^4}{4}+\frac{abe\,x^4}{2}+\frac{a^2g\,x^3}{3}+\frac{2abd\,x^3}{3}+\frac{a^2f\,x^2}{2}+abc\,x^2+a^2d\ln(x)+a^2ex-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] 1/10*b^2*h*x^10+1/9*b^2*g*x^9+1/8*b^2*f*x^8+2/7*x^7*a*b*h+1/7*b^2*e*x^7+1/3
*x^6*a*b*g+1/6*b^2*d*x^6+2/5*x^5*a*b*f+1/5*b^2*c*x^5+1/4*x^4*a^2*h+1/2*a*b*
e*x^4+1/3*x^3*a^2*g+2/3*a*b*d*x^3+1/2*a^2*f*x^2+a*b*c*x^2+a^2*e*x-a^2*c/x+a
^2*d*ln(x)
```

maxima [A] time = 1.40, size = 146, normalized size = 0.99

$$\frac{1}{10}\,b^2hx^{10}+\frac{1}{9}\,b^2gx^9+\frac{1}{8}\,b^2fx^8+\frac{1}{7}\left(b^2e+2\,abh\right)x^7+\frac{1}{6}\left(b^2d+2\,abg\right)x^6+\frac{1}{5}\left(b^2c+2\,abf\right)x^5+\frac{1}{4}\left(2\,abe+a^2h\right)x^4+a^2ex+\frac{1}{3}\left(2\,abd+a^2g\right)x^3+a^2d\log(x)+\frac{1}{2}\left(2\,abc+a^2f\right)x^2-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="max
ima")
```

```
[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 1/7*(b^2*e + 2*a*b*h)*x^7
 + 1/6*(b^2*d + 2*a*b*g)*x^6 + 1/5*(b^2*c + 2*a*b*f)*x^5 + 1/4*(2*a*b*e + a
^2*h)*x^4 + a^2*e*x + 1/3*(2*a*b*d + a^2*g)*x^3 + a^2*d*log(x) + 1/2*(2*a*b
*c + a^2*f)*x^2 - a^2*c/x
```

mupad [B] time = 0.10, size = 145, normalized size = 0.99

$$x^2\left(\frac{f\,a^2}{2}+b\,c\,a\right)+x^5\left(\frac{c\,b^2}{5}+\frac{2\,a\,f\,b}{5}\right)+x^3\left(\frac{g\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^6\left(\frac{d\,b^2}{6}+\frac{a\,g\,b}{3}\right)+x^4\left(\frac{h\,a^2}{4}+\frac{b\,e\,a}{2}\right)+x^7\left(\frac{e\,b^2}{7}+\frac{2\,a\,h\,b}{7}\right)-\frac{a^2\,c}{x}+\frac{b^2\,f\,x^8}{8}+\frac{b^2\,g\,x^9}{9}+\frac{b^2\,h\,x^{10}}{10}+a^2\,d\,\ln(x)+a^2\,e\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

```
[Out] x^2*((a^2*f)/2 + a*b*c) + x^5*((b^2*c)/5 + (2*a*b*f)/5) + x^3*((a^2*g)/3 +
(2*a*b*d)/3) + x^6*((b^2*d)/6 + (a*b*g)/3) + x^4*((a^2*h)/4 + (a*b*e)/2) +
x^7*((b^2*e)/7 + (2*a*b*h)/7) - (a^2*c)/x + (b^2*f*x^8)/8 + (b^2*g*x^9)/9 +
 (b^2*h*x^10)/10 + a^2*d*log(x) + a^2*e*x
```

sympy [A] time = 0.36, size = 156, normalized size = 1.06

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{b^2fx^8}{8}+\frac{b^2gx^9}{9}+\frac{b^2hx^{10}}{10}+x^7\left(\frac{2abh}{7}+\frac{b^2e}{7}\right)+x^6\left(\frac{abg}{3}+\frac{b^2d}{6}\right)+x^5\left(\frac{2abf}{5}+\frac{b^2c}{5}\right)+x^4\left(\frac{a^2h}{4}+\frac{abe}{2}\right)+x^3\left(\frac{a^2g}{3}+\frac{2abd}{3}\right)+x^2\left(\frac{a^2f}{2}+abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + b**2*f*x**8/8 + b**2*g*x**9/9 + b**2
*h*x**10/10 + x**7*(2*a*b*h/7 + b**2*e/7) + x**6*(a*b*g/3 + b**2*d/6) + x**
5*(2*a*b*f/5 + b**2*c/5) + x**4*(a**2*h/4 + a*b*e/2) + x**3*(a**2*g/3 + 2*a
*b*d/3) + x**2*(a**2*f/2 + a*b*c)
```

3.337 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+\frac{1}{4}bx^4(2af+bc)+ax(af+2bc)+\frac{1}{5}bx^5(2ag+bd)+\frac{1}{2}ax^2(ag+2bd)+\frac{2}{3}abex^3+\frac{h\left(a+bx^3\right)^3}{9b}+\frac{1}{6}b$$

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1583, 1820}

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+\frac{1}{4}bx^4(2af+bc)+ax(af+2bc)+\frac{1}{5}bx^5(2ag+bd)+\frac{1}{2}ax^2(ag+2bd)+\frac{2}{3}abex^3+\frac{h\left(a+bx^3\right)^3}{9b}+\frac{1}{6}b^2ex^6+\frac{1}{7}b^2fx^7+\frac{1}{8}b^2gx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -(a^2*c)/(2*x^2) - (a^2*d)/x + a*(2*b*c + a*f)*x + (a*(2*b*d + a*g)*x^2)/2
+ (2*a*b*e*x^3)/3 + (b*(b*c + 2*a*f)*x^4)/4 + (b*(b*d + 2*a*g)*x^5)/5 + (b^
2*e*x^6)/6 + (b^2*f*x^7)/7 + (b^2*g*x^8)/8 + (h*(a + b*x^3)^3)/(9*b) + a^2*
e*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^3}\,dx &= \frac{h\left(a+bx^3\right)^3}{9b}+\int \frac{\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx\\ &= \frac{h\left(a+bx^3\right)^3}{9b}+\int \left(a(2bc+af)+\frac{a^2c}{x^3}+\frac{a^2d}{x^2}+\frac{a^2e}{x}+a(2bd\right.\\ &= -\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a(2bc+af)x+\frac{1}{2}a(2bd+ag)x^2+\frac{2}{3}abex^3+\end{aligned}$$

Mathematica [A] time = 0.10, size = 127, normalized size = 0.86

$$\frac{a^2\left(-3c-6dx+x^3\left(6f+3gx+2hx^2\right)\right)}{6x^2}+a^2e\log(x)+\frac{1}{30}abx\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+\frac{b^2x^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] (a^2*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (a*b*x*(60*c +
 x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/30 + (b^2*x^4*(630*c
```

```
+ x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/2520 + a^2*e*Log[
x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^3, x]
```

fricas [A] time = 0.40, size = 153, normalized size = 1.04

$$\frac{280\, b^2hx^{11} + 315\, b^2gx^{10} + 360\, b^2fx^9 + 420\, (b^2e + 2\, abh)x^8 + 504\, (b^2d + 2\, abg)x^7 + 630\, (b^2c + 2\, abf)x^6 + 840\, (2\, abe + a^2h)x^5 + 2520\, a^2ex^2\log(x) + 1260\, (2\, abd + a^2g)x^4 - 2520\, a^2dx + 2520\, (2\, abc + a^2f)x^3 - 1260\, a^2c}{2520\, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(280*b^2*h*x^11 + 315*b^2*g*x^10 + 360*b^2*f*x^9 + 420*(b^2*e + 2*a*
b*h)*x^8 + 504*(b^2*d + 2*a*b*g)*x^7 + 630*(b^2*c + 2*a*b*f)*x^6 + 840*(2*a
*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2*log(x) + 1260*(2*a*b*d + a^2*g)*x^4 - 25
20*a^2*d*x + 2520*(2*a*b*c + a^2*f)*x^3 - 1260*a^2*c)/x^2
```

giac [A] time = 0.15, size = 153, normalized size = 1.04

$$\frac{1}{9}\, b^2hx^9 + \frac{1}{8}\, b^2gx^8 + \frac{1}{7}\, b^2fx^7 + \frac{1}{3}\, abhx^6 + \frac{1}{6}\, b^2x^6e + \frac{1}{5}\, b^2dx^5 + \frac{2}{5}\, abgx^5 + \frac{1}{4}\, b^2cx^4 + \frac{1}{2}\, abfx^4 + \frac{1}{3}\, a^2hx^3 + \frac{2}{3}\, abx^3e + abdx^2 + \frac{1}{2}\, a^2gx^2 + 2\, abcx + a^2fx + a^2e\log(|x|) - \frac{2\, a^2dx + a^2c}{2\, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/3*a*b*h*x^6 + 1/6*b^2*x^6
*e + 1/5*b^2*d*x^5 + 2/5*a*b*g*x^5 + 1/4*b^2*c*x^4 + 1/2*a*b*f*x^4 + 1/3*a^
2*h*x^3 + 2/3*a*b*x^3*e + a*b*d*x^2 + 1/2*a^2*g*x^2 + 2*a*b*c*x + a^2*f*x +
 a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

maple [A] time = 0.05, size = 150, normalized size = 1.02

$$\frac{b^2h\,x^9}{9} + \frac{b^2g\,x^8}{8} + \frac{b^2f\,x^7}{7} + \frac{abh\,x^6}{3} + \frac{b^2e\,x^6}{6} + \frac{2abg\,x^5}{5} + \frac{b^2d\,x^5}{5} + \frac{abf\,x^4}{2} + \frac{b^2c\,x^4}{4} + \frac{a^2h\,x^3}{3} + \frac{2abe\,x^3}{3} + \frac{a^2g\,x^2}{2} + abd\,x^2 + a^2e\ln(x) + a^2fx + 2abcx - \frac{a^2d}{x} - \frac{a^2c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] 1/9*b^2*h*x^9+1/8*b^2*g*x^8+1/7*b^2*f*x^7+1/3*x^6*a*b*h+1/6*b^2*e*x^6+2/5*x
^5*a*b*g+1/5*b^2*d*x^5+1/2*x^4*a*b*f+1/4*b^2*c*x^4+1/3*x^3*a^2*h+2/3*a*b*e*
x^3+1/2*x^2*a^2*g+a*b*d*x^2+a^2*f*x+2*a*b*c*x-1/2*a^2*c/x^2-a^2*d/x+a^2*e*l
n(x)
```

maxima [A] time = 1.35, size = 146, normalized size = 0.99

$$\frac{1}{9}\, b^2hx^9 + \frac{1}{8}\, b^2gx^8 + \frac{1}{7}\, b^2fx^7 + \frac{1}{6}\, (b^2e + 2\, abh)x^6 + \frac{1}{5}\, (b^2d + 2\, abg)x^5 + \frac{1}{4}\, (b^2c + 2\, abf)x^4 + \frac{1}{3}\, (2\, abe + a^2h)x^3 + a^2e\log(x) + \frac{1}{2}\, (2\, abd + a^2g)x^2 + (2\, abc + a^2f)x - \frac{2\, a^2dx + a^2c}{2\, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="max
ima")
```

```
[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/6*(b^2*e + 2*a*b*h)*x^6 +
 1/5*(b^2*d + 2*a*b*g)*x^5 + 1/4*(b^2*c + 2*a*b*f)*x^4 + 1/3*(2*a*b*e + a^2
*h)*x^3 + a^2*e*log(x) + 1/2*(2*a*b*d + a^2*g)*x^2 + (2*a*b*c + a^2*f)*x -
1/2*(2*a^2*d*x + a^2*c)/x^2
```

mupad [**B**] time = 5.01, size = 145, normalized size = 0.99

$$x\left(f\,a^2+2\,b\,c\,a\right)-\frac{\frac{a^2\,c}{2}+a^2\,d\,x}{x^2}+x^4\,\left(\frac{c\,b^2}{4}+\frac{a\,f\,b}{2}\right)+x^2\,\left(\frac{g\,a^2}{2}+b\,d\,a\right)+x^5\,\left(\frac{d\,b^2}{5}+\frac{2\,a\,g\,b}{5}\right)+x^3\,\left(\frac{h\,a^2}{3}+\frac{2\,b\,e\,a}{3}\right)+x^6\,\left(\frac{e\,b^2}{6}+\frac{a\,h\,b}{3}\right)+\frac{b^2\,f\,x^7}{7}+\frac{b^2\,g\,x^8}{8}+\frac{b^2\,h\,x^9}{9}+a^2\,e\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

```
[Out] x*(a^2*f + 2*a*b*c) - ((a^2*c)/2 + a^2*d*x)/x^2 + x^4*((b^2*c)/4 + (a*b*f)/
2) + x^2*((a^2*g)/2 + a*b*d) + x^5*((b^2*d)/5 + (2*a*b*g)/5) + x^3*((a^2*h)
/3 + (2*a*b*e)/3) + x^6*((b^2*e)/6 + (a*b*h)/3) + (b^2*f*x^7)/7 + (b^2*g*x^
8)/8 + (b^2*h*x^9)/9 + a^2*e*log(x)
```

sympy [**A**] time = 0.45, size = 158, normalized size = 1.07

$$a^2e\log(x)+\frac{b^2fx^7}{7}+\frac{b^2gx^8}{8}+\frac{b^2hx^9}{9}+x^6\left(\frac{abh}{3}+\frac{b^2e}{6}\right)+x^5\left(\frac{2abg}{5}+\frac{b^2d}{5}\right)+x^4\left(\frac{abf}{2}+\frac{b^2c}{4}\right)+x^3\left(\frac{a^2h}{3}+\frac{2abe}{3}\right)+x^2\left(\frac{a^2g}{2}+abd\right)+x\left(a^2f+2abc\right)+\frac{-a^2c-2a^2dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**2*e*log(x) + b**2*f*x**7/7 + b**2*g*x**8/8 + b**2*h*x**9/9 + x**6*(a*b*h
/3 + b**2*e/6) + x**5*(2*a*b*g/5 + b**2*d/5) + x**4*(a*b*f/2 + b**2*c/4) +
x**3*(a**2*h/3 + 2*a*b*e/3) + x**2*(a**2*g/2 + a*b*d) + x*(a**2*f + 2*a*b*c
) + (-a**2*c - 2*a**2*d*x)/(2*x**2)
```

3.338 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

Optimal. Leaf size=152

$$-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+\frac{1}{3}bx^3(2af+bc)+a\log(x)(af+2bc)+\frac{1}{4}bx^4(2ag+bd)+ax(ag+2bd)+\frac{1}{5}bx^5(2ah+be)+\frac{1}{2}ax^2(ah+2$$

Rubi [A] time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+\frac{1}{3}bx^3(2af+bc)+a\log(x)(af+2bc)+\frac{1}{4}bx^4(2ag+bd)+ax(ag+2bd)+\frac{1}{5}bx^5(2ah+be)+\frac{1}{2}ax^2(ah+2be)+\frac{1}{6}b^2fx^6+\frac{1}{7}b^2gx^7+\frac{1}{8}b^2hx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a^2*c)/(3*x^3) - (a^2*d)/(2*x^2) - (a^2*e)/x + a*(2*b*d + a*g)*x + (a*(2*
b*e + a*h)*x^2)/2 + (b*(b*c + 2*a*f)*x^3)/3 + (b*(b*d + 2*a*g)*x^4)/4 + (b*
(b*e + 2*a*h)*x^5)/5 + (b^2*f*x^6)/6 + (b^2*g*x^7)/7 + (b^2*h*x^8)/8 + a*(2
*b*c + a*f)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = \int \left(a(2bd+ag)+\frac{a^2c}{x^4}+\frac{a^2d}{x^3}+\frac{a^2e}{x^2}+\frac{a(2bc+af)}{x}+a(2$$

$$= -\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+a(2bd+ag)x+\frac{1}{2}a(2be+ah)x^2+\frac{1}{3}$$

Mathematica [A] time = 0.10, size = 123, normalized size = 0.81

$$-\frac{a^2\left(2c+3x\left(d+2ex-\left(x^3(2g+hx)\right)\right)\right)}{6x^3}+a\log(x)(af+2bc)+\frac{1}{30}abx\left(60d+x\left(30e+x\left(20f+15gx+12hx^2\right)\right)\right)+\frac{1}{840}b^2x^3\left(280c+x\left(210d+x\left(168e+140fx+120gx^2+105hx^3\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/6*(a^2*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (a*b*x*(60*d + x
*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/30 + (b^2*x^3*(280*c + x*(210*d +
x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3))))/840 + a*(2*b*c + a*f)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^4, x]
```

fricas [A] time = 0.40, size = 153, normalized size = 1.01

$$\frac{105\, b^2 h x^{11} + 120\, b^2 g x^{10} + 140\, b^2 f x^9 + 168\, \left(b^2 e + 2\, abh\right) x^8 + 210\, \left(b^2 d + 2\, abg\right) x^7 + 280\, \left(b^2 c + 2\, abf\right) x^6 + 420\, \left(2\, abe + a^2 h\right) x^5 - 840\, a^2 e x^2 + 840\, \left(2\, abd + a^2 g\right) x^4 + 840\, \left(2\, abc + a^2 f\right) x^3 \log\left(x\right) - 420\, a^2 dx - 280\, a^2 c}{840\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/840*(105*b^2*h*x^11 + 120*b^2*g*x^10 + 140*b^2*f*x^9 + 168*(b^2*e + 2*a*b
*h)*x^8 + 210*(b^2*d + 2*a*b*g)*x^7 + 280*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*
b*e + a^2*h)*x^5 - 840*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 840*(2*a*b*c
 + a^2*f)*x^3*log(x) - 420*a^2*d*x - 280*a^2*c)/x^3
```

giac [A] time = 0.17, size = 153, normalized size = 1.01

$$\frac{1}{8}\, b^2 h x^8 + \frac{1}{7}\, b^2 g x^7 + \frac{1}{6}\, b^2 f x^6 + \frac{2}{5}\, abhx^5 + \frac{1}{5}\, b^2 x^5 e + \frac{1}{4}\, b^2 dx^4 + \frac{1}{2}\, abgx^4 + \frac{1}{3}\, b^2 cx^3 + \frac{2}{3}\, abfx^3 + \frac{1}{2}\, a^2 hx^2 + abx^2 e + 2\, abdx + a^2 gx + \left(2\, abc + a^2 f\right) \log\left(|x|\right) - \frac{6\, a^2 x^2 e + 3\, a^2 dx + 2\, a^2 c}{6\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gia
c")
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 2/5*a*b*h*x^5 + 1/5*b^2*x^5
*e + 1/4*b^2*d*x^4 + 1/2*a*b*g*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*f*x^3 + 1/2*a^
2*h*x^2 + a*b*x^2*e + 2*a*b*d*x + a^2*g*x + (2*a*b*c + a^2*f)*log(abs(x)) -
 1/6*(6*a^2*x^2*e + 3*a^2*d*x + 2*a^2*c)/x^3
```

maple [A] time = 0.05, size = 149, normalized size = 0.98

$$\frac{b^2 h\, x^8}{8} + \frac{b^2 g\, x^7}{7} + \frac{b^2 f\, x^6}{6} + \frac{2abh\, x^5}{5} + \frac{b^2 e\, x^5}{5} + \frac{abg\, x^4}{2} + \frac{b^2 d\, x^4}{4} + \frac{2abf\, x^3}{3} + \frac{b^2 c\, x^3}{3} + \frac{a^2 h\, x^2}{2} + abe\, x^2 + a^2 f \ln\left(x\right) + a^2 gx + 2abc \ln\left(x\right) + 2abdx - \frac{a^2 e}{x} - \frac{a^2 d}{2x^2} - \frac{a^2 c}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] 1/8*b^2*h*x^8+1/7*b^2*g*x^7+1/6*b^2*f*x^6+2/5*x^5*a*b*h+1/5*x^5*b^2*e+1/2*x
^4*a*b*g+1/4*x^4*b^2*d+2/3*x^3*a*b*f+1/3*b^2*c*x^3+1/2*x^2*a^2*h+a*b*e*x^2+
a^2*g*x+2*b*d*a*x-1/3*a^2*c/x^3-1/2*a^2*d/x^2-a^2*e/x+ln(x)*a^2*f+2*ln(x)*a
*b*c
```

maxima [A] time = 1.32, size = 147, normalized size = 0.97

$$\frac{1}{8}\, b^2 h x^8 + \frac{1}{7}\, b^2 g x^7 + \frac{1}{6}\, b^2 f x^6 + \frac{1}{5}\, \left(b^2 e + 2\, abh\right) x^5 + \frac{1}{4}\, \left(b^2 d + 2\, abg\right) x^4 + \frac{1}{3}\, \left(b^2 c + 2\, abf\right) x^3 + \frac{1}{2}\, \left(2\, abe + a^2 h\right) x^2 + \left(2\, abd + a^2 g\right) x + \left(2\, abc + a^2 f\right) \log\left(x\right) - \frac{6\, a^2 e x^2 + 3\, a^2 dx + 2\, a^2 c}{6\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="max
ima")
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 1/5*(b^2*e + 2*a*b*h)*x^5 +
 1/4*(b^2*d + 2*a*b*g)*x^4 + 1/3*(b^2*c + 2*a*b*f)*x^3 + 1/2*(2*a*b*e + a^2
*h)*x^2 + (2*a*b*d + a^2*g)*x + (2*a*b*c + a^2*f)*log(x) - 1/6*(6*a^2*e*x^2
 + 3*a^2*d*x + 2*a^2*c)/x^3
```

mupad [B] time = 0.08, size = 145, normalized size = 0.95

$$x\, \left(g\, a^2 + 2\, b\, d\, a\right) - \frac{e\, a^2\, x^2 + \frac{d\, a^2\, x}{2} + \frac{c\, a^2}{3}}{x^3} + x^3\, \left(\frac{c\, b^2}{3} + \frac{2\, a\, f\, b}{3}\right) + x^4\, \left(\frac{d\, b^2}{4} + \frac{a\, g\, b}{2}\right) + x^2\, \left(\frac{h\, a^2}{2} + b\, e\, a\right) + x^5\, \left(\frac{e\, b^2}{5} + \frac{2\, a\, h\, b}{5}\right) + \ln\left(x\right)\, \left(f\, a^2 + 2\, b\, c\, a\right) + \frac{b^2\, f\, x^6}{6} + \frac{b^2\, g\, x^7}{7} + \frac{b^2\, h\, x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

```
[Out] x*(a^2*g + 2*a*b*d) - ((a^2*c)/3 + a^2*e*x^2 + (a^2*d*x)/2)/x^3 + x^3*((b^2
*c)/3 + (2*a*b*f)/3) + x^4*((b^2*d)/4 + (a*b*g)/2) + x^2*((a^2*h)/2 + a*b*e
) + x^5*((b^2*e)/5 + (2*a*b*h)/5) + log(x)*(a^2*f + 2*a*b*c) + (b^2*f*x^6)/
6 + (b^2*g*x^7)/7 + (b^2*h*x^8)/8
```

sympy [A] time = 0.88, size = 158, normalized size = 1.04

$$a\left(af+2bc\right)\log\left(x\right)+\frac{b^2fx^6}{6}+\frac{b^2gx^7}{7}+\frac{b^2hx^8}{8}+x^5\left(\frac{2abh}{5}+\frac{b^2e}{5}\right)+x^4\left(\frac{abg}{2}+\frac{b^2d}{4}\right)+x^3\left(\frac{2abf}{3}+\frac{b^2c}{3}\right)+x^2\left(\frac{a^2h}{2}+abe\right)+x\left(a^2g+2abd\right)+\frac{-2a^2c-3a^2dx-6a^2ex^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a*(a*f + 2*b*c)*log(x) + b**2*f*x**6/6 + b**2*g*x**7/7 + b**2*h*x**8/8 + x*
*5*(2*a*b*h/5 + b**2*e/5) + x**4*(a*b*g/2 + b**2*d/4) + x**3*(2*a*b*f/3 + b
**2*c/3) + x**2*(a**2*h/2 + a*b*e) + x*(a**2*g + 2*a*b*d) + (-2*a**2*c - 3*
a**2*d*x - 6*a**2*e*x**2)/(6*x**3)
```

3.339 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$

Optimal. Leaf size=152

$$-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}+\frac{1}{2}bx^2(2af+bc)-\frac{a(af+2bc)}{x}+\frac{1}{3}bx^3(2ag+bd)+a\log(x)(ag+2bd)+\frac{1}{4}bx^4(2ah+be)+ax(ah+2be)+$$

Rubi [A] time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}+\frac{1}{2}bx^2(2af+bc)-\frac{a(af+2bc)}{x}+\frac{1}{3}bx^3(2ag+bd)+a\log(x)(ag+2bd)+\frac{1}{4}bx^4(2ah+be)+ax(ah+2be)+\frac{1}{5}b^2fx^5+\frac{1}{6}b^2gx^6+\frac{1}{7}b^2hx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -(a^2*c)/(4*x^4) - (a^2*d)/(3*x^3) - (a^2*e)/(2*x^2) - (a*(2*b*c + a*f))/x
+ a*(2*b*e + a*h)*x + (b*(b*c + 2*a*f)*x^2)/2 + (b*(b*d + 2*a*g)*x^3)/3 + (
b*(b*e + 2*a*h)*x^4)/4 + (b^2*f*x^5)/5 + (b^2*g*x^6)/6 + (b^2*h*x^7)/7 + a*
(2*b*d + a*g)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = \int \left(a(2be+ah)+\frac{a^2c}{x^5}+\frac{a^2d}{x^4}+\frac{a^2e}{x^3}+\frac{a(2bc+af)}{x^2}+\frac{a(2bd}{x}\right.$$
$$= -\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}-\frac{a(2bc+af)}{x}+a(2be+ah)x+\frac{1}{2}b(bc+$$

Mathematica [A] time = 0.12, size = 125, normalized size = 0.82

$$-\frac{a^2\left(3c+4dx+6x^2\left(e+2fx-2hx^3\right)\right)}{12x^4}-\frac{2abc}{x}+a\log(x)(ag+2bd)+\frac{1}{6}abx(12e+x(6f+x(4g+3hx)))+\frac{1}{420}b^2x^2\left(210c+x\left(140d+x\left(105e+84fx+70gx^2+60hx^3\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] (-2*a*b*c)/x - (a^2*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) +
 (a*b*x*(12*e + x*(6*f + x*(4*g + 3*h*x))))/6 + (b^2*x^2*(210*c + x*(140*d
+ x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/420 + a*(2*b*d + a*g)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^5,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^5, x]
```

fricas [A] time = 0.41, size = 153, normalized size = 1.01

$$\frac{60\,b^2hx^{11}+70\,b^2gx^{10}+84\,b^2fx^9+105\left(b^2e+2\,abh\right)x^8+140\left(b^2d+2\,abg\right)x^7+210\left(b^2c+2\,abf\right)x^6+420\left(2\,abe+a^2h\right)x^5+420\left(2\,abd+a^2g\right)x^4\log(x)-210\,a^2ex^2-140\,a^2dx-420\left(2\,abc+a^2f\right)x^3-105\,a^2c}{420\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/420*(60*b^2*h*x^11 + 70*b^2*g*x^10 + 84*b^2*f*x^9 + 105*(b^2*e + 2*a*b*h)
*x^8 + 140*(b^2*d + 2*a*b*g)*x^7 + 210*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*b*e
 + a^2*h)*x^5 + 420*(2*a*b*d + a^2*g)*x^4*log(x) - 210*a^2*e*x^2 - 140*a^2*
d*x - 420*(2*a*b*c + a^2*f)*x^3 - 105*a^2*c)/x^4
```

giac [A] time = 0.17, size = 152, normalized size = 1.00

$$\frac{1}{7}\,b^2hx^7+\frac{1}{6}\,b^2gx^6+\frac{1}{5}\,b^2fx^5+\frac{1}{2}\,abhx^4+\frac{1}{4}\,b^2x^4e+\frac{1}{3}\,b^2dx^3+\frac{2}{3}\,abgx^3+\frac{1}{2}\,b^2cx^2+abfx^2+a^2hx+2\,abxe+\left(2\,abd+a^2g\right)\log\left(|x|\right)-\frac{6\,a^2x^2e+4\,a^2dx+12\left(2\,abc+a^2f\right)x^3+3\,a^2c}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/2*a*b*h*x^4 + 1/4*b^2*x^4
*e + 1/3*b^2*d*x^3 + 2/3*a*b*g*x^3 + 1/2*b^2*c*x^2 + a*b*f*x^2 + a^2*h*x +
2*a*b*x*e + (2*a*b*d + a^2*g)*log(abs(x)) - 1/12*(6*a^2*x^2*e + 4*a^2*d*x +
 12*(2*a*b*c + a^2*f)*x^3 + 3*a^2*c)/x^4
```

maple [A] time = 0.05, size = 149, normalized size = 0.98

$$\frac{b^2hx^7}{7}+\frac{b^2gx^6}{6}+\frac{b^2fx^5}{5}+\frac{abhx^4}{2}+\frac{b^2ex^4}{4}+\frac{2abgx^3}{3}+\frac{b^2dx^3}{3}+abfx^2+\frac{b^2cx^2}{2}+a^2g\ln(x)+a^2hx+2abd\ln(x)+2abex-\frac{a^2f}{x}-\frac{2abc}{x}-\frac{a^2e}{2x^2}-\frac{a^2d}{3x^3}-\frac{a^2c}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] 1/7*b^2*h*x^7+1/6*b^2*g*x^6+1/5*b^2*f*x^5+1/2*x^4*a*b*h+1/4*x^4*b^2*e+2/3*x
^3*a*b*g+1/3*x^3*b^2*d+x^2*a*b*f+1/2*b^2*c*x^2+a^2*h*x+2*a*b*e*x-1/4*a^2*c/
x^4-1/3*a^2*d/x^3-1/2*a^2*e/x^2-a^2/x*f-2*a/x*b*c+ln(x)*a^2*g+2*ln(x)*a*b*d
```

maxima [A] time = 1.37, size = 147, normalized size = 0.97

$$\frac{1}{7}\,b^2hx^7+\frac{1}{6}\,b^2gx^6+\frac{1}{5}\,b^2fx^5+\frac{1}{4}\left(b^2e+2\,abh\right)x^4+\frac{1}{3}\left(b^2d+2\,abg\right)x^3+\frac{1}{2}\left(b^2c+2\,abf\right)x^2+\left(2\,abe+a^2h\right)x+\left(2\,abd+a^2g\right)\log(x)-\frac{6\,a^2ex^2+4\,a^2dx+12\left(2\,abc+a^2f\right)x^3+3\,a^2c}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="max
ima")
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/4*(b^2*e + 2*a*b*h)*x^4 +
 1/3*(b^2*d + 2*a*b*g)*x^3 + 1/2*(b^2*c + 2*a*b*f)*x^2 + (2*a*b*e + a^2*h)*
x + (2*a*b*d + a^2*g)*log(x) - 1/12*(6*a^2*e*x^2 + 4*a^2*d*x + 12*(2*a*b*c
+ a^2*f)*x^3 + 3*a^2*c)/x^4
```

mupad [B] time = 0.07, size = 145, normalized size = 0.95

$$x\,\left(h\,a^2+2\,b\,e\,a\right)-\frac{\frac{a^2\,c}{4}+x^3\,\left(f\,a^2+2\,b\,c\,a\right)+\frac{a^2\,e\,x^2}{2}+\frac{a^2\,d\,x}{3}}{x^4}+x^2\,\left(\frac{c\,b^2}{2}+a\,f\,b\right)+x^3\,\left(\frac{d\,b^2}{3}+\frac{2\,a\,g\,b}{3}\right)+x^4\,\left(\frac{e\,b^2}{4}+\frac{a\,h\,b}{2}\right)+\ln(x)\,\left(g\,a^2+2\,b\,d\,a\right)+\frac{b^2\,f\,x^5}{5}+\frac{b^2\,g\,x^6}{6}+\frac{b^2\,h\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

```
[Out] x*(a^2*h + 2*a*b*e) - ((a^2*c)/4 + x^3*(a^2*f + 2*a*b*c) + (a^2*e*x^2)/2 +
(a^2*d*x)/3)/x^4 + x^2*((b^2*c)/2 + a*b*f) + x^3*((b^2*d)/3 + (2*a*b*g)/3)
+ x^4*((b^2*e)/4 + (a*b*h)/2) + log(x)*(a^2*g + 2*a*b*d) + (b^2*f*x^5)/5 +
(b^2*g*x^6)/6 + (b^2*h*x^7)/7
```

sympy [A] time = 3.23, size = 156, normalized size = 1.03

$$a\left(ag+2bd\right)\log\left(x\right)+\frac{b^2fx^5}{5}+\frac{b^2gx^6}{6}+\frac{b^2hx^7}{7}+x^4\left(\frac{abh}{2}+\frac{b^2e}{4}\right)+x^3\left(\frac{2abg}{3}+\frac{b^2d}{3}\right)+x^2\left(abf+\frac{b^2c}{2}\right)+x\left(a^2h+2abe\right)+\frac{-3a^2c-4a^2dx-6a^2ex^2+x^3\left(-12a^2f-24abc\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a*(a*g + 2*b*d)*log(x) + b**2*f*x**5/5 + b**2*g*x**6/6 + b**2*h*x**7/7 + x*
*4*(a*b*h/2 + b**2*e/4) + x**3*(2*a*b*g/3 + b**2*d/3) + x**2*(a*b*f + b**2*
c/2) + x*(a**2*h + 2*a*b*e) + (-3*a**2*c - 4*a**2*d*x - 6*a**2*e*x**2 + x**
3*(-12*a**2*f - 24*a*b*c))/(12*x**4)
```

3.340 $\int x^4 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=223

$$\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x^{15}(3$$

Rubi [A] time = 0.29, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x^{15}(3ag+bd)+\frac{1}{16}b^2x^{16}(3ah+be)+\frac{3}{11}abx^{11}(af+bc)+\frac{1}{4}abx^{12}(ag+bd)+\frac{3}{13}abx^{13}(ah+be)+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 +
 (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c + a*
f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (b^2
*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*
x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int x^4 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx &= \int \left(a^3cx^4+a^3dx^5+a^3ex^6+a^2(3bc+af)x^7+a^2(3bd\right. \\ &= \frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2(3bc+af)x^8+\frac{1}{9}a^2 \end{aligned}$$

Mathematica [A] time = 0.06, size = 223, normalized size = 1.00

$$\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x^{15}(3ag+bd)+\frac{1}{16}b^2x^{16}(3ah+be)+\frac{3}{11}abx^{11}(af+bc)+\frac{1}{4}abx^{12}(ag+bd)+\frac{3}{13}abx^{13}(ah+be)+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 +
 (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c + a*
f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (b^2
*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*
x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5),x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5), x]
```

fricas [A] time = 0.36, size = 229, normalized size = 1.03

$$\frac{1}{19}x^{19}hb^3 + \frac{1}{18}x^{18}gb^3 + \frac{1}{17}x^{17}fb^3 + \frac{1}{16}x^{16}eb^3 + \frac{3}{16}x^{16}hb^2a + \frac{1}{15}x^{15}db^3 + \frac{1}{5}x^{15}gb^2a + \frac{1}{14}x^{14}cb^3 + \frac{3}{14}x^{14}fb^2a + \frac{3}{13}x^{13}eb^2a + \frac{3}{13}x^{13}hba^2 + \frac{1}{4}x^{12}db^2a + \frac{1}{4}x^{12}gba^2 + \frac{3}{11}x^{11}cb^2a + \frac{3}{11}x^{11}fba^2 + \frac{3}{10}x^{10}eba^2 + \frac{1}{10}x^{10}ha^3 + \frac{1}{3}x^9dba^2 + \frac{1}{9}x^9ga^3 + \frac{3}{8}x^8cba^2 + \frac{1}{8}x^8fa^3 + \frac{1}{7}x^7ea^3 + \frac{1}{6}x^6da^3 + \frac{1}{5}x^5ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/19*x^19*h*b^3 + 1/18*x^18*g*b^3 + 1/17*x^17*f*b^3 + 1/16*x^16*e*b^3 + 3/1
6*x^16*h*b^2*a + 1/15*x^15*d*b^3 + 1/5*x^15*g*b^2*a + 1/14*x^14*c*b^3 + 3/1
4*x^14*f*b^2*a + 3/13*x^13*e*b^2*a + 3/13*x^13*h*b*a^2 + 1/4*x^12*d*b^2*a +
 1/4*x^12*g*b*a^2 + 3/11*x^11*c*b^2*a + 3/11*x^11*f*b*a^2 + 3/10*x^10*e*b*a
^2 + 1/10*x^10*h*a^3 + 1/3*x^9*d*b*a^2 + 1/9*x^9*g*a^3 + 3/8*x^8*c*b*a^2 +
1/8*x^8*f*a^3 + 1/7*x^7*e*a^3 + 1/6*x^6*d*a^3 + 1/5*x^5*c*a^3
```

giac [A] time = 0.15, size = 233, normalized size = 1.04

$$\frac{1}{19}b^3hx^{19} + \frac{1}{18}b^3gx^{18} + \frac{1}{17}b^3fx^{17} + \frac{3}{16}ab^2hx^{16} + \frac{1}{16}b^3x^{16}e + \frac{1}{15}b^3dx^{15} + \frac{1}{5}ab^2gx^{15} + \frac{1}{14}b^3cx^{14} + \frac{3}{14}ab^2fx^{14} + \frac{3}{13}a^2bhx^{13} + \frac{3}{13}ab^2x^{13}e + \frac{1}{4}ab^2dx^{12} + \frac{1}{4}a^2bgx^{12} + \frac{3}{11}ab^2cx^{11} + \frac{3}{11}a^2bfx^{11} + \frac{1}{10}a^3hx^{10} + \frac{3}{10}a^2bx^{10}e + \frac{1}{3}a^2bdx^9 + \frac{1}{9}a^3gx^9 + \frac{3}{8}a^2bcx^8 + \frac{1}{8}a^3fx^8 + \frac{1}{7}a^3x^7e + \frac{1}{6}a^3dx^6 + \frac{1}{5}a^3cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 3/16*a*b^2*h*x^16 + 1
/16*b^3*x^16*e + 1/15*b^3*d*x^15 + 1/5*a*b^2*g*x^15 + 1/14*b^3*c*x^14 + 3/1
4*a*b^2*f*x^14 + 3/13*a^2*b*h*x^13 + 3/13*a*b^2*x^13*e + 1/4*a*b^2*d*x^12 +
 1/4*a^2*b*g*x^12 + 3/11*a*b^2*c*x^11 + 3/11*a^2*b*f*x^11 + 1/10*a^3*h*x^10
 + 3/10*a^2*b*x^10*e + 1/3*a^2*b*d*x^9 + 1/9*a^3*g*x^9 + 3/8*a^2*b*c*x^8 +
1/8*a^3*f*x^8 + 1/7*a^3*x^7*e + 1/6*a^3*d*x^6 + 1/5*a^3*c*x^5
```

maple [A] time = 0.04, size = 224, normalized size = 1.00

$$\frac{b^3hx^{19}}{19} + \frac{b^3gx^{18}}{18} + \frac{b^3fx^{17}}{17} + \frac{\left(3ab^2h + b^3e\right)x^{16}}{16} + \frac{\left(3ab^2g + b^3d\right)x^{15}}{15} + \frac{\left(3ab^2f + b^3c\right)x^{14}}{14} + \frac{\left(3a^2bh + 3aeb^2\right)x^{13}}{13} + \frac{\left(3a^2bg + 3ab^2d\right)x^{12}}{12} + \frac{a^3ex^7}{7} + \frac{\left(3a^2bf + 3ab^2c\right)x^{11}}{11} + \frac{a^3dx^6}{6} + \frac{\left(a^3h + 3a^2be\right)x^{10}}{10} + \frac{a^3cx^5}{5} + \frac{\left(a^3g + 3a^2db\right)x^9}{9} + \frac{\left(a^3f + 3a^2cb\right)x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/19*b^3*h*x^19+1/18*b^3*g*x^18+1/17*b^3*f*x^17+1/16*(3*a*b^2*h+b^3*e)*x^16
+1/15*(3*a*b^2*g+b^3*d)*x^15+1/14*(3*a*b^2*f+b^3*c)*x^14+1/13*(3*a^2*b*h+3*
a*b^2*e)*x^13+1/12*(3*a^2*b*g+3*a*b^2*d)*x^12+1/11*(3*a^2*b*f+3*a*b^2*c)*x^
11+1/10*(a^3*h+3*a^2*b*e)*x^10+1/9*(a^3*g+3*a^2*b*d)*x^9+1/8*(a^3*f+3*a^2*b
*c)*x^8+1/7*a^3*e*x^7+1/6*a^3*d*x^6+1/5*a^3*c*x^5
```

maxima [A] time = 1.37, size = 217, normalized size = 0.97

$$\frac{1}{19}b^3hx^{19} + \frac{1}{18}b^3gx^{18} + \frac{1}{17}b^3fx^{17} + \frac{1}{16}\left(b^3e + 3ab^2h\right)x^{16} + \frac{1}{15}\left(b^3d + 3ab^2g\right)x^{15} + \frac{1}{14}\left(b^3c + 3ab^2f\right)x^{14} + \frac{3}{13}\left(ab^2e + a^2bh\right)x^{13} + \frac{1}{4}\left(ab^2d + a^2bg\right)x^{12} + \frac{3}{11}\left(ab^2c + a^2bf\right)x^{11} + \frac{1}{7}a^3ex^7 + \frac{1}{10}\left(3a^2be + a^3h\right)x^{10} + \frac{1}{6}a^3dx^6 + \frac{1}{9}\left(3a^2bd + a^3g\right)x^9 + \frac{1}{5}a^3cx^5 + \frac{1}{8}\left(3a^2bc + a^3f\right)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*(b^3*e + 3*a*b^2
*h)*x^16 + 1/15*(b^3*d + 3*a*b^2*g)*x^15 + 1/14*(b^3*c + 3*a*b^2*f)*x^14 +
3/13*(a*b^2*e + a^2*b*h)*x^13 + 1/4*(a*b^2*d + a^2*b*g)*x^12 + 3/11*(a*b^2*
c + a^2*b*f)*x^11 + 1/7*a^3*e*x^7 + 1/10*(3*a^2*b*e + a^3*h)*x^10 + 1/6*a^3
```

```
*d*x^6 + 1/9*(3*a^2*b*d + a^3*g)*x^9 + 1/5*a^3*c*x^5 + 1/8*(3*a^2*b*c + a^3
*f)*x^8
```

mupad [B] time = 0.17, size = 205, normalized size = 0.92

$$x^8\left(\frac{fa^3}{8}+\frac{3bca^2}{8}\right)+x^{14}\left(\frac{cb^3}{14}+\frac{3afb^2}{14}\right)+x^9\left(\frac{ga^3}{9}+\frac{bda^2}{3}\right)+x^{15}\left(\frac{db^3}{15}+\frac{agb^2}{5}\right)+x^{10}\left(\frac{ha^3}{10}+\frac{3bea^2}{10}\right)+x^{16}\left(\frac{eb^3}{16}+\frac{3ahb^2}{16}\right)+\frac{a^3cx^5}{5}+\frac{a^3dx^6}{6}+\frac{a^3ex^7}{7}+\frac{b^3fx^{17}}{17}+\frac{b^3gx^{18}}{18}+\frac{b^3hx^{19}}{19}+\frac{3abx^{11}\left(bc+af\right)}{11}+\frac{abx^{12}\left(bd+ag\right)}{4}+\frac{3abx^{13}\left(be+ah\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^8*((a^3*f)/8 + (3*a^2*b*c)/8) + x^14*((b^3*c)/14 + (3*a*b^2*f)/14) + x^9*
((a^3*g)/9 + (a^2*b*d)/3) + x^15*((b^3*d)/15 + (a*b^2*g)/5) + x^10*((a^3*h)
/10 + (3*a^2*b*e)/10) + x^16*((b^3*e)/16 + (3*a*b^2*h)/16) + (a^3*c*x^5)/5
+ (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*
h*x^19)/19 + (3*a*b*x^11*(b*c + a*f))/11 + (a*b*x^12*(b*d + a*g))/4 + (3*a*
b*x^13*(b*e + a*h))/13
```

sympy [A] time = 0.12, size = 246, normalized size = 1.10

$$\frac{a^3cx^5}{5}+\frac{a^3dx^6}{6}+\frac{a^3ex^7}{7}+\frac{b^3fx^{17}}{17}+\frac{b^3gx^{18}}{18}+\frac{b^3hx^{19}}{19}+x^{16}\left(\frac{3ab^2h}{16}+\frac{b^3e}{16}\right)+x^{15}\left(\frac{ab^2g}{5}+\frac{b^3d}{15}\right)+x^{14}\left(\frac{3ab^2f}{14}+\frac{b^3c}{14}\right)+x^{13}\left(\frac{3a^2bh}{13}+\frac{3ab^2e}{13}\right)+x^{12}\left(\frac{a^2bg}{4}+\frac{ab^2d}{4}\right)+x^{11}\left(\frac{3a^2bf}{11}+\frac{3ab^2c}{11}\right)+x^{10}\left(\frac{a^3h}{10}+\frac{3a^2be}{10}\right)+x^9\left(\frac{a^3g}{9}+\frac{a^2bd}{3}\right)+x^8\left(\frac{a^3f}{8}+\frac{3a^2bc}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**5/5 + a**3*d*x**6/6 + a**3*e*x**7/7 + b**3*f*x**17/17 + b**3*g*x*
*18/18 + b**3*h*x**19/19 + x**16*(3*a*b**2*h/16 + b**3*e/16) + x**15*(a*b**
2*g/5 + b**3*d/15) + x**14*(3*a*b**2*f/14 + b**3*c/14) + x**13*(3*a**2*b*h/
13 + 3*a*b**2*e/13) + x**12*(a**2*b*g/4 + a*b**2*d/4) + x**11*(3*a**2*b*f/1
1 + 3*a*b**2*c/11) + x**10*(a**3*h/10 + 3*a**2*b*e/10) + x**9*(a**3*g/9 + a
**2*b*d/3) + x**8*(a**3*f/8 + 3*a**2*b*c/8)
```

3.341 $\int x^3 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=223

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3ag+b$$

Rubi [A] time = 0.23, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3ag+bd)+\frac{1}{15}b^2x^{15}(3ah+be)+\frac{3}{10}abx^{10}(af+bc)+\frac{3}{11}abx^{11}(ag+bd)+\frac{1}{4}abx^{12}(ah+be)+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7 +
 (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*f)
*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^2*(
b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^
15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^3 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \int \left(a^3cx^3+a^3dx^4+a^3ex^5+a^2(3bc+af)x^6+a^2(3bd+a\right.$$
$$=\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2(3bc+af)x^7+\frac{1}{8}a^2(3b$$

Mathematica [A] time = 0.05, size = 223, normalized size = 1.00

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3ag+bd)+\frac{1}{15}b^2x^{15}(3ah+be)+\frac{3}{10}abx^{10}(af+bc)+\frac{3}{11}abx^{11}(ag+bd)+\frac{1}{4}abx^{12}(ah+be)+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7 +
 (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*f)
*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^2*(
b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^
15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5),x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5), x]
```

fricas [A] time = 0.37, size = 229, normalized size = 1.03

$$\frac{1}{18}x^{18}hb^3 + \frac{1}{17}x^{17}gb^3 + \frac{1}{16}x^{16}fb^3 + \frac{1}{15}x^{15}eb^3 + \frac{1}{5}x^{15}hb^2a + \frac{1}{14}x^{14}db^3 + \frac{3}{14}x^{14}gb^2a + \frac{1}{13}x^{13}cb^3 + \frac{3}{13}x^{13}fb^2a + \frac{1}{4}x^{12}eb^2a + \frac{1}{4}x^{12}hba^2 + \frac{3}{11}x^{11}db^2a + \frac{3}{11}x^{11}gba^2 + \frac{3}{10}x^{10}cb^2a + \frac{3}{10}x^{10}fba^2 + \frac{1}{3}x^9eba^2 + \frac{1}{9}x^9ha^3 + \frac{3}{8}x^8dba^2 + \frac{1}{8}x^8ga^3 + \frac{3}{7}x^7cba^2 + \frac{1}{7}x^7fa^3 + \frac{1}{6}x^6ea^3 + \frac{1}{5}x^5da^3 + \frac{1}{4}x^4ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/18*x^18*h*b^3 + 1/17*x^17*g*b^3 + 1/16*x^16*f*b^3 + 1/15*x^15*e*b^3 + 1/5
*x^15*h*b^2*a + 1/14*x^14*d*b^3 + 3/14*x^14*g*b^2*a + 1/13*x^13*c*b^3 + 3/1
3*x^13*f*b^2*a + 1/4*x^12*e*b^2*a + 1/4*x^12*h*b*a^2 + 3/11*x^11*d*b^2*a +
3/11*x^11*g*b*a^2 + 3/10*x^10*c*b^2*a + 3/10*x^10*f*b*a^2 + 1/3*x^9*e*b*a^2
 + 1/9*x^9*h*a^3 + 3/8*x^8*d*b*a^2 + 1/8*x^8*g*a^3 + 3/7*x^7*c*b*a^2 + 1/7*
x^7*f*a^3 + 1/6*x^6*e*a^3 + 1/5*x^5*d*a^3 + 1/4*x^4*c*a^3
```

giac [A] time = 0.17, size = 233, normalized size = 1.04

$$\frac{1}{18}b^3hx^{18} + \frac{1}{17}b^3gx^{17} + \frac{1}{16}b^3fx^{16} + \frac{1}{5}ab^2hx^{15} + \frac{1}{15}b^3x^{15}e + \frac{1}{14}b^3dx^{14} + \frac{3}{14}ab^2gx^{14} + \frac{1}{13}b^3cx^{13} + \frac{3}{13}ab^2fx^{13} + \frac{1}{4}a^2bhx^{12} + \frac{1}{4}ab^2x^{12}e + \frac{3}{11}ab^2dx^{11} + \frac{3}{11}a^2bgx^{11} + \frac{3}{10}ab^2cx^{10} + \frac{3}{10}a^2bfx^{10} + \frac{1}{9}a^3hx^9 + \frac{1}{3}a^2bx^9e + \frac{3}{8}a^2bdx^8 + \frac{1}{8}a^3gx^8 + \frac{3}{7}a^2bcx^7 + \frac{1}{7}a^3fx^7 + \frac{1}{6}a^3x^6e + \frac{1}{5}a^3dx^5 + \frac{1}{4}a^3cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/5*a*b^2*h*x^15 + 1/
15*b^3*x^15*e + 1/14*b^3*d*x^14 + 3/14*a*b^2*g*x^14 + 1/13*b^3*c*x^13 + 3/1
3*a*b^2*f*x^13 + 1/4*a^2*b*h*x^12 + 1/4*a*b^2*x^12*e + 3/11*a*b^2*d*x^11 +
3/11*a^2*b*g*x^11 + 3/10*a*b^2*c*x^10 + 3/10*a^2*b*f*x^10 + 1/9*a^3*h*x^9 +
 1/3*a^2*b*x^9*e + 3/8*a^2*b*d*x^8 + 1/8*a^3*g*x^8 + 3/7*a^2*b*c*x^7 + 1/7*
a^3*f*x^7 + 1/6*a^3*x^6*e + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

maple [A] time = 0.04, size = 224, normalized size = 1.00

$$\frac{b^3hx^{18}}{18} + \frac{b^3gx^{17}}{17} + \frac{b^3fx^{16}}{16} + \frac{\left(3ab^2h + b^3e\right)x^{15}}{15} + \frac{\left(3ab^2g + b^3d\right)x^{14}}{14} + \frac{\left(3ab^2f + b^3c\right)x^{13}}{13} + \frac{\left(3a^2bh + 3aeb^2\right)x^{12}}{12} + \frac{\left(3a^2bg + 3ab^2d\right)x^{11}}{11} + \frac{a^3ex^6}{6} + \frac{\left(3a^2bf + 3ab^2c\right)x^{10}}{10} + \frac{a^3dx^5}{5} + \frac{\left(a^3h + 3a^2be\right)x^9}{9} + \frac{a^3cx^4}{4} + \frac{\left(a^3g + 3a^2db\right)x^8}{8} + \frac{\left(a^3f + 3a^2cb\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/18*b^3*h*x^18+1/17*b^3*g*x^17+1/16*b^3*f*x^16+1/15*(3*a*b^2*h+b^3*e)*x^15
+1/14*(3*a*b^2*g+b^3*d)*x^14+1/13*(3*a*b^2*f+b^3*c)*x^13+1/12*(3*a^2*b*h+3*
a*b^2*e)*x^12+1/11*(3*a^2*b*g+3*a*b^2*d)*x^11+1/10*(3*a^2*b*f+3*a*b^2*c)*x^
10+1/9*(a^3*h+3*a^2*b*e)*x^9+1/8*(a^3*g+3*a^2*b*d)*x^8+1/7*(a^3*f+3*a^2*b*c
)*x^7+1/6*a^3*e*x^6+1/5*a^3*d*x^5+1/4*a^3*c*x^4
```

maxima [A] time = 1.37, size = 217, normalized size = 0.97

$$\frac{1}{18}b^3hx^{18} + \frac{1}{17}b^3gx^{17} + \frac{1}{16}b^3fx^{16} + \frac{1}{15}\left(b^3e + 3ab^2h\right)x^{15} + \frac{1}{14}\left(b^3d + 3ab^2g\right)x^{14} + \frac{1}{13}\left(b^3c + 3ab^2f\right)x^{13} + \frac{1}{4}\left(ab^2e + a^2bh\right)x^{12} + \frac{3}{11}\left(ab^2d + a^2bg\right)x^{11} + \frac{3}{10}\left(ab^2c + a^2bf\right)x^{10} + \frac{1}{6}a^3ex^6 + \frac{1}{9}\left(3a^2be + a^3h\right)x^9 + \frac{1}{5}a^3dx^5 + \frac{1}{8}\left(3a^2bd + a^3g\right)x^8 + \frac{1}{4}a^3cx^4 + \frac{1}{7}\left(3a^2bc + a^3f\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*(b^3*e + 3*a*b^2
*h)*x^15 + 1/14*(b^3*d + 3*a*b^2*g)*x^14 + 1/13*(b^3*c + 3*a*b^2*f)*x^13 +
1/4*(a*b^2*e + a^2*b*h)*x^12 + 3/11*(a*b^2*d + a^2*b*g)*x^11 + 3/10*(a*b^2*
c + a^2*b*f)*x^10 + 1/6*a^3*e*x^6 + 1/9*(3*a^2*b*e + a^3*h)*x^9 + 1/5*a^3*d
```

```
*x^5 + 1/8*(3*a^2*b*d + a^3*g)*x^8 + 1/4*a^3*c*x^4 + 1/7*(3*a^2*b*c + a^3*f
)*x^7
```

mupad [B] time = 5.16, size = 205, normalized size = 0.92

$$x^7\left(\frac{f\,a^3}{7}+\frac{3\,b\,c\,a^2}{7}\right)+x^{13}\left(\frac{c\,b^3}{13}+\frac{3\,a\,f\,b^2}{13}\right)+x^8\left(\frac{g\,a^3}{8}+\frac{3\,b\,d\,a^2}{8}\right)+x^{14}\left(\frac{d\,b^3}{14}+\frac{3\,a\,g\,b^2}{14}\right)+x^9\left(\frac{h\,a^3}{9}+\frac{b\,e\,a^2}{3}\right)+x^{15}\left(\frac{e\,b^3}{15}+\frac{a\,h\,b^2}{5}\right)+\frac{a^3\,c\,x^4}{4}+\frac{a^3\,d\,x^5}{5}+\frac{a^3\,e\,x^6}{6}+\frac{b^3\,f\,x^{16}}{16}+\frac{b^3\,g\,x^{17}}{17}+\frac{b^3\,h\,x^{18}}{18}+\frac{3\,a\,b\,x^{10}\,\left(b\,c+a\,f\right)}{10}+\frac{3\,a\,b\,x^{11}\,\left(b\,d+a\,g\right)}{11}+\frac{a\,b\,x^{12}\,\left(b\,e+a\,h\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^7*((a^3*f)/7 + (3*a^2*b*c)/7) + x^13*((b^3*c)/13 + (3*a*b^2*f)/13) + x^8*
((a^3*g)/8 + (3*a^2*b*d)/8) + x^14*((b^3*d)/14 + (3*a*b^2*g)/14) + x^9*((a^
3*h)/9 + (a^2*b*e)/3) + x^15*((b^3*e)/15 + (a*b^2*h)/5) + (a^3*c*x^4)/4 + (
a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x
^18)/18 + (3*a*b*x^10*(b*c + a*f))/10 + (3*a*b*x^11*(b*d + a*g))/11 + (a*b*
x^12*(b*e + a*h))/4
```

sympy [A] time = 0.12, size = 246, normalized size = 1.10

$$\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{b^3fx^{16}}{16}+\frac{b^3gx^{17}}{17}+\frac{b^3hx^{18}}{18}+x^{15}\left(\frac{ab^2h}{5}+\frac{b^3e}{15}\right)+x^{14}\left(\frac{3ab^2g}{14}+\frac{b^3d}{14}\right)+x^{13}\left(\frac{3ab^2f}{13}+\frac{b^3c}{13}\right)+x^{12}\left(\frac{a^2bh}{4}+\frac{ab^2e}{4}\right)+x^{11}\left(\frac{3a^2bg}{11}+\frac{3ab^2d}{11}\right)+x^{10}\left(\frac{3a^2bf}{10}+\frac{3ab^2c}{10}\right)+x^9\left(\frac{a^3h}{9}+\frac{a^2be}{3}\right)+x^8\left(\frac{a^3g}{8}+\frac{3a^2bd}{8}\right)+x^7\left(\frac{a^3f}{7}+\frac{3a^2bc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + b**3*f*x**16/16 + b**3*g*x*
*17/17 + b**3*h*x**18/18 + x**15*(a*b**2*h/5 + b**3*e/15) + x**14*(3*a*b**2
*g/14 + b**3*d/14) + x**13*(3*a*b**2*f/13 + b**3*c/13) + x**12*(a**2*b*h/4
+ a*b**2*e/4) + x**11*(3*a**2*b*g/11 + 3*a*b**2*d/11) + x**10*(3*a**2*b*f/1
0 + 3*a*b**2*c/10) + x**9*(a**3*h/9 + a**2*b*e/3) + x**8*(a**3*g/8 + 3*a**2
*b*d/8) + x**7*(a**3*f/7 + 3*a**2*b*c/7)
```

3.342 $\int x^2 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=212

$$\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{3}a^2bfx^9+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+be)+$$

Rubi [A] time = 0.18, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1582, 1850}

$$\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{3}a^2bfx^9+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+be)+\frac{1}{4}ab^2fx^{12}+\frac{c\left(a+bx^3\right)^4}{12b}+\frac{3}{10}abx^{10}(ag+bd)+\frac{3}{11}abx^{11}(ah+be)+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^3*f*x^6)/6 + (a^2*(3*b*d + a*g)*x^7)/7 +
 (a^2*(3*b*e + a*h)*x^8)/8 + (a^2*b*f*x^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10
+ (3*a*b*(b*e + a*h)*x^11)/11 + (a*b^2*f*x^12)/4 + (b^2*(b*d + 3*a*g)*x^13)
/13 + (b^2*(b*e + 3*a*h)*x^14)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^
3*h*x^17)/17 + (c*(a + b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx &= \frac{c\left(a+bx^3\right)^4}{12b}+\int \left(a+bx^3\right)^3 \left(-cx^2+x^2\left(c+dx+ex^2\right.\right.\\ &= \frac{c\left(a+bx^3\right)^4}{12b}+\int \left(a^3dx^3+a^3ex^4+a^3fx^5+a^2(3bd+\right.\\ &= \frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2(3bd+ag)x^7+\frac{1}{8}a^2\end{aligned}$$

Mathematica [A] time = 0.06, size = 223, normalized size = 1.05

$$\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^2x^6(af+3bc)+\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{12}b^2x^{12}(3af+bc)+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+be)+\frac{1}{3}abx^9(af+bc)+\frac{3}{10}abx^{10}(ag+bd)+\frac{3}{11}abx^{11}(ah+be)+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*(3*b*c + a*f)*x^6)/6 +
 (a^2*(3*b*d + a*g)*x^7)/7 + (a^2*(3*b*e + a*h)*x^8)/8 + (a*b*(b*c + a*f)*x
^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10 + (3*a*b*(b*e + a*h)*x^11)/11 + (b^2*(b
*c + 3*a*f)*x^12)/12 + (b^2*(b*d + 3*a*g)*x^13)/13 + (b^2*(b*e + 3*a*h)*x^1
4)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3*h*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^3\right)^3 \left(c + dx + ex^2 + fx^3 + gx^4 + hx^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x
^5), x]
```

fricas [A] time = 0.38, size = 229, normalized size = 1.08

$$\frac{1}{17}x^{17}hb^3 + \frac{1}{16}x^{16}gb^3 + \frac{1}{15}x^{15}fb^3 + \frac{1}{14}x^{14}eb^3 + \frac{3}{14}x^{14}hb^2a + \frac{1}{13}x^{13}db^3 + \frac{3}{13}x^{13}gb^2a + \frac{1}{12}x^{12}cb^3 + \frac{1}{4}x^{12}fb^2a + \frac{3}{11}x^{11}eb^2a + \frac{3}{11}x^{11}hba^2 + \frac{3}{10}x^{10}db^2a + \frac{3}{10}x^{10}gba^2 + \frac{1}{3}x^9cb^2a + \frac{1}{3}x^9fba^2 + \frac{3}{8}x^8eba^2 + \frac{1}{8}x^8ha^3 + \frac{3}{7}x^7dba^2 + \frac{1}{7}x^7ga^3 + \frac{1}{2}x^6cba^2 + \frac{1}{6}x^6fa^3 + \frac{1}{5}x^5ea^3 + \frac{1}{4}x^4da^3 + \frac{1}{3}x^3ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/17*x^17*h*b^3 + 1/16*x^16*g*b^3 + 1/15*x^15*f*b^3 + 1/14*x^14*e*b^3 + 3/1
4*x^14*h*b^2*a + 1/13*x^13*d*b^3 + 3/13*x^13*g*b^2*a + 1/12*x^12*c*b^3 + 1/
4*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/11*x^11*h*b*a^2 + 3/10*x^10*d*b^2*a
+ 3/10*x^10*g*b*a^2 + 1/3*x^9*c*b^2*a + 1/3*x^9*f*b*a^2 + 3/8*x^8*e*b*a^2 +
 1/8*x^8*h*a^3 + 3/7*x^7*d*b*a^2 + 1/7*x^7*g*a^3 + 1/2*x^6*c*b*a^2 + 1/6*x^
6*f*a^3 + 1/5*x^5*e*a^3 + 1/4*x^4*d*a^3 + 1/3*x^3*c*a^3
```

giac [A] time = 0.18, size = 233, normalized size = 1.10

$$\frac{1}{17}b^3hx^{17} + \frac{1}{16}b^3gx^{16} + \frac{1}{15}b^3fx^{15} + \frac{3}{14}ab^2hx^{14} + \frac{1}{14}b^3x^{14}e + \frac{1}{13}b^3dx^{13} + \frac{3}{13}ab^2gx^{13} + \frac{1}{12}b^3cx^{12} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}a^2bhx^{11} + \frac{3}{11}ab^2x^{11}e + \frac{3}{10}ab^2dx^{10} + \frac{3}{10}a^2bgx^{10} + \frac{1}{3}ab^2cx^9 + \frac{1}{3}a^2bfx^9 + \frac{1}{8}a^3hx^8 + \frac{3}{8}a^2bx^8e + \frac{3}{7}a^2bdx^7 + \frac{1}{7}a^3gx^7 + \frac{1}{2}a^2bcx^6 + \frac{1}{6}a^3fx^6 + \frac{1}{5}a^3x^5e + \frac{1}{4}a^3dx^4 + \frac{1}{3}a^3cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 3/14*a*b^2*h*x^14 + 1
/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 3/13*a*b^2*g*x^13 + 1/12*b^3*c*x^12 + 1/
4*a*b^2*f*x^12 + 3/11*a^2*b*h*x^11 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10
+ 3/10*a^2*b*g*x^10 + 1/3*a*b^2*c*x^9 + 1/3*a^2*b*f*x^9 + 1/8*a^3*h*x^8 + 3
/8*a^2*b*x^8*e + 3/7*a^2*b*d*x^7 + 1/7*a^3*g*x^7 + 1/2*a^2*b*c*x^6 + 1/6*a^
3*f*x^6 + 1/5*a^3*x^5*e + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

maple [A] time = 0.05, size = 224, normalized size = 1.06

$$\frac{b^3h\,x^{17}}{17} + \frac{b^3g\,x^{16}}{16} + \frac{b^3f\,x^{15}}{15} + \frac{\left(3a\,b^2h + b^3e\right)x^{14}}{14} + \frac{\left(3a\,b^2g + b^3d\right)x^{13}}{13} + \frac{\left(3a\,b^2f + b^3c\right)x^{12}}{12} + \frac{\left(3a^2bh + 3ae\,b^2\right)x^{11}}{11} + \frac{\left(3a^2bg + 3a\,b^2d\right)x^{10}}{10} + \frac{a^3e\,x^5}{5} + \frac{\left(3a^2bf + 3a\,b^2c\right)x^9}{9} + \frac{a^3d\,x^4}{4} + \frac{\left(a^3h + 3a^2be\right)x^8}{8} + \frac{a^3c\,x^3}{3} + \frac{\left(a^3g + 3a^2db\right)x^7}{7} + \frac{\left(a^3f + 3a^2cb\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/17*b^3*h*x^17+1/16*b^3*g*x^16+1/15*b^3*f*x^15+1/14*(3*a*b^2*h+b^3*e)*x^14
+1/13*(3*a*b^2*g+b^3*d)*x^13+1/12*(3*a*b^2*f+b^3*c)*x^12+1/11*(3*a^2*b*h+3*
a*b^2*e)*x^11+1/10*(3*a^2*b*g+3*a*b^2*d)*x^10+1/9*(3*a^2*b*f+3*a*b^2*c)*x^9
+1/8*(a^3*h+3*a^2*b*e)*x^8+1/7*(a^3*g+3*a^2*b*d)*x^7+1/6*(a^3*f+3*a^2*b*c)*
x^6+1/5*a^3*e*x^5+1/4*a^3*d*x^4+1/3*a^3*c*x^3
```

maxima [A] time = 1.36, size = 217, normalized size = 1.02

$$\frac{1}{17}b^3hx^{17}+\frac{1}{16}b^3gx^{16}+\frac{1}{15}b^3fx^{15}+\frac{1}{14}\left(b^3e+3\,ab^2h\right)x^{14}+\frac{1}{13}\left(b^3d+3\,ab^2g\right)x^{13}+\frac{1}{12}\left(b^3c+3\,ab^2f\right)x^{12}+\frac{3}{11}\left(ab^2e+a^2bh\right)x^{11}+\frac{3}{10}\left(ab^2d+a^2bg\right)x^{10}+\frac{1}{3}\left(ab^2c+a^2bf\right)x^9+\frac{1}{5}a^3ex^5+\frac{1}{8}\left(3\,a^2be+a^3h\right)x^8+\frac{1}{4}a^3dx^4+\frac{1}{7}\left(3\,a^2bd+a^3g\right)x^7+\frac{1}{3}a^3cx^3+\frac{1}{6}\left(3\,a^2bc+a^3f\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*(b^3*e + 3*a*b^2
*h)*x^14 + 1/13*(b^3*d + 3*a*b^2*g)*x^13 + 1/12*(b^3*c + 3*a*b^2*f)*x^12 +
3/11*(a*b^2*e + a^2*b*h)*x^11 + 3/10*(a*b^2*d + a^2*b*g)*x^10 + 1/3*(a*b^2*
c + a^2*b*f)*x^9 + 1/5*a^3*e*x^5 + 1/8*(3*a^2*b*e + a^3*h)*x^8 + 1/4*a^3*d*
x^4 + 1/7*(3*a^2*b*d + a^3*g)*x^7 + 1/3*a^3*c*x^3 + 1/6*(3*a^2*b*c + a^3*f)
*x^6
```

mupad [B] time = 0.16, size = 205, normalized size = 0.97

$$x^6\left(\frac{f\,a^3}{6}+\frac{b\,c\,a^2}{2}\right)+x^{12}\left(\frac{c\,b^3}{12}+\frac{a\,f\,b^2}{4}\right)+x^7\left(\frac{g\,a^3}{7}+\frac{3\,b\,d\,a^2}{7}\right)+x^{13}\left(\frac{d\,b^3}{13}+\frac{3\,a\,g\,b^2}{13}\right)+x^8\left(\frac{h\,a^3}{8}+\frac{3\,b\,e\,a^2}{8}\right)+x^{14}\left(\frac{e\,b^3}{14}+\frac{3\,a\,h\,b^2}{14}\right)+\frac{a^3\,c\,x^3}{3}+\frac{a^3\,d\,x^4}{4}+\frac{a^3\,e\,x^5}{5}+\frac{b^3\,f\,x^{15}}{15}+\frac{b^3\,g\,x^{16}}{16}+\frac{b^3\,h\,x^{17}}{17}+\frac{a\,b\,x^9\,\left(b\,c+a\,f\right)}{3}+\frac{3\,a\,b\,x^{10}\,\left(b\,d+a\,g\right)}{10}+\frac{3\,a\,b\,x^{11}\,\left(b\,e+a\,h\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^6*((a^3*f)/6 + (a^2*b*c)/2) + x^12*((b^3*c)/12 + (a*b^2*f)/4) + x^7*((a^3
*g)/7 + (3*a^2*b*d)/7) + x^13*((b^3*d)/13 + (3*a*b^2*g)/13) + x^8*((a^3*h)/
8 + (3*a^2*b*e)/8) + x^14*((b^3*e)/14 + (3*a*b^2*h)/14) + (a^3*c*x^3)/3 + (
a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3*h*x
^17)/17 + (a*b*x^9*(b*c + a*f))/3 + (3*a*b*x^10*(b*d + a*g))/10 + (3*a*b*x^
11*(b*e + a*h))/11
```

sympy [A] time = 0.12, size = 246, normalized size = 1.16

$$\frac{a^3cx^3}{3}+\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{b^3fx^{15}}{15}+\frac{b^3gx^{16}}{16}+\frac{b^3hx^{17}}{17}+x^{14}\left(\frac{3ab^2h}{14}+\frac{b^3e}{14}\right)+x^{13}\left(\frac{3ab^2g}{13}+\frac{b^3d}{13}\right)+x^{12}\left(\frac{ab^2f}{4}+\frac{b^3c}{12}\right)+x^{11}\left(\frac{3a^2bh}{11}+\frac{3ab^2e}{11}\right)+x^{10}\left(\frac{3a^2bg}{10}+\frac{3ab^2d}{10}\right)+x^9\left(\frac{a^2bf}{3}+\frac{ab^2c}{3}\right)+x^8\left(\frac{a^3h}{8}+\frac{3a^2be}{8}\right)+x^7\left(\frac{a^3g}{7}+\frac{3a^2bd}{7}\right)+x^6\left(\frac{a^3f}{6}+\frac{a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + b**3*f*x**15/15 + b**3*g*x*
*16/16 + b**3*h*x**17/17 + x**14*(3*a*b**2*h/14 + b**3*e/14) + x**13*(3*a*b
**2*g/13 + b**3*d/13) + x**12*(a*b**2*f/4 + b**3*c/12) + x**11*(3*a**2*b*h/
11 + 3*a*b**2*e/11) + x**10*(3*a**2*b*g/10 + 3*a*b**2*d/10) + x**9*(a**2*b*
f/3 + a*b**2*c/3) + x**8*(a**3*h/8 + 3*a**2*b*e/8) + x**7*(a**3*g/7 + 3*a**
2*b*d/7) + x**6*(a**3*f/6 + a**2*b*c/2)
```

3.343 $\int x\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=212

$$\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{6}a^3gx^6+\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{3}a^2bgx^9+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{13}b^2x^{13}(3ah+be)+\frac{1}{4}al$$

Rubi [A] time = 0.18, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1582, 1850}

$$\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{3}a^2bgx^9+\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{6}a^3gx^6+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{13}b^2x^{13}(3ah+be)+\frac{1}{4}ab^2gx^{12}+\frac{3}{8}abx^8(af+bc)+\frac{d\left(a+bx^3\right)^4}{12b}+\frac{3}{10}abx^{10}(ah+be)+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 + (a^3*g*x^6)/6 +
 (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)*x^8)/8 + (a^2*b*g*x^9)/3 +
(3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c + 3*a*f)*x^11)/11 + (a*b^2*g*x^12)/
4 + (b^2*(b*e + 3*a*h)*x^13)/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*
h*x^16)/16 + (d*(a + b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{d\left(a+bx^3\right)^4}{12b}+\int \left(a+bx^3\right)^3\left(-dx^2+x\left(c+dx+ex^2+fx\right.\right.\\ &= \frac{d\left(a+bx^3\right)^4}{12b}+\int \left(a^3cx+a^3ex^3+a^2(3bc+af)x^4+a^3gx^5\right.\\ &= \frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2(3bc+af)x^5+\frac{1}{6}a^3gx^6+\frac{1}{7}a^2(3be\end{aligned}$$

Mathematica [A] time = 0.04, size = 223, normalized size = 1.05

$$\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{6}a^2x^6(ag+3bd)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{12}b^2x^{12}(3ag+bd)+\frac{1}{13}b^2x^{13}(3ah+be)+\frac{3}{8}abx^8(af+bc)+\frac{1}{3}abx^9(ag+bd)+\frac{3}{10}abx^{10}(ah+be)+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 +
 (a^2*(3*b*d + a*g)*x^6)/6 + (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)
*x^8)/8 + (a*b*(b*d + a*g)*x^9)/3 + (3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c
 + 3*a*f)*x^11)/11 + (b^2*(b*d + 3*a*g)*x^12)/12 + (b^2*(b*e + 3*a*h)*x^13)
/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5
),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5
), x]
```

fricas [A] time = 0.36, size = 229, normalized size = 1.08

$$\frac{1}{16}x^{16}hb^3+\frac{1}{15}x^{15}gb^3+\frac{1}{14}x^{14}fb^3+\frac{1}{13}x^{13}eb^3+\frac{3}{13}x^{13}hb^2a+\frac{1}{12}x^{12}db^3+\frac{1}{4}x^{12}gb^2a+\frac{1}{11}x^{11}cb^3+\frac{3}{11}x^{11}fb^2a+\frac{3}{10}x^{10}eb^2a+\frac{3}{10}x^{10}hba^2+\frac{1}{3}x^9db^2a+\frac{1}{3}x^9gba^2+\frac{3}{8}x^8cb^2a+\frac{3}{8}x^8fba^2+\frac{3}{7}x^7eba^2+\frac{1}{7}x^7ha^3+\frac{1}{2}x^6dba^2+\frac{1}{6}x^6ga^3+\frac{3}{5}x^5cba^2+\frac{1}{5}x^5fa^3+\frac{1}{4}x^4ea^3+\frac{1}{3}x^3da^3+\frac{1}{2}x^2ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/16*x^16*h*b^3 + 1/15*x^15*g*b^3 + 1/14*x^14*f*b^3 + 1/13*x^13*e*b^3 + 3/1
3*x^13*h*b^2*a + 1/12*x^12*d*b^3 + 1/4*x^12*g*b^2*a + 1/11*x^11*c*b^3 + 3/1
1*x^11*f*b^2*a + 3/10*x^10*e*b^2*a + 3/10*x^10*h*b*a^2 + 1/3*x^9*d*b^2*a +
1/3*x^9*g*b*a^2 + 3/8*x^8*c*b^2*a + 3/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/7
*x^7*h*a^3 + 1/2*x^6*d*b*a^2 + 1/6*x^6*g*a^3 + 3/5*x^5*c*b*a^2 + 1/5*x^5*f*
a^3 + 1/4*x^4*e*a^3 + 1/3*x^3*d*a^3 + 1/2*x^2*c*a^3
```

giac [A] time = 0.17, size = 233, normalized size = 1.10

$$\frac{1}{16}\,b^3hx^{16}+\frac{1}{15}\,b^3gx^{15}+\frac{1}{14}\,b^3fx^{14}+\frac{3}{13}\,ab^2hx^{13}+\frac{1}{13}\,b^3x^{13}e+\frac{1}{12}\,b^3dx^{12}+\frac{1}{4}\,ab^2gx^{12}+\frac{1}{11}\,b^3cx^{11}+\frac{3}{11}\,ab^2fx^{11}+\frac{3}{10}\,a^2bhx^{10}+\frac{3}{10}\,ab^2x^{10}e+\frac{1}{3}\,ab^2dx^9+\frac{1}{3}\,a^2bgx^9+\frac{3}{8}\,ab^2cx^8+\frac{3}{8}\,a^2bfx^8+\frac{1}{7}\,a^3hx^7+\frac{3}{7}\,a^2bx^7e+\frac{1}{2}\,a^2bdx^6+\frac{1}{6}\,a^3gx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{5}\,a^3fx^5+\frac{1}{4}\,a^3x^4e+\frac{1}{3}\,a^3dx^3+\frac{1}{2}\,a^3cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 3/13*a*b^2*h*x^13 + 1
/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/4*a*b^2*g*x^12 + 1/11*b^3*c*x^11 + 3/1
1*a*b^2*f*x^11 + 3/10*a^2*b*h*x^10 + 3/10*a*b^2*x^10*e + 1/3*a*b^2*d*x^9 +
1/3*a^2*b*g*x^9 + 3/8*a*b^2*c*x^8 + 3/8*a^2*b*f*x^8 + 1/7*a^3*h*x^7 + 3/7*a
^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 1/6*a^3*g*x^6 + 3/5*a^2*b*c*x^5 + 1/5*a^3*f*
x^5 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

maple [A] time = 0.04, size = 224, normalized size = 1.06

$$\frac{b^3h\,x^{16}}{16}+\frac{b^3g\,x^{15}}{15}+\frac{b^3f\,x^{14}}{14}+\frac{\left(3a\,b^2h+b^3e\right)x^{13}}{13}+\frac{\left(3a\,b^2g+b^3d\right)x^{12}}{12}+\frac{\left(3a\,b^2f+b^3c\right)x^{11}}{11}+\frac{\left(3a^2bh+3ae\,b^2\right)x^{10}}{10}+\frac{\left(3a^2bg+3a\,b^2d\right)x^9}{9}+\frac{a^3e\,x^4}{4}+\frac{\left(3a^2bf+3a\,b^2c\right)x^8}{8}+\frac{a^3d\,x^3}{3}+\frac{\left(a^3h+3a^2be\right)x^7}{7}+\frac{a^3c\,x^2}{2}+\frac{\left(a^3g+3a^2db\right)x^6}{6}+\frac{\left(a^3f+3a^2cb\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/16*b^3*h*x^16+1/15*b^3*g*x^15+1/14*b^3*f*x^14+1/13*(3*a*b^2*h+b^3*e)*x^13
+1/12*(3*a*b^2*g+b^3*d)*x^12+1/11*(3*a*b^2*f+b^3*c)*x^11+1/10*(3*a^2*b*h+3*
a*b^2*e)*x^10+1/9*(3*a^2*b*g+3*a*b^2*d)*x^9+1/8*(3*a^2*b*f+3*a*b^2*c)*x^8+1
/7*(a^3*h+3*a^2*b*e)*x^7+1/6*(a^3*g+3*a^2*b*d)*x^6+1/5*(a^3*f+3*a^2*b*c)*x^
5+1/4*a^3*e*x^4+1/3*a^3*d*x^3+1/2*a^3*c*x^2
```

maxima [A] time = 1.35, size = 217, normalized size = 1.02

$$\frac{1}{16}\,b^3hx^{16}+\frac{1}{15}\,b^3gx^{15}+\frac{1}{14}\,b^3fx^{14}+\frac{1}{13}\left(b^3e+3\,ab^2h\right)x^{13}+\frac{1}{12}\left(b^3d+3\,ab^2g\right)x^{12}+\frac{1}{11}\left(b^3c+3\,ab^2f\right)x^{11}+\frac{3}{10}\left(ab^2e+a^2bh\right)x^{10}+\frac{1}{3}\left(ab^2d+a^2bg\right)x^9+\frac{3}{8}\left(ab^2c+a^2bf\right)x^8+\frac{1}{4}\,a^3ex^4+\frac{1}{7}\left(3\,a^2be+a^3h\right)x^7+\frac{1}{3}\,a^3dx^3+\frac{1}{6}\left(3\,a^2bd+a^3g\right)x^6+\frac{1}{2}\,a^3cx^2+\frac{1}{5}\left(3\,a^2bc+a^3f\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*(b^3*e + 3*a*b^2
*h)*x^13 + 1/12*(b^3*d + 3*a*b^2*g)*x^12 + 1/11*(b^3*c + 3*a*b^2*f)*x^11 +
3/10*(a*b^2*e + a^2*b*h)*x^10 + 1/3*(a*b^2*d + a^2*b*g)*x^9 + 3/8*(a*b^2*c
+ a^2*b*f)*x^8 + 1/4*a^3*e*x^4 + 1/7*(3*a^2*b*e + a^3*h)*x^7 + 1/3*a^3*d*x^
3 + 1/6*(3*a^2*b*d + a^3*g)*x^6 + 1/2*a^3*c*x^2 + 1/5*(3*a^2*b*c + a^3*f)*x
^5
```

mupad [B] time = 0.16, size = 205, normalized size = 0.97

$$x^5\left(\frac{f\,a^3}{5}+\frac{3\,b\,c\,a^2}{5}\right)+x^{11}\left(\frac{c\,b^3}{11}+\frac{3\,a\,f\,b^2}{11}\right)+x^6\left(\frac{g\,a^3}{6}+\frac{b\,d\,a^2}{2}\right)+x^{12}\left(\frac{d\,b^3}{12}+\frac{a\,g\,b^2}{4}\right)+x^7\left(\frac{h\,a^3}{7}+\frac{3\,b\,e\,a^2}{7}\right)+x^{13}\left(\frac{e\,b^3}{13}+\frac{3\,a\,h\,b^2}{13}\right)+\frac{a^3\,c\,x^2}{2}+\frac{a^3\,d\,x^3}{3}+\frac{a^3\,e\,x^4}{4}+\frac{b^3\,f\,x^{14}}{14}+\frac{b^3\,g\,x^{15}}{15}+\frac{b^3\,h\,x^{16}}{16}+\frac{3\,a\,b\,x^8\,\left(b\,c+a\,f\right)}{8}+\frac{a\,b\,x^9\,\left(b\,d+a\,g\right)}{3}+\frac{3\,a\,b\,x^{10}\,\left(b\,e+a\,h\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^5*((a^3*f)/5 + (3*a^2*b*c)/5) + x^11*((b^3*c)/11 + (3*a*b^2*f)/11) + x^6*
((a^3*g)/6 + (a^2*b*d)/2) + x^12*((b^3*d)/12 + (a*b^2*g)/4) + x^7*((a^3*h)/
7 + (3*a^2*b*e)/7) + x^13*((b^3*e)/13 + (3*a*b^2*h)/13) + (a^3*c*x^2)/2 + (
a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x
^16)/16 + (3*a*b*x^8*(b*c + a*f))/8 + (a*b*x^9*(b*d + a*g))/3 + (3*a*b*x^10
*(b*e + a*h))/10
```

sympy [A] time = 0.11, size = 246, normalized size = 1.16

$$\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{b^3fx^{14}}{14}+\frac{b^3gx^{15}}{15}+\frac{b^3hx^{16}}{16}+x^{13}\left(\frac{3ab^2h}{13}+\frac{b^3e}{13}\right)+x^{12}\left(\frac{ab^2g}{4}+\frac{b^3d}{12}\right)+x^{11}\left(\frac{3ab^2f}{11}+\frac{b^3c}{11}\right)+x^{10}\left(\frac{3a^2bh}{10}+\frac{3ab^2e}{10}\right)+x^9\left(\frac{a^2bg}{3}+\frac{ab^2d}{3}\right)+x^8\left(\frac{3a^2bf}{8}+\frac{3ab^2c}{8}\right)+x^7\left(\frac{a^3h}{7}+\frac{3a^2be}{7}\right)+x^6\left(\frac{a^3g}{6}+\frac{a^2bd}{2}\right)+x^5\left(\frac{a^3f}{5}+\frac{3a^2bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + b**3*f*x**14/14 + b**3*g*x*
*15/15 + b**3*h*x**16/16 + x**13*(3*a*b**2*h/13 + b**3*e/13) + x**12*(a*b**
2*g/4 + b**3*d/12) + x**11*(3*a*b**2*f/11 + b**3*c/11) + x**10*(3*a**2*b*h/
10 + 3*a*b**2*e/10) + x**9*(a**2*b*g/3 + a*b**2*d/3) + x**8*(3*a**2*b*f/8 +
 3*a*b**2*c/8) + x**7*(a**3*h/7 + 3*a**2*b*e/7) + x**6*(a**3*g/6 + a**2*b*d
/2) + x**5*(a**3*f/5 + 3*a**2*b*c/5)
```

3.344 $\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\, dx$

Optimal. Leaf size=207

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{6}a^3hx^6+\frac{1}{4}a^2x^4(af+3bc)+\frac{1}{5}a^2x^5(ag+3bd)+\frac{1}{3}a^2bhx^9+\frac{1}{10}b^2x^{10}(3af+bc)+\frac{1}{11}b^2x^{11}(3ag+bd)+\frac{1}{4}$$

Rubi [A] time = 0.18, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1582, 1850}

$$\frac{1}{4}a^2x^4(af+3bc)+\frac{1}{5}a^2x^5(ag+3bd)+\frac{1}{3}a^2bhx^9+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{6}a^3hx^6+\frac{1}{10}b^2x^{10}(3af+bc)+\frac{1}{11}b^2x^{11}(3ag+bd)+\frac{1}{4}ab^2hx^{12}+\frac{3}{7}abx^7(af+bc)+\frac{3}{8}abx^8(ag+bd)+\frac{e\left(a+bx^3\right)^4}{12b}+\frac{1}{13}b^3fx^{13}+\frac{1}{14}b^3gx^{14}+\frac{1}{15}b^3hx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^2*(3*b*c + a*f)*x^4)/4 + (a^2*(3*b*d + a*g)*x^
5)/5 + (a^3*h*x^6)/6 + (3*a*b*(b*c + a*f)*x^7)/7 + (3*a*b*(b*d + a*g)*x^8)/
8 + (a^2*b*h*x^9)/3 + (b^2*(b*c + 3*a*f)*x^10)/10 + (b^2*(b*d + 3*a*g)*x^11
)/11 + (a*b^2*h*x^12)/4 + (b^3*f*x^13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/
15 + (e*(a + b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\, dx &= \frac{e\left(a+bx^3\right)^4}{12b}+\int (a+bx^3)^3 (c+dx+fx^3+gx^4+hx^5)\\ &= \frac{e\left(a+bx^3\right)^4}{12b}+\int \left(a^3c+a^3dx+a^2(3bc+af)x^3+a^2(3ba\right.\\ &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{4}a^2(3bc+af)x^4+\frac{1}{5}a^2(3bd+ag)x^5+\end{aligned}$$

Mathematica [A] time = 0.11, size = 170, normalized size = 0.82

$$\frac{x\left(2002a^3\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+143a^2bx^3(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))+13ab^2x^6\left(3960c+7x\left(495d+440ex+6x^2\left(66f+60gx+55hx^2\right)\right)\right)+2b^3x^9\left(6006c+x\left(5460d+11x\left(455e+420fx+390gx^2+364hx^3\right)\right)\right)\right)}{120120}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (x*(13*a*b^2*x^6*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55
*h*x^2))) + 2002*a^3*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x
^3))) + 2*b^3*x^9*(6006*c + x*(5460*d + 11*x*(455*e + 420*f*x + 390*g*x^2 +
 364*h*x^3))) + 143*a^2*b*x^3*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x
*(9*g + 8*h*x)))))))/120120
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),
x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),
 x]
```

fricas [A] time = 0.37, size = 226, normalized size = 1.09

$$\frac{1}{15}x^{15}hb^3+\frac{1}{14}x^{14}gb^3+\frac{1}{13}x^{13}fb^3+\frac{1}{12}x^{12}eb^3+\frac{1}{4}x^{12}hb^2a+\frac{1}{11}x^{11}db^3+\frac{3}{11}x^{11}gb^2a+\frac{1}{10}x^{10}cb^3+\frac{3}{10}x^{10}fb^2a+\frac{1}{3}x^9eb^2a+\frac{1}{3}x^9hba^2+\frac{3}{8}x^8db^2a+\frac{3}{8}x^8gba^2+\frac{3}{7}x^7cb^2a+\frac{3}{7}x^7fba^2+\frac{1}{2}x^6eba^2+\frac{1}{6}x^6ha^3+\frac{3}{5}x^5dba^2+\frac{1}{5}x^5ga^3+\frac{3}{4}x^4cba^2+\frac{1}{4}x^4fa^3+\frac{1}{3}x^3ea^3+\frac{1}{2}x^2da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/15*x^15*h*b^3 + 1/14*x^14*g*b^3 + 1/13*x^13*f*b^3 + 1/12*x^12*e*b^3 + 1/4
*x^12*h*b^2*a + 1/11*x^11*d*b^3 + 3/11*x^11*g*b^2*a + 1/10*x^10*c*b^3 + 3/1
0*x^10*f*b^2*a + 1/3*x^9*e*b^2*a + 1/3*x^9*h*b*a^2 + 3/8*x^8*d*b^2*a + 3/8*
x^8*g*b*a^2 + 3/7*x^7*c*b^2*a + 3/7*x^7*f*b*a^2 + 1/2*x^6*e*b*a^2 + 1/6*x^6
*h*a^3 + 3/5*x^5*d*b*a^2 + 1/5*x^5*g*a^3 + 3/4*x^4*c*b*a^2 + 1/4*x^4*f*a^3
+ 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3
```

giac [A] time = 0.15, size = 230, normalized size = 1.11

$$\frac{1}{15}b^3hx^{15}+\frac{1}{14}b^3gx^{14}+\frac{1}{13}b^3fx^{13}+\frac{1}{4}ab^2hx^{12}+\frac{1}{12}b^3x^{12}e+\frac{1}{11}b^3dx^{11}+\frac{3}{11}ab^2gx^{11}+\frac{1}{10}b^3cx^{10}+\frac{3}{10}ab^2fx^{10}+\frac{1}{3}a^2bhx^9+\frac{1}{3}ab^2x^9e+\frac{3}{8}ab^2dx^8+\frac{3}{8}a^2bgx^8+\frac{3}{7}ab^2cx^7+\frac{3}{7}a^2bfx^7+\frac{1}{6}a^3hx^6+\frac{1}{2}a^2bx^6e+\frac{3}{5}a^2bdx^5+\frac{1}{5}a^3gx^5+\frac{3}{4}a^2bcx^4+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3x^3e+\frac{1}{2}a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/4*a*b^2*h*x^12 + 1/
12*b^3*x^12*e + 1/11*b^3*d*x^11 + 3/11*a*b^2*g*x^11 + 1/10*b^3*c*x^10 + 3/1
0*a*b^2*f*x^10 + 1/3*a^2*b*h*x^9 + 1/3*a*b^2*x^9*e + 3/8*a*b^2*d*x^8 + 3/8*
a^2*b*g*x^8 + 3/7*a*b^2*c*x^7 + 3/7*a^2*b*f*x^7 + 1/6*a^3*h*x^6 + 1/2*a^2*b
*x^6*e + 3/5*a^2*b*d*x^5 + 1/5*a^3*g*x^5 + 3/4*a^2*b*c*x^4 + 1/4*a^3*f*x^4
+ 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

maple [A] time = 0.04, size = 221, normalized size = 1.07

$$\frac{b^3hx^{15}}{15}+\frac{b^3gx^{14}}{14}+\frac{b^3fx^{13}}{13}+\frac{\left(3ab^2h+b^3e\right)x^{12}}{12}+\frac{\left(3ab^2g+b^3d\right)x^{11}}{11}+\frac{\left(3ab^2f+b^3c\right)x^{10}}{10}+\frac{\left(3a^2bh+3aeb^2\right)x^9}{9}+\frac{\left(3a^2bg+3ab^2d\right)x^8}{8}+\frac{a^3ex^3}{3}+\frac{\left(3a^2bf+3ab^2c\right)x^7}{7}+\frac{a^3dx^2}{2}+\frac{\left(a^3h+3a^2be\right)x^6}{6}+a^3cx+\frac{\left(a^3g+3a^2db\right)x^5}{5}+\frac{\left(a^3f+3a^2cb\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/15*b^3*h*x^15+1/14*b^3*g*x^14+1/13*b^3*f*x^13+1/12*(3*a*b^2*h+b^3*e)*x^12
+1/11*(3*a*b^2*g+b^3*d)*x^11+1/10*(3*a*b^2*f+b^3*c)*x^10+1/9*(3*a^2*b*h+3*a
*b^2*e)*x^9+1/8*(3*a^2*b*g+3*a*b^2*d)*x^8+1/7*(3*a^2*b*f+3*a*b^2*c)*x^7+1/6
*(a^3*h+3*a^2*b*e)*x^6+1/5*(a^3*g+3*a^2*b*d)*x^5+1/4*(a^3*f+3*a^2*b*c)*x^4+
1/3*a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

maxima [A] time = 1.34, size = 214, normalized size = 1.03

$$\frac{1}{15}b^3hx^{15}+\frac{1}{14}b^3gx^{14}+\frac{1}{13}b^3fx^{13}+\frac{1}{12}\left(b^3e+3ab^2h\right)x^{12}+\frac{1}{11}\left(b^3d+3ab^2g\right)x^{11}+\frac{1}{10}\left(b^3c+3ab^2f\right)x^{10}+\frac{1}{3}\left(ab^2e+a^2bh\right)x^9+\frac{3}{8}\left(ab^2d+a^2bg\right)x^8+\frac{3}{7}\left(ab^2c+a^2bf\right)x^7+\frac{1}{3}a^3ex^3+\frac{1}{6}\left(3a^2be+a^3h\right)x^6+\frac{1}{2}a^3dx^2+\frac{1}{5}\left(3a^2bd+a^3g\right)x^5+a^3cx+\frac{1}{4}\left(3a^2bc+a^3f\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*(b^3*e + 3*a*b^2
*h)*x^12 + 1/11*(b^3*d + 3*a*b^2*g)*x^11 + 1/10*(b^3*c + 3*a*b^2*f)*x^10 +
1/3*(a*b^2*e + a^2*b*h)*x^9 + 3/8*(a*b^2*d + a^2*b*g)*x^8 + 3/7*(a*b^2*c +
a^2*b*f)*x^7 + 1/3*a^3*e*x^3 + 1/6*(3*a^2*b*e + a^3*h)*x^6 + 1/2*a^3*d*x^2
+ 1/5*(3*a^2*b*d + a^3*g)*x^5 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*f)*x^4
```

mupad [B] time = 0.16, size = 202, normalized size = 0.98

$$x^4\left(\frac{f\,a^3}{4}+\frac{3\,b\,c\,a^2}{4}\right)+x^{10}\left(\frac{c\,b^3}{10}+\frac{3\,a\,f\,b^2}{10}\right)+x^5\left(\frac{g\,a^3}{5}+\frac{3\,b\,d\,a^2}{5}\right)+x^{11}\left(\frac{d\,b^3}{11}+\frac{3\,a\,g\,b^2}{11}\right)+x^6\left(\frac{h\,a^3}{6}+\frac{b\,e\,a^2}{2}\right)+x^{12}\left(\frac{e\,b^3}{12}+\frac{a\,h\,b^2}{4}\right)+\frac{a^3\,d\,x^2}{2}+\frac{a^3\,e\,x^3}{3}+\frac{b^3\,f\,x^{13}}{13}+\frac{b^3\,g\,x^{14}}{14}+\frac{b^3\,h\,x^{15}}{15}+a^3\,c\,x+\frac{3\,a\,b\,x^7\,\left(b\,c+a\,f\right)}{7}+\frac{3\,a\,b\,x^8\,\left(b\,d+a\,g\right)}{8}+\frac{a\,b\,x^9\,\left(b\,e+a\,h\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

```
[Out] x^4*((a^3*f)/4 + (3*a^2*b*c)/4) + x^10*((b^3*c)/10 + (3*a*b^2*f)/10) + x^5*
((a^3*g)/5 + (3*a^2*b*d)/5) + x^11*((b^3*d)/11 + (3*a*b^2*g)/11) + x^6*((a^
3*h)/6 + (a^2*b*e)/2) + x^12*((b^3*e)/12 + (a*b^2*h)/4) + (a^3*d*x^2)/2 + (
a^3*e*x^3)/3 + (b^3*f*x^13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/15 + a^3*c*
x + (3*a*b*x^7*(b*c + a*f))/7 + (3*a*b*x^8*(b*d + a*g))/8 + (a*b*x^9*(b*e +
 a*h))/3
```

sympy [A] time = 0.12, size = 243, normalized size = 1.17

$$a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{b^3fx^{13}}{13}+\frac{b^3gx^{14}}{14}+\frac{b^3hx^{15}}{15}+x^{12}\left(\frac{ab^2h}{4}+\frac{b^3e}{12}\right)+x^{11}\left(\frac{3ab^2g}{11}+\frac{b^3d}{11}\right)+x^{10}\left(\frac{3ab^2f}{10}+\frac{b^3c}{10}\right)+x^9\left(\frac{a^2bh}{3}+\frac{ab^2e}{3}\right)+x^8\left(\frac{3a^2bg}{8}+\frac{3ab^2d}{8}\right)+x^7\left(\frac{3a^2bf}{7}+\frac{3ab^2c}{7}\right)+x^6\left(\frac{a^3h}{6}+\frac{a^2be}{2}\right)+x^5\left(\frac{a^3g}{5}+\frac{3a^2bd}{5}\right)+x^4\left(\frac{a^3f}{4}+\frac{3a^2bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + b**3*f*x**13/13 + b**3*g*x**14/1
4 + b**3*h*x**15/15 + x**12*(a*b**2*h/4 + b**3*e/12) + x**11*(3*a*b**2*g/11
 + b**3*d/11) + x**10*(3*a*b**2*f/10 + b**3*c/10) + x**9*(a**2*b*h/3 + a*b*
*2*e/3) + x**8*(3*a**2*b*g/8 + 3*a*b**2*d/8) + x**7*(3*a**2*b*f/7 + 3*a*b**
2*c/7) + x**6*(a**3*h/6 + a**2*b*e/2) + x**5*(a**3*g/5 + 3*a**2*b*d/5) + x*
*4*(a**3*f/4 + 3*a**2*b*c/4)
```

3.345 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal. Leaf size=200

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+\frac{1}{2}ab^2cx^6+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3ah+$$

Rubi [A] time = 0.15, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1583, 1820}

$$a^2bcx^3+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{2}ab^2cx^6+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3ah+be)+\frac{3}{7}abx^7(ag+bd)+\frac{3}{8}abx^8(ah+be)+\frac{f\left(a+bx^3\right)^4}{12b}+\frac{1}{9}b^3cx^9+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (a^2*(3*b*d + a*g)*x^4)/4 + (a^2*(3
*b*e + a*h)*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b*(b*d + a*g)*x^7)/7 + (3*a*b*(
b*e + a*h)*x^8)/8 + (b^3*c*x^9)/9 + (b^2*(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e
 + 3*a*h)*x^11)/11 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + (f*(a + b*x^3)^4)/
(12*b) + a^3*c*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x}\,dx &= \frac{f\left(a+bx^3\right)^4}{12b}+\int \frac{\left(a+bx^3\right)^3\left(c+dx+ex^2+gx^4+hx^5\right)}{x}\,dx\\ &= \frac{f\left(a+bx^3\right)^4}{12b}+\int \left(a^3d+\frac{a^3c}{x}+a^3ex+3a^2bcx^2+a^2(3bd+\right.\\ &= a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2(3bd+ag)x^4+\frac{1}{5}a^2(3be+ah\end{aligned}$$

Mathematica [A] time = 0.13, size = 214, normalized size = 1.07

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{3}a^2x^3(af+3bc)+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+\frac{1}{9}b^2x^9(3af+bc)+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3ah+be)+\frac{1}{2}abx^6(af+bc)+\frac{3}{7}abx^7(ag+bd)+\frac{3}{8}abx^8(ah+be)+\frac{1}{12}b^3fx^{12}+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*c + a*f)*x^3)/3 + (a^2*(3*b*d + a*g)*x^
4)/4 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b*(b*c + a*f)*x^6)/2 + (3*a*b*(b*d +
a*g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^2*(b*c + 3*a*f)*x^9)/9 + (b^2*
(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e + 3*a*h)*x^11)/11 + (b^3*f*x^12)/12 + (b
^3*g*x^13)/13 + (b^3*h*x^14)/14 + a^3*c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x, x]
```

fricas [A] time = 0.42, size = 212, normalized size = 1.06

$$\frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{1}{11}\left(b^3e+3\,ab^2h\right)x^{11}+\frac{1}{10}\left(b^3d+3\,ab^2g\right)x^{10}+\frac{1}{9}\left(b^3c+3\,ab^2f\right)x^9+\frac{3}{8}\left(ab^2e+a^2bh\right)x^8+\frac{3}{7}\left(ab^2d+a^2bg\right)x^7+\frac{1}{2}\left(ab^2c+a^2bf\right)x^6+\frac{1}{2}\,a^3ex^2+\frac{1}{5}\left(3\,a^2be+a^3h\right)x^5+a^3dx+\frac{1}{4}\left(3\,a^2bd+a^3g\right)x^4+a^3c\log\left(x\right)+\frac{1}{3}\left(3\,a^2bc+a^3f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="frica
s")
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e + 3*a*b^2
*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c + 3*a*b^2*f)*x^9 + 3/
8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c + a^
2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3
*a^2*b*d + a^3*g)*x^4 + a^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

giac [A] time = 0.16, size = 228, normalized size = 1.14

$$\frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{3}{11}\,ab^2hx^{11}+\frac{1}{11}\,b^3x^{11}e+\frac{1}{10}\,b^3dx^{10}+\frac{3}{10}\,ab^2gx^{10}+\frac{1}{9}\,b^3cx^9+\frac{1}{3}\,ab^2fx^9+\frac{3}{8}\,a^2bhx^8+\frac{3}{8}\,ab^2x^8e+\frac{3}{7}\,ab^2dx^7+\frac{3}{7}\,a^2bgx^7+\frac{1}{2}\,ab^2cx^6+\frac{1}{2}\,a^2bfx^6+\frac{1}{5}\,a^3hx^5+\frac{3}{5}\,a^2bx^5e+\frac{3}{4}\,a^2bdx^4+\frac{1}{4}\,a^3gx^4+a^2bcx^3+\frac{1}{3}\,a^3fx^3+\frac{1}{2}\,a^3x^2e+a^3dx+a^3c\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac"
)
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 3/11*a*b^2*h*x^11 + 1
/11*b^3*x^11*e + 1/10*b^3*d*x^10 + 3/10*a*b^2*g*x^10 + 1/9*b^3*c*x^9 + 1/3*
a*b^2*f*x^9 + 3/8*a^2*b*h*x^8 + 3/8*a*b^2*x^8*e + 3/7*a*b^2*d*x^7 + 3/7*a^2
*b*g*x^7 + 1/2*a*b^2*c*x^6 + 1/2*a^2*b*f*x^6 + 1/5*a^3*h*x^5 + 3/5*a^2*b*x^
5*e + 3/4*a^2*b*d*x^4 + 1/4*a^3*g*x^4 + a^2*b*c*x^3 + 1/3*a^3*f*x^3 + 1/2*a
^3*x^2*e + a^3*d*x + a^3*c*log(abs(x))
```

maple [A] time = 0.05, size = 224, normalized size = 1.12

$$\frac{b^3hx^{14}}{14}+\frac{b^3gx^{13}}{13}+\frac{b^3fx^{12}}{12}+\frac{3ab^2hx^{11}}{11}+\frac{b^3ex^{11}}{11}+\frac{3ab^2gx^{10}}{10}+\frac{b^3dx^{10}}{10}+\frac{ab^2fx^9}{3}+\frac{b^3cx^9}{9}+\frac{3a^2bhx^8}{8}+\frac{3ab^2ex^8}{8}+\frac{3a^2bgx^7}{7}+\frac{3ab^2dx^7}{7}+\frac{a^2bfx^6}{2}+\frac{ab^2cx^6}{2}+\frac{a^3hx^5}{5}+\frac{3a^2bex^5}{5}+\frac{a^3gx^4}{4}+\frac{3a^2bdx^4}{4}+\frac{a^3fx^3}{3}+a^2bcx^3+\frac{a^3ex^2}{2}+a^3c\ln\left(x\right)+a^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] 1/14*b^3*h*x^14+1/13*b^3*g*x^13+1/12*x^12*f*b^3+3/11*x^11*a*b^2*h+1/11*b^3*
e*x^11+3/10*x^10*a*b^2*g+1/10*b^3*d*x^10+1/3*x^9*a*b^2*f+1/9*b^3*c*x^9+3/8*
x^8*a^2*b*h+3/8*a*b^2*e*x^8+3/7*x^7*a^2*b*g+3/7*a*b^2*d*x^7+1/2*x^6*a^2*b*f
+1/2*a*b^2*c*x^6+1/5*x^5*a^3*h+3/5*a^2*b*e*x^5+1/4*x^4*a^3*g+3/4*a^2*b*d*x^
4+1/3*x^3*a^3*f+a^2*b*c*x^3+1/2*a^3*e*x^2+a^3*d*x+a^3*c*ln(x)
```

maxima [A] time = 1.42, size = 212, normalized size = 1.06

$$\frac{1}{14}\,b^{3}hx^{14}+\frac{1}{13}\,b^{3}gx^{13}+\frac{1}{12}\,b^{3}fx^{12}+\frac{1}{11}\,\left(b^{3}e+3\,ab^{2}h\right)x^{11}+\frac{1}{10}\,\left(b^{3}d+3\,ab^{2}g\right)x^{10}+\frac{1}{9}\,\left(b^{3}c+3\,ab^{2}f\right)x^{9}+\frac{3}{8}\,\left(ab^{2}e+a^{2}bh\right)x^{8}+\frac{3}{7}\,\left(ab^{2}d+a^{2}bg\right)x^{7}+\frac{1}{2}\,\left(ab^{2}c+a^{2}bf\right)x^{6}+\frac{1}{2}\,a^{3}ex^{2}+\frac{1}{5}\,\left(3\,a^{2}be+a^{3}h\right)x^{5}+a^{3}dx+\frac{1}{4}\,\left(3\,a^{2}bd+a^{3}g\right)x^{4}+a^{3}c\log(x)+\frac{1}{3}\,\left(3\,a^{2}bc+a^{3}f\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxim
a")
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e + 3*a*b^2
*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c + 3*a*b^2*f)*x^9 + 3/
8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c + a^
2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3
*a^2*b*d + a^3*g)*x^4 + a^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

mupad [B] time = 5.11, size = 199, normalized size = 1.00

$$x^3\,\left(\frac{f\,a^3}{3}+b\,c\,a^2\right)+x^9\,\left(\frac{c\,b^3}{9}+\frac{a\,f\,b^2}{3}\right)+x^4\,\left(\frac{g\,a^3}{4}+\frac{3\,b\,d\,a^2}{4}\right)+x^{10}\,\left(\frac{d\,b^3}{10}+\frac{3\,a\,g\,b^2}{10}\right)+x^5\,\left(\frac{h\,a^3}{5}+\frac{3\,b\,e\,a^2}{5}\right)+x^{11}\,\left(\frac{e\,b^3}{11}+\frac{3\,a\,h\,b^2}{11}\right)+\frac{a^3\,e\,x^2}{2}+\frac{b^3\,f\,x^{12}}{12}+\frac{b^3\,g\,x^{13}}{13}+\frac{b^3\,h\,x^{14}}{14}+a^3\,c\,\ln(x)+a^3\,d\,x+\frac{a\,b\,x^6\,\left(b\,c+a\,f\right)}{2}+\frac{3\,a\,b\,x^7\,\left(b\,d+a\,g\right)}{7}+\frac{3\,a\,b\,x^8\,\left(b\,e+a\,h\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

```
[Out] x^3*((a^3*f)/3 + a^2*b*c) + x^9*((b^3*c)/9 + (a*b^2*f)/3) + x^4*((a^3*g)/4
+ (3*a^2*b*d)/4) + x^10*((b^3*d)/10 + (3*a*b^2*g)/10) + x^5*((a^3*h)/5 + (3
*a^2*b*e)/5) + x^11*((b^3*e)/11 + (3*a*b^2*h)/11) + (a^3*e*x^2)/2 + (b^3*f*
x^12)/12 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + a^3*c*log(x) + a^3*d*x + (a*
b*x^6*(b*c + a*f))/2 + (3*a*b*x^7*(b*d + a*g))/7 + (3*a*b*x^8*(b*e + a*h))/
8
```

sympy [A] time = 0.54, size = 240, normalized size = 1.20

$$a^{3}c\log{\left(x \right)}+a^{3}dx+\frac{a^{3}ex^{2}}{2}+\frac{b^{3}fx^{12}}{12}+\frac{b^{3}gx^{13}}{13}+\frac{b^{3}hx^{14}}{14}+x^{11}\left(\frac{3ab^{2}h}{11}+\frac{b^{3}e}{11}\right)+x^{10}\left(\frac{3ab^{2}g}{10}+\frac{b^{3}d}{10}\right)+x^{9}\left(\frac{ab^{2}f}{3}+\frac{b^{3}c}{9}\right)+x^{8}\left(\frac{3a^{2}bh}{8}+\frac{3ab^{2}e}{8}\right)+x^{7}\left(\frac{3a^{2}bg}{7}+\frac{3ab^{2}d}{7}\right)+x^{6}\left(\frac{a^{2}bf}{2}+\frac{ab^{2}c}{2}\right)+x^{5}\left(\frac{a^{3}h}{5}+\frac{3a^{2}be}{5}\right)+x^{4}\left(\frac{a^{3}g}{4}+\frac{3a^{2}bd}{4}\right)+x^{3}\left(\frac{a^{3}f}{3}+a^{2}bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + b**3*f*x**12/12 + b**3*g*x**13/1
3 + b**3*h*x**14/14 + x**11*(3*a*b**2*h/11 + b**3*e/11) + x**10*(3*a*b**2*g
/10 + b**3*d/10) + x**9*(a*b**2*f/3 + b**3*c/9) + x**8*(3*a**2*b*h/8 + 3*a*
b**2*e/8) + x**7*(3*a**2*b*g/7 + 3*a*b**2*d/7) + x**6*(a**2*b*f/2 + a*b**2*
c/2) + x**5*(a**3*h/5 + 3*a**2*b*e/5) + x**4*(a**3*g/4 + 3*a**2*b*d/4) + x*
*3*(a**3*f/3 + a**2*b*c)
```

3.346 $\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

Optimal. Leaf size=198

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be)+\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3ah+$$

Rubi [A] time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1583, 1820}

$$\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be)-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3ah+be)+\frac{3}{5}abx^5(af+bc)+\frac{3}{7}abx^7(ah+be)+\frac{g\left(a+bx^3\right)^4}{12b}+\frac{1}{9}b^3dx^9+\frac{1}{11}b^3fx^{11}+\frac{1}{13}b^3hx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (a^2*(3*b*c + a*f)*x^2)/2 + a^2*b*d*x^3 + (a^2*(3*
b*e + a*h)*x^4)/4 + (3*a*b*(b*c + a*f)*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b*(b
*e + a*h)*x^7)/7 + (b^2*(b*c + 3*a*f)*x^8)/8 + (b^3*d*x^9)/9 + (b^2*(b*e +
3*a*h)*x^10)/10 + (b^3*f*x^11)/11 + (b^3*h*x^13)/13 + (g*(a + b*x^3)^4)/(12
*b) + a^3*d*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^2}\,dx &= \frac{g\left(a+bx^3\right)^4}{12b}+\int \frac{\left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+hx^5\right)}{x^2} \\
&= \frac{g\left(a+bx^3\right)^4}{12b}+\int \left(a^3e+\frac{a^3c}{x^2}+\frac{a^3d}{x}+a^2(3bc+af)x+3 \right. \\
&= -\frac{a^3c}{x}+a^3ex+\frac{1}{2}a^2(3bc+af)x^2+a^2bdx^3+\frac{1}{4}a^2(3be+a
\end{aligned}$$

Mathematica [A] time = 0.21, size = 172, normalized size = 0.87

$$a^3\left(-\frac{c}{x}+ex+\frac{1}{12}x^2\left(6f+4gx+3hx^2\right)\right)+a^3d\log(x)+\frac{1}{140}a^2bx^2\left(210c+x\left(140d+x\left(105e+84fx+70gx^2+60hx^3\right)\right)\right)+\frac{1}{840}ab^2x^5\left(504c+x\left(420d+x\left(360e+315fx+280gx^2+252hx^3\right)\right)\right)+\frac{b^3x^8\left(6435c+5720dx+6x^2\left(858e+780fx+715gx^2+660hx^3\right)\right)}{51480}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] a^3*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) + (b^3*x^8*(6435*c +
5720*d*x + 6*x^2*(858*e + 780*f*x + 715*g*x^2 + 660*h*x^3)))/51480 + (a^2*b
*x^2*(210*c + x*(140*d + x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/140 +
(a*b^2*x^5*(504*c + x*(420*d + x*(360*e + 315*f*x + 280*g*x^2 + 252*h*x^3))
))/840 + a^3*d*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^2, x]
```

fricas [A] time = 0.41, size = 219, normalized size = 1.11

$$\frac{27720\,b^3hx^{14}+30030\,b^3gx^{13}+32760\,b^3fx^{12}+36036\left(b^3e+3\,ab^2h\right)x^{11}+40040\left(b^3d+3\,ab^2g\right)x^{10}+45045\left(b^3c+3\,ab^2f\right)x^9+154440\left(ab^2e+a^2bh\right)x^8+180180\left(ab^2d+a^2bg\right)x^7+216216\left(ab^2c+a^2bf\right)x^6+360360\,a^3ex^2+90090\left(3\,a^2be+a^3h\right)x^5+360360\,a^3dx\log(x)+120120\left(3\,a^2bd+a^3g\right)x^4-360360\,a^3c+180180\left(3\,a^2bc+a^3f\right)x^3}{360360\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/360360*(27720*b^3*h*x^14 + 30030*b^3*g*x^13 + 32760*b^3*f*x^12 + 36036*(b
^3*e + 3*a*b^2*h)*x^11 + 40040*(b^3*d + 3*a*b^2*g)*x^10 + 45045*(b^3*c + 3*
a*b^2*f)*x^9 + 154440*(a*b^2*e + a^2*b*h)*x^8 + 180180*(a*b^2*d + a^2*b*g)*
x^7 + 216216*(a*b^2*c + a^2*b*f)*x^6 + 360360*a^3*e*x^2 + 90090*(3*a^2*b*e
+ a^3*h)*x^5 + 360360*a^3*d*x*log(x) + 120120*(3*a^2*b*d + a^3*g)*x^4 - 360
360*a^3*c + 180180*(3*a^2*b*c + a^3*f)*x^3)/x
```

giac [A] time = 0.15, size = 228, normalized size = 1.15

$$\frac{1}{13}\,b^3hx^{13}+\frac{1}{12}\,b^3gx^{12}+\frac{1}{11}\,b^3fx^{11}+\frac{3}{10}\,ab^2hx^{10}+\frac{1}{10}\,b^3x^{10}e+\frac{1}{9}\,b^3dx^9+\frac{1}{3}\,ab^2gx^9+\frac{1}{8}\,b^3cx^8+\frac{3}{8}\,ab^2fx^8+\frac{3}{7}\,a^2bhx^7+\frac{3}{7}\,ab^2x^7e+\frac{1}{2}\,ab^2dx^6+\frac{1}{2}\,a^2bgx^6+\frac{3}{5}\,ab^2cx^5+\frac{3}{5}\,a^2bfx^5+\frac{1}{4}\,a^3hx^4+\frac{3}{4}\,a^2bx^4e+a^2bdx^3+\frac{1}{3}\,a^3gx^3+\frac{3}{2}\,a^2bcx^2+\frac{1}{2}\,a^3fx^2+a^3xe+a^3d\log\left(|x|\right)-\frac{a^3c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="gia
c")
```

```
[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 3/10*a*b^2*h*x^10 + 1
/10*b^3*x^10*e + 1/9*b^3*d*x^9 + 1/3*a*b^2*g*x^9 + 1/8*b^3*c*x^8 + 3/8*a*b^
2*f*x^8 + 3/7*a^2*b*h*x^7 + 3/7*a*b^2*x^7*e + 1/2*a*b^2*d*x^6 + 1/2*a^2*b*g
*x^6 + 3/5*a*b^2*c*x^5 + 3/5*a^2*b*f*x^5 + 1/4*a^3*h*x^4 + 3/4*a^2*b*x^4*e
+ a^2*b*d*x^3 + 1/3*a^3*g*x^3 + 3/2*a^2*b*c*x^2 + 1/2*a^3*f*x^2 + a^3*x*e +
 a^3*d*log(abs(x)) - a^3*c/x
```

maple [A] time = 0.05, size = 224, normalized size = 1.13

$$\frac{b^3hx^{13}}{13}+\frac{b^3gx^{12}}{12}+\frac{b^3fx^{11}}{11}+\frac{3ab^2hx^{10}}{10}+\frac{b^3ex^{10}}{10}+\frac{ab^2gx^9}{3}+\frac{b^3dx^9}{9}+\frac{3ab^2fx^8}{8}+\frac{b^3cx^8}{8}+\frac{3a^2bhx^7}{7}+\frac{3ab^2ex^7}{7}+\frac{a^2bgx^6}{2}+\frac{ab^2dx^6}{2}+\frac{3a^2bfx^5}{5}+\frac{3ab^2cx^5}{5}+\frac{a^3hx^4}{4}+\frac{3a^2bex^4}{4}+\frac{a^3gx^3}{3}+a^2bdx^3+\frac{a^3fx^2}{2}+\frac{3a^2bcx^2}{2}+a^3d\ln(x)+a^3ex-\frac{a^3c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] 1/13*b^3*h*x^13+1/12*b^3*g*x^12+1/11*b^3*f*x^11+3/10*x^10*a*b^2*h+1/10*b^3*
e*x^10+1/3*x^9*a*b^2*g+1/9*b^3*d*x^9+3/8*x^8*a*b^2*f+1/8*b^3*c*x^8+3/7*x^7*
a^2*b*h+3/7*a*b^2*e*x^7+1/2*x^6*a^2*b*g+1/2*a*b^2*d*x^6+3/5*x^5*a^2*b*f+3/5
```

```
*a*b^2*c*x^5+1/4*x^4*a^3*h+3/4*a^2*b*e*x^4+1/3*x^3*a^3*g+a^2*b*d*x^3+1/2*x^
2*a^3*f+3/2*a^2*b*c*x^2+a^3*e*x-a^3*c/x+a^3*d*ln(x)
```

maxima [A] time = 1.29, size = 212, normalized size = 1.07

$$\frac{1}{13}\,b^3hx^{13}+\frac{1}{12}\,b^3gx^{12}+\frac{1}{11}\,b^3fx^{11}+\frac{1}{10}\left(b^3e+3\,ab^2h\right)x^{10}+\frac{1}{9}\left(b^3d+3\,ab^2g\right)x^9+\frac{1}{8}\left(b^3c+3\,ab^2f\right)x^8+\frac{3}{7}\left(ab^2e+a^2bh\right)x^7+\frac{1}{2}\left(ab^2d+a^2bg\right)x^6+\frac{3}{5}\left(ab^2c+a^2bf\right)x^5+a^3ex+\frac{1}{4}\left(3\,a^2be+a^3h\right)x^4+a^3d\log(x)+\frac{1}{3}\left(3\,a^2bd+a^3g\right)x^3-\frac{a^3c}{x}+\frac{1}{2}\left(3\,a^2bc+a^3f\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="max
ima")
```

```
[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 1/10*(b^3*e + 3*a*b^2
*h)*x^10 + 1/9*(b^3*d + 3*a*b^2*g)*x^9 + 1/8*(b^3*c + 3*a*b^2*f)*x^8 + 3/7*
(a*b^2*e + a^2*b*h)*x^7 + 1/2*(a*b^2*d + a^2*b*g)*x^6 + 3/5*(a*b^2*c + a^2*
b*f)*x^5 + a^3*e*x + 1/4*(3*a^2*b*e + a^3*h)*x^4 + a^3*d*log(x) + 1/3*(3*a^
2*b*d + a^3*g)*x^3 - a^3*c/x + 1/2*(3*a^2*b*c + a^3*f)*x^2
```

mupad [B] time = 5.05, size = 199, normalized size = 1.01

$$x^2\left(\frac{f\,a^3}{2}+\frac{3\,b\,c\,a^2}{2}\right)+x^8\left(\frac{c\,b^3}{8}+\frac{3\,a\,f\,b^2}{8}\right)+x^3\left(\frac{g\,a^3}{3}+b\,d\,a^2\right)+x^9\left(\frac{d\,b^3}{9}+\frac{a\,g\,b^2}{3}\right)+x^4\left(\frac{h\,a^3}{4}+\frac{3\,b\,e\,a^2}{4}\right)+x^{10}\left(\frac{e\,b^3}{10}+\frac{3\,a\,h\,b^2}{10}\right)-\frac{a^3\,c}{x}+\frac{b^3\,f\,x^{11}}{11}+\frac{b^3\,g\,x^{12}}{12}+\frac{b^3\,h\,x^{13}}{13}+a^3\,d\,\ln(x)+a^3\,e\,x+\frac{3\,a\,b\,x^5\,\left(b\,c+a\,f\right)}{5}+\frac{a\,b\,x^6\,\left(b\,d+a\,g\right)}{2}+\frac{3\,a\,b\,x^7\,\left(b\,e+a\,h\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

```
[Out] x^2*((a^3*f)/2 + (3*a^2*b*c)/2) + x^8*((b^3*c)/8 + (3*a*b^2*f)/8) + x^3*((a
^3*g)/3 + a^2*b*d) + x^9*((b^3*d)/9 + (a*b^2*g)/3) + x^4*((a^3*h)/4 + (3*a^
2*b*e)/4) + x^10*((b^3*e)/10 + (3*a*b^2*h)/10) - (a^3*c)/x + (b^3*f*x^11)/1
1 + (b^3*g*x^12)/12 + (b^3*h*x^13)/13 + a^3*d*log(x) + a^3*e*x + (3*a*b*x^5
*(b*c + a*f))/5 + (a*b*x^6*(b*d + a*g))/2 + (3*a*b*x^7*(b*e + a*h))/7
```

sympy [A] time = 0.51, size = 236, normalized size = 1.19

$$-\frac{a^3c}{x}+a^3d\log{(x)}+a^3ex+\frac{b^3fx^{11}}{11}+\frac{b^3gx^{12}}{12}+\frac{b^3hx^{13}}{13}+x^{10}\left(\frac{3ab^2h}{10}+\frac{b^3e}{10}\right)+x^9\left(\frac{ab^2g}{3}+\frac{b^3d}{9}\right)+x^8\left(\frac{3ab^2f}{8}+\frac{b^3c}{8}\right)+x^7\left(\frac{3a^2bh}{7}+\frac{3ab^2e}{7}\right)+x^6\left(\frac{a^2bg}{2}+\frac{ab^2d}{2}\right)+x^5\left(\frac{3a^2bf}{5}+\frac{3ab^2c}{5}\right)+x^4\left(\frac{a^3h}{4}+\frac{3a^2be}{4}\right)+x^3\left(\frac{a^3g}{3}+a^2bd\right)+x^2\left(\frac{a^3f}{2}+\frac{3a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + b**3*f*x**11/11 + b**3*g*x**12/12 +
b**3*h*x**13/13 + x**10*(3*a*b**2*h/10 + b**3*e/10) + x**9*(a*b**2*g/3 + b*
*3*d/9) + x**8*(3*a*b**2*f/8 + b**3*c/8) + x**7*(3*a**2*b*h/7 + 3*a*b**2*e/
7) + x**6*(a**2*b*g/2 + a*b**2*d/2) + x**5*(3*a**2*b*f/5 + 3*a*b**2*c/5) +
x**4*(a**3*h/4 + 3*a**2*b*e/4) + x**3*(a**3*g/3 + a**2*b*d) + x**2*(a**3*f/
2 + 3*a**2*b*c/2)
```

3.347 $\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} \, dx$

Optimal. Leaf size=198

$$-\frac{a^3 c}{2x^2}-\frac{a^3 d}{x}+a^3 e \log(x)+a^2 x(af+3bc)+\frac{1}{2}a^2 x^2(ag+3bd)+a^2 bex^3+\frac{1}{7}b^2 x^7(3af+bc)+\frac{1}{8}b^2 x^8(3ag+bd)+\frac{1}{2}ab^2 ex^6+\frac{3}{4}ab$$

Rubi [A] time = 0.20, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1583, 1820}

$$a^2 x(af+3bc)+\frac{1}{2}a^2 x^2(ag+3bd)+a^2 bex^3-\frac{a^3 c}{2x^2}-\frac{a^3 d}{x}+a^3 e \log(x)+\frac{1}{7}b^2 x^7(3af+bc)+\frac{1}{8}b^2 x^8(3ag+bd)+\frac{1}{2}ab^2 ex^6+\frac{3}{4}abx^4(af+bc)+\frac{3}{5}abx^5(ag+bd)+\frac{h\left(a+bx^3\right)^4}{12b}+\frac{1}{9}b^3 ex^9+\frac{1}{10}b^3 fx^{10}+\frac{1}{11}b^3 gx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -(a^3*c)/(2*x^2) - (a^3*d)/x + a^2*(3*b*c + a*f)*x + (a^2*(3*b*d + a*g)*x^2
)/2 + a^2*b*e*x^3 + (3*a*b*(b*c + a*f)*x^4)/4 + (3*a*b*(b*d + a*g)*x^5)/5 +
 (a*b^2*e*x^6)/2 + (b^2*(b*c + 3*a*f)*x^7)/7 + (b^2*(b*d + 3*a*g)*x^8)/8 +
(b^3*e*x^9)/9 + (b^3*f*x^10)/10 + (b^3*g*x^11)/11 + (h*(a + b*x^3)^4)/(12*b
) + a^3*e*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^3} \, dx = \frac{h\left(a+bx^3\right)^4}{12b}+\int \frac{\left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4\right)}{x^3} \, dx$$

$$= \frac{h\left(a+bx^3\right)^4}{12b}+\int \left(a^2(3bc+af)+\frac{a^3 c}{x^3}+\frac{a^3 d}{x^2}+\frac{a^3 e}{x}+a^2(3l\right.$$

$$= -\frac{a^3 c}{2x^2}-\frac{a^3 d}{x}+a^2(3bc+af)x+\frac{1}{2}a^2(3bd+ag)x^2+a^2 bex^3+$$

Mathematica [A] time = 0.15, size = 174, normalized size = 0.88

$$\frac{a^3\left(-3c-6dx+x^3\left(6f+3gx+2hx^2\right)\right)}{6x^2}+a^3 e \log(x)+\frac{1}{20}a^2 bx\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+\frac{1}{840}ab^2 x^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))+\frac{b^3 x^7\left(3960c+7x\left(495d+440ex+6x^2\left(66f+60gx+55hx^2\right)\right)\right)}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] (a^3*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (b^3*x^7*(3960
*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2))))/27720 + (a^
2*b*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/20 + (a*
b^2*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/84
0 + a^3*e*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^3, x]
```

fricas [A] time = 0.42, size = 219, normalized size = 1.11

$$\frac{2310\,b^3hx^{14}+2520\,b^3gx^{13}+2772\,b^3fx^{12}+3080\left(b^3e+3\,ab^2h\right)x^{11}+3465\left(b^3d+3\,ab^2g\right)x^{10}+3960\left(b^3c+3\,ab^2f\right)x^9+13860\left(ab^2e+a^2bh\right)x^8+16632\left(ab^2d+a^2bg\right)x^7+20790\left(ab^2c+a^2bf\right)x^6+27720\,a^3ex^2\log(x)+9240\left(3\,a^2be+a^3h\right)x^5-27720\,a^3dx+13860\left(3\,a^2bd+a^3g\right)x^4-13860\,a^3c+27720\left(3\,a^2bc+a^3f\right)x^3}{27720\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/27720*(2310*b^3*h*x^14 + 2520*b^3*g*x^13 + 2772*b^3*f*x^12 + 3080*(b^3*e
+ 3*a*b^2*h)*x^11 + 3465*(b^3*d + 3*a*b^2*g)*x^10 + 3960*(b^3*c + 3*a*b^2*f
)*x^9 + 13860*(a*b^2*e + a^2*b*h)*x^8 + 16632*(a*b^2*d + a^2*b*g)*x^7 + 207
90*(a*b^2*c + a^2*b*f)*x^6 + 27720*a^3*e*x^2*log(x) + 9240*(3*a^2*b*e + a^3
*h)*x^5 - 27720*a^3*d*x + 13860*(3*a^2*b*d + a^3*g)*x^4 - 13860*a^3*c + 277
20*(3*a^2*b*c + a^3*f)*x^3)/x^2
```

giac [A] time = 0.16, size = 226, normalized size = 1.14

$$\frac{1}{12}\,b^3hx^{12}+\frac{1}{11}\,b^3gx^{11}+\frac{1}{10}\,b^3fx^{10}+\frac{1}{3}\,ab^2hx^9+\frac{1}{9}\,b^3x^9e+\frac{1}{8}\,b^3dx^8+\frac{3}{8}\,ab^2gx^8+\frac{1}{7}\,b^3cx^7+\frac{3}{7}\,ab^2fx^7+\frac{1}{2}\,a^2bhx^6+\frac{1}{2}\,ab^2x^6e+\frac{3}{5}\,ab^2dx^5+\frac{3}{5}\,a^2bgx^5+\frac{3}{4}\,ab^2cx^4+\frac{3}{4}\,a^2bfx^4+\frac{1}{3}\,a^3hx^3+a^2bx^3e+\frac{3}{2}\,a^2bdx^2+\frac{1}{2}\,a^3gx^2+3\,a^2bcx+a^3fx+a^3e\log\left(|x|\right)-\frac{2\,a^3dx+a^3c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/3*a*b^2*h*x^9 + 1/9
*b^3*x^9*e + 1/8*b^3*d*x^8 + 3/8*a*b^2*g*x^8 + 1/7*b^3*c*x^7 + 3/7*a*b^2*f*
x^7 + 1/2*a^2*b*h*x^6 + 1/2*a*b^2*x^6*e + 3/5*a*b^2*d*x^5 + 3/5*a^2*b*g*x^5
 + 3/4*a*b^2*c*x^4 + 3/4*a^2*b*f*x^4 + 1/3*a^3*h*x^3 + a^2*b*x^3*e + 3/2*a^
2*b*d*x^2 + 1/2*a^3*g*x^2 + 3*a^2*b*c*x + a^3*f*x + a^3*e*log(abs(x)) - 1/2
*(2*a^3*d*x + a^3*c)/x^2
```

maple [A] time = 0.06, size = 222, normalized size = 1.12

$$\frac{b^3h\,x^{12}}{12}+\frac{b^3g\,x^{11}}{11}+\frac{b^3f\,x^{10}}{10}+\frac{a\,b^2h\,x^9}{3}+\frac{b^3e\,x^9}{9}+\frac{3a\,b^2g\,x^8}{8}+\frac{b^3d\,x^8}{8}+\frac{3a\,b^2f\,x^7}{7}+\frac{b^3c\,x^7}{7}+\frac{a^2bh\,x^6}{2}+\frac{a\,b^2e\,x^6}{2}+\frac{3a^2bg\,x^5}{5}+\frac{3a\,b^2d\,x^5}{5}+\frac{3a^2bf\,x^4}{4}+\frac{3a\,b^2c\,x^4}{4}+\frac{a^3h\,x^3}{3}+a^2be\,x^3+\frac{a^3g\,x^2}{2}+\frac{3a^2bd\,x^2}{2}+a^3e\ln(x)+a^3fx+3a^2bcx-\frac{a^3d}{x}-\frac{a^3c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] 1/12*b^3*h*x^12+1/11*b^3*g*x^11+1/10*b^3*f*x^10+1/3*x^9*a*b^2*h+1/9*b^3*e*x
^9+3/8*x^8*a*b^2*g+1/8*b^3*d*x^8+3/7*x^7*a*b^2*f+1/7*b^3*c*x^7+1/2*x^6*a^2*
b*h+1/2*a*b^2*e*x^6+3/5*x^5*a^2*b*g+3/5*a*b^2*d*x^5+3/4*x^4*a^2*b*f+3/4*a*b
```

```
^2*c*x^4+1/3*x^3*a^3*h+a^2*b*e*x^3+1/2*x^2*a^3*g+3/2*a^2*b*d*x^2+a^3*f*x+3*
a^2*b*c*x-1/2*a^3*c/x^2-a^3*d/x+a^3*e*ln(x)
```

maxima [A] time = 1.38, size = 212, normalized size = 1.07

$$\frac{1}{12}b^3hx^{12}+\frac{1}{11}b^3gx^{11}+\frac{1}{10}b^3fx^{10}+\frac{1}{9}\left(b^3e+3\,ab^2h\right)x^9+\frac{1}{8}\left(b^3d+3\,ab^2g\right)x^8+\frac{1}{7}\left(b^3c+3\,ab^2f\right)x^7+\frac{1}{2}\left(ab^2e+a^2bh\right)x^6+\frac{3}{5}\left(ab^2d+a^2bg\right)x^5+\frac{3}{4}\left(ab^2c+a^2bf\right)x^4+a^3e\log(x)+\frac{1}{3}\left(3\,a^2be+a^3h\right)x^3+\frac{1}{2}\left(3\,a^2bd+a^3g\right)x^2+\left(3\,a^2bc+a^3f\right)x-\frac{2\,a^3dx+a^3c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="max
ima")
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/9*(b^3*e + 3*a*b^2*
h)*x^9 + 1/8*(b^3*d + 3*a*b^2*g)*x^8 + 1/7*(b^3*c + 3*a*b^2*f)*x^7 + 1/2*(a
*b^2*e + a^2*b*h)*x^6 + 3/5*(a*b^2*d + a^2*b*g)*x^5 + 3/4*(a*b^2*c + a^2*b*
f)*x^4 + a^3*e*log(x) + 1/3*(3*a^2*b*e + a^3*h)*x^3 + 1/2*(3*a^2*b*d + a^3*
g)*x^2 + (3*a^2*b*c + a^3*f)*x - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

mupad [B] time = 0.14, size = 199, normalized size = 1.01

$$x^7\left(\frac{c\,b^3}{7}+\frac{3\,a\,f\,b^2}{7}\right)+x^2\left(\frac{g\,a^3}{2}+\frac{3\,b\,d\,a^2}{2}\right)+x^8\left(\frac{d\,b^3}{8}+\frac{3\,a\,g\,b^2}{8}\right)+x^3\left(\frac{h\,a^3}{3}+b\,e\,a^2\right)+x^9\left(\frac{e\,b^3}{9}+\frac{a\,h\,b^2}{3}\right)-\frac{\frac{a^3\,c}{2}+a^3\,d\,x}{x^2}+x\,\left(f\,a^3+3\,b\,c\,a^2\right)+\frac{b^3\,f\,x^{10}}{10}+\frac{b^3\,g\,x^{11}}{11}+\frac{b^3\,h\,x^{12}}{12}+a^3\,e\,\ln(x)+\frac{3\,a\,b\,x^4\,\left(b\,c+a\,f\right)}{4}+\frac{3\,a\,b\,x^5\,\left(b\,d+a\,g\right)}{5}+\frac{a\,b\,x^6\,\left(b\,e+a\,h\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

```
[Out] x^7*((b^3*c)/7 + (3*a*b^2*f)/7) + x^2*((a^3*g)/2 + (3*a^2*b*d)/2) + x^8*((b
^3*d)/8 + (3*a*b^2*g)/8) + x^3*((a^3*h)/3 + a^2*b*e) + x^9*((b^3*e)/9 + (a*
b^2*h)/3) - ((a^3*c)/2 + a^3*d*x)/x^2 + x*(a^3*f + 3*a^2*b*c) + (b^3*f*x^10
)/10 + (b^3*g*x^11)/11 + (b^3*h*x^12)/12 + a^3*e*log(x) + (3*a*b*x^4*(b*c +
 a*f))/4 + (3*a*b*x^5*(b*d + a*g))/5 + (a*b*x^6*(b*e + a*h))/2
```

sympy [A] time = 0.59, size = 238, normalized size = 1.20

$$a^3e\log(x)+\frac{b^3fx^{10}}{10}+\frac{b^3gx^{11}}{11}+\frac{b^3hx^{12}}{12}+x^9\left(\frac{ab^2h}{3}+\frac{b^3e}{9}\right)+x^8\left(\frac{3ab^2g}{8}+\frac{b^3d}{8}\right)+x^7\left(\frac{3ab^2f}{7}+\frac{b^3c}{7}\right)+x^6\left(\frac{a^2bh}{2}+\frac{ab^2e}{2}\right)+x^5\left(\frac{3a^2bg}{5}+\frac{3ab^2d}{5}\right)+x^4\left(\frac{3a^2bf}{4}+\frac{3ab^2c}{4}\right)+x^3\left(\frac{a^3h}{3}+a^2be\right)+x^2\left(\frac{a^3g}{2}+\frac{3a^2bd}{2}\right)+x\left(a^3f+3a^2bc\right)+\frac{-a^3c-2a^3dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**3*e*log(x) + b**3*f*x**10/10 + b**3*g*x**11/11 + b**3*h*x**12/12 + x**9*
(a*b**2*h/3 + b**3*e/9) + x**8*(3*a*b**2*g/8 + b**3*d/8) + x**7*(3*a*b**2*f
/7 + b**3*c/7) + x**6*(a**2*b*h/2 + a*b**2*e/2) + x**5*(3*a**2*b*g/5 + 3*a*
b**2*d/5) + x**4*(3*a**2*b*f/4 + 3*a*b**2*c/4) + x**3*(a**3*h/3 + a**2*b*e)
 + x**2*(a**3*g/2 + 3*a**2*b*d/2) + x*(a**3*f + 3*a**2*b*c) + (-a**3*c - 2*
a**3*d*x)/(2*x**2)
```

3.348 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

Optimal. Leaf size=209

$$-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2\log(x)(af+3bc)+a^2x(ag+3bd)+\frac{1}{2}a^2x^2(ah+3be)+\frac{1}{6}b^2x^6(3af+bc)+\frac{1}{7}b^2x^7(3ag+bd)+\frac{1}{8}b^2x^8$$

Rubi [A] time = 0.18, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1820}

$$a^2\log(x)(af+3bc)+a^2x(ag+3bd)+\frac{1}{2}a^2x^2(ah+3be)-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+\frac{1}{6}b^2x^6(3af+bc)+\frac{1}{7}b^2x^7(3ag+bd)+\frac{1}{8}b^2x^8(3ah+be)+abx^3(af+bc)+\frac{3}{4}abx^4(ag+bd)+\frac{3}{5}abx^5(ah+be)+\frac{1}{9}b^3fx^9+\frac{1}{10}b^3gx^{10}+\frac{1}{11}b^3hx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a^3*c)/(3*x^3) - (a^3*d)/(2*x^2) - (a^3*e)/x + a^2*(3*b*d + a*g)*x + (a^2
*(3*b*e + a*h)*x^2)/2 + a*b*(b*c + a*f)*x^3 + (3*a*b*(b*d + a*g)*x^4)/4 + (
3*a*b*(b*e + a*h)*x^5)/5 + (b^2*(b*c + 3*a*f)*x^6)/6 + (b^2*(b*d + 3*a*g)*x
^7)/7 + (b^2*(b*e + 3*a*h)*x^8)/8 + (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*
h*x^11)/11 + a^2*(3*b*c + a*f)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = \int \left(a^2(3bd+ag)+\frac{a^3c}{x^4}+\frac{a^3d}{x^3}+\frac{a^3e}{x^2}+\frac{a^2(3bc+af)}{x}+a\right.$$

$$= -\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2(3bd+ag)x+\frac{1}{2}a^2(3be+ah)x^2+$$

Mathematica [A] time = 0.15, size = 172, normalized size = 0.82

$$-\frac{a^3\left(2c+3x\left(d+2ex-\left(x^3(2g+hx)\right)\right)\right)}{6x^3}+a^2\log(x)(af+3bc)+\frac{1}{20}a^2bx\left(60d+x\left(30e+x\left(20f+15gx+12hx^2\right)\right)\right)+\frac{1}{280}ab^2x^3\left(280c+x\left(210d+x\left(168e+140fx+120gx^2+105hx^3\right)\right)\right)+\frac{b^3x^6\left(4620c+x\left(3960d+7x\left(495e+4x\left(110f+99gx+90hx^2\right)\right)\right)\right)}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -1/6*(a^3*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (a^2*b*x*(60*d +
 x*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/20 + (a*b^2*x^3*(280*c + x*(210*
d + x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3))))/280 + (b^3*x^6*(4620*c +
 x*(3960*d + 7*x*(495*e + 4*x*(110*f + 99*g*x + 90*h*x^2)))))/27720 + a^2*(
3*b*c + a*f)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^4, x]
```

fricas [A] time = 0.42, size = 219, normalized size = 1.05

$$\frac{2520\,b^3hx^{14}+2772\,b^3gx^{13}+3080\,b^3fx^{12}+3465\left(b^3e+3\,ab^2h\right)x^{11}+3960\left(b^3d+3\,ab^2g\right)x^{10}+4620\left(b^3c+3\,ab^2f\right)x^9+16632\left(ab^2e+a^2bh\right)x^8+20790\left(ab^2d+a^2bg\right)x^7+27720\left(ab^2c+a^2bf\right)x^6-27720\,a^3ex^2+13860\left(3\,a^2be+a^3h\right)x^5-13860\,a^3dx+27720\left(3\,a^2bd+a^3g\right)x^4+27720\left(3\,a^2bc+a^3f\right)x^3\log(x)-9240\,a^3c}{27720\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/27720*(2520*b^3*h*x^14 + 2772*b^3*g*x^13 + 3080*b^3*f*x^12 + 3465*(b^3*e
+ 3*a*b^2*h)*x^11 + 3960*(b^3*d + 3*a*b^2*g)*x^10 + 4620*(b^3*c + 3*a*b^2*f
)*x^9 + 16632*(a*b^2*e + a^2*b*h)*x^8 + 20790*(a*b^2*d + a^2*b*g)*x^7 + 277
20*(a*b^2*c + a^2*b*f)*x^6 - 27720*a^3*e*x^2 + 13860*(3*a^2*b*e + a^3*h)*x^
5 - 13860*a^3*d*x + 27720*(3*a^2*b*d + a^3*g)*x^4 + 27720*(3*a^2*b*c + a^3*
f)*x^3*log(x) - 9240*a^3*c)/x^3
```

giac [A] time = 0.19, size = 225, normalized size = 1.08

$$\frac{1}{11}\,b^3hx^{11}+\frac{1}{10}\,b^3gx^{10}+\frac{1}{9}\,b^3fx^9+\frac{3}{8}\,ab^2hx^8+\frac{1}{8}\,b^3x^8e+\frac{1}{7}\,b^3dx^7+\frac{3}{7}\,ab^2gx^7+\frac{1}{6}\,b^3cx^6+\frac{1}{2}\,ab^2fx^6+\frac{3}{5}\,a^2bhx^5+\frac{3}{5}\,ab^2x^5e+\frac{3}{4}\,ab^2dx^4+\frac{3}{4}\,a^2bgx^4+ab^2cx^3+a^2bfx^3+\frac{1}{2}\,a^3hx^2+\frac{3}{2}\,a^2bx^2e+3\,a^2bdx+a^3gx+\left(3\,a^2bc+a^3f\right)\log\left(|x|\right)-\frac{6\,a^3x^2e+3\,a^3dx+2\,a^3c}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gia
c")
```

```
[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 3/8*a*b^2*h*x^8 + 1/8*b
^3*x^8*e + 1/7*b^3*d*x^7 + 3/7*a*b^2*g*x^7 + 1/6*b^3*c*x^6 + 1/2*a*b^2*f*x^
6 + 3/5*a^2*b*h*x^5 + 3/5*a*b^2*x^5*e + 3/4*a*b^2*d*x^4 + 3/4*a^2*b*g*x^4 +
 a*b^2*c*x^3 + a^2*b*f*x^3 + 1/2*a^3*h*x^2 + 3/2*a^2*b*x^2*e + 3*a^2*b*d*x
+ a^3*g*x + (3*a^2*b*c + a^3*f)*log(abs(x)) - 1/6*(6*a^3*x^2*e + 3*a^3*d*x
+ 2*a^3*c)/x^3
```

maple [A] time = 0.05, size = 220, normalized size = 1.05

$$\frac{b^3h\,x^{11}}{11}+\frac{b^3g\,x^{10}}{10}+\frac{b^3f\,x^9}{9}+\frac{3a\,b^2h\,x^8}{8}+\frac{b^3e\,x^8}{8}+\frac{3a\,b^2g\,x^7}{7}+\frac{b^3d\,x^7}{7}+\frac{a\,b^2f\,x^6}{2}+\frac{b^3c\,x^6}{6}+\frac{3a^2bh\,x^5}{5}+\frac{3a\,b^2e\,x^5}{5}+\frac{3a^2bg\,x^4}{4}+\frac{3a\,b^2d\,x^4}{4}+a^2bf\,x^3+a\,b^2c\,x^3+\frac{a^3h\,x^2}{2}+\frac{3a^2be\,x^2}{2}+a^3f\ln(x)+a^3gx+3a^2bc\ln(x)+3a^2bdx-\frac{a^3e}{x}-\frac{a^3d}{2x^2}-\frac{a^3c}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] 1/11*b^3*h*x^11+1/10*b^3*g*x^10+1/9*b^3*f*x^9+3/8*x^8*a*b^2*h+1/8*x^8*b^3*e
+3/7*x^7*a*b^2*g+1/7*x^7*b^3*d+1/2*x^6*a*b^2*f+1/6*x^6*b^3*c+3/5*x^5*a^2*b*
h+3/5*x^5*a*b^2*e+3/4*x^4*a^2*b*g+3/4*x^4*a*b^2*d+x^3*a^2*b*f+a*b^2*c*x^3+1
/2*x^2*a^3*h+3/2*x^2*a^2*b*e+a^3*g*x+3*a^2*d*b*x-1/3*a^3*c/x^3-1/2*a^3*d/x^
2-a^3*e/x+ln(x)*a^3*f+3*ln(x)*a^2*b*c
```

maxima [A] time = 1.36, size = 212, normalized size = 1.01

$$\frac{1}{11}\,b^3hx^{11}+\frac{1}{10}\,b^3gx^{10}+\frac{1}{9}\,b^3fx^9+\frac{1}{8}\left(b^3e+3\,ab^2h\right)x^8+\frac{1}{7}\left(b^3d+3\,ab^2g\right)x^7+\frac{1}{6}\left(b^3c+3\,ab^2f\right)x^6+\frac{3}{5}\left(ab^2e+a^2bh\right)x^5+\frac{3}{4}\left(ab^2d+a^2bg\right)x^4+\left(ab^2c+a^2bf\right)x^3+\frac{1}{2}\left(3\,a^2be+a^3h\right)x^2+\left(3\,a^2bd+a^3g\right)x+\left(3\,a^2bc+a^3f\right)\log(x)-\frac{6\,a^3ex^2+3\,a^3dx+2\,a^3c}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="max
ima")
```

```
[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 1/8*(b^3*e + 3*a*b^2*h)
*x^8 + 1/7*(b^3*d + 3*a*b^2*g)*x^7 + 1/6*(b^3*c + 3*a*b^2*f)*x^6 + 3/5*(a*b
^2*e + a^2*b*h)*x^5 + 3/4*(a*b^2*d + a^2*b*g)*x^4 + (a*b^2*c + a^2*b*f)*x^3
```

```
 + 1/2*(3*a^2*b*e + a^3*h)*x^2 + (3*a^2*b*d + a^3*g)*x + (3*a^2*b*c + a^3*f
)*log(x) - 1/6*(6*a^3*e*x^2 + 3*a^3*d*x + 2*a^3*c)/x^3
```

mupad [B] time = 0.12, size = 199, normalized size = 0.95

$$x^6\left(\frac{c\,b^3}{6}+\frac{a\,f\,b^2}{2}\right)+x^7\left(\frac{d\,b^3}{7}+\frac{3\,a\,g\,b^2}{7}\right)+x^2\left(\frac{h\,a^3}{2}+\frac{3\,b\,e\,a^2}{2}\right)+x^8\left(\frac{e\,b^3}{8}+\frac{3\,a\,h\,b^2}{8}\right)+\ln(x)\left(f\,a^3+3\,b\,c\,a^2\right)-\frac{e\,a^3\,x^2+\frac{d\,a^3\,x}{2}+\frac{c\,a^3}{3}}{x^3}+x\left(g\,a^3+3\,b\,d\,a^2\right)+\frac{b^3\,f\,x^9}{9}+\frac{b^3\,g\,x^{10}}{10}+\frac{b^3\,h\,x^{11}}{11}+a\,b\,x^3\left(b\,c+a\,f\right)+\frac{3\,a\,b\,x^4\left(b\,d+a\,g\right)}{4}+\frac{3\,a\,b\,x^5\,(b\,e+a\,h)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

```
[Out] x^6*((b^3*c)/6 + (a*b^2*f)/2) + x^7*((b^3*d)/7 + (3*a*b^2*g)/7) + x^2*((a^3
*h)/2 + (3*a^2*b*e)/2) + x^8*((b^3*e)/8 + (3*a*b^2*h)/8) + log(x)*(a^3*f +
3*a^2*b*c) - ((a^3*c)/3 + a^3*e*x^2 + (a^3*d*x)/2)/x^3 + x*(a^3*g + 3*a^2*b
*d) + (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*h*x^11)/11 + a*b*x^3*(b*c + a*
f) + (3*a*b*x^4*(b*d + a*g))/4 + (3*a*b*x^5*(b*e + a*h))/5
```

sympy [A] time = 1.04, size = 236, normalized size = 1.13

$$a^2\left(af+3bc\right)\log(x)+\frac{b^3fx^9}{9}+\frac{b^3gx^{10}}{10}+\frac{b^3hx^{11}}{11}+x^8\left(\frac{3ab^2h}{8}+\frac{b^3e}{8}\right)+x^7\left(\frac{3ab^2g}{7}+\frac{b^3d}{7}\right)+x^6\left(\frac{ab^2f}{2}+\frac{b^3c}{6}\right)+x^5\left(\frac{3a^2bh}{5}+\frac{3ab^2e}{5}\right)+x^4\left(\frac{3a^2bg}{4}+\frac{3ab^2d}{4}\right)+x^3\left(a^2bf+ab^2c\right)+x^2\left(\frac{a^3h}{2}+\frac{3a^2be}{2}\right)+x\left(a^3g+3a^2bd\right)+\frac{-2a^3c-3a^3dx-6a^3ex^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a**2*(a*f + 3*b*c)*log(x) + b**3*f*x**9/9 + b**3*g*x**10/10 + b**3*h*x**11/
11 + x**8*(3*a*b**2*h/8 + b**3*e/8) + x**7*(3*a*b**2*g/7 + b**3*d/7) + x**6
*(a*b**2*f/2 + b**3*c/6) + x**5*(3*a**2*b*h/5 + 3*a*b**2*e/5) + x**4*(3*a**
2*b*g/4 + 3*a*b**2*d/4) + x**3*(a**2*b*f + a*b**2*c) + x**2*(a**3*h/2 + 3*a
**2*b*e/2) + x*(a**3*g + 3*a**2*b*d) + (-2*a**3*c - 3*a**3*d*x - 6*a**3*e*x
**2)/(6*x**3)
```

3.349 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$

Optimal. Leaf size=209

$$-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(af+3bc)}{x}+a^2\log(x)(ag+3bd)+a^2x(ah+3be)+\frac{1}{5}b^2x^5(3af+bc)+\frac{1}{6}b^2x^6(3ag+bd)+\frac{1}{7}b^2x^7(3ah$$

Rubi [A] time = 0.18, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$-\frac{a^2(af+3bc)}{x}+a^2\log(x)(ag+3bd)+a^2x(ah+3be)-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}+\frac{1}{5}b^2x^5(3af+bc)+\frac{1}{6}b^2x^6(3ag+bd)+\frac{1}{7}b^2x^7(3ah+be)+\frac{3}{2}abx^2(af+bc)+abx^3(ag+bd)+\frac{3}{4}abx^4(ah+be)+\frac{1}{8}b^3fx^8+\frac{1}{9}b^3gx^9+\frac{1}{10}b^3hx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -(a^3*c)/(4*x^4) - (a^3*d)/(3*x^3) - (a^3*e)/(2*x^2) - (a^2*(3*b*c + a*f))/
x + a^2*(3*b*e + a*h)*x + (3*a*b*(b*c + a*f)*x^2)/2 + a*b*(b*d + a*g)*x^3 +
 (3*a*b*(b*e + a*h)*x^4)/4 + (b^2*(b*c + 3*a*f)*x^5)/5 + (b^2*(b*d + 3*a*g)
*x^6)/6 + (b^2*(b*e + 3*a*h)*x^7)/7 + (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*
h*x^10)/10 + a^2*(3*b*d + a*g)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = \int \left(a^2(3be+ah)+\frac{a^3c}{x^5}+\frac{a^3d}{x^4}+\frac{a^3e}{x^3}+\frac{a^2(3bc+af)}{x^2}+\frac{a^2(3$$
$$= -\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(3bc+af)}{x}+a^2(3be+ah)x+\frac{3}{2}ab($$

Mathematica [A] time = 0.16, size = 170, normalized size = 0.81

$$a^2\log(x)(ag+3bd)+\frac{-210a^3\left(3c+4dx+6x^2\left(e+2fx-2hx^3\right)\right)+630a^2bx^3\left(x^2\left(12e+6fx+4gx^2+3hx^3\right)-12c\right)+18ab^2x^6\left(210c+x\left(140d+105ex+84fx^2+70gx^3+60hx^4\right)\right)+b^3x^9\left(504c+x\left(420d+360ex+315fx^2+280gx^3+252hx^4\right)\right)}{2520x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] (-210*a^3*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)) + 630*a^2*b*x^3*(-12*
c + x^2*(12*e + 6*f*x + 4*g*x^2 + 3*h*x^3)) + 18*a*b^2*x^6*(210*c + x*(140*
d + 105*e*x + 84*f*x^2 + 70*g*x^3 + 60*h*x^4)) + b^3*x^9*(504*c + x*(420*d
+ 360*e*x + 315*f*x^2 + 280*g*x^3 + 252*h*x^4)))/(2520*x^4) + a^2*(3*b*d +
a*g)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^5,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)
)/x^5, x]
```

fricas [A] time = 0.40, size = 219, normalized size = 1.05

$$\frac{252\,b^{3}hx^{14}+280\,b^{3}gx^{13}+315\,b^{3}fx^{12}+360\,\left(b^{3}e+3\,ab^{2}h\right)x^{11}+420\,\left(b^{3}d+3\,ab^{2}g\right)x^{10}+504\,\left(b^{3}c+3\,ab^{2}f\right)x^{9}+1890\,\left(ab^{2}e+a^{2}bh\right)x^{8}+2520\,\left(ab^{2}d+a^{2}bg\right)x^{7}+3780\,\left(ab^{2}c+a^{2}bf\right)x^{6}-1260\,a^{3}ex^{2}+2520\,\left(3\,a^{2}be+a^{3}h\right)x^{5}+2520\,\left(3\,a^{2}bd+a^{3}g\right)x^{4}\log\left(x\right)-840\,a^{3}dx-630\,a^{3}c-2520\,\left(3\,a^{2}bc+a^{3}f\right)x^{3}}{2520\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(252*b^3*h*x^14 + 280*b^3*g*x^13 + 315*b^3*f*x^12 + 360*(b^3*e + 3*a
*b^2*h)*x^11 + 420*(b^3*d + 3*a*b^2*g)*x^10 + 504*(b^3*c + 3*a*b^2*f)*x^9 +
 1890*(a*b^2*e + a^2*b*h)*x^8 + 2520*(a*b^2*d + a^2*b*g)*x^7 + 3780*(a*b^2*
c + a^2*b*f)*x^6 - 1260*a^3*e*x^2 + 2520*(3*a^2*b*e + a^3*h)*x^5 + 2520*(3*
a^2*b*d + a^3*g)*x^4*log(x) - 840*a^3*d*x - 630*a^3*c - 2520*(3*a^2*b*c + a
^3*f)*x^3)/x^4
```

giac [A] time = 0.15, size = 224, normalized size = 1.07

$$\frac{1}{10}\,b^{3}hx^{10}+\frac{1}{9}\,b^{3}gx^{9}+\frac{1}{8}\,b^{3}fx^{8}+\frac{3}{7}\,ab^{2}hx^{7}+\frac{1}{7}\,b^{3}x^{7}e+\frac{1}{6}\,b^{3}dx^{6}+\frac{1}{2}\,ab^{2}gx^{6}+\frac{1}{5}\,b^{3}cx^{5}+\frac{3}{5}\,ab^{2}fx^{5}+\frac{3}{4}\,a^{2}bhx^{4}+\frac{3}{4}\,ab^{2}x^{4}e+ab^{2}dx^{3}+a^{2}bgx^{3}+\frac{3}{2}\,ab^{2}cx^{2}+\frac{3}{2}\,a^{2}bfx^{2}+a^{3}hx+3\,a^{2}bxe+\left(3\,a^{2}bd+a^{3}g\right)\log\left(|x|\right)-\frac{6\,a^{3}x^{2}e+4\,a^{3}dx+3\,a^{3}c+12\,\left(3\,a^{2}bc+a^{3}f\right)x^{3}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 3/7*a*b^2*h*x^7 + 1/7*b^3
*x^7*e + 1/6*b^3*d*x^6 + 1/2*a*b^2*g*x^6 + 1/5*b^3*c*x^5 + 3/5*a*b^2*f*x^5
+ 3/4*a^2*b*h*x^4 + 3/4*a*b^2*x^4*e + a*b^2*d*x^3 + a^2*b*g*x^3 + 3/2*a*b^2
*c*x^2 + 3/2*a^2*b*f*x^2 + a^3*h*x + 3*a^2*b*x*e + (3*a^2*b*d + a^3*g)*log(
abs(x)) - 1/12*(6*a^3*x^2*e + 4*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*
x^3)/x^4
```

maple [A] time = 0.05, size = 220, normalized size = 1.05

$$\frac{b^{3}h\,x^{10}}{10}+\frac{b^{3}g\,x^{9}}{9}+\frac{b^{3}f\,x^{8}}{8}+\frac{3a\,b^{2}h\,x^{7}}{7}+\frac{b^{3}e\,x^{7}}{7}+\frac{a\,b^{2}g\,x^{6}}{2}+\frac{b^{3}d\,x^{6}}{6}+\frac{3a\,b^{2}f\,x^{5}}{5}+\frac{b^{3}c\,x^{5}}{5}+\frac{3a^{2}bh\,x^{4}}{4}+\frac{3a\,b^{2}e\,x^{4}}{4}+a^{2}bg\,x^{3}+a\,b^{2}d\,x^{3}+\frac{3a^{2}bf\,x^{2}}{2}+\frac{3a\,b^{2}c\,x^{2}}{2}+a^{3}g\ln\left(x\right)+a^{3}hx+3a^{2}bd\ln\left(x\right)+3a^{2}bex-\frac{a^{3}f}{x}-\frac{3a^{2}bc}{x}-\frac{a^{3}e}{2x^{2}}-\frac{a^{3}d}{3x^{3}}-\frac{a^{3}c}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] 1/10*b^3*h*x^10+1/9*b^3*g*x^9+1/8*b^3*f*x^8+3/7*x^7*a*b^2*h+1/7*x^7*b^3*e+1
/2*x^6*a*b^2*g+1/6*x^6*b^3*d+3/5*x^5*a*b^2*f+1/5*x^5*b^3*c+3/4*x^4*a^2*b*h+
3/4*x^4*a*b^2*e+x^3*a^2*b*g+x^3*a*b^2*d+3/2*x^2*a^2*b*f+3/2*a*b^2*c*x^2+a^3
*h*x+3*a^2*b*e*x-1/4*a^3*c/x^4-1/3*a^3*d/x^3-1/2*a^3*e/x^2-a^3/x*f-3*a^2/x*
b*c+ln(x)*a^3*g+3*ln(x)*a^2*b*d
```

maxima [A] time = 1.39, size = 212, normalized size = 1.01

$$\frac{1}{10}\,b^{3}hx^{10}+\frac{1}{9}\,b^{3}gx^{9}+\frac{1}{8}\,b^{3}fx^{8}+\frac{1}{7}\,\left(b^{3}e+3\,ab^{2}h\right)x^{7}+\frac{1}{6}\,\left(b^{3}d+3\,ab^{2}g\right)x^{6}+\frac{1}{5}\,\left(b^{3}c+3\,ab^{2}f\right)x^{5}+\frac{3}{4}\,\left(ab^{2}e+a^{2}bh\right)x^{4}+\left(ab^{2}d+a^{2}bg\right)x^{3}+\frac{3}{2}\,\left(ab^{2}c+a^{2}bf\right)x^{2}+\left(3\,a^{2}be+a^{3}h\right)x+\left(3\,a^{2}bd+a^{3}g\right)\log\left(x\right)-\frac{6\,a^{3}ex^{2}+4\,a^{3}dx+3\,a^{3}c+12\,\left(3\,a^{2}bc+a^{3}f\right)x^{3}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="max
ima")
```

```
[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 1/7*(b^3*e + 3*a*b^2*h)*x
^7 + 1/6*(b^3*d + 3*a*b^2*g)*x^6 + 1/5*(b^3*c + 3*a*b^2*f)*x^5 + 3/4*(a*b^2
*e + a^2*b*h)*x^4 + (a*b^2*d + a^2*b*g)*x^3 + 3/2*(a*b^2*c + a^2*b*f)*x^2 +
```

```
 (3*a^2*b*e + a^3*h)*x + (3*a^2*b*d + a^3*g)*log(x) - 1/12*(6*a^3*e*x^2 + 4
*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*x^3)/x^4
```

mupad [B] time = 5.03, size = 199, normalized size = 0.95

$$x^5\left(\frac{c\,b^3}{5}+\frac{3\,a\,f\,b^2}{5}\right)+x^6\left(\frac{d\,b^3}{6}+\frac{a\,g\,b^2}{2}\right)+x^7\left(\frac{e\,b^3}{7}+\frac{3\,a\,h\,b^2}{7}\right)+\ln(x)\,\left(g\,a^3+3\,b\,d\,a^2\right)-\frac{x^3\,\left(f\,a^3+3\,b\,c\,a^2\right)+\frac{a^3\,c}{4}+\frac{a^3\,e\,x^2}{2}+\frac{a^3\,d\,x}{3}}{x^4}+x\,\left(h\,a^3+3\,b\,e\,a^2\right)+\frac{b^3\,f\,x^8}{8}+\frac{b^3\,g\,x^9}{9}+\frac{b^3\,h\,x^{10}}{10}+\frac{3\,a\,b\,x^2\,\left(b\,c+a\,f\right)}{2}+a\,b\,x^3\,\left(b\,d+a\,g\right)+\frac{3\,a\,b\,x^4\,\left(b\,e+a\,h\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

```
[Out] x^5*((b^3*c)/5 + (3*a*b^2*f)/5) + x^6*((b^3*d)/6 + (a*b^2*g)/2) + x^7*((b^3
*e)/7 + (3*a*b^2*h)/7) + log(x)*(a^3*g + 3*a^2*b*d) - (x^3*(a^3*f + 3*a^2*b
*c) + (a^3*c)/4 + (a^3*e*x^2)/2 + (a^3*d*x)/3)/x^4 + x*(a^3*h + 3*a^2*b*e)
+ (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*h*x^10)/10 + (3*a*b*x^2*(b*c + a*f))
/2 + a*b*x^3*(b*d + a*g) + (3*a*b*x^4*(b*e + a*h))/4
```

sympy [A] time = 3.14, size = 235, normalized size = 1.12

$$a^2\left(ag+3bd\right)\log\left(x\right)+\frac{b^3fx^8}{8}+\frac{b^3gx^9}{9}+\frac{b^3hx^{10}}{10}+x^7\left(\frac{3ab^2h}{7}+\frac{b^3e}{7}\right)+x^6\left(\frac{ab^2g}{2}+\frac{b^3d}{6}\right)+x^5\left(\frac{3ab^2f}{5}+\frac{b^3c}{5}\right)+x^4\left(\frac{3a^2bh}{4}+\frac{3ab^2e}{4}\right)+x^3\left(a^2bg+ab^2d\right)+x^2\left(\frac{3a^2bf}{2}+\frac{3ab^2c}{2}\right)+x\left(a^3h+3a^2be\right)+\frac{-3a^3c-4a^3dx-6a^3ex^2+x^3\left(-12a^3f-36a^2bc\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a**2*(a*g + 3*b*d)*log(x) + b**3*f*x**8/8 + b**3*g*x**9/9 + b**3*h*x**10/10
 + x**7*(3*a*b**2*h/7 + b**3*e/7) + x**6*(a*b**2*g/2 + b**3*d/6) + x**5*(3*
a*b**2*f/5 + b**3*c/5) + x**4*(3*a**2*b*h/4 + 3*a*b**2*e/4) + x**3*(a**2*b*
g + a*b**2*d) + x**2*(3*a**2*b*f/2 + 3*a*b**2*c/2) + x*(a**3*h + 3*a**2*b*e
) + (-3*a**3*c - 4*a**3*d*x - 6*a**3*e*x**2 + x**3*(-12*a**3*f - 36*a**2*b*
c))/(12*x**4)
```

3.350 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=331

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6b^{10/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(\right.}{3b^{10/3}}$$

Rubi [A] time = 1.07, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6b^{10/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3b^{10/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}b^{10/3}}+\frac{x^2(bc-af)}{2b^2}+\frac{x^3(bd-ag)}{3b^2}-\frac{a(bd-ag)\log\left(a+bx^3\right)}{3b^3}+\frac{x^4(be-ah)}{4b^2}-\frac{ax(be-ah)}{b^3}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+\frac{hx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] -((a*(b*e - a*h)*x)/b^3) + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(3
*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^7)
/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(10/3)) + (a^(2/3)
*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*b
^(10/3)) - (a^(2/3)*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) - (a*(b*d - a*g)*Log[a +
b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx = \frac{hx^7}{7b} + \frac{\int \frac{x^4\left(7bc+7bdx+7(be-ah)x^2+7bfx^3+7bgx^4\right)}{a+bx^3}\,dx}{7b}$$

$$= \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \frac{x^4\left(42b^2c+42b(bd-ag)x+42b(be-ah)x^2+42b^2fx^3\right)}{a+bx^3}\,dx}{42b^2}$$

$$= \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \frac{x^4\left(210b^2(bc-af)+210b^2(bd-ag)x+210b^2(be-ah)x^2\right)}{a+bx^3}\,dx}{210b^3}$$

$$= \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \Big(-210a(be-ah) + 210b(bc-af)x + 210b}{}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b}$$

$$= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b}$$

Mathematica [A] time = 0.56, size = 334, normalized size = 1.01

$$\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^{2/3}be+a^{5/3}h+ab^{2/3}f-b^{5/3}c\right)}{6b^{10/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+b^{5/3}c\right)}{3b^{10/3}} + \frac{a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}b^{10/3}} + \frac{a(ag-bd)\log\left(a+bx^3\right)}{3b^3} + \frac{ax(ah-be)}{b^3} + \frac{x^2(bc-af)}{2b^2} + \frac{x^3(bd-ag)}{3b^2} + \frac{x^4(be-ah)}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (a*(-(b*e) + a*h)*x)/b^3 + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(3
*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^7)
/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(10/3)) + (a^(2/3)*(b^(5
/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/(3
*b^(10/3)) + (a^(2/3)*(-(b^(5/3)*c) - a^(2/3)*b*e + a*b^(2/3)*f + a^(5/3)*h
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) + (a*(-(b*d)
 + a*g)*Log[a + b*x^3])/(3*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [A] time = 0.20, size = 380, normalized size = 1.15

$$-\frac{(abd-a^2g)\log\left(|bx^3+a|\right)}{3\,b^3}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h-\left(-ab^2\right)^{\frac{1}{3}}abe-\left(-ab^2\right)^{\frac{2}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h-\left(-ab^2\right)^{\frac{1}{3}}abe+\left(-ab^2\right)^{\frac{2}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4}+\frac{60\,b^6hx^7+70\,b^6gx^6+84\,b^6fx^5-105\,ab^5hx^4+105\,b^6x^4e+140\,b^6dx^3-140\,ab^5gx^3+210\,b^6cx^2-210\,ab^5fx^2+420\,a^2b^4hx-420\,ab^5xe}{420\,b^7}+\frac{\left(ab^{14}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^{13}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3b^{12}h-a^2b^{13}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] -1/3*(a*b*d - a^2*g)*log(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*
a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3
)*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/420*(60*b^6*h*x^7 + 70*b^6*
g*x^6 + 84*b^6*f*x^5 - 105*a*b^5*h*x^4 + 105*b^6*x^4*e + 140*b^6*d*x^3 - 14
0*a*b^5*g*x^3 + 210*b^6*c*x^2 - 210*a*b^5*f*x^2 + 420*a^2*b^4*h*x - 420*a*b
^5*x*e)/b^7 + 1/3*(a*b^14*c*(-a/b)^(1/3) - a^2*b^13*f*(-a/b)^(1/3) + a^3*b^
12*h - a^2*b^13*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^15)
```

maple [B] time = 0.05, size = 533, normalized size = 1.61

$$\frac{hx^7}{7b}+\frac{gx^6}{6b}+\frac{fx^5}{5b}-\frac{ahx^4}{4b^2}+\frac{ex^4}{4b}-\frac{agx^3}{3b^2}+\frac{dx^3}{3b}-\frac{afx^2}{2b^2}+\frac{cx^2}{2b}-\frac{\sqrt{3}\,a^3h\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{a^3h\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{a^3h\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{\sqrt{3}\,a^2e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{a^2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{a^2e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{\sqrt{3}\,a^2f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{a^2f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{a^2f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{a^2g\ln\left(bx^3+a\right)}{3b^3}+\frac{a^2hx}{b^3}-\frac{\sqrt{3}\,ac\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{ac\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{ac\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{ad\ln\left(bx^3+a\right)}{3b^2}-\frac{aex}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*a^2/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1
/3*a^3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h-1/
3*a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/3*a
^2/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/2*a/
b^2*f*x^2+1/b^3*a^2*h*x-1/b^2*a*e*x-1/3/b^2*x^3*a*g-1/3*a/b^2*ln(b*x^3+a)*d
-1/4/b^2*x^4*a*h+1/3*a^2/b^3*ln(b*x^3+a)*g-1/6*a/b^2/(a/b)^(1/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))*c-1/6*a^2/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))*e-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f+1/3*a/b^2/(a/b)^(1/3
)*ln(x+(a/b)^(1/3))*c+1/6*a^2/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))*f+1/6*a^3/b^4/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*h-1/3*a^3/
b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*h+1/3*a^2/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*e+1/4/b*x^4*e+1/3/b*x^3*d+1/2/b*c*x^2+1/5/b*f*x^5+1/6*g*x^6/b+1/7*h*x^7/
b
```

maxima [A] time = 2.98, size = 378, normalized size = 1.14

$$-\frac{\sqrt{3}\left(ab^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2bf\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2be\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^3h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}+\frac{60\,b^2hx^7+70\,b^2gx^6+84\,b^2fx^5+105\left(b^2e-abh\right)x^4+140\left(b^2d-abg\right)x^3+210\left(b^2c-abf\right)x^2-420\left(abe-a^2h\right)x}{420\,b^3}-\frac{\left(2\,ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2bg\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2bf\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2be-a^3h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2bg\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2bf\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2be+a^3h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] -1/3*sqrt(3)*(a*b^2*c*(a/b)^(2/3) - a^2*b*f*(a/b)^(2/3) - a^2*b*e*(a/b)^(1/
3) + a^3*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b^3) + 1/420*(60*b^2*h*x^7 + 70*b^2*g*x^6 + 84*b^2*f*x^5 + 105*(b^2*e -
 a*b*h)*x^4 + 140*(b^2*d - a*b*g)*x^3 + 210*(b^2*c - a*b*f)*x^2 - 420*(a*b*
e - a^2*h)*x)/b^3 - 1/6*(2*a*b^2*d*(a/b)^(2/3) - 2*a^2*b*g*(a/b)^(2/3) + a*
b^2*c*(a/b)^(1/3) - a^2*b*f*(a/b)^(1/3) + a^2*b*e - a^3*h)*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 1/3*(a*b^2*d*(a/b)^(2/3) - a^2*b
*g*(a/b)^(2/3) - a*b^2*c*(a/b)^(1/3) + a^2*b*f*(a/b)^(1/3) - a^2*b*e + a^3*
h)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

mupad [B] time = 5.09, size = 1271, normalized size = 3.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] x^2*(c/(2*b) - (a*f)/(2*b^2)) + x^3*(d/(3*b) - (a*g)/(3*b^2)) + x^4*(e/(4*b
) - (a*h)/(4*b^2)) + symsum(log(root(27*b^10*z^3 + 27*a*b^8*d*z^2 - 27*a^2*
b^7*g*z^2 - 9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z + 9*a^3*b^5*e*f*z + 9*a^3*b^
5*c*h*z - 9*a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a^2*b^6*d^2*z + 3*a^6*b*f*g
*h - 3*a^5*b^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^2*c*g*h + 3*a^4*b^3*d*e*f
+ 3*a^4*b^3*c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c*d*e - 3*a^6*b*e*h^2 + 3*a
^5*b^2*e^2*h + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g - 3*a^4*b^3*c*f^2 + 3*a^3*
b^4*c^2*f + a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 - a^4*b^3*e^3 - a^2*b^5*c^3
 - a^6*b*g^3, z, k)*((6*a^2*b^4*d - 6*a^3*b^3*g)/b^4 + (x*(3*a^2*b^4*e - 3*
a^3*b^3*h))/b^4 + 9*root(27*b^10*z^3 + 27*a*b^8*d*z^2 - 27*a^2*b^7*g*z^2 -
9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z + 9*a^3*b^5*e*f*z + 9*a^3*b^5*c*h*z - 9*
a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a^2*b^6*d^2*z + 3*a^6*b*f*g*h - 3*a^5*b
^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^2*c*g*h + 3*a^4*b^3*d*e*f + 3*a^4*b^3*
c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c*d*e - 3*a^6*b*e*h^2 + 3*a^5*b^2*e^2*h
 + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g - 3*a^4*b^3*c*f^2 + 3*a^3*b^4*c^2*f +
a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 - a^4*b^3*e^3 - a^2*b^5*c^3 - a^6*b*g^3
, z, k)*a*b^2) + (a^5*g^2 + a^3*b^2*d^2 - a^5*f*h + a^4*b*c*h - 2*a^4*b*d*g
 + a^4*b*e*f - a^3*b^2*c*e)/b^4 + (x*(a^4*b*f^2 + a^2*b^3*c^2 + a^5*g*h - a
^4*b*d*h - a^4*b*e*g - 2*a^3*b^2*c*f + a^3*b^2*d*e))/b^4)*root(27*b^10*z^3
+ 27*a*b^8*d*z^2 - 27*a^2*b^7*g*z^2 - 9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z +
9*a^3*b^5*e*f*z + 9*a^3*b^5*c*h*z - 9*a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a
^2*b^6*d^2*z + 3*a^6*b*f*g*h - 3*a^5*b^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^
2*c*g*h + 3*a^4*b^3*d*e*f + 3*a^4*b^3*c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c
*d*e - 3*a^6*b*e*h^2 + 3*a^5*b^2*e^2*h + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g
- 3*a^4*b^3*c*f^2 + 3*a^3*b^4*c^2*f + a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 -
 a^4*b^3*e^3 - a^2*b^5*c^3 - a^6*b*g^3, z, k), k, 1, 3) + (f*x^5)/(5*b) + (
g*x^6)/(6*b) + (h*x^7)/(7*b) - (a*x*(e/b - (a*h)/b^2))/b
```

sympy [B] time = 60.52, size = 881, normalized size = 2.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] x**4*(-a*h/(4*b**2) + e/(4*b)) + x**3*(-a*g/(3*b**2) + d/(3*b)) + x**2*(-a*
f/(2*b**2) + c/(2*b)) + x*(a**2*h/b**3 - a*e/b**2) + RootSum(27*_t**3*b**10
 + _t**2*(-27*a**2*b**7*g + 27*a*b**8*d) + _t*(-9*a**4*b**4*f*h + 9*a**4*b*
*4*g**2 + 9*a**3*b**5*c*h - 18*a**3*b**5*d*g + 9*a**3*b**5*e*f - 9*a**2*b**
6*c*e + 9*a**2*b**6*d**2) + a**7*h**3 - 3*a**6*b*e*h**2 + 3*a**6*b*f*g*h -
```

```
a**6*b*g**3 - 3*a**5*b**2*c*g*h - 3*a**5*b**2*d*f*h + 3*a**5*b**2*d*g**2 +
3*a**5*b**2*e**2*h - 3*a**5*b**2*e*f*g + a**5*b**2*f**3 + 3*a**4*b**3*c*d*h
 + 3*a**4*b**3*c*e*g - 3*a**4*b**3*c*f**2 - 3*a**4*b**3*d**2*g + 3*a**4*b**
3*d*e*f - a**4*b**3*e**3 + 3*a**3*b**4*c**2*f - 3*a**3*b**4*c*d*e + a**3*b*
*4*d**3 - a**2*b**5*c**3, Lambda(_t, _t*log(x + (-9*_t**2*a*b**7*f + 9*_t**
2*b**8*c - 3*_t*a**4*b**3*h**2 + 6*_t*a**3*b**4*e*h + 6*_t*a**3*b**4*f*g -
6*_t*a**2*b**5*c*g - 6*_t*a**2*b**5*d*f - 3*_t*a**2*b**5*e**2 + 6*_t*a*b**6
*c*d + a**6*g*h**2 - a**5*b*d*h**2 - 2*a**5*b*e*g*h + 2*a**5*b*f**2*h - a**
5*b*f*g**2 - 4*a**4*b**2*c*f*h + a**4*b**2*c*g**2 + 2*a**4*b**2*d*e*h + 2*a
**4*b**2*d*f*g + a**4*b**2*e**2*g - 2*a**4*b**2*e*f**2 + 2*a**3*b**3*c**2*h
 - 2*a**3*b**3*c*d*g + 4*a**3*b**3*c*e*f - a**3*b**3*d**2*f - a**3*b**3*d*e
**2 - 2*a**2*b**4*c**2*e + a**2*b**4*c*d**2)/(a**6*h**3 - 3*a**5*b*e*h**2 +
 3*a**4*b**2*e**2*h - a**4*b**2*f**3 + 3*a**3*b**3*c*f**2 - a**3*b**3*e**3
- 3*a**2*b**4*c**2*f + a*b**5*c**3)))) + f*x**5/(5*b) + g*x**6/(6*b) + h*x*
*7/(7*b)
```

3.351 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=313

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\cdots\right.}{\sqrt{3}b^{8/3}}$$

Rubi [A] time = 0.99, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}b^{8/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3b^{8/3}}+\frac{x(bc-af)}{b^2}+\frac{x^2(bd-ag)}{2b^2}+\frac{x^3(be-ah)}{3b^2}-\frac{a(be-ah)\log\left(a+bx^3\right)}{3b^3}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*c - a*f)*x)/b^2 + ((b*d - a*g)*x^2)/(2*b^2) + ((b*e - a*h)*x^3)/(3*b^2)
 + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6*b) + (a^(1/3)*(b^(4/3)*c + a^
(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b
*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*c -
 a*f) - a^(1/3)*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(8/3)) - (a*(b*e - a*h)*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx = \frac{hx^6}{6b}+\frac{\int \frac{x^3\left(6bc+6bdx+6(be-ah)x^2+6bfx^3+6bgx^4\right)}{a+bx^3}dx}{6b}$$

$$= \frac{gx^5}{5b}+\frac{hx^6}{6b}+\frac{\int \frac{x^3\left(30b^2c+30b(bd-ag)x+30b(be-ah)x^2+30b^2fx^3\right)}{a+bx^3}dx}{30b^2}$$

$$= \frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}+\frac{\int \frac{x^3\left(120b^2(bc-af)+120b^2(bd-ag)x+120b^2(be-ah)x^2\right)}{a+bx^3}d}{120b^3}$$

$$= \frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}+\frac{\int \Big(120b(bc-af)+120b(bd-ag)x+120b(}{}$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}-\int$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}-\int$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}-\frac{a}{}$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}-\sqrt[3]{}$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}-\sqrt[3]{}$$

$$= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}+\sqrt[3]{}$$

Mathematica [A] time = 0.29, size = 299, normalized size = 0.96

$$\frac{10\sqrt[3]{a}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)-20\sqrt[3]{a}\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)-20\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}g-\sqrt[3]{a}bd+a\sqrt[3]{b}f-b^{4/3}c\right)+60bx(bc-af)+30bx^2(bd-ag)+20bx^3(be-ah)+20a(ah-be)\log\left(a+bx^3\right)+15b^2fx^4+12b^2gx^5+10b^2hx^6}{60b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (60*b*(b*c - a*f)*x + 30*b*(b*d - a*g)*x^2 + 20*b*(b*e - a*h)*x^3 + 15*b^2*
f*x^4 + 12*b^2*g*x^5 + 10*b^2*h*x^6 - 20*Sqrt[3]*a^(1/3)*b^(1/3)*(-(b^(4/3)
*c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] - 20*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f +
 a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1
/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2] + 20*a*(-(b*e) + a*h)*Log[a + b*x^3])/(60*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [A] time = 0.18, size = 353, normalized size = 1.13

$$\frac{\left(a^{2} h - a b e\right) \log\left(\left|b x^{3} + a\right|\right)}{3 \, b^{3}} - \frac{\sqrt{3} \left(\left(-a b^{2}\right)^{\frac{1}{3}} b^{2} c - \left(-a b^{2}\right)^{\frac{1}{3}} a b f - \left(-a b^{2}\right)^{\frac{2}{3}} b d + \left(-a b^{2}\right)^{\frac{2}{3}} a g\right) \arctan\left(\frac{\sqrt{3} \left(2 \, x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 \, b^{4}} - \frac{\left(\left(-a b^{2}\right)^{\frac{1}{3}} b^{2} c - \left(-a b^{2}\right)^{\frac{1}{3}} a b f + \left(-a b^{2}\right)^{\frac{2}{3}} b d - \left(-a b^{2}\right)^{\frac{2}{3}} a g\right) \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \, b^{4}} + \frac{10 \, b^{5} h x^{6} + 12 \, b^{5} g x^{5} + 15 \, b^{5} f x^{4} - 20 \, a b^{4} h x^{3} + 20 \, b^{5} x^{3} e + 30 \, b^{5} d x^{2} - 30 \, a b^{4} g x^{2} + 60 \, b^{5} c x - 60 \, a b^{4} f x}{60 \, b^{6}} + \frac{\left(a b^{12} d \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^{2} b^{11} g \left(-\frac{a}{b}\right)^{\frac{1}{3}} + a b^{12} c - a^{2} b^{11} f\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 \, a b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] 1/3*(a^2*h - a*b*e)*log(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b
^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)
*b^2*c - (-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*lo
g(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(10*b^5*h*x^6 + 12*b^5*g*
x^5 + 15*b^5*f*x^4 - 20*a*b^4*h*x^3 + 20*b^5*x^3*e + 30*b^5*d*x^2 - 30*a*b^
4*g*x^2 + 60*b^5*c*x - 60*a*b^4*f*x)/b^6 + 1/3*(a*b^12*d*(-a/b)^(1/3) - a^2
*b^11*g*(-a/b)^(1/3) + a*b^12*c - a^2*b^11*f)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a*b^13)
```

maple [B] time = 0.05, size = 505, normalized size = 1.61

$$\frac{h x^{6}}{6 b}+\frac{g x^{5}}{5 b}+\frac{f x^{4}}{4 b}-\frac{a h x^{3}}{3 b^{2}}+\frac{e x^{3}}{3 b}-\frac{a g x^{2}}{2 b^{2}}+\frac{d x^{2}}{2 b}+\frac{\sqrt{3}\, a^{2} f \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}+\frac{a^{2} f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}-\frac{a^{2} f \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}+\frac{\sqrt{3}\, a^{2} g \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{3}}-\frac{a^{2} g \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{3}}+\frac{a^{2} g \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{3}}+\frac{a^{2} h \ln\left(b x^{3}+a\right)}{3 b^{3}}-\frac{\sqrt{3}\, a c \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}-\frac{a c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}+\frac{a c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}-\frac{\sqrt{3}\, a d \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}+\frac{a d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}-\frac{a d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}-\frac{a e \ln\left(b x^{3}+a\right)}{3 b^{2}}-\frac{a f x}{b^{2}}+\frac{c x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/6*h*x^6/b+1/5*g*x^5/b+1/4/b*f*x^4-1/3/b^2*x^3*a*h+1/3/b*e*x^3-1/2/b^2*x^2
*a*g+1/2/b*d*x^2-a/b^2*f*x+1/b*c*x+1/3/(a/b)^(2/3)*a^2/b^3*f*ln(x+(a/b)^(1/
3))-1/3/(a/b)^(2/3)*a/b^2*c*ln(x+(a/b)^(1/3))-1/6/(a/b)^(2/3)*a^2/b^3*f*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/6/(a/b)^(2/3)*a/b^2*c*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3/(a/b)^(2/3)*3^(1/2)*a^2/b^3*f*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))-1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x-1))*c-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*g+1/3*a/b^2/(a/b)^(1/3)
*ln(x+(a/b)^(1/3))*d+1/6*a^2/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*g-1/6*a/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/3*a^2/b^3
*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*g-1/3*3^(1/2)/
(a/b)^(1/3)*a/b^2*d*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*a^2/b^3*ln(
b*x^3+a)*h-1/3*a/b^2*e*ln(b*x^3+a)
```

maxima [A] time = 2.90, size = 332, normalized size = 1.06

$$\frac{10 \, b h x^{6} + 12 \, b g x^{5} + 15 \, b f x^{4} + 20 \, {\left(b e - a h\right)} x^{3} + 30 \, {\left(b d - a g\right)} x^{2} + 60 \, {\left(b c - a f\right)} x}{60 \, b^{2}} - \frac{\sqrt{3} {\left(a b^{2} d \left(\frac{a}{b}\right)^{\frac{2}{3}} - a^{2} b g \left(\frac{a}{b}\right)^{\frac{2}{3}} + a b^{2} c \left(\frac{a}{b}\right)^{\frac{1}{3}} - a^{2} b f \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} \arctan\left(\frac{\sqrt{3} {\left(2 \, x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}}{3 \, \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 \, a b^{3}} - \frac{{\left(2 \, a b e \left(\frac{a}{b}\right)^{\frac{2}{3}} - 2 \, a^{2} h \left(\frac{a}{b}\right)^{\frac{2}{3}} + a b d \left(\frac{a}{b}\right)^{\frac{1}{3}} - a^{2} g \left(\frac{a}{b}\right)^{\frac{1}{3}} - a b c + a^{2} f\right)} \log\left(x^{2} - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \, b^{3} \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{{\left(a b e \left(\frac{a}{b}\right)^{\frac{2}{3}} - a^{2} h \left(\frac{a}{b}\right)^{\frac{2}{3}} - a b d \left(\frac{a}{b}\right)^{\frac{1}{3}} + a^{2} g \left(\frac{a}{b}\right)^{\frac{1}{3}} + a b c - a^{2} f\right)} \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \, b^{3} \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] 1/60*(10*b*h*x^6 + 12*b*g*x^5 + 15*b*f*x^4 + 20*(b*e - a*h)*x^3 + 30*(b*d -
 a*g)*x^2 + 60*(b*c - a*f)*x)/b^2 - 1/3*sqrt(3)*(a*b^2*d*(a/b)^(2/3) - a^2*
b*g*(a/b)^(2/3) + a*b^2*c*(a/b)^(1/3) - a^2*b*f*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/6*(2*a*b*e*(a/b)^(2/3) -
2*a^2*h*(a/b)^(2/3) + a*b*d*(a/b)^(1/3) - a^2*g*(a/b)^(1/3) - a*b*c + a^2*f
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 1/3*(a*b*e*(a/
b)^(2/3) - a^2*h*(a/b)^(2/3) - a*b*d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3) + a*b*
c - a^2*f)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 4.99, size = 1236, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] x^2*(d/(2*b) - (a*g)/(2*b^2)) + x^3*(e/(3*b) - (a*h)/(3*b^2)) + symsum(log(
root(27*b^9*z^3 + 27*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b^6*c*d*z - 18*a^
3*b^4*e*h*z + 9*a^3*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5*c*g*z + 9*a^4*b
^3*h^2*z + 9*a^2*b^5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*g + 3*a^4*b^2*d*
f*h + 3*a^4*b^2*c*g*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g - 3*a^3*b^3*c*d*h
 + 3*a^2*b^4*c*d*e + 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^4*b^2*d*g^2 + 3*
a^3*b^3*d^2*g + 3*a^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3*e^3 + a^5*b*g^3
 + a*b^5*c^3 - a^4*b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k)*((6*a^2*b^4*e - 6
*a^3*b^3*h)/b^4 + (x*(3*a^2*b^3*f - 3*a*b^4*c))/b^3 + 9*root(27*b^9*z^3 + 2
7*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b^6*c*d*z - 18*a^3*b^4*e*h*z + 9*a^3
*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5*c*g*z + 9*a^4*b^3*h^2*z + 9*a^2*b^
5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*g + 3*a^4*b^2*d*f*h + 3*a^4*b^2*c*g
*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g - 3*a^3*b^3*c*d*h + 3*a^2*b^4*c*d*e
+ 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^4*b^2*d*g^2 + 3*a^3*b^3*d^2*g + 3*a
^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3*e^3 + a^5*b*g^3 + a*b^5*c^3 - a^4*
b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k)*a*b^2) + (a^5*h^2 + a^3*b^2*e^2 - 2*
a^4*b*e*h + a^4*b*f*g + a^2*b^3*c*d - a^3*b^2*c*g - a^3*b^2*d*f)/b^4 + (x*(
a^4*g^2 + a^2*b^2*d^2 - a^4*f*h + a^3*b*c*h - 2*a^3*b*d*g + a^3*b*e*f - a^2
*b^2*c*e))/b^3)*root(27*b^9*z^3 + 27*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b
^6*c*d*z - 18*a^3*b^4*e*h*z + 9*a^3*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5
*c*g*z + 9*a^4*b^3*h^2*z + 9*a^2*b^5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*
g + 3*a^4*b^2*d*f*h + 3*a^4*b^2*c*g*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g -
 3*a^3*b^3*c*d*h + 3*a^2*b^4*c*d*e + 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^
4*b^2*d*g^2 + 3*a^3*b^3*d^2*g + 3*a^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3
*e^3 + a^5*b*g^3 + a*b^5*c^3 - a^4*b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k),
k, 1, 3) + x*(c/b - (a*f)/b^2) + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6
*b)
```

sympy [B] time = 73.53, size = 845, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] x**3*(-a*h/(3*b**2) + e/(3*b)) + x**2*(-a*g/(2*b**2) + d/(2*b)) + x*(-a*f/b
**2 + c/b) + RootSum(27*_t**3*b**9 + _t**2*(-27*a**2*b**6*h + 27*a*b**7*e)
+ _t*(9*a**4*b**3*h**2 - 18*a**3*b**4*e*h + 9*a**3*b**4*f*g - 9*a**2*b**5*c
*g - 9*a**2*b**5*d*f + 9*a**2*b**5*e**2 + 9*a*b**6*c*d) - a**6*h**3 + 3*a**
5*b*e*h**2 - 3*a**5*b*f*g*h + a**5*b*g**3 + 3*a**4*b**2*c*g*h + 3*a**4*b**2
*d*f*h - 3*a**4*b**2*d*g**2 - 3*a**4*b**2*e**2*h + 3*a**4*b**2*e*f*g - a**4
*b**2*f**3 - 3*a**3*b**3*c*d*h - 3*a**3*b**3*c*e*g + 3*a**3*b**3*c*f**2 + 3
*a**3*b**3*d**2*g - 3*a**3*b**3*d*e*f + a**3*b**3*e**3 - 3*a**2*b**4*c**2*f
```

```
 + 3*a**2*b**4*c*d*e - a**2*b**4*d**3 + a*b**5*c**3, Lambda(_t, _t*log(x +
(9*_t**2*a*b**6*g - 9*_t**2*b**7*d - 6*_t*a**3*b**3*g*h + 6*_t*a**2*b**4*d*
h + 6*_t*a**2*b**4*e*g + 3*_t*a**2*b**4*f**2 - 6*_t*a*b**5*c*f - 6*_t*a*b**
5*d*e + 3*_t*b**6*c**2 + a**5*g*h**2 - a**4*b*d*h**2 - 2*a**4*b*e*g*h - a**
4*b*f**2*h + 2*a**4*b*f*g**2 + 2*a**3*b**2*c*f*h - 2*a**3*b**2*c*g**2 + 2*a
**3*b**2*d*e*h - 4*a**3*b**2*d*f*g + a**3*b**2*e**2*g + a**3*b**2*e*f**2 -
a**2*b**3*c**2*h + 4*a**2*b**3*c*d*g - 2*a**2*b**3*c*e*f + 2*a**2*b**3*d**2
*f - a**2*b**3*d*e**2 + a*b**4*c**2*e - 2*a*b**4*c*d**2)/(a**4*b*g**3 - 3*a
**3*b**2*d*g**2 + a**3*b**2*f**3 - 3*a**2*b**3*c*f**2 + 3*a**2*b**3*d**2*g
+ 3*a*b**4*c**2*f - a*b**4*d**3 - b**5*c**3)))) + f*x**4/(4*b) + g*x**5/(5*
b) + h*x**6/(6*b)
```

3.352 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$

Optimal. Leaf size=294

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a$$

Rubi [A] time = 0.98, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}b^{8/3}}+\frac{(bc-af)\log\left(a+bx^3\right)}{3b^2}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3b^{8/3}}+\frac{x(bd-ag)}{b^2}+\frac{x^2(be-ah)}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*d - a*g)*x)/b^2 + ((b*e - a*h)*x^2)/(2*b^2) + (f*x^3)/(3*b) + (g*x^4)/(
4*b) + (h*x^5)/(5*b) + (a^(1/3)*(b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3
)) - (a^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(
1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3)) + ((b*c - a*f)
*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx = \frac{hx^5}{5b} + \frac{\int \frac{x^2\left(5bc+5bdx+5(be-ah)x^2+5bfx^3+5bgx^4\right)}{a+bx^3}dx}{5b}$$

$$= \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \frac{x^2\left(20b^2c+20b(bd-ag)x+20b(be-ah)x^2+20b^2fx^3\right)}{a+bx^3}dx}{20b^2}$$

$$= \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \frac{x^2\left(60b^2(bc-af)+60b^2(bd-ag)x+60b^2(be-ah)x^2\right)}{a+bx^3}dx}{60b^3}$$

$$= \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \Bigl(60b(bd-ag)+60b(be-ah)x - \frac{60\left(ab(bd-a \cdots\right.}{\cdots}}{60b^3}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\int \frac{ab(bd-ag)+ab(be- \cdots}{a+ \cdots}}{\cdots}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\int \frac{ab(bd-ag)+ab(be- \cdots}{a+bx^3}}{b^3}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{(bc-af)\log\left(a \cdots\right.}{3b^2}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-a \cdots\right.}{\cdots}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-a \cdots\right.}{\cdots}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\sqrt[3]{a}\left(b^{4/3}d+\sqrt[3]{a} \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.31, size = 290, normalized size = 0.99

$$\frac{10\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)+20\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be+a\sqrt[3]{b}g-b^{4/3}d\right)-20\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}h-\sqrt[3]{a}be+a\sqrt[3]{b}g-b^{4/3}d\right)+20b^{2/3}(bc-af)\log\left(a+bx^3\right)+60b^{2/3}x(bd-ag)+30b^{2/3}x^2(be-ah)+20b^{5/3}fx^3+15b^{5/3}gx^4+12b^{5/3}hx^5}{60b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (60*b^(2/3)*(b*d - a*g)*x + 30*b^(2/3)*(b*e - a*h)*x^2 + 20*b^(5/3)*f*x^3 +
 15*b^(5/3)*g*x^4 + 12*b^(5/3)*h*x^5 - 20*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a
^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] + 20*a^(1/3)*(-(b^(4/3)*d) + a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*
Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*
g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(2/3)*
(b*c - a*f)*Log[a + b*x^3])/(60*b^(8/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [A] time = 0.24, size = 333, normalized size = 1.13

$$\frac{(bc-af)\log\left(|bx^3+a|\right)}{3\,b^2}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d-\left(-ab^2\right)^{\frac{1}{3}}abg+\left(-ab^2\right)^{\frac{2}{3}}ah-\left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d-\left(-ab^2\right)^{\frac{1}{3}}abg-\left(-ab^2\right)^{\frac{2}{3}}ah+\left(-ab^2\right)^{\frac{2}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4}+\frac{12\,b^4hx^5+15\,b^4gx^4+20\,b^4fx^3-30\,ab^3hx^2+30\,b^4x^2e+60\,b^4dx-60\,ab^3gx}{60\,b^5}-\frac{\left(a^2b^9h\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{10}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-ab^{10}d+a^2b^9g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] 1/3*(b*c - a*f)*log(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*d
 - (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1
/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*b^2
*d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(12*b^4*h*x^5 + 15*b^4*g*x^4
+ 20*b^4*f*x^3 - 30*a*b^3*h*x^2 + 30*b^4*x^2*e + 60*b^4*d*x - 60*a*b^3*g*x)
/b^5 - 1/3*(a^2*b^9*h*(-a/b)^(1/3) - a*b^10*(-a/b)^(1/3)*e - a*b^10*d + a^2
*b^9*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^11)
```

maple [B] time = 0.05, size = 483, normalized size = 1.64

$$\frac{hx^5}{5b}+\frac{gx^4}{4b}+\frac{fx^3}{3b}-\frac{ahx^2}{2b^2}+\frac{ex^2}{2b}+\frac{\sqrt{3}\,a^2g\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{a^2g\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{a^2g\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{\sqrt{3}\,a^2h\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{a^2h\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{a^2h\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{ad\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{ad\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{\sqrt{3}\,ae\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{ae\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{ae\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{af\ln\left(bx^3+a\right)}{3b^2}-\frac{agx}{b^2}+\frac{c\ln\left(bx^3+a\right)}{3b}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/5*h*x^5/b+1/4*g*x^4/b+1/3/b*f*x^3-1/2/b^2*x^2*a*h+1/2/b*e*x^2-1/b^2*a*g*x
+1/b*d*x+1/3/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^2*g-1/3/b^2/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))*a*d-1/6/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a
^2*g+1/6/(a/b)^(2/3)*a/b^2*d*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b^3/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*g-1/3/(a/b)^(2/
3)*3^(1/2)*a/b^2*d*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/b^3/(a/b)^(1
/3)*ln(x+(a/b)^(1/3))*a^2*h+1/3/b^2*a*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b
^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*h-1/6/b^2*a*e/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*h-1/3/b^2*a*e*3^(1/2)/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/b^2*ln(b*x^3+a)*a*f+1/3/b*c*ln(b*x^3+
a)
```

maxima [A] time = 3.00, size = 313, normalized size = 1.06

$$-\frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}+\frac{12\,bhx^5+15\,bgx^4+20\,bfx^3+30\,(be-ah)x^2+60\,(bd-ag)x}{60\,b^2}+\frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+abd-a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd+a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] -1/3*sqrt(3)*(a*b*e*(a/b)^(2/3) - a^2*h*(a/b)^(2/3) + a*b*d*(a/b)^(1/3) - a
^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^
2) + 1/60*(12*b*h*x^5 + 15*b*g*x^4 + 20*b*f*x^3 + 30*(b*e - a*h)*x^2 + 60*(
b*d - a*g)*x)/b^2 + 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) - a*b*e*
(a/b)^(1/3) + a^2*h*(a/b)^(1/3) + a*b*d - a^2*g)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3)
 + a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - a*b*d + a^2*g)*log(x + (a/b)^(1/
3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 5.02, size = 1170, normalized size = 3.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] x^2*(e/(2*b) - (a*h)/(2*b^2)) + symsum(log(root(27*b^8*z^3 + 27*a*b^6*f*z^2
 - 27*b^7*c*z^2 - 18*a*b^5*c*f*z + 9*a*b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*
b^4*e*g*z - 9*a^2*b^4*d*h*z + 9*a^2*b^4*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*h
 - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*
a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4
*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*
f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3
, z, k)*((6*a^2*b^3*f - 6*a*b^4*c)/b^3 + (x*(3*a^2*b^3*g - 3*a*b^4*d))/b^3
+ 9*root(27*b^8*z^3 + 27*a*b^6*f*z^2 - 27*b^7*c*z^2 - 18*a*b^5*c*f*z + 9*a*
b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*b^4*e*g*z - 9*a^2*b^4*d*h*z + 9*a^2*b^4
*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*
a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2
*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*
g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3
 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k)*a*b^2) + (a*b^3*c^2 + a^3*b*f^2
 + a^4*g*h - a^3*b*d*h - a^3*b*e*g - 2*a^2*b^2*c*f + a^2*b^2*d*e)/b^3 + (x*
(a^4*h^2 + a^2*b^2*e^2 + a*b^3*c*d - 2*a^3*b*e*h + a^3*b*f*g - a^2*b^2*c*g
- a^2*b^2*d*f))/b^3)*root(27*b^8*z^3 + 27*a*b^6*f*z^2 - 27*b^7*c*z^2 - 18*a
*b^5*c*f*z + 9*a*b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*b^4*e*g*z - 9*a^2*b^4*
d*h*z + 9*a^2*b^4*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a
^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*
b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2
*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*
b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k), k, 1, 3) + x*
(d/b - (a*g)/b^2) + (f*x^3)/(3*b) + (g*x^4)/(4*b) + (h*x^5)/(5*b)
```

sympy [B] time = 88.70, size = 790, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] x**2*(-a*h/(2*b**2) + e/(2*b)) + x*(-a*g/b**2 + d/b) + RootSum(27*_t**3*b**
8 + _t**2*(27*a*b**6*f - 27*b**7*c) + _t*(9*a**3*b**3*g*h - 9*a**2*b**4*d*h
 - 9*a**2*b**4*e*g + 9*a**2*b**4*f**2 - 18*a*b**5*c*f + 9*a*b**5*d*e + 9*b*
*6*c**2) + a**5*h**3 - 3*a**4*b*e*h**2 + 3*a**4*b*f*g*h - a**4*b*g**3 - 3*a
**3*b**2*c*g*h - 3*a**3*b**2*d*f*h + 3*a**3*b**2*d*g**2 + 3*a**3*b**2*e**2*
h - 3*a**3*b**2*e*f*g + a**3*b**2*f**3 + 3*a**2*b**3*c*d*h + 3*a**2*b**3*c*
e*g - 3*a**2*b**3*c*f**2 - 3*a**2*b**3*d**2*g + 3*a**2*b**3*d*e*f - a**2*b*
*3*e**3 + 3*a*b**4*c**2*f - 3*a*b**4*c*d*e + a*b**4*d**3 - b**5*c**3, Lambd
a(_t, _t*log(x + (9*_t**2*a*b**5*h - 9*_t**2*b**6*e + 6*_t*a**2*b**3*f*h +
3*_t*a**2*b**3*g**2 - 6*_t*a*b**4*c*h - 6*_t*a*b**4*d*g - 6*_t*a*b**4*e*f +
```

```
 6*_t*b**5*c*e + 3*_t*b**5*d**2 + 2*a**4*g*h**2 - 2*a**3*b*d*h**2 - 4*a**3*
b*e*g*h + a**3*b*f**2*h + a**3*b*f*g**2 - 2*a**2*b**2*c*f*h - a**2*b**2*c*g
**2 + 4*a**2*b**2*d*e*h - 2*a**2*b**2*d*f*g + 2*a**2*b**2*e**2*g - a**2*b**
2*e*f**2 + a*b**3*c**2*h + 2*a*b**3*c*d*g + 2*a*b**3*c*e*f + a*b**3*d**2*f
- 2*a*b**3*d*e**2 - b**4*c**2*e - b**4*c*d**2)/(a**4*h**3 - 3*a**3*b*e*h**2
 + a**3*b*g**3 - 3*a**2*b**2*d*g**2 + 3*a**2*b**2*e**2*h + 3*a*b**3*d**2*g
- a*b**3*e**3 - b**4*d**3)))) + f*x**3/(3*b) + g*x**4/(4*b) + h*x**5/(5*b)
```

3.353 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=275

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6\sqrt[3]{a}\,b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3\sqrt[3]{a}\,b^{7/3}}-$$

Rubi [A] time = 0.92, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6\sqrt[3]{a}\,b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3\sqrt[3]{a}\,b^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}\sqrt[3]{a}\,b^{7/3}}+\frac{(bd-ag)\log\left(a+bx^3\right)}{3b^2}+\frac{x(be-ah)}{b^2}+\frac{fx^2}{2b}+\frac{gx^3}{3b}+\frac{hx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*e - a*h)*x)/b^2 + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*b) - ((b^(
5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(7/3)) - ((b^(2/3)*(b*c - a*f) +
 a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(7/3)) + ((b^(
2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*a^(1/3)*b^(7/3)) + ((b*d - a*g)*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx = \frac{hx^4}{4b} + \frac{\int \frac{x\left(4bc+4bdx+4(be-ah)x^2+4bfx^3+4bgx^4\right)}{a+bx^3}dx}{4b}$$

$$= \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \frac{x\left(12b^2c+12b(bd-ag)x+12b(be-ah)x^2+12b^2fx^3\right)}{a+bx^3}dx}{12b^2}$$

$$= \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \frac{x\left(24b^2(bc-af)+24b^2(bd-ag)x+24b^2(be-ah)x^2\right)}{a+bx^3}dx}{24b^3}$$

$$= \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \Big(24b(be-ah) - \frac{24\left(ab(be-ah)-b^2(bc-af)x-b^2(bd-\cdots\right.}{a+bx^3}}{24b^3}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\int \frac{ab(be-ah)-b^2(bc-af)x-b^2(bd-ag)x^2}{a+bx^3} d\cdots}{b^3}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\int \frac{ab(be-ah)-b^2(bc-af)x}{a+bx^3}dx}{b^3} + \frac{(bd-\cdots}{\cdots}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{(bd-ag)\log\left(a+bx^3\right)}{3b^2} - \frac{\int \frac{\sqrt[3]{a}\cdots}{\cdots}}{\cdots}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\mathrm{l}\cdots}{3\sqrt[3]{a}\,b^{7/3}}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\mathrm{l}\cdots}{3\sqrt[3]{a}\,b^{7/3}}$$

$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\cdots}{\sqrt{3}\,\sqrt[3]{a}\,b^{7/3}}$$

Mathematica [A] time = 0.48, size = 272, normalized size = 0.99

$$\frac{\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+b^{5/3}c\right)}{\sqrt[3]{a}} + \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-a^{2/3}be+a^{5/3}h+ab^{2/3}f-b^{5/3}c\right)}{\sqrt[3]{a}} - \frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt[3]{a}} + 4\sqrt[3]{b}(bd-ag)\log\left(a+bx^3\right) + 12\sqrt[3]{b}x(be-ah) + 6b^{4/3}fx^2 + 4b^{4/3}gx^3 + 3b^{4/3}hx^4}{12b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (12*b^(1/3)*(b*e - a*h)*x + 6*b^(4/3)*f*x^2 + 4*b^(4/3)*g*x^3 + 3*b^(4/3)*h
*x^4 - (4*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (4*(-(b^(5/3)*c) - a^(2/3
)*b*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (2*(b^
(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(1/3) + 4*b^(1/3)*(b*d - a*g)*Log[a + b*x^3])/(12*b
^(7/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3
),x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3
), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [A] time = 0.28, size = 295, normalized size = 1.07

$$-\frac{\sqrt{3}\left(a^{2}h-abe-\left(-ab^{2}\right)^{\frac{1}{3}}bc+\left(-ab^{2}\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}b}-\frac{\left(a^{2}h-abe+\left(-ab^{2}\right)^{\frac{1}{3}}bc-\left(-ab^{2}\right)^{\frac{1}{3}}af\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}b}+\frac{(bd-ag)\log\left(|bx^{3}+a|\right)}{3b^{2}}+\frac{3b^{3}hx^{4}+4b^{3}gx^{3}+6b^{3}fx^{2}-12ab^{2}hx+12b^{3}xe}{12b^{4}}-\frac{\left(b^{9}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{8}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}b^{7}h-ab^{8}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(a^2*h - a*b*e - (-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*
(a^2*h - a*b*e + (-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) + 1/3*(b*d - a*g)*log(abs(b*x^3
+ a))/b^2 + 1/12*(3*b^3*h*x^4 + 4*b^3*g*x^3 + 6*b^3*f*x^2 - 12*a*b^2*h*x +
12*b^3*x*e)/b^4 - 1/3*(b^9*c*(-a/b)^(1/3) - a*b^8*f*(-a/b)^(1/3) + a^2*b^7*
h - a*b^8*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^9)
```

maple [B] time = 0.04, size = 455, normalized size = 1.65

$$\frac{hx^{4}}{4b}+\frac{gx^{3}}{3b}+\frac{fx^{2}}{2b}+\frac{\sqrt{3}\,a^{2}h\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{a^{2}h\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{a^{2}h\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{\sqrt{3}\,ae\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{ae\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{ae\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{\sqrt{3}\,af\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{af\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{af\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{ag\ln\left(bx^{3}+a\right)}{3b^{2}}-\frac{ahx}{b^{2}}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{d\ln\left(bx^{3}+a\right)}{3b}+\frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/4*h*x^4/b+1/3*g*x^3/b+1/2/b*f*x^2-1/b^2*a*h*x+1/b*e*x+1/3/b^3/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))*a^2*h-1/3/(a/b)^(2/3)*a/b^2*e*ln(x+(a/b)^(1/3))-1/6/b^3/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*h+1/6/(a/b)^(2/3)*a/b^2*e
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*h-1/3/(a/b)^(2/3)*3^(1/2)*a/b^2*e*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/(a/b)^(1/3)*a/b^2*f*ln(x+(a/b)^(1/3))-1/
3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c-1/6/(a/b)^(1/3)*a/b^2*f*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c-1/
3*3^(1/2)/(a/b)^(1/3)*a/b^2*f*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b
*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/3/b^2*ln(b
*x^3+a)*a*g+1/3/b*d*ln(b*x^3+a)
```

maxima [A] time = 3.03, size = 300, normalized size = 1.09

$$\frac{\sqrt{3}\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^{2}}+\frac{3bhx^{4}+4bgx^{3}+6bfx^{2}+12(be-ah)x}{12b^{2}}+\frac{\left(2b^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+abe-a^{2}h\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-abe+a^{2}h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(3)*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3) - a*b*e*(a/b)^(1/3) + a^
2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2
) + 1/12*(3*b*h*x^4 + 4*b*g*x^3 + 6*b*f*x^2 + 12*(b*e - a*h)*x)/b^2 + 1/6*(
2*b^2*d*(a/b)^(2/3) - 2*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)
^(1/3) + a*b*e - a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(
2/3)) + 1/3*(b^2*d*(a/b)^(2/3) - a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) + a*
b*f*(a/b)^(1/3) - a*b*e + a^2*h)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 4.99, size = 1161, normalized size = 4.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

```
[Out] symsum(log(root(27*a*b^7*z^3 - 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5*
c*e*z - 9*a^3*b^3*f*h*z - 18*a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*c*
h*z + 9*a*b^5*d^2*z + 9*a^3*b^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a
^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*
b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2
*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^
4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*((6*a^2*b^2*g
- 6*a*b^3*d)/b^2 + (x*(3*a^2*b^2*h - 3*a*b^3*e))/b^2 + 9*root(27*a*b^7*z^3
- 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5*c*e*z - 9*a^3*b^3*f*h*z - 18*
a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*c*h*z + 9*a*b^5*d^2*z + 9*a^3*b
^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*
h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h +
 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*
b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f
^3 - a*b^4*d^3 - a^5*h^3, z, k)*a*b^2) + (a^3*g^2 + a*b^2*d^2 - a^3*f*h - a
*b^2*c*e + a^2*b*c*h - 2*a^2*b*d*g + a^2*b*e*f)/b^2 + (x*(b^3*c^2 + a^2*b*f
^2 + a^3*g*h - 2*a*b^2*c*f + a*b^2*d*e - a^2*b*d*h - a^2*b*e*g))/b^2)*root(
27*a*b^7*z^3 - 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5*c*e*z - 9*a^3*b^
3*f*h*z - 18*a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*c*h*z + 9*a*b^5*d^
2*z + 9*a^3*b^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3
*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^
2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d
*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^
3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k), k, 1, 3) + x*(e/b - (a*h)/b^2
) + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*b)
```

sympy [B] time = 63.00, size = 811, normalized size = 2.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] x*(-a*h/b**2 + e/b) + RootSum(27*_t**3*a*b**7 + _t**2*(27*a**2*b**5*g - 27*
a*b**6*d) + _t*(-9*a**3*b**3*f*h + 9*a**3*b**3*g**2 + 9*a**2*b**4*c*h - 18*
a**2*b**4*d*g + 9*a**2*b**4*e*f - 9*a*b**5*c*e + 9*a*b**5*d**2) - a**5*h**3
 + 3*a**4*b*e*h**2 - 3*a**4*b*f*g*h + a**4*b*g**3 + 3*a**3*b**2*c*g*h + 3*a
**3*b**2*d*f*h - 3*a**3*b**2*d*g**2 - 3*a**3*b**2*e**2*h + 3*a**3*b**2*e*f*
g - a**3*b**2*f**3 - 3*a**2*b**3*c*d*h - 3*a**2*b**3*c*e*g + 3*a**2*b**3*c*
f**2 + 3*a**2*b**3*d**2*g - 3*a**2*b**3*d*e*f + a**2*b**3*e**3 - 3*a*b**4*c
**2*f + 3*a*b**4*c*d*e - a*b**4*d**3 + b**5*c**3, Lambda(_t, _t*log(x + (-9
*_t**2*a**2*b**5*f + 9*_t**2*a*b**6*c + 3*_t*a**4*b**2*h**2 - 6*_t*a**3*b**
3*e*h - 6*_t*a**3*b**3*f*g + 6*_t*a**2*b**4*c*g + 6*_t*a**2*b**4*d*f + 3*_t
*a**2*b**4*e**2 - 6*_t*a*b**5*c*d + a**5*g*h**2 - a**4*b*d*h**2 - 2*a**4*b*
```

```
e*g*h + 2*a**4*b*f**2*h - a**4*b*f*g**2 - 4*a**3*b**2*c*f*h + a**3*b**2*c*g
**2 + 2*a**3*b**2*d*e*h + 2*a**3*b**2*d*f*g + a**3*b**2*e**2*g - 2*a**3*b**
2*e*f**2 + 2*a**2*b**3*c**2*h - 2*a**2*b**3*c*d*g + 4*a**2*b**3*c*e*f - a**
2*b**3*d**2*f - a**2*b**3*d*e**2 - 2*a*b**4*c**2*e + a*b**4*c*d**2)/(a**5*h
**3 - 3*a**4*b*e*h**2 + 3*a**3*b**2*e**2*h - a**3*b**2*f**3 + 3*a**2*b**3*c
*f**2 - a**2*b**3*e**3 - 3*a*b**4*c**2*f + b**5*c**3)))) + f*x**2/(2*b) + g
*x**3/(3*b) + h*x**4/(4*b)
```

3.354 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3} dx$

Optimal. Leaf size=259

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6a^{2/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{2/3}b^{5/3}}$$

Rubi [A] time = 0.37, antiderivative size = 257, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{2/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}a^{2/3}b^{5/3}}+\frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}+\frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] (f*x)/b + (g*x^2)/(2*b) + (h*x^3)/(3*b) - ((b^(4/3)*c + a^(1/3)*b*d - a*b^(
1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqr
t[3]*a^(2/3)*b^(5/3)) + ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^
(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*c - a*f - (a^(1/3)*(b*d - a*g
))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4
/3)) + ((b*e - a*h)*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx &= \int \left(\frac{f}{b}+\frac{gx}{b}+\frac{hx^2}{b}+\frac{bc-af+(bd-ag)x+(be-ah)x^2}{b\left(a+bx^3\right)}\right)dx \\
&= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\int \frac{bc-af+(bd-ag)x+(be-ah)x^2}{a+bx^3}\,dx}{b} \\
&= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\int \frac{bc-af+(bd-ag)x}{a+bx^3}\,dx}{b}+\frac{(be-ah)\int \frac{x^2}{a+bx^3}\,dx}{b} \\
&= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(bc-af)+\sqrt[3]{a}(bd-ag)\right)\cdots}{a^{2/3}-\sqrt[3]{a}\cdots}}{3a^2\cdots} \\
&= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}+\frac{(be-ah)\,\mathrm{l}\cdots}{3\cdots} \\
&= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{\left(bc-af-\cdots\right.}{\cdots} \\
&= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}-\frac{\left(b^{4/3}c+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{5/3}}+\cdots
\end{aligned}
$$

Mathematica [A] time = 0.39, size = 254, normalized size = 0.98

$$
\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}g-\sqrt[3]{a}\,bd+a\sqrt[3]{b}\,f-b^{4/3}c\right)}{a^{2/3}}+\frac{2(be-ah)\log\left(a+bx^3\right)}{\sqrt[3]{b}}+6b^{2/3}fx+3b^{2/3}gx^2+2b^{2/3}hx^3}{6b^{5/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*f*x + 3*b^(2/3)*g*x^2 + 2*b^(2/3)*h*x^3 + (2*Sqrt[3]*(-(b^(4/3)*
c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(2/3) + (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)
*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - ((b^(4/3)*c - a^(1/3)*b*d - a*b^(1/
3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) +
 (2*(b*e - a*h)*Log[a + b*x^3])/b^(1/3))/(6*b^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{a + bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3), x
]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 272, normalized size = 1.05

$$-\frac{\sqrt{3} \left(b^2 c - abf - \left(-ab^2\right)^{\frac{1}{3}} bd + \left(-ab^2\right)^{\frac{1}{3}} ag\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}} b} - \frac{\left(b^2 c - abf + \left(-ab^2\right)^{\frac{1}{3}} bd - \left(-ab^2\right)^{\frac{1}{3}} ag\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}} b} - \frac{(ah - be)\log\left(\left|bx^3 + a\right|\right)}{3b^2} + \frac{2b^2hx^3 + 3b^2gx^2 + 6b^2fx}{6b^3} - \frac{\left(b^7 d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^6 g\left(-\frac{a}{b}\right)^{\frac{1}{3}} + b^7 c - ab^6 f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c - a*b*f - (-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*
(b^2*c - a*b*f + (-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*(a*h - b*e)*log(abs(b*x^3
+ a))/b^2 + 1/6*(2*b^2*h*x^3 + 3*b^2*g*x^2 + 6*b^2*f*x)/b^3 - 1/3*(b^7*d*(-
a/b)^(1/3) - a*b^6*g*(-a/b)^(1/3) + b^7*c - a*b^6*f)*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b^7)
```

maple [B] time = 0.05, size = 429, normalized size = 1.66

$$\frac{hx^3}{3b} + \frac{gx^2}{2b} - \frac{\sqrt{3}\, af \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} - \frac{af \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} + \frac{af \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} - \frac{\sqrt{3}\, ag \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b^2} + \frac{ag \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b^2} - \frac{ag \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b^2} - \frac{ah \ln\left(bx^3 + a\right)}{3b^2} + \frac{\sqrt{3}\, c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} + \frac{c \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{c \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b} + \frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b} - \frac{d \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b} + \frac{d \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b} + \frac{e \ln\left(bx^3 + a\right)}{3b} + \frac{fx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*h*x^3/b+1/2*g*x^2/b+1/b*f*x-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*f+1
/3/(a/b)^(2/3)/b*c*ln(x+(a/b)^(1/3))+1/6/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))*a*f-1/6/(a/b)^(2/3)/b*c*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1
/3/(a/b)^(2/3)*3^(1/2)*a/b^2*f*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/
(a/b)^(2/3)*3^(1/2)/b*c*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b^2/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))*a*g-1/3/(a/b)^(1/3)/b*d*ln(x+(a/b)^(1/3))-1/6/b^
2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*g+1/6/(a/b)^(1/3)/b*d*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/
```

```
2)*(2/(a/b)^(1/3)*x-1))*a*g+1/3*3^(1/2)/(a/b)^(1/3)/b*d*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))-1/3/b^2*ln(b*x^3+a)*a*h+1/3/b*e*ln(b*x^3+a)
```

maxima [A] time = 3.04, size = 266, normalized size = 1.03

$$\frac{2\,hx^3+3\,gx^2+6\,fx}{6\,b}+\frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}+\frac{\left(2\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc+af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(be\left(\frac{a}{b}\right)^{\frac{2}{3}}-ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc-af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*h*x^3 + 3*g*x^2 + 6*f*x)/b + 1/3*sqrt(3)*(b^2*d*(a/b)^(2/3) - a*b*g*
(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) + 1/6*(2*b*e*(a/b)^(2/3) - 2*a*h*(a/b
)^(2/3) + b*d*(a/b)^(1/3) - a*g*(a/b)^(1/3) - b*c + a*f)*log(x^2 - x*(a/b)^
(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*e*(a/b)^(2/3) - a*h*(a/b)^(
2/3) - b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) + b*c - a*f)*log(x + (a/b)^(1/3))/
(b^2*(a/b)^(2/3))
```

mupad [B] time = 5.03, size = 1150, normalized size = 4.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x)
```

```
[Out] symsum(log((a^3*h^2 + a*b^2*e^2 + b^3*c*d - a*b^2*c*g - a*b^2*d*f - 2*a^2*b
*e*h + a^2*b*f*g)/b^2 + root(27*a^2*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5
*e*z^2 + 9*a*b^5*c*d*z - 18*a^3*b^3*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f
*z - 9*a^2*b^4*c*g*z + 9*a^4*b^2*h^2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h -
3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2
*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^
2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2
 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z
, k)*((6*a^2*b^2*h - 6*a*b^3*e)/b^2 + (x*(3*b^3*c - 3*a*b^2*f))/b + 9*root(
27*a^2*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5*e*z^2 + 9*a*b^5*c*d*z - 18*a
^3*b^3*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f*z - 9*a^2*b^4*c*g*z + 9*a^4*
b^2*h^2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f
*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g
+ 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3
*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 +
a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k)*a*b^2) + (x*(b^2*d^2 + a
^2*g^2 - b^2*c*e - a^2*f*h + a*b*c*h - 2*a*b*d*g + a*b*e*f))/b)*root(27*a^2
*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5*e*z^2 + 9*a*b^5*c*d*z - 18*a^3*b^3
*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f*z - 9*a^2*b^4*c*g*z + 9*a^4*b^2*h^
2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3
*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^
2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d
*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^
3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k), k, 1, 3) + (g*x^2)/(2*b) + (h
*x^3)/(3*b) + (f*x)/b
```

sympy [B] time = 59.39, size = 804, normalized size = 3.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**6 + _t**2*(27*a**3*b**4*h - 27*a**2*b**5*e) + _t*(
9*a**4*b**2*h**2 - 18*a**3*b**3*e*h + 9*a**3*b**3*f*g - 9*a**2*b**4*c*g - 9
*a**2*b**4*d*f + 9*a**2*b**4*e**2 + 9*a*b**5*c*d) + a**5*h**3 - 3*a**4*b*e*
h**2 + 3*a**4*b*f*g*h - a**4*b*g**3 - 3*a**3*b**2*c*g*h - 3*a**3*b**2*d*f*h
 + 3*a**3*b**2*d*g**2 + 3*a**3*b**2*e**2*h - 3*a**3*b**2*e*f*g + a**3*b**2*
f**3 + 3*a**2*b**3*c*d*h + 3*a**2*b**3*c*e*g - 3*a**2*b**3*c*f**2 - 3*a**2*
b**3*d**2*g + 3*a**2*b**3*d*e*f - a**2*b**3*e**3 + 3*a*b**4*c**2*f - 3*a*b*
*4*c*d*e + a*b**4*d**3 - b**5*c**3, Lambda(_t, _t*log(x + (9*_t**2*a**3*b**
4*g - 9*_t**2*a**2*b**5*d + 6*_t*a**4*b**2*g*h - 6*_t*a**3*b**3*d*h - 6*_t*
a**3*b**3*e*g - 3*_t*a**3*b**3*f**2 + 6*_t*a**2*b**4*c*f + 6*_t*a**2*b**4*d
*e - 3*_t*a*b**5*c**2 + a**5*g*h**2 - a**4*b*d*h**2 - 2*a**4*b*e*g*h - a**4
*b*f**2*h + 2*a**4*b*f*g**2 + 2*a**3*b**2*c*f*h - 2*a**3*b**2*c*g**2 + 2*a*
*3*b**2*d*e*h - 4*a**3*b**2*d*f*g + a**3*b**2*e**2*g + a**3*b**2*e*f**2 - a
**2*b**3*c**2*h + 4*a**2*b**3*c*d*g - 2*a**2*b**3*c*e*f + 2*a**2*b**3*d**2*
f - a**2*b**3*d*e**2 + a*b**4*c**2*e - 2*a*b**4*c*d**2)/(a**4*b*g**3 - 3*a*
*3*b**2*d*g**2 + a**3*b**2*f**3 - 3*a**2*b**3*c*f**2 + 3*a**2*b**3*d**2*g +
 3*a*b**4*c**2*f - a*b**4*d**3 - b**5*c**3)))) + f*x/b + g*x**2/(2*b) + h*x
**3/(3*b)
```

3.355 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx$

Optimal. Leaf size=258

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6a^{2/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{2/3}b^{5/3}}-$$

Rubi [A] time = 0.47, antiderivative size = 256, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{2/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{2/3}b^{5/3}}-\frac{(bc-af)\log\left(a+bx^3\right)}{3ab}+\frac{c\log(x)}{a}+\frac{gx}{b}+\frac{hx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) - ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^(4/3)
*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(
5/3)) + (c*Log[x])/a + ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(
1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*d - a*g - (a^(1/3)*(b*e - a*h)
)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/
3)) - ((b*c - a*f)*Log[a + b*x^3])/(3*a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)}\,dx = \int \left(\frac{g}{b}+\frac{c}{ax}+\frac{hx}{b}+\frac{a(bd-ag)+a(be-ah)x-b(bc-af)x^2}{ab\left(a+bx^3\right)}\right)dx$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\int \frac{a(bd-ag)+a(be-ah)x-b(bc-af)x^2}{a+bx^3}\,dx}{ab}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\int \frac{a(bd-ag)+a(be-ah)x}{a+bx^3}\,dx}{ab}-\frac{(bc-af)\int \frac{x^2}{a+bx^3}\,dx}{a}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}-\frac{(bc-af)\log\left(a+bx^3\right)}{3ab}+\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}(bd-ag)+a^{4/\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc\cdots}{\cdots}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{\left(bd\cdots\right.}{\cdots}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}-\frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}}+\frac{c\log\cdots}{a}$$

Mathematica [A] time = 0.31, size = 258, normalized size = 1.00

$$\frac{-\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)+2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)+2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}h-\sqrt[3]{a}be+a\sqrt[3]{b}g-b^{4/3}d\right)-2b^{2/3}(bc-af)\log\left(a+bx^3\right)+6ab^{2/3}gx+3ab^{2/3}hx^2+6b^{5/3}c\log(x)}{6ab^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

```
[Out] (6*a*b^(2/3)*g*x + 3*a*b^(2/3)*h*x^2 + 2*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a^
(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + 6*b^(5/3)*c*Log[x] + 2*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)
*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e
 - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
- 2*b^(2/3)*(b*c - a*f)*Log[a + b*x^3])/(6*a*b^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)
),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)
), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 281, normalized size = 1.09

$$\frac{c\log\left(|x|\right)}{a}-\frac{\sqrt{3}\left(b^2d-abg+\left(-ab^2\right)^{\frac{1}{3}}ah-\left(-ab^2\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b}-\frac{\left(b^2d-abg-\left(-ab^2\right)^{\frac{1}{3}}ah+\left(-ab^2\right)^{\frac{1}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b}-\frac{\left(bc-af\right)\log\left(|bx^3+a|\right)}{3ab}+\frac{bhx^2+2bgx}{2b^2}+\frac{\left(a^3b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-a^2b^3d+a^3b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a - 1/3*sqrt(3)*(b^2*d - a*b*g + (-a*b^2)^(1/3)*a*h - (-a*b^2
)^(1/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2
)^(2/3)*b) - 1/6*(b^2*d - a*b*g - (-a*b^2)^(1/3)*a*h + (-a*b^2)^(1/3)*b*e)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*(b*c - a*
f)*log(abs(b*x^3 + a))/(a*b) + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 + 1/3*(a^3*b^2*h
*(-a/b)^(1/3) - a^2*b^3*(-a/b)^(1/3)*e - a^2*b^3*d + a^3*b^2*g)*(-a/b)^(1/3
)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

maple [B] time = 0.05, size = 426, normalized size = 1.65

$$\frac{hx^2}{2b}-\frac{\sqrt{3}\,ag\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{ag\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{ag\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{\sqrt{3}\,ah\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{ah\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{ah\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{c\ln\left(x\right)}{a}-\frac{c\ln\left(bx^3+a\right)}{3a}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{f\ln\left(bx^3+a\right)}{3b}+\frac{gx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x)
```

```
[Out] 1/2/b*h*x^2+1/b*g*x-1/3/b^2*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*g+1/3/(a/b)^(2/
3)/b*d*ln(x+(a/b)^(1/3))+1/6/b^2*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))*g-1/6/(a/b)^(2/3)/b*d*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/b^2*a/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*g+1/3/(a/b)^(2/3)*
3^(1/2)/b*d*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b^2*a/(a/b)^(1/3)*l
```

```
n(x+(a/b)^(1/3))*h-1/3/(a/b)^(1/3)/b*e*ln(x+(a/b)^(1/3))-1/6/b^2*a/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*h+1/6/(a/b)^(1/3)/b*e*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))-1/3/b^2*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*h+1/3*3^(1/2)/(a/b)^(1/3)/b*e*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))+1/3/b*f*ln(b*x^3+a)-1/3/a*c*ln(b*x^3+a)+1/a*c*ln(x)
```

maxima [A] time = 3.02, size = 290, normalized size = 1.12

$$\frac{c\log(x)}{a}+\frac{hx^2+2gx}{2b}+\frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b}-\frac{\left(2b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+abd-a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd+a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima"
)
```

```
[Out] c*log(x)/a + 1/2*(h*x^2 + 2*g*x)/b + 1/3*sqrt(3)*(a*b*e*(a/b)^(2/3) - a^2*h
*(a/b)^(2/3) + a*b*d*(a/b)^(1/3) - a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b) - 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f
*(a/b)^(2/3) - a*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3) + a*b*d - a^2*g)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/3*(b^2*c*(a/b)^(2
/3) - a*b*f*(a/b)^(2/3) + a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - a*b*d + a
^2*g)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [B] time = 5.10, size = 1731, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x)
```

```
[Out] symsum(log(b^2*c*d^2 - root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*
c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c*
f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h + 3
*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*
b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2
*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2
+ a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z,
 k)*(a^3*g^2 - root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*c*z^2 +
9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c*f*z + 9*
a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c
*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*
f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a
^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^
3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*((x*
(33*a^2*b^4*f - 24*a*b^5*c))/b^2 + 3*a^2*b^2*e - 3*a^3*b*h - 36*root(27*a^3
*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^
3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c*f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^
2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3
*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^
2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d
*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^
3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*a^2*b^3*x) + (x*(4*b^5*c^2 + 1
0*a^2*b^3*f^2 - 14*a*b^4*c*f + 10*a*b^4*d*e - 10*a^2*b^3*d*h - 10*a^2*b^3*e
*g + 10*a^3*b^2*g*h))/b^2 + a*b^2*d^2 - a^3*f*h + 2*a*b^2*c*e - 2*a^2*b*c*h
 - 2*a^2*b*d*g + a^2*b*e*f) - b^2*c^2*e + a^2*c*g^2 + (x*(b^4*d^3 + a^4*h^3
 - a*b^3*e^3 - a^3*b*g^3 + b^4*c^2*f + a^2*b^2*f^3 + 3*a^2*b^2*d*g^2 + 3*a^
2*b^2*e^2*h - 2*b^4*c*d*e - 2*a*b^3*c*f^2 - 3*a*b^3*d^2*g - 3*a^3*b*e*h^2 -
 2*a^2*b^2*c*g*h - 3*a^2*b^2*d*f*h - 3*a^2*b^2*e*f*g + 2*a*b^3*c*d*h + 2*a*
b^3*c*e*g + 3*a*b^3*d*e*f + 3*a^3*b*f*g*h))/b^2 + a*b*c^2*h - a^2*c*f*h - 2
*a*b*c*d*g + a*b*c*e*f)*root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5
```

```
*c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c
*f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h +
3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2
*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^
2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2
 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z
, k), k, 1, 3) + (h*x^2)/(2*b) + (c*log(x))/a + (g*x)/b
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.356 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=253

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}}$$

Rubi [A] time = 0.45, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}a^{4/3}b^{4/3}}-\frac{(bd-ag)\log\left(a+bx^3\right)}{3ab}-\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{hx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + (h*x)/b + ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(4/3
)) + (d*Log[x])/a + ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(4/3)*b^(4/3)) - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e
- a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(4/3))
 - ((b*d - a*g)*Log[a + b*x^3])/(3*a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)}\,dx = \int \left(\frac{h}{b}+\frac{c}{ax^2}+\frac{d}{ax}+\frac{a(be-ah)-b(bc-af)x-b(bd-ag)x^2}{ab\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\int \frac{a(be-ah)-b(bc-af)x-b(bd-ag)x^2}{a+bx^3}\,dx}{ab}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\int \frac{a(be-ah)-b(bc-af)x}{a+bx^3}\,dx}{ab}-\frac{(bd-ag)\int \frac{x^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}-\frac{(bd-ag)\log\left(a+bx^3\right)}{3ab}+\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}\,b(bc-af)+2a\sqrt[3]{b}\ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}b^{4/3}}\ldots$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}b^{4/3}}\ldots$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{4/3}}+\frac{d\log(\ldots}{a}$$

Mathematica [A] time = 0.32, size = 257, normalized size = 1.02

$$\frac{1}{6}\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-a^{2/3}be+a^{5/3}h+ab^{2/3}f-b^{5/3}c\right)}{a^{4/3}b^{4/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+b^{5/3}c\right)}{a^{4/3}b^{4/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{a^{4/3}b^{4/3}}+\frac{2(ag-bd)\log\left(a+bx^3\right)}{ab}-\frac{6c}{ax}+\frac{6d\log(x)}{a}+\frac{6hx}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```

```
[Out] ((-6*c)/(a*x) + (6*h*x)/b + (2*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)
*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(4/3)*b^(4/
3)) + (6*d*Log[x])/a + (2*(b^(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*
h)*Log[a^(1/3) + b^(1/3)*x])/(a^(4/3)*b^(4/3)) + ((-(b^(5/3)*c) - a^(2/3)*b
*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(a^(4/3)*b^(4/3)) + (2*(-(b*d) + a*g)*Log[a + b*x^3])/(a*b))/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^2 \left(a + bx^3\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^
3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^
3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 277, normalized size = 1.09

$$\frac{hx}{b} + \frac{d \log\left(|x|\right)}{a} + \frac{\sqrt{3} \left(a^2 h - abe - \left(-ab^2\right)^{\frac{1}{3}} bc + \left(-ab^2\right)^{\frac{1}{3}} af\right) \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 \left(-ab^2\right)^{\frac{2}{3}} a} + \frac{\left(a^2 h - abe + \left(-ab^2\right)^{\frac{1}{3}} bc - \left(-ab^2\right)^{\frac{1}{3}} af\right) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(-ab^2\right)^{\frac{2}{3}} a} - \frac{(bd - ag) \log\left(|bx^3 + a|\right)}{3ab} - \frac{c}{ax} + \frac{\left(ab^4 c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2 b^3 f \left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^3 b^2 h - a^2 b^3 e\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 a^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] h*x/b + d*log(abs(x))/a + 1/3*sqrt(3)*(a^2*h - a*b*e - (-a*b^2)^(1/3)*b*c +
 (-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/
((-a*b^2)^(2/3)*a) + 1/6*(a^2*h - a*b*e + (-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/
3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/3*(
b*d - a*g)*log(abs(b*x^3 + a))/(a*b) - c/(a*x) + 1/3*(a*b^4*c*(-a/b)^(1/3)
- a^2*b^3*f*(-a/b)^(1/3) + a^3*b^2*h - a^2*b^3*e)*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/(a^3*b^3)
```

maple [B] time = 0.06, size = 423, normalized size = 1.67

$$-\frac{\sqrt{3}\, a h \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} - \frac{a h \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} + \frac{a h \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} - \frac{\sqrt{3}\, c \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a} + \frac{c \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} a} - \frac{c \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} a} + \frac{d \ln(x)}{a} - \frac{d \ln\left(b x^3 + a\right)}{3a} + \frac{\sqrt{3}\, e \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b} + \frac{e \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{e \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b} + \frac{\sqrt{3}\, f \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b} - \frac{f \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b} + \frac{f \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} b} + \frac{g \ln\left(b x^3 + a\right)}{3b} + \frac{hx}{b} - \frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x)
```

```
[Out] h*x/b-1/3/b^2*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*h+1/3/b/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))*e+1/6/b^2*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*h-1/6/b/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-1/3/b^2*a/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h+1/3/b/(a/b)^(2/3)*3^(1/2)*arct
```

```
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f
+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))*f-1/6/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/3
/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-1/3/a*3^(1
/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/3/b*ln(b*x^3+a)
*g-1/3/a*d*ln(b*x^3+a)-1/a*c/x+1/a*d*ln(x)
```

maxima [A] time = 3.02, size = 290, normalized size = 1.15

$$\frac{hx}{b}+\frac{d\log(x)}{a}-\frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b}-\frac{c}{ax}-\frac{\left(2b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+abe-a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-abe+a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] h*x/b + d*log(x)/a - 1/3*sqrt(3)*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3) - a
*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(a^2*b) - c/(a*x) - 1/6*(2*b^2*d*(a/b)^(2/3) - 2*a*b*g*(a/b)
^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3) + a*b*e - a^2*h)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/3*(b^2*d*(a/b)^(2/3) -
a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) - a*b*e + a^2*h)*
log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [B] time = 5.09, size = 1802, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x)
```

```
[Out] symsum(log((b^3*c*d^2 + a^3*d*h^2 + a*b^2*d*e^2 - a*b^2*d^2*f - a*b^2*c*d*g
 - 2*a^2*b*d*e*h + a^2*b*d*f*g)/a - root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2
+ 27*a^3*b^4*d*z^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z +
 9*a^3*b^3*c*h*z - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*
a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2
*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^
2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3
*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^
3 + a^5*h^3, z, k)*(root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^3*b^4*d*z
^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3*b^3*c*h*z
 - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b*f*g*h - 3*
a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b
^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*
f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 -
 a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z,
k)*((3*a^2*b^3*c - 3*a^3*b^2*f)/a + (x*(24*a^3*b^4*d - 33*a^4*b^3*g))/(a^2*
b) + 36*root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^3*b^4*d*z^2 - 9*a^4*b
^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3*b^3*c*h*z - 9*a^2*b^4
*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e
- 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3
*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^
2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3
 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*a^2*b^3*x
) + (a^4*h^2 + a^2*b^2*e^2 - 2*a*b^3*c*d - 2*a^3*b*e*h + a^3*b*f*g - a^2*b^
2*c*g + 2*a^2*b^2*d*f)/a + (x*(4*a^2*b^4*d^2 + 10*a^4*b^2*g^2 - 10*a^2*b^4*
c*e + 10*a^3*b^3*c*h - 14*a^3*b^3*d*g + 10*a^3*b^3*e*f - 10*a^4*b^2*f*h))/(
a^2*b)) + (x*(b^5*c^3 - a^5*h^3 + a^4*b*g^3 + a^2*b^3*e^3 - a^3*b^2*f^3 + 3
*a^2*b^3*c*f^2 + a^2*b^3*d^2*g - 2*a^3*b^2*d*g^2 - 3*a^3*b^2*e^2*h - 3*a*b^
4*c^2*f + 3*a^4*b*e*h^2 - 2*a^2*b^3*c*d*h - 3*a^2*b^3*c*e*g - 2*a^2*b^3*d*e
```

```
*f + 3*a^3*b^2*c*g*h + 2*a^3*b^2*d*f*h + 3*a^3*b^2*e*f*g + 2*a*b^4*c*d*e -
3*a^4*b*f*g*h))/(a^2*b))*root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^3*b^
4*d*z^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3*b^3*
c*h*z - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b*f*g*h
 - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*
a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4
*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*
f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3
, z, k), k, 1, 3) + (h*x)/b - c/(a*x) + (d*log(x))/a
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a),x)
[Out] Timed out
```

3.357 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=260

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6a^{5/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{5/3}b^{2/3}}+\frac{\text{ta}}{}$$

Rubi [A] time = 0.38, antiderivative size = 258, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{5/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}a^{5/3}b^{2/3}}-\frac{(be-ah)\log\left(a+bx^3\right)}{3ab}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

```
[Out] -c/(2*a*x^2) - d/(a*x) + ((b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*
g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(2
/3)) + (e*Log[x])/a - ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(1
/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b*c - a*f - (a^(1/3)*(b*d - a*g))
/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(1/3
)) - ((b*e - a*h)*Log[a + b*x^3])/(3*a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)}\,dx=\int\left(\frac{c}{ax^3}+\frac{d}{ax^2}+\frac{e}{ax}+\frac{-bc+af-(bd-ag)x-(be-ah)x^2}{a\left(a+bx^3\right)}\right)dx$$

$$=-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}+\frac{\int\frac{-bc+af-(bd-ag)x-(be-ah)x^2}{a+bx^3}\,dx}{a}$$

$$=-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}+\frac{\int\frac{-bc+af+(-bd+ag)x}{a+bx^3}\,dx}{a}+\frac{(-be+ah)\int\frac{x^2}{a+bx^3}\,\cdots}{a}$$

$$=-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{(be-ah)\log\left(a+bx^3\right)}{3ab}+\frac{\int\frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(-bc+af)\cdots\right.}{\cdots}}{\cdots}$$

$$=-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}-\cdots$$

$$=-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\cdots$$

$$=-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\left(b^{4/3}c+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}b^{2/3}}+\frac{e\cdots}{\cdots}$$

Mathematica [A] time = 0.46, size = 257, normalized size = 0.99

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+b^{4/3}c\right)}{b^{2/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}\,bd-a\sqrt[3]{b}\,f+b^{4/3}c\right)}{b^{2/3}}+\frac{2a^{2/3}(ah-be)\log\left(a+bx^3\right)}{b}-\frac{3a^{2/3}c}{x^2}-\frac{6a^{2/3}d}{x}+6a^{2/3}e\log(x)}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(2/3)*c)/x^2 - (6*a^(2/3)*d)/x + (2*Sqrt[3]*(b^(4/3)*c + a^(1/3)*b*d
 - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(2/3) + 6*a^(2/3)*e*Log[x] - (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^
(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((b^(4/3)*c - a^(1/3)*b*d - a*
b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2
/3) + (2*a^(2/3)*(-(b*e) + a*h)*Log[a + b*x^3])/b)/(6*a^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^
3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^
3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [A] time = 0.28, size = 269, normalized size = 1.03

$$\frac{e\log\left(|x|\right)}{a}+\frac{\sqrt{3}\left(b^2c-abf-\left(-ab^2\right)^{\frac{1}{3}}bd+\left(-ab^2\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}a}+\frac{\left(b^2c-abf+\left(-ab^2\right)^{\frac{1}{3}}bd-\left(-ab^2\right)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}a}+\frac{(ah-be)\log\left(|bx^3+a|\right)}{3ab}+\frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2c-a^2bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b}-\frac{2dx+c}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] e*log(abs(x))/a + 1/3*sqrt(3)*(b^2*c - a*b*f - (-a*b^2)^(1/3)*b*d + (-a*b^2
)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2
)^(2/3)*a) + 1/6*(b^2*c - a*b*f + (-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) + 1/3*(a*h - b*
e)*log(abs(b*x^3 + a))/(a*b) + 1/3*(a*b^2*d*(-a/b)^(1/3) - a^2*b*g*(-a/b)^(
1/3) + a*b^2*c - a^2*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) -
 1/2*(2*d*x + c)/(a*x^2)
```

maple [B] time = 0.05, size = 423, normalized size = 1.63

$$-\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{e\ln(x)}{a}-\frac{e\ln\left(bx^3+a\right)}{3a}+\frac{\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{\sqrt{3}\,g\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{g\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{g\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{h\ln\left(bx^3+a\right)}{3b}-\frac{d}{ax}-\frac{c}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x)
```

```
[Out] 1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*f-1/3/(a/b)^(2/3)/a*c*ln(x+(a/b)^(1/3))
-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f+1/6/(a/b)^(2/3)/a*c*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))*f-1/3/(a/b)^(2/3)*3^(1/2)/a*c*arctan(1/3*3^(1/2)*(
```

```
2/(a/b)^(1/3)*x-1))-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*g+1/3/(a/b)^(1/3)/a
*d*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*g-
1/6/(a/b)^(1/3)/a*d*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*g-1/3*3^(1/2)/(a/b)^(1/3)/a*d*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b*ln(b*x^3+a)*h-1/3/a*e*ln(b*x^
3+a)+1/a*e*ln(x)-1/2/a*c/x^2-1/a*d/x
```

maxima [A] time = 3.00, size = 271, normalized size = 1.04

$$\frac{e\log(x)}{a}-\frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b}-\frac{\left(2be\left(\frac{a}{b}\right)^{\frac{2}{3}}-2ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc+af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(be\left(\frac{a}{b}\right)^{\frac{2}{3}}-ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc-af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2dx+c}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] e*log(x)/a - 1/3*sqrt(3)*(b^2*d*(a/b)^(2/3) - a*b*g*(a/b)^(2/3) + b^2*c*(a/
b)^(1/3) - a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/(a^2*b) - 1/6*(2*b*e*(a/b)^(2/3) - 2*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/
3) - a*g*(a/b)^(1/3) - b*c + a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
*b*(a/b)^(2/3)) - 1/3*(b*e*(a/b)^(2/3) - a*h*(a/b)^(2/3) - b*d*(a/b)^(1/3)
+ a*g*(a/b)^(1/3) + b*c - a*f)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/2
*(2*d*x + c)/(a*x^2)
```

mupad [B] time = 5.20, size = 6948, normalized size = 26.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x)
```

```
[Out] symsum(log(-(b^5*c^3*x - a^5*h^3*x - a^2*b^3*d*e^2 + 36*root(27*a^5*b^3*z^3
 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z
 - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*
a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2
*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*
d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3
*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*
b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^3*a^5*b^3*x - a^3*b^2*e*f^2 + a^3*b^2*
e^2*g - a^3*b^2*f^3*x - a*b^4*c^2*e - a*b^4*d^3*x + a^4*b*g^3*x + root(27*a
^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4
*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*
h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g +
 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*
a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2
*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*
h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^2*b^4*c^2 + 3*root(27*a^5*
b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^
2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2
*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*
a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2
*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*
g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3
 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^2*a^4*b^3*d + root(27*a^5*b^3*z
^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g
*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z +
9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b
^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*
c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 +
 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^
```

2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^4*b^2*f^2 - 3*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^2*a^5*b^2*g + 2*a^2*b^3*c*e*f + a^3*b^2*d*e*h + 10*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^5*b*h^2*x - 3*a*b^4*c^2*f*x + 2*a^4*b*e*h^2*x + 4*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*e^2*x + 24*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^2*a^4*b^3*e*x - 33*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^2*a^5*b^2*h*x + 3*a^2*b^3*c*f^2*x + 3*a^2*b^3*d^2*g*x - 3*a^3*b^2*d*g^2*x - a^3*b^2*e^2*h*x + root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^5*b*g*h - a^4*b*e*g*h - 2*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*c*f - 2*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*d*e - root(27*a^5*b^3*z^3 - 27*a^5*

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b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b
^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^
2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3
*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^
4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d
^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 +
a^4*b*g^3 + b^5*c^3, z, k)*a^4*b^2*d*h + 2*root(27*a^5*b^3*z^3 - 27*a^5*b^2
*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*
d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z
 - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^
3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b
*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*
g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4
*b*g^3 + b^5*c^3, z, k)*a^4*b^2*e*g + 2*a*b^4*c*d*e*x - 3*a^4*b*f*g*h*x + 1
0*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*
h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z
 + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*
b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3
*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h
- 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4
*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^2*b^4*c*d*x - 1
0*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*
h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z
 + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*
b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3
*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h
- 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4
*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*c*g*x - 1
0*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*
h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z
 + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*
b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3
*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h
- 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4
*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^3*b^3*d*f*x - 1
4*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*
h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z
 + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*
b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3
*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h
- 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4
*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^4*b^2*e*h*x + 1
0*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*
h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z
 + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*
b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3
*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h
- 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4
*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^4*b^2*f*g*x - 3
*a^2*b^3*c*d*h*x - 2*a^2*b^3*c*e*g*x - 2*a^2*b^3*d*e*f*x + 3*a^3*b^2*c*g*h*
x + 3*a^3*b^2*d*f*h*x + 2*a^3*b^2*e*f*g*x)/a^3)*root(27*a^5*b^3*z^3 - 27*a^
5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3
*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*
e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h +
 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*
a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3
*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3
+ a^4*b*g^3 + b^5*c^3, z, k), k, 1, 3) - c/(2*a*x^2) - d/(a*x) + (e*log(x))
/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a),x)
[Out] Timed out
```

3.358 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=276

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6a^{5/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{5/3}b^{2/3}}+$$

Rubi [A] time = 0.44, antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{5/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{5/3}b^{2/3}}+\frac{(bc-af)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(bc-af)}{a^2}-\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

```
[Out] -c/(3*a*x^3) - d/(2*a*x^2) - e/(a*x) + ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3
)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*a^(5/3)*b^(2/3)) - ((b*c - a*f)*Log[x])/a^2 - ((b^(1/3)*(b*d - a*g) - a^(
1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b*d - a
*g - (a^(1/3)*(b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(6*a^(5/3)*b^(1/3)) + ((b*c - a*f)*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^4}+\frac{d}{ax^3}+\frac{e}{ax^2}+\frac{-bc+af}{a^2x}+\frac{-a(bd-ag)-a(be-ah)x+b(bc-\ldots}{a^2\left(a+bx^3\right)}\right.$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}+\frac{\int \frac{-a(bd-ag)-a(be-ah)x+b(bc-af)x^2}{a+bx^3}\,d\ldots}{a^2}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}+\frac{\int \frac{-a(bd-ag)-a(be-ah)x}{a+bx^3}\,dx}{a^2}+\frac{(b(bc\ldots}{}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}+\frac{(bc-af)\log\left(a+bx^3\right)}{3a^2}+\frac{\int \ldots}{}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}-\frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}\ldots\right.}{3a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}-\frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}\ldots\right.}{3a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}+\frac{\left(b^{4/3}d+\sqrt[3]{a}\,be-a\sqrt[3]{b}\,g-a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}b^{2/3}}\ldots$$

Mathematica [A] time = 0.55, size = 264, normalized size = 0.96

$$-\frac{-\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{b^{2/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{b^{2/3}}+\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}h-\sqrt[3]{a}be+a\sqrt[3]{b}g-b^{4/3}d\right)}{b^{2/3}}-2(bc-af)\log\left(a+bx^3\right)+6\log(x)(bc-af)+\frac{2ac}{x^3}+\frac{3ad}{x^2}+\frac{6ae}{x}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/6*((2*a*c)/x^3 + (3*a*d)/x^2 + (6*a*e)/x + (2*Sqrt[3]*a^(1/3)*(-(b^(4/3)
*d) - a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(2/3) + 6*(b*c - a*f)*Log[x] + (2*a^(1/3)*(b^(4/3)*d - a^(
1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^
(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 2*(b*c - a*f)*Log[a + b*x^3])/a^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^4\left(a + bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^
3)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^
3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [A] time = 0.23, size = 291, normalized size = 1.05

$$\frac{\sqrt{3}\left(b^{2} d - a b g + \left(-a b^{2}\right)^{\frac{1}{3}} a h - \left(-a b^{2}\right)^{\frac{1}{3}} b e\right) \arctan\left(\frac{\sqrt{3}\left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-a b^{2}\right)^{\frac{2}{3}} a} + \frac{\left(b^{2} d - a b g - \left(-a b^{2}\right)^{\frac{1}{3}} a h + \left(-a b^{2}\right)^{\frac{1}{3}} b e\right) \log\left(x^{2} + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-a b^{2}\right)^{\frac{2}{3}} a} + \frac{\left(b c - a f\right) \log\left(\left|b x^{3} + a\right|\right)}{3 a^{2}} - \frac{\left(b c - a f\right) \log\left(\left|x\right|\right)}{a^{2}} - \frac{\left(a^{4} b h\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^{3} b^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}} e - a^{3} b^{2} d + a^{4} b g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 a^{5} b} - \frac{6 a x^{2} e + 3 a d x + 2 a c}{6 a^{2} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] 1/3*sqrt(3)*(b^2*d - a*b*g + (-a*b^2)^(1/3)*a*h - (-a*b^2)^(1/3)*b*e)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) + 1/6*(
b^2*d - a*b*g - (-a*b^2)^(1/3)*a*h + (-a*b^2)^(1/3)*b*e)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) + 1/3*(b*c - a*f)*log(abs(b*x^3 +
 a))/a^2 - (b*c - a*f)*log(abs(x))/a^2 - 1/3*(a^4*b*h*(-a/b)^(1/3) - a^3*b^
2*(-a/b)^(1/3)*e - a^3*b^2*d + a^4*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/
3)))/(a^5*b) - 1/6*(6*a*x^2*e + 3*a*d*x + 2*a*c)/(a^2*x^3)
```

maple [B] time = 0.06, size = 442, normalized size = 1.60

$$-\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} a} - \frac{d \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} a} + \frac{d \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} a} - \frac{\sqrt{3}\, e \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} a} + \frac{e \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} a} - \frac{e \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} a} + \frac{f \ln(x)}{a} - \frac{f \ln\left(b x^{3} + a\right)}{3a} - \frac{bc \ln(x)}{a^{2}} + \frac{bc \ln\left(b x^{3} + a\right)}{3a^{2}} + \frac{\sqrt{3}\, g \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} + \frac{g \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b} - \frac{g \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b} + \frac{\sqrt{3}\, h \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b} - \frac{h \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} b} + \frac{h \ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} b} - \frac{e}{ax} - \frac{d}{2a x^{2}} - \frac{c}{3a x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x)
```

```
[Out] 1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*g-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d
-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*g+1/6/a/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))*g-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
```

```
(a/b)^(1/3)*x-1))*d-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*h+1/3/(a/b)^(1/3)/a
*e*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*h-
1/6/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e+1/3*3^(1/2)/b/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h-1/3/a*3^(1/2)/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*e-1/3/a*ln(b*x^3+a)*f+1/3/a^2*b*ln(b*
x^3+a)*c-1/a*e/x-1/3/a*c/x^3-1/2/a*d/x^2+1/a*ln(x)*f-1/a^2*ln(x)*b*c
```

maxima [A] time = 3.08, size = 302, normalized size = 1.09

$$-\frac{\left(bc-af\right)\log\left(x\right)}{a^{2}}-\frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{2}h\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^{3}}+\frac{\left(2b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}+abd-a^{2}g\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd+a^{2}g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{6ex^{2}+3dx+2c}{6ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] -(b*c - a*f)*log(x)/a^2 - 1/3*sqrt(3)*(a*b*e*(a/b)^(2/3) - a^2*h*(a/b)^(2/3
) + a*b*d*(a/b)^(1/3) - a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/a^3 + 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) -
a*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3) + a*b*d - a^2*g)*log(x^2 - x*(a/b)^(1
/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 1/3*(b^2*c*(a/b)^(2/3) - a*b*f*(a/
b)^(2/3) + a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - a*b*d + a^2*g)*log(x + (
a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 1/6*(6*e*x^2 + 3*d*x + 2*c)/(a*x^3)
```

mupad [B] time = 5.87, size = 1842, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x)
```

```
[Out] symsum(log(- (b^5*c*d^2 - b^5*c^2*e + a^2*b^3*c*g^2 - a^2*b^3*e*f^2 - a^3*b
^2*f*g^2 + a^3*b^2*f^2*h - a*b^4*d^2*f + a*b^4*c^2*h - 2*a^2*b^3*c*f*h + 2*
a^2*b^3*d*f*g - 2*a*b^4*c*d*g + 2*a*b^4*c*e*f)/a^3 - root(27*a^6*b^2*z^3 +
27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e*g*z - 9*a
^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2*z + 9*a^2
*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*
f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h
 - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^
2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2
*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*((a^2*b^4*d^2 + a^4*b^2*g^2 + 2*a^2*b^4*c
*e - 2*a^3*b^3*c*h - 2*a^3*b^3*d*g - 2*a^3*b^3*e*f + 2*a^4*b^2*f*h)/a^3 + r
oot(27*a^6*b^2*z^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z -
9*a^4*b^2*e*g*z - 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*
a^4*b^2*f^2*z + 9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2
*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*
e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3
*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^
3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*((3*a^4*b^3*e - 3*a^
5*b^2*h)/a^3 - (x*(24*a^3*b^4*c - 24*a^4*b^3*f))/a^3 + 36*root(27*a^6*b^2*z
^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e*g*z
- 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2*z +
9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b
^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*
c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 -
 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^
3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*a^2*b^3*x) + (x*(4*a*b^5*c^2 + 4*a^3
*b^3*f^2 - 8*a^2*b^4*c*f + 10*a^2*b^4*d*e - 10*a^3*b^3*d*h - 10*a^3*b^3*e*g
 + 10*a^4*b^2*g*h))/a^3) - (x*(b^5*d^3 - a*b^4*e^3 + a^4*b*h^3 - a^3*b^2*g^
3 + 3*a^2*b^3*d*g^2 + 3*a^2*b^3*e^2*h - 3*a^3*b^2*e*h^2 - 2*b^5*c*d*e - 3*a
*b^4*d^2*g - 2*a^2*b^3*c*g*h - 2*a^2*b^3*d*f*h - 2*a^2*b^3*e*f*g + 2*a^3*b^
```

```
2*f*g*h + 2*a*b^4*c*d*h + 2*a*b^4*c*e*g + 2*a*b^4*d*e*f))/a^3)*root(27*a^6*
b^2*z^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e
*g*z - 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2
*z + 9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*
a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2
*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*
g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3
 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/
a + (d*x)/(2*a))/x^3 - (log(x)*(b*c - a*f))/a^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.359 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=337

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{18\sqrt[3]{a}\,b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-\right.}{9\sqrt[3]{a}\,b^{10/3}}$$

Rubi [A] time = 0.72, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1828, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{18\sqrt[3]{a}\,b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{9\sqrt[3]{a}\,b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-4a^{2/3}be+7a^{5/3}h-5ab^{2/3}f+2b^{5/3}c\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{10/3}}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3b^3\left(a+bx^3\right)}+\frac{(bd-2ag)\log\left(a+bx^3\right)}{3b^3}+\frac{x(be-2ah)}{b^3}+\frac{fx^2}{2b^2}+\frac{gx^3}{3b^2}+\frac{hx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*h)*x)/b^3 + (f*x^2)/(2*b^2) + (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2)
 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*b^3*(a + b*
x^3)) - ((2*b^(5/3)*c - 4*a^(2/3)*b*e - 5*a*b^(2/3)*f + 7*a^(5/3)*h)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(10/3)) -
 ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)
*x])/(9*a^(1/3)*b^(10/3)) + ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*
a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(10/3))
 + ((b*d - 2*a*g)*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}dx = \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3b^3\left(a+bx^3\right)} - \int \frac{a^2(be-ah)-2ab(bc-af)x-}{}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3b^3\left(a+bx^3\right)} - \frac{\int\left(-3a(be-2ah)-3\right.}{}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(b\right.}{3b^3\left(a+bx^3\right)}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(b\right.}{3b^3\left(a+bx^3\right)}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(b\right.}{3b^3\left(a+bx^3\right)}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(b\right.}{3b^3\left(a+bx^3\right)}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(b\right.}{3b^3\left(a+bx^3\right)}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(b\right.}{3b^3\left(a+bx^3\right)}$$

Mathematica [A] time = 0.57, size = 334, normalized size = 0.99

$$\frac{\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(4a^{2/3}b^{4/3}e-7a^{5/3}\sqrt[3]{b}h-5abf+2b^2c\right)}{\sqrt[3]{a}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-4a^{2/3}b^{4/3}e+7a^{5/3}\sqrt[3]{b}h+5abf-2b^2c\right)}{\sqrt[3]{a}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-4a^{2/3}b^{4/3}e+7a^{5/3}\sqrt[3]{b}h-5abf+2b^2c\right)}{\sqrt[3]{a}}-\frac{12b^{2/3}\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{a+bx^3}+12b^{2/3}(bd-2ag)\log\left(a+bx^3\right)+36b^{2/3}x(be-2ah)+18b^{5/3}fx^2+12b^{5/3}gx^3+9b^{5/3}hx^4}{36b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(2/3)*(b*e - 2*a*h)*x + 18*b^(5/3)*f*x^2 + 12*b^(5/3)*g*x^3 + 9*b^(5/
3)*h*x^4 - (12*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))))
/(a + b*x^3) - (4*Sqrt[3]*(2*b^2*c - 4*a^(2/3)*b^(4/3)*e - 5*a*b*f + 7*a^(5
/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (4*(-
2*b^2*c - 4*a^(2/3)*b^(4/3)*e + 5*a*b*f + 7*a^(5/3)*b^(1/3)*h)*Log[a^(1/3)
+ b^(1/3)*x])/a^(1/3) + (2*(2*b^2*c + 4*a^(2/3)*b^(4/3)*e - 5*a*b*f - 7*a^(
5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3) + 1
2*b^(2/3)*(b*d - 2*a*g)*Log[a + b*x^3])/(36*b^(11/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [A] time = 0.20, size = 357, normalized size = 1.06

$$-\frac{\sqrt{3}\left(7\,a^{2}h-4\,abe-2\,\left(-ab^{2}\right)^{\frac{1}{3}}bc+5\,\left(-ab^{2}\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}-\frac{\left(7\,a^{2}h-4\,abe+2\,\left(-ab^{2}\right)^{\frac{1}{3}}bc-5\,\left(-ab^{2}\right)^{\frac{1}{3}}af\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}+\frac{\left(bd-2\,ag\right)\log\left(\left|bx^{3}+a\right|\right)}{3\,b^{3}}+\frac{abd-a^{2}g-\left(b^{2}c-abf\right)x^{2}-\left(a^{2}h-abe\right)x}{3\left(bx^{3}+a\right)b^{3}}-\frac{\left(2\,b^{6}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^{5}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^{2}b^{4}h-4\,ab^{5}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{7}}+\frac{3\,b^{6}hx^{4}+4\,b^{6}gx^{3}+6\,b^{6}fx^{2}-24\,ab^{5}hx+12\,b^{6}xe}{12\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/9*sqrt(3)*(7*a^2*h - 4*a*b*e - 2*(-a*b^2)^(1/3)*b*c + 5*(-a*b^2)^(1/3)*a
*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b
^2) - 1/18*(7*a^2*h - 4*a*b*e + 2*(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*f
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + 1/3*(b*d
- 2*a*g)*log(abs(b*x^3 + a))/b^3 + 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2
 - (a^2*h - a*b*e)*x)/((b*x^3 + a)*b^3) - 1/9*(2*b^6*c*(-a/b)^(1/3) - 5*a*b
^5*f*(-a/b)^(1/3) + 7*a^2*b^4*h - 4*a*b^5*e)*(-a/b)^(1/3)*log(abs(x - (-a/b
)^(1/3)))/(a*b^7) + 1/12*(3*b^6*h*x^4 + 4*b^6*g*x^3 + 6*b^6*f*x^2 - 24*a*b^
5*h*x + 12*b^6*x*e)/b^8
```

maple [B] time = 0.06, size = 562, normalized size = 1.67

$$\frac{h\,x^{4}}{4b^{2}}+\frac{af\,x^{2}}{3\left(b\,x^{3}+a\right)b^{2}}-\frac{c\,x^{2}}{3\left(b\,x^{3}+a\right)b}+\frac{g\,x^{3}}{3b^{2}}-\frac{a^{2}hx}{3\left(b\,x^{3}+a\right)b^{3}}+\frac{aex}{3\left(b\,x^{3}+a\right)b^{2}}+\frac{f\,x^{2}}{2b^{2}}-\frac{a^{2}g}{3\left(b\,x^{3}+a\right)b^{3}}+\frac{7\sqrt{3}\,a^{2}h\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{4}}+\frac{7a^{2}h\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{4}}-\frac{7a^{2}h\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{4}}+\frac{ad}{3\left(b\,x^{3}+a\right)b^{2}}-\frac{4\sqrt{3}\,ae\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{4ae\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{2ae\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{5\sqrt{3}\,af\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{5af\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{5af\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{2ag\ln\left(b\,x^{3}+a\right)}{3b^{3}}-\frac{2ahx}{b^{3}}+\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{d\ln\left(b\,x^{3}+a\right)}{3b^{2}}+\frac{ex}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] -5/9/b^3*a*f*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-4/
9/b^3*e*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+7/9/b
^4*a^2*h/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-7/18/b
^4*a^2*h/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/b^3*e*a/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))+1/b^2*e*x-1/3/b/(b*x^3+a)*c*x^2-1/3/b^3/(b*x^3+a)*a^2
*g+1/3/b^2/(b*x^3+a)*d*a-2/b^3*a*h*x-2/9/b^2*c/(a/b)^(1/3)*ln(x+(a/b)^(1/3)
)+1/9/b^2*c/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-2/3/b^3*ln(b*x^3+
a)*a*g+2/9/b^2*c*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)+2/9/b^3*e*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+5/9/b^3*a*f/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))-5/18/b^3*a*f/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))+1/3/b^2/(b*x^3+a)*x^2*a*f-1/3/b^3/(b*x^3+a)*a^2*h*x+1/3/b^2/(b*x
^3+a)*a*e*x+7/9/b^4*a^2*h/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/3/b^2*ln(b*x^3+a)
*d+1/2/b^2*f*x^2+1/3*g*x^3/b^2+1/4*h*x^4/b^2
```

maxima [A] time = 3.05, size = 364, normalized size = 1.08

$$\frac{abd-a^{2}g-\left(b^{2}c-abf\right)x^{2}+\left(abe-a^{2}h\right)x}{3\left(b^{4}x^{3}+ab^{3}\right)}+\frac{\sqrt{3}\left(2\,b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{3}}+\frac{3\,bhx^{4}+4\,bgx^{3}+6\,bfx^{2}+12\,(be-2\,ah)x}{12\,b^{3}}+\frac{\left(6\,b^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-12\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+4\,abe-7\,a^{2}h\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(3\,b^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-6\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,abe+7\,a^{2}h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 + (a*b*e - a^2*h)*x)/(b^4*x^3 + a*
b^3) + 1/9*sqrt(3)*(2*b^2*c*(a/b)^(2/3) - 5*a*b*f*(a/b)^(2/3) - 4*a*b*e*(a/
b)^(1/3) + 7*a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(a*b^3) + 1/12*(3*b*h*x^4 + 4*b*g*x^3 + 6*b*f*x^2 + 12*(b*e - 2*a*
h)*x)/b^3 + 1/18*(6*b^2*d*(a/b)^(2/3) - 12*a*b*g*(a/b)^(2/3) + 2*b^2*c*(a/b
)^(1/3) - 5*a*b*f*(a/b)^(1/3) + 4*a*b*e - 7*a^2*h)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/9*(3*b^2*d*(a/b)^(2/3) - 6*a*b*g*(a/b)
^(2/3) - 2*b^2*c*(a/b)^(1/3) + 5*a*b*f*(a/b)^(1/3) - 4*a*b*e + 7*a^2*h)*log
(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

mupad [B] time = 5.11, size = 1241, normalized size = 3.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log(root(729*a*b^10*z^3 - 729*a*b^8*d*z^2 + 1458*a^2*b^7*g*z^2 - 216
*a*b^6*c*e*z - 945*a^3*b^4*f*h*z - 972*a^2*b^5*d*g*z + 540*a^2*b^5*e*f*z +
378*a^2*b^5*c*h*z + 243*a*b^6*d^2*z + 972*a^3*b^4*g^2*z - 630*a^4*b*f*g*h +
 72*a*b^4*c*d*e + 360*a^3*b^2*e*f*g + 315*a^3*b^2*d*f*h + 252*a^3*b^2*c*g*h
 - 180*a^2*b^3*d*e*f - 144*a^2*b^3*c*e*g - 126*a^2*b^3*c*d*h + 588*a^4*b*e*
h^2 - 60*a*b^4*c^2*f - 336*a^3*b^2*e^2*h - 324*a^3*b^2*d*g^2 + 162*a^2*b^3*
d^2*g + 150*a^2*b^3*c*f^2 - 125*a^3*b^2*f^3 + 64*a^2*b^3*e^3 + 216*a^4*b*g^
3 - 27*a*b^4*d^3 - 343*a^5*h^3 + 8*b^5*c^3, z, k)*((108*a^2*b^3*g - 54*a*b^
4*d)/(9*b^4) + (x*(63*a^2*b^3*h - 36*a*b^4*e))/(9*b^4) + 9*root(729*a*b^10*
z^3 - 729*a*b^8*d*z^2 + 1458*a^2*b^7*g*z^2 - 216*a*b^6*c*e*z - 945*a^3*b^4*
f*h*z - 972*a^2*b^5*d*g*z + 540*a^2*b^5*e*f*z + 378*a^2*b^5*c*h*z + 243*a*b
^6*d^2*z + 972*a^3*b^4*g^2*z - 630*a^4*b*f*g*h + 72*a*b^4*c*d*e + 360*a^3*b
^2*e*f*g + 315*a^3*b^2*d*f*h + 252*a^3*b^2*c*g*h - 180*a^2*b^3*d*e*f - 144*
a^2*b^3*c*e*g - 126*a^2*b^3*c*d*h + 588*a^4*b*e*h^2 - 60*a*b^4*c^2*f - 336*
a^3*b^2*e^2*h - 324*a^3*b^2*d*g^2 + 162*a^2*b^3*d^2*g + 150*a^2*b^3*c*f^2 -
 125*a^3*b^2*f^3 + 64*a^2*b^3*e^3 + 216*a^4*b*g^3 - 27*a*b^4*d^3 - 343*a^5*
h^3 + 8*b^5*c^3, z, k)*a*b^2) + (36*a^3*g^2 + 9*a*b^2*d^2 - 35*a^3*f*h - 8*
a*b^2*c*e + 14*a^2*b*c*h - 36*a^2*b*d*g + 20*a^2*b*e*f)/(9*b^4) + (x*(4*b^3
*c^2 + 25*a^2*b*f^2 + 42*a^3*g*h - 20*a*b^2*c*f + 12*a*b^2*d*e - 21*a^2*b*d
*h - 24*a^2*b*e*g))/(9*b^4))*root(729*a*b^10*z^3 - 729*a*b^8*d*z^2 + 1458*a
^2*b^7*g*z^2 - 216*a*b^6*c*e*z - 945*a^3*b^4*f*h*z - 972*a^2*b^5*d*g*z + 54
0*a^2*b^5*e*f*z + 378*a^2*b^5*c*h*z + 243*a*b^6*d^2*z + 972*a^3*b^4*g^2*z -
 630*a^4*b*f*g*h + 72*a*b^4*c*d*e + 360*a^3*b^2*e*f*g + 315*a^3*b^2*d*f*h +
 252*a^3*b^2*c*g*h - 180*a^2*b^3*d*e*f - 144*a^2*b^3*c*e*g - 126*a^2*b^3*c*
d*h + 588*a^4*b*e*h^2 - 60*a*b^4*c^2*f - 336*a^3*b^2*e^2*h - 324*a^3*b^2*d*
g^2 + 162*a^2*b^3*d^2*g + 150*a^2*b^3*c*f^2 - 125*a^3*b^2*f^3 + 64*a^2*b^3*
e^3 + 216*a^4*b*g^3 - 27*a*b^4*d^3 - 343*a^5*h^3 + 8*b^5*c^3, z, k), k, 1,
3) + x*(e/b^2 - (2*a*h)/b^3) - (x*((a^2*h)/3 - (a*b*e)/3) + (a^2*g)/3 + x^2
*((b^2*c)/3 - (a*b*f)/3) - (a*b*d)/3)/(a*b^3 + b^4*x^3) + (f*x^2)/(2*b^2) +
 (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.360 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx$

Optimal. Leaf size=311

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{18a^{2/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd\right.}{9a^{2/3}b^{8/3}}$$

Rubi [A] time = 0.64, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1828, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{18a^{2/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{9a^{2/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{4/3}g+2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{3\sqrt{3}a^{2/3}b^{8/3}}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3b^2\left(a+bx^3\right)}+\frac{(be-2ah)\log\left(a+bx^3\right)}{3b^3}+\frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (f*x)/b^2 + (g*x^2)/(2*b^2) + (h*x^3)/(3*b^2) - (x*(b*c - a*f + (b*d - a*g)
*x + (b*e - a*h)*x^2))/(3*b^2*(a + b*x^3)) - ((b^(4/3)*c + 2*a^(1/3)*b*d -
4*a*b^(1/3)*f - 5*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(3*Sqrt[3]*a^(2/3)*b^(8/3)) + ((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d
 - 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b^(1/3)*(b*c -
 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(2/3)*b^(8/3)) + ((b*e - 2*a*h)*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)} - \frac{\int \frac{-ab(bc-af)-2ab(bd-ag)x}{}}{}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)} - \frac{\int \left(-3abf-3abgx-\right.}{}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}+\frac{\int}{}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}+\frac{\int}{}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}+\frac{(be}{}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}+\frac{\left(\sqrt[3]{b}\right.}{}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}+\frac{\left(\sqrt[3]{b}\right.}{}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}-\frac{\left(b^{4/}\right.}{}$$

Mathematica [A] time = 0.22, size = 294, normalized size = 0.95

$$\frac{-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}g-2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^{4/3}g-2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}g-2\sqrt[3]{a}bd+4a\sqrt[3]{b}f-b^{4/3}c\right)}{a^{2/3}}-\frac{6\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{a+bx^3}+6(be-2ah)\log\left(a+bx^3\right)+18bfx+9bgx^2+6bhx^3}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b*f*x + 9*b*g*x^2 + 6*b*h*x^3 - (6*(a^2*h + b^2*x*(c + d*x) - a*b*(e +
x*(f + g*x))))/(a + b*x^3) + (2*Sqrt[3]*b^(1/3)*(-(b^(4/3)*c) - 2*a^(1/3)*b
*d + 4*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/a^(2/3) + (2*b^(1/3)*(b^(4/3)*c - 2*a^(1/3)*b*d - 4*a*b^(1/3)*f + 5*a^(
4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - (b^(1/3)*(b^(4/3)*c - 2*a^(1/3)
*b*d - 4*a*b^(1/3)*f + 5*a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(2/3) + 6*(b*e - 2*a*h)*Log[a + b*x^3])/(18*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 330, normalized size = 1.06

$$-\frac{\sqrt{3}\left(b^{2}c-4\,abf-2\,\left(-ab^{2}\right)^{\frac{1}{3}}bd+5\,\left(-ab^{2}\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}-\frac{\left(b^{2}c-4\,abf+2\,\left(-ab^{2}\right)^{\frac{1}{3}}bd-5\,\left(-ab^{2}\right)^{\frac{1}{3}}ag\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}-\frac{(2\,ah-be)\log\left(\left|bx^{3}+a\right|\right)}{3\,b^{3}}-\frac{a^{2}h+\left(b^{2}d-abg\right)x^{2}-abe+\left(b^{2}c-abf\right)x}{3\left(bx^{3}+a\right)b^{3}}-\frac{\left(2\,b^{4}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^{3}g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^{4}c-4\,ab^{3}f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{5}}+\frac{2\,b^{4}hx^{3}+3\,b^{4}gx^{2}+6\,b^{4}fx}{6\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/9*sqrt(3)*(b^2*c - 4*a*b*f - 2*(-a*b^2)^(1/3)*b*d + 5*(-a*b^2)^(1/3)*a*g
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2
) - 1/18*(b^2*c - 4*a*b*f + 2*(-a*b^2)^(1/3)*b*d - 5*(-a*b^2)^(1/3)*a*g)*lo
g(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) - 1/3*(2*a*h -
b*e)*log(abs(b*x^3 + a))/b^3 - 1/3*(a^2*h + (b^2*d - a*b*g)*x^2 - a*b*e + (
b^2*c - a*b*f)*x)/((b*x^3 + a)*b^3) - 1/9*(2*b^4*d*(-a/b)^(1/3) - 5*a*b^3*g
*(-a/b)^(1/3) + b^4*c - 4*a*b^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^5) + 1/6*(2*b^4*h*x^3 + 3*b^4*g*x^2 + 6*b^4*f*x)/b^6
```

maple [B] time = 0.06, size = 533, normalized size = 1.71

$$\frac{ag\,x^{2}}{3\left(b\,x^{3}+a\right)b^{2}}-\frac{d\,x^{2}}{3\left(b\,x^{3}+a\right)b}+\frac{h\,x^{3}}{3b^{2}}+\frac{af\,x}{3\left(b\,x^{3}+a\right)b^{2}}-\frac{cx}{3\left(b\,x^{3}+a\right)b}+\frac{g\,x^{2}}{2b^{2}}-\frac{a^{2}h}{3\left(b\,x^{3}+a\right)b^{3}}+\frac{ae}{3\left(b\,x^{3}+a\right)b^{2}}-\frac{4\sqrt{3}\,af\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{4af\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{2af\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{5\sqrt{3}\,ag\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{5ag\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{5ag\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{2ah\ln\left(b\,x^{3}+a\right)}{3b^{3}}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{2\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{e\ln\left(b\,x^{3}+a\right)}{3b^{2}}+\frac{fx}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*h*x^3/b^2+1/2*g*x^2/b^2+1/b^2*f*x+1/3/b^2/(b*x^3+a)*x^2*a*g-1/3/b/(b*x^
3+a)*x^2*d+1/3/b^2/(b*x^3+a)*a*f*x-1/3/(b*x^3+a)/b*c*x-1/3/b^3/(b*x^3+a)*a^
2*h+1/3/b^2/(b*x^3+a)*a*e-4/9/b^3*a*f/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9/b^3
*a*f/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/b^3*a*f/(a/b)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9/b^2*c/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))-1/18/b^2*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/9/b
^2*c/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+5/9/b^3*a*
g/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18/b^3*a*g/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))-5/9/b^3*a*g*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))-2/9/b^2*d/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2*d/(a/b)^(1/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/b^2*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/3/b^3*ln(b*x^3+a)*a*h+1/3/b^2*ln(b*x^3+a)*e
```

maxima [A] time = 3.14, size = 329, normalized size = 1.06

$$\frac{abe-a^{2}h-\left(b^{2}d-abg\right)x^{2}-\left(b^{2}c-abf\right)x}{3\left(b^{4}x^{3}+ab^{3}\right)}+\frac{2\,hx^{3}+3\,gx^{2}+6\,fx}{6\,b^{2}}+\frac{\sqrt{3}\left(2\,b^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{3}}+\frac{\left(6\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-12\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc+4\,af\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(3\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-6\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc-4\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] 1/3*(a*b*e - a^2*h - (b^2*d - a*b*g)*x^2 - (b^2*c - a*b*f)*x)/(b^4*x^3 + a*
b^3) + 1/6*(2*h*x^3 + 3*g*x^2 + 6*f*x)/b^2 + 1/9*sqrt(3)*(2*b^2*d*(a/b)^(2/
3) - 5*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - 4*a*b*f*(a/b)^(1/3))*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) + 1/18*(6*b*e*(a/b)^(2
/3) - 12*a*h*(a/b)^(2/3) + 2*b*d*(a/b)^(1/3) - 5*a*g*(a/b)^(1/3) - b*c + 4*
a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/9*(3*b*e*
(a/b)^(2/3) - 6*a*h*(a/b)^(2/3) - 2*b*d*(a/b)^(1/3) + 5*a*g*(a/b)^(1/3) + b
*c - 4*a*f)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 0.15, size = 1229, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((36*a^3*h^2 + 9*a*b^2*e^2 + 2*b^3*c*d - 5*a*b^2*c*g - 8*a*b^2*d*
f - 36*a^2*b*e*h + 20*a^2*b*f*g)/(9*b^4) + root(729*a^2*b^9*z^3 + 1458*a^3*
b^6*h*z^2 - 729*a^2*b^7*e*z^2 + 54*a*b^6*c*d*z - 972*a^3*b^4*e*h*z + 540*a^
3*b^4*f*g*z - 216*a^2*b^5*d*f*z - 135*a^2*b^5*c*g*z + 972*a^4*b^3*h^2*z + 2
43*a^2*b^5*e^2*z + 360*a^4*b*f*g*h - 18*a*b^4*c*d*e - 180*a^3*b^2*e*f*g - 1
44*a^3*b^2*d*f*h - 90*a^3*b^2*c*g*h + 72*a^2*b^3*d*e*f + 45*a^2*b^3*c*e*g +
 36*a^2*b^3*c*d*h - 324*a^4*b*e*h^2 + 12*a*b^4*c^2*f + 162*a^3*b^2*e^2*h +
150*a^3*b^2*d*g^2 - 60*a^2*b^3*d^2*g - 48*a^2*b^3*c*f^2 + 64*a^3*b^2*f^3 -
27*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8*a*b^4*d^3 + 216*a^5*h^3 - b^5*c^3, z, k)
*((108*a^2*b^3*h - 54*a*b^4*e)/(9*b^4) + (x*(9*b^4*c - 36*a*b^3*f))/(9*b^3)
 + 9*root(729*a^2*b^9*z^3 + 1458*a^3*b^6*h*z^2 - 729*a^2*b^7*e*z^2 + 54*a*b
^6*c*d*z - 972*a^3*b^4*e*h*z + 540*a^3*b^4*f*g*z - 216*a^2*b^5*d*f*z - 135*
a^2*b^5*c*g*z + 972*a^4*b^3*h^2*z + 243*a^2*b^5*e^2*z + 360*a^4*b*f*g*h - 1
8*a*b^4*c*d*e - 180*a^3*b^2*e*f*g - 144*a^3*b^2*d*f*h - 90*a^3*b^2*c*g*h +
72*a^2*b^3*d*e*f + 45*a^2*b^3*c*e*g + 36*a^2*b^3*c*d*h - 324*a^4*b*e*h^2 +
12*a*b^4*c^2*f + 162*a^3*b^2*e^2*h + 150*a^3*b^2*d*g^2 - 60*a^2*b^3*d^2*g -
 48*a^2*b^3*c*f^2 + 64*a^3*b^2*f^3 - 27*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8*a*b
^4*d^3 + 216*a^5*h^3 - b^5*c^3, z, k)*a*b^2) + (x*(4*b^2*d^2 + 25*a^2*g^2 -
 3*b^2*c*e - 24*a^2*f*h + 6*a*b*c*h - 20*a*b*d*g + 12*a*b*e*f))/(9*b^3))*ro
ot(729*a^2*b^9*z^3 + 1458*a^3*b^6*h*z^2 - 729*a^2*b^7*e*z^2 + 54*a*b^6*c*d*
z - 972*a^3*b^4*e*h*z + 540*a^3*b^4*f*g*z - 216*a^2*b^5*d*f*z - 135*a^2*b^5
*c*g*z + 972*a^4*b^3*h^2*z + 243*a^2*b^5*e^2*z + 360*a^4*b*f*g*h - 18*a*b^4
*c*d*e - 180*a^3*b^2*e*f*g - 144*a^3*b^2*d*f*h - 90*a^3*b^2*c*g*h + 72*a^2*
b^3*d*e*f + 45*a^2*b^3*c*e*g + 36*a^2*b^3*c*d*h - 324*a^4*b*e*h^2 + 12*a*b^
4*c^2*f + 162*a^3*b^2*e^2*h + 150*a^3*b^2*d*g^2 - 60*a^2*b^3*d^2*g - 48*a^2
*b^3*c*f^2 + 64*a^3*b^2*f^3 - 27*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8*a*b^4*d^3
+ 216*a^5*h^3 - b^5*c^3, z, k), k, 1, 3) - (x*((b*c)/3 - (a*f)/3) + (a^2*h
- a*b*e)/(3*b) + x^2*((b*d)/3 - (a*g)/3))/(a*b^2 + b^3*x^3) + (g*x^2)/(2*b^
2) + (h*x^3)/(3*b^2) + (f*x)/b^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.361 $\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=290

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)}{18a^{2/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5\right.}{9a^{2/3}b^{8/3}}$$

Rubi [A] time = 0.50, antiderivative size = 288, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1823, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}-4ag+bd\right)}{18a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)}{9a^{2/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{4/3}h+2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{3\sqrt{3}a^{2/3}b^{8/3}}+\frac{f\log\left(a+bx^3\right)}{3b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (4*g*x)/(3*b^2) + (5*h*x^2)/(6*b^2) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*
x^5)/(3*b*(a + b*x^3)) - ((b^(4/3)*d + 2*a^(1/3)*b*e - 4*a*b^(1/3)*g - 5*a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2
/3)*b^(8/3)) + ((b^(1/3)*(b*d - 4*a*g) - a^(1/3)*(2*b*e - 5*a*h))*Log[a^(1/
3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b*d - 4*a*g - (a^(1/3)*(2*b*e - 5*
a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*
b^(7/3)) + (f*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{a+bx^3}dx}{3b}$$

$$= -\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{\int\left(\frac{4g}{b}+\frac{5hx}{b}+\frac{bd-4ag+(2be-5ah)x+}{b\left(a+bx^3\right)}\right.}{3b}$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{\int\frac{bd-4ag+(2be-5ah)x}{a+bx^3}}{3b^2}$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{\int\frac{bd-4ag+(2be-5ah)x}{a+bx^3}}{3b^2}$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{f\log\left(a+bx^3\right)}{3b^2}+$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{\left(bd-4ag-\frac{\sqrt[3]{a}(2b}{\sqrt[3]{}}\right.}{9}$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}+\frac{\left(bd-4ag-\frac{\sqrt[3]{a}(2b}{\sqrt[3]{}}\right.}{9}$$

$$= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}-\frac{\left(b^{4/3}d+2\sqrt[3]{a}\,be-\right.}{}$$

Mathematica [A] time = 0.25, size = 280, normalized size = 0.97

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}h-2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{a^{2/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^{4/3}h-2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{a^{2/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}h-2\sqrt[3]{a}be+4a\sqrt[3]{b}g-b^{4/3}d\right)}{a^{2/3}}-\frac{6b^{2/3}(b(c+x(d+ex))-a(f+x(g+hx)))}{a+bx^3}+6b^{2/3}f\log\left(a+bx^3\right)+18b^{2/3}gx+9b^{2/3}hx^2}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(2/3)*g*x + 9*b^(2/3)*h*x^2 - (6*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f
+ x*(g + h*x))))/(a + b*x^3) + (2*Sqrt[3]*(-(b^(4/3)*d) - 2*a^(1/3)*b*e + 4
*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^
(2/3) + (2*(b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^
(1/3) + b^(1/3)*x])/a^(2/3) - ((b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3)*g +
 5*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) + 6*b
^(2/3)*f*Log[a + b*x^3])/(18*b^(8/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^2, x]
```

fricas [C] time = 1.81, size = 12153, normalized size = 41.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] 1/36*(18*b*h*x^5 + 36*b*g*x^4 - 6*(2*b*e - 5*a*h)*x^2 - 2*(b^3*x^3 + a*b^2)
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*
f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h
+ (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^
3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*
h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*
h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)
*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1
/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*
g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^
2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h
^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a
^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4
*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)*log(-8*a*b^3*d
*e^2 + 3*a*b^3*d^2*f - 18*a^2*b^2*e*f^2 + 48*a^3*b*f*g^2 - 1/4*(2*a^2*b^6*e
 - 5*a^3*b^5*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 2
0*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*
e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^
3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h
 + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(3
2*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g
^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(
1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h
+ (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^
3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*
h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*
h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)
*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)
^2 - 50*(a^3*b*d - 4*a^4*g)*h^2 + 1/2*(a*b^5*d^2 - 12*a^2*b^4*e*f - 8*a^2*b
^4*d*g + 16*a^3*b^3*g^2 + 30*a^3*b^3*f*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(
9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))
/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(
a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^
3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^
4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 2
4*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*
d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 -
9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d
^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^
2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4
*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^
2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(
a^2*b^8))^(1/3) - 6*f/b^2) + 8*(4*a^2*b^2*e^2 - 3*a^2*b^2*d*f)*g + 5*(8*a^2
*b^2*d*e + 9*a^3*b*f^2 - 32*a^3*b*e*g)*h - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^
3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*
h^2 - 125*a^4*h^3)*x) - 12*b*c + 12*a*f - 12*(b*d - 4*a*g)*x + (18*b*f*x^3
+ (b^3*x^3 + a*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e
 + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^
2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*
```

a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e
^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 -
2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (
16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8
))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*
g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*
a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*
b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*
f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h
)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/
b^2) + 18*a*f - 3*sqrt(1/3)*(b^3*x^3 + a*b^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*
b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*
h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*
g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2
*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3
*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9
*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54
f^3/b^6 - 9(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^
7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*
g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^
3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*
f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*
g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)^2*a*b^5 + 12*(2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)
/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)
*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^
2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b
^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(
9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d
*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f
^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7)
 - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^
3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3
+ 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*
g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)
*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)*a*b^3*f + 32*b^2*d*e + 36*a*b*f^2 - 128
*a*b*e*g - 80*(a*b*d - 4*a^2*g)*h)/(a*b^5)))*log(8*a*b^3*d*e^2 - 3*a*b^3*d^
2*f + 18*a^2*b^2*e*f^2 - 48*a^3*b*f*g^2 + 1/4*(2*a^2*b^6*e - 5*a^3*b^5*h)*(
2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^
2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h +
(9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*
d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^
2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h
+ 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a
^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3
)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g
- 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*
b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3
)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3
b + 3(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e
^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)^2 + 50*(a^3*b*d
- 4*a^4*g)*h^2 - 1/2*(a*b^5*d^2 - 12*a^2*b^4*e*f - 8*a^2*b^4*d*g + 16*a^3*b
^3*g^2 + 30*a^3*b^3*f*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^
2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*
(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3
 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*
b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h
^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*

h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2) - 8*(4*a^2*b^2*e^2 - 3*a^2*b^2*d*f)*g - 5*(8*a^2*b^2*d*e + 9*a^3*b*f^2 - 32*a^3*b*e*g)*h - 2*(b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)*x + 3/4*sqrt(1/3)*(2*a*b^5*d^2 + 12*a^2*b^4*e*f - 16*a^2*b^4*d*g + 32*a^3*b^3*g^2 - 30*a^3*b^3*f*h + (2*a^2*b^6*e - 5*a^3*b^5*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2))*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)^2*a*b^5 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)*a*b^3*f + 32*b^2*d*e + 36*a*b*f^2 - 128*a*b*e*g - 80*(a*b*d - 4*a^2*g)*h)/(a*b^5))) + (18*b*f*x^3 + (b^3*x^3 + a*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(

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1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e
*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a
^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*
h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*
a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(
4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2) + 18*a*f + 3*
sqrt(1/3)*(b^3*x^3 + a*b^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b
^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^
3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7)
- (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3
 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 +
 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g
 + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*
a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^
2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*
a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e
^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 -
2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (
16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8
))^(1/3) - 6*f/b^2)^2*a*b^5 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4
 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/
b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) -
(b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 -
 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 1
25*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g +
 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*
b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*
d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*
b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2
*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*
(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16
*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))
^(1/3) - 6*f/b^2)*a*b^3*f + 32*b^2*d*e + 36*a*b*f^2 - 128*a*b*e*g - 80*(a*b
*d - 4*a^2*g)*h)/(a*b^5)))*log(8*a*b^3*d*e^2 - 3*a*b^3*d^2*f + 18*a^2*b^2*e
*f^2 - 48*a^3*b*f*g^2 + 1/4*(2*a^2*b^6*e - 5*a^3*b^5*h)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)
*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5
*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2
*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(
a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b
+ 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3
- 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*
(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a
*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3
*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4
*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24
*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d
^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)^2 + 50*(a^3*b*d - 4*a^4*g)*h^2 - 1
/2*(a*b^5*d^2 - 12*a^2*b^4*e*f - 8*a^2*b^4*d*g + 16*a^3*b^3*g^2 + 30*a^3*b^
3*f*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h
 + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^
2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 1
2*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^
3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 9
0*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f
*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1
/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2
- 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g +
 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125
```

```
*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*
h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2
- 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2) - 8*(4*a
^2*b^2*e^2 - 3*a^2*b^2*d*f)*g - 5*(8*a^2*b^2*d*e + 9*a^3*b*f^2 - 32*a^3*b*e
*g)*h - 2*(b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a
^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)*x - 3/4*sqrt(1
/3)*(2*a*b^5*d^2 + 12*a^2*b^4*e*f - 16*a^2*b^4*d*g + 32*a^3*b^3*g^2 - 30*a^
3*b^3*f*h + (2*a^2*b^6*e - 5*a^3*b^5*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*
f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(
54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*
b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*
b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*
d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*
e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^
2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*
(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3
 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*
b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h
^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*
h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^
2*b^8))^(1/3) - 6*f/b^2))*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4
 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/
b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) -
(b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 -
 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 1
25*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g +
 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*
b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*
d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*
b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2
*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*
(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16
*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))
^(1/3) - 6*f/b^2)^2*a*b^5 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 -
 (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^
6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b
^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 6
0*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125
*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 2
0*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^
3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*
e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^
3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h
 + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(3
2*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g
^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(
1/3) - 6*f/b^2)*a*b^3*f + 32*b^2*d*e + 36*a*b*f^2 - 128*a*b*e*g - 80*(a*b*d
 - 4*a^2*g)*h)/(a*b^5))))/(b^3*x^3 + a*b^2)
```

giac [A] time = 0.19, size = 307, normalized size = 1.06

$$\frac{f \log \left(\left|bx^{3} + a\right|\right)}{3 \, b^{2}} - \frac{\sqrt{3} \left(b^{2} d - 4 \, abg + 5 \, \left(-ab^{2}\right)^{\frac{1}{3}} ah - 2 \, \left(-ab^{2}\right)^{\frac{1}{3}} be\right) \arctan\left(\frac{\sqrt{3} \left(2 \, x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \, \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \, \left(-ab^{2}\right)^{\frac{2}{3}} b^{2}} - \frac{\left(b^{2} d - 4 \, abg - 5 \, \left(-ab^{2}\right)^{\frac{1}{3}} ah + 2 \, \left(-ab^{2}\right)^{\frac{1}{3}} be\right) \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \, \left(-ab^{2}\right)^{\frac{2}{3}} b^{2}} + \frac{(ah - be)x^{2} - bc + af - \left(bd - ag\right)x}{3 \, \left(bx^{3} + a\right)b^{2}} + \frac{b^{2} h x^{2} + 2 \, b^{2} g x}{2 \, b^{4}} + \frac{\left(5 \, ab^{3} h \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2 \, b^{4} \left(-\frac{a}{b}\right)^{\frac{1}{3}} e - b^{4} d + 4 \, ab^{3} g\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9 \, ab^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] 1/3*f*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(b^2*d - 4*a*b*g + 5*(-a*b^2)^(
1/3)*a*h - 2*(-a*b^2)^(1/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
```

```
a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/18*(b^2*d - 4*a*b*g - 5*(-a*b^2)^(1/3)
*a*h + 2*(-a*b^2)^(1/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*
b^2)^(2/3)*b^2) + 1/3*((a*h - b*e)*x^2 - b*c + a*f - (b*d - a*g)*x)/((b*x^3
 + a)*b^2) + 1/2*(b^2*h*x^2 + 2*b^2*g*x)/b^4 + 1/9*(5*a*b^3*h*(-a/b)^(1/3)
- 2*b^4*(-a/b)^(1/3)*e - b^4*d + 4*a*b^3*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b^5)
```

maple [B] time = 0.06, size = 506, normalized size = 1.74

$$\frac{ahx^2}{3(bx^3+a)b^2}-\frac{ex^2}{3(bx^3+a)b}+\frac{agx}{3(bx^3+a)b^2}-\frac{dx}{3(bx^3+a)b}+\frac{hx^2}{2b^2}+\frac{af}{3(bx^3+a)b^2}-\frac{4\sqrt{3}\,ag\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{4ag\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2ag\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{5\sqrt{3}\,ah\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{5ah\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{5ah\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{c}{3(bx^3+a)b}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{2\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{f\ln(bx^3+a)}{3b^2}+\frac{gx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/2*h*x^2/b^2+g*x/b^2+1/3/b^2/(b*x^3+a)*x^2*a*h-1/3/b/(b*x^3+a)*x^2*e+1/3/b
^2/(b*x^3+a)*a*g*x-1/3/b/(b*x^3+a)*x*d+1/3/b^2/(b*x^3+a)*a*f-1/3/b/(b*x^3+a
)*c-4/9/b^3*a*g/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9/b^3*a*g/(a/b)^(2/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/b^3*a*g/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))+1/9/b^2*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18/b^2*
d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/9/(a/b)^(2/3)*3^(1/2)/b^2
*d*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+5/9/b^3*a*h/(a/b)^(1/3)*ln(x+(a/
b)^(1/3))-5/18/b^3*a*h/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9/b^
3*a*h*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/9/b^2*e
/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+2/9/b^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))+1/3/b^2*f*ln(b*x^3+a)
```

maxima [A] time = 3.03, size = 283, normalized size = 0.98

$$-\frac{(be-ah)x^2+bc-af+(bd-ag)x}{3(b^3x^3+ab^2)}+\frac{\sqrt{3}\left(2be\left(\frac{a}{b}\right)^{\frac{2}{3}}-5ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-4ag\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^2}+\frac{hx^2+2gx}{2b^2}+\frac{\left(6bf\left(\frac{a}{b}\right)^{\frac{2}{3}}+2be\left(\frac{a}{b}\right)^{\frac{1}{3}}-5ah\left(\frac{a}{b}\right)^{\frac{1}{3}}-bd+4ag\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(3bf\left(\frac{a}{b}\right)^{\frac{2}{3}}-2be\left(\frac{a}{b}\right)^{\frac{1}{3}}+5ah\left(\frac{a}{b}\right)^{\frac{1}{3}}+bd-4ag\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] -1/3*((b*e - a*h)*x^2 + b*c - a*f + (b*d - a*g)*x)/(b^3*x^3 + a*b^2) + 1/9*
sqrt(3)*(2*b*e*(a/b)^(2/3) - 5*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) - 4*a*g*(a
/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) + 1/
2*(h*x^2 + 2*g*x)/b^2 + 1/18*(6*b*f*(a/b)^(2/3) + 2*b*e*(a/b)^(1/3) - 5*a*h
*(a/b)^(1/3) - b*d + 4*a*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/
b)^(2/3)) + 1/9*(3*b*f*(a/b)^(2/3) - 2*b*e*(a/b)^(1/3) + 5*a*h*(a/b)^(1/3)
+ b*d - 4*a*g)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 0.14, size = 816, normalized size = 2.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((9*a*b*f^2 + 2*b^2*d*e + 20*a^2*g*h - 5*a*b*d*h - 8*a*b*e*g)/(9*
b^3) + root(729*a^2*b^8*z^3 - 729*a^2*b^6*f*z^2 + 54*a*b^5*d*e*z + 540*a^3*
b^3*g*h*z - 216*a^2*b^4*e*g*z - 135*a^2*b^4*d*h*z + 243*a^2*b^4*f^2*z - 180
*a^3*b*f*g*h - 18*a*b^3*d*e*f + 72*a^2*b^2*e*f*g + 45*a^2*b^2*d*f*h + 150*a
^3*b*e*h^2 + 12*a*b^3*d^2*g - 60*a^2*b^2*e^2*h - 48*a^2*b^2*d*g^2 - 27*a^2*
b^2*f^3 + 64*a^3*b*g^3 + 8*a*b^3*e^3 - 125*a^4*h^3 - b^4*d^3, z, k)*((x*(9*
b^4*d - 36*a*b^3*g))/(9*b^3) - 6*a*f + 9*root(729*a^2*b^8*z^3 - 729*a^2*b^6
```

```
*f*z^2 + 54*a*b^5*d*e*z + 540*a^3*b^3*g*h*z - 216*a^2*b^4*e*g*z - 135*a^2*b
^4*d*h*z + 243*a^2*b^4*f^2*z - 180*a^3*b*f*g*h - 18*a*b^3*d*e*f + 72*a^2*b^
2*e*f*g + 45*a^2*b^2*d*f*h + 150*a^3*b*e*h^2 + 12*a*b^3*d^2*g - 60*a^2*b^2*
e^2*h - 48*a^2*b^2*d*g^2 - 27*a^2*b^2*f^3 + 64*a^3*b*g^3 + 8*a*b^3*e^3 - 12
5*a^4*h^3 - b^4*d^3, z, k)*a*b^2) + (x*(4*b^2*e^2 + 25*a^2*h^2 - 3*b^2*d*f
- 20*a*b*e*h + 12*a*b*f*g))/(9*b^3))*root(729*a^2*b^8*z^3 - 729*a^2*b^6*f*z
^2 + 54*a*b^5*d*e*z + 540*a^3*b^3*g*h*z - 216*a^2*b^4*e*g*z - 135*a^2*b^4*d
*h*z + 243*a^2*b^4*f^2*z - 180*a^3*b*f*g*h - 18*a*b^3*d*e*f + 72*a^2*b^2*e*
f*g + 45*a^2*b^2*d*f*h + 150*a^3*b*e*h^2 + 12*a*b^3*d^2*g - 60*a^2*b^2*e^2*
h - 48*a^2*b^2*d*g^2 - 27*a^2*b^2*f^3 + 64*a^3*b*g^3 + 8*a*b^3*e^3 - 125*a^
4*h^3 - b^4*d^3, z, k), k, 1, 3) - ((b*c)/3 - (a*f)/3 + x*((b*d)/3 - (a*g)/
3) + x^2*((b*e)/3 - (a*h)/3))/(a*b^2 + b^3*x^3) + (h*x^2)/(2*b^2) + (g*x)/b
^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

3.362 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=289

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{18a^{4/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{9a^{4/3}b^{7/3}}$$

Rubi [A] time = 0.51, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1828, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{18a^{4/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{9a^{4/3}b^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{2/3}be-4a^{5/3}h+2ab^{2/3}f+b^{5/3}c\right)}{3\sqrt{3}\,a^{4/3}b^{7/3}}-\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3ab^2\left(a+bx^3\right)}+\frac{g\log\left(a+bx^3\right)}{3b^2}+\frac{hx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (h*x)/b^2 - (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*a*
b^2*(a + b*x^3)) - ((b^(5/3)*c + a^(2/3)*b*e + 2*a*b^(2/3)*f - 4*a^(5/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(7
/3)) - ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(1/3) + b^(1/
3)*x])/(9*a^(4/3)*b^(7/3)) + ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h
))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(7/3)) + (
g*Log[a + b*x^3])/(3*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} - \frac{\int \frac{-a(be-ah)-b(bc+2af)x-}{a+bx^3}}{3ab^2}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} - \frac{\int\left(-3ah-\frac{a(be-4ah)+b}{}\right.}{3ab}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} + \frac{\int \frac{a(be-4ah)+b(bc+2a}{a+bx^3}}{3ab^2}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} + \frac{\int \frac{a(be-4ah)+b(bc+2a}{a+bx^3}}{3ab^2}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} + \frac{g\log\left(a+bx^3\right)}{3b^2} -$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} - \frac{\left(b^{2/3}(bc+2af)-\right.}{}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} - \frac{\left(b^{2/3}(bc+2af)-\right.}{}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} - \frac{\left(b^{5/3}c+a^{2/3}be+\right.}{}$$

Mathematica [A] time = 0.26, size = 285, normalized size = 0.99

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^{2/3}b^{4/3}e+4a^{5/3}\sqrt[3]{b}h+2abf+b^2c\right)}{a^{4/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-a^{2/3}b^{4/3}e+4a^{5/3}\sqrt[3]{b}h+2abf+b^2c\right)}{a^{4/3}} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{2/3}b^{4/3}e-4a^{5/3}\sqrt[3]{b}h+2abf+b^2c\right)}{a^{4/3}} + \frac{6b^{2/3}\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{a\left(a+bx^3\right)} + 6b^{2/3}g\log\left(a+bx^3\right)+18b^{2/3}hx}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(2/3)*h*x + (6*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f
*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + a^(2/3)*b^(4/3)*e + 2*a*b*f -
4*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) -
 (2*(b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(5/3)*b^(1/3)*h)*Log[a^(1/3)
 + b^(1/3)*x])/a^(4/3) + ((b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(5/3)*
b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) + 6*b^(2
/3)*g*Log[a + b*x^3])/(18*b^(8/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3
)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3
)^2, x]
```

fricas [C] time = 2.03, size = 12617, normalized size = 43.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="frica
s")
```

```
[Out] 1/36*(36*a*b*h*x^4 - 12*a*b*d + 12*a^2*g + 12*(b^2*c - a*b*f)*x^2 - 2*(a*b^
3*x^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9
*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*
e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2
*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^
2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 6
4*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*
e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*
f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*
b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*
e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(
9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*
h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^
2)*log(-2*a*b^4*c^2*e - 8*a^2*b^3*c*e*f - 8*a^3*b^2*e*f^2 + 3*a^3*b^2*e^2*g
 + 48*a^5*g*h^2 - 1/4*(a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^
2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*
b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2
+ 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7)
 - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*
a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2
- 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3
/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6)
- (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3
 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6
*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^
3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2
*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)^2 - 9*(a^3*b^2*c + 2*a^4*b*f)*g^2 + 1/2*(
a^3*b^4*e^2 - 8*a^4*b^3*e*h + 16*a^5*b^2*h^2 - 6*(a^3*b^4*c + 2*a^4*b^3*f)*
g)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*
a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8
*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a
*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b
*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*
(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g
*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(
e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12
*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5
*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g
*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (
e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2) + 8*(a^2*b^
3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2 - 3*a^4*b*e*g)*h - (b^5*c^3 + a^2*b^3*e
^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h +
48*a^4*b*e*h^2 - 64*a^5*h^3)*x) - 12*(a*b*e - 4*a^2*h)*x + (18*a*b*g*x^3 +
18*a^2*g + (a*b^3*x^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4
 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3
/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6)
```

- (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3
 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6
*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^
3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2
*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c
*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^
2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e
^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f +
64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6
*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7)
)^(1/3) - 6*g/b^2) - 3*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2)*sqrt(-((2*(1/2)^(2/3
)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2
*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e
*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*
a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*
h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*
h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e
^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3
) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b
)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 +
 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7)
- (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a
^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 -
 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)^2*a^2*b^4 + 12*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f -
2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(
e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12
*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5
*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g
*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (
e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(
3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*
b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2
+ 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7)
 - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*
a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2
- 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)*a^2*b^2*g + 16*b^2*c*e + 3
2*a*b*e*f + 36*a^2*g^2 - 64*(a*b*c + 2*a^2*f)*h)/(a^2*b^4)))*log(2*a*b^4*c^
2*e + 8*a^2*b^3*c*e*f + 8*a^3*b^2*e*f^2 - 3*a^3*b^2*e^2*g - 48*a^5*g*h^2 +
1/4*(a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 -
(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^
6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (
b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 -
12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*
b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 -
 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^
3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e
+ (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b
^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*
h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*
a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^
2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(
1/3) - 6*g/b^2)^2 + 9*(a^3*b^2*c + 2*a^4*b*f)*g^2 - 1/2*(a^3*b^4*e^2 - 8*a^
4*b^3*e*h + 16*a^5*b^2*h^2 - 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*(2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c
*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f
 - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^
2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^
3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h

```
+ 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3
 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*
g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8
*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) -
(b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4
*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3
*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2) - 8*(a^2*b^3*c^2 + 4*a^3*b^2*
c*f + 4*a^4*b*f^2 - 3*a^4*b*e*g)*h - 2*(b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2
*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 -
 64*a^5*h^3)*x + 3/4*sqrt(1/3)*(2*a^3*b^4*e^2 - 16*a^4*b^3*e*h + 32*a^5*b^2
*h^2 + (a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4
 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3
/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6)
- (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3
 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6
*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^
3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2
*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c
*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^
2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e
^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f +
64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6
*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7)
)^(1/3) - 6*g/b^2) + 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*sqrt(-((2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*
h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f
- 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2
*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3
)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h +
 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3
- 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g
/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*
a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (
b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*
b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*
e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)^2*a^2*b^4 + 12*(2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c
*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f
 - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^
2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^
3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h
+ 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3
 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*
g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8
*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) -
(b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4
*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3
*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)*a^2*b^2*g + 16*b^2*c*e + 32*a
*b*e*f + 36*a^2*g^2 - 64*(a*b*c + 2*a^2*f)*h)/(a^2*b^4))) + (18*a*b*g*x^3 +
 18*a^2*g + (a*b^3*x^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^
4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^
3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6)
 - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^
3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 +
6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f
^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^
```

```
2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*
c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a
^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*
e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f +
 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g +
6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7
))^(1/3) - 6*g/b^2) + 3*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2)*sqrt(-((2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f -
2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(
e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12
*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5
*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g
*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (
e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(
3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*
b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2
+ 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7)
 - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*
a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2
- 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)^2*a^2*b^4 + 12*(2*(1/2)^(2
/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f -
 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*
(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 1
2*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^
5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*
g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 -
(e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a
*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2
 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7
) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)
*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2
 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)*a^2*b^2*g + 16*b^2*c*e +
32*a*b*e*f + 36*a^2*g^2 - 64*(a*b*c + 2*a^2*f)*h)/(a^2*b^4)))*log(2*a*b^4*c
^2*e + 8*a^2*b^3*c*e*f + 8*a^3*b^2*e*f^2 - 3*a^3*b^2*e^2*g - 48*a^5*g*h^2 +
 1/4*(a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 -
 (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b
^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) -
(b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 -
 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a
*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3
- 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b
^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e
 + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*
b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2
*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64
*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e
^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^
(1/3) - 6*g/b^2)^2 + 9*(a^3*b^2*c + 2*a^4*b*f)*g^2 - 1/2*(a^3*b^4*e^2 - 8*a
^4*b^3*e*h + 16*a^5*b^2*h^2 - 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*(2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*
c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*
f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a
^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h
^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h
 + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^
3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
 + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)
*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 +
```

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8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) -
 (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^
4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 -
3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2) - 8*(a^2*b^3*c^2 + 4*a^3*b^2
*c*f + 4*a^4*b*f^2 - 3*a^4*b*e*g)*h - 2*(b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^
2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2
- 64*a^5*h^3)*x - 3/4*sqrt(1/3)*(2*a^3*b^4*e^2 - 16*a^4*b^3*e*h + 32*a^5*b^
2*h^2 + (a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^
4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^
3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6)
 - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^
3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 +
6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f
^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^
2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*
c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a
^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*
e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f +
 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g +
6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7
))^(1/3) - 6*g/b^2) + 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*sqrt(-((2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c
*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f
 - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^
2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^
3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h
+ 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3
 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*
g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8
*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) -
(b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4
*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3
*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)^2*a^2*b^4 + 12*(2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*
c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*
f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a
^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h
^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h
 + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^
3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
 + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)
*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 +
8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) -
 (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^
4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 -
3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)*a^2*b^2*g + 16*b^2*c*e + 32*
a*b*e*f + 36*a^2*g^2 - 64*(a*b*c + 2*a^2*f)*h)/(a^2*b^4))))/(a*b^3*x^3 + a^
2*b^2)
```

giac [A] time = 0.20, size = 318, normalized size = 1.10

$$\frac{hx}{b^2} + \frac{g\log\left(\left|bx^3+a\right|\right)}{3\,b^2} + \frac{\sqrt{3}\left(4\,a^2h - abe + \left(-ab^2\right)^{\frac{1}{3}}bc + 2\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab} + \frac{\left(4\,a^2h - abe - \left(-ab^2\right)^{\frac{1}{3}}bc - 2\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{abd - a^2g - \left(b^2c - abf\right)x^2 - \left(a^2h - abe\right)x}{3\left(bx^3+a\right)ab^2} - \frac{\left(ab^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,a^2b^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4\,a^3b^3h + a^2b^4e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac"
)
```

```
[Out] h*x/b^2 + 1/3*g*log(abs(b*x^3 + a))/b^2 + 1/9*sqrt(3)*(4*a^2*h - a*b*e + (-
```

```
a*b^2)^(1/3)*b*c + 2*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) + 1/18*(4*a^2*h - a*b*e - (-a*b^2)
^(1/3)*b*c - 2*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(2/3)*a*b) - 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 - (a^2*h -
 a*b*e)*x)/((b*x^3 + a)*a*b^2) - 1/9*(a*b^5*c*(-a/b)^(1/3) + 2*a^2*b^4*f*(-
a/b)^(1/3) - 4*a^3*b^3*h + a^2*b^4*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
))/(a^3*b^5)
```

maple [B] time = 0.05, size = 502, normalized size = 1.74

$$\frac{c x^2}{3\left(b x^3+a\right)a}-\frac{f x^2}{3\left(b x^3+a\right)b}+\frac{ahx}{3\left(b x^3+a\right)b^2}-\frac{ex}{3\left(b x^3+a\right)b}+\frac{ag}{3\left(b x^3+a\right)b^2}-\frac{4\sqrt{3}\,ah\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{4ah\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2ah\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{d}{3\left(b x^3+a\right)b}+\frac{\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{2\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{2f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{g\ln\left(b x^3+a\right)}{3b^2}+\frac{hx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] h*x/b^2-1/3/(b*x^3+a)/b*f*x^2+1/3/(b*x^3+a)/a*x^2*c+1/3/b^2/(b*x^3+a)*a*h*x
-1/3/b/(b*x^3+a)*e*x+1/3/b^2/(b*x^3+a)*a*g-1/3/b/(b*x^3+a)*d-4/9/b^3*a/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))*h+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*e+2/9/b^3
*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*h-1/18/b^2/(a/b)^(2/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*e-4/9/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*h+1/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1))*e-2/9/(a/b)^(1/3)/b^2*f*ln(x+(a/b)^(1/3))-1/9/b/a/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/9/(a/b)^(1/3)/b^2*f*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))+1/18/(a/b)^(1/3)/a/b*c*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9*3^(
1/2)/(a/b)^(1/3)/b^2*f*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9/b/a*3^(1
/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/3*g*ln(b*x^3+a)
/b^2
```

maxima [A] time = 2.96, size = 311, normalized size = 1.08

$$-\frac{abd-a^2g-\left(b^2c-abf\right)x^2+\left(abe-a^2h\right)x}{3\left(ab^3x^3+a^2b^2\right)}+\frac{hx}{b^2}+\frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}+\frac{\left(6\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-abe+4\,a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(3\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+abe-4\,a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxim
a")
```

```
[Out] -1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 + (a*b*e - a^2*h)*x)/(a*b^3*x^3 +
 a^2*b^2) + h*x/b^2 + 1/9*sqrt(3)*(b^2*c*(a/b)^(2/3) + 2*a*b*f*(a/b)^(2/3)
+ a*b*e*(a/b)^(1/3) - 4*a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(a^2*b^2) + 1/18*(6*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/
3) + 2*a*b*f*(a/b)^(1/3) - a*b*e + 4*a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)
^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/9*(3*a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3)
- 2*a*b*f*(a/b)^(1/3) + a*b*e - 4*a^2*h)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^
(2/3))
```

mupad [B] time = 5.39, size = 827, normalized size = 2.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((9*a^2*g^2 + b^2*c*e - 8*a^2*f*h - 4*a*b*c*h + 2*a*b*e*f)/(9*a*b
^2) - root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4*b^3*f*h*z - 108*a^
3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243*a^4*b^3*g^2*z + 72*
a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*a^2*b^3*c*e*g - 48*a^
4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2*b^3*c*f^2 + 8*a^3*b^2
```

```
*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3*e^3, z, k)*(6*a*g - b*
e*x + 4*a*h*x - 9*root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4*b^3*f*
h*z - 108*a^3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243*a^4*b^3
*g^2*z + 72*a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*a^2*b^3*c
*e*g - 48*a^4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2*b^3*c*f^2
 + 8*a^3*b^2*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3*e^3, z, k)
*a*b^2) + (x*(b^3*c^2 + 4*a^2*b*f^2 + 12*a^3*g*h + 4*a*b^2*c*f - 3*a^2*b*e*
g))/(9*a^2*b^2))*root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4*b^3*f*h
*z - 108*a^3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243*a^4*b^3*
g^2*z + 72*a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*a^2*b^3*c*
e*g - 48*a^4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2*b^3*c*f^2
+ 8*a^3*b^2*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3*e^3, z, k),
 k, 1, 3) - ((b*d)/3 - (a*g)/3 + x*((b*e)/3 - (a*h)/3) - (b*x^2*(b*c - a*f)
)/(3*a))/(a*b^2 + b^3*x^3) + (h*x)/b^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

3.363 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=276

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{18a^{5/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+b\right.}{9a^{5/3}b^{5/3}}$$

Rubi [A] time = 0.37, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1858, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{18a^{5/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{9a^{5/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)}{3\sqrt{3}a^{5/3}b^{5/3}}+\frac{h\log\left(a+bx^3\right)}{3b^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a*b*(a + b*x^3)) - ((2
*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*(2*b*
c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(5
/3)) - ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(5/3)) + (h*Log[a + b*x^3])/(3*b
^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^2}\,dx = \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} - \frac{\int \frac{-b(2bc+af)-b(bd+2ag)x-3abhx^2}{a+bx^3}}{3ab^2}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} - \frac{\int \frac{-b(2bc+af)-b(bd+2ag)x}{a+bx^3}\,dx}{3ab^2} + \cdots$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} + \frac{h\log\left(a+bx^3\right)}{3b^2} - \int \frac{\sqrt[3]{a}\left(-2b^4\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} + \frac{\left(\sqrt[3]{b}\,(2bc+af)-\sqrt[3]{a}\,(bd+\cdots\right.}{9a^{5/3}b^{5\cdots}}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} + \frac{\left(\sqrt[3]{b}\,(2bc+af)-\sqrt[3]{a}\,(bd+\cdots\right.}{9a^{5/3}b^{5\cdots}}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} - \frac{\left(2b^{4/3}c+\sqrt[3]{a}\,bd+a\sqrt[3]{b}\,f+2\cdots\right.}{3\sqrt{3}\,a^{5/3}\cdots}$$

Mathematica [A] time = 0.21, size = 268, normalized size = 0.97

$$\frac{\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}g+\sqrt[3]{a}bd-a\sqrt[3]{b}f-2b^{4/3}c\right)}{a^{5/3}}+\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^{4/3}g-\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)}{a^{5/3}}-\frac{2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)}{a^{5/3}}+\frac{6\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{a\left(a+bx^3\right)}+6h\log\left(a+bx^3\right)}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

```
[Out] ((6*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a*(a + b*x^3)) - (2
*Sqrt[3]*b^(1/3)*(2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(2*b^(4/3)*
c - a^(1/3)*b*d + a*b^(1/3)*f - 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(5
/3) + (b^(1/3)*(-2*b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f + 2*a^(4/3)*g)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 6*h*Log[a + b*x^3])/
(18*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{\left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,
x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,
 x]
```

fricas [C] time = 1.87, size = 12636, normalized size = 45.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas"
)
```

```
[Out] -1/36*(12*a*b*e - 12*a^2*h - 12*(b^2*d - a*b*g)*x^2 + 2*(a*b^3*x^3 + a^2*b^
2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g +
9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*
a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b
^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 1
2*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 -
 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g
 - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1
/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*
h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*
c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*
a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b +
(f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c
)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)*log(4*a*b^4
*c*d^2 + 2*a^2*b^3*d^2*f + 1/4*(a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*
c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2
 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2
*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4
*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^
3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a
^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a
*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f
^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) +
(8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(
4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12
*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)^2 + 8*(2*a^3*b^2*c + a^4*b*f)*g^
```

```
2 + 9*(a^4*b*d + 2*a^5*g)*h^2 - 1/2*(4*a^2*b^5*c^2 + 4*a^3*b^4*c*f + a^4*b^
3*f^2 - 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h
^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b
^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a
*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f
^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) +
(8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(
4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12
*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 -
 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6)
+ (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6
*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^
5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*
a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^
5*b^6))^(1/3) - 6*h/b^2) + 8*(2*a^2*b^3*c*d + a^3*b^2*d*f)*g - 3*(4*a^2*b^3
*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + (8*b^5*c^3 + a*b^4*d^3 + 12*a*b^4*c^2
*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^2*b^3*d^2*g + 12*a^3*b^2*d*g^2 + 8
*a^4*b*g^3)*x) - 12*(b^2*c - a*b*f)*x - (18*a*b*h*x^3 + 18*a^2*h + (a*b^3*x
^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a
^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b
^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^
4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^
2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 -
 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2
 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f
*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3
+ 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b
*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*
h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2
 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)
+ 3*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a
^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*
g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2
*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5
) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h -
 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3
- 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b
^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b
^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3
 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 2
7*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)
*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)
/(a^5*b^6))^(1/3) - 6*h/b^2)^2*a^3*b^4 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(
a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c
*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^
2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^
5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h
- 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3
 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/
b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*
b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^
3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 +
27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h
)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4
)/(a^5*b^6))^(1/3) - 6*h/b^2)*a^3*b^2*h + 32*b^3*c*d + 16*a*b^2*d*f + 36*a^
3*h^2 + 32*(2*a*b^2*c + a^2*b*f)*g)/(a^3*b^4)))*log(-4*a*b^4*c*d^2 - 2*a^2*
```

```
b^3*d^2*f - 1/4*(a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(
9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^
3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g
)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*
c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5)
 + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h -
3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 -
 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^
6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^
6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3
+ 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27
*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*
d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/
(a^5*b^6))^(1/3) - 6*h/b^2)^2 - 8*(2*a^3*b^2*c + a^4*b*f)*g^2 - 9*(a^4*b*d
+ 2*a^5*g)*h^2 + 1/2*(4*a^2*b^5*c^2 + 4*a^3*b^4*c*f + a^4*b^3*f^2 - 6*(a^4*
b^3*d + 2*a^5*b^2*g)*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3
*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^
6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^
6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3
+ 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27
*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*
d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/
(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d +
 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 +
a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g
+ 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^
3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^
2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) -
 6*h/b^2) - 8*(2*a^2*b^3*c*d + a^3*b^2*d*f)*g + 3*(4*a^2*b^3*c^2 + 4*a^3*b^
2*c*f + a^4*b*f^2)*h + 2*(8*b^5*c^3 + a*b^4*d^3 + 12*a*b^4*c^2*f + 6*a^2*b^
3*c*f^2 + a^3*b^2*f^3 + 6*a^2*b^3*d^2*g + 12*a^3*b^2*d*g^2 + 8*a^4*b*g^3)*x
 + 3/4*sqrt(1/3)*(8*a^2*b^5*c^2 + 8*a^3*b^4*c*f + 2*a^4*b^3*f^2 + (a^4*b^5*
d + 2*a^5*b^4*g)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d +
2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(
2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8
*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2
*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^
3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*
b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^
6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*
b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d
^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^
3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f
*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (
f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^
2) + 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a
^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*
g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2
*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5
) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h -
 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3
- 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b
^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b
^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3
 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 2
7*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)
*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)
/(a^5*b^6))^(1/3) - 6*h/b^2)^2*a^3*b^4 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)
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*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(
a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c
*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^
2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^
5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h
- 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3
 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/
b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*
b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^
3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 +
27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h
)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4
)/(a^5*b^6))^(1/3) - 6*h/b^2)*a^3*b^2*h + 32*b^3*c*d + 16*a*b^2*d*f + 36*a^
3*h^2 + 32*(2*a*b^2*c + a^2*b*f)*g)/(a^3*b^4))) - (18*a*b*h*x^3 + 18*a^2*h
+ (a*b^3*x^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3
*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^
6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^
6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3
+ 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27
*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*
d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/
(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d +
 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 +
a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g
+ 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^
3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^
2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) -
 6*h/b^2) - 3*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)
*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (
d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f +
 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3
)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 +
36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b
^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2
)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 +
 a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b
^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^
2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2
*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)^2*a^3*b^4 + 12*(2*(1/2)^(2/3)*(-I*sq
rt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g
)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 +
(d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f
+ 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^
3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 +
 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*
b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1
)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^
2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2
+ a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*
b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g
^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^
2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)*a^3*b^2*h + 32*b^3*c*d + 16*a*b^2*d
*f + 36*a^3*h^2 + 32*(2*a*b^2*c + a^2*b*f)*g)/(a^3*b^4)))*log(-4*a*b^4*c*d^
2 - 2*a^2*b^3*d^2*f - 1/4*(a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^(2/3)*(-I*sqrt
(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*
a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d
*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f +
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6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)
/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 3
6*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^
3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*
(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)
*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 +
a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^
5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2
 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*
f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)^2 - 8*(2*a^3*b^2*c + a^4*b*f)*g^2 - 9
*(a^4*b*d + 2*a^5*g)*h^2 + 1/2*(4*a^2*b^5*c^2 + 4*a^3*b^4*c*f + a^4*b^3*f^2
 - 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^
4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/
(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)
*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 +
a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^
5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2
 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*
f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2
*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*
b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*
b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3
 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b
^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6
))^(1/3) - 6*h/b^2) - 8*(2*a^2*b^3*c*d + a^3*b^2*d*f)*g + 3*(4*a^2*b^3*c^2
+ 4*a^3*b^2*c*f + a^4*b*f^2)*h + 2*(8*b^5*c^3 + a*b^4*d^3 + 12*a*b^4*c^2*f
+ 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^2*b^3*d^2*g + 12*a^3*b^2*d*g^2 + 8*a^
4*b*g^3)*x - 3/4*sqrt(1/3)*(8*a^2*b^5*c^2 + 8*a^3*b^4*c*f + 2*a^4*b^3*f^2 +
 (a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*
b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3
/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3
*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f
^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 +
 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*
h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^
4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*
d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3
 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2
*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4
*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*
(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3
) - 6*h/b^2) + 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*sqrt(-((2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)
*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (
d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f +
 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3
)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 +
36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b
^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2
)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 +
 a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b
^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^
2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2
*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)^2*a^3*b^4 + 12*(2*(1/2)^(2/3)*(-I*sq
rt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g
)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 +
(d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f
+ 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^
```

```
3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 +
 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*
b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1
)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^
2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2
+ a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*
b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g
^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^
2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)*a^3*b^2*h + 32*b^3*c*d + 16*a*b^2*d
*f + 36*a^3*h^2 + 32*(2*a*b^2*c + a^2*b*f)*g)/(a^3*b^4))))/(a*b^3*x^3 + a^2
*b^2)
```

giac [**A**] time = 0.19, size = 302, normalized size = 1.09

$$\frac{h\log\left(\left|bx^3+a\right|\right)}{3b^2}-\frac{\sqrt{3}\left(2b^2c+abf-\left(-ab^2\right)^{\frac{1}{3}}bd-2\left(-ab^2\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab}-\frac{\left(2b^2c+abf+\left(-ab^2\right)^{\frac{1}{3}}bd+2\left(-ab^2\right)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab}+\frac{\left(bd-ag\right)x^2+\left(bc-af\right)x+\frac{a^2h-abe}{b}}{3\left(bx^3+a\right)ab}-\frac{\left(ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2a^2b^2g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2ab^3c+a^2b^2f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*h*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(2*b^2*c + a*b*f - (-a*b^2)^(1/
3)*b*d - 2*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/18*(2*b^2*c + a*b*f + (-a*b^2)^(1/3)*b*d
 + 2*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)
^(2/3)*a*b) + 1/3*((b*d - a*g)*x^2 + (b*c - a*f)*x + (a^2*h - a*b*e)/b)/((b
*x^3 + a)*a*b) - 1/9*(a*b^3*d*(-a/b)^(1/3) + 2*a^2*b^2*g*(-a/b)^(1/3) + 2*a
*b^3*c + a^2*b^2*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

maple [**B**] time = 0.05, size = 462, normalized size = 1.67

$$\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{2\sqrt{3}\,g\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{2g\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{g\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{h\ln\left(bx^3+a\right)}{3b^2}+\frac{-\frac{\left(ag-bd\right)x^2}{3ab}-\frac{\left(af-bc\right)x}{3ab}+\frac{ah-be}{3b^2}}{bx^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (-1/3*(a*g-b*d)/a/b*x^2-1/3*(a*f-b*c)/a/b*x+1/3*(a*h-b*e)/b^2)/(b*x^3+a)+1/
9/(a/b)^(2/3)/b^2*f*ln(x+(a/b)^(1/3))+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))
*c-1/18/(a/b)^(2/3)/b^2*f*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/9/b/a/(a/b)^(
2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/9/(a/b)^(2/3)*3^(1/2)/b^2*f*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-2/9/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*g-1/9/
b/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*d+1/9/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))*g+1/18/b/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d+2
/9/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*g+1/9/b/
a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+1/3*h*ln(b*
x^3+a)/b^2
```

maxima [**A**] time = 2.95, size = 292, normalized size = 1.06

$$-\frac{abe-a^2h-\left(b^2d-abg\right)x^2-\left(b^2c-abf\right)x}{3\left(ab^3x^3+a^2b^2\right)}+\frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+2abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+2b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^2}+\frac{\left(6ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+2ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-2bc-af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(3ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-2ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+2bc+af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima"
)
```

```
[Out] -1/3*(a*b*e - a^2*h - (b^2*d - a*b*g)*x^2 - (b^2*c - a*b*f)*x)/(a*b^3*x^3 +
 a^2*b^2) + 1/9*sqrt(3)*(b^2*d*(a/b)^(2/3) + 2*a*b*g*(a/b)^(2/3) + 2*b^2*c*
```

```
(a/b)^(1/3) + a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(a^2*b^2) + 1/18*(6*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) + 2*a*g*(a/
b)^(1/3) - 2*b*c - a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)
^(2/3)) + 1/9*(3*a*h*(a/b)^(2/3) - b*d*(a/b)^(1/3) - 2*a*g*(a/b)^(1/3) + 2*
b*c + a*f)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [B] time = 5.54, size = 835, normalized size = 3.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x)
```

```
[Out] symsum(log((root(729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3*f*g*z + 1
08*a^3*b^4*c*g*z + 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*b^2*h^2*z
- 18*a^4*b*f*g*h - 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^3*c*d*h -
12*a*b^4*c^2*f + 12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c*f^2 + 8*a
^4*b*g^3 + a*b^4*d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k)*(9*root(
729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3*f*g*z + 108*a^3*b^4*c*g*z
+ 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*b^2*h^2*z - 18*a^4*b*f*g*h
- 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^3*c*d*h - 12*a*b^4*c^2*f +
12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c*f^2 + 8*a^4*b*g^3 + a*b^4*
d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k)*a^2*b^2 - 6*a^2*h + 2*b^2
*c*x + a*b*f*x))/a + (9*a^3*h^2 + 2*b^3*c*d + 4*a*b^2*c*g + a*b^2*d*f + 2*a
^2*b*f*g)/(9*a^2*b^2) + (x*(b^2*d^2 + 4*a^2*g^2 - 3*a^2*f*h - 6*a*b*c*h + 4
*a*b*d*g))/(9*a^2*b))*root(729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3
*f*g*z + 108*a^3*b^4*c*g*z + 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*
b^2*h^2*z - 18*a^4*b*f*g*h - 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^
3*c*d*h - 12*a*b^4*c^2*f + 12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c
*f^2 + 8*a^4*b*g^3 + a*b^4*d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k
), k, 1, 3) + ((x*(b*c - a*f))/(3*a*b) - (b*e - a*h)/(3*b^2) + (x^2*(b*d -
a*g))/(3*a*b))/(a + b*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.364 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2} dx$

Optimal. Leaf size=289

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(ag+2bd)-\sqrt[3]{a}(2ah+be)\right)}{18a^{5/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+2bd)-\sqrt[3]{a}(2ah\right.}{9a^{5/3}b^{5/3}}$$

Rubi [A] time = 0.56, antiderivative size = 287, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2ah+be)}{\sqrt[3]{b}}+ag+2bd\right)}{18a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+2bd)-\sqrt[3]{a}(2ah+be)\right)}{9a^{5/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{4/3}h+\sqrt[3]{a}be+a\sqrt[3]{b}g+2b^{4/3}d\right)}{3\sqrt{3}a^{5/3}b^{5/3}}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{3a^2b\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^2}+\frac{c\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(3*a^2*b*(a + b*x
^3)) - ((2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + (c*Lo
g[x])/a^2 + ((b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e + 2*a*h))*Log[a^(1/3) +
b^(1/3)*x])/(9*a^(5/3)*b^(5/3)) - ((2*b*d + a*g - (a^(1/3)*(b*e + 2*a*h))/b
^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3)
) - (c*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}\,dx = \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-b(2bd+ag)x-b(be+2\cdots}{x\left(a+bx^3\right)}}{3ab^2}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} - \frac{\int\left(-\frac{3b^2c}{ax}+\frac{b\left(-a(2bd+ag)\cdots\right.}{a\cdots}\right.}{3ab\cdots}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{\int \frac{-a(2bd+ag)\cdots}{\cdots}}{\cdots}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{\int \frac{-a(2bd+ag)\cdots}{a+\cdots}}{3a\cdots}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{c\log\left(a+b\cdots\right.}{3a^2}\cdots$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2bd+ag-\cdots\right.}{\cdots}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2bd+ag-\cdots\right.}{\cdots}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} - \frac{\left(2b^{4/3}d+\sqrt[3]{a}\,be+a\sqrt[3]{b}\cdots\right.}{3\sqrt{\cdots}}$$

Mathematica [A] time = 0.22, size = 269, normalized size = 0.93

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}h+\sqrt[3]{a}be-a\sqrt[3]{b}g-2b^{4/3}d\right)}{b^{5/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^{4/3}h-\sqrt[3]{a}be+a\sqrt[3]{b}g+2b^{4/3}d\right)}{b^{5/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2a^{4/3}h+\sqrt[3]{a}be+a\sqrt[3]{b}g+2b^{4/3}d\right)}{b^{5/3}}-\frac{6a(a(f+x(g+hx))-b(c+x(d+ex)))}{b\left(a+bx^3\right)}-6c\log\left(a+bx^3\right)+18c\log(x)}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)) - (2
*Sqrt[3]*a^(1/3)*(2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) + 18*c*Log[x] + (2*a^(1/
3)*(2*b^(4/3)*d - a^(1/3)*b*e + a*b^(1/3)*g - 2*a^(4/3)*h)*Log[a^(1/3) + b^
(1/3)*x])/b^(5/3) + (a^(1/3)*(-2*b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g + 2*
a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 6*c*Lo
g[a + b*x^3])/(18*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)
^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)
^2), x]
```

fricas [C] time = 35.29, size = 12541, normalized size = 43.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="frica
s")
```

```
[Out] 1/324*(108*a*b*c - 108*a^2*f + 108*(a*b*e - a^2*h)*x^2 - 2*(a^2*b^2*x^3 + a
^3*b)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h +
 (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^
2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 +
a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h
+ 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 -
(g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^
2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1
/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e +
2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^
3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3
*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 1
2*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*
(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54
*c/a^2)*log(12*b^4*c*d^2 + 9*b^4*c^2*e + 4*a*b^3*d*e^2 + 3*a^2*b^2*c*g^2 +
1/324*(a^4*b^4*e + 2*a^5*b^3*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 +
 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 +
 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b
^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^
3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*
(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g
*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c
*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(
9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/
1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3
+ 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*
c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*
b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*
b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2 + 8*(2*a^3*b*d + a^4*g)*h^2 - 1/18*(4*a
^2*b^4*d^2 + 6*a^2*b^4*c*e + 4*a^3*b^3*d*g + a^4*b^2*g^2 + 12*a^3*b^3*c*h)*
((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g
+ 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e
+ 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*
e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a
^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 -
 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g +
9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) +
81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*
g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12
*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h
^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^
2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g +
 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2
) + 2*(6*a*b^3*c*d + a^2*b^2*e^2)*g + 2*(9*a*b^3*c^2 + 8*a^2*b^2*d*e + 4*a^
3*b*e*g)*h + (8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^
3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)*x) + 108*(a*b*d - a
^2*g)*x - (162*b^2*c*x^3 + 162*a*b*c - (a^2*b^2*x^3 + a^3*b)*((-I*sqrt(3) +
 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b
)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h +
```

```
(e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3
*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8
*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4
*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h
)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3)
+ 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4
*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g +
 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3
)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(
d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2
*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2) - 3*sqrt(1/3
)*(a^2*b^2*x^3 + a^3*b)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 +
2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 +
1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^
3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3
*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(
27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*
h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*
d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9
*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1
458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 +
 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c
^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b
^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b
^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2*a^4*b^3 - 108*((-I*sqrt(3) + 1)*(9*c^2/a
^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))
/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h
)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a
^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a
^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^
2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3
 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*
c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)
*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d
*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5)
- 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*
h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*
d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)*a^2*b^3*c + 2916*b^3*c^2
 + 2592*a*b^2*d*e + 1296*a^2*b*e*g + 2592*(2*a^2*b*d + a^3*g)*h)/(a^4*b^3))
)*log(-12*b^4*c*d^2 - 9*b^4*c^2*e - 4*a*b^3*d*e^2 - 3*a^2*b^2*c*g^2 - 1/324
*(a^4*b^4*e + 2*a^5*b^3*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*
b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/16
2*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) +
 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g
^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b
^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a
^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)
*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3
*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*
(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a
^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 +
 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 +
 (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/
(a^6*b^5))^(1/3) + 54*c/a^2)^2 - 8*(2*a^3*b*d + a^4*g)*h^2 + 1/18*(4*a^2*b^
4*d^2 + 6*a^2*b^4*c*e + 4*a^3*b^3*d*g + a^4*b^2*g^2 + 12*a^3*b^3*c*h)*((-I*
sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d
*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a
^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 +
 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*
```

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e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e
*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*
g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I
*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h +
 (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^
3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 +
8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^
4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*
h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2) - 2
*(6*a*b^3*c*d + a^2*b^2*e^2)*g - 2*(9*a*b^3*c^2 + 8*a^2*b^2*d*e + 4*a^3*b*e
*g)*h + 2*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b
*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)*x + 1/108*sqrt(1/3)*(7
2*a^2*b^4*d^2 - 54*a^2*b^4*c*e + 72*a^3*b^3*d*g + 18*a^4*b^2*g^2 - 108*a^3*
b^3*c*h + (a^4*b^4*e + 2*a^5*b^3*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c
^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a
^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a
^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2
+ a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1
458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3
*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 -
 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/1
62*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3)
+ 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*
g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*
b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*
a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e
)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2))*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4
- (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-
1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a
^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*
b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*
b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 -
 e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 -
2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3
/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/
(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^
2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1
/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h -
 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3
 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2*a^4*b^3 - 108*((-I*sqrt(3
) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^
2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h
 + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*
b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2
+ 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*
a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*
d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(
3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g
+ 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*
g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*
h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b -
6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*
a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)*a^2*b^3*c
 + 2916*b^3*c^2 + 2592*a*b^2*d*e + 1296*a^2*b*e*g + 2592*(2*a^2*b*d + a^3*g
)*h)/(a^4*b^3))) - (162*b^2*c*x^3 + 162*a*b*c - (a^2*b^2*x^3 + a^3*b)*((-I*
sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d
*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a
^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 +
 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*
```

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e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e
*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*
g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I
*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h +
 (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^
3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 +
8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^
4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*
h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2) + 3
*sqrt(1/3)*(a^2*b^2*x^3 + a^3*b)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b
^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c
^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*
c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*
g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) -
 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h
 - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d
^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 +
 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b
^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^
3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*
(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g
*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c
*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2*a^4*b^3 - 108*((-I*sqrt(3) + 1)
*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(
a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*
g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^
2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^
4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b
- 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c
)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1
)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*
h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*
a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(
a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g
^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^
3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)*a^2*b^3*c + 291
6*b^3*c^2 + 2592*a*b^2*d*e + 1296*a^2*b*e*g + 2592*(2*a^2*b*d + a^3*g)*h)/(
a^4*b^3)))*log(-12*b^4*c*d^2 - 9*b^4*c^2*e - 4*a*b^3*d*e^2 - 3*a^2*b^2*c*g^
2 - 1/324*(a^4*b^4*e + 2*a^5*b^3*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c
^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a
^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a
^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2
+ a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1
458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3
*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 -
 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/1
62*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3)
+ 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*
g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*
b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*
a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e
)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2 - 8*(2*a^3*b*d + a^4*g)*h^2 + 1/18*
(4*a^2*b^4*d^2 + 6*a^2*b^4*c*e + 4*a^3*b^3*d*g + a^4*b^2*g^2 + 12*a^3*b^3*c
*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (
e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*
d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*
b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h +
12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g
^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*
```

```
g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3
) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*
a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3
+ 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b
*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*
e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e
*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c
/a^2) - 2*(6*a*b^3*c*d + a^2*b^2*e^2)*g - 2*(9*a*b^3*c^2 + 8*a^2*b^2*d*e +
4*a^3*b*e*g)*h + 2*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^
2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)*x - 1/108*sqr
t(1/3)*(72*a^2*b^4*d^2 - 54*a^2*b^4*c*e + 72*a^3*b^3*d*g + 18*a^4*b^2*g^2 -
 108*a^3*b^3*c*h + (a^4*b^4*e + 2*a^5*b^3*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 -
 (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1
/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^
2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b
^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b
^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 -
e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2
*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/
a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(
a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2
 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/
1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h -
3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3
- 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2))*sqrt(-(((-I*sqrt(3) + 1)*(9
*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4
*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g +
 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g
 + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h
^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6
*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a
^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(
-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*
a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2
*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5
*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2
- e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 -
 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2*a^4*b^3 - 108*((
-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g +
4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e +
2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^
3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3
*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 1
2*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*
(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81
*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*
h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a
*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2
 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)
*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4
*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)*
a^2*b^3*c + 2916*b^3*c^2 + 2592*a*b^2*d*e + 1296*a^2*b*e*g + 2592*(2*a^2*b*
d + a^3*g)*h)/(a^4*b^3))) + 324*(b^2*c*x^3 + a*b*c)*log(x))/(a^2*b^2*x^3 +
a^3*b)
```

giac [A] time = 0.20, size = 319, normalized size = 1.10

$$-\frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{c\log\left(|x|\right)}{a^2}-\frac{\sqrt{3}\left(2\,b^2d+abg-2\,\left(-ab^2\right)^{\frac{1}{3}}ah-\left(-ab^2\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}ab}-\frac{\left(2\,b^2d+abg+2\,\left(-ab^2\right)^{\frac{1}{3}}ah+\left(-ab^2\right)^{\frac{1}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}ab}+\frac{abc-a^2f-\left(a^2h-abe\right)x^2+\left(abd-a^2g\right)x}{3\left(bx^3+a\right)a^2b}-\frac{\left(2\,a^4b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+2\,a^3b^3d+a^4b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac"
)
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^2 + c*log(abs(x))/a^2 - 1/9*sqrt(3)*(2*b^2*d +
 a*b*g - 2*(-a*b^2)^(1/3)*a*h - (-a*b^2)^(1/3)*b*e)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/18*(2*b^2*d + a*b*g
 + 2*(-a*b^2)^(1/3)*a*h + (-a*b^2)^(1/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) + 1/3*(a*b*c - a^2*f - (a^2*h - a*b*e)*x^2
 + (a*b*d - a^2*g)*x)/((b*x^3 + a)*a^2*b) - 1/9*(2*a^4*b^2*h*(-a/b)^(1/3) +
 a^3*b^3*(-a/b)^(1/3)*e + 2*a^3*b^3*d + a^4*b^2*g)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^5*b^3)
```

maple [B]   time = 0.06, size = 507, normalized size = 1.75

$$\frac{e x^{2}}{3 \left(b x^{3}+a\right) a}-\frac{h x^{2}}{3 \left(b x^{3}+a\right) b}+\frac{d x}{3 \left(b x^{3}+a\right) a}-\frac{g x}{3 \left(b x^{3}+a\right) b}+\frac{2 \sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} a b}+\frac{2 d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} a b}-\frac{d \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} a b}+\frac{\sqrt{3}\, e \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a b}-\frac{e \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} a b}+\frac{e \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{a}{b}\right)^{\frac{1}{3}} a b}+\frac{c}{3 \left(b x^{3}+a\right) a}+\frac{c \ln(x)}{a^{2}}-\frac{c \ln\left(b x^{3}+a\right)}{3 a^{2}}-\frac{f}{3 \left(b x^{3}+a\right) b}+\frac{\sqrt{3}\, g \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}+\frac{g \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}-\frac{g \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}+\frac{2 \sqrt{3}\, h \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}-\frac{2 h \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}+\frac{h \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x)
```

```
[Out] -1/3/(b*x^3+a)/b*x^2*h+1/3/(b*x^3+a)/a*e*x^2-1/3/(b*x^3+a)/b*x*g+1/3/a*x/(b
*x^3+a)*d-1/3/(b*x^3+a)/b*f+1/3/a/(b*x^3+a)*c+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))*g+2/9/a/b*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18/b^2/(a/b)^(2/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*g-1/9/(a/b)^(2/3)/a/b*d*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))*g+2/9/a/b*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))-2/9/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*h-1/9/(a/b)^(1/3)/a/b*e*ln(x+(a/b
)^(1/3))+1/9/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*h+1/18/(a/b)
^(1/3)/a/b*e*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/b^2*3^(1/2)/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h+1/9*3^(1/2)/(a/b)^(1/3)/a/b*e*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/a^2*c*ln(b*x^3+a)+1/a^2*c*ln(x)
```

maxima [A]   time = 3.04, size = 302, normalized size = 1.04

$$\frac{(be-ah)x^{2}+bc-af+\left(bd-ag\right)x}{3\left(ab^{2}x^{3}+a^{2}b\right)}+\frac{c\log(x)}{a^{2}}+\frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,a^{2}h\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}b}-\frac{\left(6\,b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abd+a^{2}g\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\,b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abd-a^{2}g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxim
a")
```

```
[Out] 1/3*((b*e - a*h)*x^2 + b*c - a*f + (b*d - a*g)*x)/(a*b^2*x^3 + a^2*b) + c*l
og(x)/a^2 + 1/9*sqrt(3)*(a*b*e*(a/b)^(2/3) + 2*a^2*h*(a/b)^(2/3) + 2*a*b*d*
(a/b)^(1/3) + a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(a^3*b) - 1/18*(6*b^2*c*(a/b)^(2/3) - a*b*e*(a/b)^(1/3) - 2*a^2*h
*(a/b)^(1/3) + 2*a*b*d + a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2
*b^2*(a/b)^(2/3)) - 1/9*(3*b^2*c*(a/b)^(2/3) + a*b*e*(a/b)^(1/3) + 2*a^2*h*
(a/b)^(1/3) - 2*a*b*d - a^2*g)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

mupad [B]   time = 5.60, size = 1660, normalized size = 5.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x)
```

```
[Out] ((b*c - a*f)/(3*a*b) + (x*(b*d - a*g))/(3*a*b) + (x^2*(b*e - a*h))/(3*a*b))
/(a + b*x^3) + symsum(log((c*(4*b^2*d^2 + a^2*g^2 - 3*b^2*c*e - 6*a*b*c*h +
 4*a*b*d*g))/(9*a^3) - (root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b
^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^
2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*
```

```
b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2
*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z,
 k)*(a^3*g^2 + 4*a*b^2*d^2 + 36*b^3*c^2*x + 324*root(729*a^6*b^5*z^3 + 729*
a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 5
4*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 3
6*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a
^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b
^5*c^3 + a^2*b^3*e^3, z, k)^2*a^4*b^3*x - 18*root(729*a^6*b^5*z^3 + 729*a^4
*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a
^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a
^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*
b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*
c^3 + a^2*b^3*e^3, z, k)*a^4*b*h + 6*a*b^2*c*e + 12*a^2*b*c*h + 4*a^2*b*d*g
 + 20*a^3*g*h*x - 9*root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g
*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^
5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*
c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g
^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)*
a^3*b^2*e + 216*root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z
 + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^5*c^
2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*
g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 -
 a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)*a^2*
b^3*c*x + 20*a*b^2*d*e*x + 40*a^2*b*d*h*x + 10*a^2*b*e*g*x))/(9*a^2) - (x*(
8*a^4*h^3 - 8*b^4*d^3 + a*b^3*e^3 - a^3*b*g^3 - 6*a^2*b^2*d*g^2 + 6*a^2*b^2
*e^2*h + 12*b^4*c*d*e - 12*a*b^3*d^2*g + 12*a^3*b*e*h^2 + 12*a^2*b^2*c*g*h
+ 24*a*b^3*c*d*h + 6*a*b^3*c*e*g))/(27*a^3*b^2))*root(729*a^6*b^5*z^3 + 729
*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z +
54*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h +
36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*
a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*
b^5*c^3 + a^2*b^3*e^3, z, k), k, 1, 3) + (c*log(x))/a^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.365 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=301

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{18a^{7/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc\right.}{9a^{7/3}b^{4/3}}$$

Rubi [A] time = 0.59, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{18a^{7/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{9a^{7/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+4b^{5/3}c\right)}{3\sqrt{3}a^{7/3}b^{4/3}}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3a^2b\left(a+bx^3\right)}-\frac{d\log\left(a+bx^3\right)}{3a^2}-\frac{c}{a^2x}+\frac{d\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(c/(a^2*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3
*a^2*b*(a + b*x^3)) + ((4*b^(5/3)*c - 2*a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)
*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b
^(4/3)) + (d*Log[x])/a^2 + ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(4/3)) - ((b^(2/3)*(4*b*c - a*f) +
a^(2/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*
a^(7/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx = \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-3b^2dx-b(2be+ah)x^2 \cdots}{x^2\left(a+bx^3\right)}}{3ab^2 \cdots}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^2c}{ax^2}-\frac{3b^2d}{ax}+\frac{b\left(-a(2 \cdots\right.}{\cdots}\right.}{\cdots}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \frac{\int \cdots}{\cdots}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \frac{\int \cdots}{\cdots}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \frac{d\,\mathrm{lo}\cdots}{\cdots}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{\left(b^2 \cdots\right.}{\cdots}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{\left(b^2 \cdots\right.}{\cdots}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{\left(4b^{5/3}c-2a^{2/3} \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.40, size = 285, normalized size = 0.95

$$-\frac{\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{2/3}be+a^{5/3}h-ab^{2/3}f+4b^{5/3}c\right)}{b^{4/3}} - \frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^{2/3}be+a^{5/3}h-ab^{2/3}f+4b^{5/3}c\right)}{b^{4/3}} + \frac{2\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2a^{2/3}be+a^{5/3}h+ab^{2/3}f-4b^{5/3}c\right)}{b^{4/3}} + \frac{6a\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{b\left(a+bx^3\right)} + 6ad\log\left(a+bx^3\right) + \frac{18ac}{x} - 18ad\log(x)}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -1/18*((18*a*c)/x + (6*a*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))
))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-4*b^(5/3)*c + 2*a^(2/3)*b*e + a*b
^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/3)
- 18*a*d*Log[x] - (2*a^(2/3)*(4*b^(5/3)*c + 2*a^(2/3)*b*e - a*b^(2/3)*f + a
^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(4*b^(5/3)*c + 2*a^(
2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/b^(4/3) + 6*a*d*Log[a + b*x^3])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^
3)^2),x]
```

[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2), x]

fricas [C] time = 35.59, size = 12556, normalized size = 41.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")

[Out] -1/324*(108*(4*b^2*c - a*b*f)*x^3 + 324*a*b*c - 108*(a*b*e - a^2*h)*x^2 + 2*(a^2*b^2*x^4 + a^3*b*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2)*log(-36*a*b^4*c*d^2 + 64*a*b^4*c^2*e + 12*a^2*b^3*d*e^2 + 4*a^3*b^2*e*f^2 + 3*a^4*b*d*h^2 - 1/324*(4*a^5*b^4*c - a^6*b^3*f)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2)^2 + 1/18*(24*a^3*b^4*c*d - 4*a^4*b^3*e^2 - 6*a^4*b^3*d*f - 4*a^5*b^2*e*h - a^6*b*h^2)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2) + (9*a^2*b^3*d^2 - 32*a^2*b^3*c*e)*f + 2*(16*a^2*b^3*c^2 + 6*a^3*b^2*d*e - 8*a^3*b^2*c*f + a^4*b*f^2)*h - (64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)*x) - 108*(a*b*d - a^2*g)*x + (162*b^2*d*x^4 + 162*a*b*d*x - (a^2*b^2*x^4 + a^3*b*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(

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a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(
64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^
3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*
c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e
^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)
*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f
*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^
5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 1
2*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 +
 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 -
9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^
4)/(a^7*b^4))^(1/3) + 54*d/a^2) - 3*sqrt(1/3)*(a^2*b^2*x^4 + a^3*b*x)*sqrt(
-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 -
8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*
b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 -
 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4
*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^
3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*
h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 8
1*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (
9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a
*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*
h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (
f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)
*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a
^2)^2*a^4*b^2 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f - 2*c*
h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a^2*f*h +
2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3
 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3
*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^
4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e
*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a
^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*
f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*
a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*
e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e
*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f +
6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^
4))^(1/3) + 54*d/a^2)*a^2*b^2*d + 2916*b^2*d^2 - 10368*b^2*c*e + 2592*a*b*e
*f - 1296*(4*a*b*c - a^2*f)*h)/(a^4*b^2)))*log(36*a*b^4*c*d^2 - 64*a*b^4*c^
2*e - 12*a^2*b^3*d*e^2 - 4*a^3*b^2*e*f^2 - 3*a^4*b*d*h^2 + 1/324*(4*a^5*b^4
*c - a^6*b^3*f)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f - 2*c*h)*a
*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e
*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8
*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2
*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*
e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f +
 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b
^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f -
2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*
b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*
h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2
 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f
^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^
(1/3) + 54*d/a^2)^2 - 1/18*(24*a^3*b^4*c*d - 4*a^4*b^3*e^2 - 6*a^4*b^3*d*f
- 4*a^5*b^2*e*h - a^6*b*h^2)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e
*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a
^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(6
4*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3
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 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c
^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^
3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*
a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*
h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5
*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12
*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 +
6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9
*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4
)/(a^7*b^4))^(1/3) + 54*d/a^2) - (9*a^2*b^3*d^2 - 32*a^2*b^3*c*e)*f - 2*(16
*a^2*b^3*c^2 + 6*a^3*b^2*d*e - 8*a^3*b^2*c*f + a^4*b*f^2)*h - 2*(64*b^5*c^3
 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3
*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)*x + 1/108*sqrt(1/3)*(216*a^3*b^4*c*d
+ 72*a^4*b^3*e^2 - 54*a^4*b^3*d*f + 72*a^5*b^2*e*h + 18*a^6*b*h^2 - (4*a^5*
b^4*c - a^6*b^3*f)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f - 2*c*h
)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2
*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3
- 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*
b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4
*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*
f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^
7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f
 - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a
^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e
^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*
h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6
*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4
))^(1/3) + 54*d/a^2))*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e
*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a
^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(6
4*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3
 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c
^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^
3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*
a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*
h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5
*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12
*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 +
6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9
*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4
)/(a^7*b^4))^(1/3) + 54*d/a^2)^2*a^4*b^2 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4
 - (a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*
d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^
6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c
*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4)
+ 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)
*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*
c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a
^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2
) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2
- a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1
458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*
b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e
 + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2)*a^2*b^2*d + 2916*b^2*d^2 -
 10368*b^2*c*e + 2592*a*b*e*f - 1296*(4*a*b*c - a^2*f)*h)/(a^4*b^2))) + (16
2*b^2*d*x^4 + 162*a*b*d*x - (a^2*b^2*x^4 + a^3*b*x)*((-I*sqrt(3) + 1)*(9*d^
2/a^4 - (a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-
1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*
d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*
```

b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*
b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d
*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3
- 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*
d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^
6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c
*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4)
+ 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)
*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*
c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2) + 3*sqrt(1/3)*(a^2*b^
2*x^4 + a^3*b*x)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f -
2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a^2*f*
h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5
*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12
*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 +
6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9
*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4
)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2
*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3
- 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*
b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4
*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*
f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^
7*b^4))^(1/3) + 54*d/a^2)^2*a^4*b^2 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a
^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a
^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2
) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2
- a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1
458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*
b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e
 + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 +
1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1
/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3
*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(
64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 +
 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16
*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2)*a^2*b^2*d + 2916*b^2*d^2 - 1036
8*b^2*c*e + 2592*a*b*e*f - 1296*(4*a*b*c - a^2*f)*h)/(a^4*b^2)))*log(36*a*b
^4*c*d^2 - 64*a*b^4*c^2*e - 12*a^2*b^3*d*e^2 - 4*a^3*b^2*e*f^2 - 3*a^4*b*d*
h^2 + 1/324*(4*a^5*b^4*c - a^6*b^3*f)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^2*f
h + 2(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^6 +
 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) -
1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^
3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*
(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2
+ 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 1
6*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/16
2*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/145
8*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2
*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b
^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(
4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2
*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2)^2 - 1/18*(24*a^3*b^4*c*d - 4*a^4*b^
3*e^2 - 6*a^4*b^3*d*f - 4*a^5*b^2*e*h - a^6*b*h^2)*((-I*sqrt(3) + 1)*(9*d^2
/a^4 - (a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1
/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d
/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b
^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b
^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*

```
f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 -
 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d
^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6
*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*
f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) +
 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*
a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c
*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2) - (9*a^2*b^3*d^2 - 32*
a^2*b^3*c*e)*f - 2*(16*a^2*b^3*c^2 + 6*a^3*b^2*d*e - 8*a^3*b^2*c*f + a^4*b*
f^2)*h - 2*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2
- a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)*x - 1/108*sqrt(
1/3)*(216*a^3*b^4*c*d + 72*a^4*b^3*e^2 - 54*a^4*b^3*d*f + 72*a^5*b^2*e*h +
18*a^6*b*h^2 - (4*a^5*b^4*c - a^6*b^3*f)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (a^
2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1/27*d^3/a^
6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2)
 - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 -
 a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/14
58*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b
^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e
+ 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1
/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/
1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*
b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) + 1/1458*(6
4*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*a^3*b^2 +
2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*
c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2))*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2
/a^4 - (a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)/(a^4*b^2))/(-1
/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d
/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b
^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b
^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*
f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 -
 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d
^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b^2)*d/(a^6
*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2*f + 12*a^2*b^3*c*
f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^5*h^3)/(a^7*b^4) +
 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12*e^2*h + 9*d*f*h)*
a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3 - 3*(9*d^3 - 24*c
*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2)^2*a^4*b^2 - 108*((-I*s
qrt(3) + 1)*(9*d^2/a^4 - (a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 - 8*c*e)*b
^2)/(a^4*b^2))/(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d
^2 - 8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^
4*c^2*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2
 - a^5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3
 - 12*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^
2*b^3 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 81*(I*sqr
t(3) + 1)*(-1/27*d^3/a^6 + 1/162*(a^2*f*h + 2*(e*f - 2*c*h)*a*b + (9*d^2 -
8*c*e)*b^2)*d/(a^6*b^2) - 1/1458*(64*b^5*c^3 - 8*a^2*b^3*e^3 - 48*a*b^4*c^2
*f + 12*a^2*b^3*c*f^2 - a^3*b^2*f^3 - 12*a^3*b^2*e^2*h - 6*a^4*b*e*h^2 - a^
5*h^3)/(a^7*b^4) + 1/1458*(64*b^5*c^3 + 6*a^4*b*e*h^2 + a^5*h^3 - (f^3 - 12
*e^2*h + 9*d*f*h)*a^3*b^2 + 2*(4*e^3 - 9*d*e*f + 6*(f^2 + 3*d*h)*c)*a^2*b^3
 - 3*(9*d^3 - 24*c*d*e + 16*c^2*f)*a*b^4)/(a^7*b^4))^(1/3) + 54*d/a^2)*a^2*
b^2*d + 2916*b^2*d^2 - 10368*b^2*c*e + 2592*a*b*e*f - 1296*(4*a*b*c - a^2*f
)*h)/(a^4*b^2))) - 324*(b^2*d*x^4 + a*b*d*x)*log(x))/(a^2*b^2*x^4 + a^3*b*x
)
```

giac [A] time = 0.20, size = 328, normalized size = 1.09

$$-\frac{d\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{2}}+\frac{d\log\left(\left|x\right|\right)}{a^{2}}-\frac{\sqrt{3}\left(a^{2}h+2\,abe+4\,\left(-ab^{2}\right)^{\frac{1}{3}}bc-\left(-ab^{2}\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{\left(a^{2}h+2\,abe-4\,\left(-ab^{2}\right)^{\frac{1}{3}}bc+\left(-ab^{2}\right)^{\frac{1}{3}}af\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}-\frac{4\,b^{2}cx^{3}-abfx^{3}+a^{2}hx^{2}-abx^{2}e-abdx+a^{2}gx+3\,abc}{3\left(bx^{4}+ax\right)a^{2}b}+\frac{\left(4\,a^{2}b^{4}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}b^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{4}b^{2}h-2\,a^{3}b^{3}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{5}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 - 1/9*sqrt(3)*(a^2*h + 2
*a*b*e + 4*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/18*(a^2*h + 2*a*b*e
 - 4*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/3*(4*b^2*c*x^3 - a*b*f*x^3 + a^2*h*x^2
 - a*b*x^2*e - a*b*d*x + a^2*g*x + 3*a*b*c)/((b*x^4 + a*x)*a^2*b) + 1/9*(4*
a^2*b^4*c*(-a/b)^(1/3) - a^3*b^3*f*(-a/b)^(1/3) - a^4*b^2*h - 2*a^3*b^3*e)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b^3)
```

maple [B] time = 0.06, size = 517, normalized size = 1.72

$$\frac{f x^2}{3\left(b x^3+a\right)a}-\frac{b c x^2}{3\left(b x^3+a\right)a^2}+\frac{e x}{3\left(b x^3+a\right)a}-\frac{h x}{3\left(b x^3+a\right)b}+\frac{2\sqrt{3}\, e \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} a b}+\frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} a b}-\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} a b}+\frac{\sqrt{3}\, f \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a b}-\frac{f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a b}+\frac{f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}} a b}+\frac{d}{3\left(b x^3+a\right)a}-\frac{4\sqrt{3}\, c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2}+\frac{4c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2}-\frac{2c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a^2}+\frac{d\ln(x)}{a^2}-\frac{d\ln\left(b x^3+a\right)}{3a^2}-\frac{g}{3\left(b x^3+a\right)b}+\frac{\sqrt{3}\, h \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} b^2}+\frac{h\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} b^2}-\frac{h\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}} b^2}-\frac{c}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a/(b*x^3+a)*x^2*f-1/3/a^2/(b*x^3+a)*b*c*x^2-1/3/(b*x^3+a)/b*x*h+1/3/(b*
x^3+a)/a*e*x-1/3/(b*x^3+a)/b*g+1/3/a/(b*x^3+a)*d+1/9/b^2/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*h+2/9/(a/b)^(2/3)/a/b*e*ln(x+(a/b)^(1/3))-1/18/b^2/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*h-1/9/(a/b)^(2/3)/a/b*e*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))*h+2/9/(a/b)^(2/3)*3^(1/2)/a/b*e*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))-1/9/(a/b)^(1/3)/a/b*f*ln(x+(a/b)^(1/3))+4/9/a^2/(a/b)^(1/3)*ln(x+(a/
b)^(1/3))*c+1/18/a/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-2/9/a^
2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1/9*3^(1/2)/(a/b)^(1/3)/a
/b*f*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-4/9/a^2*3^(1/2)/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-1/3/a^2*d*ln(b*x^3+a)-1/a^2*c/x+1/a
^2*d*ln(x)
```

maxima [A] time = 3.13, size = 329, normalized size = 1.09

$$-\frac{\left(4 b^2 c-a b f\right)x^3+3 a b c-\left(a b e-a^2 h\right)x^2-\left(a b d-a^2 g\right)x}{3\left(a^2 b^2 x^4+a^3 b x\right)}+\frac{d\log(x)}{a^2}-\frac{\sqrt{3}\left(4 b^2 c\left(\frac{a}{b}\right)^{\frac{2}{3}}-a b f\left(\frac{a}{b}\right)^{\frac{2}{3}}-2 a b e\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2 h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 a^3 b}-\frac{\left(6 b^2 d\left(\frac{a}{b}\right)^{\frac{2}{3}}+4 b^2 c\left(\frac{a}{b}\right)^{\frac{1}{3}}-a b f\left(\frac{a}{b}\right)^{\frac{1}{3}}+2 a b e+a^2 h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 a^2 b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3 b^2 d\left(\frac{a}{b}\right)^{\frac{2}{3}}-4 b^2 c\left(\frac{a}{b}\right)^{\frac{1}{3}}+a b f\left(\frac{a}{b}\right)^{\frac{1}{3}}-2 a b e-a^2 h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 a^2 b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] -1/3*((4*b^2*c - a*b*f)*x^3 + 3*a*b*c - (a*b*e - a^2*h)*x^2 - (a*b*d - a^2*
g)*x)/(a^2*b^2*x^4 + a^3*b*x) + d*log(x)/a^2 - 1/9*sqrt(3)*(4*b^2*c*(a/b)^(
2/3) - a*b*f*(a/b)^(2/3) - 2*a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3))*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b) - 1/18*(6*b^2*d*(a/b)^
(2/3) + 4*b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3) + 2*a*b*e + a^2*h)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) - 1/9*(3*b^2*d*(a/b)^(
2/3) - 4*b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) - 2*a*b*e - a^2*h)*log(x + (
a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

mupad [B] time = 5.77, size = 1684, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((d*(a^3*h^2 + 4*a*b^2*e^2 + 12*b^3*c*d - 3*a*b^2*d*f + 4*a^2*b*e
*h))/(9*a^4) - (root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h*z
```

```
 - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^4*d
^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3*d*e
*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f^2 -
 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z, k)*(
a^3*h^2 + 4*a*b^2*e^2 + 36*b^3*d^2*x - 24*b^3*c*d + 324*root(729*a^7*b^4*z^
3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e
*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d
*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f
 - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5
*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z, k)^2*a^4*b^3*x + 6*a*b^2*d*f + 4*a^2*b*
e*h - 80*b^3*c*e*x + 36*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b
^2*f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a
^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2
*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3
*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3,
 z, k)*a^2*b^3*c - 9*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*
f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*
b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^
3*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*
f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z,
 k)*a^3*b^2*f + 216*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f
*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b
^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3
*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f
^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z,
k)*a^2*b^3*d*x - 40*a*b^2*c*h*x + 20*a*b^2*e*f*x + 10*a^2*b*f*h*x))/(9*a^2)
 + (x*(64*b^5*c^3 + a^5*h^3 + 8*a^2*b^3*e^3 - a^3*b^2*f^3 + 12*a^2*b^3*c*f^
2 + 12*a^3*b^2*e^2*h - 48*a*b^4*c^2*f + 6*a^4*b*e*h^2 + 24*a^2*b^3*c*d*h -
12*a^2*b^3*d*e*f - 6*a^3*b^2*d*f*h + 48*a*b^4*c*d*e))/(27*a^5*b))*root(729*
a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h*z - 108*a^4*b^3*c*h*z + 54
*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9
*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a
*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^4
*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z, k), k, 1, 3) - (c/a + (x^3*(4
*b*c - a*f))/(3*a^2) - (x*(b*d - a*g))/(3*a*b) - (x^2*(b*e - a*h))/(3*a*b))
/(a*x + b*x^4) + (d*log(x))/a^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.366 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=306

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)}{18a^{8/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd\right.}{9a^{8/3}b^{2/3}}$$

Rubi [A] time = 0.58, antiderivative size = 304, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}-2af+5bc\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)}{9a^{8/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/3}c\right)}{3\sqrt{3}a^{8/3}b^{2/3}}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3a^2\left(a+bx^3\right)}-\frac{e\log\left(a+bx^3\right)}{3a^2}-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^
2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*c + 4*a^(1/3)*b*d - 2*a*b^(1/3)*f - a
^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
8/3)*b^(2/3)) + (e*Log[x])/a^2 - ((b^(1/3)*(5*b*c - 2*a*f) - a^(1/3)*(4*b*d
 - a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^(2/3)) + ((5*b*c - 2*a*f -
(a^(1/3)*(4*b*d - a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^(8/3)*b^(1/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-3b^2dx-3b^2ex^2+2b^2\left(\frac{bc}{a}-\right.}{x^3\left(a+bx^3\right)}}{3ab^2}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^2c}{ax^3} - \frac{3b^2d}{ax^2} - \frac{3b^2e}{ax} + \frac{b^2}{}\right.}{3a}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} -$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} -$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} -$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} -$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} -$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)} + \frac{\left(5b^{4/3}c+4\right.}{}$$

Mathematica [A] time = 0.54, size = 292, normalized size = 0.95

$$-\frac{-\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}g-4\sqrt[3]{a}bd+2a\sqrt[3]{b}f-5b^{4/3}c\right)}{b^{2/3}}+\frac{6a\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{b\left(a+bx^3\right)}+6ae\log\left(a+bx^3\right)+\frac{9ac}{x^2}+\frac{18ad}{x}-18ae\log(x)}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/18*((9*a*c)/x^2 + (18*a*d)/x + (6*a*(a^2*h + b^2*x*(c + d*x) - a*b*(e +
x*(f + g*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(1/3)*(-5*b^(4/3)*c - 4*a^(1/
3)*b*d + 2*a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/b^(2/3) - 18*a*e*Log[x] + (2*a^(1/3)*(5*b^(4/3)*c - 4*a^(1/3)*b*d - 2
*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)*(5*b
^(4/3)*c - 4*a^(1/3)*b*d - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*a*e*Log[a + b*x^3])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^
3)^2),x]
```

[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2), x]

fricas [C] time = 24.67, size = 12231, normalized size = 39.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")

[Out] -1/324*(108*(4*b^2*d - a*b*g)*x^4 + 324*a*b*d*x + 54*(5*b^2*c - 2*a*b*f)*x^3 + 162*a*b*c - 108*(a*b*e - a^2*h)*x^2 + 2*(a^2*b^2*x^5 + a^3*b*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)*log(-160*a*b^3*c*d^2 + 75*a*b^3*c^2*e - 36*a^2*b^2*d*e^2 + 12*a^3*b*e*f^2 - 1/324*(4*a^6*b^2*d - a^7*b*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2 - 2*(5*a^3*b*c - 2*a^4*f)*g^2 - 1/18*(25*a^3*b^3*c^2 - 24*a^4*b^2*d*e - 20*a^4*b^2*c*f + 4*a^5*b*f^2 + 6*a^5*b*e*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2) + 4*(16*a^2*b^2*d^2 - 15*a^2*b^2*c*e)*f + (80*a^2*b^2*c*d + 9*a^3*b*e^2 - 32*a^3*b*d*f)*g - (125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)*x) + (162*b^2*e*x^5 + 162*a*b*e*x^2 - (a^2*b^2*x^5 + a^3*b*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b

)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2) - 3*sqrt(1/3)*(a^2*b^2*x^5 + a^3*b*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2*a^5*b - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)*a^3*b*e + 25920*b^2*c*d + 2916*a*b*e^2 - 10368*a*b*d*f - 1296*(5*a*b*c - 2*a^2*f)*g)/(a^5*b)))*log(160*a*b^3*c*d^2 - 75*a*b^3*c^2*e + 36*a^2*b^2*d*e^2 - 12*a^3*b*e*f^2 + 1/324*(4*a^6*b^2*d - a^7*b*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2 + 2*(5*a^3*b*c - 2*a^4*f)*g^2 + 1/18*(25*a^3*b^3*c^2 - 24*a^4*b^2*d*e - 20*a^4*b^2*c*f + 4*a^5*b*f^2 + 6*a^5*b*e*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*

a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d
^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3
- 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*
(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b
^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*
f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b
^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*
g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2
*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*
f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2)
)^(1/3) + 54*e/a^2) - 4*(16*a^2*b^2*d^2 - 15*a^2*b^2*c*e)*f - (80*a^2*b^2*c
*d + 9*a^3*b*e^2 - 32*a^3*b*d*f)*g - 2*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*
b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*
g^2 - a^4*g^3)*x + 1/108*sqrt(1/3)*(450*a^3*b^3*c^2 + 216*a^4*b^2*d*e - 360
*a^4*b^2*c*f + 72*a^5*b*f^2 - 54*a^5*b*e*g - (4*a^6*b^2*d - a^7*b*g)*((-I*s
qrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*
a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d
*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3
*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2
 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g
+ 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^
2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sq
rt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f
- 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^
2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 -
a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6
*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b
^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2))*
sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*
d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g +
 (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^
3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 1
2*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f
^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 -
 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/
3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9
*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 -
 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a
^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3
- 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*
e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3)
+ 54*e/a^2)^2*a^5*b - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^
2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^
2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^
4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 4
8*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3
 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16
*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*
b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c
*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c
^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a
^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 +
a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^
2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3
)/(a^8*b^2))^(1/3) + 54*e/a^2)*a^3*b*e + 25920*b^2*c*d + 2916*a*b*e^2 - 103
68*a*b*d*f - 1296*(5*a*b*c - 2*a^2*f)*g)/(a^5*b))) + (162*b^2*e*x^5 + 162*a
*b*e*x^2 - (a^2*b^2*x^5 + a^3*b*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2
*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1
/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1

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458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^
3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(
125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24
*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 7
5*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/16
2*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458
*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b
*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125
*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*
e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c
^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2) + 3*sqrt(1/3)*(a^2*b^2*x^5 + a^3*
b*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e
^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^
2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a
*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^
2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 -
 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(
4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^
2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f
*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^
3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g
 + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*
(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f
^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))
^(1/3) + 54*e/a^2)^2*a^5*b - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d
 + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162
*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*
(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*
f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*
b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e
*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^
2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(2
0*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(12
5*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3
 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4
*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f
+ 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f
)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)*a^3*b*e + 25920*b^2*c*d + 2916*a*b*e^
2 - 10368*a*b*d*f - 1296*(5*a*b*c - 2*a^2*f)*g)/(a^5*b)))*log(160*a*b^3*c*d
^2 - 75*a*b^3*c^2*e + 36*a^2*b^2*d*e^2 - 12*a^3*b*e*f^2 + 1/324*(4*a^6*b^2*
d - a^7*b*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^
2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2
*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*
b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2
*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 -
2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4
*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2
))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*
g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3
*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g
+ 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(
4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^
2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^
(1/3) + 54*e/a^2)^2 + 2*(5*a^3*b*c - 2*a^4*f)*g^2 + 1/18*(25*a^3*b^3*c^2 -
24*a^4*b^2*d*e - 20*a^4*b^2*c*f + 4*a^5*b*f^2 + 6*a^5*b*e*g)*((-I*sqrt(3) +
 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^
5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c
*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f +
 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g
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^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^
2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 -
2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) +
1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)
*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60
*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)
/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*
a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(
32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2) - 4*(16*a
^2*b^2*d^2 - 15*a^2*b^2*c*e)*f - (80*a^2*b^2*c*d + 9*a^3*b*e^2 - 32*a^3*b*d
*f)*g - 2*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2
- 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)*x - 1/108*sqrt
(1/3)*(450*a^3*b^3*c^2 + 216*a^4*b^2*d*e - 360*a^4*b^2*c*f + 72*a^5*b*f^2 -
 54*a^5*b*e*g - (4*a^6*b^2*d - a^7*b*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*
b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6
+ 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) -
1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8
*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/145
8*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 -
 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e
+ 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1
/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1
458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^
3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(
125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24
*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 7
5*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2))*sqrt(-(((-I*sqrt(3) + 1)*(9*e
^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-
1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)
*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*
b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8
*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b
 + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^
3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/2
7*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/
(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2
*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^
2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b +
3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 -
 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2*a^5*b - 108*((-
I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*
g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 -
8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*
b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*
g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f
*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)
*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I
*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d
*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3
*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2
 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g
+ 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^
2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2
)*a^3*b*e + 25920*b^2*c*d + 2916*a*b*e^2 - 10368*a*b*d*f - 1296*(5*a*b*c -
2*a^2*f)*g)/(a^5*b))) - 324*(b^2*e*x^5 + a*b*e*x^2)*log(x))/(a^2*b^2*x^5 +
a^3*b*x^2)
```

giac [A]   time = 0.19, size = 336, normalized size = 1.10

$$-\frac{e\log\left(\left|bx^{3}+a\right|\right)}{3a^{2}}+\frac{e\log\left(\left|x\right|\right)}{a^{2}}+\frac{\sqrt{3}\left(5b^{2}c-2abf-4\left(-ab^{2}\right)^{\frac{1}{3}}bd+\left(-ab^{2}\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}+\frac{\left(5b^{2}c-2abf+4\left(-ab^{2}\right)^{\frac{1}{3}}bd-\left(-ab^{2}\right)^{\frac{1}{3}}ag\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}+\frac{\left(4a^{2}b^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5a^{2}b^{2}c-2a^{3}bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^{5}b}-\frac{2\left(4b^{2}d-abg\right)x^{4}+6abdx+\left(5b^{2}c-2abf\right)x^{3}+3abc+2\left(a^{2}h-abe\right)x^{2}}{6\left(bx^{3}+a\right)a^{2}bx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 + 1/9*sqrt(3)*(5*b^2*c -
 2*a*b*f - 4*(-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) + 1/18*(5*b^2*c - 2*a
*b*f + 4*(-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) + 1/9*(4*a^2*b^2*d*(-a/b)^(1/3) - a^3*
b*g*(-a/b)^(1/3) + 5*a^2*b^2*c - 2*a^3*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^5*b) - 1/6*(2*(4*b^2*d - a*b*g)*x^4 + 6*a*b*d*x + (5*b^2*c - 2*
a*b*f)*x^3 + 3*a*b*c + 2*(a^2*h - a*b*e)*x^2)/((b*x^3 + a)*a^2*b*x^2)
```

maple [B]   time = 0.07, size = 527, normalized size = 1.72

$$\frac{gx^{2}}{3\left(bx^{3}+a\right)a}-\frac{bdx^{2}}{3\left(bx^{3}+a\right)a^{2}}+\frac{fx}{3\left(bx^{3}+a\right)a}-\frac{bcx}{3\left(bx^{3}+a\right)a^{2}}+\frac{2\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,g\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{g\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{g\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{e}{3\left(bx^{3}+a\right)a}-\frac{5\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{5c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{5c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{4\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}+\frac{4d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}-\frac{2d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}+\frac{e\ln\left(x\right)}{a^{2}}-\frac{e\ln\left(bx^{3}+a\right)}{3a^{2}}-\frac{h}{3\left(bx^{3}+a\right)b}-\frac{d}{a^{2}x}-\frac{c}{2a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a/(b*x^3+a)*x^2*g-1/3/(b*x^3+a)/a^2*b*d*x^2+1/3/a/(b*x^3+a)*f*x-1/3/(b*
x^3+a)/a^2*b*c*x-1/3/(b*x^3+a)/b*h+1/3/(b*x^3+a)/a*e-5/9/(a/b)^(2/3)/a^2*c*
ln(x+(a/b)^(1/3))+5/18/(a/b)^(2/3)/a^2*c*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-
5/9/(a/b)^(2/3)*3^(1/2)/a^2*c*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/9/(
a/b)^(2/3)/a/b*f*ln(x+(a/b)^(1/3))-1/9/a*f/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+2/9/a*f/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))+4/9/(a/b)^(1/3)/a^2*d*ln(x+(a/b)^(1/3))-2/9/(a/b)^(1/3)/a^2*d*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9*3^(1/2)/(a/b)^(1/3)/a^2*d*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/a*g/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/18/a*g
/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/9/a*g*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/a^2*e*ln(b*x^3+a)-1/a^2*d/x
+1/a^2*e*ln(x)-1/2/a^2*c/x^2
```

maxima [A]   time = 3.06, size = 316, normalized size = 1.03

$$-\frac{2\left(4b^{2}d-abg\right)x^{4}+6abdx+\left(5b^{2}c-2abf\right)x^{3}+3abc-2\left(abe-a^{2}h\right)x^{2}}{6\left(a^{2}b^{2}x^{5}+a^{3}bx^{2}\right)}+\frac{e\log\left(x\right)}{a^{2}}-\frac{\sqrt{3}\left(4bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-ag\left(\frac{a}{b}\right)^{\frac{2}{3}}+5bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-2af\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^{3}}-\frac{\left(6be\left(\frac{a}{b}\right)^{\frac{2}{3}}+4bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5bc+2af\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3be\left(\frac{a}{b}\right)^{\frac{2}{3}}-4bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+5bc-2af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] -1/6*(2*(4*b^2*d - a*b*g)*x^4 + 6*a*b*d*x + (5*b^2*c - 2*a*b*f)*x^3 + 3*a*b
*c - 2*(a*b*e - a^2*h)*x^2)/(a^2*b^2*x^5 + a^3*b*x^2) + e*log(x)/a^2 - 1/9*
sqrt(3)*(4*b*d*(a/b)^(2/3) - a*g*(a/b)^(2/3) + 5*b*c*(a/b)^(1/3) - 2*a*f*(a
/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/18*(
6*b*e*(a/b)^(2/3) + 4*b*d*(a/b)^(1/3) - a*g*(a/b)^(1/3) - 5*b*c + 2*a*f)*lo
g(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/9*(3*b*e*(a/b)
^(2/3) - 4*b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) + 5*b*c - 2*a*f)*log(x + (a/b)
^(1/3))/(a^2*b*(a/b)^(2/3))
```

mupad [B]   time = 5.71, size = 1632, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] symsum(log((b^2*e*(25*b^2*c^2 + 4*a^2*f^2 - 3*a^2*e*g - 20*a*b*c*f + 12*a*b
*d*e))/(9*a^5) - (root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z
 - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*
e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*
e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^
2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z,
 k)*b^2*(25*b^2*c^2 + 4*a^2*f^2 - 9*root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^
2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*
d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e
*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g
 + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4
*c^3 + a^4*g^3, z, k)*a^4*g + 6*a^2*e*g + 36*root(729*a^8*b^2*z^3 + 729*a^6
*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a
^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^
2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*
b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3
+ 125*b^4*c^3 + a^4*g^3, z, k)*a^3*b*d + 36*a*b*e^2*x + 200*b^2*c*d*x + 20*
a^2*f*g*x + 324*root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z -
 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^
2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*
g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2
+ 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k
)^2*a^5*b*x - 20*a*b*c*f - 24*a*b*d*e - 50*a*b*c*g*x - 80*a*b*d*f*x + 216*r
oot(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*
z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*
f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^
2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3
- 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k)*a^3*b*e*x))/(9*
a^3) - (b*x*(125*b^4*c^3 + a^4*g^3 - 64*a*b^3*d^3 - 8*a^3*b*f^3 + 60*a^2*b^
2*c*f^2 + 48*a^2*b^2*d^2*g - 150*a*b^3*c^2*f - 12*a^3*b*d*g^2 - 30*a^2*b^2*
c*e*g - 48*a^2*b^2*d*e*f + 120*a*b^3*c*d*e + 12*a^3*b*e*f*g))/(27*a^6))*roo
t(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z
- 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*
g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2
- 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 -
8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k), k, 1, 3) - (c/(2
*a) + (x^3*(5*b*c - 2*a*f))/(6*a^2) + (x^4*(4*b*d - a*g))/(3*a^2) + (d*x)/a
 - (x^2*(b*e - a*h))/(3*a*b))/(a*x^2 + b*x^5) + (e*log(x))/a^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.367 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=338

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(\sqrt[3]{b}\,(5bd-2ag)-\sqrt[3]{a}\,(4be-ah)\right)}{18a^{8/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(5bd-2ag)-\sqrt[3]{a}\,(4be\right.}{9a^{8/3}b^{2/3}}$$

Rubi [A] time = 0.73, antiderivative size = 336, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}\,(4be-ah)}{\sqrt[3]{b}}-2ag+5bd\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\sqrt[3]{b}\,(5bd-2ag)-\sqrt[3]{a}\,(4be-ah)\right)}{9a^{8/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+4\sqrt[3]{a}\,be-2a\sqrt[3]{b}\,g+5b^{4/3}d\right)}{3\sqrt{3}\,a^{8/3}b^{2/3}}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{3a^2\left(a+bx^3\right)}+\frac{(2bc-af)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2bc-af)}{a^3}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d - a*g + (b*e - a*h)*x
- b*((b*c)/a - f)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*d + 4*a^(1/3)*b*e
 - 2*a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(3*Sqrt[3]*a^(8/3)*b^(2/3)) - ((2*b*c - a*f)*Log[x])/a^3 - ((b^(1/3)*
(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/
3)*b^(2/3)) + ((5*b*d - 2*a*g - (a^(1/3)*(4*b*e - a*h))/b^(1/3))*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(1/3)) + ((2*b*c - a*f)
*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-3b^2dx-3b^2ex^2+3b^2\left(\frac{bc}{a}\right.}{x^4(} }{3}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int\left(-\frac{3b^2c}{ax^4}-\frac{3b^2d}{ax^3}-\frac{3b^2e}{ax^2}-\right.}{}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - ($$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - ($$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - ($$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - ($$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - ($$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} + -($$

Mathematica [A] time = 0.62, size = 303, normalized size = 0.90

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-4\sqrt[3]{a}be-2a\sqrt[3]{b}g+5b^{4/3}d\right)}{b^{2/3}}-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}h-4\sqrt[3]{a}be-2a\sqrt[3]{b}g+5b^{4/3}d\right)}{b^{2/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}h-4\sqrt[3]{a}be+2a\sqrt[3]{b}g-5b^{4/3}d\right)}{b^{2/3}}+\frac{a(6a(f+x(g+hx))-6b(c+x(d+ex)))}{a+bx^3}+6(2bc-af)\log\left(a+bx^3\right)+18\log(x)(af-2bc)-\frac{6ac}{x^3}-\frac{9ad}{x^2}-\frac{18ae}{x}}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x + (a*(-6*b*(c + x*(d + e*x)) + 6*a
*(f + x*(g + h*x))))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(-5*b^(4/3)*d - 4*a^(
1/3)*b*e + 2*a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(2/3) + 18*(-2*b*c + a*f)*Log[x] - (2*a^(1/3)*(5*b^(4/3)*d - 4*a^
(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) +
(a^(1/3)*(5*b^(4/3)*d - 4*a^(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*(2*b*c - a*f)*Log[a + b
*x^3])/(18*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^
3)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^
3)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [A] time = 0.23, size = 363, normalized size = 1.07

$$\frac{\sqrt{3}\left(5\,b^{2}d-2\,abg+\left(-ab^{2}\right)^{\frac{1}{3}}ah-4\,\left(-ab^{2}\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}+\frac{\left(5\,b^{2}d-2\,abg-\left(-ab^{2}\right)^{\frac{1}{3}}ah+4\,\left(-ab^{2}\right)^{\frac{1}{3}}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}}+\frac{\left(2\,bc-af\right)\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{3}}-\frac{\left(2\,bc-af\right)\log\left(\left|x\right|\right)}{a^{3}}-\frac{\left(a^{5}bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^{4}b^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-5\,a^{4}b^{2}d+2\,a^{5}bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{7}b}+\frac{2\left(a^{2}h-4\,abe\right)x^{5}-\left(5\,abd-2\,a^{2}g\right)x^{4}-6\,a^{2}x^{2}e-3\,a^{2}dx-2\left(2\,abc-a^{2}f\right)x^{3}-2\,a^{2}c}{6\left(bx^{3}+a\right)a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] 1/9*sqrt(3)*(5*b^2*d - 2*a*b*g + (-a*b^2)^(1/3)*a*h - 4*(-a*b^2)^(1/3)*b*e)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2)
 + 1/18*(5*b^2*d - 2*a*b*g - (-a*b^2)^(1/3)*a*h + 4*(-a*b^2)^(1/3)*b*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) + 1/3*(2*b*c - a
*f)*log(abs(b*x^3 + a))/a^3 - (2*b*c - a*f)*log(abs(x))/a^3 - 1/9*(a^5*b*h*
(-a/b)^(1/3) - 4*a^4*b^2*(-a/b)^(1/3)*e - 5*a^4*b^2*d + 2*a^5*b*g)*(-a/b)^(
1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b) + 1/6*(2*(a^2*h - 4*a*b*e)*x^5 - (5
*a*b*d - 2*a^2*g)*x^4 - 6*a^2*x^2*e - 3*a^2*d*x - 2*(2*a*b*c - a^2*f)*x^3 -
 2*a^2*c)/((b*x^3 + a)*a^3*x^3)
```

maple [B] time = 0.06, size = 561, normalized size = 1.66

$$\frac{hx^{2}}{3\left(bx^{3}+a\right)a}-\frac{bex^{2}}{3\left(bx^{3}+a\right)a^{2}}+\frac{gx}{3\left(bx^{3}+a\right)a}-\frac{bdx}{3\left(bx^{3}+a\right)a^{2}}+\frac{2\sqrt{3}\,g\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2g\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{g\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,h\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{h\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{h\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{f}{3\left(bx^{3}+a\right)a}-\frac{bc}{3\left(bx^{3}+a\right)a^{2}}-\frac{5\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{5d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{5d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{4\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}+\frac{4e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}-\frac{2e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}+\frac{f\ln\left(x\right)}{a^{2}}-\frac{f\ln\left(bx^{3}+a\right)}{3a^{2}}-\frac{2bc\ln\left(x\right)}{a^{3}}+\frac{2bc\ln\left(bx^{3}+a\right)}{3a^{3}}-\frac{e}{a^{2}x}-\frac{d}{2a^{2}x^{2}}-\frac{c}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x)
```

```
[Out] -1/2/a^2*d/x^2-1/a^2*e/x+1/3/a/(b*x^3+a)*x^2*h+1/3/a/(b*x^3+a)*g*x+4/9/(a/b
)^(1/3)/a^2*e*ln(x+(a/b)^(1/3))+5/18/(a/b)^(2/3)/a^2*d*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))-5/9/(a/b)^(2/3)/a^2*d*ln(x+(a/b)^(1/3))-2/9/(a/b)^(1/3)/a^2*e
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/a^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/a^2*ln(x)*f+1/3/a/(b*x^3+a)*f-1/3/a^2*ln(b*
x^3+a)*f-1/3/a^2*c/x^3-1/3/(b*x^3+a)/a^2*b*c+2/9/a*g/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/9/a*h*3^(1/2)/b/(a/b)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/a*h/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+
1/18/a*h/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/a*g/b/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))-5/9/(a/b)^(2/3)*3^(1/2)/a^2*d*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))-1/9/a*g/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/
3/a^2/(b*x^3+a)*b*e*x^2-1/3/(b*x^3+a)/a^2*b*d*x-2/a^3*b*c*ln(x)+2/3/a^3*b*c
*ln(b*x^3+a)
```

maxima [A] time = 3.08, size = 365, normalized size = 1.08

$$-\frac{2\,(4\,be-ah)x^{5}+\left(5\,bd-2\,ag\right)x^{4}+6\,aex^{2}+2\left(2\,bc-af\right)x^{3}+3\,adx+2\,ac}{6\left(a^{2}bx^{6}+a^{3}x^{3}\right)}-\frac{\left(2\,bc-af\right)\log(x)}{a^{3}}-\frac{\sqrt{3}\left(4\,abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{2}h\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^{2}g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{4}}+\frac{\left(12\,b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-6\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abd-2\,a^{2}g\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{3}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(6\,b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abd+2\,a^{2}g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{3}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] -1/6*(2*(4*b*e - a*h)*x^5 + (5*b*d - 2*a*g)*x^4 + 6*a*e*x^2 + 2*(2*b*c - a*
f)*x^3 + 3*a*d*x + 2*a*c)/(a^2*b*x^6 + a^3*x^3) - (2*b*c - a*f)*log(x)/a^3
- 1/9*sqrt(3)*(4*a*b*e*(a/b)^(2/3) - a^2*h*(a/b)^(2/3) + 5*a*b*d*(a/b)^(1/3
) - 2*a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/a^4 + 1/18*(12*b^2*c*(a/b)^(2/3) - 6*a*b*f*(a/b)^(2/3) - 4*a*b*e*(a/b)^(1
/3) + a^2*h*(a/b)^(1/3) + 5*a*b*d - 2*a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*b*(a/b)^(2/3)) + 1/9*(6*b^2*c*(a/b)^(2/3) - 3*a*b*f*(a/b)^(2/
3) + 4*a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - 5*a*b*d + 2*a^2*g)*log(x + (
a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

mupad [B] time = 5.96, size = 1924, normalized size = 5.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] symsum(log(- (50*b^5*c*d^2 - 48*b^5*c^2*e + 8*a^2*b^3*c*g^2 - 12*a^2*b^3*e*
f^2 - 4*a^3*b^2*f*g^2 + 3*a^3*b^2*f^2*h - 25*a*b^4*d^2*f + 12*a*b^4*c^2*h -
 12*a^2*b^3*c*f*h + 20*a^2*b^3*d*f*g - 40*a*b^4*c*d*g + 48*a*b^4*c*e*f)/(9*
a^6) - root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c*z^2 + 54*a
^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3*c*f*z + 54
0*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4*b*f*g*h -
360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b^2*c*g*h +
180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^4*b*e*h^2 +
 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3*b^2*d*g^2
- 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*g^3 + 125*a
*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k)*((25*a^3*b^4*d^2 + 4*a^5*b^2*g^2 +
48*a^3*b^4*c*e - 12*a^4*b^3*c*h - 20*a^4*b^3*d*g - 24*a^4*b^3*e*f + 6*a^5*b
^2*f*h)/(9*a^6) + root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c
*z^2 + 54*a^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3
*c*f*z + 540*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4
*b*f*g*h - 360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b
^2*c*g*h + 180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^
4*b*e*h^2 + 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3
*b^2*d*g^2 - 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*
g^3 + 125*a*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k)*((36*a^6*b^3*e - 9*a^7*b
^2*h)/(9*a^6) - (x*(1296*a^5*b^4*c - 648*a^6*b^3*f))/(27*a^6) + 36*root(729
*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c*z^2 + 54*a^6*b*g*h*z - 21
6*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3*c*f*z + 540*a^4*b^3*d*e*z
 + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4*b*f*g*h - 360*a*b^4*c*d*e
 - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b^2*c*g*h + 180*a^2*b^3*d*e
*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^4*b*e*h^2 + 324*a*b^4*c^2*
f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3*b^2*d*g^2 - 162*a^2*b^3*c
*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*g^3 + 125*a*b^4*d^3 - 216*
b^5*c^3 + a^5*h^3, z, k)*a^2*b^3*x) + (x*(432*a^2*b^5*c^2 + 108*a^4*b^3*f^2
 - 432*a^3*b^4*c*f + 600*a^3*b^4*d*e - 150*a^4*b^3*d*h - 240*a^4*b^3*e*g +
60*a^5*b^2*g*h))/(27*a^6)) - (x*(125*b^5*d^3 - 64*a*b^4*e^3 + a^4*b*h^3 - 8
*a^3*b^2*g^3 + 60*a^2*b^3*d*g^2 + 48*a^2*b^3*e^2*h - 12*a^3*b^2*e*h^2 - 240
*b^5*c*d*e - 150*a*b^4*d^2*g - 24*a^2*b^3*c*g*h - 30*a^2*b^3*d*f*h - 48*a^2
*b^3*e*f*g + 12*a^3*b^2*f*g*h + 60*a*b^4*c*d*h + 96*a*b^4*c*e*g + 120*a*b^4
*d*e*f))/(27*a^6))*root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*
c*z^2 + 54*a^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^
3*c*f*z + 540*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^
4*b*f*g*h - 360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*
b^2*c*g*h + 180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a
```

```
^4*b*e*h^2 + 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^
3*b^2*d*g^2 - 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b
*g^3 + 125*a*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k), k, 1, 3) - (c/(3*a) +
(e*x^2)/a + (x^3*(2*b*c - a*f))/(3*a^2) + (x^4*(5*b*d - 2*a*g))/(6*a^2) + (
x^5*(4*b*e - a*h))/(3*a^2) + (d*x)/(2*a))/(a*x^3 + b*x^6) - (log(x)*(2*b*c
- a*f))/a^3
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.368 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=345

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{54a^{4/3}b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-\right.}{27a^{4/3}b^{10/3}}$$

Rubi [A] time = 0.89, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {1828, 1858, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{54a^{4/3}b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{27a^{4/3}b^{10/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{2/3}be-14a^{5/3}h+5ab^{2/3}f+b^{5/3}c\right)}{9\sqrt{3}a^{4/3}b^{10/3}}-\frac{x\left(-2bx(bc-4af)-3bx^2(bd-3ag)+a(7be-13ah)\right)}{18ab^3\left(a+bx^3\right)}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6b^3\left(a+bx^3\right)^2}+\frac{g\log\left(a+bx^3\right)}{3b^3}+\frac{hx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (h*x)/b^3 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*b^
3*(a + b*x^3)^2) - (x*(a*(7*b*e - 13*a*h) - 2*b*(b*c - 4*a*f)*x - 3*b*(b*d
- 3*a*g)*x^2))/(18*a*b^3*(a + b*x^3)) - ((b^(5/3)*c + 2*a^(2/3)*b*e + 5*a*b
^(2/3)*f - 14*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(9*Sqrt[3]*a^(4/3)*b^(10/3)) - ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e -
7*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(10/3)) + ((b^(2/3)*(b*c +
5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(4/3)*b^(10/3)) + (g*Log[a + b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}dx = \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \int \frac{a^2(be-ah)-2ab(bc-a\ldots}{\ldots}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-13ah)-\ldots\right.}{\ldots}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-13ah)-\ldots\right.}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-1\ldots\right.}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-1\ldots\right.}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-1\ldots\right.}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-1\ldots\right.}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-1\ldots\right.}{\ldots}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-1\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.38, size = 342, normalized size = 0.99

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^{2/3}b^{4/3}e+14a^{5/3}\sqrt[3]{b}h+5abf+b^2c\right)}{a^{4/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^{2/3}b^{4/3}e+14a^{5/3}\sqrt[3]{b}h+5abf+b^2c\right)}{a^{4/3}} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2a^{2/3}b^{4/3}e-14a^{5/3}\sqrt[3]{b}h+5abf+b^2c\right)}{a^{4/3}} - \frac{9b^{2/3}\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{\left(a+bx^3\right)^2} + \frac{3b^{2/3}\left(a^2(12g+13hx)-ab(6d+x(7e+8fx))+2b^2cx^2\right)}{a\left(a+bx^3\right)} + 18b^{2/3}g\log\left(a+bx^3\right) + 54b^{2/3}hx}{54b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(2/3)*h*x - (9*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f
*x))))/(a + b*x^3)^2 + (3*b^(2/3)*(2*b^2*c*x^2 + a^2*(12*g + 13*h*x) - a*b*
(6*d + x*(7*e + 8*f*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + 2*a^(2/3)*b
^(4/3)*e + 5*a*b*f - 14*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/a^(4/3) - (2*(b^2*c - 2*a^(2/3)*b^(4/3)*e + 5*a*b*f + 14*a^(5/
3)*b^(1/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + ((b^2*c - 2*a^(2/3)*b^(4/
3)*e + 5*a*b*f + 14*a^(5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/a^(4/3) + 18*b^(2/3)*g*Log[a + b*x^3])/(54*b^(11/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^3, x]
```

fricas [C] time = 2.69, size = 12967, normalized size = 37.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/108*(108*a*b^2*h*x^7 + 12*(b^3*c - 4*a*b^2*f)*x^5 - 42*(a*b^2*e - 7*a^2*b
*h)*x^4 - 18*a^2*b*d + 54*a^3*g - 36*(a*b^2*d - 2*a^2*b*g)*x^3 - 6*(a*b^2*c
 + 5*a^2*b*f)*x^2 - 2*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*
f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h
)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*
a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176
*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*
a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g
+ 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)
/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*
c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3
 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 16
8*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 +
15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b +
 (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2
 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)*log(-4*a*b^4*c^2*e - 4
0*a^2*b^3*c*e*f - 100*a^3*b^2*e*f^2 + 36*a^3*b^2*e^2*g + 1764*a^5*g*h^2 - 1
/4*(a^3*b^8*c + 5*a^4*b^7*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 -
(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(145
8*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*
g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2
+ 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a
^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*
h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^
2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3
)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2
*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2
*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*
h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 -
 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2
*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^1
0))^(1/3) - 18*g/b^3)^2 - 81*(a^3*b^2*c + 5*a^4*b*f)*g^2 + (2*a^3*b^5*e^2 -
 28*a^4*b^4*e*h + 98*a^5*b^3*h^2 - 9*(a^3*b^5*c + 5*a^4*b^4*f)*g)*(2*(1/2)^
(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2
*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 -
70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3
 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h
+ 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f +
 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*
e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^
2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(
2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b
^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^
3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*
```

c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a
^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(
25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) + 28*(a^2*b^3*c^
2 + 10*a^3*b^2*c*f + 25*a^4*b*f^2 - 18*a^4*b*e*g)*h - (b^5*c^3 + 8*a^2*b^3*
e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2
*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)*x) - 24*(a^2*b*e - 7*a^3*h)*x + (54*a
*b^2*g*x^6 + 108*a^2*b*g*x^3 + 54*a^3*g + (a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*
b^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 7
0*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*
e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 +
 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*
a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15
*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (
125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 -
 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*
g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/
(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 +
125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4
*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h
+ 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2
- (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) -
3*sqrt(1/3)*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*sqrt(-((2*(1/2)^(2/3)*(-I
sqrt(3) + 1)(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f -
 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a
^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b
^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^
4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5
h^3 - 3(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 1
68*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a
^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e
 + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 +
8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a
^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*
a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (1
25*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 -
18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)^2*a^2*b^6 + 36*(2*(1/2)^(
2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*
(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 7
0*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3
+ 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h +
 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f +
2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e
*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2
*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2
*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^
5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3
 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c
^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^
4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(2
5*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)*a^2*b^3*g + 32*b^
2*c*e + 160*a*b*e*f + 324*a^2*g^2 - 224*(a*b*c + 5*a^2*f)*h)/(a^2*b^6)))*lo
g(4*a*b^4*c^2*e + 40*a^2*b^3*c*e*f + 100*a^3*b^2*e*f^2 - 36*a^3*b^2*e^2*g -
 1764*a^5*g*h^2 + 1/4*(a^3*b^8*c + 5*a^4*b^7*f)*(2*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a
*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5
*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f
+ 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2
 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*
(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h

+ 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^
2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3
*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^
2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2
*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 -
270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c
)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)^2 + 81*(a^3*b^2*c + 5*a^4*b*f)*g^2
 - (2*a^3*b^5*e^2 - 28*a^4*b^4*e*h + 98*a^5*b^3*h^2 - 9*(a^3*b^5*c + 5*a^4*
b^4*f)*g)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^
2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b
^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*
c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 -
 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3
 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*
b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*
f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*
b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f
^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)
/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f
*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3
*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^
3) - 28*(a^2*b^3*c^2 + 10*a^3*b^2*c*f + 25*a^4*b*f^2 - 18*a^4*b*e*g)*h - 2*
(b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*
f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3))*x + 3/4*sqrt(1/3
)*(8*a^3*b^5*e^2 - 112*a^4*b^4*e*h + 392*a^5*b^3*h^2 + (a^3*b^8*c + 5*a^4*b
^7*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 -
70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c
*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3
+ 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168
*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 1
5*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b +
(125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2
- 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458
g^3/b^9 - 27(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g
/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 +
 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^
4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h
 + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2
 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) +
 18*(a^3*b^5*c + 5*a^4*b^4*f)*g)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81
*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2
*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7
*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2
*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*
a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3
 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*
g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) +
 (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f
*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 1
5*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 11
76*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 274
4*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*
g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^
3)/(a^4*b^10))^(1/3) - 18*g/b^3)^2*a^2*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a
*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5
*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f

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+ 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2
 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*
(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h
+ 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^
2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3
*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^
2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2
*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 -
270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c
)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)*a^2*b^3*g + 32*b^2*c*e + 160*a*b*e
*f + 324*a^2*g^2 - 224*(a*b*c + 5*a^2*f)*h)/(a^2*b^6))) + (54*a*b^2*g*x^6 +
 108*a^2*b*g*x^3 + 54*a^3*g + (a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2
)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 +
 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2
- 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e
^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*
h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f
 + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 27
0*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*
a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27
*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) -
(b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*
f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^
5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)
*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3
*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) + 3*sqrt(1/3)*
(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1
)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)
/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*
f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 7
5*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 -
2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(24
3*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 3
78*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1
/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 -
 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^
3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h
 + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f
+ 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270
*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a
^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)^2*a^2*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c
*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 +
 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c
^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*
e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3
 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e
^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b
^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (
81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^
2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b
^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^
4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f
^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e
*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)*a^2*b^3*g + 32*b^2*c*e + 160*
a*b*e*f + 324*a^2*g^2 - 224*(a*b*c + 5*a^2*f)*h)/(a^2*b^6)))*log(4*a*b^4*c^
2*e + 40*a^2*b^3*c*e*f + 100*a^3*b^2*e*f^2 - 36*a^3*b^2*e^2*g - 1764*a^5*g*
h^2 + 1/4*(a^3*b^8*c + 5*a^4*b^7*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2
```

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/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6
))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h
)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3
*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*
h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 6
30*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)
*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*
a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*
b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a
^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^
5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g +
168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(
a^4*b^10))^(1/3) - 18*g/b^3)^2 + 81*(a^3*b^2*c + 5*a^4*b*f)*g^2 - (2*a^3*b^
5*e^2 - 28*a^4*b^4*e*h + 98*a^5*b^3*h^2 - 9*(a^3*b^5*c + 5*a^4*b^4*f)*g)*(2
*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*
a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81
*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*
b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2
*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*
c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3
 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g
)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9
 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^
9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3
*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10)
- (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e
*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^
3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) - 28*(a^2
*b^3*c^2 + 10*a^3*b^2*c*f + 25*a^4*b*f^2 - 18*a^4*b*e*g)*h - 2*(b^5*c^3 + 8
*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^
3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)*x - 3/4*sqrt(1/3)*(8*a^3*b^5
*e^2 - 112*a^4*b^4*e*h + 392*a^5*b^3*h^2 + (a^3*b^8*c + 5*a^4*b^7*f)*(2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2
+ 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2
 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*
e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2
*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*
f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 2
70*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)
*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 2
7*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) -
 (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2
*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b
^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2
)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 -
3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) + 18*(a^3*b^5
*c + 5*a^4*b^4*f)*g)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (
2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458
*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g
/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 +
 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^
4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h
 + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2
 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)
*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*
(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*
f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h
^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 -
```

3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)^2*a^2*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)*a^2*b^3*g + 32*b^2*c*e + 160*a*b*e*f + 324*a^2*g^2 - 224*(a*b*c + 5*a^2*f)*h)/(a^2*b^6))))/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)

giac [A] time = 0.21, size = 385, normalized size = 1.12

$$\frac{hx}{b^3}+\frac{g\log\left(\left|bx^3+a\right|\right)}{3\,b^3}+\frac{\sqrt{3}\left(14\,a^2h-2\,abe+\left(-ab^2\right)^{\frac{1}{3}}bc+5\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab^2}+\frac{\left(14\,a^2h-2\,abe-\left(-ab^2\right)^{\frac{1}{3}}bc-5\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab^2}+\frac{2\left(b^3c-4\,ab^2f\right)x^5+\left(13\,a^2bh-7\,ab^2e\right)x^4-3\,a^2bd+9\,a^3g-6\left(ab^2d-2\,a^2bg\right)x^3-\left(ab^2c+5\,a^2bf\right)x^2+2\left(5\,a^3h-2\,a^2be\right)x}{18\left(bx^3+a\right)^2ab^3}-\frac{\left(ab^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^3b^4h+2\,a^2b^5e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] h*x/b^3 + 1/3*g*log(abs(b*x^3 + a))/b^3 + 1/27*sqrt(3)*(14*a^2*h - 2*a*b*e + (-a*b^2)^(1/3)*b*c + 5*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) + 1/54*(14*a^2*h - 2*a*b*e - (-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) + 1/18*(2*(b^3*c - 4*a*b^2*f)*x^5 + (13*a^2*b*h - 7*a*b^2*e)*x^4 - 3*a^2*b*d + 9*a^3*g - 6*(a*b^2*d - 2*a^2*b*g)*x^3 - (a*b^2*c + 5*a^2*b*f)*x^2 + 2*(5*a^3*h - 2*a^2*b*e)*x)/((b*x^3 + a)^2*a*b^3) - 1/27*(a*b^6*c*(-a/b)^(1/3) + 5*a^2*b^5*f*(-a/b)^(1/3) - 14*a^3*b^4*h + 2*a^2*b^5*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^7)

maple [B] time = 0.06, size = 619, normalized size = 1.79

$$\frac{cx^5}{9\left(bx^3+a\right)^2a}-\frac{4fx^5}{9\left(bx^3+a\right)^2b}+\frac{13ahx^4}{18\left(bx^3+a\right)^2b^2}-\frac{7ex^4}{18\left(bx^3+a\right)^2b}+\frac{2agx^3}{3\left(bx^3+a\right)^2b^2}-\frac{dx^3}{3\left(bx^3+a\right)^2b}-\frac{5afx^2}{18\left(bx^3+a\right)^2b^2}-\frac{cx^2}{18\left(bx^3+a\right)^2b}+\frac{5a^2hx}{9\left(bx^3+a\right)^2b^3}-\frac{2aex}{9\left(bx^3+a\right)^2b^2}+\frac{a^2g}{2\left(bx^3+a\right)^2b^3}-\frac{ad}{6\left(bx^3+a\right)^2b^2}-\frac{14\sqrt{3}\,ah\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{14ah\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{7ah\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2}-\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2}+\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2}+\frac{2\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{5\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{5f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{5f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{g\ln\left(bx^3+a\right)}{3b^3}+\frac{hx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)

[Out] 1/27/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c+1/2/b^3/(b*x^3+a)^2*a^2*g+5/54/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f-5/27/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f-4/9/b/(b*x^3+a)^2*f*x^5-1/3/b/(b*x^3+a)^2*x^3*d-1/6/b^2/(b*x^3+a)^2*d*a-1/18/b/(b*x^3+a)^2*x^2*c-14/27/b^4*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h+1/9/(b*x^3+a)^2/a*c*x^5-1/27/(a/b)^(2/3)/b^3*e*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-7/18/(b*x^3+a)^2/b*e*x^4+2/27/(a/b)^(2/3)/b^3*e*ln(x+(a/b)^(1/3))+1/3*g*ln(b*x^3+a)/b^3-5/18/b^2/(b*x^3+a)^2*x^2*a*f+2/3/b^2/(b*x^3+a)^2*x^3*a*g+5/27/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f-14/27/b^4*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*h-1/27/b^2/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*c+1/54/b^2/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+5/9/b^3/(b*x^3+a)^2*a^2*h*x+13/18/b^2/(b*x^3+a)^2*x^4*a*h+7/27/b^4*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*h-2/9/(b*x^3+a)^2*a/b^2*e*x+2/27/(a/b)^(2/3)*3^(1/2)/b^3*e*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+h*x/b^3

maxima [A] time = 3.13, size = 391, normalized size = 1.13

$$\frac{2\left(b^{3}c-4\,ab^{2}f\right)x^{5}-\left(7\,ab^{2}e-13\,a^{2}bh\right)x^{4}-3\,a^{2}bd+9\,a^{3}g-6\left(ab^{2}d-2\,a^{2}bg\right)x^{3}-\left(ab^{2}c+5\,a^{2}bf\right)x^{2}-2\left(2\,a^{2}be-5\,a^{3}h\right)x}{18\left(ab^{5}x^{6}+2\,a^{2}b^{4}x^{3}+a^{3}b^{3}\right)}+\frac{hx}{b^{3}}+\frac{\sqrt{3}\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^{2}h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{2}b^{3}}+\frac{\left(18\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abe+14\,a^{2}h\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(9\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abe-14\,a^{2}h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(2*(b^3*c - 4*a*b^2*f)*x^5 - (7*a*b^2*e - 13*a^2*b*h)*x^4 - 3*a^2*b*d
+ 9*a^3*g - 6*(a*b^2*d - 2*a^2*b*g)*x^3 - (a*b^2*c + 5*a^2*b*f)*x^2 - 2*(2*
a^2*b*e - 5*a^3*h)*x)/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3) + h*x/b^3 + 1/2
7*sqrt(3)*(b^2*c*(a/b)^(2/3) + 5*a*b*f*(a/b)^(2/3) + 2*a*b*e*(a/b)^(1/3) -
14*a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(
a^2*b^3) + 1/54*(18*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) + 5*a*b*f*(a/b)^(
1/3) - 2*a*b*e + 14*a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^4*(a
/b)^(2/3)) + 1/27*(9*a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) - 5*a*b*f*(a/b)^
(1/3) + 2*a*b*e - 14*a^2*h)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

mupad [B] time = 0.58, size = 916, normalized size = 2.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log(root(19683*a^4*b^10*z^3 - 19683*a^4*b^7*g*z^2 - 5670*a^4*b^4*f*h
*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z + 162*a^2*b^6*c*e*z + 6561*a^4*
b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c*g*h - 270*a^3*b^2*e*f*g - 54*a
^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c^2*f + 168*a^3*b^2*e^2*h + 75*a
^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e^3 - 729*a^4*b*g^3 + 2744*a^5*h
^3 + b^5*c^3, z, k)*(9*root(19683*a^4*b^10*z^3 - 19683*a^4*b^7*g*z^2 - 5670
*a^4*b^4*f*h*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z + 162*a^2*b^6*c*e*z
 + 6561*a^4*b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c*g*h - 270*a^3*b^2*
e*f*g - 54*a^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c^2*f + 168*a^3*b^2*
e^2*h + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e^3 - 729*a^4*b*g^3
+ 2744*a^5*h^3 + b^5*c^3, z, k)*a*b^2 - (6*a*g)/b + (x*(54*a^2*b^4*e - 378*
a^3*b^3*h))/(81*a^2*b^4)) + (81*a^2*g^2 + 2*b^2*c*e - 70*a^2*f*h - 14*a*b*c
*h + 10*a*b*e*f)/(81*a*b^4) + (x*(b^3*c^2 + 25*a^2*b*f^2 + 126*a^3*g*h + 10
*a*b^2*c*f - 18*a^2*b*e*g))/(81*a^2*b^4))*root(19683*a^4*b^10*z^3 - 19683*a
^4*b^7*g*z^2 - 5670*a^4*b^4*f*h*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z
+ 162*a^2*b^6*c*e*z + 6561*a^4*b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c
*g*h - 270*a^3*b^2*e*f*g - 54*a^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c
^2*f + 168*a^3*b^2*e^2*h + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e
^3 - 729*a^4*b*g^3 + 2744*a^5*h^3 + b^5*c^3, z, k), k, 1, 3) - (x^2*((b^2*c
)/18 + (5*a*b*f)/18) - (a^2*g)/2 - x*((5*a^2*h)/9 - (2*a*b*e)/9) + x^3*((b^
2*d)/3 - (2*a*b*g)/3) + (b*x^4*(7*b*e - 13*a*h))/18 + (a*b*d)/6 - (b*x^5*(b
^2*c - 4*a*b*f))/(9*a))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (h*x)/b^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.369 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=325

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag\right.}{27a^{5/3}b^{8/3}}$$

Rubi [A] time = 0.64, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1828, 1858, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{27a^{5/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)}{9\sqrt{3}a^{5/3}b^{8/3}}+\frac{x\left(2x(bd-4ag)+3x^2(be-3ah)-7af+bc\right)}{18ab^2\left(a+bx^3\right)}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}+\frac{h\log\left(a+bx^3\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] -(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*b^2*(a + b*x^3)^2) +
(x*(b*c - 7*a*f + 2*(b*d - 4*a*g)*x + 3*(b*e - 3*a*h)*x^2))/(18*a*b^2*(a +
b*x^3)) - ((b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(
a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((
b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(5/3)*b^(8/3)) - ((b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3)) + (h*Log[a
+ b*x^3])/(3*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}dx = -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} - \frac{\int \frac{-ab(bc-af)-2ab(bd-ag)x}{}}{}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-\right.}{18ab^2\left(\right.}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-\right.}{18ab^2\left(\right.}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-\right.}{18ab^2\left(\right.}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-\right.}{18ab^2\left(\right.}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-\right.}{18ab^2\left(\right.}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-\right.}{18ab^2\left(\right.}$$

Mathematica [A] time = 0.34, size = 315, normalized size = 0.97

$$\frac{\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}g+\sqrt[3]{a}bd-2a\sqrt[3]{b}f-b^{4/3}c\right)}{a^{5/3}}+\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^{4/3}g-\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{5/3}}-\frac{2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{5/3}}-\frac{9\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{\left(a+bx^3\right)^2}+\frac{36a^2h-3ab(6e+x(7f+8gx))+3b^2x(c+2dx)}{a\left(a+bx^3\right)}+18h\log\left(a+bx^3\right)}{54b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x^3)^2 + (36
*a^2*h + 3*b^2*x*(c + 2*d*x) - 3*a*b*(6*e + x*(7*f + 8*g*x)))/(a*(a + b*x^3
)) - (2*Sqrt[3]*b^(1/3)*(b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3
)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(b^(
4/3)*c - a^(1/3)*b*d + 2*a*b^(1/3)*f - 5*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x
])/a^(5/3) + (b^(1/3)*(-(b^(4/3)*c) + a^(1/3)*b*d - 2*a*b^(1/3)*f + 5*a^(4/
3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 18*h*Log[a
+ b*x^3])/(54*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^3,x]
```

[Out] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3, x]

fricas [C] time = 2.47, size = 12939, normalized size = 39.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/108*(12*(b^3*d - 4*a*b^2*g)*x^5 + 6*(b^3*c - 7*a*b^2*f)*x^4 - 18*a^2*b*e
+ 54*a^3*h - 36*(a*b^2*e - 2*a^2*b*h)*x^3 - 6*(a*b^2*d + 5*a^2*b*g)*x^2 - 2
*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*
h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^
3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f +
5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b
^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(
a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 +
135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c
)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3
) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5
*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^
2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a
^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 1
35*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)
*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*log(2*a*b^4*
c*d^2 + 4*a^2*b^3*d^2*f + 1/4*(a^4*b^7*d + 5*a^5*b^6*g)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f +
5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^
3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3
*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2
 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*
a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g -
(4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3
*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*
c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2
+ 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a
^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (
4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/
b^3)^2 + 50*(a^3*b^2*c + 2*a^4*b*f)*g^2 + 81*(a^4*b*d + 5*a^5*g)*h^2 - 1/2*
(a^2*b^6*c^2 + 4*a^3*b^5*c*f + 4*a^4*b^4*f^2 - 18*(a^4*b^4*d + 5*a^5*b^3*g)
*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g +
 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d
 + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^
3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2
*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 -
 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*
a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/
(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d
+ 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3
 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*
d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 -
5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a
^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(
a^5*b^9))^(1/3) - 18*h/b^3) + 20*(a^2*b^3*c*d + 2*a^3*b^2*d*f)*g - 9*(a^2*b
^3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2)*h + (b^5*c^3 + a*b^4*d^3 + 6*a*b^4*c^
2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 + 15*a^2*b^3*d^2*g + 75*a^3*b^2*d*g^
2 + 125*a^4*b*g^3)*x) - 12*(a*b^2*c + 2*a^2*b*f)*x + (54*a*b^2*h*x^6 + 108*
```

```
a^2*b*h*x^3 + 54*a^3*h + (a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2
*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g
+ 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 +
6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*
b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*
f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d
^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3)
 + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g +
 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6
*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b
*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f
*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^
2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3)
- 18*h/b^3) + 3*sqrt(1/3)*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*sqrt(-((2*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*
h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^
2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^
3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g +
 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g
^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
- 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9
))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2
*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3
*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g +
75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^
3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 -
 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9)
)^(1/3) - 18*h/b^3)^2*a^3*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/
b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^
6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*
g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c
*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*
b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*
c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^
2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g
)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*
f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b
^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c
*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2
*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*a^3*b^3*h + 16*b
^3*c*d + 32*a*b^2*d*f + 324*a^3*h^2 + 80*(a*b^2*c + 2*a^2*b*f)*g)/(a^3*b^6)
))*log(-2*a*b^4*c*d^2 - 4*a^2*b^3*d^2*f - 1/4*(a^4*b^7*d + 5*a^5*b^6*g)*(2*
(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3
*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a
^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b
^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g
+ 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*
g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^
9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^
2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^
3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g +
 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g
^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
- 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9
))^(1/3) - 18*h/b^3)^2 - 50*(a^3*b^2*c + 2*a^4*b*f)*g^2 - 81*(a^4*b*d + 5*a
^5*g)*h^2 + 1/2*(a^2*b^6*c^2 + 4*a^3*b^5*c*f + 4*a^4*b^4*f^2 - 18*(a^4*b^4*
```

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d + 5*a^5*b^3*g)*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d
+ 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b
^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^
3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*
f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3
 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g
^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 -
6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^
9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3
*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f
^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3
+ 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^
2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6
*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3) - 20*(a^2*b^3*c*d + 2*a^3*b^2*d
*f)*g + 9*(a^2*b^3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2)*h + 2*(b^5*c^3 + a*b^
4*d^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 + 15*a^2*b^3*d^2*g
 + 75*a^3*b^2*d*g^2 + 125*a^4*b*g^3)*x + 3/4*sqrt(1/3)*(2*a^2*b^6*c^2 + 8*a
^3*b^5*c*f + 8*a^4*b^4*f^2 + (a^4*b^7*d + 5*a^5*b^6*g)*(2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5
*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3
*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*
c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2
+ 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a
^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (
4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*
h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c
^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 +
 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^
4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4
*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b
^3) + 18*(a^4*b^4*d + 5*a^5*b^3*g)*h)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1
)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^
2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*
d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12
*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*
g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*
f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*
d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*
sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d
*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*
a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g
^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f
^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d
*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)^2*a^3*
b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*
f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^
3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b
^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^
2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*
h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h
)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*
b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3
*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^
4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2
*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h
^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)
*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b
^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*a^3*b^3*h + 16*b^3*c*d + 32*a*b^2*d*f + 32
```

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4*a^3*h^2 + 80*(a*b^2*c + 2*a^2*b*f)*g)/(a^3*b^6))) + (54*a*b^2*h*x^6 + 108
*a^2*b*h*x^3 + 54*a^3*h + (a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2
/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (
2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g
 + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 +
 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3
*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54
*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*
d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3
) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g
+ 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 +
6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*
b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*
f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d
^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3)
 - 18*h/b^3) - 3*sqrt(1/3)*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*sqrt(-((2*
(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3
*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a
^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b
^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g
+ 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*
g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^
9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^
2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^
3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g +
 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g
^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
- 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9
))^(1/3) - 18*h/b^3)^2*a^3*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2
/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b
^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c
*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*
c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5
*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135
*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a
^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*
g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c
*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*
b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*
c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^
2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*a^3*b^3*h + 16*
b^3*c*d + 32*a*b^2*d*f + 324*a^3*h^2 + 80*(a*b^2*c + 2*a^2*b*f)*g)/(a^3*b^6
)))*log(-2*a*b^4*c*d^2 - 4*a^2*b^3*d^2*f - 1/4*(a^4*b^7*d + 5*a^5*b^6*g)*(2
*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^
3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*
a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*
b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g
 + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25
*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^
2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b
^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a
^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b
^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g
+ 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*
g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^
9))^(1/3) - 18*h/b^3)^2 - 50*(a^3*b^2*c + 2*a^4*b*f)*g^2 - 81*(a^4*b*d + 5*
```

```
a^5*g)*h^2 + 1/2*(a^2*b^6*c^2 + 4*a^3*b^5*c*f + 4*a^4*b^4*f^2 - 18*(a^4*b^4
*d + 5*a^5*b^3*g)*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d
 + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/
b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a
^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b
*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^
3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*
g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 -
 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b
^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^
3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*
f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3
 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g
^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 -
6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3) - 20*(a^2*b^3*c*d + 2*a^3*b^2*
d*f)*g + 9*(a^2*b^3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2)*h + 2*(b^5*c^3 + a*b
^4*d^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 + 15*a^2*b^3*d^2*
g + 75*a^3*b^2*d*g^2 + 125*a^4*b*g^3)*x - 3/4*sqrt(1/3)*(2*a^2*b^6*c^2 + 8*
a^3*b^5*c*f + 8*a^4*b^4*f^2 + (a^4*b^7*d + 5*a^5*b^6*g)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f +
5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^
3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3
*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2
 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*
a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g -
(4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3
*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*
c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2
+ 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a
^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (
4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/
b^3) + 18*(a^4*b^4*d + 5*a^5*b^3*g)*h)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b
^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2
*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 1
2*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4
*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8
*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9
*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*
d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12
*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*
g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*
f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*
d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)^2*a^3
*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b
*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b
^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (
b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a
^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5
*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*
h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a
*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^
3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b
^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^
2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*
h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h
)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*
```

```
b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*a^3*b^3*h + 16*b^3*c*d + 32*a*b^2*d*f + 3
24*a^3*h^2 + 80*(a*b^2*c + 2*a^2*b*f)*g)/(a^3*b^6))))/(a*b^5*x^6 + 2*a^2*b^
4*x^3 + a^3*b^3)
```

giac [A] time = 0.78, size = 363, normalized size = 1.12

$$\frac{h\log\left(|bx^3+a|\right)}{3\,b^3}-\frac{\sqrt{3}\left(b^2c+2\,abf-\left(-ab^2\right)^{\frac{1}{3}}bd-5\left(-ab^2\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab^2}-\frac{\left(b^2c+2\,abf+\left(-ab^2\right)^{\frac{1}{3}}bd+5\left(-ab^2\right)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab^2}+\frac{2\left(b^2d-4\,abg\right)x^5+\left(b^2c-7\,abf\right)x^4+6\left(2\,a^2h-abe\right)x^3-\left(abd+5\,a^2g\right)x^2-2\left(abc+2\,a^2f\right)x+\frac{3\left(3\,a^3h-a^2be\right)}{b}}{18\left(bx^3+a\right)^2ab^2}-\frac{\left(ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^3g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^4c+2\,a^2b^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}|\right)}{27\,a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] 1/3*h*log(abs(b*x^3 + a))/b^3 - 1/27*sqrt(3)*(b^2*c + 2*a*b*f - (-a*b^2)^(1
/3)*b*d - 5*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1/54*(b^2*c + 2*a*b*f + (-a*b^2)^(1/3)*
b*d + 5*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b
^2)^(2/3)*a*b^2) + 1/18*(2*(b^2*d - 4*a*b*g)*x^5 + (b^2*c - 7*a*b*f)*x^4 +
6*(2*a^2*h - a*b*e)*x^3 - (a*b*d + 5*a^2*g)*x^2 - 2*(a*b*c + 2*a^2*f)*x + 3
*(3*a^3*h - a^2*b*e)/b)/((b*x^3 + a)^2*a*b^2) - 1/27*(a*b^4*d*(-a/b)^(1/3)
+ 5*a^2*b^3*g*(-a/b)^(1/3) + a*b^4*c + 2*a^2*b^3*f)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^3*b^5)
```

maple [A] time = 0.06, size = 515, normalized size = 1.58

$$\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}+\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}-\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^2}-\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^2}+\frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^2}+\frac{2\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{5\sqrt{3}\,g\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{5g\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{5g\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{h\ln\left(b\,x^3+a\right)}{3b^3}+\frac{-\frac{(4ag-bd)x^5}{9ab}-\frac{(7af-bc)x^4}{18ab}+\frac{(2ah-be)x^3}{3b^2}-\frac{(5ag+bd)x^2}{18b^2}-\frac{(2af+bc)x}{9b^2}+\frac{(3ah-be)a}{6b^3}}{\left(b\,x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (-1/9*(4*a*g-b*d)/a/b*x^5-1/18*(7*a*f-b*c)/a/b*x^4+1/3*(2*a*h-b*e)/b^2*x^3-
1/18*(5*a*g+b*d)/b^2*x^2-1/9*(2*a*f+b*c)/b^2*x+1/6*a*(3*a*h-b*e)/b^3)/(b*x^
3+a)^2+2/27/(a/b)^(2/3)/b^3*f*ln(x+(a/b)^(1/3))+1/27/a/b^2/(a/b)^(2/3)*ln(x
+(a/b)^(1/3))*c-1/27/(a/b)^(2/3)/b^3*f*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/
54/a/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+2/27/(a/b)^(2/3)*3
^(1/2)/b^3*f*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/27/a/b^2/(a/b)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c-5/27/b^3/(a/b)^(1/3)*ln(
x+(a/b)^(1/3))*g-1/27/(a/b)^(1/3)/a/b^2*d*ln(x+(a/b)^(1/3))+5/54/b^3/(a/b)^
(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*g+1/54/(a/b)^(1/3)/a/b^2*d*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))+5/27/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))*g+1/27*3^(1/2)/(a/b)^(1/3)/a/b^2*d*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))+1/3*h*ln(b*x^3+a)/b^3
```

maxima [A] time = 3.12, size = 366, normalized size = 1.13

$$\frac{2\left(b^3d-4\,ab^2g\right)x^5+\left(b^3c-7\,ab^2f\right)x^4-3\,a^2be+9\,a^3h-6\left(ab^2e-2\,a^2bh\right)x^3-\left(ab^2d+5\,a^2bg\right)x^2-2\left(ab^2c+2\,a^2bf\right)x}{18\left(ab^5x^6+2\,a^2b^4x^3+a^3b^3\right)}+\frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3}+\frac{\left(18\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc-2\,af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(9\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc+2\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(2*(b^3*d - 4*a*b^2*g)*x^5 + (b^3*c - 7*a*b^2*f)*x^4 - 3*a^2*b*e + 9*a
^3*h - 6*(a*b^2*e - 2*a^2*b*h)*x^3 - (a*b^2*d + 5*a^2*b*g)*x^2 - 2*(a*b^2*c
 + 2*a^2*b*f)*x)/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3) + 1/27*sqrt(3)*(b^2*
d*(a/b)^(2/3) + 5*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) + 2*a*b*f*(a/b)^(1/
3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3) + 1/54*(1
8*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) + 5*a*g*(a/b)^(1/3) - b*c - 2*a*f)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/27*(9*a*h*(a/b)^
```

```
(2/3) - b*d*(a/b)^(1/3) - 5*a*g*(a/b)^(1/3) + b*c + 2*a*f)*log(x + (a/b)^(1
/3))/(a*b^3*(a/b)^(2/3))
```

mupad [B] time = 5.66, size = 908, normalized size = 2.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] ((3*a^2*h - a*b*e)/(6*b^3) - (x*(b*c + 2*a*f))/(9*b^2) - (x^2*(b*d + 5*a*g)
)/(18*b^2) - (x^3*(b*e - 2*a*h))/(3*b^2) + (x^4*(b*c - 7*a*f))/(18*a*b) + (
x^5*(b*d - 4*a*g))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log(root(1
9683*a^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*b^5*c*
g*z + 162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 270*a^4*b
*f*g*h - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h - 6*a*b^4*
c^2*f + 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8*a^3*b^2*
f^3 + 125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k)*(9*root(1968
3*a^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*b^5*c*g*z
 + 162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 270*a^4*b*f*
g*h - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h - 6*a*b^4*c^2
*f + 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3
 + 125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k)*a*b^2 - (6*a*h)
/b + (x*(54*a^2*b^3*f + 27*a*b^4*c))/(81*a^2*b^3)) + (81*a^3*h^2 + b^3*c*d
+ 5*a*b^2*c*g + 2*a*b^2*d*f + 10*a^2*b*f*g)/(81*a^2*b^4) + (x*(b^2*d^2 + 25
*a^2*g^2 - 18*a^2*f*h - 9*a*b*c*h + 10*a*b*d*g))/(81*a^2*b^3))*root(19683*a
^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*b^5*c*g*z +
162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 270*a^4*b*f*g*h
 - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h - 6*a*b^4*c^2*f
+ 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 +
125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k), k, 1, 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.370 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3} dx$

Optimal. Leaf size=297

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah\right.}{27a^{5/3}b^{8/3}}$$

Rubi [A] time = 0.43, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1823, 1858, 1860, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{27a^{5/3}b^{8/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)}{9\sqrt{3}a^{5/3}b^{8/3}}+\frac{x\left(x(2be-5ah)-4ag+bd+3bfx^2\right)}{18ab^2\left(a+bx^3\right)}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (x*(b*d - 4*a*g + (2*b*e - 5*a*h)*x + 3*b*f*x^2))/(18*a*b^2*(a + b*x^3)) -
(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(6*b*(a + b*x^3)^2) - ((b^(4/3)*d
 + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((b^(1/3)*(b*d + 2*a*g)
 - a^(1/3)*(b*e + 5*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/3)) -
((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx = -\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}+\frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= \frac{x\left(bd-4ag+(2be-5ah)x+3bfx^2\right)}{18ab^2\left(a+bx^3\right)}-\frac{c+dx+ex^2+fx^3+gx^4+}{6b\left(a+bx^3\right)^2}$$

$$= \frac{x\left(bd-4ag+(2be-5ah)x+3bfx^2\right)}{18ab^2\left(a+bx^3\right)}-\frac{c+dx+ex^2+fx^3+gx^4+}{6b\left(a+bx^3\right)^2}$$

$$= \frac{x\left(bd-4ag+(2be-5ah)x+3bfx^2\right)}{18ab^2\left(a+bx^3\right)}-\frac{c+dx+ex^2+fx^3+gx^4+}{6b\left(a+bx^3\right)^2}$$

$$= \frac{x\left(bd-4ag+(2be-5ah)x+3bfx^2\right)}{18ab^2\left(a+bx^3\right)}-\frac{c+dx+ex^2+fx^3+gx^4+}{6b\left(a+bx^3\right)^2}$$

$$= \frac{x\left(bd-4ag+(2be-5ah)x+3bfx^2\right)}{18ab^2\left(a+bx^3\right)}-\frac{c+dx+ex^2+fx^3+gx^4+}{6b\left(a+bx^3\right)^2}$$

Mathematica [A]   time = 0.30, size = 287, normalized size = 0.97

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}h+\sqrt[3]{a}be-2a\sqrt[3]{b}g-b^{4/3}d\right)}{a^{5/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^{4/3}h-\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)}{a^{5/3}}-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)}{a^{5/3}}-\frac{9b^{2/3}(b(c+x(d+ex))-a(f+x(g+hx)))}{\left(a+bx^3\right)^2}+\frac{3b^{2/3}(bx(d+2ex)-a(6f+x(7g+8hx)))}{a\left(a+bx^3\right)}}{54b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (
3*b^(2/3)*(b*x*(d + 2*e*x) - a*(6*f + x*(7*g + 8*h*x))))/(a*(a + b*x^3)) -
(2*Sqrt[3]*(b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(4/3)*d - a^(1/3)*b*e
+ 2*a*b^(1/3)*g - 5*a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) + ((-(b^(4
/3)*d) + a^(1/3)*b*e - 2*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(8/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x
^3)^3, x]
```

fricas [C] time = 1.94, size = 6926, normalized size = 23.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] -1/108*(36*a*b*f*x^3 - 12*(b^2*e - 4*a*b*h)*x^5 - 6*(b^2*d - 7*a*b*g)*x^4 +
 18*a*b*c + 18*a^2*f + 6*(a*b*e + 5*a^2*h)*x^2 + 2*(a*b^4*x^6 + 2*a^2*b^3*x
^3 + a^3*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*
d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2
+ 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*
b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)
 - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) +
 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a
^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^
4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*
b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))*log(2*a*b^3*d*e^2 + 4*a^2*b
^2*e^2*g + 1/4*(a^4*b^6*e + 5*a^5*b^5*h)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4
*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*
b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^
3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2
*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g
+ 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2
*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 1
25*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b +
 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))^
2 + 50*(a^3*b*d + 2*a^4*g)*h^2 - 1/2*(a^2*b^5*d^2 + 4*a^3*b^4*d*g + 4*a^4*b
^3*g^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g
+ 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*
a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*
(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(
1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a
^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g
^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3
- 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 -
(e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))) + 20*(a^2*b^2*d*e + 2*a^3*b*e*g)*
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h + (b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 +
 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)*x) + 12*(a*b*d + 2*a^2*g)
*x - ((a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((
b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a
^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4
*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*
d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e
*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*
d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2
+ 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*
b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)
)) + 3*sqrt(1/3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*sqrt(-(((1/2)^(1/3)*
(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 +
8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) +
(b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a
^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e +
 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*
b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h
 + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^
3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3
)/(a^5*b^8))^(1/3)))^2*a^3*b^5 + 16*b^2*d*e + 32*a*b*e*g + 80*(a*b*d + 2*a^
2*g)*h)/(a^3*b^5)))*log(-2*a*b^3*d*e^2 - 4*a^2*b^2*e^2*g - 1/4*(a^4*b^6*e +
 5*a^5*b^5*h)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*
d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2
+ 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*
b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)
 - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) +
 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a
^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^
4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*
b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))^2 - 50*(a^3*b*d + 2*a^4*g)*
h^2 + 1/2*(a^2*b^5*d^2 + 4*a^3*b^4*d*g + 4*a^4*b^3*g^2)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*
b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d
^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2
 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^
2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^
3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*
a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75
*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5
*b^8))^(1/3))) - 20*(a^2*b^2*d*e + 2*a^3*b*e*g)*h + 2*(b^4*d^3 + a*b^3*e^3
+ 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^
3*b*e*h^2 + 125*a^4*h^3)*x + 3/4*sqrt(1/3)*(2*a^2*b^5*d^2 + 8*a^3*b^4*d*g +
 8*a^4*b^3*g^2 + (a^4*b^6*e + 5*a^5*b^5*h)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b
^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^
2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*
h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d
^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*
g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d
^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 +
 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b
 + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))
))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*
g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 12
5*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b +
3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2
*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/
(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b
*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^
```

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3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2
- (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))^2*a^3*b^5 + 16*b^2*d*e + 32*a*b
*e*g + 80*(a*b*d + 2*a^2*g)*h)/(a^3*b^5))) - ((a*b^4*x^6 + 2*a^2*b^3*x^3 +
a^3*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g
 + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125
*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3
*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*
(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(
a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*
g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3
 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 -
 (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))) - 3*sqrt(1/3)*(a*b^4*x^6 + 2*a^2
*b^3*x^3 + a^3*b^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e
^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75
*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 7
5*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^
5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*
(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^
2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a
^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 -
5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))^2*a^3*b^5 + 16
*b^2*d*e + 32*a*b*e*g + 80*(a*b*d + 2*a^2*g)*h)/(a^3*b^5)))*log(-2*a*b^3*d*
e^2 - 4*a^2*b^2*e^2*g - 1/4*(a^4*b^6*e + 5*a^5*b^5*h)*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*
g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3
 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 -
 (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*
g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3
+ 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^
3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e
*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b
^8))^(1/3)))^2 - 50*(a^3*b*d + 2*a^4*g)*h^2 + 1/2*(a^2*b^5*d^2 + 4*a^3*b^4*
d*g + 4*a^4*b^3*g^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6
*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b
*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^
2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8)
)^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sq
rt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^
2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8
) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*
h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))) - 20*(a^2*b^2*d*e +
2*a^3*b*e*g)*h + 2*(b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2
+ 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)*x - 3/4*sq
rt(1/3)*(2*a^2*b^5*d^2 + 8*a^3*b^4*d*g + 8*a^4*b^3*g^2 + (a^4*b^6*e + 5*a^5
*b^5*h)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g
+ 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*
a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*
(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(
1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a
^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g
^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3
- 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 -
(e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) +
 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3
+ 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 1
25*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^
3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h
+ (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*
a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*
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```
e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2
)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))
^(1/3)))^2*a^3*b^5 + 16*b^2*d*e + 32*a*b*e*g + 80*(a*b*d + 2*a^2*g)*h)/(a^3
*b^5))))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)
```

giac [A] time = 0.23, size = 320, normalized size = 1.08

$$-\frac{\sqrt{3}\left(b^2 d + 2\,abg - 5\left(-ab^2\right)^{\frac{1}{3}} ah - \left(-ab^2\right)^{\frac{1}{3}} be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}} ab^2} - \frac{\left(b^2 d + 2\,abg + 5\left(-ab^2\right)^{\frac{1}{3}} ah + \left(-ab^2\right)^{\frac{1}{3}} be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}} ab^2} - \frac{\left(5\,ah\left(-\frac{a}{b}\right)^{\frac{1}{3}} + b\left(-\frac{a}{b}\right)^{\frac{1}{3}} e + bd + 2\,ag\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2 b^2} - \frac{8\,abhx^5 - 2\,b^2x^5e - b^2dx^4 + 7\,abgx^4 + 6\,abfx^3 + 5\,a^2hx^2 + abx^2e + 2\,abdx + 4\,a^2gx + 3\,abc + 3\,a^2 f}{18\left(bx^3 + a\right)^2 ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/27*sqrt(3)*(b^2*d + 2*a*b*g - 5*(-a*b^2)^(1/3)*a*h - (-a*b^2)^(1/3)*b*e)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^
2) - 1/54*(b^2*d + 2*a*b*g + 5*(-a*b^2)^(1/3)*a*h + (-a*b^2)^(1/3)*b*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) - 1/27*(5*a*h*
(-a/b)^(1/3) + b*(-a/b)^(1/3)*e + b*d + 2*a*g)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a^2*b^2) - 1/18*(8*a*b*h*x^5 - 2*b^2*x^5*e - b^2*d*x^4 + 7*a*b
*g*x^4 + 6*a*b*f*x^3 + 5*a^2*h*x^2 + a*b*x^2*e + 2*a*b*d*x + 4*a^2*g*x + 3*
a*b*c + 3*a^2*f)/((b*x^3 + a)^2*a*b^2)
```

maple [A] time = 0.06, size = 490, normalized size = 1.65

$$\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} a b^2} + \frac{d \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} a b^2} - \frac{d \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}} a b^2} + \frac{\sqrt{3}\, e \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^2} - \frac{e \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^2} + \frac{e \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^2} + \frac{2\sqrt{3}\, g \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} + \frac{2g \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} - \frac{g \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} + \frac{5\sqrt{3}\, h \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} - \frac{5h \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} + \frac{5h \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} + \frac{-\frac{(4ah-be)x^5}{9ab} - \frac{f x^3}{3b} - \frac{(7ag-bd)x^4}{18ab} - \frac{(5ah+be)x^2}{18b^2} - \frac{(2ag+bd)x}{9b^2} - \frac{af+bc}{6b^2}}{\left(b x^3 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (-1/9*(4*a*h-b*e)/a/b*x^5-1/18*(7*a*g-b*d)/a/b*x^4-1/3/b*f*x^3-1/18*(5*a*h+
b*e)/b^2*x^2-1/9*(2*a*g+b*d)/b^2*x-1/6*(a*f+b*c)/b^2)/(b*x^3+a)^2+2/27/b^3/
(a/b)^(2/3)*ln(x+(a/b)^(1/3))*g+1/27/(a/b)^(2/3)/a/b^2*d*ln(x+(a/b)^(1/3))-
1/27/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*g-1/54/(a/b)^(2/3)/a
/b^2*d*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/27/b^3/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*g+1/27/(a/b)^(2/3)*3^(1/2)/a/b^2*d*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-5/27/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*h
-1/27/(a/b)^(1/3)/a/b^2*e*ln(x+(a/b)^(1/3))+5/54/b^3/(a/b)^(1/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))*h+1/54/(a/b)^(1/3)/a/b^2*e*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))+5/27/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))*h+1/27*3^(1/2)/(a/b)^(1/3)/a/b^2*e*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))
```

maxima [A] time = 3.05, size = 308, normalized size = 1.04

$$-\frac{6\,abfx^3 - 2\left(b^2e - 4\,abh\right)x^5 - \left(b^2d - 7\,abg\right)x^4 + 3\,abc + 3\,a^2f + \left(abe + 5\,a^2h\right)x^2 + 2\left(abd + 2\,a^2g\right)x}{18\left(ab^4x^6 + 2\,a^2b^3x^3 + a^3b^2\right)} + \frac{\sqrt{3}\left(be\left(\frac{a}{b}\right)^{\frac{1}{3}} + 5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}} + bd + 2\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(be\left(\frac{a}{b}\right)^{\frac{1}{3}} + 5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}} - bd - 2\,ag\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(be\left(\frac{a}{b}\right)^{\frac{1}{3}} + 5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}} - bd - 2\,ag\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] -1/18*(6*a*b*f*x^3 - 2*(b^2*e - 4*a*b*h)*x^5 - (b^2*d - 7*a*b*g)*x^4 + 3*a*
b*c + 3*a^2*f + (a*b*e + 5*a^2*h)*x^2 + 2*(a*b*d + 2*a^2*g)*x)/(a*b^4*x^6 +
 2*a^2*b^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(b*e*(a/b)^(1/3) + 5*a*h*(a/b)^(1/
3) + b*d + 2*a*g)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^
3*(a/b)^(2/3)) + 1/54*(b*e*(a/b)^(1/3) + 5*a*h*(a/b)^(1/3) - b*d - 2*a*g)*l
og(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) - 1/27*(b*e*(a/b)
```

```
^(1/3) + 5*a*h*(a/b)^(1/3) - b*d - 2*a*g)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)
^(2/3))
```

mupad [B] time = 5.69, size = 627, normalized size = 2.11

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(19683\,a^5\,b^8\,z^3+810\,a^4\,b^3\,g\,h\,z+405\,a^3\,b^4\,d\,h\,z+162\,a^3\,b^4\,e\,g\,z+81\,a^2\,b^5\,d\,e\,z+75\,a^3\,b\,e\,h^2-6\,a\,b^3\,d^2\,g+15\,a^2\,b^2\,e^2\,h-12\,a^2\,b^2\,d\,g^2-8\,a^3\,b\,g^3+a\,b^3\,e^3+125\,a^4\,h^3-b^4\,d^3,z,k\right)\left(\mathrm{root}\left(19683\,a^5\,b^8\,z^3+810\,a^4\,b^3\,g\,h\,z+405\,a^3\,b^4\,d\,h\,z+162\,a^3\,b^4\,e\,g\,z+81\,a^2\,b^5\,d\,e\,z+75\,a^3\,b\,e\,h^2-6\,a\,b^3\,d^2\,g+15\,a^2\,b^2\,e^2\,h-12\,a^2\,b^2\,d\,g^2-8\,a^3\,b\,g^3+a\,b^3\,e^3+125\,a^4\,h^3-b^4\,d^3,z,k\right)\,a\,b^2\,9+\frac{x\,\left(54\,g\,a^2\,b^3+27\,d\,a\,b^4\right)}{81\,a^2\,b^3}\right)+\frac{b^2\,d\,e+10\,a^2\,g\,h+5\,a\,b\,d\,h+2\,a\,b\,e\,g}{81\,a^2\,b^3}+\frac{x\,\left(25\,a^2\,h^2+10\,a\,b\,e\,h+b^2\,e^2\right)}{81\,a^2\,b^3}\right)\,\mathrm{root}\left(19683\,a^5\,b^8\,z^3+810\,a^4\,b^3\,g\,h\,z+405\,a^3\,b^4\,d\,h\,z+162\,a^3\,b^4\,e\,g\,z+81\,a^2\,b^5\,d\,e\,z+75\,a^3\,b\,e\,h^2-6\,a\,b^3\,d^2\,g+15\,a^2\,b^2\,e^2\,h-12\,a^2\,b^2\,d\,g^2-8\,a^3\,b\,g^3+a\,b^3\,e^3+125\,a^4\,h^3-b^4\,d^3,z,k\right)\right)-\frac{\frac{b\,c+a\,f}{6\,b^2}+\frac{x\,\left(b\,d+2\,a\,g\right)}{9\,b^2}+\frac{f\,x^3}{3\,b}+\frac{x^2\,\left(b\,e+5\,a\,h\right)}{18\,b^2}-\frac{x^4\,\left(b\,d-7\,a\,g\right)}{18\,a\,b}-\frac{x^5\,\left(b\,e-4\,a\,h\right)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log(root(19683*a^5*b^8*z^3 + 810*a^4*b^3*g*h*z + 405*a^3*b^4*d*h*z +
 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z + 75*a^3*b*e*h^2 - 6*a*b^3*d^2*g + 15
*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^3*b*g^3 + a*b^3*e^3 + 125*a^4*h^3 -
 b^4*d^3, z, k)*(9*root(19683*a^5*b^8*z^3 + 810*a^4*b^3*g*h*z + 405*a^3*b^4
*d*h*z + 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z + 75*a^3*b*e*h^2 - 6*a*b^3*d^
2*g + 15*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^3*b*g^3 + a*b^3*e^3 + 125*a
^4*h^3 - b^4*d^3, z, k)*a*b^2 + (x*(54*a^2*b^3*g + 27*a*b^4*d))/(81*a^2*b^3
)) + (b^2*d*e + 10*a^2*g*h + 5*a*b*d*h + 2*a*b*e*g)/(81*a^2*b^3) + (x*(b^2*
e^2 + 25*a^2*h^2 + 10*a*b*e*h))/(81*a^2*b^3))*root(19683*a^5*b^8*z^3 + 810*
a^4*b^3*g*h*z + 405*a^3*b^4*d*h*z + 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z +
75*a^3*b*e*h^2 - 6*a*b^3*d^2*g + 15*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^
3*b*g^3 + a*b^3*e^3 + 125*a^4*h^3 - b^4*d^3, z, k), k, 1, 3) - ((b*c + a*f)
/(6*b^2) + (x*(b*d + 2*a*g))/(9*b^2) + (f*x^3)/(3*b) + (x^2*(b*e + 5*a*h))/
(18*b^2) - (x^4*(b*d - 7*a*g))/(18*a*b) - (x^5*(b*e - 4*a*h))/(9*a*b))/(a^2
 + b^2*x^6 + 2*a*b*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.371 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=323

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{54a^{7/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be\right.}{27a^{7/3}b^{7/3}}$$

Rubi [A] time = 0.48, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1828, 1858, 1860, 31, 634, 617, 204, 628}

$$\frac{x\left(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah)\right)}{18a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{54a^{7/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{27a^{7/3}b^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)}{9\sqrt{3}a^{7/3}b^{7/3}}-\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] -(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*a*b^2*(a + b*
x^3)^2) + (x*(a*(b*e - 7*a*h) + 2*b*(2*b*c + a*f)*x + 3*b*(b*d + a*g)*x^2))
/(18*a^2*b^2*(a + b*x^3)) - ((2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/3)*f + 2*a
^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(
7/3)*b^(7/3)) - ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(1/3
) + b^(1/3)*x])/(27*a^(7/3)*b^(7/3)) + ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b
*e + 2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^
(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx = -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} - \frac{\int \frac{-a(be-ah)-2b(2bc+\cdots}{(\cdots}}{\cdots}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} + \frac{x\left(a(be-7ah)+2\cdots\right.}{18\cdots}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} + \frac{x\left(a(be-7ah)+2\cdots\right.}{18\cdots}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} + \frac{x\left(a(be-7ah)+2\cdots\right.}{18\cdots}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} + \frac{x\left(a(be-7ah)+2\cdots\right.}{18\cdots}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} + \frac{x\left(a(be-7ah)+2\cdots\right.}{18\cdots}$$

Mathematica [A] time = 0.36, size = 297, normalized size = 0.92

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^{2/3}be-2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}be+2a^{5/3}h-ab^{2/3}f-2b^{5/3}c\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)-\frac{3\sqrt[3]{a}\sqrt[3]{b}\left(a^2(6g+7hx)-abx(e+2fx)-4b^2cx^2\right)}{a+bx^3}+\frac{9a^{4/3}\sqrt[3]{b}\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{\left(a+bx^3\right)^2}}{54a^{7/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-3*a^(1/3)*b^(1/3)*(-4*b^2*c*x^2 - a*b*x*(e + 2*f*x) + a^2*(6*g + 7*h*x))
)/(a + b*x^3) + (9*a^(4/3)*b^(1/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*
(e + f*x))))/(a + b*x^3)^2 - 2*Sqrt[3]*(2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/
3)*f + 2*a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*b^(
5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f + 2*a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x]
+ (2*b^(5/3)*c - a^(2/3)*b*e + a*b^(2/3)*f - 2*a^(5/3)*h)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(7/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3
)^3,x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3
)^3, x]
```

fricas [C] time = 2.23, size = 7190, normalized size = 22.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="frica
s")
```

```
[Out] -1/108*(36*a^2*b*g*x^3 - 12*(2*b^3*c + a*b^2*f)*x^5 - 6*(a*b^2*e - 7*a^2*b*
h)*x^4 + 18*a^2*b*d + 18*a^3*g - 6*(7*a*b^2*c - a^2*b*f)*x^2 + 2*(a^2*b^4*x
^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^
2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*
h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 1
2*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b
^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)
*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f
+ 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*
h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 +
 (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))*log(
8*a*b^4*c^2*e + 8*a^2*b^3*c*e*f + 2*a^3*b^2*e*f^2 + 1/4*(2*a^5*b^6*c + a^6*
b^5*f)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^
2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*
a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h
^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) -
2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/
(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3
*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5
*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^
2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2 - 1/2*(a^4*b^4*e^2 + 4*a^
5*b^3*e*h + 4*a^6*b^2*h^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b
^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h +
```

```
 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a
^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)
/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*
b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6
*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3
)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f
^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) + 4*(4*
a^2*b^3*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + (8*b^5*c^3 + a^2*b^3*e^3 + 12*
a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*
h^2 + 8*a^5*h^3)*x) + 12*(a^2*b*e + 2*a^3*h)*x - ((a^2*b^4*x^6 + 2*a^3*b^3*
x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*
a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*
h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 -
 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^
(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(
3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f
^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7)
 - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h
)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) + 3*sqrt(1/3)*(a^2*
b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*
b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*
a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b
^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*
c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (
e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*
a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*
h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 -
 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^
(1/3)))^2*a^4*b^4 + 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c + a^2*f)*h)/(a^4*
b^4)))*log(-8*a*b^4*c^2*e - 8*a^2*b^3*c*e*f - 2*a^3*b^2*e*f^2 - 1/4*(2*a^5*
b^6*c + a^6*b^5*f)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 +
 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*
b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h
^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^
7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*s
qrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3
*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b
^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e
^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2 + 1/2*(a^4*b^
4*e^2 + 4*a^5*b^3*e*h + 4*a^6*b^2*h^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5
*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3
*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*
c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f
^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f
 + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b
^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2
 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*
a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/
3))) - 4*(4*a^2*b^3*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + 2*(8*b^5*c^3 + a^2
*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h
 + 12*a^4*b*e*h^2 + 8*a^5*h^3)*x + 3/4*sqrt(1/3)*(2*a^4*b^4*e^2 + 8*a^5*b^3
*e*h + 8*a^6*b^2*h^2 + (2*a^5*b^6*c + a^6*b^5*f)*((1/2)^(1/3)*(I*sqrt(3) +
1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f
^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 +
 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e
^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*
f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^
3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a
^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*
```

e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7
*b^7))^(1/3))))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e
^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*
a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b
*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^
7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(
-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2
*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a
^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 -
 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2*a^4*b^4 +
 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c + a^2*f)*h)/(a^4*b^4))) - ((a^2*b^4*
x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a
^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2
*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f -
12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*
b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h
)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f
 + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5
*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3
+ (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) - 3
sqrt(1/3)(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*sqrt(-(((1/2)^(1/3)*(I*s
qrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 +
a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*
b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3
*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e
 + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a
^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2
*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f -
12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*
b^3)/(a^7*b^7))^(1/3)))^2*a^4*b^4 + 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c +
 a^2*f)*h)/(a^4*b^4)))*log(-8*a*b^4*c^2*e - 8*a^2*b^3*c*e*f - 2*a^3*b^2*e*f
^2 - 1/4*(2*a^5*b^6*c + a^6*b^5*f)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3
 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2
*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*
f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*
a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4
*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c
^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8
*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*
h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))
^2 + 1/2*(a^4*b^4*e^2 + 4*a^5*b^3*e*h + 4*a^6*b^2*h^2)*((1/2)^(1/3)*(I*sqrt
(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3
*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5
*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^
2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e +
2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*
b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h
+ 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*
a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3
)/(a^7*b^7))^(1/3))) - 4*(4*a^2*b^3*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + 2*
(8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 +
 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)*x - 3/4*sqrt(1/3)*(2*a^4*b^4
*e^2 + 8*a^5*b^3*e*h + 8*a^6*b^2*h^2 + (2*a^5*b^6*c + a^6*b^5*f)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c
*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7
) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2
*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*
b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*
c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*

```
b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c
^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^
2)*a^2*b^3)/(a^7*b^7))^(1/3))))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5
*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3
*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*
c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f
^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f
 + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b
^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2
 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*
a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/
3)))^2*a^4*b^4 + 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c + a^2*f)*h)/(a^4*b^4
))))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
```

giac [A] time = 0.21, size = 340, normalized size = 1.05

$$-\frac{\sqrt{3}\left(2 a^{2} h+a b e-2\left(-a b^{2}\right)^{\frac{1}{3}} b c-\left(-a b^{2}\right)^{\frac{1}{3}} a f\right) \arctan\left(\frac{\sqrt{3}\left(2 x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-a b^{2}\right)^{\frac{2}{3}} a^{2} b}-\frac{\left(2 a^{2} h+a b e+2\left(-a b^{2}\right)^{\frac{1}{3}} b c+\left(-a b^{2}\right)^{\frac{1}{3}} a f\right) \log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-a b^{2}\right)^{\frac{2}{3}} a^{2} b}-\frac{\left(2 b^{2} c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a b f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2 a^{2} h+a b e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27 a^{3} b^{2}}+\frac{4 b^{3} c x^{5}+2 a b^{2} f x^{5}-7 a^{2} b h x^{4}+a b^{2} x^{4} e-6 a^{2} b g x^{3}+7 a b^{2} c x^{2}-a^{2} b f x^{2}-4 a^{3} h x-2 a^{2} b x e-3 a^{2} b d-3 a^{3} g}{18\left(b x^{3}+a\right)^{2} a^{2} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac"
)
```

```
[Out] -1/27*sqrt(3)*(2*a^2*h + a*b*e - 2*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*
b) - 1/54*(2*a^2*h + a*b*e + 2*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1/27*(2*b^2*
c*(-a/b)^(1/3) + a*b*f*(-a/b)^(1/3) + 2*a^2*h + a*b*e)*(-a/b)^(1/3)*log(abs
(x - (-a/b)^(1/3)))/(a^3*b^2) + 1/18*(4*b^3*c*x^5 + 2*a*b^2*f*x^5 - 7*a^2*b
*h*x^4 + a*b^2*x^4*e - 6*a^2*b*g*x^3 + 7*a*b^2*c*x^2 - a^2*b*f*x^2 - 4*a^3*
h*x - 2*a^2*b*x*e - 3*a^2*b*d - 3*a^3*g)/((b*x^3 + a)^2*a^2*b^2)
```

maple [A] time = 0.06, size = 498, normalized size = 1.54

$$\frac{\sqrt{3}\, e \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} a b^{2}}+\frac{e \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} a b^{2}}-\frac{e \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}} a b^{2}}+\frac{\sqrt{3}\, f \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^{2}}-\frac{f \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^{2}}+\frac{f \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} a b^{2}}+\frac{2\sqrt{3}\, c \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{2} b}-\frac{2 c \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{2} b}+\frac{c \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{2} b}+\frac{2\sqrt{3}\, h \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}+\frac{2 h \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}-\frac{h \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}+\frac{\frac{(a f+2 b c) x^{5}}{9 a^{2}}-\frac{g x^{3}}{3 b}-\frac{(7 a h-b e) x^{4}}{18 a b}-\frac{(a f-7 b c) x^{2}}{18 a b}-\frac{(2 a h+b e) x}{9 b^{2}}-\frac{a g+b d}{6 b^{2}}}{\left(b x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(a*f+2*b*c)/a^2*x^5-1/18*(7*a*h-b*e)/a/b*x^4-1/3/b*g*x^3-1/18*(a*f-7*b
*c)/a/b*x^2-1/9*(2*a*h+b*e)/b^2*x-1/6*(a*g+b*d)/b^2)/(b*x^3+a)^2+2/27/b^3/(
a/b)^(2/3)*ln(x+(a/b)^(1/3))*h+1/27/(a/b)^(2/3)/a/b^2*e*ln(x+(a/b)^(1/3))-1
/27/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*h-1/54/(a/b)^(2/3)/a/
b^2*e*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/27/b^3/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h+1/27/(a/b)^(2/3)*3^(1/2)/a/b^2*e*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/27/(a/b)^(1/3)/a/b^2*f*ln(x+(a/b)^(1/3)
)-2/27/(a/b)^(1/3)/a^2/b*c*ln(x+(a/b)^(1/3))+1/54/b^2/a/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))*f+1/27/(a/b)^(1/3)/a^2/b*c*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))+1/27/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))*f+2/27*3^(1/2)/(a/b)^(1/3)/a^2/b*c*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))
```

maxima [A] time = 3.07, size = 344, normalized size = 1.07

$$-\frac{6 a^{2} b g x^{3}-2\left(2 b^{3} c+a b^{2} f\right) x^{5}-\left(a b^{2} e-7 a^{2} b h\right) x^{4}+3 a^{2} b d+3 a^{3} g-\left(7 a b^{2} c-a^{2} b f\right) x^{2}+2\left(a^{2} b e+2 a^{3} h\right) x}{18\left(a^{2} b^{4} x^{6}+2 a^{3} b^{3} x^{3}+a^{4} b^{2}\right)}+\frac{\sqrt{3}\left(2 b^{2} c\left(\frac{a}{b}\right)^{\frac{1}{3}}+a b f\left(\frac{a}{b}\right)^{\frac{1}{3}}+a b e+2 a^{2} h\right) \arctan\left(\frac{\sqrt{3}\left(2 x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 a^{2} b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2 b^{2} c\left(\frac{a}{b}\right)^{\frac{1}{3}}+a b f\left(\frac{a}{b}\right)^{\frac{1}{3}}-a b e-2 a^{2} h\right) \log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 a^{2} b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2 b^{2} c\left(\frac{a}{b}\right)^{\frac{1}{3}}+a b f\left(\frac{a}{b}\right)^{\frac{1}{3}}-a b e-2 a^{2} h\right) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 a^{2} b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxim
a")
```

```
[Out] -1/18*(6*a^2*b*g*x^3 - 2*(2*b^3*c + a*b^2*f)*x^5 - (a*b^2*e - 7*a^2*b*h)*x^
4 + 3*a^2*b*d + 3*a^3*g - (7*a*b^2*c - a^2*b*f)*x^2 + 2*(a^2*b*e + 2*a^3*h)
*x)/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2) + 1/27*sqrt(3)*(2*b^2*c*(a/b)^(
1/3) + a*b*f*(a/b)^(1/3) + a*b*e + 2*a^2*h)*arctan(1/3*sqrt(3)*(2*x - (a/b)
^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)) + 1/54*(2*b^2*c*(a/b)^(1/3) + a*
b*f*(a/b)^(1/3) - a*b*e - 2*a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
a^2*b^3*(a/b)^(2/3)) - 1/27*(2*b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) - a*b*
e - 2*a^2*h)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

mupad [B] time = 5.36, size = 640, normalized size = 1.98

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(19683\,a^7\,b^7\,z^3+162\,a^5\,b^3\,f\,h\,z+324\,a^4\,b^4\,c\,h\,z+81\,a^4\,b^4\,e\,f\,z+162\,a^3\,b^5\,c\,e\,z-12\,a^4\,b\,e\,h^2+12\,a\,b^4\,c^2\,f-6\,a^3\,b^2\,e^2\,h+6\,a^2\,b^3\,c\,f^2+a^3\,b^2\,f^3-8\,a^5\,h^3+8\,b^5\,c^3-a^2\,b^3\,e^3,z,k\right)\left(\mathrm{root}\left(19683\,a^7\,b^7\,z^3+162\,a^5\,b^3\,f\,h\,z+324\,a^4\,b^4\,c\,h\,z+81\,a^4\,b^4\,e\,f\,z+162\,a^3\,b^5\,c\,e\,z-12\,a^4\,b\,e\,h^2+12\,a\,b^4\,c^2\,f-6\,a^3\,b^2\,e^2\,h+6\,a^2\,b^3\,c\,f^2+a^3\,b^2\,f^3-8\,a^5\,h^3+8\,b^5\,c^3-a^2\,b^3\,e^3,z,k\right)\,a\,b^2\,9+\frac{x\,\left(54\,h\,a^4\,b+27\,e\,a^3\,b^2\right)}{81\,a^4\,b}\right)+\frac{2\,b^2\,c\,e+2\,a^2\,f\,h+4\,a\,b\,c\,h+a\,b\,e\,f}{81\,a^3\,b^2}+\frac{x\,\left(a^2\,f^2+4\,a\,b\,c\,f+4\,b^2\,c^2\right)}{81\,a^4\,b}\right)\,\mathrm{root}\left(19683\,a^7\,b^7\,z^3+162\,a^5\,b^3\,f\,h\,z+324\,a^4\,b^4\,c\,h\,z+81\,a^4\,b^4\,e\,f\,z+162\,a^3\,b^5\,c\,e\,z-12\,a^4\,b\,e\,h^2+12\,a\,b^4\,c^2\,f-6\,a^3\,b^2\,e^2\,h+6\,a^2\,b^3\,c\,f^2+a^3\,b^2\,f^3-8\,a^5\,h^3+8\,b^5\,c^3-a^2\,b^3\,e^3,z,k\right)\right)-\frac{\frac{b\,d+a\,g}{6\,b^2}+\frac{x\,\left(b\,e+2\,a\,h\right)}{9\,b^2}+\frac{g\,x^3}{3\,b}-\frac{x^5\,\left(2\,b\,c+a\,f\right)}{9\,a^2}-\frac{x^2\,\left(7\,b\,c-a\,f\right)}{18\,a\,b}-\frac{x^4\,\left(b\,e-7\,a\,h\right)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

```
[Out] symsum(log(root(19683*a^7*b^7*z^3 + 162*a^5*b^3*f*h*z + 324*a^4*b^4*c*h*z +
 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^4*b*e*h^2 + 12*a*b^4*c^2*f - 6
*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 - 8*a^5*h^3 + 8*b^5*c^3 - a^
2*b^3*e^3, z, k)*(9*root(19683*a^7*b^7*z^3 + 162*a^5*b^3*f*h*z + 324*a^4*b^
4*c*h*z + 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^4*b*e*h^2 + 12*a*b^4*
c^2*f - 6*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 - 8*a^5*h^3 + 8*b^5
*c^3 - a^2*b^3*e^3, z, k)*a*b^2 + (x*(27*a^3*b^2*e + 54*a^4*b*h))/(81*a^4*b
)) + (2*b^2*c*e + 2*a^2*f*h + 4*a*b*c*h + a*b*e*f)/(81*a^3*b^2) + (x*(4*b^2
*c^2 + a^2*f^2 + 4*a*b*c*f))/(81*a^4*b))*root(19683*a^7*b^7*z^3 + 162*a^5*b
^3*f*h*z + 324*a^4*b^4*c*h*z + 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^
4*b*e*h^2 + 12*a*b^4*c^2*f - 6*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^
3 - 8*a^5*h^3 + 8*b^5*c^3 - a^2*b^3*e^3, z, k), k, 1, 3) - ((b*d + a*g)/(6*
b^2) + (x*(b*e + 2*a*h))/(9*b^2) + (g*x^3)/(3*b) - (x^5*(2*b*c + a*f))/(9*a
^2) - (x^2*(7*b*c - a*f))/(18*a*b) - (x^4*(b*e - 7*a*h))/(18*a*b))/(a^2 + b
^2*x^6 + 2*a*b*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.372 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=313

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{54a^{8/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+\right.}{27a^{8/3}b^{5/3}}$$

Rubi [A] time = 0.43, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1858, 1854, 1860, 31, 634, 617, 204, 628}

$$-\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{18a^2b^2\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{54a^{8/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{27a^{8/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)}{9\sqrt{3}a^{8/3}b^{5/3}}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a*b*(a + b*x^3)^2) - (
3*a*(b*e + a*h) - b*x*(5*b*c + a*f + 2*(2*b*d + a*g)*x))/(18*a^2*b^2*(a + b
*x^3)) - ((5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + ((b
^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(1/3) + b^(1/3)*x])/(27
*a^(8/3)*b^(5/3)) - ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^3}\,dx = \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{\int \frac{-b(5bc+af)-2b(2bd+ag)x-3b(be+ah)}{\left(a+bx^3\right)^2}}{6ab^2}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(}{18a^2b^2\left(a+bx^3\right)}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(}{18a^2b^2\left(a+bx^3\right)}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(}{18a^2b^2\left(a+bx^3\right)}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(}{18a^2b^2\left(a+bx^3\right)}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(}{18a^2b^2\left(a+bx^3\right)}$$

Mathematica [A] time = 0.28, size = 295, normalized size = 0.94

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g+2\sqrt[3]{a}bd-a\sqrt[3]{b}f-5b^{4/3}c\right)+2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}(-g)-2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)-2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)+\frac{9a^{5/3}\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{\left(a+bx^3\right)^2}+\frac{3a^{2/3}\left(-6a^2h+abx(f+2gx)+b^2x(5c+4dx)\right)}{a+bx^3}}{54a^{8/3}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```

```
[Out] ((3*a^(2/3)*(-6*a^2*h + b^2*x*(5*c + 4*d*x) + a*b*x*(f + 2*g*x)))/(a + b*x^
3) + (9*a^(5/3)*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x
^3)^2 - 2*Sqrt[3]*b^(1/3)*(5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4
/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*b^(4/3)*c
 - 2*a^(1/3)*b*d + a*b^(1/3)*f - a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + b^(1
/3)*(-5*b^(4/3)*c + 2*a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{\left(a + bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,
x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,
 x]
```

fricas [C] time = 1.94, size = 6984, normalized size = 22.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas"
)
```

```
[Out] -1/108*(36*a^2*b*h*x^3 - 12*(2*b^3*d + a*b^2*g)*x^5 - 6*(5*b^3*c + a*b^2*f)
*x^4 + 18*a^2*b*e + 18*a^3*h - 6*(7*a*b^2*d - a^2*b*g)*x^2 + 2*(a^2*b^4*x^6
 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*
a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*
g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*
d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a
^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)
*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 1
5*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(
a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*
d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))*log(40*a*b^3*
c*d^2 + 8*a^2*b^2*d^2*f + 1/4*(2*a^6*b^4*d + a^7*b^3*g)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 +
 a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b
^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 -
(8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^
2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*
b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g
+ 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*
g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8
*b^5))^(1/3)))^2 + 2*(5*a^3*b*c + a^4*f)*g^2 - 1/2*(25*a^3*b^4*c^2 + 10*a^4
*b^3*c*f + a^5*b^2*f^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^
3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g +
6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^
2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b
^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I
*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^
2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*
b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*
g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3))) + 8*(5*a^2*b^2*c*
```

```
d + a^3*b*d*f)*g + (125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2
*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)*x) - 12*(4
*a*b^2*c - a^2*b*f)*x - ((a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^
2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5)
 + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a
^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2
*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c
^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b
^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f
^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*
b^3)/(a^8*b^5))^(1/3))) + 3*sqrt(1/3)*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^
2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b
^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2
 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(
5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2
*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/
(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 +
a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^
4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (
8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 160*b^2*c*d + 32*a*
b*d*f + 16*(5*a*b*c + a^2*f)*g)/(a^5*b^3)))*log(-40*a*b^3*c*d^2 - 8*a^2*b^2
*d^2*f - 1/4*(2*a^6*b^4*d + a^7*b^3*g)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a
^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3
 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f
)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f +
5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^
3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 +
 a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5
*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2
 - 2*(5*a^3*b*c + a^4*f)*g^2 + 1/2*(25*a^3*b^4*c^2 + 10*a^4*b^3*c*f + a^5*b^
2*f^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*
c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a
^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c
*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1
/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^
5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3
*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c
^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d
^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3))) - 8*(5*a^2*b^2*c*d + a^3*b*d*f)*g
 + 2*(125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*
f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)*x + 3/4*sqrt(1/3)*(50*a^3
*b^4*c^2 + 20*a^4*b^3*c*f + 2*a^5*b^2*f^2 + (2*a^6*b^4*d + a^7*b^3*g)*((1/2
)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a
^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8
*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2
*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(1
0*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*
b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*
a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3
 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*
f)*a*b^3)/(a^8*b^5))^(1/3))))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4
*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2
*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 +
(f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*
a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*
c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*
c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a
```

^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c
*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2*a^
5*b^3 + 160*b^2*c*d + 32*a*b*d*f + 16*(5*a*b*c + a^2*f)*g)/(a^5*b^3))) - ((
a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a
^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3
+ (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f
)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f +
5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^
3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 +
a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5
*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3))) -
3*sqrt(1/3)*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*sqrt(-(((1/2)^(1/3)*(I*
sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^
2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (12
5*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2
- (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d +
a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8
*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2
*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6
*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(
a^8*b^5))^(1/3)))^2*a^5*b^3 + 160*b^2*c*d + 32*a*b*d*f + 16*(5*a*b*c + a^2*
f)*g)/(a^5*b^3)))*log(-40*a*b^3*c*d^2 - 8*a^2*b^2*d^2*f - 1/4*(2*a^6*b^4*d
+ a^7*b^3*g)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*
a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g
^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b +
3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)
- 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) +
1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2
+ a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125
*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2
- (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2 - 2*(5*a^3*b*c + a^4*f)*g^
2 + 1/2*(25*a^3*b^4*c^2 + 10*a^4*b^3*c*f + a^5*b^2*f^2)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 +
a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b
^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 -
(8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^
2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*
b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g
+ 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*
g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8
b^5))^(1/3))) - 8(5*a^2*b^2*c*d + a^3*b*d*f)*g + 2*(125*b^4*c^3 + 8*a*b^3
*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6
*a^3*b*d*g^2 + a^4*g^3)*x - 3/4*sqrt(1/3)*(50*a^3*b^4*c^2 + 20*a^4*b^3*c*f
+ 2*a^5*b^2*f^2 + (2*a^6*b^4*d + a^7*b^3*g)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((
125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 +
12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4
*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*
c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d
*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75
*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*
g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b +
3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)
)))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*
b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2
+ a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*
(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) -
2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)
/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 +

```
a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b
^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 -
(8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 160*b^2*c*d + 32*a
*b*d*f + 16*(5*a*b*c + a^2*f)*g)/(a^5*b^3))))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3
+ a^4*b^2)
```

giac [A] time = 0.22, size = 330, normalized size = 1.05

$$-\frac{\sqrt{3}\left(5\,b^{2}c+abf-2\,\left(-ab^{2}\right)^{\frac{1}{3}}bd-\left(-ab^{2}\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b}-\frac{\left(5\,b^{2}c+abf+2\,\left(-ab^{2}\right)^{\frac{1}{3}}bd+\left(-ab^{2}\right)^{\frac{1}{3}}ag\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^{2}\right)^{\frac{2}{3}}a^{2}b}-\frac{\left(2\,bd\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc+af\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{3}b}+\frac{4\,b^{3}dx^{5}+2\,ab^{2}gx^{5}+5\,b^{3}cx^{4}+ab^{2}fx^{4}-6\,a^{2}bhx^{3}+7\,ab^{2}dx^{2}-a^{2}bgx^{2}+8\,ab^{2}cx-2\,a^{2}bfx-3\,a^{3}h-3\,a^{2}be}{18\,\left(bx^{3}+a\right)^{2}a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(5*b^2*c + a*b*f - 2*(-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*
b) - 1/54*(5*b^2*c + a*b*f + 2*(-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1/27*(2*b*d*
(-a/b)^(1/3) + a*g*(-a/b)^(1/3) + 5*b*c + a*f)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a^3*b) + 1/18*(4*b^3*d*x^5 + 2*a*b^2*g*x^5 + 5*b^3*c*x^4 + a*b
^2*f*x^4 - 6*a^2*b*h*x^3 + 7*a*b^2*d*x^2 - a^2*b*g*x^2 + 8*a*b^2*c*x - 2*a^
2*b*f*x - 3*a^3*h - 3*a^2*b*e)/((b*x^3 + a)^2*a^2*b^2)
```

maple [A] time = 0.06, size = 506, normalized size = 1.62

$$\frac{\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}-\frac{f\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{\sqrt{3}\,g\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}-\frac{g\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{g\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{5\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{5c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{5c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{2\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{\frac{\left(ag+2bd\right)x^{5}}{9a^{2}}-\frac{h\,x^{3}}{3b}+\frac{\left(af+5bc\right)x^{4}}{18a^{2}}-\frac{\left(ag-7bd\right)x^{2}}{18ab}-\frac{\left(af-4bc\right)x}{9ab}-\frac{ah+be}{6b^{2}}}{\left(b\,x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(a*g+2*b*d)/a^2*x^5+1/18*(a*f+5*b*c)/a^2*x^4-1/3/b*h*x^3-1/18*(a*g-7*b
*d)/a/b*x^2-1/9*(a*f-4*b*c)/a/b*x-1/6*(a*h+b*e)/b^2)/(b*x^3+a)^2+1/27/(a/b)
^(2/3)/a/b^2*f*ln(x+(a/b)^(1/3))+5/27/(a/b)^(2/3)/a^2/b*c*ln(x+(a/b)^(1/3))
-1/54/(a/b)^(2/3)/a/b^2*f*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/54/(a/b)^(2/3
)/a^2/b*c*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/27/(a/b)^(2/3)*3^(1/2)/a/b^2*
f*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+5/27/(a/b)^(2/3)*3^(1/2)/a^2/b*c*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))*g-2/27/(a/b)^(1/3)/a^2/b*d*ln(x+(a/b)^(1/3))+1/54/a/b^2/(a/b)^(1/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*g+1/27/(a/b)^(1/3)/a^2/b*d*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))*g+2/27*3^(1/2)/(a/b)^(1/3)/a^2/b*d*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))
```

maxima [A] time = 3.11, size = 327, normalized size = 1.04

$$-\frac{6\,a^{2}bhx^{3}-2\left(2\,b^{3}d+ab^{2}g\right)x^{5}-\left(5\,b^{3}c+ab^{2}f\right)x^{4}+3\,a^{2}be+3\,a^{3}h-\left(7\,ab^{2}d-a^{2}bg\right)x^{2}-2\left(4\,ab^{2}c-a^{2}bf\right)x}{18\left(a^{2}b^{4}x^{6}+2\,a^{3}b^{3}x^{3}+a^{4}b^{2}\right)}+\frac{\sqrt{3}\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc+af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,bc-af\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,bc-af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima"
)
```

```
[Out] -1/18*(6*a^2*b*h*x^3 - 2*(2*b^3*d + a*b^2*g)*x^5 - (5*b^3*c + a*b^2*f)*x^4
+ 3*a^2*b*e + 3*a^3*h - (7*a*b^2*d - a^2*b*g)*x^2 - 2*(4*a*b^2*c - a^2*b*f)
*x)/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2) + 1/27*sqrt(3)*(2*b*d*(a/b)^(1/
3) + a*g*(a/b)^(1/3) + 5*b*c + a*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/54*(2*b*d*(a/b)^(1/3) + a*g*(a/b)^(1
/3) - 5*b*c - a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2
```

```
/3)) - 1/27*(2*b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) - 5*b*c - a*f)*log(x + (a/
b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

mupad [B] time = 0.43, size = 630, normalized size = 2.01

$$\frac{\frac{x^4\,\left(5\,b\,c+a\,f\right)}{18\,a^2}-\frac{h\,x^3}{3\,b}-\frac{b\,e+a\,h}{6\,b^2}+\frac{x^5\,\left(2\,b\,d+a\,g\right)}{9\,a^2}+\frac{x\,\left(4\,b\,c-a\,f\right)}{9\,a\,b}+\frac{x^2\,\left(7\,b\,d-a\,g\right)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(19683\,a^8\,b^5\,z^3+81\,a^5\,b^2\,f\,g\,z+405\,a^4\,b^3\,c\,g\,z+162\,a^4\,b^3\,d\,f\,z+810\,a^3\,b^4\,c\,d\,z+6\,a^3\,b\,d\,g^2-75\,a\,b^3\,c^2\,f+12\,a^2\,b^2\,d^2\,g-15\,a^2\,b^2\,c\,f^2+8\,a\,b^3\,d^3+a^4\,g^3-125\,b^4\,c^3-a^3\,b\,f^3,z,k\right)\,\left(\mathrm{root}\left(19683\,a^8\,b^5\,z^3+81\,a^5\,b^2\,f\,g\,z+405\,a^4\,b^3\,c\,g\,z+162\,a^4\,b^3\,d\,f\,z+810\,a^3\,b^4\,c\,d\,z+6\,a^3\,b\,d\,g^2-75\,a\,b^3\,c^2\,f+12\,a^2\,b^2\,d^2\,g-15\,a^2\,b^2\,c\,f^2+8\,a\,b^3\,d^3+a^4\,g^3-125\,b^4\,c^3-a^3\,b\,f^3,z,k\right)\,a\,b^2\,9+\frac{x\,\left(27\,f\,a^3\,b^2+135\,c\,a^2\,b^3\right)}{81\,a^4\,b}\right)+\frac{10\,b^2\,c\,d+a^2\,f\,g+5\,a\,b\,c\,g+2\,a\,b\,d\,f}{81\,a^4\,b}+\frac{x\,\left(a^2\,g^2+4\,a\,b\,d\,g+4\,b^2\,d^2\right)}{81\,a^4\,b}\right)\,\mathrm{root}\left(19683\,a^8\,b^5\,z^3+81\,a^5\,b^2\,f\,g\,z+405\,a^4\,b^3\,c\,g\,z+162\,a^4\,b^3\,d\,f\,z+810\,a^3\,b^4\,c\,d\,z+6\,a^3\,b\,d\,g^2-75\,a\,b^3\,c^2\,f+12\,a^2\,b^2\,d^2\,g-15\,a^2\,b^2\,c\,f^2+8\,a\,b^3\,d^3+a^4\,g^3-125\,b^4\,c^3-a^3\,b\,f^3,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x)
```

```
[Out] ((x^4*(5*b*c + a*f))/(18*a^2) - (h*x^3)/(3*b) - (b*e + a*h)/(6*b^2) + (x^5*
(2*b*d + a*g))/(9*a^2) + (x*(4*b*c - a*f))/(9*a*b) + (x^2*(7*b*d - a*g))/(1
8*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log(root(19683*a^8*b^5*z^3 + 8
1*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4*b^3*d*f*z + 810*a^3*b^4*c*d*z
 + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^2*d^2*g - 15*a^2*b^2*c*f^2 + 8
*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^3, z, k)*(9*root(19683*a^8*b^5
*z^3 + 81*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4*b^3*d*f*z + 810*a^3*b
^4*c*d*z + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^2*d^2*g - 15*a^2*b^2*c
*f^2 + 8*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^3, z, k)*a*b^2 + (x*(1
35*a^2*b^3*c + 27*a^3*b^2*f))/(81*a^4*b)) + (10*b^2*c*d + a^2*f*g + 5*a*b*c
*g + 2*a*b*d*f)/(81*a^4*b) + (x*(4*b^2*d^2 + a^2*g^2 + 4*a*b*d*g))/(81*a^4*
b))*root(19683*a^8*b^5*z^3 + 81*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4
*b^3*d*f*z + 810*a^3*b^4*c*d*z + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^
2*d^2*g - 15*a^2*b^2*c*f^2 + 8*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^
3, z, k), k, 1, 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.373 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^3}\,dx$

Optimal. Leaf size=347

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(ag+5bd)-\sqrt[3]{a}(ah+2be)\right)}{54a^{8/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+5bd)-\sqrt[3]{a}(ah+2b\right.}{27a^{8/3}b^{5/3}}$$

Rubi [A] time = 0.72, antiderivative size = 345, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(ah+2be)}{\sqrt[3]{b}}+ag+5bd\right)}{54a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+5bd)-\sqrt[3]{a}(ah+2be)\right)}{27a^{8/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}h+2\sqrt[3]{a}be+a\sqrt[3]{b}g+5b^{4/3}d\right)}{9\sqrt{3}a^{8/3}b^{5/3}}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}+\frac{x\left(-3bx^2(3bc-af)+a(ag+5bd)+2ax(ah+2be)\right)}{18a^3b\left(a+bx^3\right)}-\frac{c\log\left(a+bx^3\right)}{3a^3}+\frac{c\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(6*a^2*b*(a + b*x
^3)^2) + (x*(a*(5*b*d + a*g) + 2*a*(2*b*e + a*h)*x - 3*b*(3*b*c - a*f)*x^2)
)/(18*a^3*b*(a + b*x^3)) - ((5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8
/3)*b^(5/3)) + (c*Log[x])/a^3 + ((b^(1/3)*(5*b*d + a*g) - a^(1/3)*(2*b*e +
a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3)) - ((5*b*d + a*g - (a^(
1/3)*(2*b*e + a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(54*a^(8/3)*b^(4/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx = \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-b(5bd+ag)x-2b(2be+ah\ldots}{x\left(a+bx^3\right)^2}}{6ab^2}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+\ldots\right.}{18a^3b\left(a\ldots\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+\ldots\right.}{18a^3b\left(a\ldots\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+\ldots\right.}{18a^3b\left(a\ldots\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+\ldots\right.}{18a^3b\left(a\ldots\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+\ldots\right.}{18a^3b\left(a\ldots\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+\ldots\right.}{18a^3b\left(a\ldots\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+\ldots\right.}{18a^3b\left(a\ldots\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+\ldots\right.}{18a^3b\left(a\ldots\right.}$$

Mathematica [A] time = 0.35, size = 311, normalized size = 0.90

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h+2\sqrt[3]{a}be-a\sqrt[3]{b}g-5b^{4/3}d\right)}{b^{5/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}(-h)-2\sqrt[3]{a}be+a\sqrt[3]{b}g+5b^{4/3}d\right)}{b^{5/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}h+2\sqrt[3]{a}be+a\sqrt[3]{b}g+5b^{4/3}d\right)}{b^{5/3}}-\frac{9a^2(a(f+x(g+hx))-b(c+x(d+ex)))}{b\left(a+bx^3\right)^2}+\frac{3a(ax(g+2hx)+6bc+bx(5d+4ex))}{b\left(a+bx^3\right)}-18c\log\left(a+bx^3\right)+54c\log(x)}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((3*a*(6*b*c + b*x*(5*d + 4*e*x) + a*x*(g + 2*h*x)))/(b*(a + b*x^3)) - (9*a
^2*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)^2) - (2*S
qrt[3]*a^(1/3)*(5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcT
an[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) + 54*c*Log[x] + (2*a^(1/3)
*(5*b^(4/3)*d - 2*a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*Log[a^(1/3) + b^(1
/3)*x])/b^(5/3) + (a^(1/3)*(-5*b^(4/3)*d + 2*a^(1/3)*b*e - a*b^(1/3)*g + a^
(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 18*c*Log
[a + b*x^3])/(54*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)
^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)
^3), x]
```

fricas [C] time = 35.66, size = 12815, normalized size = 36.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="frica
s")
```

```
[Out] 1/2916*(972*a*b^2*c*x^3 + 324*(2*a*b^2*e + a^2*b*h)*x^5 + 162*(5*a*b^2*d +
a^2*b*g)*x^4 + 1458*a^2*b*c - 486*a^3*f + 162*(7*a^2*b*e - a^3*h)*x^2 - 2*(
a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^
3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c
^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*
b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a
^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8
*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2
 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a
^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1
)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5
*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^
2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4
*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b -
3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5
*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^
3)*log(225*b^4*c*d^2 + 162*b^4*c^2*e + 40*a*b^3*d*e^2 + 9*a^2*b^2*c*g^2 + 1
/2916*(2*a^6*b^4*e + a^7*b^3*h)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2
 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^
9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/
(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^
2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5)
 - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*
e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^
3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1
/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)
*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g +
 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)
/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*
d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)
*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3)^2
+ 2*(5*a^3*b*d + a^4*g)*h^2 - 1/54*(25*a^3*b^4*d^2 + 36*a^3*b^4*c*e + 10*a^
4*b^3*d*g + a^5*b^2*g^2 + 18*a^4*b^3*c*h)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (
81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1
/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)
*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g +
 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)
/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*
d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)
*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3
) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*
g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b
^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2
+ a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4
*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*
```

g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486
c/a^3) + 2(45*a*b^3*c*d + 4*a^2*b^2*e^2)*g + (81*a*b^3*c^2 + 40*a^2*b^2*d
*e + 8*a^3*b*e*g)*h + (125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*
b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)*x) + 32
4*(4*a^2*b*d - a^3*g)*x - (1458*b^3*c*x^6 + 2916*a*b^2*c*x^3 + 1458*a^2*b*c
 - (a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (8
1*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/
27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*
a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g +
15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/
(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d
*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*
c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3)
 + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g
 + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^
3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 +
 a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*
b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g
 + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*
c/a^3) - 3*sqrt(1/3)*(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)*sqrt(-(((-I*sqrt
(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*
h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a
^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^
3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h +
6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3
 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^
2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5
))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b
^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^
3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^
2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h
^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e
^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4
)/(a^9*b^5))^(1/3) + 486*c/a^3)^2*a^6*b^3 - 972*((-I*sqrt(3) + 1)*(81*c^2/a
^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3
))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g +
5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d
^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^
4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b -
 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g +
5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*
sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h +
 (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 +
75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*
e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^
2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27
*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3)
 + 486*c/a^3)*a^3*b^3*c + 236196*b^3*c^2 + 116640*a*b^2*d*e + 23328*a^2*b*e
g + 11664(5*a^2*b*d + a^3*g)*h)/(a^6*b^3)))*log(-225*b^4*c*d^2 - 162*b^4*
c^2*e - 40*a*b^3*d*e^2 - 9*a^2*b^2*c*g^2 - 1/2916*(2*a^6*b^4*e + a^7*b^3*h)
*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*
e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b
^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^
3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^
2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h
^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e
^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4
)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^
2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(

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125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 +
 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c
^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*
b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*
d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3)^2 - 2*(5*a^3*b*d + a^4*g)*h^2 + 1
/54*(25*a^3*b^4*d^2 + 36*a^3*b^4*c*e + 10*a^4*b^3*d*g + a^5*b^2*g^2 + 18*a^
4*b^3*c*h)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3
*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^
2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(
125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 +
 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c
^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*
b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*
d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*
(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) +
 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a
^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*
(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c
*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d
^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3) - 2*(45*a*b^3*c*d + 4*a
^2*b^2*e^2)*g - (81*a*b^3*c^2 + 40*a^2*b^2*d*e + 8*a^3*b*e*g)*h + 2*(125*b^
4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^
2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)*x + 1/972*sqrt(1/3)*(1350*a^3*b^4*d^
2 - 972*a^3*b^4*c*e + 540*a^4*b^3*d*g + 54*a^5*b^2*g^2 - 486*a^4*b^3*c*h +
(2*a^6*b^4*e + a^7*b^3*h)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2 + 10*
a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9 + 1/
1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b
^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^
2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/3
9366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h
- 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*
(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^
3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b
)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^
2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*
b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2
- 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^
2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3))*sqrt(-(
((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e
*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^
2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3
 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2
*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^
3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^
3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)
/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2
 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(1
25*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 +
12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^
3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b
^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d
*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3)^2*a^6*b^3 - 972*((-I*sqrt(3) + 1)*
(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)
/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h +
(2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 7
5*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e
*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2
)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*
(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3)
```

+ 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3)*a^3*b^3*c + 236196*b^3*c^2 + 116640*a*b^2*d*e + 23328*a^2*b*e*g + 11664*(5*a^2*b*d + a^3*g)*h)/(a^6*b^3))) - (1458*b^3*c*x^6 + 2916*a*b^2*c*x^3 + 1458*a^2*b*c - (a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3) + 3*sqrt(1/3)*(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3)^2*a^6*b^3 - 972*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3)*a^3*b^3*c + 236196*b^3*c^2 + 116640*a*b^2*d*e + 23328*a^2*b*e*g + 11664*(5*a^2*b*d + a^3*g)*h)/(a^6*b^3)))*log(-225*b^4*c*d^2 - 162*b^4*c^2*e - 40*a*b^3*d*e^2 - 9*a^2*b^2*c*g^2 - 1/2916*(2*a^6*b^4*e + a^7*b^3*h)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*

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e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^
3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1
/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)
*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g +
 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)
/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*
d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)
*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3)^2
- 2*(5*a^3*b*d + a^4*g)*h^2 + 1/54*(25*a^3*b^4*d^2 + 36*a^3*b^4*c*e + 10*a^
4*b^3*d*g + a^5*b^2*g^2 + 18*a^4*b^3*c*h)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (
81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1
/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)
*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g +
 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)
/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*
d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)
*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3
) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*
g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b
^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2
+ a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4
*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*
g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486
*c/a^3) - 2*(45*a*b^3*c*d + 4*a^2*b^2*e^2)*g - (81*a*b^3*c^2 + 40*a^2*b^2*d
*e + 8*a^3*b*e*g)*h + 2*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^
2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)*x - 1
/972*sqrt(1/3)*(1350*a^3*b^4*d^2 - 972*a^3*b^4*c*e + 540*a^4*b^3*d*g + 54*a
^5*b^2*g^2 - 486*a^4*b^3*c*h + (2*a^6*b^4*e + a^7*b^3*h)*((-I*sqrt(3) + 1)*
(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)
/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h +
(2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 7
5*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e
*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2
)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*
(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3)
+ 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e +
a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b
^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h +
 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3
 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d
^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^
5))^(1/3) + 486*c/a^3))*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2
+ 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9
 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(
a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2
*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5)
- 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e
^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3
 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/
27*c^3/a^9 + 1/1458*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*
a^2*b)*c/(a^9*b^3) + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g +
15*a^2*b^2*d*g^2 + a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/
(a^8*b^5) - 1/39366*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d
*g^2 - 4*e^2*h - 9*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*
c)*a^2*b^3 - 5*(25*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3)^2*a
^6*b^3 - 972*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b^3*c^2 + 10*a*b^2*d*e + a
^3*g*h + (2*e*g + 5*d*h)*a^2*b)/(a^6*b^3))/(-1/27*c^3/a^9 + 1/1458*(81*b^3*
c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3) + 1/39366
*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 + a^3*b*g^3
```

```
 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/39366*(729*b^5
*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9*c*g*h)*a^
3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25*d^3 - 54*
c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/145
8*(81*b^3*c^2 + 10*a*b^2*d*e + a^3*g*h + (2*e*g + 5*d*h)*a^2*b)*c/(a^9*b^3)
 + 1/39366*(125*b^4*d^3 + 8*a*b^3*e^3 + 75*a*b^3*d^2*g + 15*a^2*b^2*d*g^2 +
 a^3*b*g^3 + 12*a^2*b^2*e^2*h + 6*a^3*b*e*h^2 + a^4*h^3)/(a^8*b^5) - 1/3936
6*(729*b^5*c^3 + a^5*h^3 - (g^3 - 6*e*h^2)*a^4*b - 3*(5*d*g^2 - 4*e^2*h - 9
*c*g*h)*a^3*b^2 + (8*e^3 - 75*d^2*g + 27*(2*e*g + 5*d*h)*c)*a^2*b^3 - 5*(25
*d^3 - 54*c*d*e)*a*b^4)/(a^9*b^5))^(1/3) + 486*c/a^3)*a^3*b^3*c + 236196*b^
3*c^2 + 116640*a*b^2*d*e + 23328*a^2*b*e*g + 11664*(5*a^2*b*d + a^3*g)*h)/(
a^6*b^3))) + 2916*(b^3*c*x^6 + 2*a*b^2*c*x^3 + a^2*b*c)*log(x))/(a^3*b^3*x^
6 + 2*a^4*b^2*x^3 + a^5*b)
```

giac [A]   time = 0.27, size = 376, normalized size = 1.08

$$-\frac{c\log\left(|bx^3+a|\right)}{3a^3}+\frac{c\log\left(|x|\right)}{a^3}-\frac{\sqrt{3}\left(5b^2d+abg-\left(-ab^2\right)^{\frac{1}{3}}ah-2\left(-ab^2\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2b}-\frac{\left(5b^2d+abg+\left(-ab^2\right)^{\frac{1}{3}}ah+2\left(-ab^2\right)^{\frac{1}{3}}be\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2b}+\frac{6ab^2cx^3+2\left(a^2bh+2ab^2e\right)x^5+\left(5ab^2d+a^2bg\right)x^4+9a^2bc-3a^3f-\left(a^3h-7a^2be\right)x^2+2\left(4a^2bd-a^3g\right)x}{18\left(bx^3+a\right)^2a^3b}-\frac{\left(a^5b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2a^4b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5a^4b^3d+a^5b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac"
)
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 - 1/27*sqrt(3)*(5*b^2*d
+ a*b*g - (-a*b^2)^(1/3)*a*h - 2*(-a*b^2)^(1/3)*b*e)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) - 1/54*(5*b^2*d + a*
b*g + (-a*b^2)^(1/3)*a*h + 2*(-a*b^2)^(1/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) + 1/18*(6*a*b^2*c*x^3 + 2*(a^2*b*h +
2*a*b^2*e)*x^5 + (5*a*b^2*d + a^2*b*g)*x^4 + 9*a^2*b*c - 3*a^3*f - (a^3*h -
 7*a^2*b*e)*x^2 + 2*(4*a^2*b*d - a^3*g)*x)/((b*x^3 + a)^2*a^3*b) - 1/27*(a^
5*b^2*h*(-a/b)^(1/3) + 2*a^4*b^3*(-a/b)^(1/3)*e + 5*a^4*b^3*d + a^5*b^2*g)*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b^3)
```

maple [B]   time = 0.07, size = 618, normalized size = 1.78

$$\frac{hx^5}{9\left(bx^3+a\right)^2a}+\frac{2bex^5}{9\left(bx^3+a\right)^2a^2}+\frac{gx^4}{18\left(bx^3+a\right)^2a}+\frac{5bdx^4}{18\left(bx^3+a\right)^2a^2}+\frac{bcx^3}{3\left(bx^3+a\right)^2a^2}+\frac{7ex^2}{18\left(bx^3+a\right)^2a}-\frac{hx^2}{18\left(bx^3+a\right)^2b}+\frac{4dx}{9\left(bx^3+a\right)^2a}-\frac{gx}{9\left(bx^3+a\right)^2b}+\frac{c}{2\left(bx^3+a\right)^2a}-\frac{f}{6\left(bx^3+a\right)^2b}+\frac{\sqrt{3}\,g\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}ab^2}+\frac{g\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}ab^2}-\frac{g\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}ab^2}+\frac{\sqrt{3}\,h\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2}-\frac{h\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2}+\frac{h\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^2}+\frac{5\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{5d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}-\frac{5d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{2\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}-\frac{2e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}+\frac{e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}+\frac{c\ln\left(x\right)}{a^3}-\frac{c\ln\left(bx^3+a\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x)
```

```
[Out] 1/9/a/(b*x^3+a)^2*x^5*h+1/18/a/(b*x^3+a)^2*x^4*g-1/18/(b*x^3+a)^2/b*x^2*h-1
/9/(b*x^3+a)^2/b*x*g+5/27/(a/b)^(2/3)*3^(1/2)/a^2/b*d*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))+2/27*3^(1/2)/(a/b)^(1/3)/a^2/b*e*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))+4/9/(b*x^3+a)^2/a*d*x+7/18/(b*x^3+a)^2/a*e*x^2+1/2/(b*x^3+
a)^2/a*c+5/27/(a/b)^(2/3)/a^2/b*d*ln(x+(a/b)^(1/3))-2/27/(a/b)^(1/3)/a^2/b*
e*ln(x+(a/b)^(1/3))+1/27/(a/b)^(1/3)/a^2/b*e*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))+1/3/(b*x^3+a)^2/a^2*b*c*x^3+2/9/(b*x^3+a)^2/a^2*b*e*x^5-1/6/(b*x^3+a)^2
/b*f+1/a^3*c*ln(x)-1/3/a^3*c*ln(b*x^3+a)+1/27/a/b^2/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*g+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h+5/18/(b*x^3+a)^2/a^2*b*d*x^4-5/54/(a/b
)^(2/3)/a^2/b*d*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/54/a/b^2/(a/b)^(1/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*h+1/27/a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*
g-1/54/a/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*g-1/27/a/b^2/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))*h
```

maxima [A]   time = 3.11, size = 368, normalized size = 1.06

$$\frac{6b^2cx^3+2\left(2b^2e+abh\right)x^5+\left(5b^2d+abg\right)x^4+9abc-3a^2f+\left(7abe-a^2h\right)x^2+2\left(4abd-a^2g\right)x}{18\left(a^2b^3x^6+2a^3b^2x^3+a^4b\right)}+\frac{c\log\left(x\right)}{a^3}+\frac{\sqrt{3}\left(2abe\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+5abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^4b}-\frac{\left(18b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+5abd+a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+2abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-5abd-a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxim
a")
```

```
[Out] 1/18*(6*b^2*c*x^3 + 2*(2*b^2*e + a*b*h)*x^5 + (5*b^2*d + a*b*g)*x^4 + 9*a*b
*c - 3*a^2*f + (7*a*b*e - a^2*h)*x^2 + 2*(4*a*b*d - a^2*g)*x)/(a^2*b^3*x^6
+ 2*a^3*b^2*x^3 + a^4*b) + c*log(x)/a^3 + 1/27*sqrt(3)*(2*a*b*e*(a/b)^(2/3)
 + a^2*h*(a/b)^(2/3) + 5*a*b*d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3))*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b) - 1/54*(18*b^2*c*(a/b)^(2/
3) - 2*a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) + 5*a*b*d + a^2*g)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) - 1/27*(9*b^2*c*(a/b)^(2/
3) + 2*a*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3) - 5*a*b*d - a^2*g)*log(x + (a/
b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
```

mupad [B] time = 5.70, size = 1716, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x)
```

```
[Out] ((3*b*c - a*f)/(6*a*b) + (x^4*(5*b*d + a*g))/(18*a^2) + (x^5*(2*b*e + a*h))
/(9*a^2) + (x*(4*b*d - a*g))/(9*a*b) + (x^2*(7*b*e - a*h))/(18*a*b) + (b*c*
x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((c*(25*b^2*d^2 + a^2
*g^2 - 18*b^2*c*e - 9*a*b*c*h + 10*a*b*d*g))/(81*a^6) - (root(19683*a^9*b^5
*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5
*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*
a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a
^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*
g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)*(a^3*g^2 + 25*a*b^2*d^2
+ 324*b^3*c^2*x + 2916*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^
6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6
561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h
+ 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g -
15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3
+ a^5*h^3, z, k)^2*a^6*b^3*x - 27*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*
z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^
4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2
*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b
^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 7
29*b^5*c^3 + a^5*h^3, z, k)*a^5*b*h + 36*a*b^2*c*e + 18*a^2*b*c*h + 10*a^2*
b*d*g + 10*a^3*g*h*x - 54*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81
*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z
+ 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d
*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g
 - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c
^3 + a^5*h^3, z, k)*a^4*b^2*e + 1944*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5
*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4
*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*
a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^
2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3
+ 729*b^5*c^3 + a^5*h^3, z, k)*a^3*b^3*c*x + 100*a*b^2*d*e*x + 50*a^2*b*d*h
*x + 20*a^2*b*e*g*x))/(81*a^4) - (x*(a^4*h^3 - 125*b^4*d^3 + 8*a*b^3*e^3 -
a^3*b*g^3 - 15*a^2*b^2*d*g^2 + 12*a^2*b^2*e^2*h + 180*b^4*c*d*e - 75*a*b^3*
d^2*g + 6*a^3*b*e*h^2 + 18*a^2*b^2*c*g*h + 90*a*b^3*c*d*h + 36*a*b^3*c*e*g)
)/(729*a^6*b^2))*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2*
g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^
3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a
^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3
*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*
h^3, z, k), k, 1, 3) + (c*log(x))/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
[Out] Timed out
```

3.374 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3}\,dx$

Optimal. Leaf size=362

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{54a^{10/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7b\right.}{27a^{10/3}b^{4/3}}$$

Rubi [A] time = 0.83, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{54a^{10/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{27a^{10/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{2/3}be+a^{5/3}(-h)-2ab^{2/3}f+14b^{5/3}c\right)}{9\sqrt{3}a^{10/3}b^{4/3}}+\frac{x\left(-2bx(5bc-2af)-3bx^2(3bd-ag)+a(ah+5be)\right)}{18a^3b\left(a+bx^3\right)}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}-\frac{d\log\left(a+bx^3\right)}{3a^3}-\frac{c}{a^3x}+\frac{d\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6
*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*e + a*h) - 2*b*(5*b*c - 2*a*f)*x - 3*b*(
3*b*d - a*g)*x^2))/(18*a^3*b*(a + b*x^3)) + ((14*b^(5/3)*c - 5*a^(2/3)*b*e
- 2*a*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(9*Sqrt[3]*a^(10/3)*b^(4/3)) + (d*Log[x])/a^3 + ((2*b^(2/3)*(7*b*c - a
*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(4/3)
) - ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a
^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx = \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-6b^2dx-b(5be+ah)x^2\cdots}{x^2(a\cdots}}{6\cdots}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)-2b(5b\cdots\right.}{18a^3\cdots}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)-2b(5b\cdots\right.}{18a^3\cdots}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)\cdots\right.}{\cdots}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)\cdots\right.}{\cdots}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)\cdots\right.}{\cdots}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)\cdots\right.}{\cdots}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)\cdots\right.}{\cdots}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.78, size = 336, normalized size = 0.93

$$-\frac{\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{2/3}be+a^{5/3}h-2ab^{2/3}f+14b^{5/3}c\right)}{b^{4/3}}-\frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^{2/3}be+a^{5/3}h-2ab^{2/3}f+14b^{5/3}c\right)}{b^{4/3}}+\frac{2\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{2/3}be+a^{5/3}h+2ab^{2/3}f-14b^{5/3}c\right)}{b^{4/3}}+\frac{9a^2\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{b\left(a+bx^3\right)^2}-\frac{3a\left(a^2hx+ab(6d+x(5e+4fx))-10b^2cx^2\right)}{b\left(a+bx^3\right)}+18ad\log\left(a+bx^3\right)+\frac{54ac}{x}-54ad\log(x)}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -1/54*((54*a*c)/x + (9*a^2*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x
))))/(b*(a + b*x^3)^2) - (3*a*(a^2*h*x - 10*b^2*c*x^2 + a*b*(6*d + x*(5*e +
 4*f*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-14*b^(5/3)*c + 5*a^(2/3)*
b*e + 2*a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/b^(4/3) - 54*a*d*Log[x] - (2*a^(2/3)*(14*b^(5/3)*c + 5*a^(2/3)*b*e - 2*a
*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(14*b^
(5/3)*c + 5*a^(2/3)*b*e - 2*a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + 18*a*d*Log[a + b*x^3])/a^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^2 \left(a + bx^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^
3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^
3)^3), x]
```

fricas [C] time = 36.81, size = 12951, normalized size = 35.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/2916*(972*a*b^2*d*x^4 - 648*(7*b^3*c - a*b^2*f)*x^6 + 162*(5*a*b^2*e + a^
2*b*h)*x^5 - 2916*a^2*b*c - 1134*(7*a*b^2*c - a^2*b*f)*x^3 + 324*(4*a^2*b*e
 - a^3*h)*x^2 - 2*(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x)*((-I*sqrt(3) + 1)
*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/
(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81
*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 -
 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h -
15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h
^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f
 + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^
4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f
*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(
2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a
^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39
366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h
)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d
^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3)*log(-1134
*a*b^4*c*d^2 + 1960*a*b^4*c^2*e + 225*a^2*b^3*d*e^2 + 40*a^3*b^2*e*f^2 + 9*
a^4*b*d*h^2 - 1/1458*(7*a^7*b^4*c - a^8*b^3*f)*((-I*sqrt(3) + 1)*(81*d^2/a^
6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/
(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c
*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4
*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*
h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^
3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^
2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^
4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e
*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^
3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3
- 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b
^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 +
 (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*
d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3)^2 + 1/54*(252*a^4*b^
4*c*d - 25*a^5*b^3*e^2 - 36*a^5*b^3*d*f - 10*a^6*b^2*e*h - a^7*b*h^2)*((-I*
sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 7
0*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*
h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a
^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*
b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 +
```

15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3
 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392
*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1
458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2)
 - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3
*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10
b^4) + 1/39366(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*
h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^
3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^
3) + 2*(81*a^2*b^3*d^2 - 280*a^2*b^3*c*e)*f + 2*(196*a^2*b^3*c^2 + 45*a^3*b
^2*d*e - 56*a^3*b^2*c*f + 4*a^4*b*f^2)*h - (2744*b^5*c^3 - 125*a^2*b^3*e^3
- 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h -
 15*a^4*b*e*h^2 - a^5*h^3)*x) + 486*(3*a^2*b*d - a^3*g)*x - (1458*b^3*d*x^7
 + 2916*a*b^2*d*x^4 + 1458*a^2*b*d*x - (a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b
x)((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (8
1*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e
*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^
3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3
- 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b
^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 +
 (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*
d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/
a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/
(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 16
8*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h
^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3
- 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*
c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) +
 486*d/a^3) - 3*sqrt(1/3)*(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x)*sqrt(-(((
-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2
- 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7
*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 12
5*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a
^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3
 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*
e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e +
392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 +
1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b
^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*
b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a
^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e
^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2
b^3 - 3(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d
/a^3)^2*a^6*b^2 - 972*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f
 - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(
2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/
39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^
2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4)
 + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 5
4*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3
*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3
) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2
 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 117
6*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a
^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 +
 a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 4
2*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(
a^10*b^4))^(1/3) + 486*d/a^3)*a^3*b^2*d + 236196*b^2*d^2 - 816480*b^2*c*e +
 116640*a*b*e*f - 23328*(7*a*b*c - a^2*f)*h)/(a^6*b^2)))*log(1134*a*b^4*c*d

```
^2 - 1960*a*b^4*c^2*e - 225*a^2*b^3*d*e^2 - 40*a^3*b^2*e*f^2 - 9*a^4*b*d*h^
2 + 1/1458*(7*a^7*b^4*c - a^8*b^3*f)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2
*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3
/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d
/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 1
68*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*
h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3
 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)
*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3)
+ 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h
)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^
2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b
^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 1
5*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3
- 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*
c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3)^2 - 1/54*(252*a^4*b^4*c*d - 25
*a^5*b^3*e^2 - 36*a^5*b^3*d*f - 10*a^6*b^2*e*h - a^7*b*h^2)*((-I*sqrt(3) +
1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2
)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (
81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3
 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h
- 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e
*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e
*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*
b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2
*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366
*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8
*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/
39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f
*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243
*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3) - 2*(81
*a^2*b^3*d^2 - 280*a^2*b^3*c*e)*f - 2*(196*a^2*b^3*c^2 + 45*a^3*b^2*d*e - 5
6*a^3*b^2*c*f + 4*a^4*b*f^2)*h - 2*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a
*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*
b*e*h^2 - a^5*h^3)*x + 1/486*sqrt(1/3)*(3402*a^4*b^4*c*d + 675*a^5*b^3*e^2
- 486*a^5*b^3*d*f + 270*a^6*b^2*e*h + 27*a^7*b*h^2 - (7*a^7*b^4*c - a^8*b^3
*f)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (8
1*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e
*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^
3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3
- 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b
^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 +
 (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*
d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/
a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/
(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 16
8*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h
^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3
- 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*
c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) +
 486*d/a^3))*sqrt(-(((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f -
 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*
a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39
366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2
- 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) +
 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*
d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(
243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3)
+ 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 -
```

70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3)^2*a^6*b^2 - 972*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3)*a^3*b^2*d + 236196*b^2*d^2 - 816480*b^2*c*e + 116640*a*b*e*f - 23328*(7*a*b*c - a^2*f)*h)/(a^6*b^2))) - (1458*b^3*d*x^7 + 2916*a*b^2*d*x^4 + 1458*a^2*b*d*x - (a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3) + 3*sqrt(1/3)*(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x)*sqrt(-(((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3)^2*a^6*b^2 - 972*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^

(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3)*a^3*b^2*d + 236196*b^2*d^2 - 816480*b^2*c*e + 116640*a*b*e*f - 23328*(7*a*b*c - a^2*f)*h)/(a^6*b^2)))*log(1134*a*b^4*c*d^2 - 1960*a*b^4*c^2*e - 225*a^2*b^3*d*e^2 - 40*a^3*b^2*e*f^2 - 9*a^4*b*d*h^2 + 1/1458*(7*a^7*b^4*c - a^8*b^3*f)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3)^2 - 1/54*(252*a^4*b^4*c*d - 25*a^5*b^3*e^2 - 36*a^5*b^3*d*f - 10*a^6*b^2*e*h - a^7*b*h^2)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3) - 2*(81*a^2*b^3*d^2 - 280*a^2*b^3*c*e)*f - 2*(196*a^2*b^3*c^2 + 45*a^3*b^2*d*e - 56*a^3*b^2*c*f + 4*a^4*b*f^2)*h - 2*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)*x - 1/486*sqrt(1/3)*(3402*a^4*b^4*c*d + 675*a^5*b^3*e^2 - 486*a^5*b^3*d*f + 270*a^6*b^2*e*h + 27*a^7*b*h^2 - (7*a^7*b^4*c - a^8*b^3*f)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3))*sqrt(-(((-I*sqrt(3) + 1)*(81*d^2/a^6 - (2

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*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)/(a^6*b^2))/(-1/27
*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^
2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f
 + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 -
a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8
*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*
d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1
/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7
*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 12
5*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a
^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3
 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*
e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e +
392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3)^2*a^6*b^2 - 972*((-I*sqrt(
3) + 1)*(81*d^2/a^6 - (2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e
)*b^2)/(a^6*b^2))/(-1/27*d^3/a^9 + 1/1458*(2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*
b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/39366*(2744*b^5*c^3 - 125*a^2*b^
3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e
^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4) + 1/39366*(2744*b^5*c^3 + 15*a^
4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 54*d*f*h)*a^3*b^2 + (125*e^3 - 27
0*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3*(243*d^3 - 630*c*d*e + 392*c^2*
f)*a*b^4)/(a^10*b^4))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(
2*a^2*f*h + 2*(5*e*f - 7*c*h)*a*b + (81*d^2 - 70*c*e)*b^2)*d/(a^9*b^2) - 1/
39366*(2744*b^5*c^3 - 125*a^2*b^3*e^3 - 1176*a*b^4*c^2*f + 168*a^2*b^3*c*f^
2 - 8*a^3*b^2*f^3 - 75*a^3*b^2*e^2*h - 15*a^4*b*e*h^2 - a^5*h^3)/(a^10*b^4)
 + 1/39366*(2744*b^5*c^3 + 15*a^4*b*e*h^2 + a^5*h^3 - (8*f^3 - 75*e^2*h + 5
4*d*f*h)*a^3*b^2 + (125*e^3 - 270*d*e*f + 42*(4*f^2 + 9*d*h)*c)*a^2*b^3 - 3
*(243*d^3 - 630*c*d*e + 392*c^2*f)*a*b^4)/(a^10*b^4))^(1/3) + 486*d/a^3)*a^
3*b^2*d + 236196*b^2*d^2 - 816480*b^2*c*e + 116640*a*b*e*f - 23328*(7*a*b*c
 - a^2*f)*h)/(a^6*b^2))) + 2916*(b^3*d*x^7 + 2*a*b^2*d*x^4 + a^2*b*d*x)*log
(x))/(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x)
```

giac [A] time = 0.22, size = 390, normalized size = 1.08

$$-\frac{d\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{d\log\left(\left|x\right|\right)}{a^3}-\frac{\sqrt{3}\left(a^2h+5\,abe+14\,\left(-ab^2\right)^{\frac{1}{3}}bc-2\,\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}a^3}-\frac{\left(a^2h+5\,abe-14\,\left(-ab^2\right)^{\frac{1}{3}}bc+2\,\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}a^3}+\frac{6\,ab^2dx^4-4\left(7\,b^3c-ab^2f\right)x^6+\left(a^2bh+5\,ab^2e\right)x^5-18\,a^2bc-7\left(7\,ab^2c-a^2bf\right)x^3-2\left(a^3h-4\,a^2be\right)x^2+3\left(3\,a^2bd-a^3g\right)x}{18\left(bx^3+a\right)^2a^3bx}+\frac{\left(14\,a^3b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^4b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^5b^2h-5\,a^4b^3e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^3 + d*log(abs(x))/a^3 - 1/27*sqrt(3)*(a^2*h +
5*a*b*e + 14*(-a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/54*(a^2*h + 5*a
*b*e - 14*(-a*b^2)^(1/3)*b*c + 2*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) + 1/18*(6*a*b^2*d*x^4 - 4*(7*b^3*c
- a*b^2*f)*x^6 + (a^2*b*h + 5*a*b^2*e)*x^5 - 18*a^2*b*c - 7*(7*a*b^2*c - a^
2*b*f)*x^3 - 2*(a^3*h - 4*a^2*b*e)*x^2 + 3*(3*a^2*b*d - a^3*g)*x)/((b*x^3 +
 a)^2*a^3*b*x) + 1/27*(14*a^3*b^4*c*(-a/b)^(1/3) - 2*a^4*b^3*f*(-a/b)^(1/3)
 - a^5*b^2*h - 5*a^4*b^3*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b^
3)
```

maple [B] time = 0.06, size = 622, normalized size = 1.72

$$\frac{2bf\,x^5}{9\left(b\,x^3+a\right)^2a^2}-\frac{5b^2c\,x^5}{9\left(b\,x^3+a\right)^2a^3}+\frac{h\,x^4}{18\left(b\,x^3+a\right)^2a}+\frac{5be\,x^4}{18\left(b\,x^3+a\right)^2a^2}+\frac{bd\,x^3}{3\left(b\,x^3+a\right)^2a^2}+\frac{7f\,x^2}{18\left(b\,x^3+a\right)^2a}-\frac{13bc\,x^2}{18\left(b\,x^3+a\right)^2a^2}+\frac{4ex}{9\left(b\,x^3+a\right)^2a}-\frac{hx}{9\left(b\,x^3+a\right)^2b}+\frac{d}{2\left(b\,x^3+a\right)^2a}-\frac{g}{6\left(b\,x^3+a\right)^2b}+\frac{\sqrt{3}\,h\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}+\frac{h\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}-\frac{h\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}+\frac{5\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{5e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}-\frac{5e\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{2\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}-\frac{2f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}+\frac{f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}-\frac{14\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{14c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{7c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{d\ln(x)}{a^3}-\frac{d\ln\left(b\,x^3+a\right)}{3a^3}-\frac{c}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x)
```

```
[Out] 1/2/(b*x^3+a)^2/a*d+1/18/a/(b*x^3+a)^2*x^4*h+7/18/a/(b*x^3+a)^2*f*x^2-1/9/(
b*x^3+a)^2/b*x*h-5/9/(b*x^3+a)^2/a^3*b^2*c*x^5+4/9/(b*x^3+a)^2/a*e*x-7/27/(
a/b)^(1/3)/a^3*c*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+14/27/(a/b)^(1/3)/a^3*c*
ln(x+(a/b)^(1/3))+5/18/(b*x^3+a)^2/a^2*b*e*x^4-5/54/(a/b)^(2/3)/a^2/b*e*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/a^3*c/x-14/27*3^(1/2)/(a/b)^(1/3)/a^3*c*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/9/a^2/(b*x^3+a)^2*x^5*b*f-1/6/(b*x^
3+a)^2/b*g+1/3/(b*x^3+a)^2/a^2*b*d*x^3+5/27/(a/b)^(2/3)*3^(1/2)/a^2/b*e*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/27/a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*h+2/27/a^2/b*3^(1/2)/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*f+1/a^3*d*ln(x)-1/3/a^3*d*ln(b*x^3+a)-13/18/
(b*x^3+a)^2/a^2*b*c*x^2+5/27/(a/b)^(2/3)/a^2/b*e*ln(x+(a/b)^(1/3))+1/27/a/b
^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*h-1/54/a/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))*h-2/27/a^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*f+1/27/a^2/b/(a
/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*f
```

maxima [A] time = 3.07, size = 400, normalized size = 1.10

$$\frac{6\,ab^2dx^4-4\left(7\,b^3c-ab^2f\right)x^6+\left(5\,ab^2e+a^2bh\right)x^5-18\,a^2bc-7\left(7\,ab^2c-a^2bf\right)x^3+2\left(4\,a^2be-a^3h\right)x^2+3\left(3\,a^2bd-a^3g\right)x}{18\left(a^3b^3x^7+2\,a^4b^2x^4+a^5bx\right)}+\frac{d\log(x)}{a^3}-\frac{\sqrt{3}\left(14\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}-\frac{\left(18\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abe+a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(9\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abe-a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(6*a*b^2*d*x^4 - 4*(7*b^3*c - a*b^2*f)*x^6 + (5*a*b^2*e + a^2*b*h)*x^5
 - 18*a^2*b*c - 7*(7*a*b^2*c - a^2*b*f)*x^3 + 2*(4*a^2*b*e - a^3*h)*x^2 + 3
*(3*a^2*b*d - a^3*g)*x)/(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x) + d*log(x)/
a^3 - 1/27*sqrt(3)*(14*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) - 5*a*b*e*(a
/b)^(1/3) - a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(a^4*b) - 1/54*(18*b^2*d*(a/b)^(2/3) + 14*b^2*c*(a/b)^(1/3) - 2*a*b
*f*(a/b)^(1/3) + 5*a*b*e + a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^3*b^2*(a/b)^(2/3)) - 1/27*(9*b^2*d*(a/b)^(2/3) - 14*b^2*c*(a/b)^(1/3) + 2*
a*b*f*(a/b)^(1/3) - 5*a*b*e - a^2*h)*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2
/3))
```

mupad [B] time = 5.75, size = 1747, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] symsum(log((d*(a^3*h^2 + 25*a*b^2*e^2 + 126*b^3*c*d - 18*a*b^2*d*f + 10*a^2
*b*e*h))/(81*a^7) - (root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^
6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z +
 6561*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d
*h + 270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2
*h - 168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 -
a^5*h^3 - 2744*b^5*c^3, z, k)*(a^3*h^2 + 25*a*b^2*e^2 + 324*b^3*d^2*x - 252
*b^3*c*d + 2916*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2
*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561
*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h +
270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h -
168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h
^3 - 2744*b^5*c^3, z, k)^2*a^6*b^3*x + 36*a*b^2*d*f + 10*a^2*b*e*h - 700*b^
3*c*e*x + 378*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2*f
*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561*a
^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 27
0*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 16
8*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3
 - 2744*b^5*c^3, z, k)*a^3*b^3*c - 54*root(19683*a^10*b^4*z^3 + 19683*a^7*b
```

```
^4*d*z^2 + 162*a^6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 567
0*a^4*b^4*c*e*z + 6561*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h
- 378*a^2*b^3*c*d*h + 270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f
 - 75*a^3*b^2*e^2*h - 168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 +
 729*a*b^4*d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)*a^4*b^2*f + 1944*root(19683*
a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2*f*h*z - 1134*a^5*b^3*c*h*z
 + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561*a^4*b^4*d^2*z - 1890*a*b^4
*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 270*a^2*b^3*d*e*f - 15*a^4*
b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 168*a^2*b^3*c*f^2 + 8*a^3*b
^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)*a^
3*b^3*d*x - 140*a*b^2*c*h*x + 100*a*b^2*e*f*x + 20*a^2*b*f*h*x))/(81*a^4) +
 (x*(2744*b^5*c^3 + a^5*h^3 + 125*a^2*b^3*e^3 - 8*a^3*b^2*f^3 + 168*a^2*b^3
*c*f^2 + 75*a^3*b^2*e^2*h - 1176*a*b^4*c^2*f + 15*a^4*b*e*h^2 + 252*a^2*b^3
*c*d*h - 180*a^2*b^3*d*e*f - 36*a^3*b^2*d*f*h + 1260*a*b^4*c*d*e))/(729*a^8
*b))*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2*f*h*z - 11
34*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*z + 6561*a^4*b^4*d^
2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3*c*d*h + 270*a^2*b^3
*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^2*e^2*h - 168*a^2*b^3
*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*d^3 - a^5*h^3 - 2744*b
^5*c^3, z, k), k, 1, 3) + ((x^5*(5*b*e + a*h))/(18*a^2) - (7*x^3*(7*b*c - a
*f))/(18*a^2) - c/a - (2*b*x^6*(7*b*c - a*f))/(9*a^3) + (x*(3*b*d - a*g))/(
6*a*b) + (x^2*(4*b*e - a*h))/(9*a*b) + (b*d*x^4)/(3*a^2))/(a^2*x + b^2*x^7
+ 2*a*b*x^4) + (d*log(x))/a^3
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.375 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=360

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)}{54a^{11/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd\right.}{27a^{11/3}b^{2/3}}$$

Rubi [A] time = 0.81, antiderivative size = 357, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}-5af+20bc\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)}{27a^{11/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}g+14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{9\sqrt{3}a^{11/3}b^{2/3}}-\frac{x\left(2x(5bd-2ag)+3x^2(3be-ah)-5af+11bc\right)}{18a^3\left(a+bx^3\right)}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6a^2\left(a+bx^3\right)^2}-\frac{e\log\left(a+bx^3\right)}{3a^3}-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^
2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c - 5*a*f + 2*(5*b*d - 2*a*g)*x + 3*(3
*b*e - a*h)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(4/3)*c + 14*a^(1/3)*b*d -
5*a*b^(1/3)*f - 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) + (e*Log[x])/a^3 - ((5*b^(1/3)*(4*b*c - a
*f) - 2*a^(1/3)*(7*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/
3)) + ((20*b*c - 5*a*f - (2*a^(1/3)*(7*b*d - a*g))/b^(1/3))*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3)) - (e*Log[a + b*x^3])
/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-6b^2dx-6b^2ex^2+5b^2\left(\frac{bc}{a}-f\right)x^3}{x^3\left(a+bx^3\right)}}{6ab^2}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af+2(5bd-2ag)\right)}{18a^3\left(a+bx\right)}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af+2(5bd-2ag)\right)}{18a^3\left(a+bx\right)}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af\right.}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af\right.}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af\right.}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af\right.}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af\right.}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af\right.}{}$$

Mathematica [A] time = 0.71, size = 337, normalized size = 0.94

$$-\frac{-\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}g-14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^{4/3}g-14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2a^{4/3}g-14\sqrt[3]{a}bd+5a\sqrt[3]{b}f-20b^{4/3}c\right)}{b^{2/3}}+\frac{9a^2\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{b\left(a+bx^3\right)^2}-\frac{3a(6ae+ax(5f+4gx)-bx(11c+10dx))}{a+bx^3}+18ae\log\left(a+bx^3\right)+\frac{27ac}{x^2}+\frac{54ad}{x}-54ae\log(x)}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -1/54*((27*a*c)/x^2 + (54*a*d)/x - (3*a*(6*a*e - b*x*(11*c + 10*d*x) + a*x*
(5*f + 4*g*x)))/(a + b*x^3) + (9*a^2*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*
(f + g*x))))/(b*(a + b*x^3)^2) + (2*Sqrt[3]*a^(1/3)*(-20*b^(4/3)*c - 14*a^(
1/3)*b*d + 5*a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(2/3) - 54*a*e*Log[x] + (2*a^(1/3)*(20*b^(4/3)*c - 14*a^(1/3)*b
*d - 5*a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1
/3)*(20*b^(4/3)*c - 14*a^(1/3)*b*d - 5*a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 18*a*e*Log[a + b*x^3])/a^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^3 \left(a + bx^3\right)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^
3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^
3)^3), x]
```

fricas [C] time = 26.93, size = 12435, normalized size = 34.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/2916*(972*a*b^2*e*x^5 - 648*(7*b^3*d - a*b^2*g)*x^7 - 810*(4*b^3*c - a*b^
2*f)*x^6 - 2916*a^2*b*d*x - 1134*(7*a*b^2*d - a^2*b*g)*x^4 - 1458*a^2*b*c -
 1296*(4*a*b^2*c - a^2*b*f)*x^3 + 486*(3*a^2*b*e - a^3*h)*x^2 - 2*(a^3*b^3*
x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c
*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 +
 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10
*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*
b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^
3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g +
168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g
)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3)
 + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g +
(81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a
*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2
*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^
3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*
d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*
e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)*log(-7840*a*b^3*c*d^2
+ 3600*a*b^3*c^2*e - 1134*a^2*b^2*d*e^2 + 225*a^3*b*e*f^2 - 1/1458*(7*a^8*b
^2*d - a^9*b*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g +
 (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*
c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(80
00*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a
^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1
/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b
+ 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*
(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3)
 + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f
- 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a
*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*
a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (
125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g
 + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*
b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)^2 - 40*(4*a^3*b*c - a^4*f)*g^2 - 1/54*(
400*a^4*b^3*c^2 - 252*a^5*b^2*d*e - 200*a^5*b^2*c*f + 25*a^6*b*f^2 + 36*a^6
*b*e*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2
 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10
*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c
^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3
```

- 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(
8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243
*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3
 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-
1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g
)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2
*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*
g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3
 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(2
5*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^
11*b^2))^(1/3) + 486*e/a^3) + 40*(49*a^2*b^2*d^2 - 45*a^2*b^2*c*e)*f + 2*(1
120*a^2*b^2*c*d + 81*a^3*b*e^2 - 280*a^3*b*d*f)*g - (8000*b^4*c^3 + 2744*a*
b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*
b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)*x) - (1458*b^3*e*x^8 + 2916*a*b^2*
e*x^5 + 1458*a^2*b*e*x^2 - (a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*((-I*s
qrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40
*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81
*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^
3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^
2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 +
 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e
*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e +
 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 +
 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10
b) - 1/39366(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*
b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^
3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g +
168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g
)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3)
 + 486*e/a^3) - 3*sqrt(1/3)*(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*sqrt(
-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*
d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f
*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2
744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 117
6*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b
^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 -
 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945
*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e
^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)
*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1
500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 -
8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*
e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2
 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2
))^(1/3) + 486*e/a^3)^2*a^7*b - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^
2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^
9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a
^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a
^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4
*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g
 + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*
e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1
/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g
 + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 274
4*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*
a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4
*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 6
30*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c
*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)*a^4*b*e + 3265920*b

```
^2*c*d + 236196*a*b*e^2 - 816480*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g)/(a^7
*b)))*log(7840*a*b^3*c*d^2 - 3600*a*b^3*c^2*e + 1134*a^2*b^2*d*e^2 - 225*a^
3*b*e*f^2 + 1/1458*(7*a^8*b^2*d - a^9*b*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 -
(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27
*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*
b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f +
 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2
- 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 27
0*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^
2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b
^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*
a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^
3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3
- 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8
000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*
e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3
- 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)^2 + 40*(4*a^
3*b*c - a^4*f)*g^2 + 1/54*(400*a^4*b^3*c^2 - 252*a^5*b^2*d*e - 200*a^5*b^2*
c*f + 25*a^6*b*f^2 + 36*a^6*b*e*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2
*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9
 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^
10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^
2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*
g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g
+ 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e
*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/
3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g
+ (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744
*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a
^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*
c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 63
0*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*
d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3) - 40*(49*a^2*b^2*d^2
 - 45*a^2*b^2*c*e)*f - 2*(1120*a^2*b^2*c*d + 81*a^3*b*e^2 - 280*a^3*b*d*f)*
g - 2*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^
2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)*x + 1
/486*sqrt(1/3)*(10800*a^4*b^3*c^2 + 3402*a^5*b^2*d*e - 5400*a^5*b^2*c*f + 6
75*a^6*b*f^2 - 486*a^6*b*e*g - (7*a^8*b^2*d - a^9*b*g)*((-I*sqrt(3) + 1)*(8
1*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^
7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f
- 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a
*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*
a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (
125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g
 + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*
b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b
^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*
(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 12
5*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2)
- 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3
*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 -
 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3))
*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2
 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10
*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c
^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3
 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(
8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243
*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3
```

- 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)^2*a^7*b - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)*a^4*b*e + 3265920*b^2*c*d + 236196*a*b*e^2 - 816480*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g)/(a^7*b))) - (1458*b^3*e*x^8 + 2916*a*b^2*e*x^5 + 1458*a^2*b*e*x^2 - (a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3) + 3*sqrt(1/3)*(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)^2*a^7*b - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000 *b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)*a^4*b*e + 3265920*b^2*c*d + 236196*a*b*e^2 - 816480*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g)/(a^7*b)))*log(7840*a*b^3*c*d^2 - 3600*a*b^3*c^2*e + 1134*a^2*b^2*d*e^2 - 225*a^3*b*e*f^2 + 1/1458*(7*a^8*b^2*d - a^9*b*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)^2 + 40*(4*a^3*b*c - a^4*f)*g^2 + 1/54*(400*a^4*b^3*c^2 - 252*a^5*b^2*d*e - 200*a^5*b^2*c*f + 25*a^6*b*f^2 + 36*a^6*b*e*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3) - 40*(49*a^2*b^2*d^2 - 45*a^2*b^2*c*e)*f - 2*(1120*a^2*b^2*c*d + 81*a^3*b*e^2 - 280*a^3*b*d*f)*g - 2*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)*x - 1/486*sqrt(1/3)*(10800*a^4*b^3*c^2 + 3402*a^5*b^2*d*e - 5400*a^5*b^2*c*f + 675*a^6*b*f^2 - 486*a^6*b*e*g - (7*a^8*b^2*d - a^9*b*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*

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g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 39
2*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^
2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3))*sqrt(-(((-I*sqrt(3) + 1)*(81*e^
2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b)
)/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40
*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3
*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*
b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*
f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 2
0*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)
/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c
*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(800
0*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^
3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/
39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b +
 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(
343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)^2*a^
7*b - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e
^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d +
10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4
*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f
^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366
*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(2
43*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d
^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*
(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c
*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c
^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*
d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^
3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*
(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(
a^11*b^2))^(1/3) + 486*e/a^3)*a^4*b*e + 3265920*b^2*c*d + 236196*a*b*e^2 -
816480*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g)/(a^7*b))) + 2916*(b^3*e*x^8 +
2*a*b^2*e*x^5 + a^2*b*e*x^2)*log(x))/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x
^2)
```

giac [**A**] time = 0.23, size = 399, normalized size = 1.11

$$-\frac{e\log\left(\left|bx^3+a\right|\right)}{3a^3}+\frac{e\log\left(|x|\right)}{a^3}+\frac{\sqrt{3}\left(20b^2c-5abf-14\left(-ab^2\right)^{\frac{1}{3}}bd+2\left(-ab^2\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^3}+\frac{\left(20b^2c-5abf+14\left(-ab^2\right)^{\frac{1}{3}}bd-2\left(-ab^2\right)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^3}-\frac{28b^3dx^7-4ab^2gx^7+20b^3cx^6-5ab^2fx^6-6ab^2x^5e+49ab^2dx^4-7a^2bgx^4+32ab^2cx^3-8a^2bfx^3+3a^3hx^2-9a^2bx^2e+18a^2bdx+9a^2bc}{18\left(bx^4+ax\right)^2a^3b}+\frac{\left(14a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2a^4bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20a^3b^2c-5a^4bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 + 1/27*sqrt(3)*(20*b^2*c
 - 5*a*b*f - 14*(-a*b^2)^(1/3)*b*d + 2*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) + 1/54*(20*b^2*c
 - 5*a*b*f + 14*(-a*b^2)^(1/3)*b*d - 2*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 1/18*(28*b^3*d*x^7 - 4*a*b^
2*g*x^7 + 20*b^3*c*x^6 - 5*a*b^2*f*x^6 - 6*a*b^2*x^5*e + 49*a*b^2*d*x^4 - 7
*a^2*b*g*x^4 + 32*a*b^2*c*x^3 - 8*a^2*b*f*x^3 + 3*a^3*h*x^2 - 9*a^2*b*x^2*e
 + 18*a^2*b*d*x + 9*a^2*b*c)/((b*x^4 + a*x)^2*a^3*b) + 1/27*(14*a^3*b^2*d*(
-a/b)^(1/3) - 2*a^4*b*g*(-a/b)^(1/3) + 20*a^3*b^2*c - 5*a^4*b*f)*(-a/b)^(1/
3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

maple [**B**] time = 0.07, size = 626, normalized size = 1.74

$$\frac{2bgx^5}{9\left(bx^3+a\right)^2a^2}-\frac{5b^2dx^5}{9\left(bx^3+a\right)^2a^3}+\frac{5bfx^4}{18\left(bx^3+a\right)^2a^2}-\frac{11b^2cx^4}{18\left(bx^3+a\right)^2a^3}+\frac{bex^3}{3\left(bx^3+a\right)^2a^2}+\frac{7gx^2}{18\left(bx^3+a\right)^2a}-\frac{13bdx^2}{18\left(bx^3+a\right)^2a^2}+\frac{4fx}{9\left(bx^3+a\right)^2a}-\frac{7bcx}{9\left(bx^3+a\right)^2a^2}+\frac{e}{2\left(bx^3+a\right)^2a}-\frac{h}{6\left(bx^3+a\right)^2b}+\frac{5\sqrt{3}\,f\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{5f\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}-\frac{5f\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{2\sqrt{3}\,g\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}-\frac{2g\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}+\frac{g\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}-\frac{20\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{20c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}+\frac{10c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{14\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{14d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{7d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{e\ln(x)}{a^3}-\frac{e\ln\left(bx^3+a\right)}{3a^3}-\frac{d}{a^3x}-\frac{c}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x)
```

```
[Out] 10/27/a^3*c/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+14/27/a^3*d/(a/b)
^(1/3)*ln(x+(a/b)^(1/3))-7/27/a^3*d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))+7/18/a/(b*x^3+a)^2*x^2*g-20/27/a^3*c/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+4
/9/(b*x^3+a)^2/a*f*x+5/27/(a/b)^(2/3)*3^(1/2)/a^2/b*f*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))-1/6/(b*x^3+a)^2/b*h+1/2/a/(b*x^3+a)^2*e+5/18/(b*x^3+a)^2
/a^2*b*f*x^4+2/27/a^2*g*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))-1/a^3*d/x+1/a^3*e*ln(x)-1/3/a^3*e*ln(b*x^3+a)-20/27/a^3*c/(a/b)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/27/a^2*g/b/(a/b)^(1
/3)*ln(x+(a/b)^(1/3))+1/27/a^2*g/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))-14/27/a^3*d*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))-1/2/a^3*c/x^2+5/27/(a/b)^(2/3)/a^2/b*f*ln(x+(a/b)^(1/3))-5/54/(a/b)^(2/3
)/a^2/b*f*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/a^2/(b*x^3+a)^2*x^5*b*g-5/9
/a^3/(b*x^3+a)^2*b^2*d*x^5-11/18/a^3/(b*x^3+a)^2*b^2*c*x^4+1/3/a^2/(b*x^3+a
)^2*x^3*b*e-13/18/a^2/(b*x^3+a)^2*b*d*x^2-7/9/a^2/(b*x^3+a)^2*b*c*x
```

maxima [A] time = 3.10, size = 390, normalized size = 1.08

$$\frac{6\,ab^2ex^5 - 4\left(7\,b^3d - ab^2g\right)x^7 - 5\left(4\,b^3c - ab^2f\right)x^6 - 18\,a^2bdx - 7\left(7\,ab^2d - a^2bg\right)x^4 - 9\,a^2bc - 8\left(4\,ab^2c - a^2bf\right)x^3 + 3\left(3\,a^2be - a^3h\right)x^2}{18\left(a^3b^3x^8 + 2\,a^4b^2x^5 + a^5bx^2\right)} + \frac{e\log(x)}{a^3} - \frac{\sqrt{3}\left(14\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,ag\left(\frac{a}{b}\right)^{\frac{2}{3}} + 20\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - 5\,af\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4} - \frac{\left(18\,be\left(\frac{a}{b}\right)^{\frac{2}{3}} + 14\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}} - 2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}} - 20\,bc + 5\,af\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(9\,be\left(\frac{a}{b}\right)^{\frac{2}{3}} - 14\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}} + 2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}} + 20\,bc - 5\,af\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] 1/18*(6*a*b^2*e*x^5 - 4*(7*b^3*d - a*b^2*g)*x^7 - 5*(4*b^3*c - a*b^2*f)*x^6
 - 18*a^2*b*d*x - 7*(7*a*b^2*d - a^2*b*g)*x^4 - 9*a^2*b*c - 8*(4*a*b^2*c -
a^2*b*f)*x^3 + 3*(3*a^2*b*e - a^3*h)*x^2)/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^
5*b*x^2) + e*log(x)/a^3 - 1/27*sqrt(3)*(14*b*d*(a/b)^(2/3) - 2*a*g*(a/b)^(2
/3) + 20*b*c*(a/b)^(1/3) - 5*a*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/a^4 - 1/54*(18*b*e*(a/b)^(2/3) + 14*b*d*(a/b)^(1/3)
- 2*a*g*(a/b)^(1/3) - 20*b*c + 5*a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^3*b*(a/b)^(2/3)) - 1/27*(9*b*e*(a/b)^(2/3) - 14*b*d*(a/b)^(1/3) + 2*a*
g*(a/b)^(1/3) + 20*b*c - 5*a*f)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

mupad [B] time = 5.66, size = 1697, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] symsum(log((b^2*e*(400*b^2*c^2 + 25*a^2*f^2 - 18*a^2*e*g - 200*a*b*c*f + 12
6*a*b*d*e))/(81*a^8) - (root(19683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810
*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c*d*
z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*b^2*
d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*
b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b
^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k)*b^2*(400*b^2*c^2 + 25*a^2*f^2 - 54
*root(19683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5
*b^2*d*f*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z
+ 270*a^3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*
e*g - 168*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^
2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 80
00*b^4*c^3, z, k)*a^5*g + 36*a^2*e*g + 378*root(19683*a^11*b^2*z^3 + 19683*
a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z +
 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*
d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^
3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a
^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k)*a^4*b*d + 324*a
*b*e^2*x + 2800*b^2*c*d*x + 100*a^2*f*g*x + 2916*root(19683*a^11*b^2*z^3 +
```

```
19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c
*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*
b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 600
0*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 -
 125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k)^2*a^7*b*x
 - 200*a*b*c*f - 252*a*b*d*e - 400*a*b*c*g*x - 700*a*b*d*f*x + 1944*root(19
683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f
*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^
3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 16
8*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2
+ 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c
^3, z, k)*a^4*b*e*x))/(81*a^5) - (b*x*(8000*b^4*c^3 + 8*a^4*g^3 - 2744*a*b^
3*d^3 - 125*a^3*b*f^3 + 1500*a^2*b^2*c*f^2 + 1176*a^2*b^2*d^2*g - 6000*a*b^
3*c^2*f - 168*a^3*b*d*g^2 - 720*a^2*b^2*c*e*g - 1260*a^2*b^2*d*e*f + 5040*a
*b^3*c*d*e + 180*a^3*b*e*f*g))/(729*a^9))*root(19683*a^11*b^2*z^3 + 19683*a
^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z +
22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*d
*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^3
*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^
3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k), k, 1, 3) - (c/(
2*a) + (4*x^3*(4*b*c - a*f))/(9*a^2) + (7*x^4*(7*b*d - a*g))/(18*a^2) + (d*
x)/a + (5*b*x^6*(4*b*c - a*f))/(18*a^3) + (2*b*x^7*(7*b*d - a*g))/(9*a^3) -
 (x^2*(3*b*e - a*h))/(6*a*b) - (b*e*x^5)/(3*a^2))/(a^2*x^2 + b^2*x^8 + 2*a*
b*x^5) + (e*log(x))/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.376 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^3}\,dx$

Optimal. Leaf size=395

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)}{54a^{11/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7\right.}{27a^{11/3}b^{2/3}}$$

Rubi [A] time = 1.01, antiderivative size = 392, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7be-ah)}{\sqrt[3]{b}}-5ag+20bd\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)}{27a^{11/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}h+14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{9\sqrt{3}a^{11/3}b^{2/3}}-\frac{x\left(-3bx^2\left(\frac{5bc}{a}-3f\right)+2x(5be-2ah)-5ag+11bd\right)}{18a^3\left(a+bx^3\right)}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2\left(a+bx^3\right)^2}+\frac{(3bc-af)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3bc-af)}{a^4}-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d - a*g + (b*e - a*h)*x
- b*((b*c)/a - f)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d - 5*a*g + 2*(5*b
*e - 2*a*h)*x - 3*b*((5*b*c)/a - 3*f)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(
4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) - ((3*b*c - a*f
)*Log[x])/a^4 - ((5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[a^
(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) + ((20*b*d - 5*a*g - (2*a^(1/3)*(
7*b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*
a^(11/3)*b^(1/3)) + ((3*b*c - a*f)*Log[a + b*x^3])/(3*a^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}dx = -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-6b^2dx-6b^2ex^2+6b^2\left(\frac{bc}{a}\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd-5ag+2(5be-\cdots\right.}{18a^3\left(a\cdots\right.}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd-5ag+2(5be-\cdots\right.}{18a^3\left(a\cdots\right.}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \cdots$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \cdots$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \cdots$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \cdots$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \cdots$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \cdots$$

Mathematica [A] time = 0.79, size = 352, normalized size = 0.89

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}h-14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{b^{2/3}} - \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^{4/3}h-14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{b^{2/3}} + \frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-2a^{4/3}h+14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{b^{2/3}} + \frac{a^2(9a(f+x(g+hx))-9b(c+x(d+ex)))}{\left(a+bx^3\right)^2} + \frac{3a(6af+ax(5g+4hx)-12bc-bx(11d+10ex))}{a+bx^3} + 18(3bc-af)\log\left(a+bx^3\right) + 54\log(x)(af-3bc) - \frac{18ac}{x^3} - \frac{27ad}{x^2} - \frac{54ae}{x}}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-18*a*c)/x^3 - (27*a*d)/x^2 - (54*a*e)/x + (3*a*(-12*b*c + 6*a*f - b*x*(1
1*d + 10*e*x) + a*x*(5*g + 4*h*x)))/(a + b*x^3) + (a^2*(-9*b*(c + x*(d + e*
x)) + 9*a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (2*Sqrt[3]*a^(1/3)*(20*b^(4/3
)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*(-3*b*c + a*f)*Log[x] - (2*a^(1/3)*(20*b
^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g + 2*a^(4/3)*h)*Log[a^(1/3) + b^(1
/3)*x])/b^(2/3) + (a^(1/3)*(20*b^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g +
 2*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 18*
(3*b*c - a*f)*Log[a + b*x^3])/(54*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^
3)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^
3)^3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [A] time = 0.20, size = 431, normalized size = 1.09

$$\frac{\sqrt{3}\left(20\,b^{2}d-5\,abg+2\left(-ab^{2}\right)^{\frac{1}{3}}ah-14\left(-ab^{2}\right)^{\frac{1}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}}+\frac{\left(20\,b^{2}d-5\,abg-2\left(-ab^{2}\right)^{\frac{1}{3}}ah+14\left(-ab^{2}\right)^{\frac{1}{3}}be\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^{2}\right)^{\frac{2}{3}}a^{3}}+\frac{\left(3\,bc-af\right)\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{4}}-\frac{\left(3\,bc-af\right)\log\left(\left|x\right|\right)}{a^{4}}-\frac{\left(2\,a^{6}bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^{5}b^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}}e-20\,a^{5}b^{2}d+5\,a^{6}bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{9}b}+\frac{4\left(a^{2}bh-7\,ab^{2}e\right)x^{8}-5\left(4\,ab^{2}d-a^{2}bg\right)x^{7}-6\left(3\,ab^{2}c-a^{2}bf\right)x^{6}+7\left(a^{3}h-7\,a^{2}be\right)x^{5}-18\,a^{3}x^{2}e-9\,a^{3}dx-8\left(4\,a^{2}bd-a^{3}g\right)x^{4}-6\,a^{3}c-9\left(3\,a^{2}bc-a^{3}f\right)x^{3}}{18\left(bx^{3}+a\right)^{2}a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] 1/27*sqrt(3)*(20*b^2*d - 5*a*b*g + 2*(-a*b^2)^(1/3)*a*h - 14*(-a*b^2)^(1/3)
*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)
*a^3) + 1/54*(20*b^2*d - 5*a*b*g - 2*(-a*b^2)^(1/3)*a*h + 14*(-a*b^2)^(1/3)
*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) + 1/3*(
3*b*c - a*f)*log(abs(b*x^3 + a))/a^4 - (3*b*c - a*f)*log(abs(x))/a^4 - 1/27
*(2*a^6*b*h*(-a/b)^(1/3) - 14*a^5*b^2*(-a/b)^(1/3)*e - 20*a^5*b^2*d + 5*a^6
*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b) + 1/18*(4*(a^2*b*h -
7*a*b^2*e)*x^8 - 5*(4*a*b^2*d - a^2*b*g)*x^7 - 6*(3*a*b^2*c - a^2*b*f)*x^6
+ 7*(a^3*h - 7*a^2*b*e)*x^5 - 18*a^3*x^2*e - 9*a^3*d*x - 8*(4*a^2*b*d - a^3
*g)*x^4 - 6*a^3*c - 9*(3*a^2*b*c - a^3*f)*x^3)/((b*x^3 + a)^2*a^4*x^3)
```

maple [B] time = 0.07, size = 680, normalized size = 1.72

$$\frac{2bh\,x^{5}}{9\left(b\,x^{3}+a\right)^{2}a^{2}}-\frac{5b^{2}e\,x^{5}}{9\left(b\,x^{3}+a\right)^{2}a^{3}}+\frac{5bg\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}a^{2}}-\frac{11b^{2}d\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}a^{3}}+\frac{bf\,x^{3}}{3\left(b\,x^{3}+a\right)^{2}a^{2}}-\frac{2b^{2}c\,x^{3}}{3\left(b\,x^{3}+a\right)^{2}a^{3}}+\frac{7h\,x^{2}}{18\left(b\,x^{3}+a\right)^{2}a}-\frac{13be\,x^{2}}{18\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{4gx}{9\left(b\,x^{3}+a\right)^{2}a}-\frac{7bdx}{9\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{f}{2\left(b\,x^{3}+a\right)^{2}a}-\frac{5bc}{6\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{5\sqrt{3}\,g\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{5g\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{5g\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{2\sqrt{3}\,h\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2h\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{h\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{20\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{20d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{10d\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{14\sqrt{3}\,e\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{14e\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}-\frac{7e\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{f\ln\left(x\right)}{a^{3}}-\frac{f\ln\left(b\,x^{3}+a\right)}{3a^{3}}-\frac{3bc\ln\left(x\right)}{a^{4}}+\frac{bc\ln\left(b\,x^{3}+a\right)}{a^{4}}-\frac{e}{a^{3}x}-\frac{d}{2a^{3}x^{2}}-\frac{c}{3a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x)
```

```
[Out] -5/9/(b*x^3+a)^2/a^3*b^2*e*x^5-14/27*3^(1/2)/(a/b)^(1/3)/a^3*e*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))-7/27/(a/b)^(1/3)/a^3*e*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))+14/27/(a/b)^(1/3)/a^3*e*ln(x+(a/b)^(1/3))-5/6/(b*x^3+a)^2/a^2*b*c
-20/27/(a/b)^(2/3)/a^3*d*ln(x+(a/b)^(1/3))+10/27/(a/b)^(2/3)/a^3*d*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))+1/a^3*ln(x)*f+1/2/a/(b*x^3+a)^2*f-1/3/a^3*ln(b*x^
3+a)*f-11/18/(b*x^3+a)^2/a^3*b^2*d*x^4-20/27/(a/b)^(2/3)*3^(1/2)/a^3*d*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/a^3*c/x^3+7/18/a/(b*x^3+a)^2*x^2*h+
4/9/a/(b*x^3+a)^2*g*x+2/27/a^2*h*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))+5/27/a^2*g/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
```

```
(a/b)^(1/3)*x-1))-1/2/a^3*d/x^2-1/a^3*e/x-13/18/(b*x^3+a)^2/a^2*b*e*x^2-5/5
4/a^2*g/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+5/27/a^2*g/b/(a/b)^
(2/3)*ln(x+(a/b)^(1/3))-7/9/(b*x^3+a)^2/a^2*b*d*x+5/18/a^2/(b*x^3+a)^2*x^4*
b*g+1/3/a^2/(b*x^3+a)^2*x^3*b*f+2/9/a^2/(b*x^3+a)^2*x^5*b*h-2/27/a^2*h/b/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))+1/27/a^2*h/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))-2/3/(b*x^3+a)^2/a^3*b^2*c*x^3-3/a^4*b*c*ln(x)+1/a^4*b*c*ln(b*x^
3+a)
```

maxima [A] time = 3.09, size = 444, normalized size = 1.12

$$-\frac{4\left(7b^2e-abh\right)x^8+5\left(4b^2d-abg\right)x^7+6\left(3b^2c-abf\right)x^6+7\left(7abe-a^2h\right)x^5+18a^2ex^2+8\left(4abd-a^2g\right)x^4+9a^2dx+9\left(3abc-a^2f\right)x^3+6a^2c}{18\left(a^3b^2x^9+2a^4bx^6+a^5x^3\right)}-\frac{\left(3bc-af\right)\log(x)}{a^4}-\frac{\sqrt{3}\left(14abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+20abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-5a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^5}+\frac{\left(54b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-18abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-14abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+2a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+20abd-5a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(27b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-9abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+14abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-2a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-20abd+5a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] -1/18*(4*(7*b^2*e - a*b*h)*x^8 + 5*(4*b^2*d - a*b*g)*x^7 + 6*(3*b^2*c - a*b
*f)*x^6 + 7*(7*a*b*e - a^2*h)*x^5 + 18*a^2*e*x^2 + 8*(4*a*b*d - a^2*g)*x^4
+ 9*a^2*d*x + 9*(3*a*b*c - a^2*f)*x^3 + 6*a^2*c)/(a^3*b^2*x^9 + 2*a^4*b*x^6
 + a^5*x^3) - (3*b*c - a*f)*log(x)/a^4 - 1/27*sqrt(3)*(14*a*b*e*(a/b)^(2/3)
 - 2*a^2*h*(a/b)^(2/3) + 20*a*b*d*(a/b)^(1/3) - 5*a^2*g*(a/b)^(1/3))*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 + 1/54*(54*b^2*c*(a/b)^(2
/3) - 18*a*b*f*(a/b)^(2/3) - 14*a*b*e*(a/b)^(1/3) + 2*a^2*h*(a/b)^(1/3) + 2
0*a*b*d - 5*a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3
)) + 1/27*(27*b^2*c*(a/b)^(2/3) - 9*a*b*f*(a/b)^(2/3) + 14*a*b*e*(a/b)^(1/3
) - 2*a^2*h*(a/b)^(1/3) - 20*a*b*d + 5*a^2*g)*log(x + (a/b)^(1/3))/(a^4*b*(
a/b)^(2/3))
```

mupad [B] time = 6.32, size = 1994, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] symsum(log(- (1200*b^5*c*d^2 - 1134*b^5*c^2*e + 75*a^2*b^3*c*g^2 - 126*a^2*
b^3*e*f^2 - 25*a^3*b^2*f*g^2 + 18*a^3*b^2*f^2*h - 400*a*b^4*d^2*f + 162*a*b
^4*c^2*h - 108*a^2*b^3*c*f*h + 200*a^2*b^3*d*f*g - 600*a*b^4*c*d*g + 756*a*
b^4*c*e*f)/(81*a^9) - root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 59049
*a^8*b^3*c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d*h*z
- 39366*a^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59049*a^
4*b^4*c^2*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f*g - 10
80*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^2*b^3*c*
e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1176*a^3*b
^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3*c*f^2 + 7
29*a^3*b^2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d^3 + 8*a^5*
h^3 - 19683*b^5*c^3, z, k)*((400*a^4*b^4*d^2 + 25*a^6*b^2*g^2 + 756*a^4*b^4
*c*e - 108*a^5*b^3*c*h - 200*a^5*b^3*d*g - 252*a^5*b^3*e*f + 36*a^6*b^2*f*h
)/(81*a^9) + root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 59049*a^8*b^3*
c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d*h*z - 39366*a
^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59049*a^4*b^4*c^2
*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f*g - 1080*a^3*b^
2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^2*b^3*c*e*g + 324
0*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1176*a^3*b^2*e^2*h
- 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3*c*f^2 + 729*a^3*b^
2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d^3 + 8*a^5*h^3 - 196
83*b^5*c^3, z, k)*((378*a^8*b^3*e - 54*a^9*b^2*h)/(81*a^9) - (x*(52488*a^7*
b^4*c - 17496*a^8*b^3*f))/(729*a^9) + 36*root(19683*a^12*b^2*z^3 + 19683*a^
9*b^2*f*z^2 - 59049*a^8*b^3*c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z -
3240*a^6*b^2*d*h*z - 39366*a^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b
```

```
^2*f^2*z + 59049*a^4*b^4*c^2*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890
*a^3*b^2*e*f*g - 1080*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*
f + 5670*a^2*b^3*c*e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4
*c^2*f + 1176*a^3*b^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 656
1*a^2*b^3*c*f^2 + 729*a^3*b^2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000
*a*b^4*d^3 + 8*a^5*h^3 - 19683*b^5*c^3, z, k)*a^2*b^3*x) + (x*(26244*a^3*b^
5*c^2 + 2916*a^5*b^3*f^2 - 17496*a^4*b^4*c*f + 25200*a^4*b^4*d*e - 3600*a^5
*b^3*d*h - 6300*a^5*b^3*e*g + 900*a^6*b^2*g*h))/(729*a^9)) - (x*(8000*b^5*d
^3 - 2744*a*b^4*e^3 + 8*a^4*b*h^3 - 125*a^3*b^2*g^3 + 1500*a^2*b^3*d*g^2 +
1176*a^2*b^3*e^2*h - 168*a^3*b^2*e*h^2 - 15120*b^5*c*d*e - 6000*a*b^4*d^2*g
 - 540*a^2*b^3*c*g*h - 720*a^2*b^3*d*f*h - 1260*a^2*b^3*e*f*g + 180*a^3*b^2
*f*g*h + 2160*a*b^4*c*d*h + 3780*a*b^4*c*e*g + 5040*a*b^4*d*e*f))/(729*a^9)
)*root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 59049*a^8*b^3*c*z^2 + 810
*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d*h*z - 39366*a^5*b^3*c*f*
z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59049*a^4*b^4*c^2*z + 270*a^
4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f*g - 1080*a^3*b^2*d*f*h - 8
10*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^2*b^3*c*e*g + 3240*a^2*b^3*c
*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1176*a^3*b^2*e^2*h - 6000*a^2*
b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3*c*f^2 + 729*a^3*b^2*f^3 - 274
4*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d^3 + 8*a^5*h^3 - 19683*b^5*c^3,
 z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (x^3*(3*b*c - a*f))/(2*a^2) + (4*
x^4*(4*b*d - a*g))/(9*a^2) + (7*x^5*(7*b*e - a*h))/(18*a^2) + (d*x)/(2*a) +
 (b*x^6*(3*b*c - a*f))/(3*a^3) + (5*b*x^7*(4*b*d - a*g))/(18*a^3) + (2*b*x^
8*(7*b*e - a*h))/(9*a^3))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (log(x)*(3*b*c
- a*f))/a^4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.377 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx$

Optimal. Leaf size=68

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1850}

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx &= \int \left(ac+adx+aex^2+afx^3+bcx^4+bdx^5+bex^6+bfx^7\right)\,dx\\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8\end{aligned}$$

Mathematica [A] time = 0.01, size = 68, normalized size = 1.00

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4), x]
```

fricas [A] time = 0.34, size = 54, normalized size = 0.79

$$\frac{1}{8}x^8fb+\frac{1}{7}x^7eb+\frac{1}{6}x^6db+\frac{1}{5}x^5cb+\frac{1}{4}x^4fa+\frac{1}{3}x^3ea+\frac{1}{2}x^2da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*f*b + 1/7*x^7*e*b + 1/6*x^6*d*b + 1/5*x^5*c*b + 1/4*x^4*f*a + 1/3*x
^3*e*a + 1/2*x^2*d*a + x*c*a
```

giac [A] time = 0.15, size = 56, normalized size = 0.82

$$\frac{1}{8}\,bfx^{8}+\frac{1}{7}\,bx^{7}e+\frac{1}{6}\,bdx^{6}+\frac{1}{5}\,bcx^{5}+\frac{1}{4}\,afx^{4}+\frac{1}{3}\,ax^{3}e+\frac{1}{2}\,adx^{2}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a
*x^3*e + 1/2*a*d*x^2 + a*c*x
```

maple [A] time = 0.04, size = 55, normalized size = 0.81

$$\frac{1}{8}bf\,x^{8}+\frac{1}{7}be\,x^{7}+\frac{1}{6}bd\,x^{6}+\frac{1}{5}bc\,x^{5}+\frac{1}{4}af\,x^{4}+\frac{1}{3}ae\,x^{3}+\frac{1}{2}ad\,x^{2}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x
^7+1/8*b*f*x^8
```

maxima [A] time = 1.32, size = 54, normalized size = 0.79

$$\frac{1}{8}\,bfx^{8}+\frac{1}{7}\,bex^{7}+\frac{1}{6}\,bdx^{6}+\frac{1}{5}\,bcx^{5}+\frac{1}{4}\,afx^{4}+\frac{1}{3}\,aex^{3}+\frac{1}{2}\,adx^{2}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a
*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

mupad [B] time = 0.04, size = 54, normalized size = 0.79

$$\frac{b\,f\,x^8}{8}+\frac{b\,e\,x^7}{7}+\frac{b\,d\,x^6}{6}+\frac{b\,c\,x^5}{5}+\frac{a\,f\,x^4}{4}+\frac{a\,e\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^5)/5 + (a*e*x^3)/3 + (b*d*x^6)/6 + (a*f*x^4)/4
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

sympy [A] time = 0.08, size = 63, normalized size = 0.93

$$acx+\frac{adx^{2}}{2}+\frac{aex^{3}}{3}+\frac{afx^{4}}{4}+\frac{bcx^{5}}{5}+\frac{bdx^{6}}{6}+\frac{bex^{7}}{7}+\frac{bfx^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + a*f*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*
e*x**7/7 + b*f*x**8/8
```

3.378 $\int x^3 \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right) dx$

Optimal. Leaf size=73

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1820}

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*
x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^3 \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right) dx &= \int \left(acx^3 + adx^4 + aex^5 + afx^6 + bcx^7 + bdx^8 + bex^9 + bfx^{10}\right) dx\\ &= \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.00

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*
x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4), x]
```

fricas [A] time = 0.36, size = 57, normalized size = 0.78

$$\frac{1}{11}x^{11}fb + \frac{1}{10}x^{10}eb + \frac{1}{9}x^9db + \frac{1}{8}x^8cb + \frac{1}{7}x^7fa + \frac{1}{6}x^6ea + \frac{1}{5}x^5da + \frac{1}{4}x^4ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f*b + 1/10*x^10*e*b + 1/9*x^9*d*b + 1/8*x^8*c*b + 1/7*x^7*f*a + 1
/6*x^6*e*a + 1/5*x^5*d*a + 1/4*x^4*c*a
```

giac [A] time = 0.16, size = 59, normalized size = 0.81

$$\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bx^{10}e+\frac{1}{9}\,bdx^{9}+\frac{1}{8}\,bcx^{8}+\frac{1}{7}\,afx^{7}+\frac{1}{6}\,ax^{6}e+\frac{1}{5}\,adx^{5}+\frac{1}{4}\,acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/11*b*f*x^11 + 1/10*b*x^10*e + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1
/6*a*x^6*e + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

maple [A] time = 0.04, size = 58, normalized size = 0.79

$$\frac{1}{11}bf\,x^{11}+\frac{1}{10}be\,x^{10}+\frac{1}{9}bd\,x^{9}+\frac{1}{8}bc\,x^{8}+\frac{1}{7}af\,x^{7}+\frac{1}{6}ae\,x^{6}+\frac{1}{5}ad\,x^{5}+\frac{1}{4}ac\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x)
```

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b*d*x^9+1/1
0*b*e*x^10+1/11*b*f*x^11
```

maxima [A] time = 1.33, size = 57, normalized size = 0.78

$$\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bex^{10}+\frac{1}{9}\,bdx^{9}+\frac{1}{8}\,bcx^{8}+\frac{1}{7}\,afx^{7}+\frac{1}{6}\,aex^{6}+\frac{1}{5}\,adx^{5}+\frac{1}{4}\,acx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1
/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

mupad [B] time = 0.03, size = 57, normalized size = 0.78

$$\frac{b\,f\,x^{11}}{11}+\frac{b\,e\,x^{10}}{10}+\frac{b\,d\,x^{9}}{9}+\frac{b\,c\,x^{8}}{8}+\frac{a\,f\,x^{7}}{7}+\frac{a\,e\,x^{6}}{6}+\frac{a\,d\,x^{5}}{5}+\frac{a\,c\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (b*c*x^8)/8 + (a*e*x^6)/6 + (b*d*x^9)/9 + (a*f*
x^7)/7 + (b*e*x^10)/10 + (b*f*x^11)/11
```

sympy [A] time = 0.07, size = 66, normalized size = 0.90

$$\frac{acx^{4}}{4}+\frac{adx^{5}}{5}+\frac{aex^{6}}{6}+\frac{afx^{7}}{7}+\frac{bcx^{8}}{8}+\frac{bdx^{9}}{9}+\frac{bex^{10}}{10}+\frac{bfx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + a*f*x**7/7 + b*c*x**8/8 + b*d*x**9/9
 + b*e*x**10/10 + b*f*x**11/11
```

3.379 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx$

Optimal. Leaf size=109

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1582, 1657}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 +
 (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(
a + b*x^4)^3)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx &= \frac{f\left(a+bx^4\right)^3}{12b}+\int \left(c+dx+ex^2\right)\left(a+bx^4\right)^2\,dx\\ &= \frac{f\left(a+bx^4\right)^3}{12b}+\int \left(a^2c+a^2dx+a^2ex^2+2abcx^4+2abdx^5+2abex^6+\right.\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9+\frac{}{1}\end{aligned}$$

Mathematica [A] time = 0.00, size = 124, normalized size = 1.14

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x
^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx + ex^2 + fx^3\right)\left(a + bx^4\right)^2\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2, x]
```

fricas [A] time = 0.34, size = 102, normalized size = 0.94

$$\frac{1}{12}x^{12}fb^2 + \frac{1}{11}x^{11}eb^2 + \frac{1}{10}x^{10}db^2 + \frac{1}{9}x^9cb^2 + \frac{1}{4}x^8fba + \frac{2}{7}x^7eba + \frac{1}{3}x^6dba + \frac{2}{5}x^5cba + \frac{1}{4}x^4fa^2 + \frac{1}{3}x^3ea^2 + \frac{1}{2}x^2da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b^2 + 1/4*x
^8*f*b*a + 2/7*x^7*e*b*a + 1/3*x^6*d*b*a + 2/5*x^5*c*b*a + 1/4*x^4*f*a^2 +
1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2
```

giac [A] time = 0.16, size = 105, normalized size = 0.96

$$\frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2x^{11}e + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abx^7e + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2x^3e + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

maple [A] time = 0.04, size = 103, normalized size = 0.94

$$\frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2ex^{11} + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abex^7 + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

```
[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*f*x^8
+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*f*a^2*x^4+1/3*a^2*e*x^3+1/2*
a^2*d*x^2+a^2*c*x
```

maxima [A] time = 1.36, size = 102, normalized size = 0.94

$$\frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2ex^{11} + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abex^7 + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

mupad [B] time = 0.08, size = 102, normalized size = 0.94

$$\frac{f\,a^2\,x^4}{4}+\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{f\,a\,b\,x^8}{4}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{f\,b^2\,x^{12}}{12}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*c*x^9)/9 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x^
4)/4 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12 + a^2*c*x + (2*a*b*c*x^5)/5 + (a*b
*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4
```

sympy [A] time = 0.09, size = 121, normalized size = 1.11

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{a^2fx^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{abfx^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{b^2fx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5 +
 a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**1
0/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

3.380 $\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^2 \, dx$

Optimal. Leaf size=114

$$\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{c\left(a+bx^4\right)^3}{12b}+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1582, 1850}

$$\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{c\left(a+bx^4\right)^3}{12b}+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 + (a*b*e*x^
10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15
)/15 + (c*(a + b*x^4)^3)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^3 \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2 \, dx &= \frac{c\left(a+bx^4\right)^3}{12b}+\int \left(a+bx^4\right)^2\left(-cx^3+x^3\left(c+dx+ex^2+fx^3\right)\right) dx\\ &= \frac{c\left(a+bx^4\right)^3}{12b}+\int \left(a^2dx^4+a^2ex^5+a^2fx^6+2abdx^8+2abex^9+2abf\right.\\ &= \frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 129, normalized size = 1.13

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{1}{4}abcx^8+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{12}b^2cx^{12}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (a*b*c*x^8)
/4 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*c*x^12)/12
 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2, x]
```

fricas [A] time = 0.36, size = 105, normalized size = 0.92

$$\frac{1}{15}x^{15}fb^2+\frac{1}{14}x^{14}eb^2+\frac{1}{13}x^{13}db^2+\frac{1}{12}x^{12}cb^2+\frac{2}{11}x^{11}fba+\frac{1}{5}x^{10}eba+\frac{2}{9}x^9dba+\frac{1}{4}x^8cba+\frac{1}{7}x^7fa^2+\frac{1}{6}x^6ea^2+\frac{1}{5}x^5da^2+\frac{1}{4}x^4ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*f*b^2 + 1/14*x^14*e*b^2 + 1/13*x^13*d*b^2 + 1/12*x^12*c*b^2 + 2/1
1*x^11*f*b*a + 1/5*x^10*e*b*a + 2/9*x^9*d*b*a + 1/4*x^8*c*b*a + 1/7*x^7*f*a
^2 + 1/6*x^6*e*a^2 + 1/5*x^5*d*a^2 + 1/4*x^4*c*a^2
```

giac [A] time = 0.15, size = 108, normalized size = 0.95

$$\frac{1}{15}b^2fx^{15}+\frac{1}{14}b^2x^{14}e+\frac{1}{13}b^2dx^{13}+\frac{1}{12}b^2cx^{12}+\frac{2}{11}abfx^{11}+\frac{1}{5}abx^{10}e+\frac{2}{9}abdx^9+\frac{1}{4}abcx^8+\frac{1}{7}a^2fx^7+\frac{1}{6}a^2x^6e+\frac{1}{5}a^2dx^5+\frac{1}{4}a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*x^14*e + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/1
1*a*b*f*x^11 + 1/5*a*b*x^10*e + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x
^7 + 1/6*a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

maple [A] time = 0.04, size = 106, normalized size = 0.93

$$\frac{1}{15}b^2fx^{15}+\frac{1}{14}b^2ex^{14}+\frac{1}{13}b^2dx^{13}+\frac{1}{12}b^2cx^{12}+\frac{2}{11}abfx^{11}+\frac{1}{5}abex^{10}+\frac{2}{9}abdx^9+\frac{1}{4}abcx^8+\frac{1}{7}a^2fx^7+\frac{1}{6}a^2ex^6+\frac{1}{5}a^2dx^5+\frac{1}{4}a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

```
[Out] 1/15*b^2*f*x^15+1/14*b^2*e*x^14+1/13*b^2*d*x^13+1/12*c*b^2*x^12+2/11*a*b*f*
x^11+1/5*a*b*e*x^10+2/9*a*b*d*x^9+1/4*a*b*c*x^8+1/7*a^2*f*x^7+1/6*a^2*e*x^6
+1/5*a^2*d*x^5+1/4*a^2*c*x^4
```

maxima [A] time = 1.33, size = 105, normalized size = 0.92

$$\frac{1}{15}b^2fx^{15}+\frac{1}{14}b^2ex^{14}+\frac{1}{13}b^2dx^{13}+\frac{1}{12}b^2cx^{12}+\frac{2}{11}abfx^{11}+\frac{1}{5}abex^{10}+\frac{2}{9}abdx^9+\frac{1}{4}abcx^8+\frac{1}{7}a^2fx^7+\frac{1}{6}a^2ex^6+\frac{1}{5}a^2dx^5+\frac{1}{4}a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/1
1*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x
^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

mupad [B] time = 0.07, size = 105, normalized size = 0.92

$$\frac{f\,a^2\,x^7}{7}+\frac{e\,a^2\,x^6}{6}+\frac{d\,a^2\,x^5}{5}+\frac{c\,a^2\,x^4}{4}+\frac{2\,f\,a\,b\,x^{11}}{11}+\frac{e\,a\,b\,x^{10}}{5}+\frac{2\,d\,a\,b\,x^9}{9}+\frac{c\,a\,b\,x^8}{4}+\frac{f\,b^2\,x^{15}}{15}+\frac{e\,b^2\,x^{14}}{14}+\frac{d\,b^2\,x^{13}}{13}+\frac{c\,b^2\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (b^2*c*x^12)/12 + (a^2*e*x^6)/6 + (b^2*d*x^
13)/13 + (a^2*f*x^7)/7 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15 + (a*b*c*x^8)/4
+ (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11
```

sympy [A] time = 0.09, size = 124, normalized size = 1.09

$$\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{a^2fx^7}{7}+\frac{abcx^8}{4}+\frac{2abdx^9}{9}+\frac{abex^{10}}{5}+\frac{2abfx^{11}}{11}+\frac{b^2cx^{12}}{12}+\frac{b^2dx^{13}}{13}+\frac{b^2ex^{14}}{14}+\frac{b^2fx^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + a**2*f*x**7/7 + a*b*c*x**8/
4 + 2*a*b*d*x**9/9 + a*b*e*x**10/5 + 2*a*b*f*x**11/11 + b**2*c*x**12/12 + b
**2*d*x**13/13 + b**2*e*x**14/14 + b**2*f*x**15/15
```

3.381 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx$

Optimal. Leaf size=151

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b^3cx$$

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1582, 1657}

$$\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)
/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e
*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b
*x^4)^4)/(16*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx &= \frac{f\left(a+bx^4\right)^4}{16b}+\int \left(c+dx+ex^2\right)\left(a+bx^4\right)^3\,dx\\ &= \frac{f\left(a+bx^4\right)^4}{16b}+\int \left(a^3c+a^3dx+a^3ex^2+3a^2bcx^4+3a^2bdx^5+3a^2bex\right.\\ &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx\end{aligned}$$

Mathematica [A] time = 0.01, size = 180, normalized size = 1.19

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx + ex^2 + fx^3\right)\left(a + bx^4\right)^3 \, dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]

[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3, x]

fricas [A] time = 0.34, size = 150, normalized size = 0.99

$$\frac{1}{16}x^{16}fb^3 + \frac{1}{15}x^{15}eb^3 + \frac{1}{14}x^{14}db^3 + \frac{1}{13}x^{13}cb^3 + \frac{1}{4}x^{12}fb^2a + \frac{3}{11}x^{11}eb^2a + \frac{3}{10}x^{10}db^2a + \frac{1}{3}x^9cb^2a + \frac{3}{8}x^8fba^2 + \frac{3}{7}x^7eba^2 + \frac{1}{2}x^6dba^2 + \frac{3}{5}x^5cba^2 + \frac{1}{4}x^4fa^3 + \frac{1}{3}x^3ea^3 + \frac{1}{2}x^2da^3 + xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")

[Out] 1/16*x^16*f*b^3 + 1/15*x^15*e*b^3 + 1/14*x^14*d*b^3 + 1/13*x^13*c*b^3 + 1/4*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/10*x^10*d*b^2*a + 1/3*x^9*c*b^2*a + 3/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*x^4*f*a^3 + 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3

giac [A] time = 0.16, size = 154, normalized size = 1.02

$$\frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3x^{15}e + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2x^{11}e + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bx^7e + \frac{1}{2}a^2bdx^6 + \frac{3}{5}a^2bcx^5 + \frac{1}{4}a^3fx^4 + \frac{1}{3}a^3x^3e + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")

[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x

maple [A] time = 0.04, size = 151, normalized size = 1.00

$$\frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3ex^{15} + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2ex^{11} + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^2bdx^6 + \frac{3}{5}a^2bcx^5 + \frac{1}{4}a^3fx^4 + \frac{1}{3}a^3ex^3 + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)

[Out] 1/16*b^3*f*x^16+1/15*b^3*e*x^15+1/14*b^3*d*x^14+1/13*b^3*c*x^13+1/4*a*b^2*f*x^12+3/11*a*b^2*e*x^11+3/10*a*b^2*d*x^10+1/3*a*b^2*c*x^9+3/8*a^2*b*f*x^8+3/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*a^3*f*x^4+1/3*a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x

maxima [A] time = 1.37, size = 150, normalized size = 0.99

$$\frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3ex^{15} + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2ex^{11} + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^2bdx^6 + \frac{3}{5}a^2bcx^5 + \frac{1}{4}a^3fx^4 + \frac{1}{3}a^3ex^3 + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")

[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3

```
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

mupad [B] time = 0.16, size = 150, normalized size = 0.99

$$\frac{f\,a^3\,x^4}{4}+\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,f\,a^2\,b\,x^8}{8}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{f\,a\,b^2\,x^{12}}{4}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{f\,b^3\,x^{16}}{16}+\frac{e\,b^3\,x^{15}}{15}+\frac{d\,b^3\,x^{14}}{14}+\frac{c\,b^3\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^3*d*x^2)/2 + (b^3*c*x^13)/13 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f*
x^4)/4 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16 + a^3*c*x + (3*a^2*b*c*x^5)/5 +
(a*b^2*c*x^9)/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7)/7
 + (3*a*b^2*e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

sympy [A] time = 0.10, size = 180, normalized size = 1.19

$$a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{a^3fx^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3a^2bfx^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{ab^2fx^{12}}{4}+\frac{b^3cx^{13}}{13}+\frac{b^3dx^{14}}{14}+\frac{b^3ex^{15}}{15}+\frac{b^3fx^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5/
5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**9
/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3*c*
x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

3.382 $\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^3 \, dx$

Optimal. Leaf size=156

$$\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{c\left(a+bx^4\right)^4}{16b}+\frac{1}{1}$$

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1582, 1850}

$$\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{c\left(a+bx^4\right)^4}{16b}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 + (3*a^2*b*
e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/1
4 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
+ (c*(a + b*x^4)^4)/(16*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx &= \frac{c\left(a+bx^4\right)^4}{16b}+\int \left(a+bx^4\right)^3\left(-cx^3+x^3\left(c+dx+ex^2+fx^3\right)\right)\,dx\\ &= \frac{c\left(a+bx^4\right)^4}{16b}+\int \left(a^3dx^4+a^3ex^5+a^3fx^6+3a^2bdx^8+3a^2bex^9+3a^2\right.\\ &= \frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{}{1}\end{aligned}$$

Mathematica [A] time = 0.02, size = 185, normalized size = 1.19

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{3}{8}a^2bcx^8+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{1}{4}ab^2cx^{12}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{1}{16}b^3cx^{16}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (3*a^2*b*c*
x^8)/8 + (a^2*b*d*x^9)/3 + (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (a*b
^2*c*x^12)/4 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15)/5
 + (b^3*c*x^16)/16 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3, x]
```

fricas [A] time = 0.35, size = 153, normalized size = 0.98

$$\frac{1}{19}x^{19}fb^3 + \frac{1}{18}x^{18}eb^3 + \frac{1}{17}x^{17}db^3 + \frac{1}{16}x^{16}cb^3 + \frac{1}{5}x^{15}fb^2a + \frac{3}{14}x^{14}eb^2a + \frac{3}{13}x^{13}db^2a + \frac{1}{4}x^{12}cb^2a + \frac{3}{11}x^{11}fba^2 + \frac{3}{10}x^{10}eba^2 + \frac{1}{3}x^9dba^2 + \frac{3}{8}x^8cba^2 + \frac{1}{7}x^7fa^3 + \frac{1}{6}x^6ea^3 + \frac{1}{5}x^5da^3 + \frac{1}{4}x^4ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*f*b^3 + 1/18*x^18*e*b^3 + 1/17*x^17*d*b^3 + 1/16*x^16*c*b^3 + 1/5
*x^15*f*b^2*a + 3/14*x^14*e*b^2*a + 3/13*x^13*d*b^2*a + 1/4*x^12*c*b^2*a +
3/11*x^11*f*b*a^2 + 3/10*x^10*e*b*a^2 + 1/3*x^9*d*b*a^2 + 3/8*x^8*c*b*a^2 +
 1/7*x^7*f*a^3 + 1/6*x^6*e*a^3 + 1/5*x^5*d*a^3 + 1/4*x^4*c*a^3
```

giac [A] time = 0.16, size = 157, normalized size = 1.01

$$\frac{1}{19}b^3fx^{19} + \frac{1}{18}b^3x^{18}e + \frac{1}{17}b^3dx^{17} + \frac{1}{16}b^3cx^{16} + \frac{1}{5}ab^2fx^{15} + \frac{3}{14}ab^2x^{14}e + \frac{3}{13}ab^2dx^{13} + \frac{1}{4}ab^2cx^{12} + \frac{3}{11}a^2bfx^{11} + \frac{3}{10}a^2bx^{10}e + \frac{1}{3}a^2bdx^9 + \frac{3}{8}a^2bcx^8 + \frac{1}{7}a^3fx^7 + \frac{1}{6}a^3x^6e + \frac{1}{5}a^3dx^5 + \frac{1}{4}a^3cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*x^18*e + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5
*a*b^2*f*x^15 + 3/14*a*b^2*x^14*e + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 +
3/11*a^2*b*f*x^11 + 3/10*a^2*b*x^10*e + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 +
 1/7*a^3*f*x^7 + 1/6*a^3*x^6*e + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

maple [A] time = 0.04, size = 154, normalized size = 0.99

$$\frac{1}{19}b^3f\,x^{19} + \frac{1}{18}b^3e\,x^{18} + \frac{1}{17}b^3d\,x^{17} + \frac{1}{16}b^3c\,x^{16} + \frac{1}{5}a\,b^2f\,x^{15} + \frac{3}{14}a\,b^2e\,x^{14} + \frac{3}{13}a\,b^2d\,x^{13} + \frac{1}{4}a\,b^2c\,x^{12} + \frac{3}{11}a^2bf\,x^{11} + \frac{3}{10}a^2be\,x^{10} + \frac{1}{3}a^2bd\,x^9 + \frac{3}{8}a^2bc\,x^8 + \frac{1}{7}a^3f\,x^7 + \frac{1}{6}a^3e\,x^6 + \frac{1}{5}a^3d\,x^5 + \frac{1}{4}a^3c\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/19*b^3*f*x^19+1/18*b^3*e*x^18+1/17*b^3*d*x^17+1/16*b^3*c*x^16+1/5*a*b^2*f
*x^15+3/14*a*b^2*e*x^14+3/13*a*b^2*d*x^13+1/4*a*b^2*c*x^12+3/11*a^2*b*f*x^1
1+3/10*a^2*b*e*x^10+1/3*a^2*b*d*x^9+3/8*a^2*c*b*x^8+1/7*a^3*f*x^7+1/6*a^3*e
*x^6+1/5*a^3*d*x^5+1/4*a^3*c*x^4
```

maxima [A] time = 1.38, size = 153, normalized size = 0.98

$$\frac{1}{19}b^3fx^{19} + \frac{1}{18}b^3ex^{18} + \frac{1}{17}b^3dx^{17} + \frac{1}{16}b^3cx^{16} + \frac{1}{5}ab^2fx^{15} + \frac{3}{14}ab^2ex^{14} + \frac{3}{13}ab^2dx^{13} + \frac{1}{4}ab^2cx^{12} + \frac{3}{11}a^2bfx^{11} + \frac{3}{10}a^2bex^{10} + \frac{1}{3}a^2bdx^9 + \frac{3}{8}a^2bcx^8 + \frac{1}{7}a^3fx^7 + \frac{1}{6}a^3ex^6 + \frac{1}{5}a^3dx^5 + \frac{1}{4}a^3cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5
*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 +
```

```
3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 +
 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

mupad [B] time = 0.16, size = 153, normalized size = 0.98

$$\frac{f\,a^3\,x^7}{7}+\frac{e\,a^3\,x^6}{6}+\frac{d\,a^3\,x^5}{5}+\frac{c\,a^3\,x^4}{4}+\frac{3\,f\,a^2\,b\,x^{11}}{11}+\frac{3\,e\,a^2\,b\,x^{10}}{10}+\frac{d\,a^2\,b\,x^9}{3}+\frac{3\,c\,a^2\,b\,x^8}{8}+\frac{f\,a\,b^2\,x^{15}}{5}+\frac{3\,e\,a\,b^2\,x^{14}}{14}+\frac{3\,d\,a\,b^2\,x^{13}}{13}+\frac{c\,a\,b^2\,x^{12}}{4}+\frac{f\,b^3\,x^{19}}{19}+\frac{e\,b^3\,x^{18}}{18}+\frac{d\,b^3\,x^{17}}{17}+\frac{c\,b^3\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (b^3*c*x^16)/16 + (a^3*e*x^6)/6 + (b^3*d*x^
17)/17 + (a^3*f*x^7)/7 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19 + (3*a^2*b*c*x^8
)/8 + (a*b^2*c*x^12)/4 + (a^2*b*d*x^9)/3 + (3*a*b^2*d*x^13)/13 + (3*a^2*b*e
*x^10)/10 + (3*a*b^2*e*x^14)/14 + (3*a^2*b*f*x^11)/11 + (a*b^2*f*x^15)/5
```

sympy [A] time = 0.10, size = 184, normalized size = 1.18

$$\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{a^3fx^7}{7}+\frac{3a^2bcx^8}{8}+\frac{a^2bdx^9}{3}+\frac{3a^2bex^{10}}{10}+\frac{3a^2bfx^{11}}{11}+\frac{ab^2cx^{12}}{4}+\frac{3ab^2dx^{13}}{13}+\frac{3ab^2ex^{14}}{14}+\frac{ab^2fx^{15}}{5}+\frac{b^3cx^{16}}{16}+\frac{b^3dx^{17}}{17}+\frac{b^3ex^{18}}{18}+\frac{b^3fx^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + a**3*f*x**7/7 + 3*a**2*b*c*
x**8/8 + a**2*b*d*x**9/3 + 3*a**2*b*e*x**10/10 + 3*a**2*b*f*x**11/11 + a*b*
*2*c*x**12/4 + 3*a*b**2*d*x**13/13 + 3*a*b**2*e*x**14/14 + a*b**2*f*x**15/5
 + b**3*c*x**16/16 + b**3*d*x**17/17 + b**3*e*x**18/18 + b**3*f*x**19/19
```

3.383 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx$

Optimal. Leaf size=193

$$a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx$$

Rubi [A] time = 0.16, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1582, 1657}

$$\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{f\left(a+bx^4\right)^5}{20b}+\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*a^3*b*d*x^
6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*
a^2*b^2*e*x^11)/11 + (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*
x^15)/15 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (f*(a + b*
x^4)^5)/(20*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx &= \frac{f\left(a+bx^4\right)^5}{20b}+\int \left(c+dx+ex^2\right)\left(a+bx^4\right)^4\,dx\\ &= \frac{f\left(a+bx^4\right)^5}{20b}+\int \left(a^4c+a^4dx+a^4ex^2+4a^3bcx^4+4a^3bdx^5+4a^3bex\right.\\ &= a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx\end{aligned}$$

Mathematica [A] time = 0.01, size = 236, normalized size = 1.22

$$a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{1}{4}a^4fx^4+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{1}{2}a^3bfx^8+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{1}{2}a^2b^2fx^{12}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{1}{4}ab^3fx^{16}+\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}+\frac{1}{20}b^4fx^{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (a^4*f*x^4)/4 + (4*a^3*b*c*x^5)/5
 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (a^3*b*f*x^8)/2 + (2*a^2*b^2*c*x
^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/2 +
 (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*x^15)/15 + (a*b^3*f*
x^16)/4 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (b^4*f*x^20
)/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx + ex^2 + fx^3\right)\left(a + bx^4\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4, x]
```

fricas [A] time = 0.36, size = 198, normalized size = 1.03

$$\frac{1}{20}x^{20}fb^4 + \frac{1}{19}x^{19}eb^4 + \frac{1}{18}x^{18}db^4 + \frac{1}{17}x^{17}cb^4 + \frac{1}{4}x^{16}fb^3a + \frac{4}{15}x^{15}eb^3a + \frac{2}{7}x^{14}db^3a + \frac{4}{13}x^{13}cb^3a + \frac{1}{2}x^{12}fb^2a^2 + \frac{6}{11}x^{11}eb^2a^2 + \frac{3}{5}x^{10}db^2a^2 + \frac{2}{3}x^9cb^2a^2 + \frac{1}{2}x^8fba^3 + \frac{4}{7}x^7eba^3 + \frac{2}{3}x^6dba^3 + \frac{4}{5}x^5cba^3 + \frac{1}{4}x^4fa^4 + \frac{1}{3}x^3ea^4 + \frac{1}{2}x^2da^4 + xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*f*b^4 + 1/19*x^19*e*b^4 + 1/18*x^18*d*b^4 + 1/17*x^17*c*b^4 + 1/4
*x^16*f*b^3*a + 4/15*x^15*e*b^3*a + 2/7*x^14*d*b^3*a + 4/13*x^13*c*b^3*a +
1/2*x^12*f*b^2*a^2 + 6/11*x^11*e*b^2*a^2 + 3/5*x^10*d*b^2*a^2 + 2/3*x^9*c*b
^2*a^2 + 1/2*x^8*f*b*a^3 + 4/7*x^7*e*b*a^3 + 2/3*x^6*d*b*a^3 + 4/5*x^5*c*b*
a^3 + 1/4*x^4*f*a^4 + 1/3*x^3*e*a^4 + 1/2*x^2*d*a^4 + x*c*a^4
```

giac [A] time = 0.17, size = 203, normalized size = 1.05

$$\frac{1}{20}b^4fx^{20} + \frac{1}{19}b^4x^{19}e + \frac{1}{18}b^4dx^{18} + \frac{1}{17}b^4cx^{17} + \frac{1}{4}ab^3fx^{16} + \frac{4}{15}ab^3x^{15}e + \frac{2}{7}ab^3dx^{14} + \frac{4}{13}ab^3cx^{13} + \frac{1}{2}a^2b^2fx^{12} + \frac{6}{11}a^2b^2x^{11}e + \frac{3}{5}a^2b^2dx^{10} + \frac{2}{3}a^2b^2cx^9 + \frac{1}{2}a^3bfx^8 + \frac{4}{7}a^3bx^7e + \frac{2}{3}a^3bdx^6 + \frac{4}{5}a^3bcx^5 + \frac{1}{4}a^4fx^4 + \frac{1}{3}a^4x^3e + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*x^19*e + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4
*a*b^3*f*x^16 + 4/15*a*b^3*x^15*e + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 +
1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*x^11*e + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2
*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x
```

maple [A] time = 0.04, size = 199, normalized size = 1.03

$$\frac{1}{20}b^4f\,x^{20} + \frac{1}{19}b^4e\,x^{19} + \frac{1}{18}b^4d\,x^{18} + \frac{1}{17}b^4c\,x^{17} + \frac{1}{4}a\,b^3f\,x^{16} + \frac{4}{15}a\,b^3e\,x^{15} + \frac{2}{7}a\,b^3d\,x^{14} + \frac{4}{13}a\,b^3c\,x^{13} + \frac{1}{2}a^2b^2f\,x^{12} + \frac{6}{11}a^2b^2e\,x^{11} + \frac{3}{5}a^2b^2d\,x^{10} + \frac{2}{3}a^2b^2c\,x^9 + \frac{1}{2}a^3bf\,x^8 + \frac{4}{7}a^3be\,x^7 + \frac{2}{3}a^3bd\,x^6 + \frac{4}{5}a^3bc\,x^5 + \frac{1}{4}a^4f\,x^4 + \frac{1}{3}a^4e\,x^3 + \frac{1}{2}a^4d\,x^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x)
```

```
[Out] 1/20*f*b^4*x^20+1/19*b^4*e*x^19+1/18*b^4*d*x^18+1/17*b^4*c*x^17+1/4*f*a*b^3
*x^16+4/15*a*b^3*e*x^15+2/7*a*b^3*d*x^14+4/13*a*b^3*c*x^13+1/2*f*b^2*a^2*x^
12+6/11*a^2*b^2*e*x^11+3/5*a^2*b^2*d*x^10+2/3*a^2*b^2*c*x^9+1/2*a^3*b*f*x^8
+4/7*a^3*b*e*x^7+2/3*a^3*b*d*x^6+4/5*a^3*b*c*x^5+1/4*a^4*f*x^4+1/3*a^4*e*x^
3+1/2*a^4*d*x^2+a^4*c*x
```

maxima [A] time = 1.33, size = 198, normalized size = 1.03

$$\frac{1}{20}b^4fx^{20} + \frac{1}{19}b^4ex^{19} + \frac{1}{18}b^4dx^{18} + \frac{1}{17}b^4cx^{17} + \frac{1}{4}ab^3fx^{16} + \frac{4}{15}ab^3ex^{15} + \frac{2}{7}ab^3dx^{14} + \frac{4}{13}ab^3cx^{13} + \frac{1}{2}a^2b^2fx^{12} + \frac{6}{11}a^2b^2ex^{11} + \frac{3}{5}a^2b^2dx^{10} + \frac{2}{3}a^2b^2cx^9 + \frac{1}{2}a^3bfx^8 + \frac{4}{7}a^3bex^7 + \frac{2}{3}a^3bdx^6 + \frac{4}{5}a^3bcx^5 + \frac{1}{4}a^4fx^4 + \frac{1}{3}a^4ex^3 + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4
*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 +
1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2
*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

mupad [B] time = 5.08, size = 198, normalized size = 1.03

$$\frac{f\,a^4\,x^4}{4}+\frac{e\,a^4\,x^3}{3}+\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{f\,a^3\,b\,x^8}{2}+\frac{4\,e\,a^3\,b\,x^7}{7}+\frac{2\,d\,a^3\,b\,x^6}{3}+\frac{4\,c\,a^3\,b\,x^5}{5}+\frac{f\,a^2\,b^2\,x^{12}}{2}+\frac{6\,e\,a^2\,b^2\,x^{11}}{11}+\frac{3\,d\,a^2\,b^2\,x^{10}}{5}+\frac{2\,c\,a^2\,b^2\,x^9}{3}+\frac{f\,a\,b^3\,x^{16}}{4}+\frac{4\,e\,a\,b^3\,x^{15}}{15}+\frac{2\,d\,a\,b^3\,x^{14}}{7}+\frac{4\,c\,a\,b^3\,x^{13}}{13}+\frac{f\,b^4\,x^{20}}{20}+\frac{e\,b^4\,x^{19}}{19}+\frac{d\,b^4\,x^{18}}{18}+\frac{c\,b^4\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^4*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^4*d*x^2)/2 + (b^4*c*x^17)/17 + (a^4*e*x^3)/3 + (b^4*d*x^18)/18 + (a^4*f*
x^4)/4 + (b^4*e*x^19)/19 + (b^4*f*x^20)/20 + a^4*c*x + (2*a^2*b^2*c*x^9)/3
+ (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/2 + (4*a^
3*b*c*x^5)/5 + (4*a*b^3*c*x^13)/13 + (2*a^3*b*d*x^6)/3 + (2*a*b^3*d*x^14)/7
 + (4*a^3*b*e*x^7)/7 + (4*a*b^3*e*x^15)/15 + (a^3*b*f*x^8)/2 + (a*b^3*f*x^1
6)/4
```

sympy [A] time = 0.10, size = 241, normalized size = 1.25

$$a^{4}cx+\frac{a^{4}dx^{2}}{2}+\frac{a^{4}ex^{3}}{3}+\frac{a^{4}fx^{4}}{4}+\frac{4a^{3}bcx^{5}}{5}+\frac{2a^{3}bdx^{6}}{3}+\frac{4a^{3}bex^{7}}{7}+\frac{a^{3}bfx^{8}}{2}+\frac{2a^{2}b^{2}cx^{9}}{3}+\frac{3a^{2}b^{2}dx^{10}}{5}+\frac{6a^{2}b^{2}ex^{11}}{11}+\frac{a^{2}b^{2}fx^{12}}{2}+\frac{4ab^{3}cx^{13}}{13}+\frac{2ab^{3}dx^{14}}{7}+\frac{4ab^{3}ex^{15}}{15}+\frac{ab^{3}fx^{16}}{4}+\frac{b^{4}cx^{17}}{17}+\frac{b^{4}dx^{18}}{18}+\frac{b^{4}ex^{19}}{19}+\frac{b^{4}fx^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**4*f*x**4/4 + 4*a**3*b*c*x**5/
5 + 2*a**3*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + a**3*b*f*x**8/2 + 2*a**2*b**2*c
*x**9/3 + 3*a**2*b**2*d*x**10/5 + 6*a**2*b**2*e*x**11/11 + a**2*b**2*f*x**1
2/2 + 4*a*b**3*c*x**13/13 + 2*a*b**3*d*x**14/7 + 4*a*b**3*e*x**15/15 + a*b*
*3*f*x**16/4 + b**4*c*x**17/17 + b**4*d*x**18/18 + b**4*e*x**19/19 + b**4*f
*x**20/20
```

3.384 $\int x^3 \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right)^4 \, dx$

Optimal. Leaf size=198

$$\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}$$

Rubi [A] time = 0.15, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1582, 1850}

$$\frac{6}{13}a^2b^2dx^{13} + \frac{3}{7}a^2b^2ex^{14} + \frac{2}{5}a^2b^2fx^{15} + \frac{4}{9}a^3bdx^9 + \frac{2}{5}a^3bex^{10} + \frac{4}{11}a^3bfx^{11} + \frac{1}{5}a^4dx^5 + \frac{1}{6}a^4ex^6 + \frac{1}{7}a^4fx^7 + \frac{4}{17}ab^3dx^{17} + \frac{2}{9}ab^3ex^{18} + \frac{4}{19}ab^3fx^{19} + \frac{c\left(a + bx^4\right)^5}{20b} + \frac{1}{21}b^4dx^{21} + \frac{1}{22}b^4ex^{22} + \frac{1}{23}b^4fx^{23}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9 + (2*a^3*
b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^
14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (
4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 +
(c*(a + b*x^4)^5)/(20*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right)^4 \, dx &= \frac{c\left(a + bx^4\right)^5}{20b} + \int \left(a + bx^4\right)^4 \left(-cx^3 + x^3\left(c + dx + ex^2 + fx^3\right)\right) dx \\
&= \frac{c\left(a + bx^4\right)^5}{20b} + \int \left(a^4dx^4 + a^4ex^5 + a^4fx^6 + 4a^3bdx^8 + 4a^3bex^9 + 4a^3\right. \\
&= \frac{1}{5}a^4dx^5 + \frac{1}{6}a^4ex^6 + \frac{1}{7}a^4fx^7 + \frac{4}{9}a^3bdx^9 + \frac{2}{5}a^3bex^{10} + \frac{4}{11}a^3bfx^{11} + \frac{6}{13}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 241, normalized size = 1.22

$$\frac{1}{4}a^4cx^4 + \frac{1}{5}a^4dx^5 + \frac{1}{6}a^4ex^6 + \frac{1}{7}a^4fx^7 + \frac{1}{2}a^3bcx^8 + \frac{4}{9}a^3bdx^9 + \frac{2}{5}a^3bex^{10} + \frac{4}{11}a^3bfx^{11} + \frac{1}{2}a^2b^2cx^{12} + \frac{6}{13}a^2b^2dx^{13} + \frac{3}{7}a^2b^2ex^{14} + \frac{2}{5}a^2b^2fx^{15} + \frac{1}{4}ab^3cx^{16} + \frac{4}{17}ab^3dx^{17} + \frac{2}{9}ab^3ex^{18} + \frac{4}{19}ab^3fx^{19} + \frac{1}{20}b^4cx^{20} + \frac{1}{21}b^4dx^{21} + \frac{1}{22}b^4ex^{22} + \frac{1}{23}b^4fx^{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (a^3*b*c*x^
8)/2 + (4*a^3*b*d*x^9)/9 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (a^2*
b^2*c*x^12)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f
*x^15)/5 + (a*b^3*c*x^16)/4 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (4
*a*b^3*f*x^19)/19 + (b^4*c*x^20)/20 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (
b^4*f*x^23)/23
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4, x]
```

fricas [A] time = 0.37, size = 201, normalized size = 1.02

$$\frac{1}{23}x^{23}fb^4 + \frac{1}{22}x^{22}eb^4 + \frac{1}{21}x^{21}db^4 + \frac{1}{20}x^{20}cb^4 + \frac{4}{19}x^{19}fb^3a + \frac{2}{9}x^{18}eb^3a + \frac{4}{17}x^{17}db^3a + \frac{1}{4}x^{16}cb^3a + \frac{2}{5}x^{15}fb^2a^2 + \frac{3}{7}x^{14}eb^2a^2 + \frac{6}{13}x^{13}db^2a^2 + \frac{1}{2}x^{12}cb^2a^2 + \frac{4}{11}x^{11}fba^3 + \frac{2}{5}x^{10}eba^3 + \frac{4}{9}x^9dba^3 + \frac{1}{2}x^8cba^3 + \frac{1}{7}x^7fa^4 + \frac{1}{6}x^6ea^4 + \frac{1}{5}x^5da^4 + \frac{1}{4}x^4ca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*f*b^4 + 1/22*x^22*e*b^4 + 1/21*x^21*d*b^4 + 1/20*x^20*c*b^4 + 4/1
9*x^19*f*b^3*a + 2/9*x^18*e*b^3*a + 4/17*x^17*d*b^3*a + 1/4*x^16*c*b^3*a +
2/5*x^15*f*b^2*a^2 + 3/7*x^14*e*b^2*a^2 + 6/13*x^13*d*b^2*a^2 + 1/2*x^12*c*
b^2*a^2 + 4/11*x^11*f*b*a^3 + 2/5*x^10*e*b*a^3 + 4/9*x^9*d*b*a^3 + 1/2*x^8*
c*b*a^3 + 1/7*x^7*f*a^4 + 1/6*x^6*e*a^4 + 1/5*x^5*d*a^4 + 1/4*x^4*c*a^4
```

giac [A] time = 0.19, size = 206, normalized size = 1.04

$$\frac{1}{23}b^4fx^{23} + \frac{1}{22}b^4x^{22}e + \frac{1}{21}b^4dx^{21} + \frac{1}{20}b^4cx^{20} + \frac{4}{19}ab^3fx^{19} + \frac{2}{9}ab^3x^{18}e + \frac{4}{17}ab^3dx^{17} + \frac{1}{4}ab^3cx^{16} + \frac{2}{5}a^2b^2fx^{15} + \frac{3}{7}a^2b^2x^{14}e + \frac{6}{13}a^2b^2dx^{13} + \frac{1}{2}a^2b^2cx^{12} + \frac{4}{11}a^3bfx^{11} + \frac{2}{5}a^3bx^{10}e + \frac{4}{9}a^3bdx^9 + \frac{1}{2}a^3bcx^8 + \frac{1}{7}a^4fx^7 + \frac{1}{6}a^4x^6e + \frac{1}{5}a^4dx^5 + \frac{1}{4}a^4cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*x^22*e + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
9*a*b^3*f*x^19 + 2/9*a*b^3*x^18*e + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*x^14*e + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*x^10*e + 4/9*a^3*b*d*x^9 + 1/2*a^3*
b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*x^6*e + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

maple [A] time = 0.04, size = 202, normalized size = 1.02

$$\frac{1}{23}b^4fx^{23} + \frac{1}{22}b^4ex^{22} + \frac{1}{21}b^4dx^{21} + \frac{1}{20}b^4cx^{20} + \frac{4}{19}ab^3fx^{19} + \frac{2}{9}ab^3ex^{18} + \frac{4}{17}ab^3dx^{17} + \frac{1}{4}ab^3cx^{16} + \frac{2}{5}a^2b^2fx^{15} + \frac{3}{7}a^2b^2ex^{14} + \frac{6}{13}a^2b^2dx^{13} + \frac{1}{2}a^2b^2cx^{12} + \frac{4}{11}a^3bfx^{11} + \frac{2}{5}a^3bex^{10} + \frac{4}{9}a^3bdx^9 + \frac{1}{2}a^3bcx^8 + \frac{1}{7}a^4fx^7 + \frac{1}{6}a^4ex^6 + \frac{1}{5}a^4dx^5 + \frac{1}{4}a^4cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x)
```

```
[Out] 1/23*b^4*f*x^23+1/22*b^4*e*x^22+1/21*b^4*d*x^21+1/20*c*b^4*x^20+4/19*a*b^3*
f*x^19+2/9*a*b^3*e*x^18+4/17*a*b^3*d*x^17+1/4*a*b^3*c*x^16+2/5*a^2*b^2*f*x^
15+3/7*a^2*b^2*e*x^14+6/13*a^2*b^2*d*x^13+1/2*c*b^2*a^2*x^12+4/11*a^3*b*f*x
^11+2/5*a^3*b*e*x^10+4/9*a^3*b*d*x^9+1/2*c*a^3*b*x^8+1/7*a^4*f*x^7+1/6*a^4*
e*x^6+1/5*a^4*d*x^5+1/4*c*a^4*x^4
```

maxima [A] time = 1.37, size = 201, normalized size = 1.02

$$\frac{1}{23}b^4fx^{23} + \frac{1}{22}b^4ex^{22} + \frac{1}{21}b^4dx^{21} + \frac{1}{20}b^4cx^{20} + \frac{4}{19}ab^3fx^{19} + \frac{2}{9}ab^3ex^{18} + \frac{4}{17}ab^3dx^{17} + \frac{1}{4}ab^3cx^{16} + \frac{2}{5}a^2b^2fx^{15} + \frac{3}{7}a^2b^2ex^{14} + \frac{6}{13}a^2b^2dx^{13} + \frac{1}{2}a^2b^2cx^{12} + \frac{4}{11}a^3bfx^{11} + \frac{2}{5}a^3bex^{10} + \frac{4}{9}a^3bdx^9 + \frac{1}{2}a^3bcx^8 + \frac{1}{7}a^4fx^7 + \frac{1}{6}a^4ex^6 + \frac{1}{5}a^4dx^5 + \frac{1}{4}a^4cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
9*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*a^3*
b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

mupad [**B**] time = 0.36, size = 201, normalized size = 1.02

$$\frac{f\,a^4\,x^7}{7}+\frac{e\,a^4\,x^6}{6}+\frac{d\,a^4\,x^5}{5}+\frac{c\,a^4\,x^4}{4}+\frac{4\,f\,a^3\,b\,x^{11}}{11}+\frac{2\,e\,a^3\,b\,x^{10}}{5}+\frac{4\,d\,a^3\,b\,x^9}{9}+\frac{c\,a^3\,b\,x^8}{2}+\frac{2\,f\,a^2\,b^2\,x^{15}}{5}+\frac{3\,e\,a^2\,b^2\,x^{14}}{7}+\frac{6\,d\,a^2\,b^2\,x^{13}}{13}+\frac{c\,a^2\,b^2\,x^{12}}{2}+\frac{4\,f\,a\,b^3\,x^{19}}{19}+\frac{2\,e\,a\,b^3\,x^{18}}{9}+\frac{4\,d\,a\,b^3\,x^{17}}{17}+\frac{c\,a\,b^3\,x^{16}}{4}+\frac{f\,b^4\,x^{23}}{23}+\frac{e\,b^4\,x^{22}}{22}+\frac{d\,b^4\,x^{21}}{21}+\frac{c\,b^4\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^4*(c + d*x + e*x^2 + f*x^3),x)
```

```
[Out] (a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (b^4*c*x^20)/20 + (a^4*e*x^6)/6 + (b^4*d*x^
21)/21 + (a^4*f*x^7)/7 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 + (a^2*b^2*c*x^1
2)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)/5
+ (a^3*b*c*x^8)/2 + (a*b^3*c*x^16)/4 + (4*a^3*b*d*x^9)/9 + (4*a*b^3*d*x^17)
/17 + (2*a^3*b*e*x^10)/5 + (2*a*b^3*e*x^18)/9 + (4*a^3*b*f*x^11)/11 + (4*a*
b^3*f*x^19)/19
```

sympy [**A**] time = 0.11, size = 245, normalized size = 1.24

$$\frac{a^4cx^4}{4}+\frac{a^4dx^5}{5}+\frac{a^4ex^6}{6}+\frac{a^4fx^7}{7}+\frac{a^3bcx^8}{2}+\frac{4a^3bdx^9}{9}+\frac{2a^3bex^{10}}{5}+\frac{4a^3bfx^{11}}{11}+\frac{a^2b^2cx^{12}}{2}+\frac{6a^2b^2dx^{13}}{13}+\frac{3a^2b^2ex^{14}}{7}+\frac{2a^2b^2fx^{15}}{5}+\frac{ab^3cx^{16}}{4}+\frac{4ab^3dx^{17}}{17}+\frac{2ab^3ex^{18}}{9}+\frac{4ab^3fx^{19}}{19}+\frac{b^4cx^{20}}{20}+\frac{b^4dx^{21}}{21}+\frac{b^4ex^{22}}{22}+\frac{b^4fx^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*c*x**4/4 + a**4*d*x**5/5 + a**4*e*x**6/6 + a**4*f*x**7/7 + a**3*b*c*x*
*8/2 + 4*a**3*b*d*x**9/9 + 2*a**3*b*e*x**10/5 + 4*a**3*b*f*x**11/11 + a**2*
b**2*c*x**12/2 + 6*a**2*b**2*d*x**13/13 + 3*a**2*b**2*e*x**14/7 + 2*a**2*b*
*2*f*x**15/5 + a*b**3*c*x**16/4 + 4*a*b**3*d*x**17/17 + 2*a*b**3*e*x**18/9
+ 4*a*b**3*f*x**19/19 + b**4*c*x**20/20 + b**4*d*x**21/21 + b**4*e*x**22/22
 + b**4*f*x**23/23
```

3.385 $\int \frac{c+dx+ex^2+fx^3}{a-bx^4} \, dx$

Optimal. Leaf size=133

$$\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1876, 1167, 205, 208, 1248, 635, 260}

$$\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
 + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^
4])/(4*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx &= \int \left(\frac{c+ex^2}{a-bx^4}+\frac{x\left(d+fx^2\right)}{a-bx^4}\right)dx \\
&= \int \frac{c+ex^2}{a-bx^4}\,dx+\int \frac{x\left(d+fx^2\right)}{a-bx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+fx}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(-\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right) \\
&= \frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{1}{2}d\operatorname{Subst}\left(\int \frac{1}{a-bx^2}\,d\right. \\
&= \frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{f\log}{}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 214, normalized size = 1.61

$$-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)\left(a^{3/4}e+\sqrt[4]{a}\,\sqrt{b}\,c+\sqrt{a}\,\sqrt[4]{b}\,d\right)}{4ab^{3/4}}-\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)\left(-a^{3/4}e-\sqrt[4]{a}\,\sqrt{b}\,c+\sqrt{a}\,\sqrt[4]{b}\,d\right)}{4ab^{3/4}}+\frac{\left(\sqrt[4]{a}\,\sqrt{b}\,c-a^{3/4}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2ab^{3/4}}+\frac{d\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{a}\,\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

```
[Out] ((a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4))
 - ((a^(1/4)*Sqrt[b]*c + Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/
4)*x])/(4*a*b^(3/4)) - ((-(a^(1/4)*Sqrt[b]*c) + Sqrt[a]*b^(1/4)*d - a^(3/4)
*e)*Log[a^(1/4) + b^(1/4)*x])/(4*a*b^(3/4)) + (d*Log[Sqrt[a] + Sqrt[b]*x^2]
)/(4*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.19, size = 280, normalized size = 2.11

$$-\frac{\sqrt{2}\left(b^2c-\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd+\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c+\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}bd-\sqrt{-ab}\,be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}\,be\right)\log\left(x^2+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}\,be\right)\log\left(x^2-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}-\frac{f\log\left(\left|bx^4-a\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*
sqrt(2)*(b^2*c + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*sqrt(
2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log(abs(b*x^4 - a))/b
```

maple [A] time = 0.05, size = 177, normalized size = 1.33

$$-\frac{d\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a}-\frac{e\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{e\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{f\ln\left(b\,x^4-a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+1/2*c*(a/b)^(1/4)/a
*arctan(1/(a/b)^(1/4)*x)-1/4/(a*b)^(1/2)*d*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(
1/2)*x^2-a))-1/2*e/b/(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x)+1/4*e/b/(a/b)^(1/4
)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4*f/b*ln(b*x^4-a)
```

maxima [A] time = 3.03, size = 174, normalized size = 1.31

$$\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}b}-\frac{\left(\sqrt{b}d+\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}b}-\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/4*(sqrt(b)*d - sqrt(a)*f)*log(sqrt(b)*
x^2 + sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(b)*d + sqrt(a)*f)*log(sqrt(b)*x^2 -
sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(b)*c + sqrt(a)*e)*log((sqrt(b)*x - sqrt(sq
rt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))*sqrt(b))
```

mupad [B] time = 5.66, size = 1970, normalized size = 14.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x)
```

```
[Out] symsum(log(b^2*c^2*e - b^2*c*d^2 - b^2*d^3*x - a*b*e^3 - a*b*c*f^2 - 16*roo
t(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2
*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^
2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2
*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*
```

```
c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)^2*a*b^3*c - 4*ro
ot(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^
2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a
^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^
2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2
*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*b^3*c^2*x - b^2
*c^2*f*x + 16*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2
 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^
2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*
b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d
^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)
^2*a*b^3*d*x - 4*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*
z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2
*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a
^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*
b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z,
 k)*a*b^2*e^2*x + 2*a*b*d*e*f - 8*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3
- 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2
*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*
a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2
*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*
e^4 - b^3*c^4, z, k)*a*b^2*c*f + 8*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3
 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^
2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16
*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^
2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b
*e^4 - b^3*c^4, z, k)*a*b^2*d*e + a*b*d*f^2*x - a*b*e^2*f*x + 2*b^2*c*d*e*x
 + 8*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z
 + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f
 - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 +
2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*a*b^2*d*
f*x)*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z
 + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f
 - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 +
2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k), k, 1, 4
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

3.386 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}\,f+\sqrt{b}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}-\frac{dx}{b}-$$

Rubi [A] time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1831, 1252, 774, 635, 208, 260, 1280, 1167, 205}

$$\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}\,f+\sqrt{b}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}-\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] -((d*x)/b) - (e*x^2)/(2*b) - (f*x^3)/(3*b) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*
f)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]
*f)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (Sqrt[a]*e*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a]])/(2*b^(3/2)) - (c*Log[a - b*x^4])/(4*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{a-bx^4}\,dx &= \int \left(\frac{x^3\left(c+ex^2\right)}{a-bx^4}+\frac{x^4\left(d+fx^2\right)}{a-bx^4}\right)dx\\
&= \int \frac{x^3\left(c+ex^2\right)}{a-bx^4}\,dx+\int \frac{x^4\left(d+fx^2\right)}{a-bx^4}\,dx\\
&= -\frac{fx^3}{3b}+\frac{1}{2}\,\text{Subst}\left(\int \frac{x(c+ex)}{a-bx^2}\,dx,x,x^2\right)+\frac{\int \frac{x^2\left(3af+3bdx^2\right)}{a-bx^4}\,dx}{3b}\\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\int \frac{3abd+3abfx^2}{a-bx^4}\,dx}{3b^2}-\frac{\text{Subst}\left(\int \frac{-ae-bcx}{a-bx^2}\,dx,x,x^2\right)}{2b}\\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{1}{2}c\,\text{Subst}\left(\int \frac{x}{a-bx^2}\,dx,x,x^2\right)+\frac{(ae)\,\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x\right.}{2b}\\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d+\sqrt{a}\,f\right)\tanh^{-}}{2b^{7/4}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 221, normalized size = 1.36

$$
\frac{-3\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)\left(a^{3/4}f+\sqrt[4]{a}\sqrt{b}\,d+\sqrt{a}\sqrt[4]{b}\,e\right)+3\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)\left(a^{3/4}f+\sqrt[4]{a}\sqrt{b}\,d-\sqrt{a}\sqrt[4]{b}\,e\right)+6\left(\sqrt[4]{a}\sqrt{b}\,d-a^{3/4}f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-3b^{3/4}c\log\left(a-bx^4\right)+3\sqrt{a}\sqrt[4]{b}\,e\log\left(\sqrt{a}+\sqrt{b}\,x^2\right)-12b^{3/4}dx-6b^{3/4}ex^2-4b^{3/4}fx^3}{12b^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*d*x - 6*b^(3/4)*e*x^2 - 4*b^(3/4)*f*x^3 + 6*(a^(1/4)*Sqrt[b]*d
 - a^(3/4)*f)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(a^(1/4)*Sqrt[b]*d + Sqrt[a]*
b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) - b^(1/4)*x] + 3*(a^(1/4)*Sqrt[b]*d - Sq
rt[a]*b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) + b^(1/4)*x] + 3*Sqrt[a]*b^(1/4)*e
*Log[Sqrt[a] + Sqrt[b]*x^2] - 3*b^(3/4)*c*Log[a - b*x^4])/(12*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{a-bx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.19, size = 328, normalized size = 2.02

$$-\frac{c\log\left(\left|bx^4-a\right|\right)}{4b}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}\,b^2e-\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b^4}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}\,b^2e-\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b^4}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8b^4}-\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8b^4}-\frac{2b^2fx^3+3b^2x^2e+6b^2dx}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*c*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a*
b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/b^4 - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a*b
^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/b^4 + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(
3/4)*f)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 - 1/8*sqrt(2)*((
-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) +
sqrt(-a/b))/b^4 - 1/6*(2*b^2*f*x^3 + 3*b^2*x^2*e + 6*b^2*d*x)/b^3
```

maple [A] time = 0.04, size = 208, normalized size = 1.28

$$-\frac{fx^3}{3b}-\frac{ae\ln\left(\frac{\sqrt{ab}\,x^2-a}{-\sqrt{ab}\,x^2-a}\right)}{4\sqrt{ab}\,b}-\frac{ex^2}{2b}-\frac{af\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}+\frac{af\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}-\frac{c\ln\left(bx^4-a\right)}{4b}-\frac{dx}{b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}d\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/3/b*f*x^3-1/2/b*e*x^2-1/b*d*x+1/2/b*d*(a/b)^(1/4)*arctan(1/(a/b)^(1/4)*x
)+1/4/b*d*(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))-1/4/b*a*e/(a*b)^(
1/2)*ln(((a*b)^(1/2)*x^2-a)/(-(a*b)^(1/2)*x^2-a))-1/2/b^2*a*f/(a/b)^(1/4)*a
rctan(1/(a/b)^(1/4)*x)+1/4/b^2*a*f/(a/b)^(1/4)*ln((x+(a/b)^(1/4))/(x-(a/b)^
(1/4)))-1/4/b*c*ln(b*x^4-a)
```

maxima [A] time = 2.98, size = 208, normalized size = 1.28

$$-\frac{2fx^3+3ex^2+6dx}{6b}+\frac{\frac{2\left(a\sqrt{b}d-a^{\frac{3}{2}}f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(\sqrt{a}bc-a\sqrt{b}e\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b}-\frac{\left(\sqrt{a}bc+a\sqrt{b}e\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b}-\frac{\left(a\sqrt{b}d+a^{\frac{3}{2}}f\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/6*(2*f*x^3 + 3*e*x^2 + 6*d*x)/b + 1/4*(2*(a*sqrt(b)*d - a^(3/2)*f)*arcta
n(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
- (sqrt(a)*b*c - a*sqrt(b)*e)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (sqr
```

```
t(a)*b*c + a*sqrt(b)*e)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (a*sqrt(b)
*d + a^(3/2)*f)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(s
qrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

mupad [B] time = 4.85, size = 846, normalized size = 5.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x)
```

```
[Out] symsum(log(- (a^4*f^3 + a^2*b^2*c^2*d + a^3*b*d*e^2 - a^3*b*d^2*f - 2*a^3*b
*c*e*f)/b^2 - root(256*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^
4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b
^3*d^2*e*z - 16*a*b^3*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*
e*f^2 - 4*a*b^2*c^2*d*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e
^2 + a^2*b*e^4 + b^3*c^4 - a*b^2*d^4 - a^3*f^4, z, k)*(root(256*b^7*z^4 + 2
56*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*
b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^2*z + 16
*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 - 4*a*b^2*c^2*d*f + 4*a*b^2*
c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 + a^2*b*e^4 + b^3*c^4 - a*b^2*d
^4 - a^3*f^4, z, k)*(16*a^2*b^2*d - 16*a^2*b^2*e*x) + (8*a^2*b^3*c*d - 8*a^
3*b^2*e*f)/b^2 + (x*(4*a^3*b*f^2 + 4*a^2*b^2*d^2 - 8*a^2*b^2*c*e))/b) - (x*
(a^3*e^3 + a^3*c*f^2 - 2*a^3*d*e*f + a^2*b*c*d^2 - a^2*b*c^2*e))/b)*root(25
6*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^4*e^2*z^2 + 96*b^5*c^
2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3
*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 - 4*a*b^2*c^2*d
*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 + a^2*b*e^4 + b^3*
c^4 - a*b^2*d^4 - a^3*f^4, z, k), k, 1, 4) - (e*x^2)/(2*b) - (f*x^3)/(3*b)
- (d*x)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

3.387 $\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx$

Optimal. Leaf size=293

$$-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}\,e+\sqrt{\right.}{}}{}$$

Rubi [A] time = 0.22, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}\,e+\sqrt{b}\,c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sq
rt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx &= \int \left(\frac{c+ex^2}{a+bx^4}+\frac{x\left(d+fx^2\right)}{a+bx^4}\right)dx \\
&= \int \frac{c+ex^2}{a+bx^4}\,dx+\int \frac{x\left(d+fx^2\right)}{a+bx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+fx}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\,\sqrt{b}+bx}{a+bx^4}}{2b} \\
&= \frac{1}{2}d\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}}}{4b} \\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)}{} \\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{2\sqrt{2}\,a^{3/4}b^{3/4}}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 296, normalized size = 1.01

$$
\frac{-\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{b}\,c-a^{3/4}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{b}\,c-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)-2\sqrt[4]{a}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(2\sqrt[4]{a}\sqrt[4]{b}\,d+\sqrt{2}\sqrt{a}\,e+\sqrt{2}\sqrt{b}\,c\right)+2\sqrt[4]{a}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(-2\sqrt[4]{a}\sqrt[4]{b}\,d+\sqrt{2}\sqrt{a}\,e+\sqrt{2}\sqrt{b}\,c\right)+2af\log\left(a+bx^4\right)}{8ab}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] (-2*a^(1/4)*b^(1/4)*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt
[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(Sqrt[2]
*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)] - Sqrt[2]*b^(1/4)*(a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(a^(1/4)
*Sqrt[b]*c - a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2] + 2*a*f*Log[a + b*x^4])/(8*a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.18, size = 290, normalized size = 0.99

$$\frac{f\log\left(\left|bx^4+a\right|\right)}{4\,b}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d-\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2d-\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*f*log(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3
)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^
(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4
))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*
e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b
^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a*b^3)
```

maple [A] time = 0.05, size = 294, normalized size = 1.00

$$\frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{2\sqrt{ab}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{f\ln\left(b\,x^4+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2
-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4*c*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)+1/4*(a/b)^(1/4)*2^(1/2)/a*c*arctan(2^(1/2)/(a/b)^(1/4
)*x-1)+1/2*d/(a*b)^(1/2)*arctan((1/a*b)^(1/2)*x^2)+1/8*e/b/(a/b)^(1/4)*2^(1
/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2)))+1/4*e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/
4*e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4/b*f*ln(b*x^4+
a)
```

maxima [A] time = 3.03, size = 277, normalized size = 0.95

$$\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f+bc-\sqrt{a}\sqrt{b}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f-bc+\sqrt{a}\sqrt{b}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{5}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{3}{4}}e-2\sqrt{a}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{5}{4}}c+\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{3}{4}}e+2\sqrt{a}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*b^(1/4)*f + b*c - sqrt(a)*sqrt(b)*e)*log(sqrt(
b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/8*sqrt(
2)*(sqrt(2)*a^(3/4)*b^(1/4)*f - b*c + sqrt(a)*sqrt(b)*e)*log(sqrt(b)*x^2 -
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/4*(sqrt(2)*a^(1/
4)*b^(5/4)*c + sqrt(2)*a^(3/4)*b^(3/4)*e - 2*sqrt(a)*b*d)*arctan(1/2*sqrt(2
)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*s
qrt(sqrt(a)*sqrt(b))*b^(5/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(5/4)*c + sqrt(2)*a^
(3/4)*b^(3/4)*e + 2*sqrt(a)*b*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5
/4))
```

mupad [B] time = 0.93, size = 1952, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x)
```

```
[Out] symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - a*b*e^3 - a*b*c*f^2 - 16*roo
t(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2
*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^
2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2
*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*
c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)^2*a*b^3*c - 4*ro
ot(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^
2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a
^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^
2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2
*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*b^3*c^2*x + b^2
*c^2*f*x + 16*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2
 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^
2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*
b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d
^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)
^2*a*b^3*d*x + 4*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*
z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2
*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a
^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*
b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z,
 k)*a*b^2*e^2*x + 2*a*b*d*e*f + 8*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3
+ 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2
*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*
a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2
*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*
f^4 + b^3*c^4, z, k)*a*b^2*c*f - 8*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3
 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^
2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16
*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^
2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3
*f^4 + b^3*c^4, z, k)*a*b^2*d*e + a*b*d*f^2*x - a*b*e^2*f*x - 2*b^2*c*d*e*x
 - 8*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z
 + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f
 + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 +
2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*a*b^2*d*
f*x)*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z
 + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f
 + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 +
2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k), k, 1, 4
)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

3.388 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$

Optimal. Leaf size=321

$$\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{a}\,(}{}$$

Rubi [A] time = 0.33, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1831, 1252, 774, 635, 205, 260, 1280, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}\,f+\sqrt{b}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}\,f+\sqrt{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,b^{7/4}}-\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{c\log\left(a+bx^4\right)}{4b}+\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt[b]*x^2)/S
qrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/4)*(
Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(4*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log[a + b*
x^4])/(4*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{a+bx^4}\,dx=\int \left(\frac{x^3\left(c+ex^2\right)}{a+bx^4}+\frac{x^4\left(d+fx^2\right)}{a+bx^4}\right)dx$$
$$=\int \frac{x^3\left(c+ex^2\right)}{a+bx^4}\,dx+\int \frac{x^4\left(d+fx^2\right)}{a+bx^4}\,dx$$
$$=\frac{fx^3}{3b}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+ex)}{a+bx^2}\,dx,x,x^2\right)-\frac{\int \frac{x^2\left(3af-3bdx^2\right)}{a+bx^4}\,dx}{3b}$$
$$=\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}+\frac{\int \frac{-3abd-3abfx^2}{a+bx^4}\,dx}{3b^2}+\frac{\operatorname{Subst}\left(\int \frac{-ae+bcx}{a+bx^2}\,dx,x,x^2\right)}{2b}$$
$$=\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}+\frac{1}{2}c\operatorname{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right)-\frac{(ae)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{\cdots}\right.}{2b}$$
$$=\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{c\log\left(a+bx^4\right)}{4b}+\frac{\left(\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\right)\int \cdots}{4\sqrt{2}\,b\cdots}$$
$$=\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\cdots\right.}{4\sqrt{2}\,b^{7/4}}$$
$$=\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{b}\,d+\sqrt{a}\,f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{7/4}}-\cdots$$

Mathematica [A] time = 0.24, size = 311, normalized size = 0.97

$$\frac{-3\sqrt{2}\left(a^{3/4}f-\sqrt[4]{a}\sqrt{b}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+3\sqrt{2}\left(a^{3/4}f-\sqrt[4]{a}\sqrt{b}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+6b^{3/4}c\log\left(a+bx^4\right)+6\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(2\sqrt[4]{a}\sqrt[4]{b}\,e+\sqrt{2}\sqrt{a}\,f+\sqrt{2}\sqrt{b}\,d\right)-6\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(-2\sqrt[4]{a}\sqrt[4]{b}\,e+\sqrt{2}\sqrt{a}\,f+\sqrt{2}\sqrt{b}\,d\right)+24b^{3/4}dx+12b^{3/4}ex^2+8b^{3/4}fx^3}{24b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*d*x + 12*b^(3/4)*e*x^2 + 8*b^(3/4)*f*x^3 + 6*a^(1/4)*(Sqrt[2]*S
qrt[b]*d + 2*a^(1/4)*b^(1/4)*e + Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)] - 6*a^(1/4)*(Sqrt[2]*Sqrt[b]*d - 2*a^(1/4)*b^(1/4)*e + Sqr
t[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(-(a^(1
/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + 3*Sqrt[2]*(-(a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 6*b^(3/4)*c*Log[a + b*x^4])/(24*b^(7/4
))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 308, normalized size = 0.96

$$\frac{c\log\left(\left|bx^4+a\right|\right)}{4\,b}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2e-\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\,b^2e-\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4}+\frac{2\,b^2fx^3+3\,b^2x^2e+6\,b^2dx}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*c*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^3
)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^3)^(1/4
)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/b^4 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x
^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^
2*d - (a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1
/6*(2*b^2*f*x^3 + 3*b^2*x^2*e + 6*b^2*d*x)/b^3
```

maple [A] time = 0.05, size = 325, normalized size = 1.01

$$\frac{f\,x^3}{3b}-\frac{ae\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{2\sqrt{ab}\,b}+\frac{e\,x^2}{2b}-\frac{\sqrt{2}\,af\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}-\frac{\sqrt{2}\,af\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}-\frac{\sqrt{2}\,af\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}+\frac{c\ln\left(b\,x^4+a\right)}{4b}+\frac{dx}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/3/b*f*x^3+1/2/b*e*x^2+1/b*d*x-1/4/b*d*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x-1)-1/8/b*d*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+
(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))-1/4/b*d*(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/2/b*a*e/(a*b)^(1/2)*arctan((1/a*b)
^(1/2)*x^2)-1/8/b^2*a*f/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))-1/4/b^2*a*f/(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/b^2*a*f/(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*c*ln(b*x^4+a)/b
```

maxima [A] time = 3.01, size = 305, normalized size = 0.95

$$\frac{2\,fx^3+3\,ex^2+6\,dx}{6\,b}+\frac{\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}c-abd+a^{\frac{3}{2}}\sqrt{b}\,f\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}c+abd-a^{\frac{3}{2}}\sqrt{b}\,f\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}d+\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{3}{4}}f-2\,a^{\frac{3}{2}}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}d+\sqrt{2}\,a^{\frac{7}{4}}b^{\frac{3}{4}}f+2\,a^{\frac{3}{2}}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{5}{4}}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*f*x^3 + 3*e*x^2 + 6*d*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c
 - a*b*d + a^(3/2)*sqrt(b)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x +
 sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c + a*b*d -
a^(3/2)*sqrt(b)*f)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(
a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(5/4)*b^(5/4)*d + sqrt(2)*a^(7/4)*b^(3/4)*f
 - 2*a^(3/2)*b*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) - 2*(sqrt(
2)*a^(5/4)*b^(5/4)*d + sqrt(2)*a^(7/4)*b^(3/4)*f + 2*a^(3/2)*b*e)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/b
```

mupad [B] time = 4.85, size = 838, normalized size = 2.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x)
```

```
[Out] symsum(log((a^4*f^3 + a^2*b^2*c^2*d - a^3*b*d*e^2 + a^3*b*d^2*f + 2*a^3*b*c
*e*f)/b^2 + root(256*b^7*z^4 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*
e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3
*d^2*e*z - 16*a*b^3*c*e^2*z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*
f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2
 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*(root(256*b^7*z^4 - 256
*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^
3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^2*z - 16*b
^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*
d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4
 + b^3*c^4, z, k)*(16*a^2*b^2*d - 16*a^2*b^2*e*x) - (8*a^2*b^3*c*d + 8*a^3*
b^2*e*f)/b^2 + (x*(4*a^3*b*f^2 - 4*a^2*b^2*d^2 + 8*a^2*b^2*c*e))/b) - (x*(a
^3*e^3 + a^3*c*f^2 - 2*a^3*d*e*f - a^2*b*c*d^2 + a^2*b*c^2*e))/b)*root(256*
b^7*z^4 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*e^2*z^2 + 96*b^5*c^2*
z^2 - 32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c
*e^2*z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f
 - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*
d^4 + a^3*f^4 + b^3*c^4, z, k), k, 1, 4) + (e*x^2)/(2*b) + (f*x^3)/(3*b) +
(d*x)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

3.389 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=318

$$-\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}\,e\right.}{}$$

Rubi [A] time = 0.27, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] -(a*f - b*x*(c + d*x + e*x^2))/(4*a*b*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sq
rt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4
)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(
3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^2}\,dx &= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}-\frac{\int \frac{-3c-2dx-ex^2}{a+bx^4}\,dx}{4a}\\
&= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}-\frac{\int \left(-\frac{2dx}{a+bx^4}+\frac{-3c-ex^2}{a+bx^4}\right)dx}{4a}\\
&= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}-\frac{\int \frac{-3c-ex^2}{a+bx^4}\,dx}{4a}+\frac{d\int \frac{x}{a+bx^4}\,dx}{2a}\\
&= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}+\frac{d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{4a}+\frac{\left(\frac{3\sqrt{b}\,c}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx}{8ab}\\
&= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{\left(\frac{3\sqrt{b}\,c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{16ab}+\\
&= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\right.}{16\sqrt{2}\,a^{7/4}b^{3/4}}\\
&= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{3/4}}+
\end{aligned}
$$

Mathematica [A] time = 0.41, size = 315, normalized size = 0.99

$$
\frac{\sqrt{2}\,\sqrt[4]{b}\left(a^{3/4}e-3\sqrt[4]{a}\,\sqrt{b}\,c\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+\sqrt{2}\,\sqrt[4]{b}\left(3\sqrt[4]{a}\,\sqrt{b}\,c-a^{3/4}e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)-\frac{8a(af-bx(c+x(d+ex)))}{a+bx^4}-2\sqrt[4]{a}\,\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(4\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{2}\,\sqrt{a}\,e+3\sqrt{2}\,\sqrt{b}\,c\right)+2\sqrt[4]{a}\,\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(-4\sqrt[4]{a}\,\sqrt[4]{b}\,d+\sqrt{2}\,\sqrt{a}\,e+3\sqrt{2}\,\sqrt{b}\,c\right)}{32a^2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a*(a*f - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2*a^(1/4)*b^(1/4)*(3*Sq
rt[2]*Sqrt[b]*c + 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4
)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] +
Sqrt[2]*b^(1/4)*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*a^(1/4)*Sqrt[b]*c - a^(
3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^2*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.18, size = 316, normalized size = 0.99

$$\frac{bx^3e + bdx^2 + bcx - af}{4\left(bx^4 + a\right)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d + 3\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2d + 3\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32a^2b^3} - \frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*s
qrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)
*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sq
rt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)
^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

maple [A] time = 0.05, size = 362, normalized size = 1.14

$$\frac{f x^4}{4\left(b x^4 + a\right)a} + \frac{e x^3}{4\left(b x^4 + a\right)a} + \frac{d x^2}{4\left(b x^4 + a\right)a} + \frac{cx}{4\left(b x^4 + a\right)a} + \frac{d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{4\sqrt{ab}\,a} + \frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16a^2} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{16a^2} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] 1/4/(b*x^4+a)/a*c*x+3/32*(a/b)^(1/4)*2^(1/2)/a^2*c*ln((x^2+(a/b)^(1/4)*2^(1
/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/16*(a/b)^(1/4
)*2^(1/2)/a^2*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/16*(a/b)^(1/4)*2^(1/2)/a^
2*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4/(b*x^4+a)/a*d*x^2+1/4/(a*b)^(1/2)/a
*d*arctan((1/a*b)^(1/2)*x^2)+1/4/(b*x^4+a)/a*e*x^3+1/32/(a/b)^(1/4)*2^(1/2)
/a/b*e*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*
x+(a/b)^(1/2)))+1/16/(a/b)^(1/4)*2^(1/2)/a/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x
+1)+1/16/(a/b)^(1/4)*2^(1/2)/a/b*e*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4*f*x^
4/a/(b*x^4+a)
```

maxima [A] time = 3.06, size = 305, normalized size = 0.96

$$\frac{bex^3 + bdx^2 + bcx - af}{4\left(ab^2x^4 + a^2b\right)} + \frac{\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-4\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}} + \frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e+4\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*e*x^3 + b*d*x^2 + b*c*x - a*f)/(a*b^2*x^4 + a^2*b) + 1/32*(sqrt(2)*(
3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt
(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2
- sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(
1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 4*sqrt(a)*sqrt(b)*d)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c + sq
rt(2)*a^(3/4)*b^(1/4)*e + 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b
)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)
*sqrt(b))*b^(3/4)))/a
```

mupad [B] time = 0.36, size = 478, normalized size = 1.50

$$\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\left(\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\left(12\,b^3\,c-8\,b^3\,d\,x\right)+\frac{x\left(36\,a\,b^3\,c^2-4\,a^2\,b^2\,e^2\right)}{16\,a^3}+\frac{b^2\,d\,e}{a}\right)-\frac{9\,b^2\,c^2\,e-12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}+\frac{x\left(2\,b^2\,d^3-3\,b^2\,c\,d\,e\right)}{16\,a^3}\right)\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\right)\right)+\frac{\frac{d\,x^2}{4\,a}-\frac{f}{4\,b}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((x*(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3) - (9*b^2*c^2*e - 12*b^2*c
*d^2 + a*b*e^3)/(64*a^3) - root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 +
2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*
d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(root(655
36*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2
*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4
 + 81*b^2*c^4 + a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 -
4*a^2*b^2*e^2))/(16*a^3) + (b^2*d*e)/a))*root(65536*a^7*b^3*z^4 + 3072*a^4*
b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2
*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z
, k), k, 1, 4) + ((d*x^2)/(4*a) - f/(4*b) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a
 + b*x^4)
```

sympy [A] time = 22.32, size = 517, normalized size = 1.63

$$\operatorname{RootSum}\left(65536t^4a^7b^3+t^2\left(3072a^4b^2ce+2048a^4b^2d^2\right)+t\left(128a^3bde^2-1152a^2b^2c^2d\right)+a^2e^4+18abc^2e^2-48abcd^2e+16abd^4+81b^2c^4,\left(t\mapsto t\log\left(x+\frac{4096t^3a^7b^2e^3-36864t^3a^6b^3c^2e+98304t^3a^6b^3cd^2+4608t^2a^5b^2cde^2-4096t^2a^5b^2d^3e+13824t^2a^4b^3c^3d+144ta^4bce^4+192ta^4bd^2e^3-1728ta^3b^2c^3e^2+5184ta^3b^2c^2d^2e+1536ta^3b^2cd^4+3888ta^2b^3c^5+6a^3de^5+120a^2bcd^3e^2-64a^2bd^5e+810ab^2c^4de-1080ab^2c^3d^3}{a^3e^6-9a^2bc^2e^4+96a^2bcd^2e^3-64a^2bd^4e^2-81ab^2c^4e^2+864ab^2c^3d^2e-576ab^2c^2d^4+729b^3c^6}\right)\right)\right)+\frac{-af+bcx+bdx^2+bex^3}{4a^2b+4ab^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2*
d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a*b
*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t*lo
g(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304*_t*
*3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*
d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*
b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e +
 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 + 120*a*
*2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**
3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 - 64*a**2*b
*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2
*d**4 + 729*b**3*c**6)))) + (-a*f + b*c*x + b*d*x**2 + b*e*x**3)/(4*a**2*b
+ 4*a*b**2*x**4)
```

3.390 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2} \, dx$

Optimal. Leaf size=310

$$-\frac{\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{3/4}b^{7/4}}-\frac{\left(3\sqrt{a}\,f\right.}{}$$

Rubi [A] time = 0.27, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1823, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{3/4}b^{7/4}}-\frac{\left(3\sqrt{a}\,f+\sqrt{b}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{\left(3\sqrt{a}\,f+\sqrt{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{3/4}b^{7/4}}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{3/2}}-\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(4*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt
[a]])/(4*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*d + 3*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d + 3*Sqrt[a]*
f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) - (
(Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^2}\,dx = -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)} + \frac{\int \frac{d+2ex+3fx^2}{a+bx^4}\,dx}{4b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)} + \frac{\int \left(\frac{2ex}{a+bx^4}+\frac{d+3fx^2}{a+bx^4}\right)dx}{4b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)} + \frac{\int \frac{d+3fx^2}{a+bx^4}\,dx}{4b} + \frac{e\int \frac{x}{a+bx^4}\,dx}{2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)} + \frac{e\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{4b} + \frac{\left(\frac{\sqrt{b}\,d}{\sqrt{a}}-3f\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{8b^2}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{3/2}} + \frac{\left(\frac{\sqrt{b}\,d}{\sqrt{a}}+3f\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{16b^2} + \left(\frac{\sqrt{b}\,d}{\sqrt{a}}\right.$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{3/2}} - \frac{\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x\right.}{16\sqrt{2}\,a^{3/4}b^{7/4}}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{3/2}} - \frac{\left(\sqrt{b}\,d+3\sqrt{a}\,f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{3/4}b^{7/4}} + \cdots$$

Mathematica [A] time = 0.38, size = 294, normalized size = 0.95

$$\frac{-\frac{2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(4\sqrt[4]{a}\sqrt[4]{b}\,e+3\sqrt{2}\sqrt{a}\,f+\sqrt{2}\sqrt{b}\,d\right)}{a^{3/4}}+\frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(-4\sqrt[4]{a}\sqrt[4]{b}\,e+3\sqrt{2}\sqrt{a}\,f+\sqrt{2}\sqrt{b}\,d\right)}{a^{3/4}}+\frac{\sqrt{2}\left(3\sqrt{a}\,f-\sqrt{b}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{3/4}}+\frac{\sqrt{2}\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{3/4}}-\frac{8b^{3/4}(c+x(d+x(e+fx)))}{a+bx^4}}{32b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

```
[Out] ((-8*b^(3/4)*(c + x*(d + x*(e + f*x))))/(a + b*x^4) - (2*(Sqrt[2]*Sqrt[b]*d
 + 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/a^(3/4) + (2*(Sqrt[2]*Sqrt[b]*d - 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[
2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*(
-(Sqrt[b]*d) + 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(3/4) + (Sqrt[2]*(Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4))/(32*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 303, normalized size = 0.98

$$-\frac{fx^3 + x^2e + dx + c}{4\left(bx^4 + a\right)b} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2e + \left(ab^3\right)^{\frac{1}{4}}b^2d + 3\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}\,b^2e + \left(ab^3\right)^{\frac{1}{4}}b^2d + 3\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d - 3\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,ab^4} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d - 3\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)*b) + 1/16*sqrt(2)*(2*sqrt(2)*sq
rt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/16*sqrt(2)*(2*sqrt(2)
*sqrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/32*sqrt(2)*((a*b^3
)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a*b^4) - 1/32*sqrt(2)*((a*b^3)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x
^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)
```

maple [A] time = 0.05, size = 334, normalized size = 1.08

$$\frac{e\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{4\sqrt{ab}\,b} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16ab} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{16ab} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32ab} + \frac{3\sqrt{2}\,f\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2} + \frac{3\sqrt{2}\,f\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2} + \frac{3\sqrt{2}\,f\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2} + \frac{-\frac{fx^3}{4b} - \frac{ex^2}{4b} - \frac{dx}{4b} - \frac{c}{4b}}{bx^4 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (-1/4/b*f*x^3-1/4/b*e*x^2-1/4/b*d*x-1/4/b*c)/(b*x^4+a)+1/32/b*d*(a/b)^(1/4)
/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1
/2)*x+(a/b)^(1/2)))+1/16/b*d*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x+1)+1/16/b*d*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/4/
b*e/(a*b)^(1/2)*arctan((1/a*b)^(1/2)*x^2)+3/32/b^2*f/(a/b)^(1/4)*2^(1/2)*ln
((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(
1/2)))+3/16/b^2*f/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/16/
b^2*f/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.02, size = 294, normalized size = 0.95

$$-\frac{fx^3 + ex^2 + dx + c}{4\left(b^2x^4 + ab\right)} + \frac{\frac{\sqrt{2}\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{b}\,d-3\sqrt{a}\,f\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f-4\sqrt{a}\sqrt{b}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}} + \frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f+4\sqrt{a}\sqrt{b}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^4 + a*b) + 1/32*(sqrt(2)*(sqrt(b)*d -
 3*sqrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(3/4)) - sqrt(2)*(sqrt(b)*d - 3*sqrt(a)*f)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(sqrt(2)*a^(1/4)*b^(3/4)*
d + 3*sqrt(2)*a^(3/4)*b^(1/4)*f - 4*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(sqrt(2)*a^(1/4)*b^(3/4)*d + 3*sqrt(2)*a^(3/
4)*b^(1/4)*f + 4*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b
^(3/4)))/b
```

mupad [B] time = 5.10, size = 559, normalized size = 1.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x)
```

```
[Out] symsum(log((x*(2*e^3 - 3*d*e*f))/(16*b) - (27*a*f^3 - 4*b*d*e^2 + 3*b*d^2*f
)/(64*b^2) - root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e
^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b
*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*(3*a*e*f + (b*d^2*x)/4
- (9*a*f^2*x)/4 + 4*root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^
2*b^4*e^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f +
 18*a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*a*b^2*d - 8*root
(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e^2*z^2 + 1152*a^2
*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b*d^2*f^2 + 16*a*b
*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*a*b^2*e*x))*root(65536*a^3*b^7*z^4 + 307
2*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3
*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*
d^4, z, k), k, 1, 4) - (c/(4*b) + (e*x^2)/(4*b) + (f*x^3)/(4*b) + (d*x)/(4*
b))/(a + b*x^4)
```

sympy [A] time = 44.23, size = 510, normalized size = 1.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**3*b**7 + _t**2*(3072*a**2*b**4*d*f + 2048*a**2*b**4*
e**2) + _t*(1152*a**2*b**2*e*f**2 - 128*a*b**3*d**2*e) + 81*a**2*f**4 + 18*
a*b*d**2*f**2 - 48*a*b*d*e**2*f + 16*a*b*e**4 + b**2*d**4, Lambda(_t, _t*lo
g(x + (110592*_t**3*a**4*b**5*f**3 - 12288*_t**3*a**3*b**6*d**2*f + 32768*_
t**3*a**3*b**6*d*e**2 + 13824*_t**2*a**3*b**4*d*e*f**2 - 12288*_t**2*a**3*b
**4*e**3*f + 512*_t**2*a**2*b**5*d**3*e + 3888*_t*a**3*b**2*d*f**4 + 5184*_
t*a**3*b**2*e**2*f**3 - 576*_t*a**2*b**3*d**3*f**2 + 1728*_t*a**2*b**3*d**2
*e**2*f + 512*_t*a**2*b**3*d*e**4 + 16*_t*a*b**4*d**5 + 1458*a**3*e*f**5 +
360*a**2*b*d*e**3*f**2 - 192*a**2*b*e**5*f + 30*a*b**2*d**4*e*f - 40*a*b**2
*d**3*e**3)/(729*a**3*f**6 - 81*a**2*b*d**2*f**4 + 864*a**2*b*d*e**2*f**3 -
 576*a**2*b*e**4*f**2 - 9*a*b**2*d**4*f**2 + 96*a*b**2*d**3*e**2*f - 64*a*b
**2*d**2*e**4 + b**3*d**6)))) + (-c - d*x - e*x**2 - f*x**3)/(4*a*b + 4*b**
2*x**4)
```

3.391 $\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^3}\,dx$

Optimal. Leaf size=351

$$-\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}}-\frac{(5}{}$$

Rubi [A] time = 0.32, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}\,c-5\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{11/4}b^{3/4}}-\frac{\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}\,e+21\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}\,a^{11/4}b^{3/4}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)}+\frac{3d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}-\frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*
x^2))/(8*a*b*(a + b*x^4)^2) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/
2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*
Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(
3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^3}\,dx = -\frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} - \frac{\int \frac{-7c-6dx-5ex^2}{\left(a+bx^4\right)^2}\,dx}{8a}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{\int \frac{21c+12dx+5ex^2}{a+bx^4}\,dx}{32a^2}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{\int \left(\frac{12dx}{a+bx^4}+\frac{21c+5ex^2}{a+bx^4}\right)dx}{32a^2}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{\int \frac{21c+5ex^2}{a+bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a+bx^4}\,dx}{8a^2}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{(3d)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{16a^2} + \cdots$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int \cdots}{128\sqrt{\cdots}}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)1\cdots}{\cdots}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}c+5\sqrt{\cdots}\right.}{64\cdots}$$

Mathematica [A] time = 0.44, size = 347, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(5a^{3/4}e-21\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}+\frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{b}c-5a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}}-\frac{32a^2(af-bx(c+x(d+ex)))}{b\left(a+bx^4\right)^2}-\frac{2\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(24\sqrt[4]{a}\sqrt[4]{b}d+5\sqrt{2}\sqrt{a}e+21\sqrt{2}\sqrt{b}c\right)}{b^{3/4}}+\frac{2\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-24\sqrt[4]{a}\sqrt[4]{b}d+5\sqrt{2}\sqrt{a}e+21\sqrt{2}\sqrt{b}c\right)}{b^{3/4}}+\frac{8ax(7c+x(6d+5ex))}{a+bx^4}}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a*x*(7*c + x*(6*d + 5*e*x)))/(a + b*x^4) - (32*a^2*(a*f - b*x*(c + x*(d
 + e*x))))/(b*(a + b*x^4)^2) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4
)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/b^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt
[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*
(-21*a^(1/4)*Sqrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 354, normalized size = 1.01

$$\frac{\sqrt{2}\left(12\,\sqrt{2}\,\sqrt{ab}\,b^2d+21\,\left(ab^3\right)^{\frac{1}{4}}b^2c+5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(12\,\sqrt{2}\,\sqrt{ab}\,b^2d+21\,\left(ab^3\right)^{\frac{1}{4}}b^2c+5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(21\,\left(ab^3\right)^{\frac{1}{4}}b^2c-5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}-\frac{\sqrt{2}\left(21\,\left(ab^3\right)^{\frac{1}{4}}b^2c-5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}+\frac{5\,b^2x^7e+6\,b^2dx^6+7\,b^2cx^5+9\,abx^3e+10\,abdx^2+11\,abcx-4\,a^2f}{32\,\left(bx^4+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b
^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*
c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)
*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*
(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 +
9*a*b*x^3*e + 10*a*b*d*x^2 + 11*a*b*c*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

maple [A] time = 0.05, size = 432, normalized size = 1.23

$$\frac{f\,x^4}{8\left(b\,x^4+a\right)^2a}+\frac{e\,x^3}{8\left(b\,x^4+a\right)^2a}+\frac{f\,x^4}{8\left(b\,x^4+a\right)a^2}+\frac{d\,x^2}{8\left(b\,x^4+a\right)^2a}+\frac{5e\,x^3}{32\left(b\,x^4+a\right)a^2}+\frac{cx}{8\left(b\,x^4+a\right)^2a}+\frac{3d\,x^2}{16\left(b\,x^4+a\right)a^2}+\frac{7cx}{32\left(b\,x^4+a\right)a^2}+\frac{3d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{16\sqrt{ab}\,a^2}+\frac{5\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b}+\frac{5\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b}+\frac{5\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128a^3}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128a^3}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*(a/b)^(1/4)*2^(1/2)/a^3
*c*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a
/b)^(1/2)))+21/128*(a/b)^(1/4)*2^(1/2)/a^3*c*arctan(2^(1/2)/(a/b)^(1/4)*x+1
)+21/128*(a/b)^(1/4)*2^(1/2)/a^3*c*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/8*d*x^
2/a/(b*x^4+a)^2+3/16*d/a^2*x^2/(b*x^4+a)+3/16/(a*b)^(1/2)/a^2*d*arctan((1/a
*b)^(1/2)*x^2)+1/8*e*x^3/a/(b*x^4+a)^2+5/32*e/a^2*x^3/(b*x^4+a)+5/256/(a/b)
^(1/4)*2^(1/2)/a^2/b*e*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b
)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+5/128/(a/b)^(1/4)*2^(1/2)/a^2/b*e*arctan(2^
(1/2)/(a/b)^(1/4)*x+1)+5/128/(a/b)^(1/4)*2^(1/2)/a^2/b*e*arctan(2^(1/2)/(a/
b)^(1/4)*x-1)+1/8*f*x^4/a/(b*x^4+a)^2+1/8*f/a^2*x^4/(b*x^4+a)
```

maxima [A] time = 3.02, size = 355, normalized size = 1.01

$$\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5+9\,abex^3+10\,abdx^2+11\,abcx-4\,a^2f}{32\left(a^2b^3x^8+2\,a^3b^2x^4+a^4b\right)}+\frac{\frac{\sqrt{2}\left(21\,\sqrt{b}\,c-5\,\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,\sqrt{b}\,c-5\,\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e-24\,\sqrt{a}\,\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}e+24\,\sqrt{a}\,\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}}{256\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2
+ 11*a*b*c*x - 4*a^2*f)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt
```

```
(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*
x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(s
qrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21
*sqrt(2)*a^(1/4)*b^(3/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e - 24*sqrt(a)*sqrt(
b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4
)*b^(3/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e + 24*sqrt(a)*sqrt(b)*d)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^2
```

mupad [B] time = 5.20, size = 832, normalized size = 2.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x)
```

```
[Out] symsum(log(-(b*(125*a*e^3 - 3024*b*c*d^2 + 2205*b*c^2*e - 1728*b*d^3*x + 34
4064*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^
2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^
2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z,
k)^2*a^5*b^2*c - 3200*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2
 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z
 - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 19
4481*b^2*c^4, z, k)*a^3*b*e^2*x + 2520*b*c*d*e*x + 56448*root(268435456*a^1
1*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3
*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2
 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^2*b^2*c^2*x - 1966
08*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*
d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*
e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)
^2*a^5*b^2*d*x + 15360*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^
2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*
z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 1
94481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435456*a^11*b^3*z^4 +
 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*
z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*
b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k), k, 1, 4) + ((5*d*x^2)/(16*a) -
 f/(8*b) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^2) + (3*b
*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4)
```

sympy [A] time = 108.47, size = 578, normalized size = 1.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**3 + _t**2*(6881280*a**6*b**2*c*e + 4718592
*a**6*b**2*d**2) + _t*(153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d) + 6
25*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4 +
194481*b**2*c**4, Lambda(_t, _t*log(x + (262144000*_t**3*a**10*b**2*e**3 -
4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 30
9657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e + 18207
86688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a**5*b
*d**2*e**3 - 118540800*_t*a**4*b**2*c**3*e**2 + 365783040*_t*a**4*b**2*c**2
*d**2*e + 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 + 112
500*a**3*d*e**5 + 4536000*a**2*b*c*d**3*e**2 - 2488320*a**2*b*d**5*e + 5834
4300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 - 275625
```

```
*a**2*b*c**2*e**4 + 3024000*a**2*b*c*d**2*e**3 - 2073600*a**2*b*d**4*e**2 -
 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b**2*c
**2*d**4 + 85766121*b**3*c**6)))) + (-4*a**2*f + 11*a*b*c*x + 10*a*b*d*x**2
 + 9*a*b*e*x**3 + 7*b**2*c*x**5 + 6*b**2*d*x**6 + 5*b**2*e*x**7)/(32*a**4*b
 + 64*a**3*b**2*x**4 + 32*a**2*b**3*x**8)
```

3.392 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=340

$$-\frac{3\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{3\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{7/4}b^{7/4}}-\frac{3(\sqrt{}}{}$$

Rubi [A] time = 0.33, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1823, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{3\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{3\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{128\sqrt{2}\,a^{7/4}b^{7/4}}-\frac{3\left(\sqrt{a}\,f+\sqrt{b}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{3\left(\sqrt{a}\,f+\sqrt{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}\,a^{7/4}b^{7/4}}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(8*b*(a + b*x^4)^2) + (x*(d + 2*e*x + 3*f*x^2))/
(32*a*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(3/2)*b^(3/2
)) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(6
4*Sqrt[2]*a^(7/4)*b^(7/4)) + (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(7/4)*b^(7/4)) - (3*(Sqrt[b]*d - Sqrt[a
]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a
^(7/4)*b^(7/4)) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^3}\,dx = -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{\int \frac{d+2ex+3fx^2}{\left(a+bx^4\right)^2}\,dx}{8b}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}-\frac{\int \frac{-3d-4ex-3fx^2}{a+bx^4}\,dx}{32ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}-\frac{\int \left(-\frac{4ex}{a+bx^4}+\frac{-3d-3fx^2}{a+bx^4}\right)dx}{32ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}-\frac{\int \frac{-3d-3fx^2}{a+bx^4}\,dx}{32ab}+\frac{e\int \frac{x}{a+bx^4}\,dx}{8ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}+\frac{e\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{16ab}+\frac{(3}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}+\frac{\left(3\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\right.}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{3\left(\sqrt{b}d-\sqrt{a}\right.}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{3\left(\sqrt{b}d+\sqrt{a}\right.}{64}$$

<u>Mathematica</u> [A] time = 0.39, size = 329, normalized size = 0.97

$$\frac{-\frac{2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f+3\sqrt{2}\sqrt{b}d\right)}{a^{7/4}}+\frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-8\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f+3\sqrt{2}\sqrt{b}d\right)}{a^{7/4}}+\frac{3\sqrt{2}\left(\sqrt{a}f-\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{3\sqrt{2}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}-\frac{32b^{3/4}(c+x(d+x(e+fx)))}{\left(a+bx^4\right)^2}+\frac{8b^{3/4}x(d+x(2e+3fx))}{a\left(a+bx^4\right)}}{256b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

```
[Out] ((8*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)) - (32*b^(3/4)*(c + x*(
d + x*(e + f*x))))/(a + b*x^4)^2 - (2*(3*Sqrt[2]*Sqrt[b]*d + 8*a^(1/4)*b^(1
/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/
4) + (2*(3*Sqrt[2]*Sqrt[b]*d - 8*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (3*Sqrt[2]*(-(Sqrt[b]*d)
+ Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4
) + (3*Sqrt[2]*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/a^(7/4))/(256*b^(7/4))
```

<u>IntegrateAlgebraic</u> [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.29, size = 338, normalized size = 0.99

$$\frac{3\,bfx^7+2\,bx^6e+bdx^5-afx^3-2\,ax^2e-3\,adx-4\,ac}{32\left(bx^4+a\right)^2ab}+\frac{\sqrt{2}\left(4\,\sqrt{2}\,\sqrt{ab}\,b^2e+3\,\left(ab^3\right)^{\frac{1}{4}}b^2d+3\,\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}+\frac{\sqrt{2}\left(4\,\sqrt{2}\,\sqrt{ab}\,b^2e+3\,\left(ab^3\right)^{\frac{1}{4}}b^2d+3\,\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4}+\frac{3\,\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}-\frac{3\,\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*b*f*x^7 + 2*b*x^6*e + b*d*x^5 - a*f*x^3 - 2*a*x^2*e - 3*a*d*x - 4*a
*c)/((b*x^4 + a)^2*a*b) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e + 3*(a*b
^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e
+ 3*(a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*
d - (a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
 - 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
```

maple [A] time = 0.06, size = 373, normalized size = 1.10

$$\frac{e\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{16\sqrt{ab}\,ab}+\frac{3\sqrt{2}\,f\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2}+\frac{3\sqrt{2}\,f\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2}+\frac{3\sqrt{2}\,f\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{128a^2b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{128a^2b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{256a^2b}+\frac{\frac{3fx^7}{32a}+\frac{ex^6}{16a}+\frac{dx^5}{32a}-\frac{fx^3}{32b}-\frac{ex^2}{16b}-\frac{3dx}{32b}-\frac{c}{8b}}{\left(b\,x^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (3/32*f/a*x^7+1/16/a*e*x^6+1/32*d/a*x^5-1/32/b*f*x^3-1/16/b*e*x^2-3/32/b*d*
x-1/8/b*c)/(b*x^4+a)^2+3/256/b/a^2*d*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4
)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/128/b/a
^2*d*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/128/b/a^2*d*(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/16/b/a*e/(a*b)^(1/2)*arct
an((1/a*b)^(1/2)*x^2)+3/256/b^2/a*f/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)
*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+3/128/b^2/
a*f/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+3/128/b^2/a*f/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.08, size = 343, normalized size = 1.01

$$\frac{3\,bfx^7+2\,bex^6+bdx^5-afx^3-2\,aex^2-3\,adx-4\,ac}{32\left(ab^3x^8+2\,a^2b^2x^4+a^3b\right)}+\frac{\frac{3\,\sqrt{2}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{3\,\sqrt{2}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f-8\,\sqrt{a}\,\sqrt{b}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(3\,\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\,\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{1}{4}}f+8\,\sqrt{a}\,\sqrt{b}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{b}}\,b^{\frac{3}{4}}}}{256\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(3*b*f*x^7 + 2*b*e*x^6 + b*d*x^5 - a*f*x^3 - 2*a*e*x^2 - 3*a*d*x - 4*a
*c)/(a*b^3*x^8 + 2*a^2*b^2*x^4 + a^3*b) + 1/256*(3*sqrt(2)*(sqrt(b)*d - sqr
t(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(
```

```
3/4)) - 3*sqrt(2)*(sqrt(b)*d - sqrt(a)*f)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*d +
3*sqrt(2)*a^(3/4)*b^(1/4)*f - 8*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sq
rt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqr
t(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*d + 3*sqrt(2)*a^(3/4)
*b^(1/4)*f + 8*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)
*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(
3/4)))/(a*b)
```

mupad [B] time = 0.40, size = 521, normalized size = 1.53

$$\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(268435456\,a^7\,b^7\,z^4+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b^3\,d^2\,e\,z-576\,a\,b\,d\,e^2\,f+162\,a\,b\,d^2\,f^2+256\,a\,b\,e^4+81\,a^2\,f^4+81\,b^2\,d^4,z,k\right)\left(\mathrm{root}\left(268435456\,a^7\,b^7\,z^4+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b^3\,d^2\,e\,z-576\,a\,b\,d\,e^2\,f+162\,a\,b\,d^2\,f^2+256\,a\,b\,e^4+81\,a^2\,f^4+81\,b^2\,d^4,z,k\right)\left(\frac{3\,b^2\,d}{2}-2\,b^2\,e\,x\right)+\frac{3\,e\,f}{32\,a}+\frac{x\,\left(144\,a\,b^2\,d^2-144\,a^2\,b\,f^2\right)}{4096\,a^3\,b}\right)-\frac{3\,\left(9\,b\,d^2\,f-16\,b\,d\,e^2+9\,a\,f^3\right)}{32768\,a^3\,b^2}+\frac{x\,\left(8\,e^3-9\,d\,e\,f\right)}{4096\,a^3\,b}\right)\mathrm{root}\left(268435456\,a^7\,b^7\,z^4+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b^3\,d^2\,e\,z-576\,a\,b\,d\,e^2\,f+162\,a\,b\,d^2\,f^2+256\,a\,b\,e^4+81\,a^2\,f^4+81\,b^2\,d^4,z,k\right)\right)-\frac{\frac{c}{8\,b}-\frac{d\,x^5}{32\,a}-\frac{e\,x^6}{16\,a}+\frac{e\,x^2}{16\,b}-\frac{3\,f\,x^7}{32\,a}+\frac{f\,x^3}{32\,b}+\frac{3\,d\,x}{32\,b}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x)
```

```
[Out] symsum(log((x*(8*e^3 - 9*d*e*f))/(4096*a^3*b) - (3*(9*a*f^3 - 16*b*d*e^2 +
9*b*d^2*f))/(32768*a^3*b^2) - root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d
*f*z^2 + 524288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2
*e*z - 576*a*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^
2*d^4, z, k)*(root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d*f*z^2 + 524288*
a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2*e*z - 576*a*b*d
*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^2*d^4, z, k)*((3
*b^2*d)/2 - 2*b^2*e*x) + (3*e*f)/(32*a) + (x*(144*a*b^2*d^2 - 144*a^2*b*f^2
))/(4096*a^3*b)))*root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d*f*z^2 + 524
288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2*e*z - 576*a
*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^2*d^4, z, k)
, k, 1, 4) - (c/(8*b) - (d*x^5)/(32*a) - (e*x^6)/(16*a) + (e*x^2)/(16*b) -
(3*f*x^7)/(32*a) + (f*x^3)/(32*b) + (3*d*x)/(32*b))/(a^2 + b^2*x^8 + 2*a*b*
x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.393 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=382

$$-\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}-\frac{(15}{}$$

Rubi [A] time = 0.41, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{15/4}b^{3/4}}-\frac{\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}\,e+77\sqrt{b}\,c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}\,a^{15/4}b^{3/4}}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)}+\frac{5d\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x +
45*e*x^2))/(384*a^3*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*x^2))/(12*a*b*(a
 + b*x^4)^3) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (
(77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256
*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c -
 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512
*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^4}\,dx = -\frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{\left(a+bx^4\right)^3}dx}{12a}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} + \frac{\int \frac{77c+60dx+45ex^2}{\left(a+bx^4\right)^2}dx}{96a^2}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} - \int \frac{-231c\cdots}{\cdots}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} - \int\left(-\frac{120\cdots}{a+b\cdots}\cdots\right.$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-231c\cdots}{a+b\cdots}}{38\cdots}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} + \frac{(5d)\,\mathrm{Su}\cdots}{\cdots}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} + \frac{5d\tan^{-\cdots}}{32a^{\cdots}}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} + \frac{5d\tan^{-\cdots}}{32a^{\cdots}}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} + \frac{5d\tan^{-\cdots}}{32a^{\cdots}}$$

Mathematica [A] time = 0.44, size = 379, normalized size = 0.99

$$\frac{\frac{3\sqrt{2}\left(15a^{3/4}e-77\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{3\sqrt{2}\left(77\sqrt[4]{a}\sqrt{b}c-15a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} - \frac{256a^3(af-bx(c+x(d+ex)))}{b\left(a+bx^4\right)^3} + \frac{32a^2x(11c+x(10d+9ex))}{\left(a+bx^4\right)^2} - \frac{6\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e+77\sqrt{2}\sqrt{b}c\right)}{b^{3/4}} + \frac{6\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e+77\sqrt{2}\sqrt{b}c\right)}{b^{3/4}} + \frac{8ax(77c+15x(4d+3ex))}{a+bx^4}}{3072a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*
d + 9*e*x)))/(a + b*x^4)^2 - (256*a^3*(a*f - b*x*(c + x*(d + e*x))))/(b*(a
+ b*x^4)^3) - (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*
Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^
(1/4)*(77*Sqrt[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*
Sqrt[b]*c + 15*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/b^(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
[Out] IntegrateAlgebraic[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.33, size = 391, normalized size = 1.02

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2 d+77\left(ab^3\right)^{\frac14}b^2c+15\left(ab^3\right)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}\,b^2 d+77\left(ab^3\right)^{\frac14}b^2c+15\left(ab^3\right)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac14}b^2c-15\left(ab^3\right)^{\frac34}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}-\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac14}b^2c-15\left(ab^3\right)^{\frac34}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}+\frac{45b^3x^{11}e+60b^3dx^{10}+77b^3cx^9+126ab^2x^7e+160ab^2dx^6+198ab^2cx^5+113a^2bx^3e+132a^2bdx^2+153a^2bcx-32a^3f}{384\left(bx^4+a\right)^3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2
*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqr
t(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*
b^3*c*x^9 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 113*a^2*b
*x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

maple [A] time = 0.06, size = 400, normalized size = 1.05

$$\frac{5d\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{32\sqrt{ab}\,a^3}+\frac{15\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}-1\right)}{512\left(\frac{a}{b}\right)^{\frac14}a^3b}+\frac{15\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)}{512\left(\frac{a}{b}\right)^{\frac14}a^3b}+\frac{15\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024\left(\frac{a}{b}\right)^{\frac14}a^3b}+\frac{77\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}-1\right)}{512a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)}{512a^4}+\frac{77\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{1024a^4}+\frac{\frac{15b^2ex^{11}}{128a^3}+\frac{5b^2dx^{10}}{32a^3}+\frac{77b^2cx^9}{384a^3}+\frac{21bex^7}{64a^2}+\frac{5bdx^6}{12a^2}+\frac{33bcx^5}{64a^2}+\frac{113ex^3}{384a}+\frac{11dx^2}{32a}+\frac{51cx}{128a}-\frac{f}{12b}}{\left(bx^4+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
[Out] (15/128/a^3*b^2*e*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64/a^2*b
*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/
128*c/a*x-1/12/b*f)/(b*x^4+a)^3+77/1024/a^4*c*(a/b)^(1/4)*2^(1/2)*ln((x^2+(
a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+
77/512/a^4*c*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+77/512/a^4
*c*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)+5/32/a^3*d/(a*b)^(1/
2)*arctan((1/a*b)^(1/2)*x^2)+15/1024/a^3*e/b/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a
/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1
5/512/a^3*e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+15/512/a^
3*e/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.09, size = 402, normalized size = 1.05

$$\frac{45b^3ex^{11}+60b^3dx^{10}+77b^3cx^9+126ab^2ex^7+160ab^2dx^6+198ab^2cx^5+113a^2bex^3+132a^2bdx^2+153a^2bcx-32a^3f}{384\left(a^3b^4x^{12}+3a^4b^3x^8+3a^5b^2x^4+a^6b\right)}+\frac{\frac{\sqrt{2}\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac34}}-\frac{\sqrt{2}\left(77\sqrt{b}\,c-15\sqrt{a}\,e\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac34}}+\frac{2\left(77\sqrt{2}\,a^{\frac14}b^{\frac34}c+15\sqrt{2}\,a^{\frac34}b^{\frac14}e-80\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac34}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac34}}+\frac{2\left(77\sqrt{2}\,a^{\frac14}b^{\frac34}c+15\sqrt{2}\,a^{\frac34}b^{\frac14}e+80\sqrt{a}\sqrt{b}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac34}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac34}}}{1024a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 + 126*a*b^2*e*x^7 + 160
*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*a^
2*b*c*x - 32*a^3*f)/(a^3*b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b)
+ 1/1024*(sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*sqrt(b)*c - 15*
sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*
b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e -
 80*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(
77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e + 80*sqrt(a)*sq
rt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^3
```

mupad [B] time = 5.25, size = 879, normalized size = 2.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x)
```

```
[Out] symsum(log(-(b*(3375*a*e^3 - 123200*b*c*d^2 + 88935*b*c^2*e - 64000*b*d^3*x
 + 20185088*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 83
8860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*
z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*
b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2*c - 115200*root(68719476736*a^15*b
^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680
*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a
*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*
b*e^2*x + 92400*b*c*d*e*x + 3035648*root(68719476736*a^15*b^3*z^4 + 1211105
280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z
 + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 256
0000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^3*b^2*c^2*x - 1048
5760*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800
*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 739
2000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4
 + 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x + 614400*root(68719476736*a^15*b^3*z^
4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*
b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*b*d*e
))/(2097152*a^9))*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^
2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*
d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 351
53041*b^2*c^4 + 50625*a^2*e^4, z, k), k, 1, 4) + ((11*d*x^2)/(32*a) - f/(12
*b) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^9)/(384*a^3) + (
5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (33*b*c*x^5)/(64*a^2)
+ (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x
^4 + 3*a*b^2*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

3.394 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=380

$$-\frac{\left(7\sqrt{b}\,d-5\sqrt{a}\,f\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{\left(7\sqrt{b}\,d-5\sqrt{a}\,f\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{11/4}b^{7/4}}-\frac{(5 \cdots}{\cdots}$$

Rubi [A] time = 0.40, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1823, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(7\sqrt{b}\,d-5\sqrt{a}\,f\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{\left(7\sqrt{b}\,d-5\sqrt{a}\,f\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{512\sqrt{2}\,a^{11/4}b^{7/4}}-\frac{\left(5\sqrt{a}\,f+7\sqrt{b}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{256\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{\left(5\sqrt{a}\,f+7\sqrt{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{256\sqrt{2}\,a^{11/4}b^{7/4}}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(12*b*(a + b*x^4)^3) + (x*(d + 2*e*x + 3*f*x^2))
/(96*a*b*(a + b*x^4)^2) + (x*(7*d + 12*e*x + 15*f*x^2))/(384*a^2*b*(a + b*x
^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(5/2)*b^(3/2)) - ((7*Sqrt[b]
*d + 5*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(
11/4)*b^(7/4)) + ((7*Sqrt[b]*d + 5*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(256*Sqrt[2]*a^(11/4)*b^(7/4)) - ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11/4
)*b^(7/4)) + ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^4}\,dx = -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{\int \frac{d+2ex+3fx^2}{\left(a+bx^4\right)^3}dx}{12b}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}-\frac{\int \frac{-7d-12ex-15fx^2}{\left(a+bx^4\right)^2}dx}{96ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\int \frac{21d\ldots}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{\int\left(\frac{24\ldots}{a+\ldots}\right.}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{\int \frac{21d\ldots}{a\ldots}}{38\ldots}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{e\,\mathrm{Sub}\ldots}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{e\tan^{-1}\ldots}{32a\ldots}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{e\tan^{-1}\ldots}{32a\ldots}$$

$$= -\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{e\tan^{-1}\ldots}{32a\ldots}$$

Mathematica [A] time = 0.46, size = 366, normalized size = 0.96

$$\frac{-\frac{6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(16\sqrt[4]{a}\sqrt[4]{b}e+5\sqrt{2}\sqrt{a}f+7\sqrt{2}\sqrt{b}d\right)}{a^{11/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-16\sqrt[4]{a}\sqrt[4]{b}e+5\sqrt{2}\sqrt{a}f+7\sqrt{2}\sqrt{b}d\right)}{a^{11/4}}+\frac{3\sqrt{2}\left(5\sqrt{a}f-7\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{11/4}}+\frac{3\sqrt{2}\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{11/4}}+\frac{8b^{3/4}x(7d+3x(4e+5fx))}{a^2\left(a+bx^4\right)}-\frac{256b^{3/4}(c+x(d+x(e+fx)))}{\left(a+bx^4\right)^3}+\frac{32b^{3/4}x(d+x(2e+3fx))}{a\left(a+bx^4\right)^2}}{3072b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

```
[Out] ((32*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)^2) + (8*b^(3/4)*x*(7*d
 + 3*x*(4*e + 5*f*x)))/(a^2*(a + b*x^4)) - (256*b^(3/4)*(c + x*(d + x*(e +
f*x))))/(a + b*x^4)^3 - (6*(7*Sqrt[2]*Sqrt[b]*d + 16*a^(1/4)*b^(1/4)*e + 5*
Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*(
7*Sqrt[2]*Sqrt[b]*d - 16*a^(1/4)*b^(1/4)*e + 5*Sqrt[2]*Sqrt[a]*f)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (3*Sqrt[2]*(-7*Sqrt[b]*d + 5*Sqr
t[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(11/4) +
(3*Sqrt[2]*(7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/a^(11/4))/(3072*b^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(c + dx + ex^2 + fx^3\right)}{\left(a + bx^4\right)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
[Out] IntegrateAlgebraic[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.19, size = 380, normalized size = 1.00

$$\frac{\sqrt{2}\left(8 \sqrt{2} \sqrt{ab}\, b^2 e + 7 \left(ab^3\right)^{\frac{1}{4}} b^2 d + 5 \left(ab^3\right)^{\frac{3}{4}} f\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\, a^3 b^4} + \frac{\sqrt{2}\left(8 \sqrt{2} \sqrt{ab}\, b^2 e + 7 \left(ab^3\right)^{\frac{1}{4}} b^2 d + 5 \left(ab^3\right)^{\frac{3}{4}} f\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\, a^3 b^4} + \frac{\sqrt{2}\left(7 \left(ab^3\right)^{\frac{1}{4}} b^2 d - 5 \left(ab^3\right)^{\frac{3}{4}} f\right) \log\left(x^2 + \sqrt{2}\, x \left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\, a^3 b^4} - \frac{\sqrt{2}\left(7 \left(ab^3\right)^{\frac{1}{4}} b^2 d - 5 \left(ab^3\right)^{\frac{3}{4}} f\right) \log\left(x^2 - \sqrt{2}\, x \left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024\, a^3 b^4} + \frac{15 b^2 f x^{11} + 12 b^2 x^{10} e + 7 b^2 d x^9 + 42 abf x^7 + 32 abx^6 e + 18 abdx^5 - 5 a^2 f x^3 - 12 a^2 x^2 e - 21 a^2 dx - 32 a^2 c}{384 \left(bx^4 + a\right)^3 a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
[Out] 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d + 5*(a*b^3
)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3
*b^4) + 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d +
5*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(a^3*b^4) + 1/1024*sqrt(2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/1024*sqrt(2)*(7*
(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(a^3*b^4) + 1/384*(15*b^2*f*x^11 + 12*b^2*x^10*e + 7*b^2*d*x^9 +
 42*a*b*f*x^7 + 32*a*b*x^6*e + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2*x^2*e -
21*a^2*d*x - 32*a^2*c)/((b*x^4 + a)^3*a^2*b)
```

maple [A] time = 0.06, size = 403, normalized size = 1.06

$$\frac{e \arctan\left(\sqrt{\frac{b}{a}}\, x^2\right)}{32 \sqrt{ab}\, a^2 b} + \frac{5\sqrt{2}\, f \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{512 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^2 b^2} + \frac{5\sqrt{2}\, f \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{512 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^2 b^2} + \frac{5\sqrt{2}\, f \ln\left(\frac{x^2 - \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{\frac{a}{b}}}{x^2 + \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{\frac{a}{b}}}\right)}{1024 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^2 b^2} + \frac{7 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{512 a^3 b} + \frac{7 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{512 a^3 b} + \frac{7 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d \ln\left(\frac{x^2 + \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{\frac{a}{b}}}{x^2 - \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{\frac{a}{b}}}\right)}{1024 a^3 b} + \frac{\frac{5bf x^{11}}{128a^2} + \frac{be x^{10}}{32a^2} + \frac{7bd x^9}{384a^2} + \frac{7f x^7}{64a} + \frac{e x^6}{12a} + \frac{3d x^5}{64a} - \frac{5f x^3}{384b} - \frac{e x^2}{32b} - \frac{7dx}{128b} - \frac{c}{12b}}{\left(b x^4 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
[Out] (5/128*f/a^2*b*x^11+1/32/a^2*b*e*x^10+7/384/a^2*d*b*x^9+7/64/a*f*x^7+1/12/a
*e*x^6+3/64/a*d*x^5-5/384/b*f*x^3-1/32/b*e*x^2-7/128/b*d*x-1/12/b*c)/(b*x^4
+a)^3+7/1024/a^3/b*d*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+7/512/a^3/b*d*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+7/512/a^3/b*d*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x-1)+1/32/a^2/b*e/(a*b)^(1/2)*arctan((1/a*b)^(1/
2)*x^2)+5/1024/a^2/b^2*f/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+
(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+5/512/a^2/b^2*f/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+5/512/a^2/b^2*f/(a/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 3.12, size = 396, normalized size = 1.04

$$\frac{15\,b^2fx^{11}+12\,b^2ex^{10}+7\,b^2dx^9+42\,abfx^7+32\,abex^6+18\,abdx^5-5\,a^2fx^3-12\,a^2ex^2-21\,a^2dx-32\,a^2c}{384\,\left(a^2b^4x^{12}+3\,a^3b^3x^8+3\,a^4b^2x^4+a^5b\right)}+\frac{\frac{\sqrt{2}\left(7\,\sqrt{b}d-5\,\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(7\,\sqrt{b}d-5\,\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}d+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-16\,\sqrt{a}\sqrt{b}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}+\frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}d+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f+16\,\sqrt{a}\sqrt{b}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{3}{4}}}}{1024\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/384*(15*b^2*f*x^11 + 12*b^2*e*x^10 + 7*b^2*d*x^9 + 42*a*b*f*x^7 + 32*a*b*
e*x^6 + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2*e*x^2 - 21*a^2*d*x - 32*a^2*c)/
(a^2*b^4*x^12 + 3*a^3*b^3*x^8 + 3*a^4*b^2*x^4 + a^5*b) + 1/1024*(sqrt(2)*(7
*sqrt(b)*d - 5*sqrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqr
t(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(7*sqrt(b)*d - 5*sqrt(a)*f)*log(sqrt(b)*x
^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(7*sqrt(2)*
a^(1/4)*b^(3/4)*d + 5*sqrt(2)*a^(3/4)*b^(1/4)*f - 16*sqrt(a)*sqrt(b)*e)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*b^(3/4)*
d + 5*sqrt(2)*a^(3/4)*b^(1/4)*f + 16*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*
(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqr
t(sqrt(a)*sqrt(b))*b^(3/4)))/(a^2*b)
```

mupad [B] time = 0.48, size = 888, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x)
```

```
[Out] ((3*d*x^5)/(64*a) - c/(12*b) + (e*x^6)/(12*a) - (e*x^2)/(32*b) + (7*f*x^7)/
(64*a) - (5*f*x^3)/(384*b) - (7*d*x)/(128*b) + (7*b*d*x^9)/(384*a^2) + (b*e
*x^10)/(32*a^2) + (5*b*f*x^11)/(128*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3
*a*b^2*x^8) + symsum(log(-(125*a*f^3 - 448*b*d*e^2 + 245*b*d^2*f - 512*b*e^
3*x + 1835008*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33
554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z -
8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2
*d^4, z, k)^2*a^5*b^4*d + 560*b*d*e*f*x + 25088*root(68719476736*a^11*b^7*z
^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e
*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 409
6*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k)*a^2*b^3*d^2*x - 2097152*root(
68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*
z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f +
2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k)^2*a^5*b
^4*e*x - 12800*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 3
3554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z -
 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^
2*d^4, z, k)*a^3*b^2*f^2*x + 40960*root(68719476736*a^11*b^7*z^4 + 36700160
*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 8028
16*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 6
25*a^2*f^4 + 2401*b^2*d^4, z, k)*a^3*b^2*e*f)/(2097152*a^6*b^2))*root(68719
476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 +
 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*
a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

3.395 $\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

```
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx &= \int \frac{1}{1-x} dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

```
[Out] -Log[1 - x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

```
[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3 + x^4)/(1 - x^5), x]
```

fricas [A] time = 0.41, size = 6, normalized size = 0.75

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="fricas")
[Out] -log(x - 1)
```

giac [A] time = 0.17, size = 7, normalized size = 0.88

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="giac")
[Out] -log(abs(x - 1))
```

maple [A] time = 0.04, size = 7, normalized size = 0.88

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3+x^2+x+1)/(-x^5+1),x)
[Out] -ln(x-1)
```

maxima [A] time = 1.30, size = 6, normalized size = 0.75

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="maxima")
[Out] -log(x - 1)
```

mupad [B] time = 0.02, size = 6, normalized size = 0.75

$$-\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2 + x^3 + x^4 + 1)/(x^5 - 1),x)
[Out] -log(x - 1)
```

sympy [A] time = 0.08, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3+x**2+x+1)/(-x**5+1),x)
[Out] -log(x - 1)
```

3.396 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} dx$

Optimal. Leaf size=10

$$\frac{1}{2}\log(2x+3)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1586, 31}

$$\frac{1}{2}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6),x]
```

```
[Out] Log[3 + 2*x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} dx &= \int \frac{1}{3+2x} dx\\ &= \frac{1}{2}\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)
,x]
```

```
[Out] Log[3 + 2*x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729
- 64*x^6),x]
```

```
[Out] IntegrateAlgebraic[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729
- 64*x^6), x]
```

fricas [A] time = 0.39, size = 8, normalized size = 0.80

$$\frac{1}{2}\,\log\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="fricas")
```

```
[Out] 1/2*log(2*x + 3)
```

giac [A] time = 0.19, size = 9, normalized size = 0.90

$$\frac{1}{2}\,\log\left(|2\,x+3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="giac")
```

```
[Out] 1/2*log(abs(2*x + 3))
```

maple [A] time = 0.04, size = 9, normalized size = 0.90

$$\frac{\ln\left(2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x)
```

```
[Out] 1/2*ln(3+2*x)
```

maxima [A] time = 1.33, size = 8, normalized size = 0.80

$$\frac{1}{2}\,\log\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="maxima")
```

```
[Out] 1/2*log(2*x + 3)
```

mupad [B] time = 0.06, size = 6, normalized size = 0.60

$$\frac{\ln\left(x+\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((162*x - 108*x^2 + 72*x^3 - 48*x^4 + 32*x^5 - 243)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/2
```

sympy [A] time = 0.08, size = 7, normalized size = 0.70

$$\frac{\log\left(2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729),x)
```

```
[Out] log(2*x + 3)/2
```

3.397 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\log(3-2x)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1586, 31}

$$-\frac{1}{2}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6),x]
[Out] -Log[3 - 2*x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx &= \int \frac{1}{3-2x}\,dx\\ &= -\frac{1}{2}\log(3-2x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{2}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)
,x]
[Out] -1/2*Log[3 - 2*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729
- 64*x^6),x]
```

```
[Out] IntegrateAlgebraic[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729
- 64*x^6), x]
```

fricas [A] time = 0.38, size = 8, normalized size = 0.80

$$-\frac{1}{2}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="fricas")
[Out] -1/2*log(2*x - 3)
```

giac [A] time = 0.18, size = 9, normalized size = 0.90

$$-\frac{1}{2}\,\log\left(|2\,x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="giac")
[Out] -1/2*log(abs(2*x - 3))
```

maple [A] time = 0.04, size = 9, normalized size = 0.90

$$-\frac{\ln\left(2x-3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x)
[Out] -1/2*ln(-3+2*x)
```

maxima [A] time = 1.39, size = 8, normalized size = 0.80

$$-\frac{1}{2}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="maxima")
[Out] -1/2*log(2*x - 3)
```

mupad [B] time = 4.99, size = 6, normalized size = 0.60

$$-\frac{\ln\left(x-\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5 + 243)/(64*x^6 - 729),x)
[Out] -log(x - 3/2)/2
```

sympy [A] time = 0.09, size = 8, normalized size = 0.80

$$-\frac{\log\left(2x-3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729),x)
[Out] -log(2*x - 3)/2
```

3.398 $\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx$

Optimal. Leaf size=10

$$\frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1586, 206}

$$\frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] ArcTanh[(2*x)/3]/6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx &= \int \frac{1}{9-4x^2}\,dx\\ &= \frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [B] time = 0.00, size = 21, normalized size = 2.10

$$\frac{1}{12}\log(2x+3)-\frac{1}{12}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] -1/12*Log[3 - 2*x] + Log[3 + 2*x]/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] IntegrateAlgebraic[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6), x]
```

fricas [B] time = 0.40, size = 17, normalized size = 1.70

$$\frac{1}{12}\,\log\left(2\,x+3\right)-\frac{1}{12}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

giac [B] time = 0.20, size = 15, normalized size = 1.50

$$\frac{1}{12}\,\log\left(\left|x+\frac{3}{2}\right|\right)-\frac{1}{12}\,\log\left(\left|x-\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="giac")
[Out] 1/12*log(abs(x + 3/2)) - 1/12*log(abs(x - 3/2))
```

maple [B] time = 0.05, size = 18, normalized size = 1.80

$$-\frac{\ln\left(2x-3\right)}{12}+\frac{\ln\left(2x+3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x^4+36*x^2+81)/(-64*x^6+729),x)
[Out] 1/12*ln(2*x+3)-1/12*ln(2*x-3)
```

maxima [B] time = 1.33, size = 17, normalized size = 1.70

$$\frac{1}{12}\,\log\left(2\,x+3\right)-\frac{1}{12}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="maxima")
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

mupad [B] time = 0.10, size = 6, normalized size = 0.60

$$\frac{\mathrm{atanh}\left(\frac{2\,x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(36*x^2 + 16*x^4 + 81)/(64*x^6 - 729),x)
[Out] atanh((2*x)/3)/6
```

sympy [B] time = 0.11, size = 15, normalized size = 1.50

$$-\frac{\log\left(x-\frac{3}{2}\right)}{12}+\frac{\log\left(x+\frac{3}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x**4+36*x**2+81)/(-64*x**6+729),x)
[Out] -log(x - 3/2)/12 + log(x + 3/2)/12
```

3.399 $\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx$

Optimal. Leaf size=24

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1586, 618, 204}

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx &= \int \frac{1}{9-6x+4x^2}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)\right)\\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{81 + 54x - 24x^3 - 16x^4}{729 - 64x^6} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
[Out] IntegrateAlgebraic[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6), x]
```

fricas [A] time = 0.38, size = 16, normalized size = 0.67

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3} (4x - 3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

giac [A] time = 0.19, size = 16, normalized size = 0.67

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3} (4x - 3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="giac")
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

maple [A] time = 0.04, size = 17, normalized size = 0.71

$$\frac{\sqrt{3}\, \arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x)
[Out] 1/9*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

maxima [A] time = 2.93, size = 16, normalized size = 0.67

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3} (4x - 3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="maxima")
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

mupad [B] time = 0.03, size = 16, normalized size = 0.67

$$\frac{\sqrt{3}\, \mathrm{atan}\left(\frac{\sqrt{3}\,(4x-3)}{9}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(54*x - 24*x^3 - 16*x^4 + 81)/(64*x^6 - 729),x)
```

```
[Out] (3^(1/2)*atan((3^(1/2)*(4*x - 3))/9))/9
```

sympy [**A**] time = 0.15, size = 24, normalized size = 1.00

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729),x)
```

```
[Out] sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/9
```

3.400 $\int \frac{3-2x}{729-64x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1586, 2058, 628, 618, 204}

$$-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*
x + 4*x^2]/972
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3-2x}{729-64x^6}\,dx &= \int \frac{1}{243+162x+108x^2+72x^3+48x^4+32x^5}\,dx\\ &= \int \left(\frac{1}{243(3+2x)}+\frac{3-4x}{486\left(9-6x+4x^2\right)}+\frac{1}{54\left(9+6x+4x^2\right)}\right)dx\\ &= \frac{1}{486}\log(3+2x)+\frac{1}{486}\int \frac{3-4x}{9-6x+4x^2}\,dx+\frac{1}{54}\int \frac{1}{9+6x+4x^2}\,dx\\ &= \frac{1}{486}\log(3+2x)-\frac{1}{972}\log\left(9-6x+4x^2\right)-\frac{1}{27}\,\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,6+8x\right)\\ &= \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}}+\frac{1}{486}\log(3+2x)-\frac{1}{972}\log\left(9-6x+4x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*
x + 4*x^2]/972
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-2x}{729-64x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - 2*x)/(729 - 64*x^6),x]
```

```
[Out] IntegrateAlgebraic[(3 - 2*x)/(729 - 64*x^6), x]
```

fricas [A] time = 0.41, size = 38, normalized size = 0.76

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x+3)\right)-\frac{1}{972}\log\left(4\,x^2-6\,x+9\right)+\frac{1}{486}\log\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(2*x + 3)
```

giac [A] time = 0.20, size = 39, normalized size = 0.78

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x+3)\right)-\frac{1}{972}\log\left(4\,x^2-6\,x+9\right)+\frac{1}{486}\log\left(|2\,x+3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(abs(2*x + 3))
```

maple [A] time = 0.05, size = 39, normalized size = 0.78

$$\frac{\sqrt{3}\ \arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{486}+\frac{\ln(2x+3)}{486}-\frac{\ln\left(4x^2-6x+9\right)}{972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)/(-64*x^6+729),x)
```

```
[Out] -1/972*ln(4*x^2-6*x+9)+1/486*ln(2*x+3)+1/486*3^(1/2)*arctan(1/18*(8*x+6)*3^
(1/2))
```

maxima [A] time = 2.89, size = 38, normalized size = 0.76

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x+3)\right)-\frac{1}{972}\log\left(4\,x^2-6\,x+9\right)+\frac{1}{486}\log(2\,x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(2*x + 3)
```

mupad [B] time = 0.13, size = 49, normalized size = 0.98

$$\frac{\ln\left(x+\frac{3}{2}\right)}{486}-\frac{\ln\left(x^2-\frac{3\,x}{2}+\frac{9}{4}\right)}{972}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{1327104\left(\frac{x}{884736}+\frac{1}{884736}\right)}-\frac{\sqrt{3}\,x}{7962624\left(\frac{x}{884736}+\frac{1}{884736}\right)}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/486 - log(x^2 - (3*x)/2 + 9/4)/972 - (3^(1/2)*atan(3^(1/2)/(13
27104*(x/884736 + 1/884736)) - (3^(1/2)*x)/(7962624*(x/884736 + 1/884736)))
)/486
```

sympy [A] time = 0.21, size = 46, normalized size = 0.92

$$\frac{\log\left(x+\frac{3}{2}\right)}{486}-\frac{\log\left(4x^2-6x+9\right)}{972}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x**6+729),x)
```

```
[Out] log(x + 3/2)/486 - log(4*x**2 - 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9 +
 sqrt(3)/3)/486
```

3.401 $\int \frac{3+2x}{729-64x^6} \, dx$

Optimal. Leaf size=50

$$\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1586, 2058, 618, 204, 628}

$$\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) - Log[3 - 2*x]/486 + Log[9 + 6
*x + 4*x^2]/972
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3+2x}{729-64x^6}\,dx &= \int \frac{1}{243-162x+108x^2-72x^3+48x^4-32x^5}\,dx \\
&= \int \left(-\frac{1}{243(-3+2x)}+\frac{1}{54\left(9-6x+4x^2\right)}+\frac{3+4x}{486\left(9+6x+4x^2\right)}\right)dx \\
&= -\frac{1}{486}\log(3-2x)+\frac{1}{486}\int \frac{3+4x}{9+6x+4x^2}\,dx+\frac{1}{54}\int \frac{1}{9-6x+4x^2}\,dx \\
&= -\frac{1}{486}\log(3-2x)+\frac{1}{972}\log\left(9+6x+4x^2\right)-\frac{1}{27}\,\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right) \\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}-\frac{1}{486}\log(3-2x)+\frac{1}{972}\log\left(9+6x+4x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.92

$$
\frac{1}{972}\left(\log\left(4x^2+6x+9\right)-2\log(3-2x)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(729 - 64*x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 2*Log[3 - 2*x] + Log[9 + 6*x +
4*x^2])/972
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{3+2x}{729-64x^6}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 2*x)/(729 - 64*x^6),x]
```

```
[Out] IntegrateAlgebraic[(3 + 2*x)/(729 - 64*x^6), x]
```

fricas [A] time = 0.41, size = 38, normalized size = 0.76

$$
\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)+\frac{1}{972}\log\left(4\,x^2+6\,x+9\right)-\frac{1}{486}\log\left(2\,x-3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(2*x - 3)
```

giac [A] time = 0.18, size = 39, normalized size = 0.78

$$
\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)+\frac{1}{972}\log\left(4\,x^2+6\,x+9\right)-\frac{1}{486}\log\left(|2\,x-3|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(abs(2*x - 3))
```

maple [A] time = 0.05, size = 39, normalized size = 0.78

$$\frac{\sqrt{3}\ \arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{486}-\frac{\ln(2x-3)}{486}+\frac{\ln\left(4x^{2}+6x+9\right)}{972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+3)/(-64*x^6+729),x)
```

```
[Out] 1/486*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/972*ln(4*x^2+6*x+9)-1/486*ln(2
*x-3)
```

maxima [A] time = 2.95, size = 38, normalized size = 0.76

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{972}\log\left(4x^{2}+6x+9\right)-\frac{1}{486}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(2*x - 3)
```

mupad [B] time = 4.99, size = 48, normalized size = 0.96

$$\frac{\ln\left(x^{2}+\frac{3x}{2}+\frac{9}{4}\right)}{972}-\frac{\ln\left(x-\frac{3}{2}\right)}{486}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{1327104\left(\frac{x}{884736}-\frac{1}{884736}\right)}+\frac{\sqrt{3}\,x}{7962624\left(\frac{x}{884736}-\frac{1}{884736}\right)}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 3)/(64*x^6 - 729),x)
```

```
[Out] log((3*x)/2 + x^2 + 9/4)/972 - log(x - 3/2)/486 - (3^(1/2)*atan(3^(1/2)/(13
27104*(x/884736 - 1/884736)) + (3^(1/2)*x)/(7962624*(x/884736 - 1/884736)))
)/486
```

sympy [A] time = 0.24, size = 46, normalized size = 0.92

$$-\frac{\log\left(x-\frac{3}{2}\right)}{486}+\frac{\log\left(4x^{2}+6x+9\right)}{972}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/486 + log(4*x**2 + 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/486
```

3.402 $\int \frac{9-6x+4x^2}{729-64x^6} \, dx$

Optimal. Leaf size=60

$$-\frac{1}{324}\log\left(4x^2+6x+9\right)-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1586, 2058, 634, 618, 204, 628}

$$-\frac{1}{324}\log\left(4x^2+6x+9\right)-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + Log[3 + 2*x
]/108 - Log[9 + 6*x + 4*x^2]/324
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{9-6x+4x^2}{729-64x^6}\,dx &= \int \frac{1}{81+54x-24x^3-16x^4}\,dx\\ &= \int \left(-\frac{1}{162(-3+2x)}+\frac{1}{54(3+2x)}+\frac{3-2x}{81\left(9+6x+4x^2\right)}\right)dx\\ &= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)+\frac{1}{81}\int \frac{3-2x}{9+6x+4x^2}\,dx\\ &= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\int \frac{6+8x}{9+6x+4x^2}\,dx+\frac{1}{18}\int \frac{1}{9+6x+4x^2}\,dx\\ &= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\log\left(9+6x+4x^2\right)-\frac{1}{9}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx\right.\\ &= \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{54\sqrt{3}}-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\log\left(9+6x+4x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.93

$$\frac{1}{324}\left(-\log\left(4x^2+6x+9\right)-\log(3-2x)+3\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - Log[3 - 2*x] + 3*Log[3 + 2*x] -
Log[9 + 6*x + 4*x^2])/324
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] IntegrateAlgebraic[(9 - 6*x + 4*x^2)/(729 - 64*x^6), x]
```

fricas [A] time = 0.42, size = 46, normalized size = 0.77

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log(2x+3)-\frac{1}{324}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

giac [A] time = 0.19, size = 48, normalized size = 0.80

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log(|2x+3|)-\frac{1}{324}\log(|2x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(abs(2*x + 3)) - 1/324*log(abs(2*x - 3))
```

maple [A] time = 0.05, size = 47, normalized size = 0.78

$$\frac{\sqrt{3}\ \arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{162}-\frac{\ln\left(2x-3\right)}{324}+\frac{\ln\left(2x+3\right)}{108}-\frac{\ln\left(4x^{2}+6x+9\right)}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-6*x+9)/(-64*x^6+729),x)
```

```
[Out] 1/108*ln(2*x+3)-1/324*ln(4*x^2+6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x+6)*3^(
1/2))-1/324*ln(2*x-3)
```

maxima [A] time = 2.94, size = 46, normalized size = 0.77

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(4x+3\right)\right)-\frac{1}{324}\log\left(4x^{2}+6x+9\right)+\frac{1}{108}\log\left(2x+3\right)-\frac{1}{324}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

mupad [B] time = 5.01, size = 52, normalized size = 0.87

$$\frac{\ln\left(x+\frac{3}{2}\right)}{108}-\frac{\ln\left(x-\frac{3}{2}\right)}{324}-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x^2 - 6*x + 9)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/108 - log(x - 3/2)/324 - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/
2)*1i)/324 + 1/324) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/324 - 1/3
24)
```

sympy [A] time = 0.23, size = 56, normalized size = 0.93

$$-\frac{\log\left(x-\frac{3}{2}\right)}{324}+\frac{\log\left(x+\frac{3}{2}\right)}{108}-\frac{\log\left(x^{2}+\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}+\frac{\sqrt{3}}{3}\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-6*x+9)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/324 + log(x + 3/2)/108 - log(x**2 + 3*x/2 + 9/4)/324 + sqrt(3
)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/162
```

3.403 $\int \frac{9+6x+4x^2}{729-64x^6}\,dx$

Optimal. Leaf size=60

$$\frac{1}{324}\log\left(4x^2-6x+9\right)-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1586, 2058, 634, 618, 204, 628}

$$\frac{1}{324}\log\left(4x^2-6x+9\right)-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/108 + Log[3 + 2*
x]/324 + Log[9 - 6*x + 4*x^2]/324
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{9+6x+4x^2}{729-64x^6}\,dx &= \int \frac{1}{81-54x+24x^3-16x^4}\,dx\\
&= \int \left(-\frac{1}{54(-3+2x)}+\frac{1}{162(3+2x)}+\frac{3+2x}{81\left(9-6x+4x^2\right)}\right)dx\\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{81}\int \frac{3+2x}{9-6x+4x^2}\,dx\\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\int \frac{-6+8x}{9-6x+4x^2}\,dx+\frac{1}{18}\int \frac{1}{9-6x+4x^2}\,dx\\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\log\left(9-6x+4x^2\right)-\frac{1}{9}\text{Subst}\left(\int \frac{1}{-108-x^2}\right.\\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\log\left(9-6x+4x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 52, normalized size = 0.87

$$\frac{1}{324}\left(\log\left(4x^2-6x+9\right)-3\log(3-2x)+\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + Log[3 + 2*x] +
 Log[9 - 6*x + 4*x^2])/324
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] IntegrateAlgebraic[(9 + 6*x + 4*x^2)/(729 - 64*x^6), x]
```

fricas [A] time = 0.41, size = 46, normalized size = 0.77

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log(2x+3)-\frac{1}{108}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

giac [A] time = 0.18, size = 48, normalized size = 0.80

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log(|2x+3|)-\frac{1}{108}\log(|2x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
1/324*log(abs(2*x + 3)) - 1/108*log(abs(2*x - 3))
```

maple [A] time = 0.05, size = 47, normalized size = 0.78

$$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{162}-\frac{\ln\left(2x-3\right)}{108}+\frac{\ln\left(2x+3\right)}{324}+\frac{\ln\left(4x^2-6x+9\right)}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+6*x+9)/(-64*x^6+729),x)
```

```
[Out] 1/324*ln(4*x^2-6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/324*ln(2
*x+3)-1/108*ln(2*x-3)
```

maxima [A] time = 3.08, size = 46, normalized size = 0.77

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(4\,x-3\right)\right)+\frac{1}{324}\log\left(4\,x^2-6\,x+9\right)+\frac{1}{324}\log\left(2\,x+3\right)-\frac{1}{108}\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

mupad [B] time = 4.98, size = 52, normalized size = 0.87

$$\frac{\ln\left(x+\frac{3}{2}\right)}{324}-\frac{\ln\left(x-\frac{3}{2}\right)}{108}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(6*x + 4*x^2 + 9)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/324 - log(x - 3/2)/108 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/
2)*1i)/324 - 1/324) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/324 + 1/3
24)
```

sympy [A] time = 0.23, size = 56, normalized size = 0.93

$$-\frac{\log\left(x-\frac{3}{2}\right)}{108}+\frac{\log\left(x+\frac{3}{2}\right)}{324}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+6*x+9)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/108 + log(x + 3/2)/324 + log(x**2 - 3*x/2 + 9/4)/324 + sqrt(3
)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/162
```

3.404 $\int \frac{27-8x^3}{729-64x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {26, 200, 31, 634, 618, 204, 628}

$$-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(27 - 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*x
 + 4*x^2]/108
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{27-8x^3}{729-64x^6}\,dx &= \int \frac{1}{27+8x^3}\,dx\\
&= \frac{1}{27}\int \frac{1}{3+2x}\,dx + \frac{1}{27}\int \frac{6-2x}{9-6x+4x^2}\,dx\\
&= \frac{1}{54}\log(3+2x) - \frac{1}{108}\int \frac{-6+8x}{9-6x+4x^2}\,dx + \frac{1}{6}\int \frac{1}{9-6x+4x^2}\,dx\\
&= \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)\\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}} + \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 - 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*x
 + 4*x^2]/108
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{27-8x^3}{729-64x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(27 - 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] IntegrateAlgebraic[(27 - 8*x^3)/(729 - 64*x^6), x]
```

fricas [A] time = 0.41, size = 38, normalized size = 0.76

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) + 1
/54*log(2*x + 3)
```

giac [A] time = 0.17, size = 35, normalized size = 0.70

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{108}\log\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)+\frac{1}{54}\log\left(\left|x+\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(x^2 - 3/2*x + 9/4) +
 1/54*log(abs(x + 3/2))
```

maple [A] time = 0.05, size = 39, normalized size = 0.78

$$\frac{\sqrt{3}\ \arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{54}+\frac{\ln\left(2x+3\right)}{54}-\frac{\ln\left(4x^{2}-6x+9\right)}{108}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-8*x^3+27)/(-64*x^6+729),x)
```

```
[Out] -1/108*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/54*ln(2*
x+3)
```

maxima [A] time = 2.93, size = 38, normalized size = 0.76

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)-\frac{1}{108}\log\left(4\,x^{2}-6\,x+9\right)+\frac{1}{54}\log\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) + 1
/54*log(2*x + 3)
```

mupad [B] time = 0.09, size = 46, normalized size = 0.92

$$\frac{\ln\left(x+\frac{3}{2}\right)}{54}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3 - 27)/(64*x^6 - 729),x)
```

```
[Out] log(x + 3/2)/54 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 + 1/108)
+ log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 - 1/108)
```

sympy [A] time = 0.16, size = 48, normalized size = 0.96

$$\frac{\log\left(x+\frac{3}{2}\right)}{54}-\frac{\log\left(x^{2}-\frac{3x}{2}+\frac{9}{4}\right)}{108}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x**3+27)/(-64*x**6+729),x)
```

```
[Out] log(x + 3/2)/54 - log(x**2 - 3*x/2 + 9/4)/108 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/54
```

3.405 $\int \frac{27+36x+24x^2+8x^3}{729-64x^6} \, dx$

Optimal. Leaf size=50

$$\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1586, 2058, 634, 618, 204, 628}

$$\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*x
 + 4*x^2]/36
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx &= \int \frac{1}{27-36x+24x^2-8x^3}\,dx \\
&= \int \left(-\frac{1}{9(-3+2x)}+\frac{2x}{9\left(9-6x+4x^2\right)}\right)dx \\
&= -\frac{1}{18}\log(3-2x)+\frac{2}{9}\int \frac{x}{9-6x+4x^2}\,dx \\
&= -\frac{1}{18}\log(3-2x)+\frac{1}{36}\int \frac{-6+8x}{9-6x+4x^2}\,dx+\frac{1}{6}\int \frac{1}{9-6x+4x^2}\,dx \\
&= -\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)-\frac{1}{3}\mathrm{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+\right. \\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)+\frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*x
 + 4*x^2]/36
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] IntegrateAlgebraic[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6), x]
```

fricas [A] time = 0.41, size = 38, normalized size = 0.76

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)+\frac{1}{36}\log\left(4\,x^2-6\,x+9\right)-\frac{1}{18}\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(2*x - 3)
```

giac [A] time = 0.18, size = 39, normalized size = 0.78

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\,(4\,x-3)\right)+\frac{1}{36}\log\left(4\,x^2-6\,x+9\right)-\frac{1}{18}\log\left(|2\,x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(abs(2*x - 3))
```

maple [A] time = 0.05, size = 39, normalized size = 0.78

$$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{54}-\frac{\ln(2x-3)}{18}+\frac{\ln\left(4x^2-6x+9\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x)
```

```
[Out] 1/36*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/18*ln(2*x-
3)
```

maxima [A] time = 2.95, size = 38, normalized size = 0.76

$$\frac{1}{54}\sqrt{3}\,\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(2*x - 3)
```

mupad [B] time = 0.10, size = 46, normalized size = 0.92

$$-\frac{\ln\left(x-\frac{3}{2}\right)}{18}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3i}{4}\right)\left(-\frac{1}{36}+\frac{\sqrt{3}\,1i}{108}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3i}{4}\right)\left(\frac{1}{36}+\frac{\sqrt{3}\,1i}{108}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(36*x + 24*x^2 + 8*x^3 + 27)/(64*x^6 - 729),x)
```

```
[Out] log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 + 1/36) - log(x - (3^(1/2)*
3i)/4 - 3/4)*((3^(1/2)*1i)/108 - 1/36) - log(x - 3/2)/18
```

sympy [A] time = 0.20, size = 48, normalized size = 0.96

$$-\frac{\log\left(x-\frac{3}{2}\right)}{18}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{36}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}\,x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/18 + log(x**2 - 3*x/2 + 9/4)/36 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/54
```

3.406 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{\left(729-64x^6\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\log\left(4x^2-6x+9\right)}{17496}-\frac{\log\left(4x^2+6x+9\right)}{17496}-\frac{1}{2916(2x+3)}-\frac{\log(3-2x)}{17496}+\frac{5\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\tan^{-1}\left(4\ldots\right)}{2916\ldots}$$

Rubi [A] time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1586, 2074, 634, 618, 204, 628}

$$-\frac{\log\left(4x^2-6x+9\right)}{17496}-\frac{\log\left(4x^2+6x+9\right)}{17496}-\frac{1}{2916(2x+3)}-\frac{\log(3-2x)}{17496}+\frac{5\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] -1/(2916*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) + ArcTan
[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/17496 + (5*Log[3 + 2*
x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[9 + 6*x + 4*x^2]/17496
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
```

```
PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{(3+2x)^2\left(243-162x+108x^2-72x^3+48x^4-32x^5\right)}\,dx$$

$$= \int \left(-\frac{1}{8748(-3+2x)} + \frac{1}{1458(3+2x)^2} + \frac{5}{8748(3+2x)} + \frac{}{437}\right.$$

$$= -\frac{1}{2916(3+2x)} - \frac{\log(3-2x)}{17496} + \frac{5\log(3+2x)}{17496} + \frac{\int \frac{3-2x}{9-6x+4x^2}\,d}{4374}$$

$$= -\frac{1}{2916(3+2x)} - \frac{\log(3-2x)}{17496} + \frac{5\log(3+2x)}{17496} - \frac{\int \frac{-6+8x}{9-6x+4x^2}\,d}{17496}$$

$$= -\frac{1}{2916(3+2x)} - \frac{\log(3-2x)}{17496} + \frac{5\log(3+2x)}{17496} - \frac{\log\left(9-6x\right.}{17496}$$

$$= -\frac{1}{2916(3+2x)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{2916\sqrt{3}} - \frac{\log(3-2x)}{17496}$$

Mathematica [A] time = 0.11, size = 100, normalized size = 0.91

$$\frac{-3\log\left(4x^2-6x+9\right)-3\log\left(4x^2+6x+9\right)-\frac{18}{2x+3}-3\log(3-2x)+15\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488}$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)
^2,x]
```

```
[Out] (-18/(3 + 2*x) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[3]*ArcTa
n[(3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + 15*Log[3 + 2*x] - 3*Log[9 - 6*x
 + 4*x^2] - 3*Log[9 + 6*x + 4*x^2])/52488
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{\left(729-64x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729
- 64*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729
- 64*x^6)^2, x]
```

fricas [A] time = 0.41, size = 115, normalized size = 1.05

$$\frac{6\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-3(2x+3)\log\left(4x^2+6x+9\right)-3(2x+3)\log\left(4x^2-6x+9\right)+15(2x+3)\log(2x+3)-3(2x+3)\log(2x-3)-18}{52488(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="fricas")
```

```
[Out] 1/52488*(6*sqrt(3)*(2*x + 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(2*x
 + 3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 3*(2*x + 3)*log(4*x^2 + 6*x + 9) - 3*
(2*x + 3)*log(4*x^2 - 6*x + 9) + 15*(2*x + 3)*log(2*x + 3) - 3*(2*x + 3)*lo
g(2*x - 3) - 18)/(2*x + 3)
```

giac [A] time = 0.19, size = 86, normalized size = 0.78

$$\frac{1}{8748}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x+3)\right)+\frac{1}{26244}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x-3)\right)-\frac{1}{2916\,(2\,x+3)}-\frac{1}{17496}\log\left(4\,x^{2}+6\,x+9\right)-\frac{1}{17496}\log\left(4\,x^{2}-6\,x+9\right)+\frac{5}{17496}\log\left(|2\,x+3|\right)-\frac{1}{17496}\log\left(|2\,x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="giac")
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/174
96*log(4*x^2 - 6*x + 9) + 5/17496*log(abs(2*x + 3)) - 1/17496*log(abs(2*x -
 3))
```

maple [A] time = 0.06, size = 85, normalized size = 0.77

$$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{26244}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{8748}-\frac{\ln(2x-3)}{17496}+\frac{5\ln(2x+3)}{17496}-\frac{\ln\left(4x^{2}-6x+9\right)}{17496}-\frac{\ln\left(4x^{2}+6x+9\right)}{17496}-\frac{1}{2916\,(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x)
```

```
[Out] -1/17496*ln(4*x^2-6*x+9)+1/26244*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/291
6/(2*x+3)+5/17496*ln(2*x+3)-1/17496*ln(4*x^2+6*x+9)+1/8748*3^(1/2)*arctan(1
/18*(8*x+6)*3^(1/2))-1/17496*ln(2*x-3)
```

maxima [A] time = 2.98, size = 84, normalized size = 0.76

$$\frac{1}{8748}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x+3)\right)+\frac{1}{26244}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x-3)\right)-\frac{1}{2916\,(2\,x+3)}-\frac{1}{17496}\log\left(4\,x^{2}+6\,x+9\right)-\frac{1}{17496}\log\left(4\,x^{2}-6\,x+9\right)+\frac{5}{17496}\log\left(2\,x+3\right)-\frac{1}{17496}\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="maxima")
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/174
96*log(4*x^2 - 6*x + 9) + 5/17496*log(2*x + 3) - 1/17496*log(2*x - 3)
```

mupad [B] time = 5.10, size = 100, normalized size = 0.91

$$\frac{5\ln\left(x+\frac{3}{2}\right)}{17496}-\frac{\ln\left(x-\frac{3}{2}\right)}{17496}-\frac{1}{5832\left(x+\frac{3}{2}\right)}-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(162*x - 108*x^2 + 72*x^3 - 48*x^4 + 32*x^5 - 243)/(64*x^6 - 729)^2,x)
```

```
[Out] (5*log(x + 3/2))/17496 - log(x - 3/2)/17496 - 1/(5832*(x + 3/2)) - log(x -
(3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/17496 + 1/17496) + log(x + (3^(1/2)*3i)
/4 + 3/4)*((3^(1/2)*1i)/17496 - 1/17496) - log(x - (3^(1/2)*3i)/4 - 3/4)*((
3^(1/2)*1i)/52488 + 1/17496) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/
52488 - 1/17496)
```

sympy [A] time = 0.43, size = 105, normalized size = 0.95

$$-\frac{\log\left(x-\frac{3}{2}\right)}{17496}+\frac{5\log\left(x+\frac{3}{2}\right)}{17496}-\frac{\log\left(x^{2}-\frac{3x}{2}+\frac{9}{4}\right)}{17496}-\frac{\log\left(x^{2}+\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{26244}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{8748}-\frac{1}{5832x+8748}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729)**2,x
)
```

```
[Out] -log(x - 3/2)/17496 + 5*log(x + 3/2)/17496 - log(x**2 - 3*x/2 + 9/4)/17496
- log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/2
6244 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/8748 - 1/(5832*x + 8748)
```

3.407 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} dx$

Optimal. Leaf size=110

$$\frac{\log(4x^2-6x+9)}{17496}+\frac{\log(4x^2+6x+9)}{17496}+\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}$$

Rubi [A] time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1586, 2074, 634, 618, 204, 628}

$$\frac{\log(4x^2-6x+9)}{17496}+\frac{\log(4x^2+6x+9)}{17496}+\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) + ArcTan[
(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x])/17496 + Log[3 + 2*
x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9 + 6*x + 4*x^2]/17496
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
```

```
PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{(3-2x)^2\left(243+162x+108x^2+72x^3+48x^4+32x^5\right)}\,dx$$

$$= \int \left(\frac{1}{1458(-3+2x)^2} - \frac{5}{8748(-3+2x)} + \frac{1}{8748(3+2x)} + \frac{}{437}\right.$$

$$= \frac{1}{2916(3-2x)} - \frac{5\log(3-2x)}{17496} + \frac{\log(3+2x)}{17496} + \frac{\int \frac{3+2x}{9-6x+4x^2}\,dx}{4374}$$

$$= \frac{1}{2916(3-2x)} - \frac{5\log(3-2x)}{17496} + \frac{\log(3+2x)}{17496} + \frac{\int \frac{-6+8x}{9-6x+4x^2}\,dx}{17496}$$

$$= \frac{1}{2916(3-2x)} - \frac{5\log(3-2x)}{17496} + \frac{\log(3+2x)}{17496} + \frac{\log\left(9-6x+\right.}{17496}$$

$$= \frac{1}{2916(3-2x)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{8748\sqrt{3}} - \frac{5\log(3-2x)}{17496}$$

Mathematica [A] time = 0.09, size = 97, normalized size = 0.88

$$\frac{3\left(\log\left(4x^2-6x+9\right)+\log\left(4x^2+6x+9\right)+\frac{6}{3-2x}-5\log(3-2x)+\log(2x+3)\right)+6\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488}$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)
^2,x]
```

```
[Out] (6*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*S
qrt[3])] + 3*(6/(3 - 2*x) - 5*Log[3 - 2*x] + Log[3 + 2*x] + Log[9 - 6*x + 4
*x^2] + Log[9 + 6*x + 4*x^2]))/52488
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{\left(729-64x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729
- 64*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729
- 64*x^6)^2, x]
```

fricas [A] time = 0.41, size = 115, normalized size = 1.05

$$\frac{2\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+6\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3(2x-3)\log\left(4x^2+6x+9\right)+3(2x-3)\log\left(4x^2-6x+9\right)+3(2x-3)\log(2x+3)-15(2x-3)\log(2x-3)-18}{52488(2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="fricas")
```

```
[Out] 1/52488*(2*sqrt(3)*(2*x - 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 6*sqrt(3)*(2*x
 - 3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(2*x - 3)*log(4*x^2 + 6*x + 9) + 3*
(2*x - 3)*log(4*x^2 - 6*x + 9) + 3*(2*x - 3)*log(2*x + 3) - 15*(2*x - 3)*lo
g(2*x - 3) - 18)/(2*x - 3)
```

giac [A] time = 0.21, size = 86, normalized size = 0.78

$$\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x-3)}+\frac{1}{17496}\log\left(4x^2+6x+9\right)+\frac{1}{17496}\log\left(4x^2-6x+9\right)+\frac{1}{17496}\log\left(|2x+3|\right)-\frac{5}{17496}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="giac")
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/174
96*log(4*x^2 - 6*x + 9) + 1/17496*log(abs(2*x + 3)) - 5/17496*log(abs(2*x -
 3))
```

maple [A] time = 0.05, size = 85, normalized size = 0.77

$$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{8748}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{26244}-\frac{5\ln\left(2x-3\right)}{17496}+\frac{\ln\left(2x+3\right)}{17496}+\frac{\ln\left(4x^2-6x+9\right)}{17496}+\frac{\ln\left(4x^2+6x+9\right)}{17496}-\frac{1}{2916\left(2x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x)
```

```
[Out] 1/17496*ln(4*x^2-6*x+9)+1/8748*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/17496
*ln(2*x+3)+1/17496*ln(4*x^2+6*x+9)+1/26244*3^(1/2)*arctan(1/18*(8*x+6)*3^(1
/2))-1/2916/(2*x-3)-5/17496*ln(2*x-3)
```

maxima [A] time = 2.94, size = 84, normalized size = 0.76

$$\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x-3)}+\frac{1}{17496}\log\left(4x^2+6x+9\right)+\frac{1}{17496}\log\left(4x^2-6x+9\right)+\frac{1}{17496}\log\left(2x+3\right)-\frac{5}{17496}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="maxima")
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/174
96*log(4*x^2 - 6*x + 9) + 1/17496*log(2*x + 3) - 5/17496*log(2*x - 3)
```

mupad [B] time = 0.19, size = 100, normalized size = 0.91

$$\frac{\ln\left(x+\frac{3}{2}\right)}{17496}-\frac{5\ln\left(x-\frac{3}{2}\right)}{17496}-\frac{1}{5832\left(x-\frac{3}{2}\right)}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3i}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1i}{17496}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3i}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1i}{17496}\right)-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3i}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1i}{52488}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3i}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1i}{52488}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5 + 243)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/17496 - (5*log(x - 3/2))/17496 - 1/(5832*(x - 3/2)) - log(x -
(3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/17496 - 1/17496) + log(x + (3^(1/2)*3i)
/4 - 3/4)*((3^(1/2)*1i)/17496 + 1/17496) - log(x - (3^(1/2)*3i)/4 + 3/4)*((
3^(1/2)*1i)/52488 - 1/17496) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/
52488 + 1/17496)
```

sympy [A] time = 0.47, size = 105, normalized size = 0.95

$$-\frac{5\log\left(x-\frac{3}{2}\right)}{17496}+\frac{\log\left(x+\frac{3}{2}\right)}{17496}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{8748}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{26244}-\frac{1}{5832x-8748}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729)**2,x)
```

```
[Out] -5*log(x - 3/2)/17496 + log(x + 3/2)/17496 + log(x**2 - 3*x/2 + 9/4)/17496
+ log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/8
748 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/26244 - 1/(5832*x - 8748)
```

3.408 $\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=81

$$\frac{1}{17496(3-2x)}-\frac{1}{17496(2x+3)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1586, 1170, 207, 618, 204}

$$\frac{1}{17496(3-2x)}-\frac{1}{17496(2x+3)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(
13122*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(13122*Sqrt[3]) + ArcTanh[(2
*x)/3]/8748
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(9-4x^2\right)^2\left(81+36x^2+16x^4\right)}\,dx$$

$$= \int \left(\frac{1}{8748(-3+2x)^2}+\frac{1}{8748(3+2x)^2}-\frac{1}{1458\left(-9+4x^2\right)}+\frac{1}{4374\left(9-6x+4x^2\right)}+\frac{1}{43\ldots}\right.$$

$$= \frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}+\frac{\int \frac{1}{9-6x+4x^2}\,dx}{4374}+\frac{\int \frac{1}{9+6x+4x^2}\,dx}{4374}-\frac{\int \frac{1}{-9+4x^2}\,dx}{1458}$$

$$= \frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}-\frac{\mathrm{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)}{2187}-\ldots$$

$$= \frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

Mathematica [C] time = 0.57, size = 122, normalized size = 1.51

$$\frac{\frac{36x}{9-4x^2}-9\log(3-2x)+9\log(2x+3)+3\sqrt{3}\tan^{-1}\left(\frac{1}{3}\left(\sqrt{3}-i\right)x\right)+4i\sqrt{3}\tanh^{-1}\left(\frac{1}{3}\left(1-i\sqrt{3}\right)x\right)+\left(-3+\frac{2}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}\right)\tanh^{-1}\left(\frac{1}{3}\left(x+i\sqrt{3}x\right)\right)}{157464}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] ((36*x)/(9 - 4*x^2) + 3*Sqrt[3]*ArcTan[((-I + Sqrt[3])*x)/3] + (4*I)*Sqrt[3
]*ArcTanh[((1 - I*Sqrt[3])*x)/3] + (-3 + 2/Sqrt[(1 + I*Sqrt[3])/6])*ArcTanh
[(x + I*Sqrt[3]*x)/3] - 9*Log[3 - 2*x] + 9*Log[3 + 2*x])/157464
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2, x]
```

fricas [A] time = 0.40, size = 91, normalized size = 1.12

$$\frac{4\sqrt{3}\left(4x^2-9\right)\arctan\left(\frac{4}{81}\sqrt{3}\left(2x^3+9x\right)\right)+4\sqrt{3}\left(4x^2-9\right)\arctan\left(\frac{2}{9}\sqrt{3}x\right)+9\left(4x^2-9\right)\log\left(2x+3\right)-9\left(4x^2-9\right)\log\left(2x-3\right)-36x}{157464\left(4x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/157464*(4*sqrt(3)*(4*x^2 - 9)*arctan(4/81*sqrt(3)*(2*x^3 + 9*x)) + 4*sqrt
(3)*(4*x^2 - 9)*arctan(2/9*sqrt(3)*x) + 9*(4*x^2 - 9)*log(2*x + 3) - 9*(4*x
^2 - 9)*log(2*x - 3) - 36*x)/(4*x^2 - 9)
```

giac [A] time = 0.17, size = 63, normalized size = 0.78

$$\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(4x+3\right)\right)+\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(4x-3\right)\right)-\frac{x}{4374\left(4x^2-9\right)}+\frac{1}{17496}\log\left(|2x+3|\right)-\frac{1}{17496}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(abs(2*x + 3)) - 1/1
7496*log(abs(2*x - 3))
```

maple [A] time = 0.06, size = 68, normalized size = 0.84

$$\frac{\sqrt{3}\ \arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{39366}+\frac{\sqrt{3}\ \arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{39366}-\frac{\ln\left(2x-3\right)}{17496}+\frac{\ln\left(2x+3\right)}{17496}-\frac{1}{17496\left(2x+3\right)}-\frac{1}{17496\left(2x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x)
```

```
[Out] 1/39366*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/17496/(2*x+3)+1/17496*ln(2*x
+3)+1/39366*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/17496/(2*x-3)-1/17496*ln
(2*x-3)
```

maxima [A] time = 3.06, size = 61, normalized size = 0.75

$$\frac{1}{39366}\,\sqrt{3}\,\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x+3)\right)+\frac{1}{39366}\,\sqrt{3}\,\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x-3)\right)-\frac{x}{4374\,\left(4\,x^{2}-9\right)}+\frac{1}{17496}\,\log\left(2\,x+3\right)-\frac{1}{17496}\,\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(2*x + 3) - 1/17496*
log(2*x - 3)
```

mupad [B] time = 4.92, size = 52, normalized size = 0.64

$$\frac{\mathrm{atanh}\left(\frac{2\,x}{3}\right)}{8748}+\frac{\sqrt{3}\,\left(2\,\mathrm{atan}\left(\frac{8\,\sqrt{3}\,x^3}{81}+\frac{4\,\sqrt{3}\,x}{9}\right)+2\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{9}\right)\right)}{78732}-\frac{x}{17496\,\left(x^2-\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((36*x^2 + 16*x^4 + 81)/(64*x^6 - 729)^2,x)
```

```
[Out] atanh((2*x)/3)/8748 + (3^(1/2)*(2*atan((4*3^(1/2)*x)/9 + (8*3^(1/2)*x^3)/81
) + 2*atan((2*3^(1/2)*x)/9)))/78732 - x/(17496*(x^2 - 9/4))
```

sympy [A] time = 0.23, size = 70, normalized size = 0.86

$$-\frac{x}{17496x^{2}-39366}+\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{9}\right)+2\,\mathrm{atan}\left(\frac{8\sqrt{3}x^{3}}{81}+\frac{4\sqrt{3}x}{9}\right)\right)}{78732}-\frac{\log\left(x-\frac{3}{2}\right)}{17496}+\frac{\log\left(x+\frac{3}{2}\right)}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x**4+36*x**2+81)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(17496*x**2 - 39366) + sqrt(3)*(2*atan(2*sqrt(3)*x/9) + 2*atan(8*sqrt(3)
*x**3/81 + 4*sqrt(3)*x/9))/78732 - log(x - 3/2)/17496 + log(x + 3/2)/17496
```

3.409 $\int \frac{81+54x-24x^3-16x^4}{\left(729-64x^6\right)^2} dx$

Optimal. Leaf size=92

$$\frac{x}{4374\left(4x^2-6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{157464}+\frac{\log\left(4x^2+6x+9\right)}{52488}-\frac{\log(3-2x)}{26244}+\frac{\log(2x+3)}{78732}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}$$

Rubi [A] time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$\frac{x}{4374\left(4x^2-6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{157464}+\frac{\log\left(4x^2+6x+9\right)}{52488}-\frac{\log(3-2x)}{26244}+\frac{\log(2x+3)}{78732}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*Sqrt[3]) -
 Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x + 4*x^2]/157464 + Lo
g[9 + 6*x + 4*x^2]/52488
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{81+54x-24x^3-16x^4}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(9-6x+4x^2\right)^2\left(81+54x-24x^3-16x^4\right)}\,dx$$

$$= \int \left(-\frac{1}{13122(-3+2x)}+\frac{1}{39366(3+2x)}+\frac{3-x}{729\left(9-6x+4x^2\right)^2}+\frac{39-}{78732\,(9-}\right.$$

$$= -\frac{\log(3-2x)}{26244}+\frac{\log(3+2x)}{78732}+\frac{\int\frac{39-4x}{9-6x+4x^2}\,dx}{78732}+\frac{\int\frac{3+4x}{9+6x+4x^2}\,dx}{26244}+\frac{1}{729}\int\frac{}{(9-}$$

$$= \frac{x}{4374\left(9-6x+4x^2\right)}-\frac{\log(3-2x)}{26244}+\frac{\log(3+2x)}{78732}+\frac{\log\left(9+6x+4x^2\right)}{52488}-\int$$

$$= \frac{x}{4374\left(9-6x+4x^2\right)}-\frac{\log(3-2x)}{26244}+\frac{\log(3+2x)}{78732}-\frac{\log\left(9-6x+4x^2\right)}{157464}+\text{lo}$$

$$= \frac{x}{4374\left(9-6x+4x^2\right)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}-\frac{\log(3-2x)}{26244}+\frac{\log(3+2x)}{78732}-\frac{\log(9-}{15}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 0.91

$$\frac{\frac{36x}{4x^2-6x+9}-\log\left(4x^2-6x+9\right)+3\log\left(4x^2+6x+9\right)-6\log(3-2x)+2\log(2x+3)+12\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{157464}$$

Antiderivative was successfully verified.

```
[In] Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] ((36*x)/(9 - 6*x + 4*x^2) + 12*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*L
og[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2
])/157464
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{81+54x-24x^3-16x^4}{\left(729-64x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2, x]
```

fricas [A] time = 0.40, size = 126, normalized size = 1.37

$$\frac{12\sqrt{3}\left(4x^2-6x+9\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3\left(4x^2-6x+9\right)\log\left(4x^2+6x+9\right)-\left(4x^2-6x+9\right)\log\left(4x^2-6x+9\right)+2\left(4x^2-6x+9\right)\log(2x+3)-6\left(4x^2-6x+9\right)\log(2x-3)+36x}{157464\left(4x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/157464*(12*sqrt(3)*(4*x^2 - 6*x + 9)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(4
*x^2 - 6*x + 9)*log(4*x^2 + 6*x + 9) - (4*x^2 - 6*x + 9)*log(4*x^2 - 6*x +
9) + 2*(4*x^2 - 6*x + 9)*log(2*x + 3) - 6*(4*x^2 - 6*x + 9)*log(2*x - 3) +
36*x)/(4*x^2 - 6*x + 9)
```

giac [A] time = 0.19, size = 76, normalized size = 0.83

$$\frac{1}{13122}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\left(4x^2-6x+9\right)}+\frac{1}{52488}\log\left(4x^2+6x+9\right)-\frac{1}{157464}\log\left(4x^2-6x+9\right)+\frac{1}{78732}\log(|2x+3|)-\frac{1}{26244}\log(|2x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9)
+ 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*lo
g(abs(2*x + 3)) - 1/26244*log(abs(2*x - 3))
```

maple [A] time = 0.06, size = 73, normalized size = 0.79

$$\frac{x}{17496x^2-26244x+39366}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{13122}-\frac{\ln(2x-3)}{26244}+\frac{\ln(2x+3)}{78732}-\frac{\ln\left(4x^2-6x+9\right)}{157464}+\frac{\ln\left(4x^2+6x+9\right)}{52488}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x)
```

```
[Out] 1/17496*x/(x^2-3/2*x+9/4)-1/157464*ln(4*x^2-6*x+9)+1/13122*3^(1/2)*arctan(1
/18*(8*x-6)*3^(1/2))+1/78732*ln(2*x+3)+1/52488*ln(4*x^2+6*x+9)-1/26244*ln(2
*x-3)
```

maxima [A] time = 2.97, size = 74, normalized size = 0.80

$$\frac{1}{13122}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\left(4x^2-6x+9\right)}+\frac{1}{52488}\log\left(4x^2+6x+9\right)-\frac{1}{157464}\log\left(4x^2-6x+9\right)+\frac{1}{78732}\log(2x+3)-\frac{1}{26244}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9)
+ 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*lo
g(2*x + 3) - 1/26244*log(2*x - 3)
```

mupad [B] time = 0.12, size = 77, normalized size = 0.84

$$\frac{\ln\left(x+\frac{3}{2}\right)}{78732}-\frac{\ln\left(x-\frac{3}{2}\right)}{26244}+\frac{\ln\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{52488}+\frac{x}{17496\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3i}{4}\right)\left(\frac{1}{157464}+\frac{\sqrt{3}\,1i}{26244}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3i}{4}\right)\left(-\frac{1}{157464}+\frac{\sqrt{3}\,1i}{26244}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((54*x - 24*x^3 - 16*x^4 + 81)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/78732 - log(x - 3/2)/26244 + log((3*x)/2 + x^2 + 9/4)/52488 +
x/(17496*(x^2 - (3*x)/2 + 9/4)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1
```

```
i)/26244 + 1/157464) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/26244 -
1/157464)
```

sympy [A] time = 0.42, size = 82, normalized size = 0.89

$$\frac{x}{17496x^2 - 26244x + 39366} - \frac{\log\left(x - \frac{3}{2}\right)}{26244} + \frac{\log\left(x + \frac{3}{2}\right)}{78732} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{157464} + \frac{\log\left(4x^2 + 6x + 9\right)}{52488} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{13122}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729)**2,x)
```

```
[Out] x/(17496*x**2 - 26244*x + 39366) - log(x - 3/2)/26244 + log(x + 3/2)/78732
- log(x**2 - 3*x/2 + 9/4)/157464 + log(4*x**2 + 6*x + 9)/52488 + sqrt(3)*at
an(4*sqrt(3)*x/9 - sqrt(3)/3)/13122
```

3.410 $\int \frac{3-2x}{\left(729-64x^6\right)^2}\,dx$

Optimal. Leaf size=148

$$\frac{3-x}{708588\left(4x^2-6x+9\right)}+\frac{x}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{8503056}-\frac{1}{708588(2x+3)}-\frac{\log(}{425}$$

Rubi [A] time = 0.17, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$\frac{3-x}{708588\left(4x^2-6x+9\right)}+\frac{x}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{8503056}-\frac{1}{708588(2x+3)}-\frac{\log(3-2x)}{4251528}+\frac{\log(2x+3)}{472392}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] -1/(708588*(3 + 2*x)) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(236196*(9 +
 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) + ArcTan[(
3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[3 - 2*x]/4251528 + Log[3 + 2*x
]/472392 - Log[9 - 6*x + 4*x^2]/944784 + Log[9 + 6*x + 4*x^2]/8503056
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{3-2x}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{(3-2x)\left(243+162x+108x^2+72x^3+48x^4+32x^5\right)^2}\,dx$$

$$= \int \left(-\frac{1}{2125764(-3+2x)} + \frac{1}{354294(3+2x)^2} + \frac{1}{236196(3+2x)} - \frac{x}{39366\left(9-6x+4x^2\right.}\right.$$

$$= -\frac{1}{708588(3+2x)} - \frac{\log(3-2x)}{4251528} + \frac{\log(3+2x)}{472392} + \frac{\int \frac{33+2x}{9+6x+4x^2}\,dx}{2125764} + \frac{\int \frac{7-6x}{9-6x+4x^2}\,dx}{708588} - \frac{\int}{(}$$

$$= -\frac{1}{708588(3+2x)} + \frac{3-x}{708588\left(9-6x+4x^2\right)} + \frac{x}{236196\left(9+6x+4x^2\right)} - \frac{\log(3-2x)}{4251528} +$$

$$= -\frac{1}{708588(3+2x)} + \frac{3-x}{708588\left(9-6x+4x^2\right)} + \frac{x}{236196\left(9+6x+4x^2\right)} - \frac{\log(3-2x)}{4251528} +$$

$$= -\frac{1}{708588(3+2x)} + \frac{3-x}{708588\left(9-6x+4x^2\right)} + \frac{x}{236196\left(9+6x+4x^2\right)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}} +$$

Mathematica [A] time = 0.08, size = 119, normalized size = 0.80

$$\frac{-9\log\left(4x^2-6x+9\right)+\log\left(4x^2+6x+9\right)+\frac{1944x}{32x^5+48x^4+72x^3+108x^2+162x+243}-2\log(3-2x)+18\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+18\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8503056}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] ((1944*x)/(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5) + 2*Sqrt[3]*Ar
cTan[(-3 + 4*x)/(3*Sqrt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 2
*Log[3 - 2*x] + 18*Log[3 + 2*x] - 9*Log[9 - 6*x + 4*x^2] + Log[9 + 6*x + 4*
x^2])/8503056
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-2x}{\left(729-64x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 - 2*x)/(729 - 64*x^6)^2, x]
```

fricas [B] time = 0.42, size = 256, normalized size = 1.73

$$\frac{18\,\sqrt{3}\left(32\,x^{5}+48\,x^{4}+72\,x^{3}+108\,x^{2}+162\,x+243\right)\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x+3)\right)+2\,\sqrt{3}\left(32\,x^{5}+48\,x^{4}+72\,x^{3}+108\,x^{2}+162\,x+243\right)\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x-3)\right)+\left(32\,x^{5}+48\,x^{4}+72\,x^{3}+108\,x^{2}+162\,x+243\right)\log\left(4\,x^{2}+6\,x+9\right)-9\left(32\,x^{5}+48\,x^{4}+72\,x^{3}+108\,x^{2}+162\,x+243\right)\log\left(4\,x^{2}-6\,x+9\right)+18\left(32\,x^{5}+48\,x^{4}+72\,x^{3}+108\,x^{2}+162\,x+243\right)\log\left(2\,x+3\right)-2\left(32\,x^{5}+48\,x^{4}+72\,x^{3}+108\,x^{2}+162\,x+243\right)\log\left(2\,x-3\right)+1944\,x}{8503056\left(32\,x^{5}+48\,x^{4}+72\,x^{3}+108\,x^{2}+162\,x+243\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/8503056*(18*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*ar
ctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2
 + 162*x + 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + (32*x^5 + 48*x^4 + 72*x^3 +
 108*x^2 + 162*x + 243)*log(4*x^2 + 6*x + 9) - 9*(32*x^5 + 48*x^4 + 72*x^3
+ 108*x^2 + 162*x + 243)*log(4*x^2 - 6*x + 9) + 18*(32*x^5 + 48*x^4 + 72*x^
3 + 108*x^2 + 162*x + 243)*log(2*x + 3) - 2*(32*x^5 + 48*x^4 + 72*x^3 + 108
*x^2 + 162*x + 243)*log(2*x - 3) + 1944*x)/(32*x^5 + 48*x^4 + 72*x^3 + 108*
x^2 + 162*x + 243)
```

giac [A] time = 0.20, size = 111, normalized size = 0.75

$$\frac{1}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x+3)\right)+\frac{1}{4251528}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x-3)\right)+\frac{x}{4374\left(4\,x^{2}+6\,x+9\right)\left(4\,x^{2}-6\,x+9\right)(2\,x+3)}+\frac{1}{8503056}\log\left(4\,x^{2}+6\,x+9\right)-\frac{1}{944784}\log\left(4\,x^{2}-6\,x+9\right)+\frac{1}{472392}\log\left(|2\,x+3|\right)-\frac{1}{4251528}\log\left(|2\,x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*x
+ 3)) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/
472392*log(abs(2*x + 3)) - 1/4251528*log(abs(2*x - 3))
```

maple [A] time = 0.07, size = 115, normalized size = 0.78

$$\frac{x}{944784x^{2}+1417176x+2125764}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{4251528}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{472392}-\frac{\ln\left(2x-3\right)}{4251528}+\frac{\ln\left(2x+3\right)}{472392}-\frac{\ln\left(4x^{2}-6x+9\right)}{944784}+\frac{\ln\left(4x^{2}+6x+9\right)}{8503056}-\frac{\frac{x}{4}-\frac{3}{4}}{708588\left(x^{2}-\frac{3}{2}x+\frac{9}{4}\right)}-\frac{1}{708588\left(2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)/(-64*x^6+729)^2,x)
```

```
[Out] -1/708588*(1/4*x-3/4)/(x^2-3/2*x+9/4)-1/944784*ln(4*x^2-6*x+9)+1/4251528*3^
(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/708588/(2*x+3)+1/472392*ln(2*x+3)+1/94
4784*x/(x^2+3/2*x+9/4)+1/8503056*ln(4*x^2+6*x+9)+1/472392*3^(1/2)*arctan(1/
18*(8*x+6)*3^(1/2))-1/4251528*ln(2*x-3)
```

maxima [A] time = 2.91, size = 105, normalized size = 0.71

$$\frac{1}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x+3)\right)+\frac{1}{4251528}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}\,(4\,x-3)\right)+\frac{x}{4374\left(32\,x^{5}+48\,x^{4}+72\,x^{3}+108\,x^{2}+162\,x+243\right)}+\frac{1}{8503056}\log\left(4\,x^{2}+6\,x+9\right)-\frac{1}{944784}\log\left(4\,x^{2}-6\,x+9\right)+\frac{1}{472392}\log\left(2\,x+3\right)-\frac{1}{4251528}\log\left(2\,x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*
x + 243) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) +
 1/472392*log(2*x + 3) - 1/4251528*log(2*x - 3)
```

mupad [B] time = 0.19, size = 120, normalized size = 0.81

$$\frac{\ln\left(x+\frac{3}{2}\right)}{472392}-\frac{\ln\left(x-\frac{3}{2}\right)}{4251528}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8503056}\right)-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\left(-\frac{1}{8503056}+\frac{\sqrt{3}\,1{}\mathrm{i}}{944784}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8503056}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)\left(\frac{1}{8503056}+\frac{\sqrt{3}\,1{}\mathrm{i}}{944784}\right)+\frac{x}{139968\left(x^{5}+\frac{3\,x^{4}}{2}+\frac{9\,x^{3}}{4}+\frac{27\,x^{2}}{8}+\frac{81\,x}{16}+\frac{243}{32}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 3)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/472392 - log(x - 3/2)/4251528 - log(x - (3^(1/2)*3i)/4 - 3/4)*
((3^(1/2)*1i)/8503056 + 1/944784) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)
*1i)/944784 - 1/8503056) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/8503
056 - 1/944784) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 + 1/85
03056) + x/(139968*((81*x)/16 + (27*x^2)/8 + (9*x^3)/4 + (3*x^4)/2 + x^5 +
243/32))
```

sympy [A] time = 0.65, size = 124, normalized size = 0.84

$$\frac{x}{139968x^5 + 209952x^4 + 314928x^3 + 472392x^2 + 708588x + 1062882} - \frac{\log\left(x - \frac{3}{2}\right)}{4251528} + \frac{\log\left(x + \frac{3}{2}\right)}{472392} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{944784} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)}{8503056} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{4251528} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9} + \frac{\sqrt{3}}{3}\right)}{472392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x**6+729)**2,x)
```

```
[Out] x/(139968*x**5 + 209952*x**4 + 314928*x**3 + 472392*x**2 + 708588*x + 10628
82) - log(x - 3/2)/4251528 + log(x + 3/2)/472392 - log(x**2 - 3*x/2 + 9/4)/
944784 + log(x**2 + 3*x/2 + 9/4)/8503056 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqr
t(3)/3)/4251528 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

3.411 $\int \frac{3+2x}{\left(729-64x^6\right)^2} dx$

Optimal. Leaf size=146

$$\frac{x}{236196\left(4x^2-6x+9\right)}-\frac{x+3}{708588\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{8503056}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{708588(3-2x)}-\frac{\log(\ldots}{47\ldots}$$

Rubi [A] time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$\frac{x}{236196\left(4x^2-6x+9\right)}-\frac{x+3}{708588\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{8503056}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{708588(3-2x)}-\frac{\log(3-2x)}{472392}+\frac{\log(2x+3)}{4251528}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{1417176\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(708588*(9 +
6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) + ArcTan[(3
+ 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3 - 2*x]/472392 + Log[3 + 2*x]/
4251528 - Log[9 - 6*x + 4*x^2]/8503056 + Log[9 + 6*x + 4*x^2]/944784
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+2x}{\left(729-64x^6\right)^2}\,dx &= \int \frac{1}{(3+2x)\left(243-162x+108x^2-72x^3+48x^4-32x^5\right)^2}\,dx \\
&= \int \left(\frac{1}{354294(-3+2x)^2}-\frac{1}{236196(-3+2x)}+\frac{1}{2125764(3+2x)}+\frac{3-x}{39366\left(9-6x+4x^2\right.}\cdots\right. \\
&= \frac{1}{708588(3-2x)}-\frac{\log(3-2x)}{472392}+\frac{\log(3+2x)}{4251528}+\frac{\int\frac{33-2x}{9-6x+4x^2}\,dx}{2125764}+\frac{\int\frac{7+6x}{9+6x+4x^2}\,dx}{708588}+\frac{\int\frac{}{(9-}\cdots}{}\cdots \\
&= \frac{1}{708588(3-2x)}+\frac{x}{236196\left(9-6x+4x^2\right)}-\frac{3+x}{708588\left(9+6x+4x^2\right)}-\frac{\log(3-2x)}{472392}+\frac{\text{lo}}{}\cdots \\
&= \frac{1}{708588(3-2x)}+\frac{x}{236196\left(9-6x+4x^2\right)}-\frac{3+x}{708588\left(9+6x+4x^2\right)}-\frac{\log(3-2x)}{472392}+\frac{\text{lo}}{}\cdots \\
&= \frac{1}{708588(3-2x)}+\frac{x}{236196\left(9-6x+4x^2\right)}-\frac{3+x}{708588\left(9+6x+4x^2\right)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\cdots
\end{aligned}$$

Mathematica [A] time = 0.09, size = 121, normalized size = 0.83

$$\frac{-\log\left(4x^2-6x+9\right)+9\log\left(4x^2+6x+9\right)+\frac{1944x}{-32x^5+48x^4-72x^3+108x^2-162x+243}-18\log(3-2x)+2\log(2x+3)+18\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8503056}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] ((1944*x)/(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5) + 18*Sqrt[3]*A
rcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 1
8*Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 9*Log[9 + 6*x + 4*
x^2])/8503056
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+2x}{\left(729-64x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 + 2*x)/(729 - 64*x^6)^2, x]
```

fricas [B] time = 0.41, size = 257, normalized size = 1.76

$$\frac{2\sqrt{3}\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+18\sqrt{3}\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+9\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)\log\left(4x^2+6x+9\right)-\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)\log\left(4x^2-6x+9\right)+2\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)\log\left(2x+3\right)-18\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)\log\left(2x-3\right)-1944x}{8503056\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/8503056*(2*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*arc
tan(1/9*sqrt(3)*(4*x + 3)) + 18*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2
 + 162*x - 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + 9*(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)*log(4*x^2 + 6*x + 9) - (32*x^5 - 48*x^4 + 72*x^3
- 108*x^2 + 162*x - 243)*log(4*x^2 - 6*x + 9) + 2*(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)*log(2*x + 3) - 18*(32*x^5 - 48*x^4 + 72*x^3 - 108
*x^2 + 162*x - 243)*log(2*x - 3) - 1944*x)/(32*x^5 - 48*x^4 + 72*x^3 - 108*
x^2 + 162*x - 243)
```

giac [A] time = 0.18, size = 111, normalized size = 0.76

$$\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(4x^2+6x+9\right)\left(4x^2-6x+9\right)(2x-3)}+\frac{1}{944784}\log\left(4x^2+6x+9\right)-\frac{1}{8503056}\log\left(4x^2-6x+9\right)+\frac{1}{4251528}\log\left(|2x+3|\right)-\frac{1}{472392}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*x
- 3)) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) + 1/
4251528*log(abs(2*x + 3)) - 1/472392*log(abs(2*x - 3))
```

maple [A] time = 0.07, size = 115, normalized size = 0.79

$$\frac{x}{944784x^2-1417176x+2125764}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}+\frac{\sqrt{3}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{4251528}-\frac{\ln(2x-3)}{472392}+\frac{\ln(2x+3)}{4251528}-\frac{\ln\left(4x^2-6x+9\right)}{8503056}+\frac{\ln\left(4x^2+6x+9\right)}{944784}+\frac{-\frac{x}{4}-\frac{3}{4}}{708588x^2+1062882x+1594323}-\frac{1}{708588(2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+3)/(-64*x^6+729)^2,x)
```

```
[Out] 1/944784/(x^2-3/2*x+9/4)*x-1/8503056*ln(4*x^2-6*x+9)+1/472392*3^(1/2)*arcta
n(1/18*(8*x-6)*3^(1/2))+1/4251528*ln(2*x+3)+1/708588*(-1/4*x-3/4)/(x^2+3/2*
x+9/4)+1/944784*ln(4*x^2+6*x+9)+1/4251528*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/
2))-1/708588/(2*x-3)-1/472392*ln(2*x-3)
```

maxima [A] time = 2.90, size = 105, normalized size = 0.72

$$\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)}+\frac{1}{944784}\log\left(4x^2+6x+9\right)-\frac{1}{8503056}\log\left(4x^2-6x+9\right)+\frac{1}{4251528}\log\left(2x+3\right)-\frac{1}{472392}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*
x - 243) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) +
 1/4251528*log(2*x + 3) - 1/472392*log(2*x - 3)
```

mupad [B] time = 5.09, size = 121, normalized size = 0.83

$$\frac{\ln\left(x+\frac{3}{2}\right)}{4251528}-\frac{\ln\left(x-\frac{3}{2}\right)}{472392}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right)-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right)-\frac{x}{139968\left(x^5-\frac{3x^4}{2}+\frac{9x^3}{4}-\frac{27x^2}{8}+\frac{81x}{16}-\frac{243}{32}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/4251528 - log(x - 3/2)/472392 - log(x - (3^(1/2)*3i)/4 - 3/4)*
((3^(1/2)*1i)/944784 + 1/8503056) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)
*1i)/8503056 - 1/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/9447
84 - 1/8503056) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/8503056 + 1/9
44784) - x/(139968*((81*x)/16 - (27*x^2)/8 + (9*x^3)/4 - (3*x^4)/2 + x^5 -
243/32))
```

sympy [A] time = 0.65, size = 124, normalized size = 0.85

$$-\frac{x}{139968x^5-209952x^4+314928x^3-472392x^2+708588x-1062882}-\frac{\log\left(x-\frac{3}{2}\right)}{472392}+\frac{\log\left(x+\frac{3}{2}\right)}{4251528}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{8503056}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{4251528}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(139968*x**5 - 209952*x**4 + 314928*x**3 - 472392*x**2 + 708588*x - 1062
882) - log(x - 3/2)/472392 + log(x + 3/2)/4251528 - log(x**2 - 3*x/2 + 9/4)
/8503056 + log(x**2 + 3*x/2 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sq
rt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/4251528
```

3.412 $\int \frac{9-6x+4x^2}{\left(729-64x^6\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{4x+3}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}-\frac{5\log\left(4x^2+6x+9\right)}{2834352}+\frac{1}{472392(3-2x)}-\frac{1}{157464(2x+3)}-\frac{\log(3-2x}{354294}$$

Rubi [A] time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1586, 2074, 634, 618, 204, 628, 614}

$$\frac{4x+3}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}-\frac{5\log\left(4x^2+6x+9\right)}{2834352}+\frac{1}{472392(3-2x)}-\frac{1}{157464(2x+3)}-\frac{\log(3-2x)}{354294}+\frac{\log(2x+3)}{118098}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(9 + 6*x +
4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) + ArcTan[(3 + 4*x)
/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/354294 + Log[3 + 2*x]/118098 -
 Log[9 - 6*x + 4*x^2]/944784 - (5*Log[9 + 6*x + 4*x^2])/2834352
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{9-6x+4x^2}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(9-6x+4x^2\right)\left(81+54x-24x^3-16x^4\right)^2}\,dx$$

$$= \int \left(\frac{1}{236196(-3+2x)^2} - \frac{1}{177147(-3+2x)} + \frac{1}{78732(3+2x)^2} + \frac{1}{59049(3+2x)} + \frac{}{2361}\right.$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} - \frac{\log(3-2x)}{354294} + \frac{\log(3+2x)}{118098} + \frac{\int \frac{21-10x}{9+6x+4x^2}\,dx}{708588} + \int$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196\left(9+6x+4x^2\right)} - \frac{\log(3-2x)}{354294} + \frac{\log(3+}{11809}$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196\left(9+6x+4x^2\right)} - \frac{\log(3-2x)}{354294} + \frac{\log(3+}{11809}$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196\left(9+6x+4x^2\right)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}} + \frac{\tan^{-1}\left(\right.}{52488}$$

Mathematica [A] time = 0.07, size = 111, normalized size = 0.78

$$\frac{-3\log\left(4x^2-6x+9\right)-5\log\left(4x^2+6x+9\right)+\frac{648x}{-16x^4-24x^3+54x+81}-8\log(3-2x)+24\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+18\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{2834352}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] ((648*x)/(81 + 54*x - 24*x^3 - 16*x^4) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqr
t[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 8*Log[3 - 2*x] + 24*Log
[3 + 2*x] - 3*Log[9 - 6*x + 4*x^2] - 5*Log[9 + 6*x + 4*x^2])/2834352
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{9-6x+4x^2}{\left(729-64x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2, x]
```

fricas [A] time = 0.41, size = 187, normalized size = 1.32

$$\frac{18\sqrt{3}\left(16x^4+24x^3-54x-81\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}\left(16x^4+24x^3-54x-81\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-5\left(16x^4+24x^3-54x-81\right)\log\left(4x^2+6x+9\right)-3\left(16x^4+24x^3-54x-81\right)\log\left(4x^2-6x+9\right)+24\left(16x^4+24x^3-54x-81\right)\log(2x+3)-8\left(16x^4+24x^3-54x-81\right)\log(2x-3)-648x}{2834352\left(16x^4+24x^3-54x-81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/2834352*(18*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
 + 3)) + 2*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x -
3)) - 5*(16*x^4 + 24*x^3 - 54*x - 81)*log(4*x^2 + 6*x + 9) - 3*(16*x^4 + 24
*x^3 - 54*x - 81)*log(4*x^2 - 6*x + 9) + 24*(16*x^4 + 24*x^3 - 54*x - 81)*l
og(2*x + 3) - 8*(16*x^4 + 24*x^3 - 54*x - 81)*log(2*x - 3) - 648*x)/(16*x^4
 + 24*x^3 - 54*x - 81)
```

giac [A] time = 0.18, size = 106, normalized size = 0.75

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(4x^2+6x+9\right)(2x+3)(2x-3)}-\frac{5}{2834352}\log\left(4x^2+6x+9\right)-\frac{1}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{118098}\log\left(|2x+3|\right)-\frac{1}{354294}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(2*x + 3)*(2*x - 3)) -
5/2834352*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*l
og(abs(2*x + 3)) - 1/354294*log(abs(2*x - 3))
```

maple [A] time = 0.06, size = 111, normalized size = 0.78

$$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{1417176}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{157464}-\frac{\ln(2x-3)}{354294}+\frac{\ln(2x+3)}{118098}-\frac{\ln\left(4x^2-6x+9\right)}{944784}-\frac{5\ln\left(4x^2+6x+9\right)}{2834352}-\frac{1}{157464\,(2x+3)}-\frac{-3x-\frac{9}{4}}{708588\left(x^2+\frac{3}{2}x+\frac{9}{4}\right)}-\frac{1}{472392\,(2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-6*x+9)/(-64*x^6+729)^2,x)
```

```
[Out] -1/944784*ln(4*x^2-6*x+9)+1/1417176*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/
157464/(2*x+3)+1/118098*ln(2*x+3)-1/708588*(-3*x-9/4)/(x^2+3/2*x+9/4)-5/283
4352*ln(4*x^2+6*x+9)+1/157464*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/472392
/(2*x-3)-1/354294*ln(2*x-3)
```

maxima [A] time = 2.98, size = 95, normalized size = 0.67

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(16x^4+24x^3-54x-81\right)}-\frac{5}{2834352}\log\left(4x^2+6x+9\right)-\frac{1}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{118098}\log(2x+3)-\frac{1}{354294}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 + 24*x^3 - 54*x - 81) - 5/2834352*
log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*log(2*x + 3
) - 1/354294*log(2*x - 3)
```

mupad [B] time = 5.08, size = 110, normalized size = 0.77

$$\frac{\ln\left(x+\frac{3}{2}\right)}{118098}-\frac{\ln\left(x-\frac{3}{2}\right)}{354294}-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right)+\frac{x}{69984\left(-x^4-\frac{3x^3}{2}+\frac{27x}{8}+\frac{81}{16}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 6*x + 9)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/118098 - log(x - 3/2)/354294 - log(x - (3^(1/2)*3i)/4 + 3/4)*(
(3^(1/2)*1i)/314928 + 5/2834352) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*
1i)/314928 - 5/2834352) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/28343
```

```
52 + 1/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/2834352 - 1/94
4784) + x/(69984*((27*x)/8 - (3*x^3)/2 - x^4 + 81/16))
```

sympy [A] time = 0.64, size = 116, normalized size = 0.82

$$-\frac{x}{69984x^4+104976x^3-236196x-354294}-\frac{\log\left(x-\frac{3}{2}\right)}{354294}+\frac{\log\left(x+\frac{3}{2}\right)}{118098}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}-\frac{5\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{2834352}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{1417176}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(69984*x**4 + 104976*x**3 - 236196*x - 354294) - log(x - 3/2)/354294 + l
og(x + 3/2)/118098 - log(x**2 - 3*x/2 + 9/4)/944784 - 5*log(x**2 + 3*x/2 +
9/4)/2834352 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/1417176 + sqrt(3)*at
an(4*sqrt(3)*x/9 + sqrt(3)/3)/157464
```

3.413 $\int \frac{9+6x+4x^2}{\left(729-64x^6\right)^2} \, dx$

Optimal. Leaf size=142

$$-\frac{3-4x}{236196\left(4x^2-6x+9\right)}+\frac{5\log\left(4x^2-6x+9\right)}{2834352}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{157464(3-2x)}-\frac{1}{472392(2x+3)}-\frac{\log(3-}{11809}$$

Rubi [A] time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1586, 2074, 614, 618, 204, 634, 628}

$$-\frac{3-4x}{236196\left(4x^2-6x+9\right)}+\frac{5\log\left(4x^2-6x+9\right)}{2834352}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{157464(3-2x)}-\frac{1}{472392(2x+3)}-\frac{\log(3-2x)}{118098}+\frac{\log(2x+3)}{354294}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{472392\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(9 - 6*x +
4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) + ArcTan[(3 + 4*x)/
(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/118098 + Log[3 + 2*x]/354294 +
 (5*Log[9 - 6*x + 4*x^2])/2834352 + Log[9 + 6*x + 4*x^2]/944784
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{9+6x+4x^2}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(9+6x+4x^2\right)\left(81-54x+24x^3-16x^4\right)^2}\,dx$$

$$= \int \left(\frac{1}{78732(-3+2x)^2} - \frac{1}{59049(-3+2x)} + \frac{1}{236196(3+2x)^2} + \frac{1}{177147(3+2x)} + \right.$$

$$= \frac{1}{157464(3-2x)} - \frac{1}{472392(3+2x)} - \frac{\log(3-2x)}{118098} + \frac{\log(3+2x)}{354294} + \frac{\int \frac{21+10x}{9-6x+4x^2}\,dx}{708588} +$$

$$= \frac{1}{157464(3-2x)} - \frac{1}{472392(3+2x)} - \frac{3-4x}{236196\left(9-6x+4x^2\right)} - \frac{\log(3-2x)}{118098} +$$

$$= \frac{1}{157464(3-2x)} - \frac{1}{472392(3+2x)} - \frac{3-4x}{236196\left(9-6x+4x^2\right)} - \frac{\log(3-2x)}{118098} +$$

$$= \frac{1}{157464(3-2x)} - \frac{1}{472392(3+2x)} - \frac{3-4x}{236196\left(9-6x+4x^2\right)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} +$$

Mathematica [A] time = 0.07, size = 111, normalized size = 0.78

$$\frac{5\log\left(4x^2-6x+9\right)+3\log\left(4x^2+6x+9\right)+\frac{648x}{-16x^4+24x^3-54x+81}-24\log(3-2x)+8\log(2x+3)+18\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{2834352}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] ((648*x)/(81 - 54*x + 24*x^3 - 16*x^4) + 18*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sq
rt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 24*Log[3 - 2*x] + 8*Log
[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/2834352
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{9+6x+4x^2}{\left(729-64x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2, x]
```

fricas [A] time = 0.42, size = 187, normalized size = 1.32

$$\frac{2\sqrt{3}\left(16x^4-24x^3+54x-81\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+18\sqrt{3}\left(16x^4-24x^3+54x-81\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3\left(16x^4-24x^3+54x-81\right)\log\left(4x^2+6x+9\right)+5\left(16x^4-24x^3+54x-81\right)\log\left(4x^2-6x+9\right)+8\left(16x^4-24x^3+54x-81\right)\log(2x+3)-24\left(16x^4-24x^3+54x-81\right)\log(2x-3)-648x}{2834352\left(16x^4-24x^3+54x-81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/2834352*(2*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
+ 3)) + 18*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x -
3)) + 3*(16*x^4 - 24*x^3 + 54*x - 81)*log(4*x^2 + 6*x + 9) + 5*(16*x^4 - 24
*x^3 + 54*x - 81)*log(4*x^2 - 6*x + 9) + 8*(16*x^4 - 24*x^3 + 54*x - 81)*lo
g(2*x + 3) - 24*(16*x^4 - 24*x^3 + 54*x - 81)*log(2*x - 3) - 648*x)/(16*x^4
 - 24*x^3 + 54*x - 81)
```

giac [A] time = 0.24, size = 106, normalized size = 0.75

$$\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(4x^2-6x+9\right)(2x+3)(2x-3)}+\frac{1}{944784}\log\left(4x^2+6x+9\right)+\frac{5}{2834352}\log\left(4x^2-6x+9\right)+\frac{1}{354294}\log\left(|2x+3|\right)-\frac{1}{118098}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x + 3)*(2*x - 3)) +
1/944784*log(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*l
og(abs(2*x + 3)) - 1/118098*log(abs(2*x - 3))
```

maple [A] time = 0.06, size = 111, normalized size = 0.78

$$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{157464}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{1417176}-\frac{\ln(2x-3)}{118098}+\frac{\ln(2x+3)}{354294}+\frac{5\ln\left(4x^2-6x+9\right)}{2834352}+\frac{\ln\left(4x^2+6x+9\right)}{944784}+\frac{3x-\frac{9}{4}}{708588x^2-1062882x+1594323}-\frac{1}{472392\,(2x+3)}-\frac{1}{157464\,(2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+6*x+9)/(-64*x^6+729)^2,x)
```

```
[Out] 1/708588*(3*x-9/4)/(x^2-3/2*x+9/4)+5/2834352*ln(4*x^2-6*x+9)+1/157464*3^(1/
2)*arctan(1/18*(8*x-6)*3^(1/2))-1/472392/(2*x+3)+1/354294*ln(2*x+3)+1/94478
4*ln(4*x^2+6*x+9)+1/1417176*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/157464/(
2*x-3)-1/118098*ln(2*x-3)
```

maxima [A] time = 2.88, size = 95, normalized size = 0.67

$$\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(16x^4-24x^3+54x-81\right)}+\frac{1}{944784}\log\left(4x^2+6x+9\right)+\frac{5}{2834352}\log\left(4x^2-6x+9\right)+\frac{1}{354294}\log(2x+3)-\frac{1}{118098}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 - 24*x^3 + 54*x - 81) + 1/944784*l
og(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*log(2*x + 3
) - 1/118098*log(2*x - 3)
```

mupad [B] time = 0.19, size = 111, normalized size = 0.78

$$\frac{\ln\left(x+\frac{3}{2}\right)}{354294}-\frac{\ln\left(x-\frac{3}{2}\right)}{118098}-\frac{x}{69984\left(x^4-\frac{3x^3}{2}+\frac{27x}{8}-\frac{81}{16}\right)}-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right)-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x + 4*x^2 + 9)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/354294 - log(x - 3/2)/118098 - x/(69984*((27*x)/8 - (3*x^3)/2
+ x^4 - 81/16)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/314928 - 5/28
34352) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/314928 + 5/2834352) -
log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/2834352 - 1/944784) + log(x + (
3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/2834352 + 1/944784)
```

sympy [A] time = 0.57, size = 116, normalized size = 0.82

$$-\frac{x}{69984x^4-104976x^3+236196x-354294}-\frac{\log\left(x-\frac{3}{2}\right)}{118098}+\frac{\log\left(x+\frac{3}{2}\right)}{354294}+\frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{2834352}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{157464}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{1417176}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(69984*x**4 - 104976*x**3 + 236196*x - 354294) - log(x - 3/2)/118098 + l
og(x + 3/2)/354294 + 5*log(x**2 - 3*x/2 + 9/4)/2834352 + log(x**2 + 3*x/2 +
 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/157464 + sqrt(3)*ata
n(4*sqrt(3)*x/9 + sqrt(3)/3)/1417176
```

3.414 $\int \frac{27-8x^3}{\left(729-64x^6\right)^2} dx$

Optimal. Leaf size=113

$$\frac{x}{4374\left(8x^3+27\right)}-\frac{7\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}-\frac{\log(3-2x)}{157464}+\frac{7\log(2x+3)}{472392}-\frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}}{5248}$$

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1404, 414, 522, 200, 31, 634, 618, 204, 628}

$$\frac{x}{4374\left(8x^3+27\right)}-\frac{7\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}-\frac{\log(3-2x)}{157464}+\frac{7\log(2x+3)}{472392}-\frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] x/(4374*(27 + 8*x^3)) - (7*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3])
+ ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/157464 + (7*
Log[3 + 2*x])/472392 - (7*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^
2]/314928
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{27-8x^3}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(27-8x^3\right)\left(27+8x^3\right)^2}\,dx$$

$$= \frac{x}{4374\left(27+8x^3\right)} - \frac{\int \frac{-1080+128x^3}{\left(27-8x^3\right)\left(27+8x^3\right)}\,dx}{34992}$$

$$= \frac{x}{4374\left(27+8x^3\right)} + \frac{\int \frac{1}{27-8x^3}\,dx}{2916} + \frac{7\int \frac{1}{27+8x^3}\,dx}{8748}$$

$$= \frac{x}{4374\left(27+8x^3\right)} + \frac{\int \frac{1}{3-2x}\,dx}{78732} + \frac{\int \frac{6+2x}{9+6x+4x^2}\,dx}{78732} + \frac{7\int \frac{1}{3+2x}\,dx}{236196} + \frac{7\int \frac{6-2x}{9-6x+4x^2}\,dx}{236196}$$

$$= \frac{x}{4374\left(27+8x^3\right)} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} + \frac{\int \frac{6+8x}{9+6x+4x^2}\,dx}{314928} - \frac{7\int \frac{-6+8x}{9-6x+4x^2}\,dx}{944784} + \ldots$$

$$= \frac{x}{4374\left(27+8x^3\right)} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} - \frac{7\log\left(9-6x+4x^2\right)}{944784} + \frac{\log\left(9+6x \ldots\right.}{314928 \ldots}$$

$$= \frac{x}{4374\left(27+8x^3\right)} - \frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} - \frac{7\,\mathrm{l}\ldots}{\ldots}$$

Mathematica [A] time = 0.07, size = 103, normalized size = 0.91

$$\frac{\frac{216x}{8x^3+27} - 7\log\left(4x^2-6x+9\right) + 3\log\left(4x^2+6x+9\right) - 6\log(3-2x) + 14\log(2x+3) + 14\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right) + 6\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{944784}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] ((216*x)/(27 + 8*x^3) + 14*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[
3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 6*Log[3 - 2*x] + 14*Log[3 + 2*x] - 7*Log
[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{27-8x^3}{\left(729-64x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(27 - 8*x^3)/(729 - 64*x^6)^2, x]
```

fricas [A] time = 0.41, size = 131, normalized size = 1.16

$$\frac{6\sqrt{3}\left(8x^3+27\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+14\sqrt{3}\left(8x^3+27\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3\left(8x^3+27\right)\log\left(4x^2+6x+9\right)-7\left(8x^3+27\right)\log\left(4x^2-6x+9\right)+14\left(8x^3+27\right)\log(2x+3)-6\left(8x^3+27\right)\log(2x-3)+216x}{944784\left(8x^3+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/944784*(6*sqrt(3)*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x + 3)) + 14*sqrt(3)
*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(8*x^3 + 27)*log(4*x^2 + 6*
x + 9) - 7*(8*x^3 + 27)*log(4*x^2 - 6*x + 9) + 14*(8*x^3 + 27)*log(2*x + 3)
 - 6*(8*x^3 + 27)*log(2*x - 3) + 216*x)/(8*x^3 + 27)
```

giac [A] time = 0.17, size = 89, normalized size = 0.79

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\left(8x^3+27\right)}+\frac{1}{314928}\log\left(4x^2+6x+9\right)-\frac{7}{944784}\log\left(4x^2-6x+9\right)+\frac{7}{472392}\log\left(|2x+3|\right)-\frac{1}{157464}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1/
9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x + 9
) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(abs(2*x + 3)) - 1/157464*l
og(abs(2*x - 3))
```

maple [A] time = 0.06, size = 102, normalized size = 0.90

$$\frac{7\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{157464}-\frac{\ln(2x-3)}{157464}+\frac{7\ln(2x+3)}{472392}-\frac{7\ln\left(4x^2-6x+9\right)}{944784}+\frac{\ln\left(4x^2+6x+9\right)}{314928}-\frac{-\frac{3x}{4}-\frac{9}{8}}{118098\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)}-\frac{1}{78732(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-8*x^3+27)/(-64*x^6+729)^2,x)
```

```
[Out] -1/118098*(-3/4*x-9/8)/(x^2-3/2*x+9/4)-7/944784*ln(4*x^2-6*x+9)+7/472392*3^
(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/78732/(2*x+3)+7/472392*ln(2*x+3)+1/314
928*ln(4*x^2+6*x+9)+1/157464*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/157464*
ln(2*x-3)
```

maxima [A] time = 2.94, size = 87, normalized size = 0.77

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\left(8x^3+27\right)}+\frac{1}{314928}\log\left(4x^2+6x+9\right)-\frac{7}{944784}\log\left(4x^2-6x+9\right)+\frac{7}{472392}\log(2x+3)-\frac{1}{157464}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1/
9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x + 9
) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(2*x + 3) - 1/157464*log(2*
x - 3)
```

mupad [B] time = 0.17, size = 102, normalized size = 0.90

$$\frac{7\ln\left(x+\frac{3}{2}\right)}{472392}-\frac{\ln\left(x-\frac{3}{2}\right)}{157464}+\frac{x}{34992\left(x^3+\frac{27}{8}\right)}-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right)+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{7}{944784}+\frac{\sqrt{3}\,7\mathrm{i}}{944784}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{7}{944784}+\frac{\sqrt{3}\,7\mathrm{i}}{944784}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(8*x^3 - 27)/(64*x^6 - 729)^2,x)
```

```
[Out] (7*log(x + 3/2))/472392 - log(x - 3/2)/157464 + x/(34992*(x^3 + 27/8)) - lo
g(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/314928 - 1/314928) + log(x + (3^(
1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/314928 + 1/314928) - log(x - (3^(1/2)*3i)/4
 - 3/4)*((3^(1/2)*7i)/944784 + 7/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((
3^(1/2)*7i)/944784 - 7/944784)
```

sympy [A] time = 0.53, size = 110, normalized size = 0.97

$$\frac{x}{34992x^3+118098}-\frac{\log\left(x-\frac{3}{2}\right)}{157464}+\frac{7\log\left(x+\frac{3}{2}\right)}{472392}-\frac{7\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928}+\frac{7\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x**3+27)/(-64*x**6+729)**2,x)
```

```
[Out] x/(34992*x**3 + 118098) - log(x - 3/2)/157464 + 7*log(x + 3/2)/472392 - 7*l
og(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)/314928 + 7*sqrt(3)*
atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(
3)/3)/157464
```

3.415 $\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} \, dx$

Optimal. Leaf size=131

$$-\frac{3-2x}{26244\left(4x^2-6x+9\right)}+\frac{17\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}+\frac{1}{26244(3-2x)}-\frac{7\log(3-2x)}{157464}+\frac{\log(2x+3)}{472392}$$

Rubi [A] time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$-\frac{3-2x}{26244\left(4x^2-6x+9\right)}+\frac{17\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}+\frac{1}{26244(3-2x)}-\frac{7\log(3-2x)}{157464}+\frac{\log(2x+3)}{472392}-\frac{11\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*ArcTan[(3 -
 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(15746
4*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3 + 2*x]/472392 + (17*Log[9 - 6*
x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{27+36x+24x^2+8x^3}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(27-36x+24x^2-8x^3\right)^2\left(27+36x+24x^2+8x^3\right)}\,dx$$

$$= \int \left(\frac{1}{13122(-3+2x)^2} - \frac{7}{78732(-3+2x)} + \frac{1}{236196(3+2x)} + \frac{3+2x}{4374\left(9-6x+\right.}\right.$$

$$= \frac{1}{26244(3-2x)} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{\int \frac{3+17x}{9-6x+4x^2}\,dx}{118098} + \frac{\int \frac{x}{9+6x+4x^2}\,d}{39366}$$

$$= \frac{1}{26244(3-2x)} - \frac{3-2x}{26244\left(9-6x+4x^2\right)} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{\int \frac{}{9+}}{3}$$

$$= \frac{1}{26244(3-2x)} - \frac{3-2x}{26244\left(9-6x+4x^2\right)} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{17\,\mathrm{lo}}{}$$

$$= \frac{1}{26244(3-2x)} - \frac{3-2x}{26244\left(9-6x+4x^2\right)} - \frac{11\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{7}{}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.85

$$\frac{17\log\left(4x^2-6x+9\right)+3\log\left(4x^2+6x+9\right)+\frac{216x}{-8x^3+24x^2-36x+27}-42\log(3-2x)+2\log(2x+3)+22\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{944784}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] ((216*x)/(27 - 36*x + 24*x^2 - 8*x^3) + 22*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqr
t[3])] - 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 42*Log[3 - 2*x] + 2*Log[
3 + 2*x] + 17*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{27+36x+24x^2+8x^3}{\left(729-64x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2, x]
```

fricas [A] time = 0.41, size = 187, normalized size = 1.43

$$-\frac{2\sqrt{3}\left(8x^3-24x^2+36x-27\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-22\sqrt{3}\left(8x^3-24x^2+36x-27\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-3\left(8x^3-24x^2+36x-27\right)\log\left(4x^2+6x+9\right)-17\left(8x^3-24x^2+36x-27\right)\log\left(4x^2-6x+9\right)-2\left(8x^3-24x^2+36x-27\right)\log(2x+3)+42\left(8x^3-24x^2+36x-27\right)\log(2x-3)+216x}{944784\left(8x^3-24x^2+36x-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] -1/944784*(2*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x +
 3)) - 22*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x - 3)
) - 3*(8*x^3 - 24*x^2 + 36*x - 27)*log(4*x^2 + 6*x + 9) - 17*(8*x^3 - 24*x^
2 + 36*x - 27)*log(4*x^2 - 6*x + 9) - 2*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*
x + 3) + 42*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*x - 3) + 216*x)/(8*x^3 - 24*
x^2 + 36*x - 27)
```

giac [A] time = 0.18, size = 99, normalized size = 0.76

$$-\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{11}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(4x^2-6x+9\right)(2x-3)}+\frac{1}{314928}\log\left(4x^2+6x+9\right)+\frac{17}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{472392}\log\left(|2x+3|\right)-\frac{7}{157464}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x - 3)) + 1/314928*
log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(abs(2*
x + 3)) - 7/157464*log(abs(2*x - 3))
```

maple [A] time = 0.06, size = 102, normalized size = 0.78

$$\frac{11\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}-\frac{\sqrt{3}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{472392}-\frac{7\ln(2x-3)}{157464}+\frac{\ln(2x+3)}{472392}+\frac{17\ln\left(4x^2-6x+9\right)}{944784}+\frac{\ln\left(4x^2+6x+9\right)}{314928}+\frac{\frac{9x}{4}-\frac{27}{8}}{118098x^2-177147x+\frac{531441}{2}}-\frac{1}{26244(2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x)
```

```
[Out] 1/118098*(9/4*x-27/8)/(x^2-3/2*x+9/4)+17/944784*ln(4*x^2-6*x+9)+11/472392*3
^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/472392*ln(2*x+3)+1/314928*ln(4*x^2+6*
x+9)-1/472392*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/26244/(2*x-3)-7/157464
*ln(2*x-3)
```

maxima [A] time = 2.99, size = 95, normalized size = 0.73

$$-\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{11}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(8x^3-24x^2+36x-27\right)}+\frac{1}{314928}\log\left(4x^2+6x+9\right)+\frac{17}{944784}\log\left(4x^2-6x+9\right)+\frac{1}{472392}\log(2x+3)-\frac{7}{157464}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(8*x^3 - 24*x^2 + 36*x - 27) + 1/314928*l
og(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(2*x + 3
) - 7/157464*log(2*x - 3)
```

mupad [B] time = 0.19, size = 111, normalized size = 0.85

$$\frac{\ln\left(x+\frac{3}{2}\right)}{472392}-\frac{7\ln\left(x-\frac{3}{2}\right)}{157464}-\frac{x}{34992\left(x^3-3x^2+\frac{9x}{2}-\frac{27}{8}\right)}+\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right)-\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{17}{944784}+\frac{\sqrt{3}\,11\mathrm{i}}{944784}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{17}{944784}+\frac{\sqrt{3}\,11\mathrm{i}}{944784}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((36*x + 24*x^2 + 8*x^3 + 27)/(64*x^6 - 729)^2,x)
```

```
[Out] log(x + 3/2)/472392 - (7*log(x - 3/2))/157464 - x/(34992*((9*x)/2 - 3*x^2 +
 x^3 - 27/8)) + log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 + 1/3149
28) - log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 - 1/314928) - log(
```

```
x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*11i)/944784 - 17/944784) + log(x + (3^(
1/2)*3i)/4 - 3/4)*((3^(1/2)*11i)/944784 + 17/944784)
```

sympy [A] time = 0.70, size = 119, normalized size = 0.91

$$-\frac{x}{34992x^3-104976x^2+157464x-118098}-\frac{7\log\left(x-\frac{3}{2}\right)}{157464}+\frac{\log\left(x+\frac{3}{2}\right)}{472392}+\frac{17\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928}+\frac{11\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{472392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(34992*x**3 - 104976*x**2 + 157464*x - 118098) - 7*log(x - 3/2)/157464 +
 log(x + 3/2)/472392 + 17*log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2
 + 9/4)/314928 + 11*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 - sqrt(3
)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

3.416 $\int \frac{x(27-2x^3)}{729-64x^6}\,dx$

Optimal. Leaf size=99

$$\frac{5}{576}\log\left(4x^2-6x+9\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(2x+3)-\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{32\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1511, 292, 31, 634, 618, 204, 628}

$$\frac{5}{576}\log\left(4x^2-6x+9\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(2x+3)-\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{32\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

```
[Out] (-5*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(96*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[
3])]/(32*Sqrt[3]) - Log[3 - 2*x]/96 - (5*Log[3 + 2*x])/288 + (5*Log[9 - 6*x
 + 4*x^2])/576 + Log[9 + 6*x + 4*x^2]/192
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1511

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[-(a*c), 2]}, -Dist[e/2 + (c*d)/(2*q), Int[(f*x)
^m/(q - c*x^n), x], x] + Dist[e/2 - (c*d)/(2*q), Int[(f*x)^m/(q + c*x^n), x
], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(27-2x^3\right)}{729-64x^6}\,dx &= 3\int \frac{x}{216-64x^3}\,dx+5\int \frac{x}{216+64x^3}\,dx\\ &= \frac{1}{24}\int \frac{1}{6-4x}\,dx-\frac{1}{24}\int \frac{6-4x}{36+24x+16x^2}\,dx-\frac{5}{72}\int \frac{1}{6+4x}\,dx+\frac{5}{72}\int \frac{6+4x}{36-24x+16x}\\ &= -\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(3+2x)+\frac{1}{192}\int \frac{24+32x}{36+24x+16x^2}\,dx+\frac{5}{576}\int \frac{-24+32}{36-24x+1}\\ &= -\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(3+2x)+\frac{5}{576}\log\left(9-6x+4x^2\right)+\frac{1}{192}\log\left(9+6x+4x^2\right)\\ &= -\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{32\sqrt{3}}-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(3+2x)+\frac{5}{576}\log\left(9-6x\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 91, normalized size = 0.92

$$\frac{1}{576}\left(5\log\left(4x^2-6x+9\right)+3\log\left(4x^2+6x+9\right)-6\log(3-2x)-10\log(2x+3)+10\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)-6\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
[Out] (10*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*Sqrt[3]*ArcTan[(3 + 4*x)/(3*
Sqrt[3])] - 6*Log[3 - 2*x] - 10*Log[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*L
og[9 + 6*x + 4*x^2])/576
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(27-2x^3\right)}{729-64x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
[Out] IntegrateAlgebraic[(x*(27 - 2*x^3))/(729 - 64*x^6), x]
```

fricas [A] time = 0.41, size = 75, normalized size = 0.76

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)+\frac{5}{576}\log\left(4x^2-6x+9\right)-\frac{5}{288}\log(2x+3)-\frac{1}{96}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9) -
5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

giac [A] time = 0.19, size = 69, normalized size = 0.70

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(x^2+\frac{3}{2}x+\frac{9}{4}\right)+\frac{5}{576}\log\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)-\frac{5}{288}\log\left(\left|x+\frac{3}{2}\right|\right)-\frac{1}{96}\log\left(\left|x-\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(x^2 + 3/2*x + 9/4) + 5/576*log(x^2 - 3/2*x + 9/4
) - 5/288*log(abs(x + 3/2)) - 1/96*log(abs(x - 3/2))
```

maple [A] time = 0.05, size = 76, normalized size = 0.77

$$\frac{5\sqrt{3}\ \arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{288}-\frac{\sqrt{3}\ \arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{96}-\frac{\ln(2x-3)}{96}-\frac{5\ln(2x+3)}{288}+\frac{5\ln\left(4x^2-6x+9\right)}{576}+\frac{\ln\left(4x^2+6x+9\right)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*x^3+27)/(-64*x^6+729),x)
```

```
[Out] 5/576*ln(4*x^2-6*x+9)+5/288*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-5/288*ln(2
*x+3)+1/192*ln(4*x^2+6*x+9)-1/96*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/96*
ln(2*x-3)
```

maxima [A] time = 2.99, size = 75, normalized size = 0.76

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)+\frac{5}{576}\log\left(4x^2-6x+9\right)-\frac{5}{288}\log(2x+3)-\frac{1}{96}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9) -
5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

mupad [B] time = 5.10, size = 91, normalized size = 0.92

$$-\frac{\ln\left(x-\frac{3}{2}\right)}{96}-\frac{5\ln\left(x+\frac{3}{2}\right)}{288}+\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{192}+\frac{\sqrt{3}\,1\mathrm{i}}{192}\right)-\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{192}+\frac{\sqrt{3}\,1\mathrm{i}}{192}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{576}+\frac{\sqrt{3}\,5\mathrm{i}}{576}\right)+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{576}+\frac{\sqrt{3}\,5\mathrm{i}}{576}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*x^3 - 27))/(64*x^6 - 729),x)
```

```
[Out] log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/192 + 1/192) - (5*log(x + 3/2))
/288 - log(x - 3/2)/96 - log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/192 -
1/192) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*5i)/576 - 5/576) + log(x +
 (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*5i)/576 + 5/576)
```

sympy [A] time = 0.40, size = 102, normalized size = 1.03

$$-\frac{\log\left(x-\frac{3}{2}\right)}{96}-\frac{5\log\left(x+\frac{3}{2}\right)}{288}+\frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{576}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{192}+\frac{5\sqrt{3}\ \mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{288}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x**3+27)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/96 - 5*log(x + 3/2)/288 + 5*log(x**2 - 3*x/2 + 9/4)/576 + log
(x**2 + 3*x/2 + 9/4)/192 + 5*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/288 -
sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/96
```

3.417 $\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx$

Optimal. Leaf size=84

$$a^3cx+\frac{3a^2bcx^{n+1}}{n+1}+\frac{3ab^2cx^{2n+1}}{2n+1}+\frac{d(a+bx^n)^4}{4bn}+\frac{b^3cx^{3n+1}}{3n+1}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1891, 244, 261}

$$\frac{3a^2bcx^{n+1}}{n+1}+a^3cx+\frac{3ab^2cx^{2n+1}}{2n+1}+\frac{d(a+bx^n)^4}{4bn}+\frac{b^3cx^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

```
[Out] a^3*c*x + (3*a^2*b*c*x^(1 + n))/(1 + n) + (3*a*b^2*c*x^(1 + 2*n))/(1 + 2*n)
 + (b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x^n)^4)/(4*b*n)
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx&=c\int (a+bx^n)^3\,dx+d\int x^{-1+n}(a+bx^n)^3\,dx\\&=\frac{d(a+bx^n)^4}{4bn}+c\int \left(a^3+3a^2bx^n+3ab^2x^{2n}+b^3x^{3n}\right)dx\\&=a^3cx+\frac{3a^2bcx^{1+n}}{1+n}+\frac{3ab^2cx^{1+2n}}{1+2n}+\frac{b^3cx^{1+3n}}{1+3n}+\frac{d(a+bx^n)^4}{4bn}\end{aligned}$$

Mathematica [A] time = 0.23, size = 108, normalized size = 1.29

$$\frac{x\left(c+dx^{n-1}\right)\left(4a^3cx+\frac{12a^2bcx^{n+1}}{n+1}+\frac{12ab^2cx^{2n+1}}{2n+1}+\frac{d(a+bx^n)^4}{bn}+\frac{4b^3cx^{3n+1}}{3n+1}\right)}{4(cx+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

```
[Out] (x*(c + d*x^(-1 + n))*(4*a^3*c*x + (12*a^2*b*c*x^(1 + n))/(1 + n) + (12*a*b
^2*c*x^(1 + 2*n))/(1 + 2*n) + (4*b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x
^n)^4)/(b*n)))/(4*(c*x + d*x^n))
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(c + dx^{-1+n}\right)(a + bx^n)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

```
[Out] a^3*c*x + Defer[IntegrateAlgebraic][x^(-1 + n)*(a^3*d + 3*a^2*b*c*x + 3*a^2
*b*d*x^n + 3*a*b^2*d*x^(2*n) + b^3*d*x^(3*n) + 3*a*b^2*c*x^(1 + n) + b^3*c*
x^(1 + 2*n)), x]
```

fricas [B] time = 0.44, size = 305, normalized size = 3.63

$$\frac{4\left(6a^3cn^4 + 11a^3cn^3 + 6a^3cn^2 + a^3cn\right)x + \left(6b^3dn^3 + 11b^3dn^2 + 6b^3dn + b^3d\right)x^{4n} + 4\left(6ab^2dn^3 + 11ab^2dn^2 + 6ab^2dn + ab^2d + \left(2b^3cn^3 + 3b^3cn^2 + b^3cn\right)x\right)x^{3n} + 6\left(6a^2bdn^3 + 11a^2bdn^2 + 6a^2bdn + a^2bd + 2\left(3ab^2cn^3 + 4ab^2cn^2 + ab^2cn\right)x\right)x^{2n} + 4\left(6a^3dn^3 + 11a^3dn^2 + 6a^3dn + a^3d + 3\left(6a^2bcn^3 + 5a^2bcn^2 + a^2bcn\right)x\right)x^n}{4\left(6n^4 + 11n^3 + 6n^2 + n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(6*a^3*c*n^4 + 11*a^3*c*n^3 + 6*a^3*c*n^2 + a^3*c*n)*x + (6*b^3*d*n^
3 + 11*b^3*d*n^2 + 6*b^3*d*n + b^3*d)*x^(4*n) + 4*(6*a*b^2*d*n^3 + 11*a*b^2
*d*n^2 + 6*a*b^2*d*n + a*b^2*d + (2*b^3*c*n^3 + 3*b^3*c*n^2 + b^3*c*n)*x)*x
^(3*n) + 6*(6*a^2*b*d*n^3 + 11*a^2*b*d*n^2 + 6*a^2*b*d*n + a^2*b*d + 2*(3*a
*b^2*c*n^3 + 4*a*b^2*c*n^2 + a*b^2*c*n)*x)*x^(2*n) + 4*(6*a^3*d*n^3 + 11*a^
3*d*n^2 + 6*a^3*d*n + a^3*d + 3*(6*a^2*b*c*n^3 + 5*a^2*b*c*n^2 + a^2*b*c*n)
*x)*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

giac [B] time = 0.26, size = 392, normalized size = 4.67

$$\frac{24a^3cn^4x + 8b^3cn^3xx^{3n} + 36ab^2cn^3xx^{2n} + 72a^2bcn^3xx^n + 44a^3cn^3x + 6b^3dn^3x^{4n} + 24ab^2dn^3x^{3n} + 12b^3cn^2xx^{3n} + 36a^2bdn^3x^{2n} + 48ab^2cn^2xx^{2n} + 24a^3dn^3x^n + 60a^2bcn^2xx^n + 24a^3cn^2x + 11b^3dn^2x^{4n} + 44ab^2dn^2x^{3n} + 4b^3cnxx^{3n} + 66a^2bdn^2x^{2n} + 12ab^2cnxx^{2n} + 44a^3dn^2x^n + 12a^2bcnxx^n + 4a^3cnx + 6b^3dnx^{4n} + 24ab^2dnx^{3n} + 36a^2bdnx^{2n} + 24a^3dnx^n + b^3dx^{4n} + 4ab^2dx^{3n} + 6a^2bdx^{2n} + 4a^3dx^n}{4\left(6n^4 + 11n^3 + 6n^2 + n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/4*(24*a^3*c*n^4*x + 8*b^3*c*n^3*x*x^(3*n) + 36*a*b^2*c*n^3*x*x^(2*n) + 72
*a^2*b*c*n^3*x*x^n + 44*a^3*c*n^3*x + 6*b^3*d*n^3*x^(4*n) + 24*a*b^2*d*n^3*
x^(3*n) + 12*b^3*c*n^2*x*x^(3*n) + 36*a^2*b*d*n^3*x^(2*n) + 48*a*b^2*c*n^2*
x*x^(2*n) + 24*a^3*d*n^3*x^n + 60*a^2*b*c*n^2*x*x^n + 24*a^3*c*n^2*x + 11*b
^3*d*n^2*x^(4*n) + 44*a*b^2*d*n^2*x^(3*n) + 4*b^3*c*n*x*x^(3*n) + 66*a^2*b*
d*n^2*x^(2*n) + 12*a*b^2*c*n*x*x^(2*n) + 44*a^3*d*n^2*x^n + 12*a^2*b*c*n*x*
x^n + 4*a^3*c*n*x + 6*b^3*d*n*x^(4*n) + 24*a*b^2*d*n*x^(3*n) + 36*a^2*b*d*n
*x^(2*n) + 24*a^3*d*n*x^n + b^3*d*x^(4*n) + 4*a*b^2*d*x^(3*n) + 6*a^2*b*d*x
^(2*n) + 4*a^3*d*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

maple [A] time = 0.06, size = 130, normalized size = 1.55

$$\frac{3a^2bcx\,e^{n\ln(x)}}{n+1} + \frac{3a\,b^2cx\,e^{2n\ln(x)}}{2n+1} + \frac{b^3cx\,e^{3n\ln(x)}}{3n+1} + a^3cx + \frac{a^3d\,e^{n\ln(x)}}{n} + \frac{3a^2bd\,e^{2n\ln(x)}}{2n} + \frac{a\,b^2d\,e^{3n\ln(x)}}{n} + \frac{b^3d\,e^{4n\ln(x)}}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(n-1))*(b*x^n+a)^3,x)
```

```
[Out] a^3*c*x+a^3*d/n*exp(n*ln(x))+a*b^2*d/n*exp(n*ln(x))^3+b^3*c/(3*n+1)*x*exp(n
*ln(x))^3+1/4*b^3*d/n*exp(n*ln(x))^4+3/2*a^2*d*b/n*exp(n*ln(x))^2+3*a*b^2*c
/(2*n+1)*x*exp(n*ln(x))^2+3*a^2*c*b/(n+1)*x*exp(n*ln(x))
```

maxima [A] time = 1.36, size = 118, normalized size = 1.40

$$a^3cx + \frac{b^3dx^{4n}}{4n} + \frac{ab^2dx^{3n}}{n} + \frac{3a^2bdx^{2n}}{2n} + \frac{b^3cx^{3n+1}}{3n+1} + \frac{3ab^2cx^{2n+1}}{2n+1} + \frac{3a^2bcx^{n+1}}{n+1} + \frac{a^3dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] a^3*c*x + 1/4*b^3*d*x^(4*n)/n + a*b^2*d*x^(3*n)/n + 3/2*a^2*b*d*x^(2*n)/n +
 b^3*c*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*c*x^(2*n + 1)/(2*n + 1) + 3*a^2*b*c*
x^(n + 1)/(n + 1) + a^3*d*x^n/n
```

mupad [B] time = 5.13, size = 115, normalized size = 1.37

$$a^3\,c\,x + \frac{a^3\,d\,x^n}{n} + \frac{b^3\,d\,x^{4n}}{4n} + \frac{b^3\,c\,x\,x^{3n}}{3n+1} + \frac{3\,a^2\,b\,d\,x^{2n}}{2n} + \frac{a\,b^2\,d\,x^{3n}}{n} + \frac{3\,a\,b^2\,c\,x\,x^{2n}}{2n+1} + \frac{3\,a^2\,b\,c\,x\,x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(n - 1))*(a + b*x^n)^3,x)
```

```
[Out] a^3*c*x + (a^3*d*x^n)/n + (b^3*d*x^(4*n))/(4*n) + (b^3*c*x*x^(3*n))/(3*n +
1) + (3*a^2*b*d*x^(2*n))/(2*n) + (a*b^2*d*x^(3*n))/n + (3*a*b^2*c*x*x^(2*n)
)/(2*n + 1) + (3*a^2*b*c*x*x^n)/(n + 1)
```

sympy [A] time = 8.17, size = 1251, normalized size = 14.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*c*x - a**3*d/x + 3*a**2*b*c*log(x) - 3*a**2*b*d/(2*x**2) -
3*a*b**2*c/x - a*b**2*d/x**3 - b**3*c/(2*x**2) - b**3*d/(4*x**4), Eq(n, -1)
), (a**3*c*x - 2*a**3*d/sqrt(x) + 6*a**2*b*c*sqrt(x) - 3*a**2*b*d/x + 3*a*b
**2*c*log(x) - 2*a*b**2*d/x**(3/2) - 2*b**3*c/sqrt(x) - b**3*d/(2*x**2), Eq
(n, -1/2)), (a**3*c*x - 3*a**3*d/x**(1/3) + 9*a**2*b*c*x**(2/3)/2 - 9*a**2*
b*d/(2*x**(2/3)) + 9*a*b**2*c*x**(1/3) - 3*a*b**2*d/x + b**3*c*log(x) - 3*b
**3*d/(4*x**(4/3)), Eq(n, -1/3)), ((a + b)**3*(c*x + d*log(x)), Eq(n, 0)),
(24*a**3*c*n**4*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*c*n**3*x/(2
4*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*c*n**2*x/(24*n**4 + 44*n**3 + 2
4*n**2 + 4*n) + 4*a**3*c*n*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*
d*n**3*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*d*n**2*x**n/(24*n
**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*d*n*x**n/(24*n**4 + 44*n**3 + 24*n
**2 + 4*n) + 4*a**3*d*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 72*a**2*b*
c*n**3*x*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 60*a**2*b*c*n**2*x*x**n
/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 12*a**2*b*c*n*x*x**n/(24*n**4 + 44*n
**3 + 24*n**2 + 4*n) + 36*a**2*b*d*n**3*x**(2*n)/(24*n**4 + 44*n**3 + 24*n*
*2 + 4*n) + 66*a**2*b*d*n**2*x**(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) +
 36*a**2*b*d*n*x**(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*a**2*b*d*x*
*(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a*b**2*c*n**3*x*x**(2*n)/(2
4*n**4 + 44*n**3 + 24*n**2 + 4*n) + 48*a*b**2*c*n**2*x*x**(2*n)/(24*n**4 +
44*n**3 + 24*n**2 + 4*n) + 12*a*b**2*c*n*x*x**(2*n)/(24*n**4 + 44*n**3 + 24
*n**2 + 4*n) + 24*a*b**2*d*n**3*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n
) + 44*a*b**2*d*n**2*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a*b*
*2*d*n*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 4*a*b**2*d*x**(3*n)/(
24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 8*b**3*c*n**3*x*x**(3*n)/(24*n**4 + 44
*n**3 + 24*n**2 + 4*n) + 12*b**3*c*n**2*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*
n**2 + 4*n) + 4*b**3*c*n*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6
```

```
*b**3*d*n**3*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 11*b**3*d*n**2*
x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*b**3*d*n*x**(4*n)/(24*n**4
 + 44*n**3 + 24*n**2 + 4*n) + b**3*d*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2
+ 4*n), True))
```

3.418 $\int \left(c+d x^{-1+n}\right) \left(a+b x^n\right)^2 \, dx$

Optimal. Leaf size=61

$$a^2 c x+\frac{2 a b c x^{n+1}}{n+1}+\frac{d \left(a+b x^n\right)^3}{3 b n}+\frac{b^2 c x^{2 n+1}}{2 n+1}$$

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1891, 244, 261}

$$a^2 c x+\frac{2 a b c x^{n+1}}{n+1}+\frac{d \left(a+b x^n\right)^3}{3 b n}+\frac{b^2 c x^{2 n+1}}{2 n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]

[Out] a^2*c*x + (2*a*b*c*x^(1 + n))/(1 + n) + (b^2*c*x^(1 + 2*n))/(1 + 2*n) + (d*
(a + b*x^n)^3)/(3*b*n)
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \left(c+d x^{-1+n}\right) \left(a+b x^n\right)^2 \, dx &= c \int \left(a+b x^n\right)^2 \, dx+d \int x^{-1+n} \left(a+b x^n\right)^2 \, dx\\ &=\frac{d \left(a+b x^n\right)^3}{3 b n}+c \int \left(a^2+2 a b x^n+b^2 x^{2 n}\right) \, dx\\ &=a^2 c x+\frac{2 a b c x^{1+n}}{1+n}+\frac{b^2 c x^{1+2 n}}{1+2 n}+\frac{d \left(a+b x^n\right)^3}{3 b n}\\ \end{aligned}$$

Mathematica [A] time = 0.15, size = 120, normalized size = 1.97

$$\frac{a^3 d \left(2 n^2+3 n+1\right)+3 a^2 b \left(2 n^2+3 n+1\right) \left(c n x+d x^n\right)+3 a b^2 (2 n+1) x^n \left(2 c n x+d (n+1) x^n\right)+b^3 (n+1) x^{2 n} \left(3 c n x+d (2 n+1) x^n\right)}{3 b n (n+1) (2 n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]

[Out] (a^3*d*(1 + 3*n + 2*n^2) + 3*a^2*b*(1 + 3*n + 2*n^2)*(c*n*x + d*x^n) + 3*a*
b^2*(1 + 2*n)*x^n*(2*c*n*x + d*(1 + n)*x^n) + b^3*(1 + n)*x^(2*n)*(3*c*n*x
+ d*(1 + 2*n)*x^n))/(3*b*n*(1 + n)*(1 + 2*n))
```

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

```
[Out] a^2*c*x + Defer[IntegrateAlgebraic][x^(-1 + n)*(a^2*d + 2*a*b*c*x + 2*a*b*d
*x^n + b^2*d*x^(2*n) + b^2*c*x^(1 + n)), x]
```

fricas [B] time = 0.44, size = 160, normalized size = 2.62

$$\frac{3\left(2\,a^2cn^3+3\,a^2cn^2+a^2cn\right)x+\left(2\,b^2dn^2+3\,b^2dn+b^2d\right)x^{3\,n}+3\left(2\,abdn^2+3\,abdn+abd+\left(b^2cn^2+b^2cn\right)x\right)x^{2\,n}+3\left(2\,a^2dn^2+3\,a^2dn+a^2d+2\left(2\,abcn^2+abcn\right)x\right)x^n}{3\left(2\,n^3+3\,n^2+n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(2*a^2*c*n^3 + 3*a^2*c*n^2 + a^2*c*n)*x + (2*b^2*d*n^2 + 3*b^2*d*n +
 b^2*d)*x^(3*n) + 3*(2*a*b*d*n^2 + 3*a*b*d*n + a*b*d + (b^2*c*n^2 + b^2*c*n
)*x)*x^(2*n) + 3*(2*a^2*d*n^2 + 3*a^2*d*n + a^2*d + 2*(2*a*b*c*n^2 + a*b*c*
n)*x)*x^n)/(2*n^3 + 3*n^2 + n)
```

giac [B] time = 0.22, size = 196, normalized size = 3.21

$$\frac{6\,a^2cn^3x+3\,b^2cn^2xx^{2\,n}+12\,abcn^2xx^n+9\,a^2cn^2x+2\,b^2dn^2x^{3\,n}+6\,abdn^2x^{2\,n}+3\,b^2cnxx^{2\,n}+6\,a^2dn^2x^n+6\,abcnxx^n+3\,a^2cnx+3\,b^2dnx^{3\,n}+9\,abdnx^{2\,n}+9\,a^2dnx^n+b^2dx^{3\,n}+3\,abdx^{2\,n}+3\,a^2dx^n}{3\left(2\,n^3+3\,n^2+n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*c*n^3*x + 3*b^2*c*n^2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 9*a^2*c*n
^2*x + 2*b^2*d*n^2*x^(3*n) + 6*a*b*d*n^2*x^(2*n) + 3*b^2*c*n*x*x^(2*n) + 6*
a^2*d*n^2*x^n + 6*a*b*c*n*x*x^n + 3*a^2*c*n*x + 3*b^2*d*n*x^(3*n) + 9*a*b*d
*n*x^(2*n) + 9*a^2*d*n*x^n + b^2*d*x^(3*n) + 3*a*b*d*x^(2*n) + 3*a^2*d*x^n)
/(2*n^3 + 3*n^2 + n)
```

maple [A] time = 0.06, size = 87, normalized size = 1.43

$$\frac{2abcx\,{\mathrm e}^{n\ln(x)}}{n+1}+\frac{b^2cx\,{\mathrm e}^{2n\ln(x)}}{2n+1}+a^2cx+\frac{a^2d\,{\mathrm e}^{n\ln(x)}}{n}+\frac{abd\,{\mathrm e}^{2n\ln(x)}}{n}+\frac{b^2d\,{\mathrm e}^{3n\ln(x)}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(n-1))*(b*x^n+a)^2,x)
```

```
[Out] a^2*c*x+a^2*d/n*exp(n*ln(x))+b*d*a/n*exp(n*ln(x))^2+c*b^2/(2*n+1)*x*exp(n*l
n(x))^2+1/3*b^2*d/n*exp(n*ln(x))^3+2*a*b*c/(n+1)*x*exp(n*ln(x))
```

maxima [A] time = 1.40, size = 78, normalized size = 1.28

$$a^2cx+\frac{b^2dx^{3\,n}}{3\,n}+\frac{abdx^{2\,n}}{n}+\frac{b^2cx^{2\,n+1}}{2\,n+1}+\frac{2\,abcx^{n+1}}{n+1}+\frac{a^2dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] a^2*c*x + 1/3*b^2*d*x^(3*n)/n + a*b*d*x^(2*n)/n + b^2*c*x^(2*n + 1)/(2*n +
1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^n/n
```

mupad [B] time = 5.06, size = 76, normalized size = 1.25

$$a^2\,c\,x+\frac{a^2\,d\,x^n}{n}+\frac{b^2\,d\,x^{3\,n}}{3\,n}+\frac{b^2\,c\,x\,x^{2\,n}}{2\,n+1}+\frac{a\,b\,d\,x^{2\,n}}{n}+\frac{2\,a\,b\,c\,x\,x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(n - 1))*(a + b*x^n)^2,x)
```

```
[Out] a^2*c*x + (a^2*d*x^n)/n + (b^2*d*x^(3*n))/(3*n) + (b^2*c*x*x^(2*n))/(2*n +
1) + (a*b*d*x^(2*n))/n + (2*a*b*c*x*x^n)/(n + 1)
```

sympy [A] time = 4.27, size = 552, normalized size = 9.05

$$\begin{cases} a^{2} c x - \frac{a^{2} d}{x} + 2 a b c \log{\left(x \right)} - \frac{a b d}{x^{2}} - \frac{b^{2} c}{x} - \frac{b^{2} d}{3 x^{3}} & \text{for}\: n = -1 \\ a^{2} c x - \frac{2 a^{2} d}{\sqrt{x}} + 4 a b c \sqrt{x} - \frac{2 a b d}{x} + b^{2} c \log{\left(x \right)} - \frac{2 b^{2} d}{3 x^{\frac{3}{2}}} & \text{for}\: n = - \frac{1}{2} \\ \left(a + b\right)^{2} \left(c x + d \log{\left(x \right)}\right) & \text{for}\: n = 0 \\ \frac{6 a^{2} c n^{3} x}{6 n^{3} + 9 n^{2} + 3 n} + \frac{9 a^{2} c n^{2} x}{6 n^{3} + 9 n^{2} + 3 n} + \frac{3 a^{2} c n x}{6 n^{3} + 9 n^{2} + 3 n} + \frac{6 a^{2} d n^{2} x^{n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{9 a^{2} d n x^{n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{3 a^{2} d x^{n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{12 a b c n^{2} x x^{n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{6 a b c n x x^{n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{6 a b d n^{2} x^{2 n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{9 a b d n x^{2 n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{3 a b d x^{2 n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{3 b^{2} c n^{2} x x^{2 n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{3 b^{2} c n x x^{2 n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{2 b^{2} d n^{2} x^{3 n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{3 b^{2} d n x^{3 n}}{6 n^{3} + 9 n^{2} + 3 n} + \frac{b^{2} d x^{3 n}}{6 n^{3} + 9 n^{2} + 3 n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n)**2,x)
```

```
[Out] Piecewise((a**2*c*x - a**2*d/x + 2*a*b*c*log(x) - a*b*d/x**2 - b**2*c/x - b
**2*d/(3*x**3), Eq(n, -1)), (a**2*c*x - 2*a**2*d/sqrt(x) + 4*a*b*c*sqrt(x)
- 2*a*b*d/x + b**2*c*log(x) - 2*b**2*d/(3*x**(3/2)), Eq(n, -1/2)), ((a + b)
**2*(c*x + d*log(x)), Eq(n, 0)), (6*a**2*c*n**3*x/(6*n**3 + 9*n**2 + 3*n) +
 9*a**2*c*n**2*x/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*c*n*x/(6*n**3 + 9*n**2 +
3*n) + 6*a**2*d*n**2*x**n/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*d*n*x**n/(6*n**3
 + 9*n**2 + 3*n) + 3*a**2*d*x**n/(6*n**3 + 9*n**2 + 3*n) + 12*a*b*c*n**2*x*
x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*c*n*x*x**n/(6*n**3 + 9*n**2 + 3*n) + 6
*a*b*d*n**2*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 9*a*b*d*n*x**(2*n)/(6*n**3 +
 9*n**2 + 3*n) + 3*a*b*d*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n**2*x
*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n*x*x**(2*n)/(6*n**3 + 9*n**2
+ 3*n) + 2*b**2*d*n**2*x**(3*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*d*n*x**(3*
n)/(6*n**3 + 9*n**2 + 3*n) + b**2*d*x**(3*n)/(6*n**3 + 9*n**2 + 3*n), True)
)
```

3.419 $\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx$

Optimal. Leaf size=41

$$acx+\frac{adx^n}{n}+\frac{bcx^{n+1}}{n+1}+\frac{bdx^{2n}}{2n}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1891, 14}

$$acx+\frac{adx^n}{n}+\frac{bcx^{n+1}}{n+1}+\frac{bdx^{2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

```
[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x^(1 + n))/(1 + n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx&=c\int (a+bx^n)\,dx+d\int x^{-1+n}(a+bx^n)\,dx\\&=acx+\frac{bcx^{1+n}}{1+n}+d\int \left(ax^{-1+n}+bx^{-1+2n}\right)dx\\&=acx+\frac{adx^n}{n}+\frac{bdx^{2n}}{2n}+\frac{bcx^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.13, size = 42, normalized size = 1.02

$$\frac{2a(cnx+dx^n)+bx^n\left(\frac{2cnx}{n+1}+dx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

```
[Out] (2*a*(c*n*x + d*x^n) + b*x^n*((2*c*n*x)/(1 + n) + d*x^n))/(2*n)
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^(-1 + n))*(a + b*x^n),x]
[Out] a*c*x + Defer[IntegrateAlgebraic][x^(-1 + n)*(a*d + b*c*x + b*d*x^n), x]
```

fricas [A] time = 0.44, size = 56, normalized size = 1.37

$$\frac{2\left(acn^{2}+acn\right)x+(bdn+bd)x^{2n}+2(bcnx+adn+ad)x^{n}}{2\left(n^{2}+n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="fricas")
[Out] 1/2*(2*(a*c*n^2 + a*c*n)*x + (b*d*n + b*d)*x^(2*n) + 2*(b*c*n*x + a*d*n + a
*d)*x^n)/(n^2 + n)
```

giac [A] time = 0.22, size = 65, normalized size = 1.59

$$\frac{2\,acn^{2}x+2\,bcnxx^{n}+2\,acnx+bdnx^{2n}+2\,adnx^{n}+bdx^{2n}+2\,adx^{n}}{2\left(n^{2}+n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="giac")
[Out] 1/2*(2*a*c*n^2*x + 2*b*c*n*x*x^n + 2*a*c*n*x + b*d*n*x^(2*n) + 2*a*d*n*x^n
+ b*d*x^(2*n) + 2*a*d*x^n)/(n^2 + n)
```

maple [A] time = 0.06, size = 45, normalized size = 1.10

$$\frac{bcx\,e^{n\ln(x)}}{n+1}+acx+\frac{ad\,e^{n\ln(x)}}{n}+\frac{bd\,e^{2n\ln(x)}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(n-1))*(b*x^n+a),x)
[Out] a*c*x+a*d/n*exp(n*ln(x))+b*c/(n+1)*x*exp(n*ln(x))+1/2*b*d/n*exp(n*ln(x))^2
```

maxima [A] time = 1.29, size = 39, normalized size = 0.95

$$acx+\frac{bdx^{2n}}{2n}+\frac{bcx^{n+1}}{n+1}+\frac{adx^{n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="maxima")
[Out] a*c*x + 1/2*b*d*x^(2*n)/n + b*c*x^(n + 1)/(n + 1) + a*d*x^n/n
```

mupad [B] time = 5.06, size = 38, normalized size = 0.93

$$a\,c\,x+\frac{a\,d\,x^{n}}{n}+\frac{b\,d\,x^{2n}}{2\,n}+\frac{b\,c\,x\,x^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(n - 1))*(a + b*x^n),x)
[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x*x^n)/(n + 1)
```

sympy [A] time = 2.03, size = 163, normalized size = 3.98

$$\begin{cases} acx - \frac{ad}{x} + bc\log(x) - \frac{bd}{2x^2} & \text{for } n = -1 \\ (a+b)\left(cx + d\log(x)\right) & \text{for } n = 0 \\ \frac{2acn^2x}{2n^2+2n} + \frac{2acnx}{2n^2+2n} + \frac{2adnx^n}{2n^2+2n} + \frac{2adx^n}{2n^2+2n} + \frac{2bcnxx^n}{2n^2+2n} + \frac{bdnx^{2n}}{2n^2+2n} + \frac{bdx^{2n}}{2n^2+2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n),x)
```

```
[Out] Piecewise((a*c*x - a*d/x + b*c*log(x) - b*d/(2*x**2), Eq(n, -1)), ((a + b)*
(c*x + d*log(x)), Eq(n, 0)), (2*a*c*n**2*x/(2*n**2 + 2*n) + 2*a*c*n*x/(2*n*
*2 + 2*n) + 2*a*d*n*x**n/(2*n**2 + 2*n) + 2*a*d*x**n/(2*n**2 + 2*n) + 2*b*c
*n*x*x**n/(2*n**2 + 2*n) + b*d*n*x**(2*n)/(2*n**2 + 2*n) + b*d*x**(2*n)/(2*
n**2 + 2*n), True))
```

3.420 $\int \left(c+dx^{-1+n}\right)\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^n}{n}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$cx+\frac{dx^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[c + d*x^(-1 + n),x]
[Out] c*x + (d*x^n)/n
```

Rubi steps

$$\int \left(c+dx^{-1+n}\right)\,dx=cx+\frac{dx^n}{n}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$cx+\frac{dx^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[c + d*x^(-1 + n),x]
[Out] c*x + (d*x^n)/n
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(c+dx^{-1+n}\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[c + d*x^(-1 + n),x]
[Out] c*x + Defer[IntegrateAlgebraic][d*x^(-1 + n), x]
```

fricas [A] time = 0.43, size = 17, normalized size = 1.42

$$\frac{cnx+dxx^{n-1}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x^(-1+n),x, algorithm="fricas")
[Out] (c*n*x + d*x*x^(n - 1))/n
```

giac [A] time = 0.17, size = 12, normalized size = 1.00

$$cx+\frac{dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x^(-1+n),x, algorithm="giac")
```

```
[Out] c*x + d*x^n/n
```

maple [A] time = 0.04, size = 13, normalized size = 1.08

$$cx + \frac{d\,x^{n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c+d*x^(n-1),x)
```

```
[Out] c*x+d*x^n/n
```

maxima [A] time = 1.32, size = 12, normalized size = 1.00

$$cx + \frac{dx^{n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x^(-1+n),x, algorithm="maxima")
```

```
[Out] c*x + d*x^n/n
```

mupad [B] time = 5.01, size = 12, normalized size = 1.00

$$c\,x + \frac{d\,x^{n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c + d*x^(n - 1),x)
```

```
[Out] c*x + (d*x^n)/n
```

sympy [A] time = 0.07, size = 15, normalized size = 1.25

$$cx + d\left(\begin{cases} \frac{x^{n}}{n} & \text{for}\: n - 1 \neq -1 \\ \log{\left(x \right)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x**(-1+n),x)
```

```
[Out] c*x + d*Piecewise((x**n/n, Ne(n - 1, -1)), (log(x), True))
```

3.421 $\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

Rubi [A] time = 0.39, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {6741, 1816}

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^(-1 +
(5*n)/4))/(a + b*x^n)^(3/2),x]
```

```
[Out] (-2*(a*g + 2*a*h*x^(n/4) - b*f*x^(n/2)))/(a*n*Sqrt[a + b*x^n])
```

Rule 1816

```
Int[((x_)^(m_.)*((e_) + (h_.)*(x_)^(n_.) + (f_.)*(x_)^(q_.) + (g_.)*(x_)^(r
_.)))/((a_) + (c_.)*(x_)^(n_.))^(3/2), x_Symbol] :> -Simp[(2*a*g + 4*a*h*x^
(n/4) - 2*c*f*x^(n/2))/(a*c*n*Sqrt[a + c*x^n]), x] /; FreeQ[{a, c, e, f, g,
 h, m, n}, x] && EqQ[q, n/4] && EqQ[r, (3*n)/4] && EqQ[4*m - n + 4, 0] && E
qQ[c*e + a*h, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\begin{aligned} \int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx &= \int \frac{x^{-1+\frac{n}{4}}\left(-ah+bfx^{n/4}+bgx^{3n/4}+bhx^n\right)}{(a+bx^n)^{3/2}}\,dx\\ &= -\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}} \end{aligned}$$

Mathematica [A] time = 0.31, size = 45, normalized size = 1.00

$$\frac{2bfx^{n/2}-2a\left(g+2hx^{n/4}\right)}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^
(-1 + (5*n)/4))/(a + b*x^n)^(3/2),x]
```

```
[Out] (2*b*f*x^(n/2) - 2*a*(g + 2*h*x^(n/4)))/(a*n*Sqrt[a + b*x^n])
```

IntegrateAlgebraic [A] time = 44.62, size = 45, normalized size = 1.00

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n)
 + b*h*x^(-1 + (5*n)/4))/(a + b*x^n)^(3/2),x]
```

```
[Out] (-2*(a*g + 2*a*h*x^(n/4) - b*f*x^(n/2)))/(a*n*Sqrt[a + b*x^n])
```

fricas [A] time = 0.44, size = 66, normalized size = 1.47

$$\frac{2\sqrt{bx^4x^{n-4}+a}\left(bfx^2x^{\frac{1}{2}n-2}-2\,ahxx^{\frac{1}{4}n-1}-ag\right)}{abnx^4x^{n-4}+a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x^4*x^(n - 4) + a)*(b*f*x^2*x^(1/2*n - 2) - 2*a*h*x*x^(1/4*n - 1)
- a*g)/(a*b*n*x^4*x^(n - 4) + a^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bhx^{\frac{5}{4}n-1}+bgx^{n-1}+bfx^{\frac{1}{2}n-1}-ahx^{\frac{1}{4}n-1}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1
/4*n - 1))/(b*x^n + a)^(3/2), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{-ah\,x^{\frac{n}{4}-1}+bf\,x^{\frac{n}{2}-1}+bg\,x^{n-1}+bh\,x^{\frac{5n}{4}-1}}{(b\,x^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*h*x^(-1+1/4*n)+b*f*x^(1/2*n-1)+b*g*x^(n-1)+b*h*x^(-1+5/4*n))/(b*x^n
+a)^(3/2),x)
```

```
[Out] int((-a*h*x^(-1+1/4*n)+b*f*x^(1/2*n-1)+b*g*x^(n-1)+b*h*x^(-1+5/4*n))/(b*x^n
+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bhx^{\frac{5}{4}n-1}+bgx^{n-1}+bfx^{\frac{1}{2}n-1}-ahx^{\frac{1}{4}n-1}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1
/4*n - 1))/(b*x^n + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b\,f\,x^{\frac{n}{2}-1}-a\,h\,x^{\frac{n}{4}-1}+b\,h\,x^{\frac{5\,n}{4}-1}+b\,g\,x^{n-1}}{{\left(a+b\,x^n\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*f*x^(n/2 - 1) - a*h*x^(n/4 - 1) + b*h*x^((5*n)/4 - 1) + b*g*x^(n - 1
))/(a + b*x^n)^(3/2),x)
```

```
[Out] int((b*f*x^(n/2 - 1) - a*h*x^(n/4 - 1) + b*h*x^((5*n)/4 - 1) + b*g*x^(n - 1
))/(a + b*x^n)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x**(-1+1/4*n)+b*f*x**(-1+1/2*n)+b*g*x**(-1+n)+b*h*x**(-1+5/
4*n))/(a+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

3.422 $\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}} dx$

Optimal. Leaf size=24

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Rubi [A] time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {1590}

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
,x]
```

```
[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}} dx = x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Mathematica [A] time = 0.18, size = 24, normalized size = 1.00

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d
*x^2]),x]
```

```
[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

IntegrateAlgebraic [F] time = 7.84, size = 0, normalized size = 0.00

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b
*x^2]*Sqrt[c + d*x^2]), x]
```

fricas [A] time = 0.43, size = 20, normalized size = 0.83

$$\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,bdx^4 + 2\,(bc + ad)x^2 + ac}{\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((3*b*d*x^4 + 2*(b*c + a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2
 + c)), x)
```

maple [A] time = 0.05, size = 21, normalized size = 0.88

$$\sqrt{b\,x^2 + a}\,\sqrt{d\,x^2 + c}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

maxima [A] time = 2.13, size = 20, normalized size = 0.83

$$\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

mupad [B] time = 5.59, size = 20, normalized size = 0.83

$$x\,\sqrt{b\,x^2 + a}\,\sqrt{d\,x^2 + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + 2*x^2*(a*d + b*c) + 3*b*d*x^4)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1
/2)),x)
```

```
[Out] x*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac + 2adx^2 + 2bcx^2 + 3bdx^4}{\sqrt{a + bx^2}\,\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x**2+3*b*d*x**4)/(b*x**2+a)**(1/2)/(d*x**2+c)**(
1/2),x)
```

```
[Out] Integral((a*c + 2*a*d*x**2 + 2*b*c*x**2 + 3*b*d*x**4)/(sqrt(a + b*x**2)*sqr
t(c + d*x**2)), x)
```

3.423 $\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1899, 377, 212, 206, 203, 444, 63, 298}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4)^(1/4)/2^
(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + Arc
Tanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1899

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)
*(x_)^(n_))^(q_.), x_Symbol] :> Dist[A, Int[(a + b*x^n)^p*(c + d*x^n)^q, x]
, x] + Dist[B, Int[x^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b,
c, d, A, B, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{\left(1-x^4\right)\sqrt[4]{1+x^4}}\,dx &= \int \frac{1}{\left(1-x^4\right)\sqrt[4]{1+x^4}}\,dx+\int \frac{x^3}{\left(1-x^4\right)\sqrt[4]{1+x^4}}\,dx\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[4]{1+x}}\,dx,x,x^4\right)+\text{Subst}\left(\int \frac{1}{1-2x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\text{Su}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx,x,\sqrt[4]{1+x^4}\right)-\frac{1}{2}\text{Subst}\Bigg(\\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] time = 0.20, size = 93, normalized size = 0.90

$$\frac{1}{4}x^4F_1\left(1;\frac{1}{4},1;2;-x^4,x^4\right)+\frac{-\log\left(1-\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)+\log\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}+1\right)+2\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] (x^4*AppellF1[1, 1/4, 1, 2, -x^4, x^4])/4 + (2*ArcTan[(2^(1/4)*x)/(1 + x^4)
^(1/4)] - Log[1 - (2^(1/4)*x)/(1 + x^4)^(1/4)] + Log[1 + (2^(1/4)*x)/(1 + x
^4)^(1/4)])/(4*2^(1/4))
```

IntegrateAlgebraic [A] time = 166.73, size = 103, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4)^(1/4)/2^
(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + Arc
Tanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3+1}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-(x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3+1}{\left(-x^4+1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

```
[Out] int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3+1}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3+1}{\left(x^4-1\right)\,\left(x^4+1\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 + 1)/((x^4 - 1)*(x^4 + 1)^(1/4)),x)
```

```
[Out] int(-(x^3 + 1)/((x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{x}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\right)dx-\int\frac{x^2}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\,dx-\int\frac{1}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(-x**4+1)/(x**4+1)**(1/4),x)
```

```
[Out] -Integral(-x/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1
)**(1/4) - (x**4 + 1)**(1/4)), x) - Integral(x**2/(x**3*(x**4 + 1)**(1/4) -
 x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(1/4) - (x**4 + 1)**(1/4)), x) - In
tegral(1/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(
1/4) - (x**4 + 1)**(1/4)), x)
```

3.424 $\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} \left(ac-bdx^{2n}\right) dx$

Optimal. Leaf size=28

$$x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

Rubi [A] time = 0.10, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1898}

$$x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2*n)),x
]
```

```
[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Rule 1898

```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) +
 (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(e*x*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(p + 1))/(a*c), x] /; FreeQ[{a, b, c, d, e, g, n, p}, x] && EqQ[n2, 2*n] &
& EqQ[n*(p + 1) + 1, 0] && EqQ[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]
```

Rubi steps

$$\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} \left(ac-bdx^{2n}\right) dx = x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

Mathematica [A] time = 0.35, size = 28, normalized size = 1.00

$$x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2
*n)),x]
```

```
[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} \left(ac-bdx^{2n}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c -
 b*d*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)
*(a*c - b*d*x^(2*n)), x]
```

fricas [B] time = 0.46, size = 61, normalized size = 2.18

$$\frac{bdxx^{2n} + acx + (bc+ad)xx^n}{(bx^n+a)^{\frac{n+1}{n}}(dx^n+c)^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="fricas")
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)/((b*x^n + a)^((n + 1)/n)*(d*x^n
 + c)^((n + 1)/n))
```

giac [B] time = 0.42, size = 228, normalized size = 8.14

$$bdxx^{2n}e^{\left(-\frac{n\log\left(bx^n+a\right)+\log\left(bx^n+a\right)}{n}-\frac{n\log\left(dx^n+c\right)+\log\left(dx^n+c\right)}{n}\right)}+bcxx^ne^{\left(-\frac{n\log\left(bx^n+a\right)+\log\left(bx^n+a\right)}{n}-\frac{n\log\left(dx^n+c\right)+\log\left(dx^n+c\right)}{n}\right)}+adxx^ne^{\left(-\frac{n\log\left(bx^n+a\right)+\log\left(bx^n+a\right)}{n}-\frac{n\log\left(dx^n+c\right)+\log\left(dx^n+c\right)}{n}\right)}+acxe^{\left(-\frac{n\log\left(bx^n+a\right)+\log\left(bx^n+a\right)}{n}-\frac{n\log\left(dx^n+c\right)+\log\left(dx^n+c\right)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="giac")
```

```
[Out] b*d*x*x^(2*n)*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c)
 + log(d*x^n + c))/n) + b*c*x*x^n*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n
 - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*d*x*x^n*e^(-(n*log(b*x^n + a)
 + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*c*x*e^(-(
n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/
n)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \left(-bd\,x^{2n}+ac\right)\left(b\,x^{n}+a\right)^{\frac{-n-1}{n}}\left(d\,x^{n}+c\right)^{\frac{-n-1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^((-n-1)/n)*(d*x^n+c)^((-n-1)/n)*(a*c-b*d*x^(2*n)),x)
```

```
[Out] int((b*x^n+a)^((-n-1)/n)*(d*x^n+c)^((-n-1)/n)*(a*c-b*d*x^(2*n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bdx^{2n}-ac}{\left(bx^n+a\right)^{\frac{n+1}{n}}\left(dx^n+c\right)^{\frac{n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="maxima")
```

```
[Out] -integrate((b*d*x^(2*n) - a*c)/((b*x^n + a)^((n + 1)/n)*(d*x^n + c)^((n + 1
)/n)), x)
```

mupad [B] time = 5.20, size = 95, normalized size = 3.39

$$\frac{\frac{a\,c\,x}{{\left(a+b\,x^n\right)}^{\frac{n+1}{n}}}+\frac{x\,x^n\,\left(a\,d+b\,c\right)}{{\left(a+b\,x^n\right)}^{\frac{n+1}{n}}}+\frac{b\,d\,x\,x^{2\,n}}{{\left(a+b\,x^n\right)}^{\frac{n+1}{n}}}}{{\left(c+d\,x^n\right)}^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*d*x^(2*n))/((a + b*x^n)^((n + 1)/n)*(c + d*x^n)^((n + 1)/n)),x
)
```

```
[Out] ((a*c*x)/(a + b*x^n)^((n + 1)/n) + (x*x^n*(a*d + b*c))/(a + b*x^n)^((n + 1)
/n) + (b*d*x*x^(2*n))/(a + b*x^n)^((n + 1)/n))/(c + d*x^n)^((n + 1)/n)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**((-1-n)/n)*(c+d*x**n)**((-1-n)/n)*(a*c-b*d*x**(2*n)),
x)
```

```
[Out] Timed out
```

3.425 $\int (hx)^{-1-n-np} (a + bx^n)^p (c + dx^n)^p \left(ac - bdx^{2n}\right) dx$

Optimal. Leaf size=45

$$-\frac{(hx)^{-n(p+1)} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{hn(p+1)}$$

Rubi [A] time = 0.16, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {1849}

$$-\frac{(hx)^{-n(p+1)} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{hn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*n)),x]

[Out] -(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n*(1 + p)*(h*x)^(n*(1 + p)))
)
```

Rule 1849

```
Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(
n_.))^(p_.)*((e_) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(e*(h*x)^(m + 1)*
(a + b*x^n)^(p + 1)*(c + d*x^n)^(p + 1))/(a*c*h*(m + 1)), x] /; FreeQ[{a, b
, c, d, e, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[m + n*(p + 1) + 1, 0]
&& EqQ[a*c*g*(m + 1) - b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int (hx)^{-1-n-np} (a + bx^n)^p (c + dx^n)^p \left(ac - bdx^{2n}\right) dx = -\frac{(hx)^{-n(1+p)} (a + bx^n)^{1+p} (c + dx^n)^{1+p}}{hn(1+p)}$$

Mathematica [A] time = 0.41, size = 46, normalized size = 1.02

$$-\frac{(hx)^{n(-p)-n} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{hnp + hn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*
n)),x]

[Out] -(((h*x)^(-n - n*p)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n + h*n*p))
```

IntegrateAlgebraic [F] time = 0.42, size = 0, normalized size = 0.00

$$\int (hx)^{-1-n-np} (a + bx^n)^p (c + dx^n)^p \left(ac - bdx^{2n}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c -
b*d*x^(2*n)),x]

[Out] Defer[IntegrateAlgebraic][(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*
(a*c - b*d*x^(2*n)), x]
```

fricas [B] time = 0.45, size = 119, normalized size = 2.64

$$-\frac{\left(bdxx^{2n}e^{\left(-\left(np+n+1\right)\log(h)-\left(np+n+1\right)\log(x)\right)} + acxe^{\left(-\left(np+n+1\right)\log(h)-\left(np+n+1\right)\log(x)\right)} + (bc+ad)xx^{n}e^{\left(-\left(np+n+1\right)\log(h)-\left(np+n+1\right)\log(x)\right)}\right)(bx^{n}+a)^{p}(dx^{n}+c)^{p}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] -(b*d*x*x^(2*n)*e^(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + a*c*x*e^
(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + (b*c + a*d)*x*x^n*e^(-(n*p
 + n + 1)*log(h) - (n*p + n + 1)*log(x)))*(b*x^n + a)^p*(d*x^n + c)^p/(n*p
+ n)
```

giac [B] time = 0.43, size = 237, normalized size = 5.27

$$-\frac{(bx^{n}+a)^{p}(dx^{n}+c)^{p}bdxx^{2n}e^{\left(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x)\right)} + (bx^{n}+a)^{p}(dx^{n}+c)^{p}bcxx^{n}e^{\left(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x)\right)} + (bx^{n}+a)^{p}(dx^{n}+c)^{p}adxx^{n}e^{\left(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x)\right)} + (bx^{n}+a)^{p}(dx^{n}+c)^{p}acxe^{\left(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x)\right)}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] -((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e^(-n*p*log(h) - n*p*log(x) - n
*log(h) - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*b*c*x*x
^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)) + (
b*x^n + a)^p*(d*x^n + c)^p*a*d*x*x^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h)
 - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*a*c*x*e^(-n*p*
log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)))/(n*p + n)
```

maple [C] time = 0.58, size = 138, normalized size = 3.07

$$-\frac{\left(ad\,x^{n}+bc\,x^{n}+bd\,x^{2n}+ac\right)x\left(b\,x^{n}+a\right)^{p}\left(d\,x^{n}+c\right)^{p}\mathrm{e}^{-\frac{\left(np+n+1\right)\left(-i\pi\,\mathrm{csgn}(ih)\mathrm{csgn}(ix)\mathrm{csgn}(ihx)+i\pi\,\mathrm{csgn}(ih)\mathrm{csgn}(ihx)^{2}+i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(ihx)^{2}-i\pi\mathrm{csgn}(ihx)^{3}+2\ln(h)+2\ln(x)\right)}{2}}}{\left(p+1\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^(-n*p-n-1)*(b*x^n+a)^p*(d*x^n+c)^p*(-b*d*x^(2*n)+a*c),x)
```

```
[Out] -(b*x^n+a)^p*exp(-1/2*(n*p+n+1)*(-I*Pi*csgn(I*h*x)^3+I*Pi*csgn(I*h*x)^2*csg
n(I*h)+I*Pi*csgn(I*h*x)^2*csgn(I*x)-I*Pi*csgn(I*h*x)*csgn(I*h)*csgn(I*x)+2*
ln(h)+2*ln(x)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*x/(p+1)/n*(d*x^n+c)^p
```

maxima [A] time = 3.04, size = 77, normalized size = 1.71

$$-\frac{\left(bdx^{2\,n}+ac+(bc+ad)x^{n}\right)h^{-np-n-1}e^{\left(-np\log(x)+p\log(bx^{n}+a)+p\log(dx^{n}+c)-n\log(x)\right)}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] -(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n)*h^(-n*p - n - 1)*e^(-n*p*log(x) + p*
log(b*x^n + a) + p*log(d*x^n + c) - n*log(x))/(n*(p + 1))
```

mupad [B] time = 5.37, size = 124, normalized size = 2.76

$$-(c+d\,x^{n})^{p}\left(\frac{a\,c\,x\,(a+b\,x^{n})^{p}}{n\,(h\,x)^{n+n\,p+1}\,\left(p+1\right)}+\frac{x\,x^{n}\,\left(a\,d+b\,c\right)\,(a+b\,x^{n})^{p}}{n\,(h\,x)^{n+n\,p+1}\,\left(p+1\right)}+\frac{b\,d\,x\,x^{2\,n}\,(a+b\,x^{n})^{p}}{n\,(h\,x)^{n+n\,p+1}\,\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*c - b*d*x^(2*n))*(a + b*x^n)^p*(c + d*x^n)^p)/(h*x)^(n + n*p + 1),x
)
```

```
[Out] -(c + d*x^n)^p*((a*c*x*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)) + (x*
x^n*(a*d + b*c)*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)) + (b*d*x*x^(
2*n)*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**(-n*p-n-1)*(a+b*x**n)**p*(c+d*x**n)**p*(a*c-b*d*x**(2*n)),
x)
```

```
[Out] Timed out
```

3.426

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx$$

Optimal. Leaf size=31

$$\frac{ex(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ac}$$

Rubi [A] time = 0.21, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.014, Rules used = {1897}

$$\frac{ex(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n)/(a*c
) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
```

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Rule 1897

```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) +
 (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(e*x*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(p + 1))/(a*c), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}
, x] && EqQ[n2, 2*n] && EqQ[a*c*f - e*(b*c + a*d)*(n*(p + 1) + 1), 0] && Eq
Q[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx = \frac{ex(a+bx^n)^{1+p}(c+dx^n}{ac}$$

Mathematica [A] time = 0.60, size = 31, normalized size = 1.00

$$\frac{ex(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n
)/(a*c) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
```

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

IntegrateAlgebraic [F] time = 0.43, size = 0, normalized size = 0.00

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n +
 n*p)*x^n)/(a*c) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(
1 + n + n*p)*x^n)/(a*c) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)), x]
```

fricas [A] time = 0.46, size = 54, normalized size = 1.74

$$\frac{\left(bdexx^{2\,n} + acex + (bc + ad)exx^{n}\right)(bx^{n} + a)^{p}(dx^{n} + c)^{p}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="fricas")
```

```
[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c + a*d)*e*x*x^n)*(b*x^n + a)^p*(d*x^n + c)
^p/(a*c)
```

giac [B] time = 0.56, size = 115, normalized size = 3.71

$$\frac{(bx^{n} + a)^{p}(dx^{n} + c)^{p}bdxx^{2\,n}e + (bx^{n} + a)^{p}(dx^{n} + c)^{p}bcxx^{n}e + (bx^{n} + a)^{p}(dx^{n} + c)^{p}adxx^{n}e + (bx^{n} + a)^{p}(dx^{n} + c)^{p}acxe}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e + (b*x^n + a)^p*(d*x^n + c)^p*
b*c*x*x^n*e + (b*x^n + a)^p*(d*x^n + c)^p*a*d*x*x^n*e + (b*x^n + a)^p*(d*x^
n + c)^p*a*c*x*e)/(a*c)
```

maple [A] time = 0.17, size = 52, normalized size = 1.68

$$\frac{\left(ad\,x^{n} + bc\,x^{n} + bd\,x^{2n} + ac\right)ex\left(b\,x^{n} + a\right)^{p}\left(d\,x^{n} + c\right)^{p}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p*(d*x^n+c)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2*n*p+2
*n+1)*x^(2*n)/a/c),x)
```

```
[Out] (b*x^n+a)^p*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c*(d*x^n+c)^p
```

maxima [A] time = 2.67, size = 59, normalized size = 1.90

$$\frac{\left(bdexx^{2\,n} + acex + (bce + ade)xx^{n}\right)e^{\left(p\log(bx^{n}+a)+p\log(dx^{n}+c)\right)}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="maxima")
```

```
[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c*e + a*d*e)*x*x^n)*e^(p*log(b*x^n + a) + p
*log(d*x^n + c))/(a*c)
```

mupad [B] time = 5.30, size = 76, normalized size = 2.45

$$(c + d\,x^{n})^{p}\left(e\,x\,(a + b\,x^{n})^{p} + \frac{e\,x\,x^{n}\,(a\,d + b\,c)\,(a + b\,x^{n})^{p}}{a\,c} + \frac{b\,d\,e\,x\,x^{2\,n}\,(a + b\,x^{n})^{p}}{a\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(c + d*x^n)^p*(e + (e*x^n*(a*d + b*c)*(n + n*p + 1))/(a*c
) + (b*d*e*x^(2*n)*(2*n + 2*n*p + 1))/(a*c)),x)
```

```
[Out] (c + d*x^n)^p*(e*x*(a + b*x^n)^p + (e*x*x^n*(a*d + b*c)*(a + b*x^n)^p)/(a*c
) + (b*d*e*x*x^(2*n)*(a + b*x^n)^p)/(a*c))
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+n+1)*x**n/a/c+b*d
*e*(2*n*p+2*n+1)*x**(2*n)/a/c),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.427

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)}+\frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right.$$

Optimal. Leaf size=45

$$\frac{e(hx)^{m+1}(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ach(m+1)}$$

Rubi [A] time = 0.55, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 86, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.012, Rules used = {1848}

$$\frac{e(hx)^{m+1}(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ach(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m + n + n*
p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*(1 + m))
),x]
```

```
[Out] (e*(h*x)^(1 + m)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*h*(1 + m))
```

Rule 1848

```
Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(
n_.))^(p_.)*((e_) + (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Sim
p[(e*(h*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(p + 1))/(a*c*h*(m + 1))
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[
a*c*f*(m + 1) - e*(b*c + a*d)*(m + n*(p + 1) + 1), 0] && EqQ[a*c*g*(m + 1)
- b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)}+\frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{e(hx)^{1+m}(a}{}$$

Mathematica [A] time = 0.89, size = 41, normalized size = 0.91

$$\frac{ex(hx)^m(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ac(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m +
n + n*p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*(1
 + m))),x]
```

```
[Out] (e*x*(h*x)^m*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*(1 + m))
```

IntegrateAlgebraic [F] time = 1.16, size = 0, normalized size = 0.00

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)}+\frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*
(1 + m + n + n*p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n)
)/(a*c*(1 + m))),x]
```

```
[Out] Defer[IntegrateAlgebraic][(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c +
a*d)*e*(1 + m + n + n*p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*
x^(2*n))/(a*c*(1 + m))), x]
```

fricas [A] time = 0.47, size = 88, normalized size = 1.96

$$\frac{\left(bdexx^{2\,n}e^{\left(m\log(h)+m\log(x)\right)}+acexe^{\left(m\log(h)+m\log(x)\right)}+(bc+ad)exx^{n}e^{\left(m\log(h)+m\log(x)\right)}\right)(bx^{n}+a)^{p}(dx^{n}+c)^{p}}{acm+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="fricas")
```

```
[Out] (b*d*e*x*x^(2*n)*e^(m*log(h) + m*log(x)) + a*c*e*x*e^(m*log(h) + m*log(x))
+ (b*c + a*d)*e*x*x^n*e^(m*log(h) + m*log(x)))*(b*x^n + a)^p*(d*x^n + c)^p/
(a*c*m + a*c)
```

giac [B] time = 0.81, size = 155, normalized size = 3.44

$$\frac{(bx^{n}+a)^{p}(dx^{n}+c)^{p}bdxx^{2\,n}e^{\left(m\log(h)+m\log(x)+1\right)}+(bx^{n}+a)^{p}(dx^{n}+c)^{p}bcxx^{n}e^{\left(m\log(h)+m\log(x)+1\right)}+(bx^{n}+a)^{p}(dx^{n}+c)^{p}adxx^{n}e^{\left(m\log(h)+m\log(x)+1\right)}+(bx^{n}+a)^{p}(dx^{n}+c)^{p}acxe^{\left(m\log(h)+m\log(x)+1\right)}}{acm+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e^(m*log(h) + m*log(x) + 1) + (b
*x^n + a)^p*(d*x^n + c)^p*b*c*x*x^n*e^(m*log(h) + m*log(x) + 1) + (b*x^n +
a)^p*(d*x^n + c)^p*a*d*x*x^n*e^(m*log(h) + m*log(x) + 1) + (b*x^n + a)^p*(d
*x^n + c)^p*a*c*x*e^(m*log(h) + m*log(x) + 1))/(a*c*m + a*c)
```

maple [C] time = 0.50, size = 136, normalized size = 3.02

$$\frac{\left(ad\,x^{n}+bc\,x^{n}+bd\,x^{2n}+ac\right)ex\,(b\,x^{n}+a)^{p}\,(d\,x^{n}+c)^{p}\,\mathrm{e}^{\frac{\left(-i\pi\,\mathrm{csgn}(ih)\mathrm{csgn}(ix)\mathrm{csgn}(ihx)+i\pi\,\mathrm{csgn}(ih)\mathrm{csgn}(ihx)^{2}+i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(ihx)^{2}-i\pi\mathrm{csgn}(ihx)^{3}+2\ln(h)+2\ln(x)\right)m}{2}}}{(m+1)\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^m*(b*x^n+a)^p*(d*x^n+c)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/c/(m+1
)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(m+1)),x)
```

```
[Out] (b*x^n+a)^p*exp(1/2*m*(-I*Pi*csgn(I*h)*csgn(I*x)*csgn(I*h*x)+I*Pi*csgn(I*h)
*csgn(I*h*x)^2+I*Pi*csgn(I*x)*csgn(I*h*x)^2-I*Pi*csgn(I*h*x)^3+2*ln(h)+2*ln
(x)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c/(m+1)*(d*x^n+c)^p
```

maxima [B] time = 3.04, size = 92, normalized size = 2.04

$$\frac{\left(aceh^{m}xx^{m}+bdeh^{m}xe^{\left(m\log(x)+2\,n\log(x)\right)}+(bceh^{m}+adeh^{m})xe^{\left(m\log(x)+n\log(x)\right)}\right)e^{\left(p\log(bx^{n}+a)+p\log(dx^{n}+c)\right)}}{ac(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="maxima")
```

```
[Out] (a*c*e*h^m*x*x^m + b*d*e*h^m*x*e^(m*log(x) + 2*n*log(x)) + (b*c*e*h^m + a*d
*e*h^m)*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a) + p*log(d*x^n + c))/
(a*c*(m + 1))
```

mupad [B] time = 5.64, size = 106, normalized size = 2.36

$$(c+d\,x^n)^p\,\left(\frac{e\,x\,(h\,x)^m\,(a+b\,x^n)^p}{m+1}+\frac{e\,x\,x^n\,(h\,x)^m\,\,(a\,d+b\,c)\,\,(a+b\,x^n)^p}{a\,c\,\,(m+1)}+\frac{b\,d\,e\,x\,x^{2\,n}\,(h\,x)^m\,(a+b\,x^n)^p}{a\,c\,\,(m+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + (e*x^n*(a*d + b*c)*(m + n + n*
p + 1))/(a*c*(m + 1)) + (b*d*e*x^(2*n)*(m + 2*n + 2*n*p + 1))/(a*c*(m + 1))
),x)
```

```
[Out] (c + d*x^n)^p*((e*x*(h*x)^m*(a + b*x^n)^p)/(m + 1) + (e*x*x^n*(h*x)^m*(a*d
+ b*c)*(a + b*x^n)^p)/(a*c*(m + 1)) + (b*d*e*x*x^(2*n)*(h*x)^m*(a + b*x^n)^
p)/(a*c*(m + 1)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**m*(a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x
**n/a/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x**(2*n)/a/c/(1+m)),x)
```

```
[Out] Timed out
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```